SS-Computer Assignment - 03 - Spring 2019

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1) a)

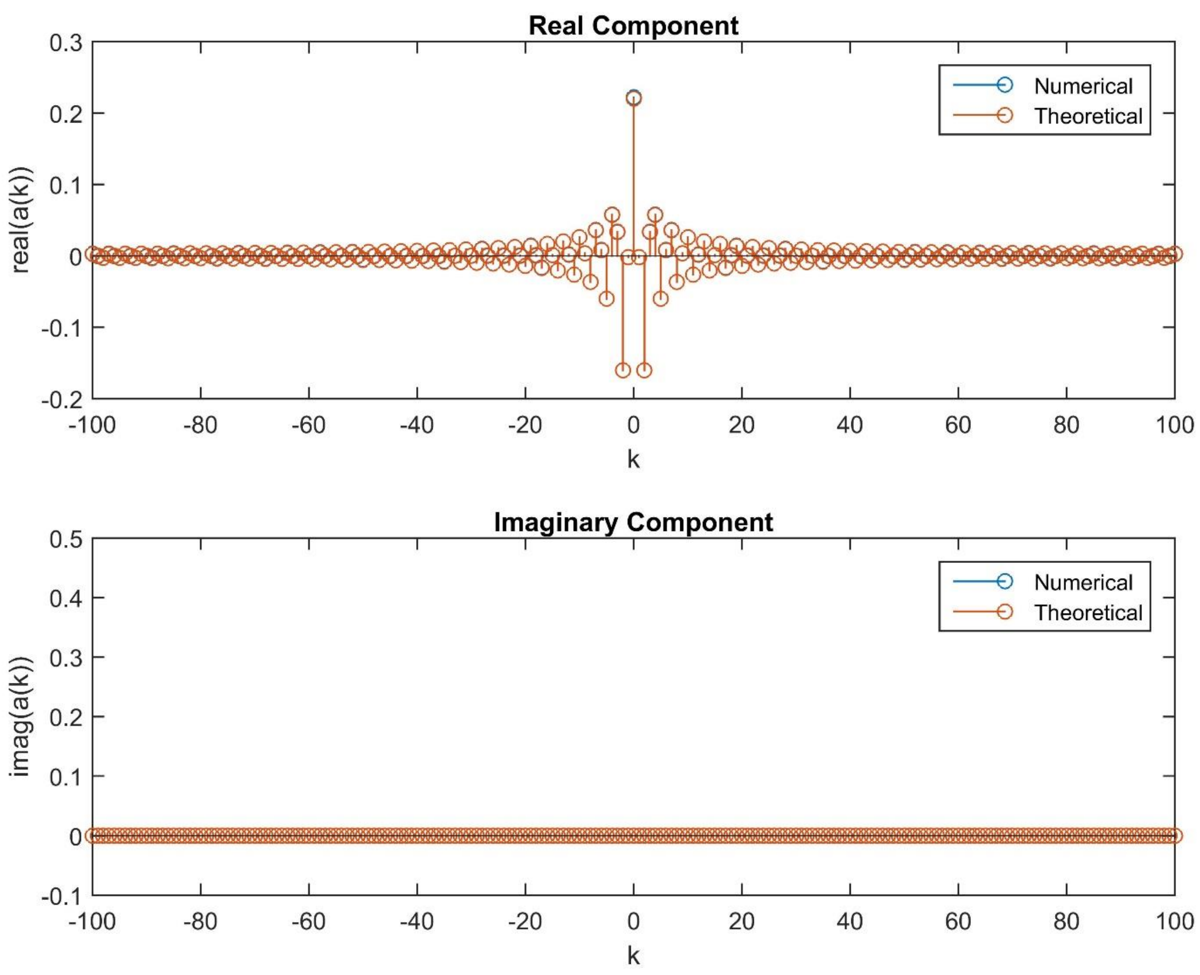
Code :

```
clear
%%
% Defining the signal
x = @(t) [t.^2 .* (t<1 & t>-1)];

%%
% Defining parameters
To = 3;
Wo = 2*pi/To;
tvec = -To/2:0.001:To/2;

%%
% Taking signal in the given time period
x_t = x(tvec);

%%
% Taking 100 samples
M=100;

for mx=1:M
% Calculating Fourier Series
kvec = -mx:mx;

for kx=1:length(kvec)
    k=kvec(kx);
    basis=exp(-1i*k*Wo*tvec);
    avec(kx) = 1/To*trapz(tvec,x_t.*basis);
end

% Reconstructing the original signal
recon=zeros(size(tvec));
for kx=1:length(kvec)
    k=kvec(kx);
    basis=exp(1i*k*Wo*tvec);
    recon=recon+avec(kx)*basis;
end
% Calculating the convergence
recon_err(mx) = mean((abs(recon-x_t)).^2);
end
%%
% Theoretical equation
a_th = 1/3*((2*sin(kvec*Wo)./(kvec*Wo)) ...
       + (4*cos(kvec*Wo)./(kvec.^2 * Wo.^2)) ...
       - (4*sin(kvec*Wo)./(kvec.^3*Wo.^3)));
a_th(kvec == 0) = 0.22;

%%
% Plotting the coefficients and comparing with the theoretical values
figure();
subplot(211); stem(kvec,real(avec));
title('Real Component');
xlabel('k');
ylabel('real(a(k))');
```

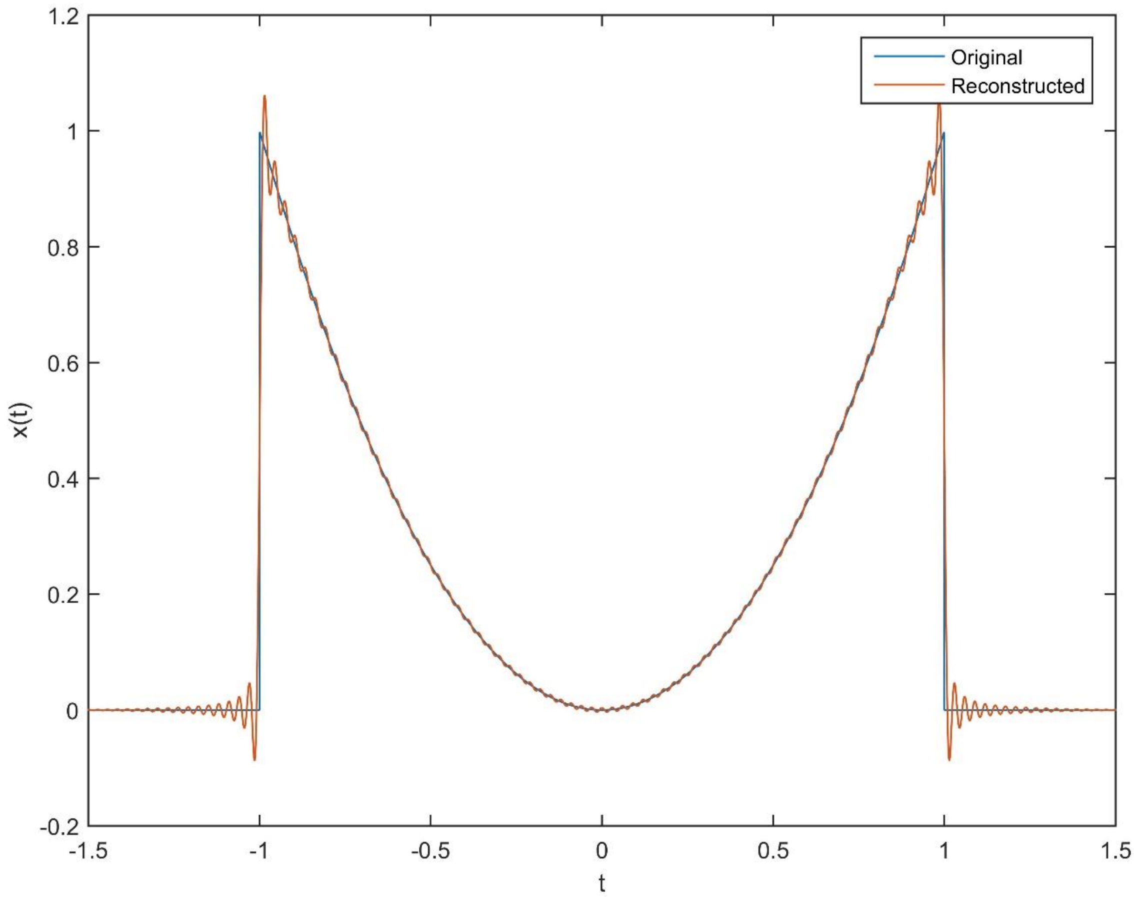
```
hold on;
stem(kvec,real(a_th));
legend('Numerical','Theoretical');

subplot(212); stem(kvec,imag(avec));
title('Imaginary Component');
xlabel('k');
ylabel('imag(a(k))');
ylim([-0.1 0.5]);
hold on;
stem(kvec,imag(a_th));
legend('Numerical','Theoretical');

% Plotting the original vs constructed signals
figure();
plot(tvec,x_t);
xlabel('t');
ylabel('x(t)');
hold on;
plot(tvec,recon);
legend('Original','Reconstructed');

% Plotting the convergence
figure();
stem(1:M, recon_err);
xlabel('M');
ylabel('Error');
legend('Error');
```

