

Set of exercises 4

Hypothesis Tests on one parameter

Exercise 1

A wine merchant is interested in the capacity of bottles of a particular vintage. He wonders if the average capacity of the bottles supplied by an operator complies with the legal capacity of 75 cl. For this purpose, he measures the content of 10 bottles taken at random in a delivery and obtains the following values (in cl): 73.2; 72.6; 74.5; 75.0; 75.5; 73.7; 74.1; 75.8; 74.8; 75.0.

Assuming the normality of the distribution of the contents and that the standard deviation is known and equal to 1cl, can the merchant reasonably have doubts as to the conformity of the capacity of the bottles, to the significance level of 5%?

The statement does not make it possible to explicitly identify the form of the alternative hypothesis. However, one must accept the idea that what would particularly upset the wine merchant is if the contents of the bottles were less than the volume specified on the label, ie 75 cl. We can therefore specify as a sequence the null and alternative hypotheses of the test:

$$\begin{cases} H_0 & \mu = 75 \\ H_1 & \mu < 75. \end{cases}$$

We must therefore perform a left unilateral test at the 5% significance level. The shape of the non-rejection region (acceptance of the null hypothesis is as follows: $RA = [-Z, +\infty[$ where Z is read from a table of the Normal probability distribution insofar as, that the sample size is less than 30, but the Bottle

capacity is assumed to follow a Normal probability distribution. It follows that the sampling distribution of means calculated from all samples from this population, regardless of their size, is also Normal. read for at the significance level for this one-sided test is obtained from the consideration of the following diagram:

Exercise 2

In the atmosphere, the rate of a noxious gas (in micrograms per cubic meter of air), for a given volume, follows a normal law of expectation μ and variance σ^2 . 10 air samples are taken, which give the following readings: 35,7 ; 44,4 ; 42,5 ; 65,3 ; 47,9 ; 45,4 ; 42,8 ; 40,0 ; 54,6 ; 61,4.

We also suppose that $\sigma^2 = 100$.

1. At the significance level of 5%, can we conclude that the level of noxious gas is equal to 50, or less?
2. Ditto for significance levels of 1% and 10%.
3. Let us imagine that the variance σ^2 is not known, but that 200 air samples were taken, giving an average rate of 51 and a corrected empirical variance equal to 100. What to conclude at the significance level of 5%: the rate is equal to or less than the maximum rate allowed?

Exercise 3

The pharmacological specifications of a drug indicate that each of its tablets should contain an average of 1.5 grams of the active substance. 100 tablets are chosen at random from the production and the measurement in grams of the quantity of active principle recorded, giving rise to the following

measurements: $\sum_{i=1}^{100} X_i = 155$ and $\sum_{i=1}^{100} X_i^2 = 248$.

At the significance level of 5%, can we say that the production of the drug complies with the indication mentioned?

Exercise 4

During the 2014-2015 season of the TOP 14 of the National Rugby League, the goalscorer of Racing Club Toulonnais, Leigh Halfpenny, managed 77 of the 102 shots on goal (penalties and conversions) attempted, making him the 10th scorer in the standings. A goal scorer is usually considered to be decisive when he has an 80% success rate on attempted kicks.

1. At the 5% materiality level, what can we say about Leigh Halfpenny, considering that the 2014-2015 season was for him a representative season of all the seasons he will play in his career?
2. If the success of the other scorers remains the same, how much (in% points) should Leigh Halfpenny's success rate increase to the minimum for him to be in the breakthrough category, if he attempts the same? number of kicks in 2014-2015 ?

Exercise 5

In Boston, anti-Vietnam War activist Dr. Benjamin Spock was tried for public incitement to desertion. The judge in charge of the case was suspected of not being fair in the selection of jurors: among the 700 people he had appointed as jurors in his previous trials, 15% were women while, on the whole of city, 29% of eligible jurors were women.

At the 5% materiality level, what can be said about the judge's impartiality in his choice of jurors?