Assignment 3 operation Research

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ICS A 21/06/2022

Question 1

⇒ Is a linear programming problem concerned with determining an optimal strategy of distributing a commodity from a group of supply centers. Each source is able to supply a fixed number of units of product and each destination has a fixed demand.

Steps of solving a transportation ~~meth~~ problem.

Step 1

⇒ First check whether the supply and demand are equal. If they are equal proceed if not add a row to balance the equation.

- Step 2

Find the intial feasible solution using the preferred method from the following

⇒ North west corner rule

⇒ matrix minimal

⇒ Vogel approximation method

Step 3

⇒ Get the optimal solution using the Modi method.

Question 2

Similarities

→ Both have an objective function

→ Both have a linear objective function

→ Both have non-negative constraints

→ Both can be solved by simplex method.

Differences.

→ Transportation problem is more often a minimization problem whilst a linear programming problem can be either.

→ Structural constraints are built up homogeneous in transportation problem, where as they are different

→ Transportation problems are solved by transportation algorithm while general linear are solved using simplex

Question 3

Differences.

→ Transportation problem is about the reduction of cost of transportation, whilst assignment problem is about assigning finite sources to finite destinations where only one destination is allocated for one source with minimum cost;

→ If total demand and total supply are not equal then the problem is said to be unbalanced. If the number of rows are ~~equal~~ not equal to the number columns then the ~~transportation~~ assignment problem is said to be unbalanced.

→ Required 2 stages to solve a transportation problem whilst an assignment problem requires only one stage.

Similarities

→ Both are special types of linear programming problems

→ Both have an objective function, structural constraints and non-negative constraints

→ The coefficients of variables in the solutio will be either 1 or 0 in both cases.

→ Both are forms of minimization problems

Question 4

→ An assignment problem is most commonly solved using the hungarian method The hungarian method uses the principle of matrix reduction which means that by adding and subtracting the appropriate number of cost we can reduce the matrix to a matrix of opportunity cost.

Step 1

→ The first step is to check that the matrix is balance by ensuring the number of columns is equal to the number of rows.

Step 2

Use the hungarian method

→ find the opportunity cost table

→ Test whether the optimal solution can be made

→ Revise the present opportunity cost table.

Question 5

Cost in Rs

Companies	A	B	C	Supply
W	10	8	9	15
X	5	2	3	20
Y	6	7	4	30
Z	7	6	9	35
Requirement	25	26	49	

a) North West corner method.

A	B	C
~~10~~	~~8~~	~~9~~
~~5~~	2	3

	A	B	C	supply
	10(15)	8	9	~~15~~ 0
	5(10)	2(10)	3	~~20~~ ~~10~~ 0
	6	7(16)	4(14)	30 14 0
	7	6	9(35)	~~35~~ 0
	25 ~~10~~ 0	26 ~~16~~ 0	49 ~~35~~ 0	100/100 → table is balanced

$x_{11} = \min(15, 25) = 15$
$x_{21} = \min(20, 10) = 10$
$x_{22} = \min(10, 26) = 10$
$x_{32} = \min(30, 16) = 16$
$x_{33} = \min(14, 49) = 14$
$x_{43} = \min(35, 35) = 35$

$10(15) + 5(10) + 2(10) + 7(16) + 4(14) + 9(35) = 703$

b) Least cost method

A	B	C	supply
10	8	9(15)	~~15~~ 0
5	2(20)	3	~~20~~ 0
6	7	4(30)	~~30~~ 0
7(25)	6(6)	9(4)	35 ~~29~~ ~~4~~ 0
~~25~~ 0	~~26~~ 6 0	~~49~~ ~~19~~ ~~4~~ 0	100/100

$x_{22} = \min(20, 26) = 20$
$x_{33} = \min(30, 49) = 30$
$x_{42} = \min(35, 6) = 6$
$x_{41} = \min(29, 25) = 25$
$x_{13} = \min(15, 19) = 15$
$x_{43} = \min(4, 4) = 4$

