

Assignment 3 Operation Research

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Question 1

- Is a linear programming problem concerned with determining an optimal strategy of distributing a commodity from a group of supply centers. Each source is able to supply a fixed number of units of product and each destination has a fixed demand.

Steps of solving a transportation ~~with~~ problem.

Step 1

- First check whether the supply and demand are equal. If they are equal proceed if not add a row to balance the equation.

→ Step 2

Find the initial feasible solution using the preferred method from the following

→ North west corner rule

→ matrix minimal

→ Vogel approximation method

Step 3

- Get the optimal solution using the Modi method.

Question 2

Similarities.

- Both have an objective function
- Both have a linear objective function
- Both have non-negative constraints
- Both can be solved by simplex method.

Differences.

- Transportation problem is more often a minimization problem whilst a linear programming problem can be either.
- Structural constraints are built up homogeneous in transportation problem, whereas they are different.
- Transportation problems are solved by transportation algorithm while general linear problems are solved using simplex.

Question 3

Differences.

- Transportation problem is about the reduction of cost of transportation, whilst assignment problem is about assigning finite sources to finite destinations where only one destination is allocated for one source with minimum cost;
- If total demand and total supply are not equal then the problem is said to be unbalanced. If the number of rows are ~~not~~ not equal to the number of columns then the ~~assignment~~ problem is said to be unbalanced.
- Requires 2 stages to solve a transportation problem whilst an assignment problem requires only one stage.

Similarities

- Both are special types of linear programming problems
- Both have an objective function, structural constraints and non-negative constraints
- The coefficients of variables in the solution will be either 1 or 0 in both cases.
- Both are forms of minimization problems

Question 4

→ An assignment problem is most commonly solved using the hungarian method
The hungarian method uses the principle of matrix reduction which means that by adding and subtracting the appropriate number of cost we can reduce the matrix to a matrix of opportunity cost.

Step 1

→ The first step is to check that the matrix is balance by ensuring the number of columns is equal to the number of rows.

Step 2

Use the hungarian method

→ find the opportunity cost table

→ Test whether the optimal solution can be made

→ Revise the present opportunity cost table.

Question 5

Companies	Cost in Rs			Supply
	A	B	C	
W	10	8	9	15
X	5	2	3	20
Y	6	7	4	30
Z	7	6	9	35
Requirement	25	26	49	

a) North West corner method.



	A	B	C	Supply
	15 (15)	8	9	15 0
	5 (10)	2 (10)	3	20 10 0
	6	7 (16)	4 (14)	30 14 0
	7	6	9 (35)	35 0
	25	26	49	100 → table is balanced
	10	16	35	
	0	0	0	

$$x_{11} = \min(15, 25) = 15$$

$$x_{21} = \min(20, 10) = 10$$

$$x_{22} = \min(10, 26) = 10$$

$$x_{32} = \min(30, 16) = 16$$

$$x_{33} = \min(14, 49) = 14$$

$$x_{43} = \min(35, 35) = 35$$

$$10(15) + 5(10) + 2(10) + 7(16) + 4(14) + 9(35) = 703$$

b) Least cost method

	A	B	C	Supply
	0	8	9 (15)	15 0
	5	2 (20)	3	20 0
	6	7	4 (35)	30 0
	7 (25)	6 (6)	9 (4)	35 29 0
	25	26	49	100
	0	5	19	
	0	0	40	

$$x_{22} = \min(20, 26) = 20$$

$$x_{33} = \min(30, 49) = 30$$

$$x_{42} = \min(35, 6) = 6$$

$$x_{41} = \min(29, 25) = 25$$

$$x_{13} = \min(15, 19) = 15$$

$$x_{43} = \min(4, 4) = 4$$

$$2(20) + 9(15) + 7(25) + 6(6) + 4(35) + 9(4) = 542$$

Vogel approximation method

	A	B	C	Supply	Pen 1
	10	8	9(15)	15	1
- - 5 -	2(20)	- 3	200	- 1	
	6	7	4(30)	30	2
	7(25)	6(6)	9(4)	35	1
Demand	25	26(6)	49	100/100	
Pen 1	1	4	1		