PLOTS :

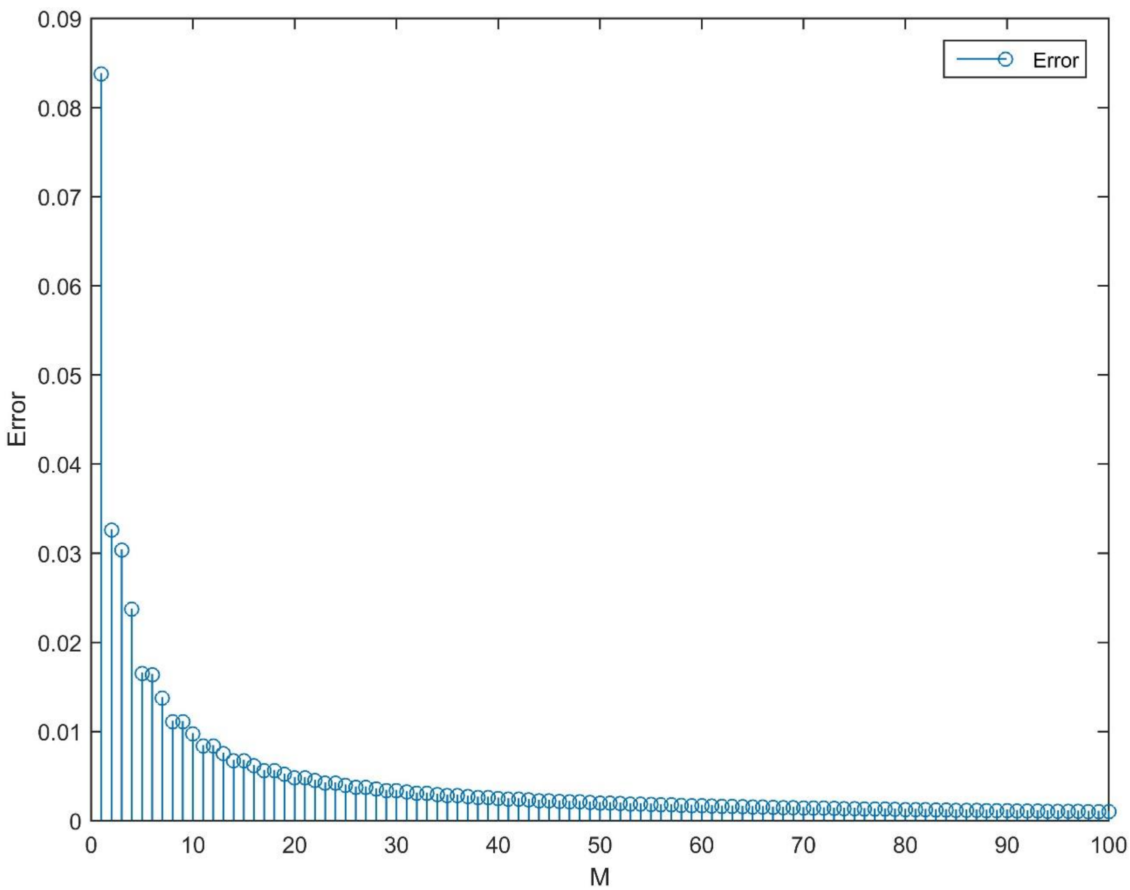
Evaluating the coefficients with the theoretical values:



Signal Recontruction:



Convergence:



1) a) $x(t) = t^2 \quad , |t| < 1 \ \& \ T_0 = 3$

$$a_K = \frac{1}{3}\int_{-1}^{1} t^2 e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{3}\left[t^2 \frac{e^{-jK\omega_0 t}}{-jK\omega_0}\Bigg|_{-1}^{1} - \int_{-1}^{1} 2t \cdot \frac{e^{-jK\omega_0 t}}{-jK\omega_0}\, dt \right]$$

$$= \frac{1}{3}\left[\frac{e^{-jK\omega_0} - e^{jK\omega_0}}{-jK\omega_0} + \frac{2}{jK\omega_0}\left(\frac{t e^{-jK\omega_0 t}}{-jK\omega_0}\Bigg|_{-1}^{1} - \int_{-1}^{1} \frac{e^{-jK\omega_0 t}}{-jK\omega_0}\, dt \right)\right]$$

$$= \frac{1}{3}\left[\frac{-2j\sin(K\omega_0)}{-jK\omega_0} + \frac{2}{jK\omega_0}\left[\frac{e^{-jK\omega_0} + e^{jK\omega_0}}{-jK\omega_0} + \frac{1}{jK\omega_0} \frac{e^{-jK\omega_0 t}}{-jK\omega_0}\Bigg|_{-1}^{1} \right]\right]$$

$$= \frac{1}{3}\left[\frac{2\sin(K\omega_0)}{K\omega_0} + \frac{2}{jK\omega_0}\left[\frac{2\cos(K\omega_0)}{-jK\omega_0} + \frac{e^{-jK\omega_0} - e^{jK\omega_0}}{-j^2K^2\omega_0^2} \right]\right]$$

$$\Rightarrow a_K = \frac{1}{3}\left[\frac{2\sin(K\omega_0)}{K\omega_0} + \frac{4\cos(K\omega_0)}{K^2\omega_0^2} - \frac{4\sin(K\omega_0)}{K^3\omega_0^3} \right]$$

$$a_0 = \frac{1}{3}\int_{-1}^{1} t^2 (1)\, dt$$

$$= \frac{1}{3} \frac{t^3}{3}\Bigg|_{-1}^{1}$$

$$= \frac{1}{3}\left[\left(\frac{1}{3}\right) - \left(-\frac{1}{3}\right) \right]$$

$$\Rightarrow a_0 = \frac{2}{9}$$

1) b)

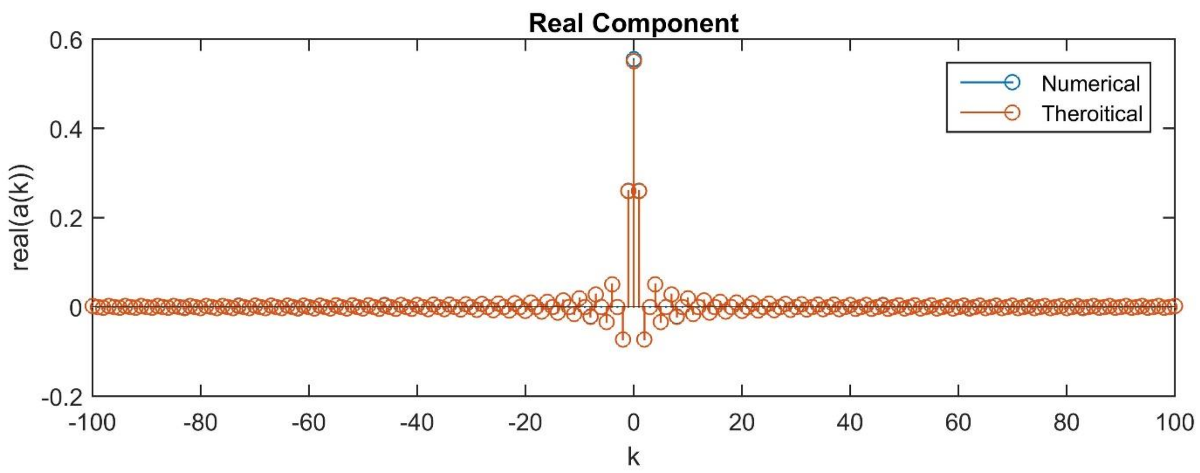
Code :