$2(20) + 9(15) + 7(25) + 6(6) + 4(30) + 9(4) = 542$

Vogel approximation method

	A	B	C	supply	Pen 1
	10	8	9(15	15	1
	~~5~~	~~2(20)~~	~~3~~	~~200~~	~~1~~
	6	7	4(30)	30	2
	7(25)	6(6)	9(4)	35	1
Demand	25	~~26~~(6)	49	100/100	
Penalty 1	1	4	1		

↑ largest penalty

$x_{22} = \min(20, 26) = 20$

	A	B	C	supply	Pen 2
	10	8	9	15	1
	~~6~~	~~7~~	~~4(30)~~	~~300~~	~~2~~
	7	6	9	35	1
Demand	25	6	~~49~~ 19		
Pen 2	1	1	5		

	A	B	C	supply	Pen3
	10	8	9	15	1
	7 (25)	6	9	~~35~~ 10	1
Demand	~~25~~ 0	6	19		
Pen 3	3	2	0		

	B	C	supply	pen 4
	8	9	15	1
	6(6)	9	~~10~~ 4	3
Demand	~~6~~ 0	19		
pen 4	2	0		

	C	supply	pen5
— —	9(15)	~~15~~ 0	~~0~~
	9	4	0
Demand	~~19~~ 4		
Pen 5	0		

	C	supply	pen 6
— —	~~9 (4)~~	~~4~~ 0	~~0~~
demand	~~4~~ 0		
pen 6	0		

$$2(20) + 4(30) + 7(25) + 6(6) + 9(4) + 9(15)$$

$$= 542$$

Question 6

Cost in Rs (*100 mills)

sources	A	B	C	D	supply
x	10	2	20	11	15
y	12	7	9	20	25
z	4	14	16	18	10
Demand	5	15	15	15	

Initial feasible using VAM

	A	B	C	D	supply	pen 1
	10	2	20	11	15	8
	12	7	9	20	25	2
	4 (5)	14	16	18	~~10~~ 5	10
Demand	~~5~~ 0	15	15	15		
Pen1	1	5	7	7		

	B	C	D	supply	Pen 2
-	~~2 (15)~~	~~20~~	~~11~~	~~15~~ 0	~~9~~
	7	9	20	25	2
	14	16	18	5	2
demand	~~15~~ 0	15	15		
P2	5	7	7		

	C	D	Supply	Pen 3
	9 (15)	20	~~25~~ 10	11
	16	18	5	2
demand	~~15~~ 0	15		
Pen 3	7	2		

	D	supply	Pen 4
—	20(10)	~~10~~ 0	20
	18	5	18
demand	~~15~~ 5		
Pen4	2		

	D	supply	pen 5
—	18(5)	~~5~~ 0	18
demand	~~5~~ 0		
pen 5	18		

Modi method

	A	B	C	D	supply
	10	2(15)	20	11	15
	12	7(0)	9(15)	20(10)	25
	4(5)	14	16	18(5)	10
demand	5	15	15	15	

$C_{ij} = U_i + V_j$ $U_2 = 0$

$C_{22} = U_2 + V_2$

$7 = 0 + V_2$

$7 = V_2$

$C_{23} = U_2 + V_3$

$9 = V_3$

$C_{24} = U_2 + V_4$

$20 = V_4$

$C_{12} = U_1 + V_2$

$2 = U_1 + 7$

$-5 = U_1$

$C_{34} = U_3 + V_4$

$18 = U_3 + 20$

$-2 = U_3$

$C_{31} = U_3 + V_1$

$4 = -2 + V_1$

$V_1 = 6$

$\Delta_{ij} = C_{ij} - (u_i + v_j)$

$\Delta_{11} = 10 - (-5+6) = 9$

$\Delta_{13} = 20 - (-5+9) = 16$

$\Delta_{14} = 11 - (-5+20) = -4$

$\Delta_{33} = 16 - (-2+9) = 9$

$\Delta_{21} = 12 - (0+6) = 6$

$\Delta_{32} = 14 - (-2+7) = 9$

A	B	C	D
10	$^{-}$2 (15)	20	$^{+}$11 (-4)
12	$^{+}$7 (0)	9 (15)	$^{-}$20 (10)
4 (5)	14	16	18 (5)

min allocation=10

A	B	C	D
10	2 (5)	20	11 (10)
12	7 (10)	9	20
4 (5)	14	16	18 (5)

