1 Set A

1. For function $f(x) = e^x$, at $x_0 = 2$, find the truncation error for backward difference ($h = 1, h = 0.1, h = 0.01, h = 0.001$) and figure out the relationship of error with the order of h. (5 marks)
   ANS: Derivative of e^x is e^x. So $f'(x) = e^x$. $f'(2) = 7.38905609893$
   Now using backward difference formula we find out derivative for each of the h value. And by subtracting the backward difference from the exact derivative at $x_0 = 2$ we find the truncation error.

h	Backward difference	Truncation error
1	4.67077427047	2.71828182846
0.1	7.03161656651	0.35743953241
0.01	7.35223366211	0.03682243682
0.001	7.38536280208	0.00369329684

   We can see that the error is linear in h.

2. A rocket has been launched, and its velocities at differet times are collected. From these data, the acceleration of the rocket, $a(t)$ at $t = 16sec$ is calculated numerically by using different methods (h=1) as shown in the table below:

Difference method	Forward	Backward	Central
a (t=16)	33.88008462692784	32.898426118911345	33.38925537291959

   Now, if the velocity of a rocket as function of time obey the equation below: where v is in m/s and t is in seconds,

   $$v(t) = 1900 \ln\left(\frac{12 * 10^4}{12 * 10^4 - 2000t}\right) - 9.8t \tag{1}$$

   find the truncation errors for the acceleration at t=16sec for Forward, Backward and Central Difference methods. (4.5 marks)
   ANS:

   $$a(t) = 1900 * 2000 \frac{12 * 10^4 - 2000t}{(12 * 10^4 - 2000t)^2} - 9.8 \tag{2}$$

Following above equation, $a(t = 16) = 33.381818$

Method	Truncation error
forward	-0.4982664451096639
backward	0.48339206290683023
central	-0.007437191101416829

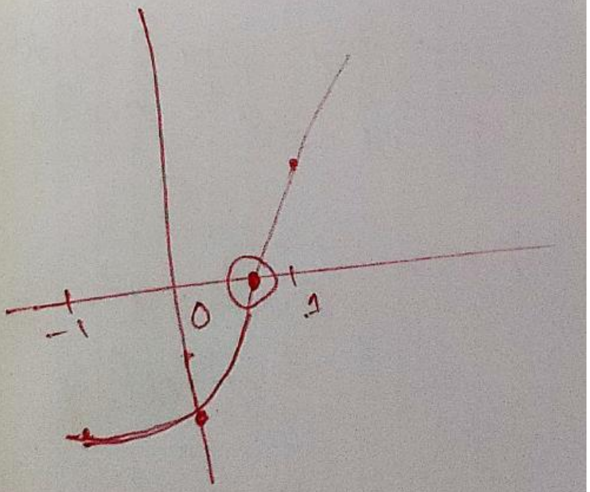
① $f(x) = 2x - \cos(x)$, $[-1, 1]$

(a) $f(-1) = -2.540302$

$f(0) = -1$

$f(1) = 1.459698$

root lies between
0 and 1.



(b) $f(x) = 0$

$\Rightarrow 2x^3 - 2x^2 - 3x + 3 = 0$

$\Rightarrow 3x = 2x^3 - 2x^2 + 3$

$$\boxed{\Rightarrow x = \frac{2x^3 - 2x^2 + 3}{3} = g_1(x)}$$

Again, $2x^3 - 2x^2 - 3x + 3 = 0$

$\Rightarrow 2x(x^2 - x) = 3x - 3$

$$\boxed{\Rightarrow x = \frac{3x - 3}{2(x^2 - x)} = g_2(x)}$$

(c) roots $\Rightarrow f(x) = 2x^3 - 2x^2 - 3x + 3$

$$\Rightarrow 0 = 2x^3 - 2x^2 - 3x + 3$$

$$\Rightarrow 2x^2(x-1) - 3(x-1) = 0$$

$$\Rightarrow (2x^2 - 3)(x-1) = 0$$

$$\therefore x = 1, \pm\sqrt{3/2}$$

$$\lambda_1 = |g_1(x)| = \left|\frac{1}{3}\left(6x^2 - 4x\right)\right|$$

$$\lambda_1 = \begin{cases} 2/3 & \text{; converge} \\ 1.367 & \text{; diverge} \\ 4.633 & \text{; diverge.} \end{cases}$$

$$\lambda_2 = |g_2(x)| = \left|\frac{3}{2} \times \frac{(x^2-x) - (x-1)(2x-1)}{(x^2-x)^2}\right|$$

$$\lambda_2 = \begin{cases} 0 & \text{; converge} \\ 1^- & \text{; diverge} \\ 1 & \text{; divenge} \end{cases}$$

3

Set 1

A

Newton's Method

K	Xk	f(Xk)
0	5	-1089.6331584
1	4.5031916	-399.0121859
2	4.0118120	-144.9610192
3	3.5348442	-51.5387238
4	3.0946337	-17.2706414
5	2.7388409	-4.9136136
6	2.5326224	-0.8872290
7	2.4763778	-0.0480822