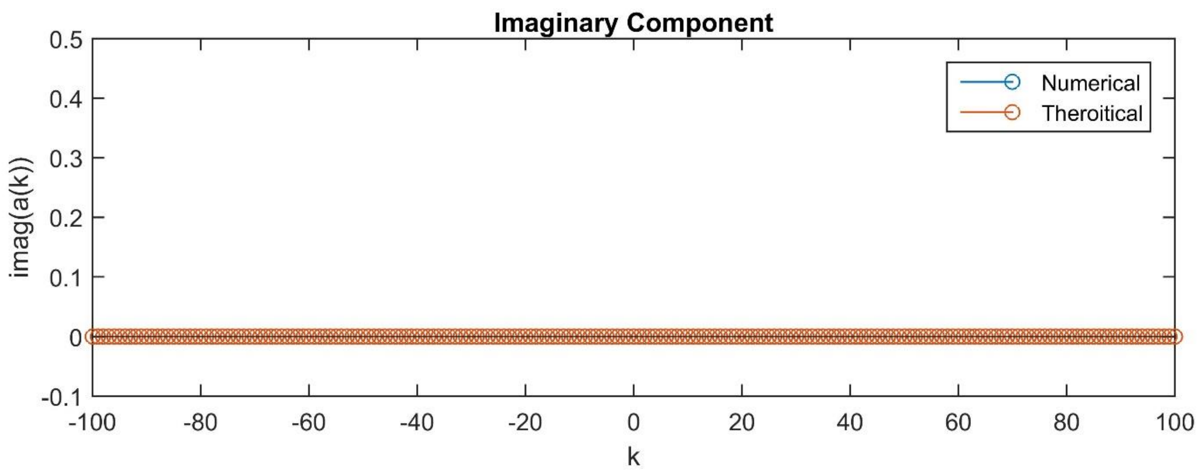
```
clear
%%
% Defining the signal
x = @(t) [(1 - abs(t)/3) .* (t<1 & t>-1)];

%%
% Defining parameters
To = 3;
Wo = 2*pi/To;
tvec = -To/2:0.001:To/2;

%%
% Taking signal in the given time period
x_t = x(tvec);

%%
% Taking 100 samples
M=100;

for mx=1:M
% Calculating Fourier Series
kvec = -mx:mx;

for kx=1:length(kvec)
    k=kvec(kx);
    basis=exp(-1i*k*Wo*tvec);
    avec(kx) = 1/To*trapz(tvec,x_t.*basis);
end

% Reconstructing the original signal
recon=zeros(size(tvec));
for kx=1:length(kvec)
    k=kvec(kx);
    basis=exp(1i*k*Wo*tvec);
    recon=recon+avec(kx)*basis;
end

% Calculating the convergence
recon_err(mx) = mean((abs(recon-x_t)).^2);
end

%%
% Theoritical equation
a_th = 2/9 * ((2*sin(kvec*Wo)./(kvec*Wo)) ...
       + (1-cos(kvec*Wo))./(kvec.^2 * Wo.^2));
a_th(kvec == 0) = 0.55;

%%
% Plotting the coefficients and comparing with the therotical values
figure();
subplot(211); stem(kvec,real(avec));
title('Real Component');
xlabel('k');
ylabel('real(a(k))');
hold on;
stem(kvec,real(a_th));
legend('Numerical','Theroitical');

subplot(212); stem(kvec,imag(avec));
title('Imaginary Component');
xlabel('k');
```

```
ylabel('imag(a(k))');
ylim([-0.1 0.5]);
hold on;
stem(kvec,imag(a_th));
legend('Numerical','Theroitical');

% Plotting the original vs constructed signals
figure();
plot(tvec,x_t);
xlabel('t');
ylabel('x(t)');
hold on;
plot(tvec,recon);
legend('Original','Reconstructed');


% Plotting the convergence
figure();
stem(1:M,(recon_err));
xlabel('M');
ylabel('Error');
legend('Error');
```

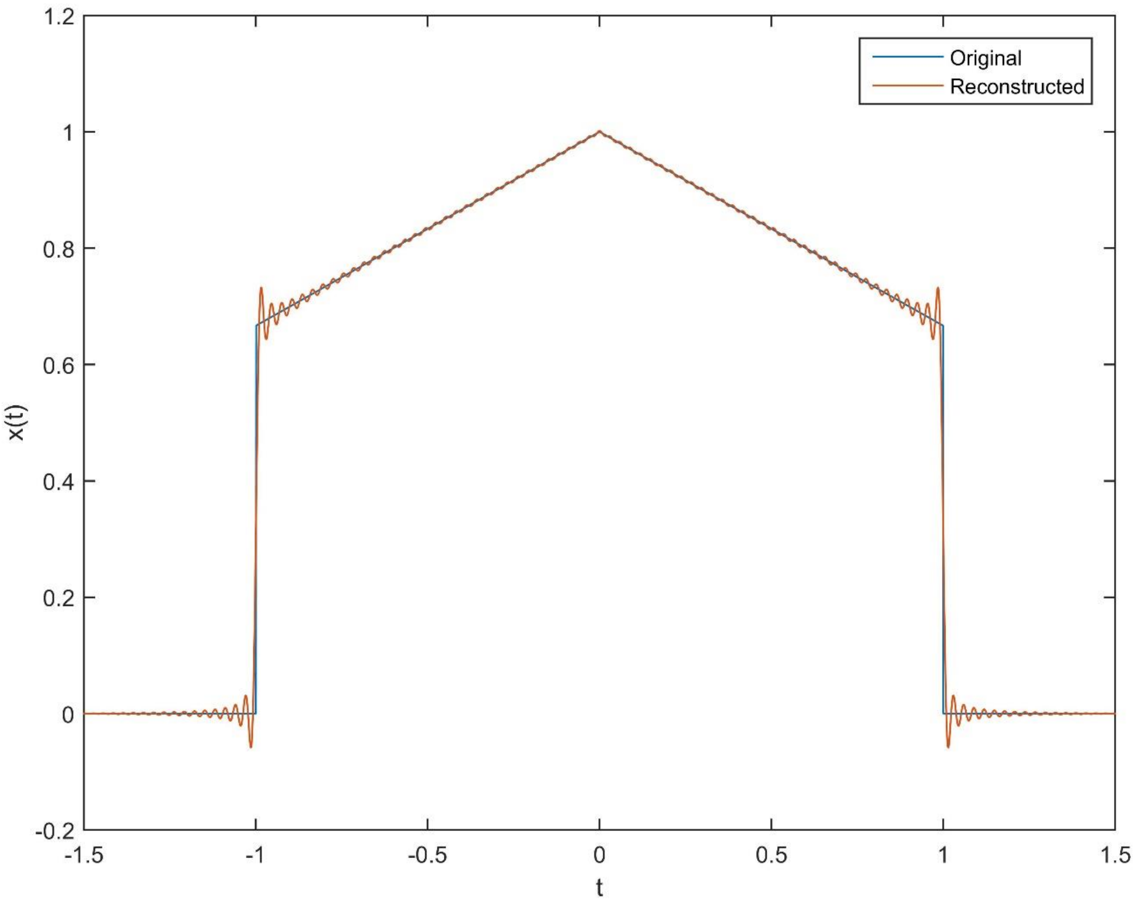
PLOTS:

Evaluating the coefficients with the theoretical values:

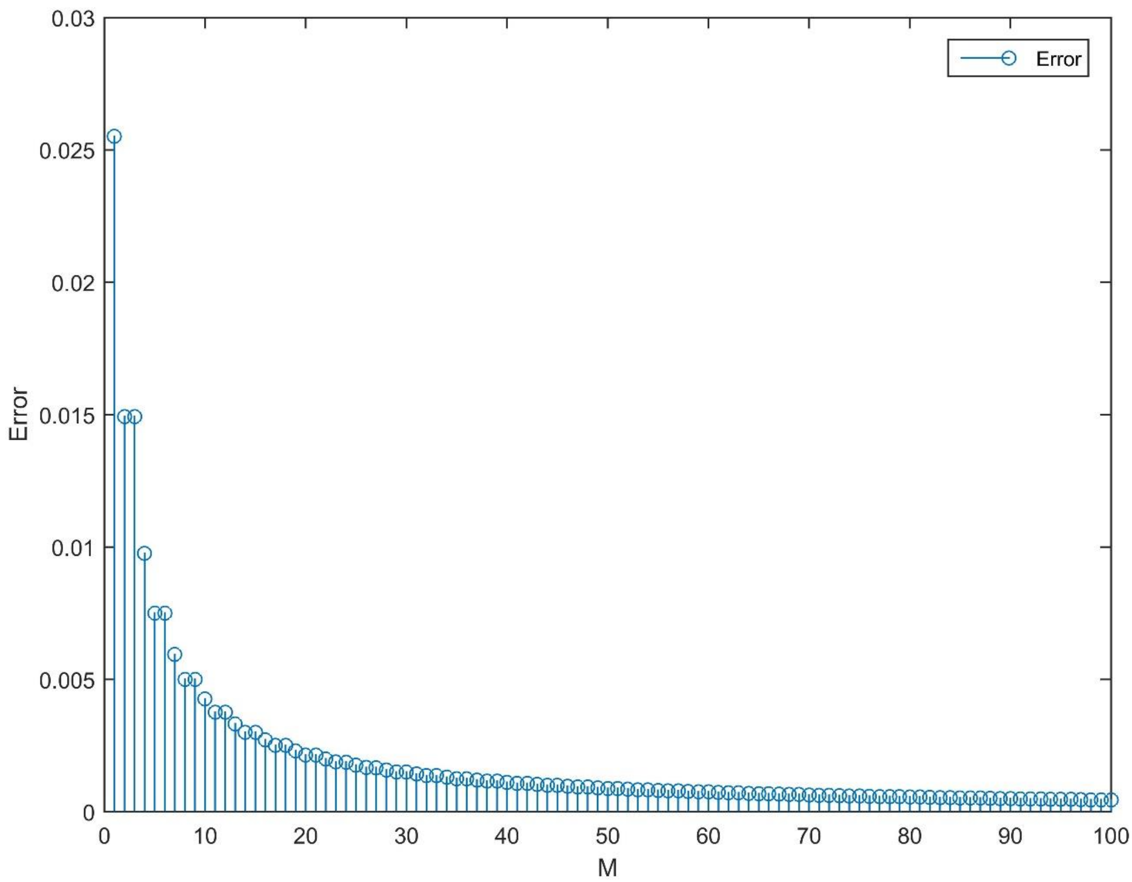




Signal Recontruction:



Convergence:



17 b) $x(t) = 1 - \frac{|t|}{3}$, $|t| < 1$, $T_0 = 3$

$$a_k = \frac{1}{3}\int_{-1}^{1}\left(1 - \frac{|t|}{3}\right) e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{3}\left[\int_{-1}^{0}\left(1+\frac{t}{3}\right) e^{-jk\omega_0 t} dt + \int_{0}^{1}\left(1-\frac{t}{3}\right) e^{-jk\omega_0 t} dt\right]$$

$$= \frac{1}{3}\left[\left(1+\frac{t}{3}\right)\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Bigg|_{-1}^{0} + \int_{-1}^{0}\frac{1}{3}\times\frac{e^{-jk\omega_0 t}}{-jk\omega_0} + \left(1-\frac{t}{3}\right)\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Bigg|_{0}^{1} + \int_{0}^{1}\frac{1}{3}\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]$$

$$= \frac{1}{3}\left[\left(1+\frac{t}{3}\right)\frac{e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{e^{-jk\omega_0 t}}{3k^2\omega_0^2}\Bigg|_{-1}^{0} + \left(1-\frac{t}{3}\right)\frac{e^{-jk\omega_0 t}}{-jk\omega_0} - \frac{e^{-jk\omega_0 t}}{3k^2\omega_0^2}\Bigg|_{0}^{1}\right]$$

$$= \frac{1}{3}\left[\left(\left(\frac{1}{-jk\omega_0} + \frac{1}{jk^2\omega_0^2}\right) - \left(\frac{2}{3}\frac{e^{jk\omega_0}}{-jk\omega_0} + \frac{e^{jk\omega_0}}{3k^2\omega_0^2}\right)\right) + \right.$$

$$\left.\left(\frac{2}{3}\frac{e^{-jk\omega_0}}{-jk\omega_0} - \frac{1}{3}\frac{e^{-jk\omega_0}}{k^2\omega_0^2}\right) - \left(\frac{1}{-jk\omega_0} - \frac{1}{3k^2\omega_0^2}\right)\right]$$

$$= \frac{1}{3}\left(\frac{2}{3}\frac{e^{jk\omega_0}}{jk\omega_0} + \frac{1 - e^{jk\omega}}{3k^2\omega_0^2} - \frac{2}{3}\frac{e^{-jk\omega_0}}{jk\omega_0} + \frac{1 - e^{-jk\omega_0}}{3k^2\omega_0^2}\right)$$

$$= \frac{1}{3}\left\{\frac{2}{3jk\omega_0}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] + \frac{2}{3k^2\omega_0^2} - \cancel{\frac{2}{3k^2\omega_0^2}\cos k\omega}\right.$$

$$\left. - \frac{1}{3}\left(\frac{e^{jk\omega_0} - e^{-jk\omega_0}}{k^2\omega_0^2}\right)\right\}$$

$$\Rightarrow a_k = \frac{1}{3}\left[\frac{2}{3k\omega}(2\sin k\omega_0) + \frac{2}{3k^2\omega_0^2} - \frac{2}{3k^2\omega_0^2}(\cos k\omega_0)\right]$$

where $\omega_0 = \frac{2\pi}{3}$

$$a_0 = \frac{1}{3}\int_{-1}^{1} 1 - \frac{|t|}{3}\, dt$$

$$= \frac{1}{3}\int_{-1}^{0} 1 + \frac{t}{3}\, dt + \int_{0}^{1} 1 - \frac{t}{3}\, dt$$

$$= \frac{1}{3}\left(t + \frac{t^2}{6}\Bigg|_{-1}^{0} + t - \frac{t^2}{6}\Bigg|_{0}^{1}\right)$$

$$= \frac{1}{3}\left[+1 - \frac{1}{6} + 1 - \frac{1}{6}\right]$$

$$\rightarrow a_0 = \frac{5}{9}$$

1) c)

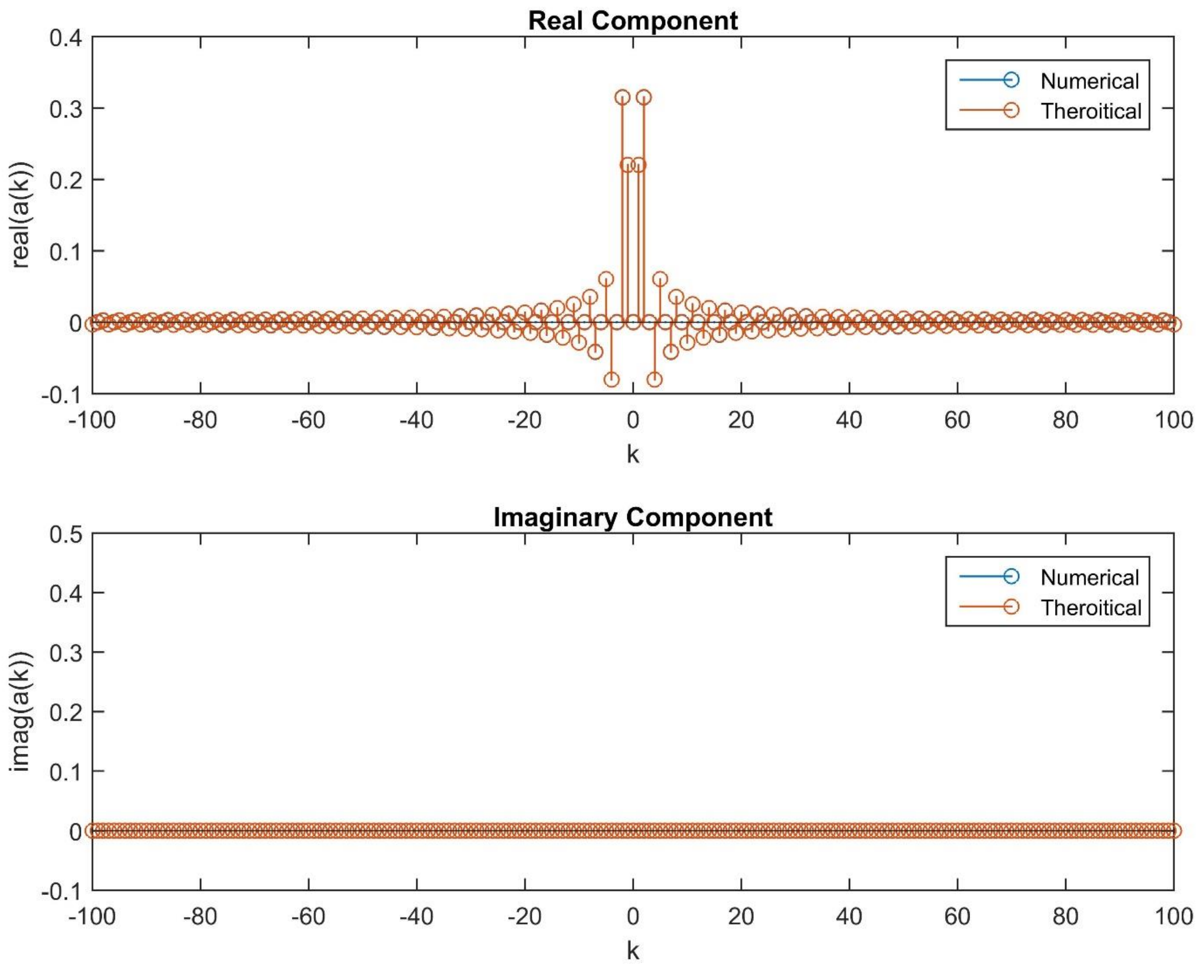
Code :

```
clear
%%
% Defining the signal
x = @(t) [cos(pi.*t) .* (t<1 & t>-1)];

%%
% Defining parameters
To = 3;
Wo = 2*pi/To;
tvec = -To/2:0.001:To/2;

%%
% Taking signal in the given time period
x_t = x(tvec);

%%
% Taking 100 samples
M=100;

for mx=1:M
% Calculating Fourier Series
kvec = -mx:mx;

for kx=1:length(kvec)
    k=kvec(kx);
    basis=exp(-1i*k*Wo*tvec);
    avec(kx) = 1/To*trapz(tvec,x_t.*basis);
end

% Reconstructing the original signal
recon=zeros(size(tvec));
for kx=1:length(kvec)
    k=kvec(kx);
    basis=exp(1i*k*Wo*tvec);
    recon=recon+avec(kx)*basis;
end

% Calculating the convergence
recon_err(mx) = mean((abs(recon-x_t)).^2);
end

%%
% Theoritical equation
a_th = sin((pi/3)*(2*kvec+3))./(pi*(2*kvec+3)) ...
        + sin((pi/3)*(3-2*kvec))./(pi*(3-2*kvec));
a_th(kvec == 0) = 0;

%%
% Plotting the coefficients and comparing with the therotical values
figure();
subplot(211); stem(kvec,real(avec));
title('Real Component');
xlabel('k');
ylabel('real(a(k))');
hold on;
stem(kvec,real(a_th));
legend('Numerical','Theroitical');

subplot(212); stem(kvec,imag(avec));
title('Imaginary Component');
```