$U_1 = u_1 = 0 \quad C_{ij} = u_i + v_j$

$C_{12} = 0 + v_2$
$2 = v_2$

$C_{14} = 0 + v_4$

$11 = v_4$

$C_{22} = u_2 + 2$

$7 = u_2 + 2$
$5 = u_2$

$C_{23} = 5 + v_3$
$9 = 5 + v_3$
$4 = v_3$

$C_{33} = u_3 + 11$

$18 = u_3 + 11$
$7 = u_3$

$C_{31} = 7 + v_1$

$4 = 7 + v_1$
$v_1 = -3$

$\Delta_{ij} = C_{ij} - (u_i + v_j)$

$\Delta_{11} = 10 - (0+3) = 7$

$\Delta_{13} = 20 - (0+4)$
$= 16$

$\Delta_{21} = 12 - (5 + (-3)) = 10$

$\Delta_{24} = 20 - (5+11) = 4$

$\Delta_{32} = 14 - (7+2) = 5$

$\Delta_{33} = 16 - (7+4)$

$= 16 - 11 = 5$

$\boxed{\text{ALL } \Delta_{ij} > 0 \text{ hence solution is optimal}}$

	A	B	C	D
x	10	2 (5)	20	11 (10)
y	12	7 (10)	9 (15)	20
z	4 (5)	14	16	18 (5)

$2(5) + 11(10) + 7(10) + 9(15) + 4(5) + 18(5) = 435 \times 100 = 43500$ Rs

Question 7

Factories	x	y	z	supply
A	400	300	200	10
B	500	600	100	8
C	600	400	300	5
D	300	500	400	6
	7	12	4	23 ~~29~~

	x	y	z	dummy A	supply	P_1
A	400	300	200	0	10	200
B	500	600	100	0	8	100
C	600	400	300	0	5	300
~~D~~	~~300~~	~~500~~	~~400~~	~~0~~ (6)	~~6~~	300
demand	7	12	4	~~6~~	~~29~~ 29	
P_1	100	100	100	0		

	x	y	z	supply	P_2
A	400	300	200	10	100
B	500	600	100 (4)	~~8~~ 4	400
C	600	400	300	5	100
demand	7	12	~~4~~0		
P_2	100	100	100		

	x	Y	Supply	P_3
	400	300	10	100
	500	600	4	100
	~~600~~	~~400(5)~~	~~50~~	~~200~~
demand	7	~~12~~ 7		
P_3	100	100		

	x	Y	Supply	P_4
	400	300(7)	~~10~~ 3	100
	500	600	4	100
demand	7	~~7~~ 0		
P_4	100	300		

	x	Supply	P_5
	400	3	400
	~~500(4)~~	~~4~~ 0	~~500~~
demand	~~7~~ 3		
P_5	100		

	x	Supply	P_6
	400(3)	3	400
demand	3		
P_6	400		

	X	Y	Z	dummy P
A	400(3)	300(7)	200	0
B	500(4)	600	100(4)	0
C	600	400(5)	300	0 (0)
D	300 (-x)+	500	400	0(6)-