B

Aitken Acceleration

K	Xk	f(Xk)
0	5	-1089.6331584
1	4.5031916	-399.0121859
2	4.0118120	-144.9610192
2(^)	-40.4646674	7
3	9.8654072 x 10^36	Math Error

Set 1 Question 4

a) $F(1) = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$ $A(2) = F(1)*A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$, $F(2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

b) $L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$

c) $A(3) = F(2)*A(2) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{pmatrix} = U$,

Ly=b $\quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ 17 \end{pmatrix}$ => $y_1 = 6, y_2 = 8, y_3 = -9$

Ux=y $\quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -9 \end{pmatrix}$ => $x_3 = 3, x_2 = 2, x_1 = 1$

Set 1

⑤ (a) Eqn of straight line = $a_0 + a_1 x$

$$\underbrace{\begin{bmatrix}1 & 100\\1 & 220\\1 & 430\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}a_0\\a_1\end{bmatrix}}_{x}=\underbrace{\begin{bmatrix}70\\180\\300\end{bmatrix}}_{b}$$

(b) $$\left[\underset{\downarrow\atop u_1}{\begin{matrix}1\\1\\1\end{matrix}}\quad\underset{\downarrow\atop u_2}{\begin{matrix}100\\220\\430\end{matrix}}\right]$$

$$P_1 = u_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$P_2 = u_2 - \frac{u_2 \cdot P_1}{P_1 \cdot P_1} P_1$$

$$= \begin{bmatrix}100\\220\\430\end{bmatrix} - \frac{(100\times1)+(220\times1)+(430\times1)}{(1\times1)+(1\times1)+(1\times1)}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= \begin{bmatrix}100\\220\\430\end{bmatrix} - 250\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$P_2 = \begin{bmatrix}-150\\-30\\180\end{bmatrix}$$

$$q_1 = \frac{P_1}{|P_1|} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{bmatrix}$$

$$q_2 = \frac{P_2}{|P_2|} = \frac{1}{30\sqrt{62}}\begin{bmatrix}-150\\-30\\180\end{bmatrix} = \begin{bmatrix}-5\sqrt{62}/62\\-\sqrt{62}/62\\3\sqrt{62}/31\end{bmatrix}$$

$$\boxed{\therefore Q = \begin{bmatrix}1/\sqrt{3} & -5\sqrt{62}/62\\1/\sqrt{3} & -\sqrt{62}/62\\1/\sqrt{3} & 3\sqrt{62}/31\end{bmatrix} = \begin{bmatrix}0.577 & -0.635\\0.577 & -0.127\\0.577 & 0.762\end{bmatrix}}$$

$$R = Q^T A$$

$$= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -5\sqrt{62}/62 & -\sqrt{62}/62 & 3\sqrt{62}/31 \end{bmatrix} \begin{bmatrix} 1 & 100 \\ 1 & 220 \\ 1 & 430 \end{bmatrix}$$

$$\boxed{R = \begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} = \begin{bmatrix} 1.732 & 433.013 \\ 0 & 236.220 \end{bmatrix}}$$

$$R x = Q^T b$$

$$\begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -5\sqrt{62}/62 & -\sqrt{62}/62 & 3\sqrt{62}/31 \end{bmatrix} \begin{bmatrix} 70 \\ 180 \\ 300 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 550/\sqrt{3} \\ 635\sqrt{2}/\sqrt{31} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 550/\sqrt{3} \\ 635\sqrt{2}/\sqrt{31} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1175/93 \\ 127/186 \end{bmatrix} = \begin{bmatrix} 12.634 \\ 0.683 \end{bmatrix}$$

$\therefore$ Eq^n of straight line = $12.634 + 0.683x$

(c) $f(x) = 12.634 + 0.683x$

$$f(1500) = 12.634 + 0.683(1500)$$

$$= 1037.134$$

$\therefore$ $\approx$ 1038 Vaccuum cleaners

6

① find out the actual Integral of $f(x)=\ln x$. within the interval $[1.4]$

ⓐ $\int_1^4 \ln x \, dx = [x\ln x - x]_1^4 = [4\ln 4 - 4 - \ln 1 + 1]$

$= \cancel{[3\ln x - 3]}\ [4\ln(4) - 3 - \ln(1)]$

$= 2.5452$

Use Composite Newton-Cotes Method. → and m=4.

ⓑ here $a = 1$, $b = 4$

$m = 4$, so, $h = \frac{b-a}{m} = \frac{3}{4}$

$x_0 = a = 1$

$x_1 = \cancel{x_0}+h = 1 + 3/4$

$= 7/4$

$x_2 = 7/4 + 3/4 = 10/4 = 5/2$

$x_3 = 10/4 + 3/4$

$= 13/4$

$x_4 = 13/4 + 3/4$

$= 4$

$C_{1,4} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$= \frac{3/4}{2}\left[1 + 2\ln(7/4) + 2\ln(5/2) + 2\ln(13/4) + \ln(4)\right]$

$= \cancel{3/R} \rightarrow 2.8858$

ⓒ Error: ?

$\left(\frac{C_{1,4} - \text{Actual}}{C_{1,4}}\right) \times 100\%$

$= \frac{2.8858 - 2.5452}{2.8858} \times 100\%$

$= 11.80\%$