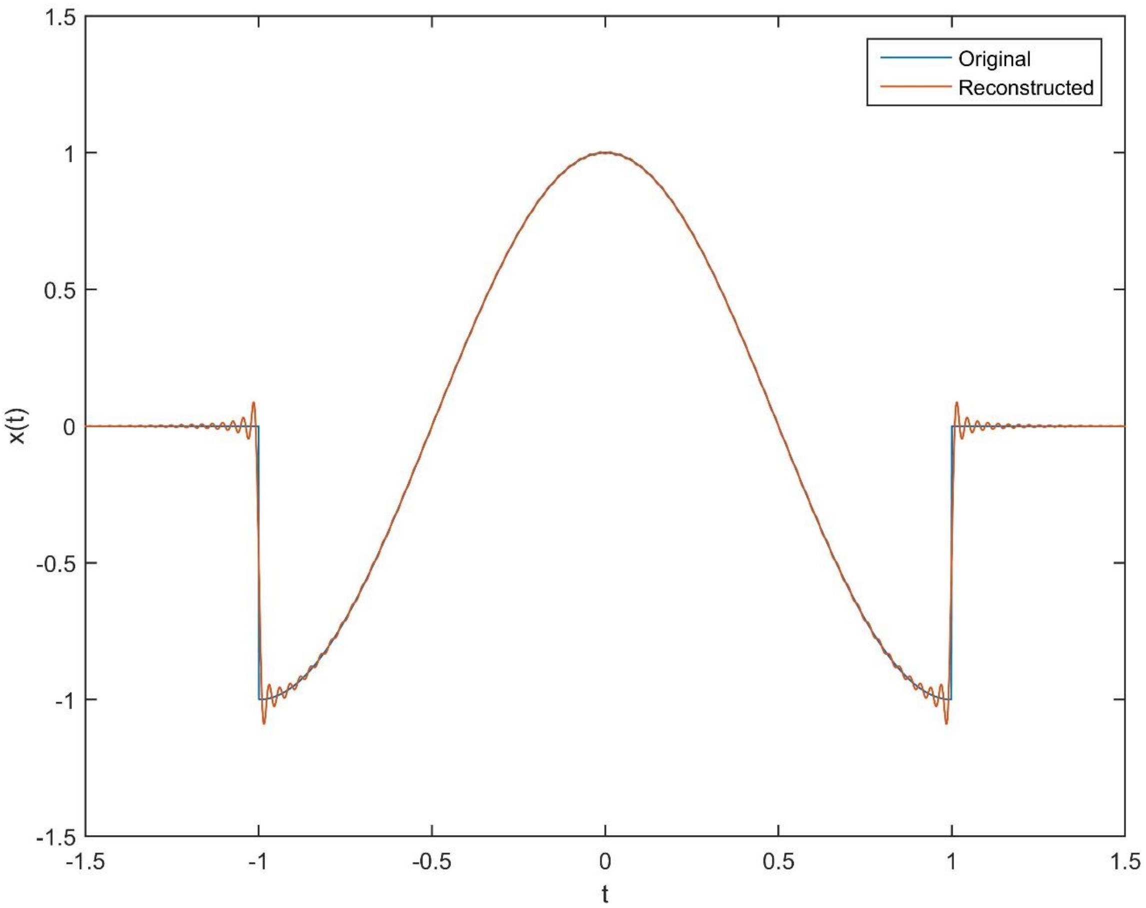
```
xlabel('k');
ylabel('imag(a(k))');
ylim([-0.1 0.5]);
hold on;
stem(kvec,imag(a_th));
legend('Numerical','Theroitical');

% Plotting the original vs constructed signals
figure();
plot(tvec,x_t);
xlabel('x(t)');
ylabel('t');
hold on;
plot(tvec,recon);
legend('Original','Reconstructed');


% Plotting the convergence
figure();
stem(1:M,(recon_err));
xlabel('M');
ylabel('Error');
legend('Error');
```

PLOTS:

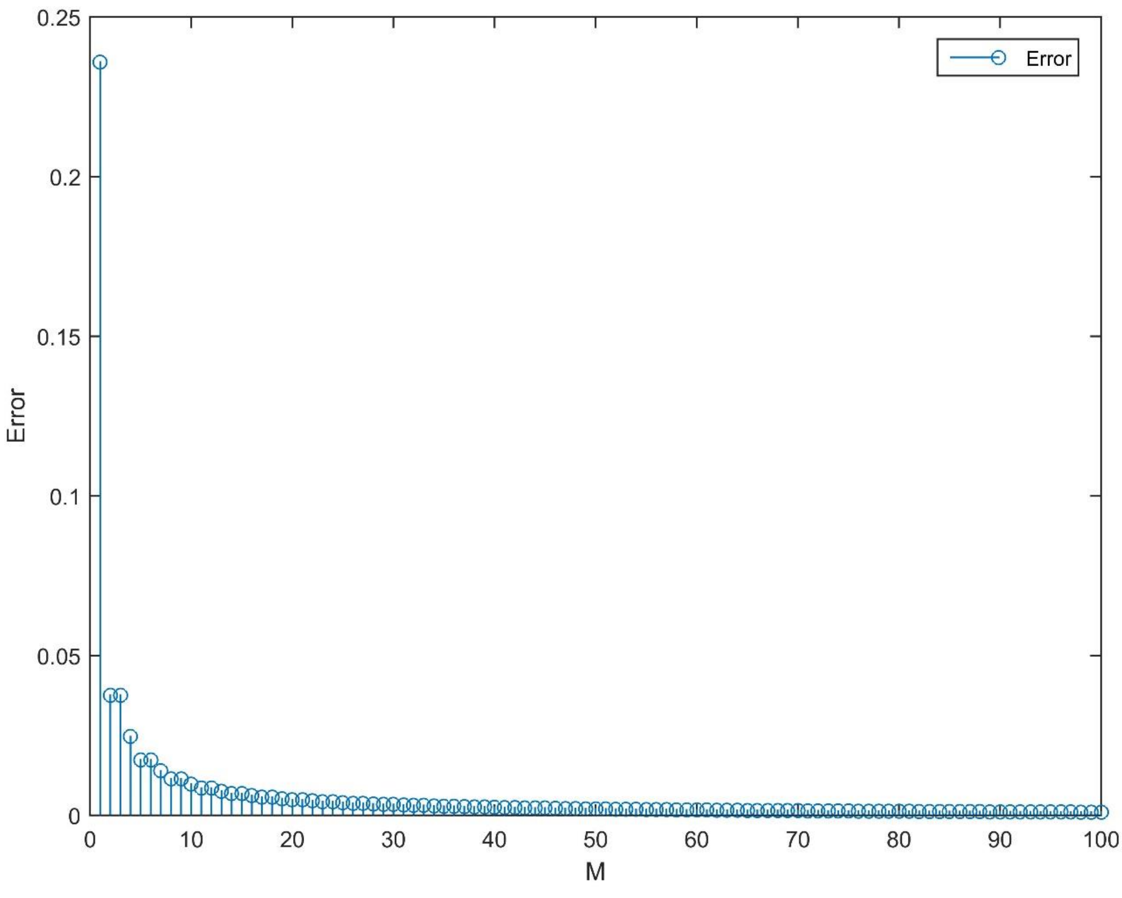
Evaluating the coefficients with the theoretical values:



Signal Recontruction:



Convergence:



17 c) $x(t) = \cos \pi t$, $T_0 = 3$ & $|t| < 1$

$$a_K = \frac{1}{T} \int_t x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{3} \int_{-1}^{1} \cos \pi t \; e^{-jk\omega_0 t} dt$$

$$= \frac{1}{3} \int_{-1}^{1} \left(\frac{e^{-j\pi t} + e^{j\pi t}}{2} \right) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{6} \int_{-1}^{1} \left[e^{-jt\left(\pi + \frac{2K\pi}{3}\right)} + e^{jt\left(\pi - \frac{2K\pi}{3}\right)} \right] dt$$

$$= \frac{1}{6} \left[\frac{e^{-jt\left(\frac{\pi}{3}(2K+3)\right)}}{-j\left(\frac{\pi}{3}(2K+3)\right)} \Bigg|_{-1}^{1} + \frac{e^{jt\left(\frac{\pi}{3}(3-2K)\right)}}{j\left(\frac{\pi}{3}(3-2K)\right)} \Bigg|_{-1}^{1} \right]$$

$$= \frac{1}{6} \left[\frac{-e^{-j\left(\frac{\pi}{3}(2K+3)\right)} + e^{j\left(\frac{\pi}{3}(2K+3)\right)}}{j\left(\frac{\pi}{3}(2K+3)\right)} + \frac{e^{j\left(\frac{\pi}{3}(3-2K)\right)} - e^{-j\left((3-2K)\frac{\pi}{3}\right)}}{j\left(\frac{\pi}{3}(3-2K)\right)} \right]$$

$$= \frac{1}{6} \left[\frac{2\sin\left(\frac{\pi}{3}(2K+3)\right)}{\frac{\pi}{3}(2K+3)} + \frac{2\sin\left(\frac{\pi}{3}(3-2K)\right)}{\frac{\pi}{3}(3-2K)} \right]$$

$$= \frac{1}{3} \times 3 \left[\frac{\sin\left(\frac{\pi}{3}(2K+3)\right)}{\pi(2K+3)} + \frac{\sin\left(\frac{\pi}{3}(3-2K)\right)}{\pi(3-2K)} \right]$$

$$\Rightarrow a_K = \frac{\sin\left(\frac{\pi}{3}(2K+3)\right)}{\pi(2K+3)} + \frac{\sin\left(\frac{\pi}{3}(3-2K)\right)}{\pi(3-2K)}$$

$$\therefore a_0 = \frac{1}{3} \int_{-1}^{1} x(t) dt$$

$$= \frac{1}{3} \int_{-1}^{1} \cos \pi t \, dt$$

$$= \frac{1}{3} \frac{\sin \pi t}{\pi} \Bigg|_{-1}^{1}$$

$$\Rightarrow a_0 = 0$$

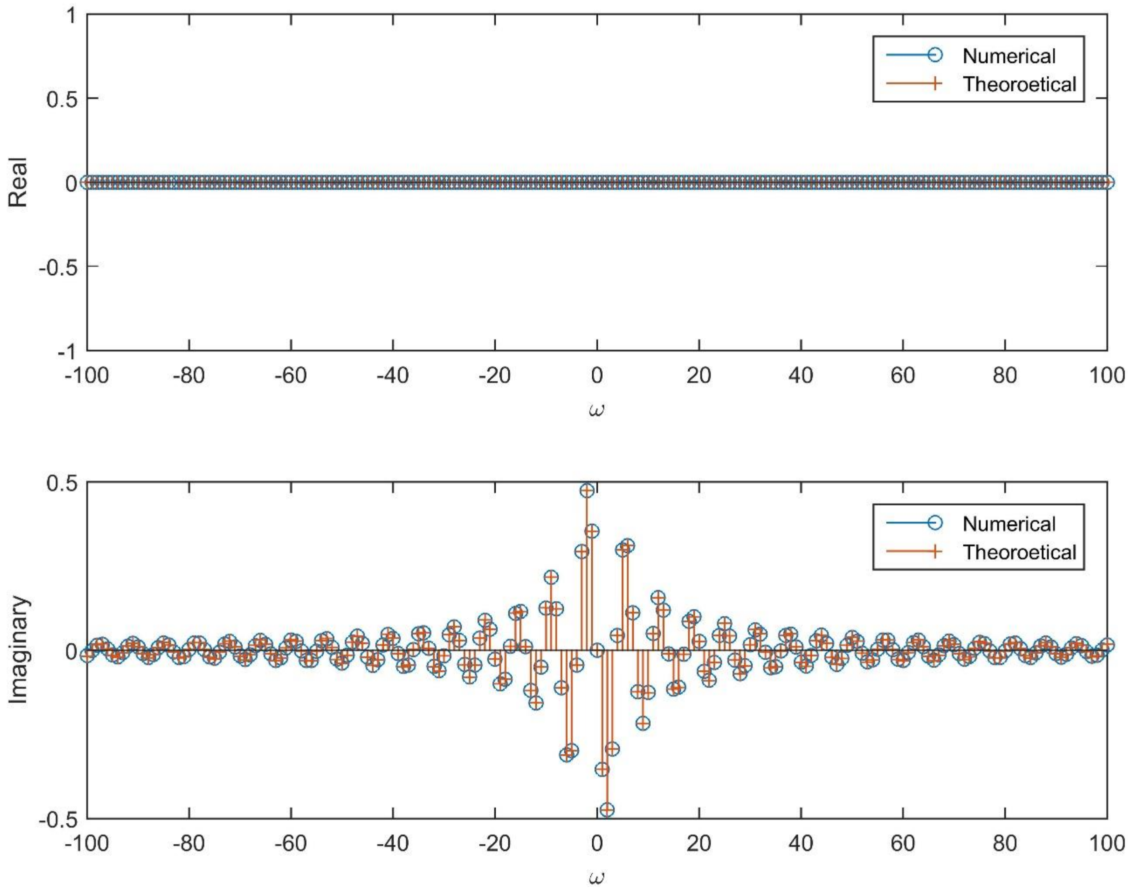
3) a)

Code :

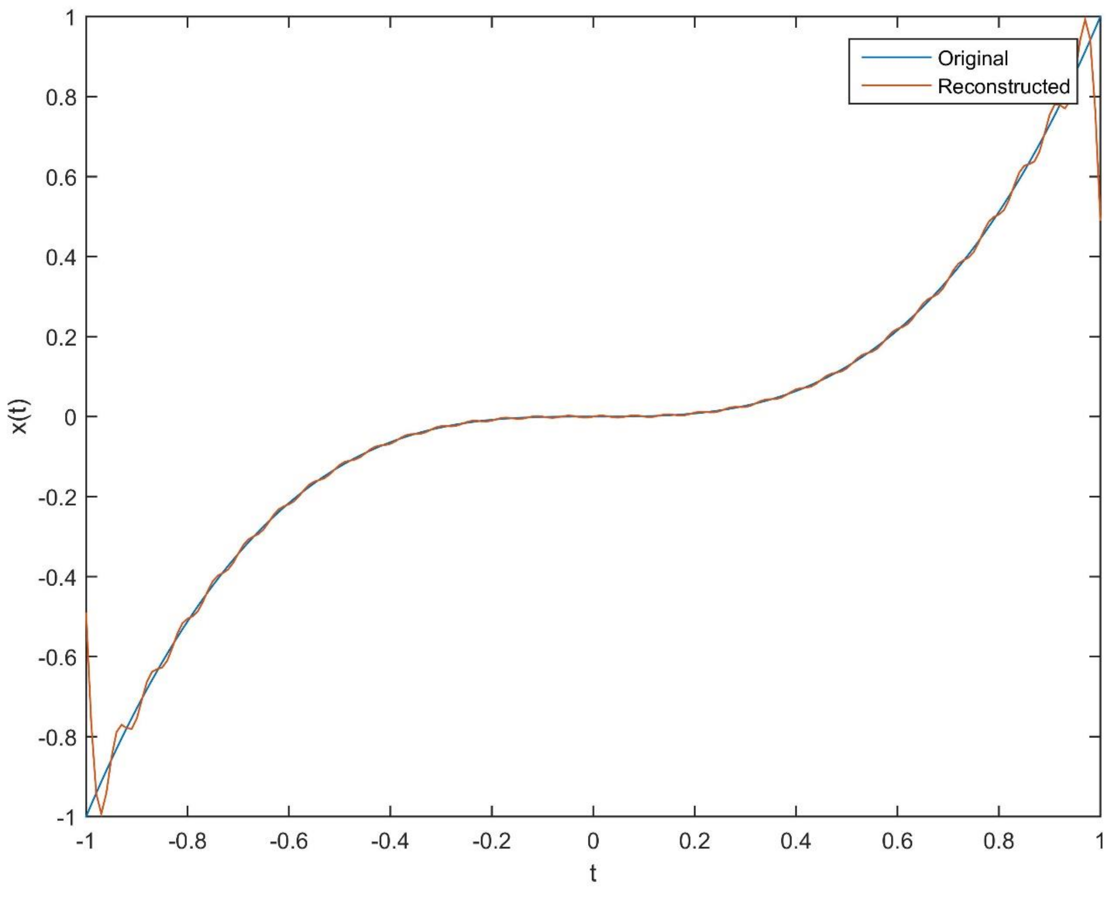
```
clear;
%Defining parameters
tvec = -1:0.01:1;
W = -100:100;

%Defining Signal
x_t = tvec.^3 ;

%Fourier Transform
for i = 1:length(W)
    basis = exp(-1i*W(i)*tvec);
    X(i) = trapz(tvec,x_t.*basis);
end

%Inverse Fourier Transform
for i = 1:length(tvec)
    basis1 = exp(1i*W*tvec(i));
    Rec(i) = (1/(2*pi))*trapz(W,basis1.*X);
end

%Theoretical approach
X_th = 2*1i*cos(W)./(W) ...
    - 6*1i*sin(W)./(W).^2  ...
    -12*1i*cos(W)./(W).^3 ...
    +12*1i*sin(W)./(W).^4;
X_th(W == 0) = 0;

%Plotting the outputs
figure();
subplot(211);
stem(W,real(X),'o');
hold on;
stem(W,real(X_th),'+');
ylim([-1 1]);
xlabel('\omega');
ylabel('Real');
legend('Numerical','Theoretical');

subplot(212);
stem(W,imag(X),'o');
hold on;
stem(W,imag(X_th),'+');
xlabel('\omega');
ylabel('Imaginary');
legend('Numerical','Theorotical');

figure();
plot(tvec,x_t);
hold on;
plot(tvec,Rec);
xlabel('t');
ylabel('x(t)');
legend('Original','Reconstructed');
```

