Question 1 solution:

a) A = $\begin{pmatrix} 2 & 1 & -1 & 2 \\ 4 & 5 & -3 & 6 \\ -2 & 5 & -2 & 6 \\ 4 & 11 & -4 & 8 \end{pmatrix}$, This is a square matrix. det(A) =-12 $\neq$ 0. So, A is non-singular.

This system has unique solution.

b) Augmented matrix = $\left(\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 5 \\ 4 & 5 & -3 & 6 & 9 \\ -2 & 5 & -2 & 6 & 4 \\ 4 & 11 & -4 & 8 & 2 \end{array}\right)$

c) $m_{21} = 2, m_{31} = -1, m_{41} = 2$. So, using these multipliers $\left(\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 5 \\ 0 & 3 & -1 & 2 & -1 \\ 0 & 6 & -3 & 8 & 9 \\ 0 & 9 & -2 & 4 & -8 \end{array}\right)$

$m_{32} = 2, m_{42} = 3$. So, using these multipliers $\left(\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 5 \\ 0 & 3 & -1 & 2 & -1 \\ 0 & 0 & -1 & 4 & 11 \\ 0 & 0 & 1 & 2 & -5 \end{array}\right)$

$m_{43} = -1$. So, using these multipliers $\left(\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 5 \\ 0 & 3 & -1 & 2 & -1 \\ 0 & 0 & -1 & 4 & 11 \\ 0 & 0 & 0 & 2 & 6 \end{array}\right)$

d) Using Back Substitution: $x_4 = 3, x_3 = 1, x_2 = -2,\ x_1 = 1$

Question 2 solution:

a) $F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$ $A^{(2)} = F^{(1)} * A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$, $F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

b) L = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$

c) $A^{(3)} = F^{(2)} * A^{(2)} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{pmatrix} = U$,

Ly=b $\quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ 17 \end{pmatrix}$ => $y_1 = 6, y_2 = 8, y_3 = -9$

Ux=y $\quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -9 \end{pmatrix}$ => $x_3 = 3, x_2 = 2, x_1 = 1$

Question 3 solution:

a) $A = \begin{pmatrix} 3 & 3 & 4 \\ 0 & 0 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ here, $m_{32} = 0$. So, Gaussian elimination method fails to solve the system.

b) Swap row 2 and 3. pivoting

4 a. if $x^T y = 0$; then the value of θ should be $90°$.
since,

$$\vec{x} \cdot \vec{y} = |x| \cdot |y| . \cos\theta = x^T y$$

$$\Rightarrow |x||y| . \cos 90° = x^T y \Rightarrow x^T y = 0$$

b. the conditions for a set to be orthonormals are:
if for 2 vectors, $\vec{x}, \vec{y}$:

(i) $x^T y = 0$ [has to be orthogonal]

(ii) $x^T x = 1$; $y^T y = 1$ [norm unity]

c. $S = \{(3,5,2)^T, (-2,2,-2)^T\}$

assuming, $x^T = (3,5,2)^T$

$y^T = (-2,2,-2)^T$

$$\therefore x^T y = [3 \ 5 \ 2] \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = [-6+10-4] = 0$$

Hence, orthogonal.

d. $S = \left\{ \frac{1}{\sqrt{36}} (-5,3)^T, \frac{1}{\sqrt{36}} (3,5)^T \right\}$

$x^T = \frac{1}{\sqrt{36}} (-5,3)^T$

$y^T = \frac{1}{\sqrt{36}} (3,5)^T$

$$\therefore x^T y = \frac{1}{36} (-5 \times 3 + 3 \times 5) = 0$$

and $x^T x = \frac{1}{36} (25+9) = \frac{34}{36} \neq 1$

$y^T y = \frac{1}{36} (9+25) = \frac{34}{36} \neq 1$

Hence, the set is not orthonormal.

e. $S = \left\{ \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)^T, \left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)^T, \left(-\frac{1}{\sqrt{66}}, -\frac{4}{\sqrt{66}}, \frac{7}{\sqrt{66}}\right)^T \right\}$

$x^T = \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)^T$

$y^T = \left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)^T$

$z^T = \left(-\frac{1}{\sqrt{66}}, -\frac{4}{\sqrt{66}}, \frac{7}{\sqrt{66}}\right)^T$

$\therefore x^Ty = \frac{3}{\sqrt{11}}\left(-\frac{1}{\sqrt{6}}\right) + \frac{1}{\sqrt{11}} \cdot \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{11}} \cdot \frac{1}{\sqrt{6}} = \frac{-3}{\sqrt{66}} + \frac{2}{\sqrt{66}} + \frac{1}{\sqrt{66}} = 0$

$\therefore y^Tz = \left(-\frac{1}{\sqrt{6}}\right)\left(\frac{-1}{\sqrt{66}}\right) + \frac{2}{\sqrt{6}}\left(-\frac{4}{\sqrt{66}}\right) + \frac{1}{\sqrt{6}} \cdot \frac{7}{\sqrt{66}} = 0$

$\therefore x^Tz / z^Tx = \left(-\frac{1}{\sqrt{66}}\right)\left(\frac{3}{\sqrt{11}}\right) + \left(-\frac{4}{\sqrt{66}}\right)\left(\frac{1}{\sqrt{11}}\right) + \frac{7}{\sqrt{66}} \cdot \frac{1}{\sqrt{11}} = 0$

Now,

$x^Tx = \left(\frac{3}{\sqrt{11}}\right)^2 + \left(\frac{1}{\sqrt{11}}\right)^2 + \left(\frac{1}{\sqrt{11}}\right)^2 = 1$

$y^Ty = \left(-\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{2}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 = 1$

$z^Tz = \left(-\frac{1}{\sqrt{66}}\right)^2 + \left(-\frac{4}{\sqrt{66}}\right)^2 + \left(\frac{7}{\sqrt{66}}\right)^2 = 1$

Since, it matches both of the conditions, it is an orthonormal set.

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