$U_i + V_j = C_{ij}$

$U_1 = 0$

$U_1 + V_j = C_{11}$
$400 = 0 + V_1$

$V_1 = 400$

$C_{21} = U_2 + V_1$

$500 = U_2 + 400$

$100 = U_2$

$C_{12} = U_1 + V_2$

$300 = 0 + V_2$

$300 = V_2$

$C_{32} = U_3 + V_2$

$400 = U_3 + 300$

$100 = U_3$

$C_{23} = U_2 + V_3$

$100 = 100 + V_3$

$0 = V_3$

$C_{34} = U_3 + V_4$

$0 = 100 + V_4$

$-100 = V_4$

$C_{44} = U_4 + V_4$

$0 = U_4 + (-100)$

$100 = U_4$

Questo

$\Delta_{ij} = C_{ij} - (u_i + v_j)$

$\Delta_{13} = 200 - (0+0)$

$= 200$

$\Delta_{14} = 0 - (0 + (-100))$

$= 100$

$\Delta_{22} = 600 - (100+500)$

$= 200$

$\Delta_{24} = 0 - (100 + (-100))$

$= 0$

$\Delta_{31} = 600 - (100 + 400)$

$= 100$

$\Delta_{33} = 300 - (100 + 0)$

$= 200$

$\Delta_{41} = 300 - (100 + 400)$

$= -200$

$\Delta_{42} = 500 - (100 + 300)$

$= 100$

$\Delta_{43} = 400 - (100 + 0)$

$= 300$

	X	Y	Z	P
A	400(3)$^-$	300(7)$^+$	200	0
B	500(4)	600	100(4)	0
C	600	400(5)$^-$	300	0 (0)$^+$
D	300$^+$	500	400	0 (6)$^-$

min allocation $= 3$

	X	Y	Z	P
A	400	300 (10)	200	0
B	500 (4)	600	100 (4)	0
C	600	400 (2)	300	0 (3)
D	300 (3)	500	400	0 (3)

$U_i + V_j = C_{ij}$ $\qquad U_2 = 0$

$C_{21} = U_2 + V_1$

$500 = 0 + V_1$

$500 = V_1$

$C_{23} = U_2 + V_3$
$100 = 0 + V_3$
$100 = V_3$

$C_{41} = U_4 + V_1$

$300 = U_4 + 500$

$-200 = U_4$

$C_{44} = U_4 + V_4$
$0 = -200 + V_4$
$200 = V_4$

$C_{34} = U_3 + V_4$
$0 = U_3 + 200$

$-200 = U_3$

$C_{32} = U_3 + V_2$

$400 = -200 + V_2$

$600 = V_2$

$C_{12} = U_1 + V_2$

$300 = U_1 + 600$

$-300 = U_1$

$\Delta_{ij} = C_{ij} - (U_i + V_j)$

$\Delta_{11} = 400 - (-300 + 500)$
$= 200$

$\Delta_{13} = 200 - (-300 + 100)$
$= 400$

$\Delta_{14} = 0 - (-300 + 200)$
$= 100$

$\Delta_{22} = 600 - (0 + 600)$

$= 0$

$\Delta_{24} = 0 - (0 + 200)$

$= -200$

$\Delta_{31} = 600 - (-200 + 500$
$= 300$

$\Delta_{33} = 300 - (-200 + 100)$
$= 400$

$\Delta_{42} = 500 - (-200 + 600)$
$= 100$

$\Delta_{43} = 400 - (-200 + 100)$

$= 500$

Question 8

	X	Y	Z	supply
A	8	7	3 (60)	~~60~~ 0
B	3(50)	8	9 (20)	~~70~~ ~~20~~ 0
C	11	3(80)	5	~~80~~ 0
demand	~~50~~	~~80~~	80	
	0	0	~~20~~ 0	

	X	Y	Z
A	8	7	3(60)
B	3(50)	8(0)	9(20)
C	11	3(80)	5

$C_{ij} = U_i + V_j$

$U_2 = 0$

$C_{21} = U_2 + V_1$
$3 = 0 + V_1$
$3 = V_1$

$C_{22} = U_2 + V_2$
$8 = 0 + V_2$
$8 = V_2$

$C_{23} = 0 + V_3$

$9 = 0 + V_3$
$9 = V_3$

$C_{13} = U_1 + V_3$
$3 = U_1 + 9$
$-6 = U_1$

$C_{32} = U_3 + V_2$
$3 = U_3 + 8$
$-5 = U_3$

$\Delta_{ij} = C_{ij} - (U_i + V_j)$

$\Delta_{11} = 8 - (-6+3)$
$= 11$

$\Delta_{12} = 7 - (-6+8)$
$= 5$

$\Delta_{31} = 11 - (-5+3)$
$= 13$

$\Delta_{33} = 5 - (-5+9)$

$= 1$

All $\Delta_{ij} > 0$, hence optimal solution is achieved

$(3 \times 50) + 8(0) + 9(20) + 3(60) + 3(80)$

$= 750$ Rs

Question 9

Persons	Jobs 1	2	3	4	Row min
A	20	25	22	28	20
B	15	18	23	17	15
C	19	17	21	24	17
D	25	23	24	24	23

Persons	Jobs 1	2	3	4
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1
Column min	0	0	1	1

	Job 1	2	3	4
A	$\boxed{0}$	5	1	7
B	0	3	7	1
C	2	$\boxed{0}$	3	6
D	2	0	0	0

minimum uncovered=1

Person	Job 1	2	3	4
A	0	5	0	6
B	0	3	6	0
C	2	$\boxed{0}$	2	5
D	3	1	0	0

Person	Job	Time
A	1	20
B	4	17
C	2	13
D	3	24

Total cost = 78

Question 10

Employees	offices W	X	Y	Z	row min
A	160	220	240	200	160
B	100	320	260	160	100
C	100	200	460	250	100
D	0	0	0	0	0

Employees	offices W	X	Y	Z
A	0	60	80	40
B	0	220	160	60
C	0	100	360	150
D	0	0	0	0
Column min	0	0	0	0

employees	W	X	Y	Z
A	$\boxed{0}$	60	80	40
B	0	220	160	60
C	0	100	360	150
D	0	$\boxed{0}$	0	0

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offices

		W	X	Y	Z
P_c	A	0	20	40	0
	B	0	180	120	20
	C	0	60	320	110
	D	40	20	0	0

		W	X	Y	Z
	A	20	20	40	0
	B	0	160	100	0
	C	0	40	300	90
	D	60	0	0	0

		W	X	Y	Z
	A	20	0	20	0
Ok	B	0	140	80	0
	C	0	20	280	90
	D	80	0	0	20

Employees	Office	cost
A	X	220
B	Z	160
C	W	100
D	Y	0

Question 11

Product

	1	2	3	4	rowmin
A	33	40	43	32	32
B	45	28	31	23	23
C	42	29	36	29	29
D	27	42	44	38	27

	1	2	3	4
A	1	8	11	0
B	22	5	8	0
C	13	0	7	0
D	0	15	17	11
Column min	0	0	7	0

	1	2	3	4
A	1	8	4	$\boxed{0}$
B	22	5	1	0
C	13	$\boxed{0}$	0	0
D	$\boxed{0}$	15	10	11

	1	2	3	4
A	1	7	3	$\boxed{0}$
B	22	4	$\boxed{0}$	0
C	14	$\boxed{0}$	0	7
D	$\boxed{0}$	14	9	11

Plant	Product	cost
A	4	32
B	3	31
C	2	29
D	1	27

Total = 119

Scanned with CamScanner

Question 12.

plant	A	B	C	D	E	row min
			Product			
1	5	11	10	12	4	
2	2	4	6	3	5	
3	3	12	5	14	6	
4	6	14	4	11	7	
5	7	9	8	12	5	

Convert to minimization matrix by subtracting the largest profit (14)

	A	B	C	D	E	row min
1	9	3	4	2	10	2
2	12	10	8	11	9	8
3	11	2	9	0	8	0
4	8	0	10	3	7	0
5	7	5	6	2	9	2

	A	B	C	D	E
1	7	1	2	0	8
2	4	2	0	3	1
3	11	2	9	0	8
4	8	0	10	3	7
5	5	3	4	0	7
column min	4	0	0	0	1

	A	B	C	D	E
1	3	1	2	$\boxed{0}$	7
2	$\boxed{0}$	2	0	3	0
3	7	2	9	0	7
4	4	$\boxed{0}$	10	3	6
5	1	3	4	0	6

	A	B	C	D	E
1	2	1	1	$\boxed{0}$	6
2	0	3	0	4	0
3	0	2	8	0	6
4	3	$\boxed{0}$	9	3	5
5	$\boxed{0}$	3	3	0	5

	A	B	C	D	E
1	2	1	0	0	5
2	1	4	0	5	$\boxed{0}$
3	6	2	7	$\boxed{0}$	5
4	3	$\boxed{0}$	8	3	4
5	$\boxed{0}$	3	2	0	4

Plant	Product	Cost
1	C	10
2	E	5
3	D	14
4	B	14
5	A	7

total = 50