PLOTS:

Evaluating the Fourier Transform with the theoretical values:



Reconstructing the signal using Inverse Fourier Transform:



7a) $x(t) = t^3 \quad , |t| < 1$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} t^3 e^{-j\omega t}\, dt$$

$$= t^3 \frac{e^{-j\omega t}}{-j\omega}\Big|_{-1}^{1} - \int_{-1}^{1} 3t^2 \frac{e^{-j\omega t}}{-j\omega}\, dt$$

$$= \left(\frac{e^{-j\omega} + e^{j\omega}}{-j\omega}\right) + \frac{3}{j\omega}\left[\frac{t^2 e^{-j\omega t}}{-j\omega}\Big|_{-1}^{1} - \int_{-1}^{1} 2t \frac{e^{-j\omega t}}{-j\omega}\, dt\right]$$

$$= \frac{2\cos\omega}{-j\omega} + \frac{3}{-j^2\omega^2}\left(\frac{e^{-j\omega} - e^{j\omega}}{1}\right) + \frac{6}{j^2\omega^2}\int_{-1}^{1} t e^{-j\omega t}\, dt$$

$$= \frac{2j\cos\omega}{\omega} + \frac{3}{\omega^2}\left[-2j\sin\omega\right] - \frac{6}{\omega^2}\left[\frac{t e^{-j\omega t}}{-j\omega}\Big|_{-1}^{1} - \int_{-1}^{1} \frac{e^{-j\omega t}}{-j\omega}\, dt\right]$$

$$= \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} + \frac{6}{j\omega^3}\left(e^{-j\omega} + e^{j\omega}\right) + \frac{6}{\omega^2(-j\omega)}\int_{-1}^{1} e^{-j\omega t}\, dt$$

$$= \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} - \frac{12j\cos\omega}{\omega^3} + \frac{6}{-j\omega^3} \cdot \frac{e^{-j\omega t}}{-j\omega}\Big|_{-1}^{1}$$

$$= \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} - \frac{12j\cos\omega}{\omega^3} - \frac{6}{\omega^4}\left(-2j\sin\omega\right)$$

$$\Rightarrow X(\omega) = \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} - \frac{12j\cos\omega}{\omega^3} + \frac{12j\sin\omega}{\omega^4}$$

$$X(0) = \int_{-1}^{1} t^3\, dt$$

$$= \frac{t^4}{4}\Big|_{-1}^{1}$$

$$\Rightarrow X(0) = 0$$

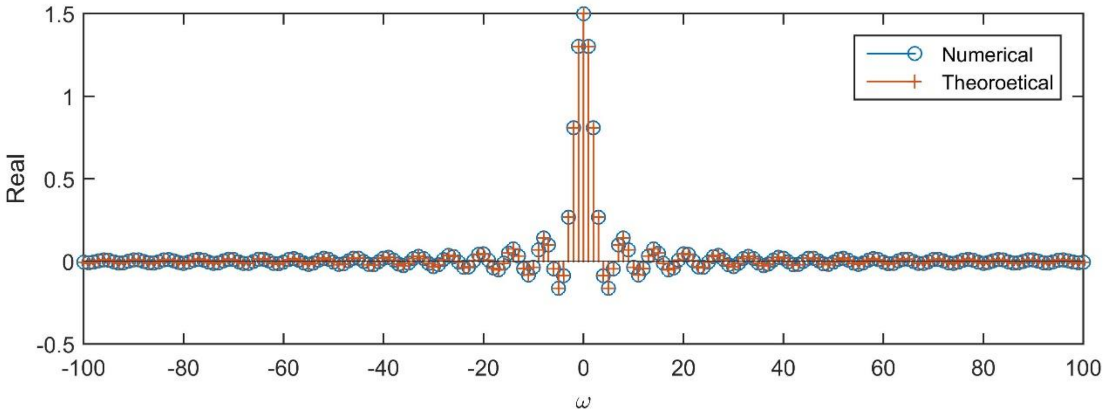
3) b)

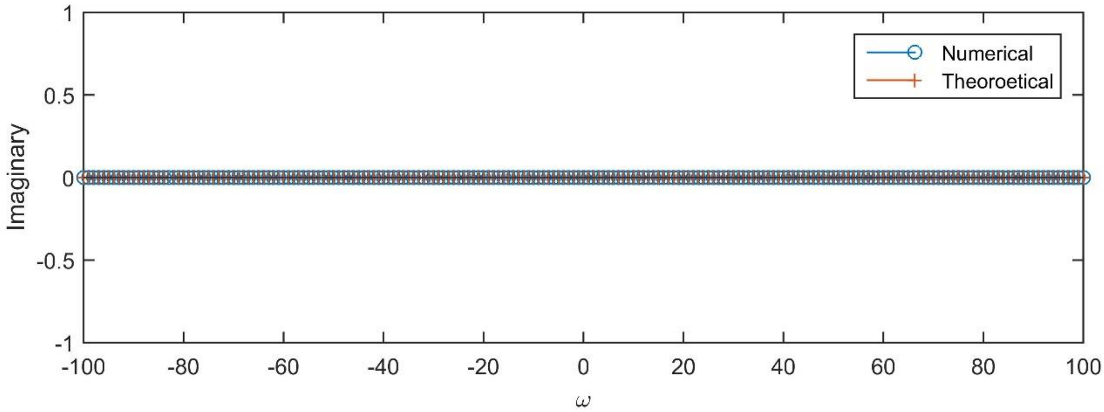
Code :

```
clear;
%Defining parameters
tvec = -1:0.01:1;
W = -100:100;

%Defining Signal
x_t = 1-abs(tvec)/2 ;

%Fourier Transform
for i = 1:length(W)
    basis = exp(-1i*W(i)*tvec);
    X(i) = trapz(tvec,x_t.*basis);
end

%Inverse Fourier Transform
for i = 1:length(tvec)
    basis1 = exp(1i*W*tvec(i));
    Rec(i) = (1/(2*pi))*trapz(W,basis1.*X);
end

%Theoretical approach
X_th = (sin(W)./W) ...
       + (1-cos(W))./(W.^2);
X_th(W == 0) = 1.5;

%Plotting the outputs
figure();
subplot(211);
stem(W,real(X),'o');
hold on;
stem(W,real(X_th),'+');
xlabel('\omega');
ylabel('Real');
legend('Numerical','Theoretical');

subplot(212);
stem(W,imag(X),'o');
hold on;
stem(W,imag(X_th),'+');
ylim([-1 1]);
xlabel('\omega');
ylabel('Imaginary');
legend('Numerical','Theoretical');

figure();
plot(tvec,x_t);
hold on;
plot(tvec,Rec);
xlabel('t');
ylabel('x(t)');
legend('Original','Reconstructed');
```

