

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
 - Write all answers sequentially.
 - Start answering a question (not the part of the question) from the top of a new page.
 - Write legibly and in orderly fashion maintaining all mathematical norms and rules.
 - Start working right away based on whatever you know. **Do not wait for the last moment and ask for time extension.**
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1. A function is given by $f(x) = 6e^{-5x}$. Now answer the following:
 - (a) (2 marks) Approximate the derivative of $f(x)$ at $x_0 = 0.2$ with step size $h = 0.5$ using the central difference method up to 6 significant figures.
 - (b) (2 marks) Approximate the derivative of $f(x)$ at $x_0 = 0.2$ with step size $h = 0.5$ using the forward difference method up to 6 significant figures.
 - (c) (3 marks) Calculate the truncation error of $f(x)$ at $x_0 = 2$ using $h = 1, 0.1, 0.01, 0.0001$ in the above mentioned two methods.
 - (d) (3 marks) Compute $D_{0.5}^{(1)}$ at $x_0 = 0.2$ using Richardson extrapolation method up to 6 significant figures and calculate the truncation error.
2. During the class, we derived in detail the first order Richardson extrapolated derivative, by using $h \rightarrow h/2$,

$$D_h^{(1)} \equiv f'(x_0) - \frac{h^4}{480} + \mathcal{O}(h^6) .$$

- (a) (5 marks) Using $h \rightarrow h/2$, derive the expression for $D_h^{(2)}$ which is the second order Richardson extrapolation.
 - (b) (5 marks) Now starting from the definition of D_h and using $h \rightarrow h/3$, derive the expression for $D_h^{(1)}$.
3. Consider the equation $f(x) = x^3 - 2x^2 - 11x + 12$ on the interval $[1.78, 8.25]$.
 - (a) (2 marks) Find the minimum number of iterations required to find the root of the given equation within the error bound of 1×10^{-3} .
 - (b) (8 marks) Show 5 iterations using the Bisection Method to find the root of the above function within the interval $[1.78, 8.25]$.
 4. Consider the function, $f(x) = x^3 - x^2 - 9x + 9$. Answer the following:
 - (a) (2 marks) State the exact roots of $f(x)$.
 - (b) (2 marks) Construct three different fixed point functions $g(x)$ such that $f(x) = 0$. (Make sure that one of the $g(x)$'s that you constructed converges to at least a root).
 - (c) (3 marks) Find the convergence rate/ratio for $g(x)$ constructed in previous part and also find which root it is converging to?
 - (d) (3 marks) Find the approximate root, x_* , of the above function using fixed point iterations up to 4 significant figures within the error bound of 1×10^{-3} using $x_0 = 0$ and any fixed point function $g(x)$ from part(b) that converges to the root(s).
 5. (10 marks) Use Newton's method to find the root, x_* , of the equation, $f(x) = x^2 e^{-x} - 0.6$, up to machine epsilon of 1×10^{-4} starting with $x_0 = 0.2$.