

Problem 5.

$$J = -\log y_1$$

$$y_1 = \frac{e^{z_1}}{e^{z_1} + e^{z_2}}$$

$$\begin{aligned} \therefore J &= -\log \left(\frac{e^{z_1}}{e^{z_1} + e^{z_2}} \right) \\ &= -[\log e^{z_1} - \log (e^{z_1} + e^{z_2})] \end{aligned}$$

$$= \underbrace{-z_1}_{\textcircled{1}} + \underbrace{\log (e^{z_1} + e^{z_2})}_{\textcircled{2}}$$

$$\frac{\partial J}{\partial w_1} = \frac{\partial \textcircled{1}}{\partial w_1} + \frac{\partial \textcircled{2}}{\partial w_1}$$

$$\begin{aligned} \frac{\partial \textcircled{1}}{\partial w_1} &= -\frac{\partial z_1}{\partial w_1} = - \left(\frac{\partial z_1}{\partial h_1} \frac{\partial h_1}{\partial g_1} \frac{\partial g_1}{\partial w_1} \right. \\ &\quad \left. + \frac{\partial z_1}{\partial h_2} \frac{\partial h_2}{\partial g_2} \frac{\partial g_2}{\partial w_1} \right) \leftarrow \textcircled{A} \end{aligned}$$

$$\frac{\partial \textcircled{2}}{\partial w_1} = \frac{\partial}{\partial w_1} \log (e^{z_1} + e^{z_2})$$

$$= \frac{1}{e^{z_1} + e^{z_2}} \cdot \frac{\partial}{\partial w_1} (e^{z_1} + e^{z_2})$$

$$= \frac{1}{e^{z_1} + e^{z_2}} \cdot \left[\underbrace{e^{z_1} \cdot \frac{\partial z_1}{\partial w_1}}_{\text{Same as } -A} + \underbrace{e^{z_2} \frac{\partial z_2}{\partial w_1}}_{\text{can be solved by a similar manner as A}} \right]$$

Same as

-A

can be solved

by a similar manner
as A