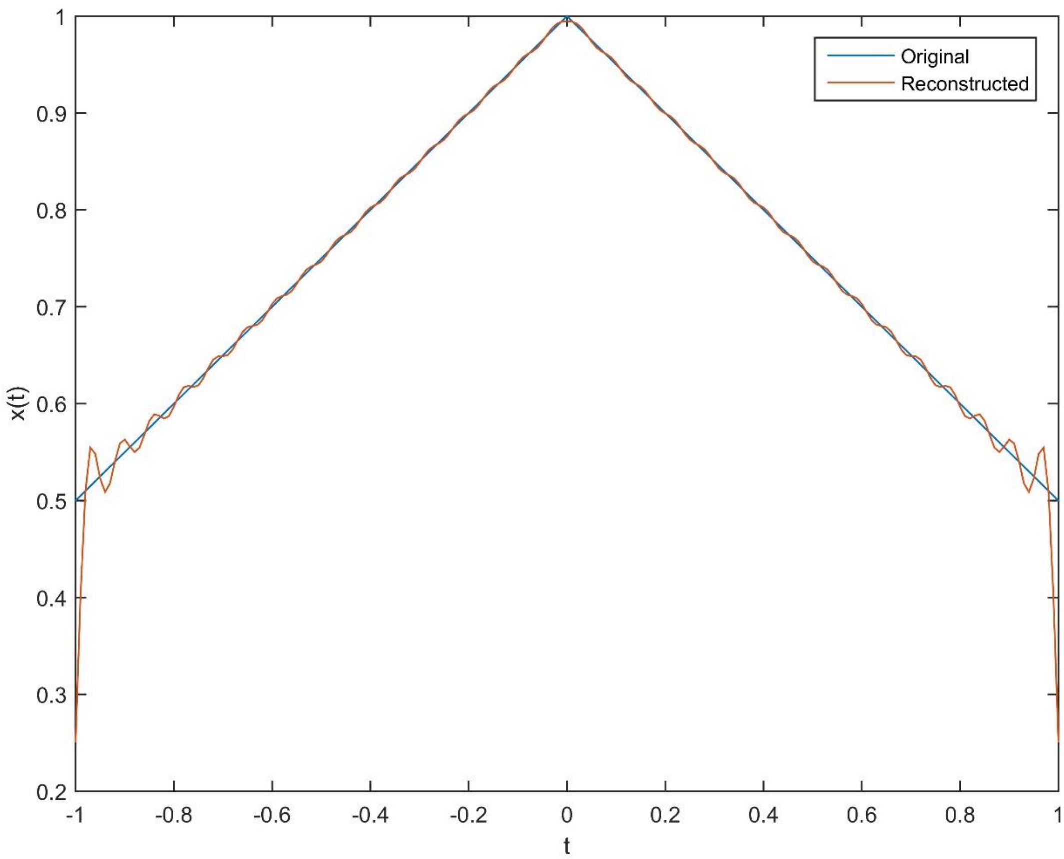
PLOTS:

Evaluating the Fourier Transform with the theoretical values:





Reconstructing the signal using Inverse Fourier Transform :



3) b) $x(t) = 1 - \frac{|t|}{2}$, $|t| < 1$

$$X(\omega) = \int_{-\infty}^{\infty} \left(1 - \frac{|t|}{2}\right) e^{-j\omega t} dt$$

$$= \int_{-1}^{0} \left(1 + \frac{t}{2}\right) e^{-j\omega t} dt + \int_{0}^{1} \left(1 - \frac{t}{2}\right) e^{-j\omega t} dt$$

$$= \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{0} + \frac{1}{2}\left[\left. t \frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{0} - \int_{-1}^{0} \frac{e^{-j\omega t}}{-j\omega} dt \right]$$

$$+ \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1} - \frac{1}{2}\left[\left. t \frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1} - \int_{0}^{1} \frac{e^{-j\omega t}}{-j\omega} dt \right]$$

$$= \frac{1 - e^{j\omega}}{-j\omega} + \frac{e^{-j\omega} - 1}{-j\omega} + \frac{1}{2}\left[\frac{e^{j\omega}}{-j\omega} + \frac{1}{j\omega} \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{0} \right]$$

$$- \frac{1}{2}\left[\frac{e^{-j\omega}}{-j\omega} + \frac{1}{j\omega} \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1} \right]$$

$$= \frac{e^{-j\omega} - e^{j\omega}}{-j\omega} + \frac{1}{2}\left[\frac{e^{j\omega} - e^{-j\omega}}{-j\omega} \right] + \frac{1}{2}\left[\frac{1}{\omega^2}(1 - e^{j\omega}) - \frac{1}{\omega^2}(e^{-j\omega} - 1) \right]$$

$$= \frac{-2j\sin\omega}{-j\omega} + \frac{1}{2}\left(\frac{2j\sin\omega}{-j\omega} \right) + \frac{1}{2}\left[\frac{1}{\omega^2}(1 - e^{-j\omega} - e^{j\omega} + 1) \right]$$

$$= \frac{2\sin\omega}{\omega} - \frac{\sin\omega}{\omega} + \frac{1}{2}\left[\frac{1}{\omega^2}(2 - 2\cos\omega) \right]$$

$$\Rightarrow X(\omega) = \frac{\sin\omega}{\omega} + \frac{1 - \cos\omega}{\omega^2}$$

$$x(0) = \int_{-1}^{0} \left(1 + \frac{t}{2}\right) dt + \int_{0}^{1} \left(1 - \frac{t}{2}\right) dt$$

$$= 1 - \frac{1}{4} + 1 - \frac{1}{4} = 2 - \frac{1}{2}$$

$$= 2 - 0.5$$

$$\Rightarrow x(0) = 1.5$$

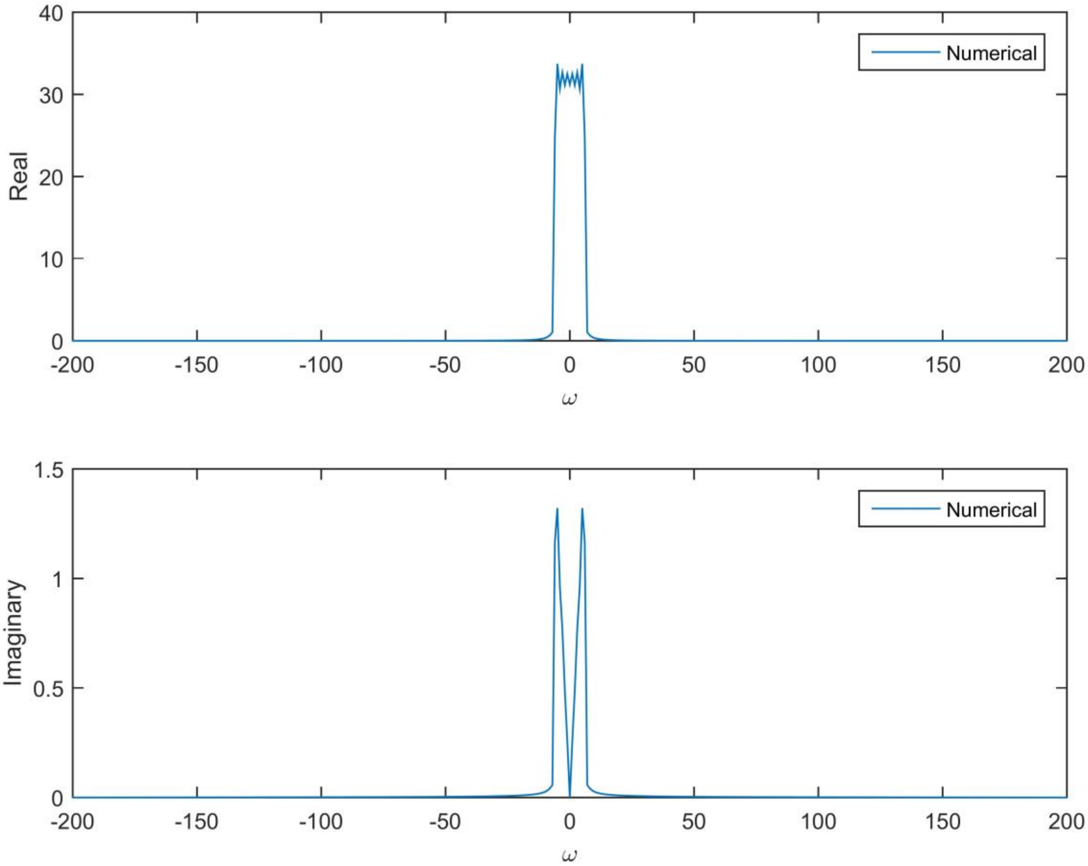
3) c)

Code :

```
clear;
%Defining parameters
tvec = -2*pi:pi/100:2*pi;
W = -200:200;

%Defining Signal
x_t = sinc(tvec) ;

%Fourier Transform
X = fftshift(fft(x_t));

%Inverse Fourier Transform
Rec = ifft(ifftshift(X));

%Plotting the outputs
figure();
subplot(211);
plot(W,abs(real(X)));
xlabel('\omega');
ylabel('Real');
legend('Numerical');

subplot(212);
plot(W,abs(imag(X)));
xlabel('\omega');
ylabel('Imaginary');
legend('Numerical');

figure();
plot(tvec,x_t);
hold on;
plot(tvec,Rec);
xlabel('t');
ylabel('x(t)');
legend('Original','Reconstructed');
```

PLOTS:

Evaluating the Fourier Transform with the theoretical values:



Reconstructing the signal using Inverse Fourier Transform: