

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules.
- Start working right away based on whatever you know. **Do not wait for the last moment and ask for time extension.**

1. Consider a set of four data points:

$$f(0) = 3, f(4) = -2, f(-1) = 2, \text{ and } f(1) = 1.$$

In the following, use these data points to find the best fit polynomial of degree 2 by using the QR-decomposition method:

- (5 marks) Identify the matrix A and b . Now, write down the linearly independent column vectors u_1 , u_2 and u_3 from the matrix A .
 - (6 marks) Using the Gram-Schmidt process construct the orthonormal column matrices (or vectors) q_1 , q_2 and q_3 from the linearly independent column vectors obtained in the previous part, and then write down the Q matrix.
 - (3 marks) Now calculate the matrix elements of R , and write down the matrix R .
 - (4 marks) Compute Rx and $Q^T b$, where $x = (a_0 \ a_1 \ a_2)^T$ which are the coefficients of the polynomial $p_2(x)$.
 - (2 marks) Using the above result, find the values of a_0 , a_1 and a_2 , and write the polynomial $p_2(x)$.
2. A function is given by $f(x) = 0.2 + 25x + 3x^2$. Now answer the following based on this function:
- (5 marks) Use the Trapezium rule to numerically integrate over the interval $[0, 2]$
 - (3 marks) Compute the exact integrated value of the given function.
 - (2 marks) Calculate the relative error in percentage.
3. Consider a function $f(x) = \frac{1}{x(\ln x)^2}$, which is continuous on the interval $[e, e+1]$. Now answer the questions below based on this function:
- (3 marks) Calculate the exact integrated value of the given function.
 - (5 marks) Find the numerical integration for $m=4$ using the Composite Newton-cotes formula.
 - (2 marks) Calculate the error in percentage from the above two parts.