↑
largest penalty

$$x_{22} = \min(20, 26) = 20$$

	A	B	C	Supply	Pen 2
	10	8	9	15	1
- - 6 -	7	- 4(3)	30	2	
	7	6	9	35	1
Demand	25	6	49	19	
Pen 2	1	1	5		

	A	B	C	Supply	Pen 3
	10	8	9	15	1
	7(25)	6	9	35	10
Demand	25	6	19		
Pen 3	3	2	0		

B	C	Supply	Pen 4
8	9	15	1
6(6)	9	10-4	3
Demand	60	19	
Pen 4	2	0	

C	Supply	Pen 5
9(5)	150	0
9	4	0
Demand	29	4
Pen 5	0	

C	Supply	Pen 6
9(4)	40	0
Demand	4	0
Pen 6	0	

$$2(20) + 4(30) + 7(25) + 6(6) + 9(4) + 9(15)$$

$$= 542$$

Question 6

sources	Cost in Rs (*100 milts)				
	A	B	C	D	Supply
X	10	2	20	11	15
Y	12	7	9	20	25
Z	4	14	16	18	10
Demand	5	15	15	15	

Initial feasible using VAM

	A	B	C	D	Supply	Pen 1
	10	2	20	11	15	8
	12	7	9	20	25	2
	4 (5)	14	16	18	10	
Demand	5	15	15	15		
Pen 1	1	5	7	7		

	B	C	D	Supply	Pen 2
-	-2 (5)	20	11	15	9
	7	9	20	25	2
	14	16	18	5	2
Demand	15	15	15		
Pen 2	5	7	7		

	C	D	Supply	Pen 3
	9 (5)	20	25	11
	16	18	5	2
Demand	15	15		
Pen 3	7	2		

	D	Supply	Pen 4
-	20(10)	100	20+
demand	18	5	18
demand	155		
Pen 4	2		

	D	Supply	Pen 5
-	18(5)	8-0	18
demand	50		
Pen 5	18		

Modi method

	A	B	C	D	Supply
-	10	2(15)	20	11	15
demand	12	7(0)	9(15)	20(10)	25
	4(5)	14	16	18(5)	10
	5	15	15	15	

$$C_{ij} = U_i + V_j$$

~~$$U_2 = 0$$~~

$$C_{22} = U_2 + V_2$$

$$C_{12} = U_1 + V_2$$

$$C_{31} = U_3 + V_1$$

$$7 = 0 + V_2$$

$$2 = U_1 + 7$$

$$4 = -2 + V_1$$

$$7 = V_2$$

$$-5 = U_1$$

$$V_1 = 6$$

$$C_{23} = U_2 + V_3$$

$$C_{34} = U_3 + V_4$$

$$9 = V_3$$

$$18 = U_3 + 20$$

$$C_{24} = U_2 + V_4$$

$$-2 = U_3$$

$$20 = V_4$$

$$\Delta_{ij} = C_{ij} - (U_i + V_j)$$

$$\Delta_{11} = 10 - (-5 + 6) = 9$$

$$\Delta_{13} = 20 - (-5 + 9) = 16$$

$$\Delta_{14} = 11 - (-5 + 10) = -4$$

$$\Delta_{33} = 16 - (-2 + 9) = 9$$

$$\Delta_{21} = 12 - (0 + 6) = 6$$

$$\Delta_{32} = 14 - (-2 + 7) = 9$$

A	B	C	D
10	-2 (15)	20	+1 (4)
12	+7 (0)	9 (15)	-20 (10)
4 (5)	14	16	18 (5)

min allocation = 10

A	B	C	D
10	2 (5)	20	11 (10)
12	7 (10)	9	20
4 (5)	14	16	18 (5)

$$U_1 = V_1 = 0 \quad C_{ij} = U_i + V_j$$

$$C_{12} = 0 + V_2 \\ 2 = V_2$$

$$C_{14} = 0 + V_4 \\ 11 = V_4$$

$$C_{22} = U_2 + 2$$

$$7 = U_2 + 2 \\ 5 = U_2$$

$$C_{23} = 5 + U_3 \\ 9 = 5 + U_3 \\ 4 = U_3$$

$$C_{33} = U_3 + 11$$

$$18 = U_3 + 11$$

$$7 = U_3$$

$$C_{31} = 7 + V_1$$

$$4 = 7 + V_1$$

$$V_1 = 3$$

$$\Delta_{ij} = C_{ij} - (U_i + V_j)$$

$$\Delta_{33} = 16 - (7 + 4)$$

$$= 16 - 11 = 5$$

$$\Delta_{13} = 20 - (0 + 4) \\ = 16$$

$$\Delta_{21} = 12 - (5 + (-3)) = 10$$

$$\Delta_{24} = 20 - (5 + 11) = 4$$

$$\Delta_{32} = 14 - (7 + 2) = 5$$

ALL $\Delta_{ij} > 0$ hence
solution is optimal

	A	B	C	D
X	10	2(5)	20	11(10)
Y	12	7(10)	9(15)	20
Z	4(5)	14	16	18(5)

$$2(5) + 11(10) + 7(10) + 9(15) + 4(5) + 18(5) = 435 \times 100 = 43500 \text{ Rs}$$

Question 7

Factories	X	Y	Z	Supply
A	400	300	200	10
B	500	600	100	8
C	600	400	300	5
D	300	500	400	6
	7	12	4	29

	X	Y	Z	Demand	Supply	P ₁
A	400	300	200	0	10	200
B	500	600	100	0	8	100
C	600	400	300	0	5	300
D	300	500	400	-0(6)	6	300
Demand	7	12	4	6	29	
P ₁	100	100	100	0		

	X	Y	Z	Supply	P ₂
A	400	300	200	10	100
B	500	600	100(4)	8	400
C	600	400	300	5	100
Demand	7	12	40		
P ₂	100	100	100		

<u>x</u>	<u>y</u>	<u>Supply</u>	<u>P₃</u>
400	300	10	100
500	600	4	100
-	-	-50	200
<u>demand</u>	7	127	
<u>P₃</u>	100	100	

<u>x</u>	<u>y</u>	<u>Supply</u>	<u>P₄</u>
400	300(7)	103	100
500	600	104	100
<u>demand</u>	7	70	
<u>P₄</u>	100	300	

<u>x</u>	<u>Supply</u>	<u>P₅</u>
400	3	400
-500(4)	40	500
	4	
<u>demand</u>	73	
<u>P₅</u>	100	

<u>x</u>	<u>Supply</u>	<u>P₆</u>
400(3)	3	400
<u>demand</u>	3	
<u>P₆</u>	400	

	X	Y	Z	P dummy
A	400(3)	300(7)	200	0
B	500(4)	600	100(4)	0
C	600	400(5)	300	0(0)
D	300(2)	500	400	0(6)

$$U_i + V_j = C_{ij}$$

$$U_1 = 0$$

$$U_1 + V_1 = C_{11}$$

$$400 = 0 + V_1$$

$$V_1 = 400$$

$$C_{21} = U_2 + V_1$$

$$C_{23} = U_2 + V_3$$

$$500 = U_2 + 400$$

$$100 = 100 + V_3$$

$$100 = U_2$$

$$U = V_3$$

$$C_{12} = U_1 + V_2$$

$$C_{34} = U_3 + V_4$$

$$300 = 0 + V_2$$

$$0 = 100 + V_4$$

$$300 = V_2$$

$$-100 = V_4$$

$$C_{32} = U_3 + V_2$$

$$C_{44} = U_4 + V_4$$

$$400 = U_3 + 300$$

$$0 = V_4 + (-100)$$

$$100 = U_3$$

$$100 = U_4$$

Quest

$$\Delta_{ij} = C_{ij} - (U_i + V_j)$$

$$\Delta_{13} = 200 - (0 + 0)$$

$$= 200$$

$$\Delta_{14} = 0 - (0 + (-100))$$

$$= 100$$

$$\Delta_{41} = 300 - (0 + 400)$$

$$= -200$$

$$\Delta_{22} = 60 - (100 + 500)$$

$$= -200$$

$$\Delta_{42} = 500 - (0 + 300)$$

$$= 200$$

$$\Delta_{24} = 0 - (100 + (-100))$$

$$= 0$$

$$\Delta_{43} = 400 - (100 + 0)$$

$$= 300$$

$$\Delta_{31} = 600 - (100 + 400)$$

$$= 100$$

$$\Delta_{33} = 300 - (100 + 0)$$

$$= 200$$

X Y Z P

A 400(3) 300(7) + 200 0

B 500(4) 600 100(4) 0

C 600 400(5) 300 0 (0) +

D 300 + 500 400 0 (6) -

min allocation = 3

X Y Z P

A 400 300(10) 200 0

B 500(4) 600 100(4) 0

C 600 400(2) 300 0(3)

D 300(3) 500 400 0(3)

$$U_i + V_j = C_{ij} \quad U_2 = 0$$

$$C_{21} = U_2 + V_1$$

$$500 = 0 + V_1$$

$$500 = V_1$$

$$\begin{aligned} C_{23} &= U_2 + V_3 \\ 100 &= 0 + V_3 \\ 100 &= V_3 \end{aligned}$$

$$C_{41} = U_4 + V_1$$

$$300 = U_4 + 500$$

$$-200 = U_4$$

$$\begin{aligned} C_{44} &= U_4 + V_4 \\ 0 &= -200 + V_4 \\ 200 &= V_4 \end{aligned}$$

$$\begin{aligned} C_{34} &= U_3 + V_4 \\ 0 &= U_3 + 200 \end{aligned}$$

$$-200 = U_3$$

$$C_{32} = U_3 + V_2$$

$$400 = -200 + V_2$$

$$600 = V_2$$

$$C_{12} = U_1 + V_2$$

$$300 = U_1 + 600$$

$$-300 = U_1$$

$$\Delta_{ij} = C_{ij} - (U_i + V_j)$$

$$\begin{aligned} \Delta_{11} &= 400 - (-300 + 500) \\ &\doteq 200 \end{aligned}$$

$$\begin{aligned} \Delta_{13} &= 200 - (-300 + 100) \\ &\doteq 400 \end{aligned}$$

$$\begin{aligned} \Delta_{14} &= 0 - (-300 + 200) \\ &= 100 \end{aligned}$$

$$\begin{aligned} \Delta_{22} &= 600 - (0 + 600) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \Delta_{24} &= 0 - (0 + 200) \\ &= -200 \end{aligned}$$

$$\begin{aligned} \Delta_{31} &= 600 - (-200 + 500) \\ &= 300 \end{aligned}$$

$$\begin{aligned} \Delta_{33} &= 300 - (-200 + 100) \\ &= 400 \end{aligned}$$

$$\begin{aligned} \Delta_{42} &= 500 - (-200 + 600) \\ &= 100 \end{aligned}$$

$$\begin{aligned} \Delta_{43} &= 400 - (-200 + 100) \\ &= 500 \end{aligned}$$

Question 8

	X	Y	Z	Supply
A	8	7	3(60)	60 0
B	3(50)	8	9(20)	70 20 0
C	11	3(60)	5	60 0
demand	50	80	80	
	0	0	20	0

	X	Y	Z
A	8	7	3(60)
B	3(50)	8(0)	9(20)
C	11	3(60)	5

$$C_{ij} = U_i + V_j$$

$$U_2 = 0$$

~~Ans~~

$$C_{21} = U_2 + V_1$$

$$3 = 0 + V_1$$

$$3 = V_1$$

$$C_{22} = U_2 + V_2$$

$$8 = 0 + V_2$$

$$8 = V_2$$

$$C_{23} = 0 + V_3$$

$$9 = 0 + V_3$$

$$9 = V_3$$

$$C_{13} = U_1 + V_3$$

$$3 = U_1 + 9$$

$$-6 = U_1$$

$$C_{32} = U_3 + V_2$$

$$3 = U_3 + 8$$

$$-5 = U_3$$

$$\Delta_{ij} = C_{ij} - (U_i + V_j)$$

$$\Delta_{11} = 8 - (-6+3) \\ = 11$$

$$\Delta_{12} = 7 - (-6+8) \\ = 5$$

$$\Delta_{31} = 11 - (-5+3) \\ = 13$$

$$\Delta_{33} = 5 - (-5+9) \\ = 1$$

All $\Delta_{ij} \geq 0$, hence optimal solution is achieved

$$(3 \times 50) + 8(0) + 9(20) + 3(60) + 3(80) \\ = 750 \text{ Rs}$$

Question 9

Persons	Jobs				Row min
	1	2	3	4	
A	20	25	22	28	20
B	15	18	23	17	15
C	19	17	21	24	17
D	25	23	24	24	23

Persons	Jobs			
	1	2	3	4
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1

Column min 0 0 1 1

0

Job

(0+0)+1=1

2

	1	2	3	4
A	1	5	1	7
B	0	3	7	1
C	2	0	3	6
D	-2	0	0	0

minimum uncovered d=1

0

Job

Person	1	2	3	4
A	0	5	0	6
B	0	3	6	0
C	2	0	2	5
D	3	1	0	0

Person	Job	Time
A	1	20
B	4	17
C	2	13
D	3	24

Total cost = 78

Question 10

Employees	offices				row min
	W	X	Y	Z	
A	160	220	240	200	160
B	100	320	260	160	100
C	100	200	460	250	100
D	0	0	0	0	0

Employees	offices				row min
	W	X	Y	Z	
A	0	60	80	40	40
B	0	220	160	60	60
C	0	100	360	150	100
D	0	0	0	0	0

Employees	W	X	Y	Z
A	0	60	80	40
B	0	220	160	60
C	0	100	360	150
D	0	0	0	0

offices

	w	x	y	z	
P ₁	A	0	-20	40	0
	B	0	180	120	20
	C	0	60	30	10
	D	10	90	0	0

w x y z

	w	x	y	z
A	20	20	40	0
B	0	160	100	0
C	0	40	300	90
D	60	0	0	0

	w	x	y	z
A	-20	0	-20	0
B	0	140	80	0
C	0	20	280	90
D	80	0	0	-20

Employees	Office	cost
A	x	220
B	z	160
C	w	100
D	y	0

Question 11

Product

	1	2	3	4	row min
A	33	40	43	32	32
B	45	28	31	23	23
C	42	29	36	29	29
D	27	42	44	38	27

	1	2	3	4
A	1	8	11	0
B	22	5	8	0
C	13	0	7	0
D	0	15	17	11

	Column min	0	0	7	0

	1	2	3	4
A	1	8	4	0
B	22	5	1	0
C	13	0	0	0
D	0	15	10	11

	1	2	3	4
A	1	7	3	0
B	22	4	0	0
C	14	0	0	7
D	0	14	9	11

Plant	Product	cost
A	4	32
B	3	31
C	2	29
D	1	27

$$\text{total} = 119$$

Question 12.

plant	Product					
	A	B	C	D	E	row min
1	5	11	10	12	4	
2	2	4	6	3	5	
3	3	12	5	14	6	
4	6	14	4	11	7	
5	7	9	8	12	5	

Convert to minimization matrix by subtracting the largest profit (14)

	A	B	C	D	E	row min
1	9	3	4	2	10	2
2	12	0	8	11	9	8
3	11	2	9	0	8	0
4	8	0	10	3	7	0
5	7	5	6	2	9	2

	A	B	C	D	E
1	7	1	2	0	8
2	4	2	0	3	1
3	11	2	9	0	8
4	8	0	10	3	7
5	5	0	4	0	7

column min 4 0 0 0 1

	A	B	C	D	E
1	3	1	2	10	7
2	0	2	0	3	0
3	7	2	9	0	7
4	4	10	3	6	
5	1	3	4	0	6

	A	B	C	D	E
1	2	1	1	6	
2	0	3	0	4	0
3	1	2	8	0	6
4	3	9	3	5	5
5	10	3	3	0	5

	A	B	C	D	E
1	2	1	0	0	5
2	1	4	0	5	10
3	6	2	7	10	5
4	3	9	8	3	4
5	10	3	12	0	4
	1	1	1	1	1

Plant	Product	Cost
1	C	10
2	E	5
3	D	14
4	B	14
5	A	7

Total = 50