

Neural Networks Basics

CE/CZ4042 – Tutorial 1

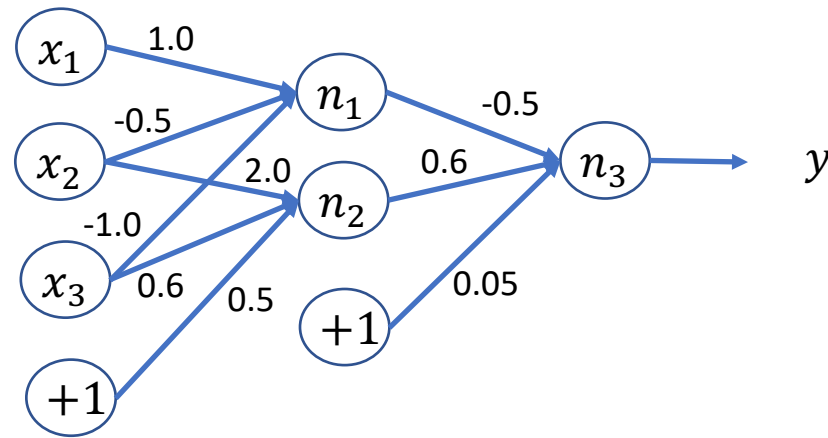
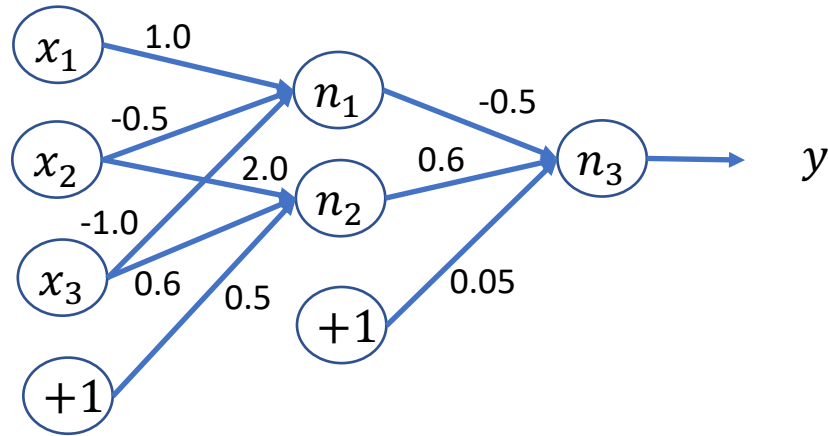


Figure 1 shows a three-layer feedforward neural network receiving 3-dimensional inputs $(x_1, x_2, x_3) \in \mathbb{R}^3$. The connection weights and biases of the neurons n_1, n_2 , and n_3 are indicated in the figure. The hidden-layer neurons have activation functions given by $g(u) = \frac{1.0}{1 + e^{-0.5u}}$ where u indicates the synaptic input to the neuron. The activation function $f(u)$ of the output neuron is a ReLU function: $f(u) = \max\{0, u\}$.

- Write weight vectors and biases connected to individual neurons, and the weight matrix and bias vector connected to the hidden layer.
- Find the synaptic inputs and activations of the neurons for the following input signals:
 - $(1.0, -0.5, 1.0)$
 - $(-1.0, 0.0, -2.0)$
 - $(2.0, 0.5, -1.0)$

Q1



Weights:

$$n_1 \text{ neuron: } \mathbf{w}_1 = \begin{pmatrix} 1.0 \\ -0.5 \\ -1.0 \end{pmatrix}; \quad n_2 \text{ neuron: } \mathbf{w}_2 = \begin{pmatrix} 0.0 \\ 2.0 \\ 0.6 \end{pmatrix}; \quad \text{and } n_3 \text{ neuron: } \mathbf{w}_3 = \begin{pmatrix} -0.5 \\ 0.6 \end{pmatrix}.$$

Biases:

$$n_1 \text{ neuron: } b_1 = 0.0; \quad n_2 \text{ neuron: } b_2 = 0.5; \quad \text{and } n_3 \text{ neuron: } b_3 = 0.05$$

To the hidden-layer

$$\text{Weight matrix } \mathbf{W} = \begin{pmatrix} 1.0 & 0.0 \\ -0.5 & 2.0 \\ -1.0 & 0.6 \end{pmatrix} \text{ and bias } \mathbf{b} = \begin{pmatrix} 0.0 \\ 0.5 \end{pmatrix}$$

Q1

Activation functions:

hidden-layer neurons: $g(u) = \frac{1.0}{1+e^{-0.5u}}$

Output neuron $f(u) = \max\{0.0, u\}$

Apply $\mathbf{x}_1 = (1.0 \quad -0.5 \quad 1.0)^T$:

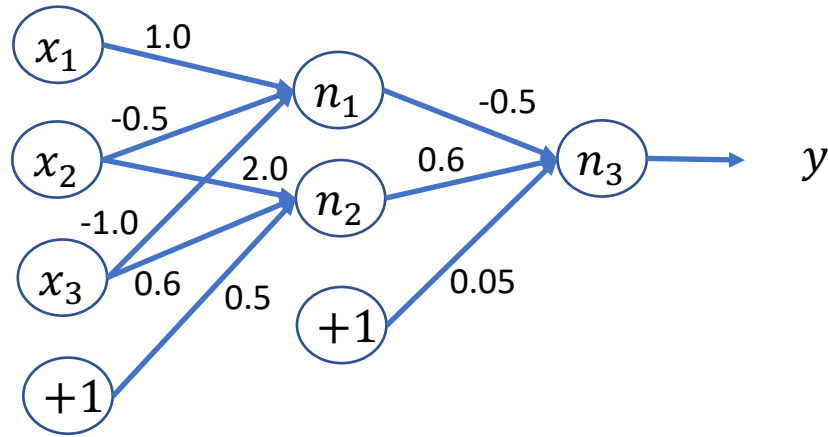
Synaptic input to neuron n_1 , $u_1 = \mathbf{x}_1^T \mathbf{w}_1 + b_1 = (1.0 \quad -0.5 \quad 1.0) \begin{pmatrix} 1.0 \\ -0.5 \\ -1.0 \end{pmatrix} + 0.0 = 0.25$

Output of neuron n_1 , $y_1 = g(u_1) = \frac{1.0}{1+e^{-0.5u_1}} = 0.531$

Synaptic input to neuron n_2 , $u_2 = \mathbf{x}_1^T \mathbf{w}_2 + b_2 = (1.0 \quad -0.5 \quad 1.0) \begin{pmatrix} 0.0 \\ 2.0 \\ 0.6 \end{pmatrix} + 0.5 = 0.1$

Output of neuron n_2 , $y_2 = g(u_2) = \frac{1.0}{1+e^{-0.5u_2}} = 0.512$

Q1



Output of the hidden layer $\mathbf{z} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0.531 \\ 0.512 \end{pmatrix}$

Synaptic input to neuron n_3 , $u_3 = \mathbf{z}^T \mathbf{w}_3 + b_3 = (0.531 \quad 0.512) \begin{pmatrix} -0.5 \\ 0.6 \end{pmatrix} + 0.05 = 0.092$

Output of neuron n_3 , $y_3 = f(u_3) = \max\{0.0, u_3\} = 0.092$

Q1

Similarly, for other two inputs:

x	u_1	y_1	u_2	y_2	z	u_3	y_3
$\begin{pmatrix} -1.0 \\ 0.0 \\ -2.0 \end{pmatrix}$	1.00	0.622	-0.700	0.413	$\begin{pmatrix} 0.622 \\ 0.413 \end{pmatrix}$	-0.013	0.0
$\begin{pmatrix} 2.0 \\ 0.5 \\ -1.0 \end{pmatrix}$	2.75	0.798	0.900	0.611	$\begin{pmatrix} 0.798 \\ 0.611 \end{pmatrix}$	0.017	0.017

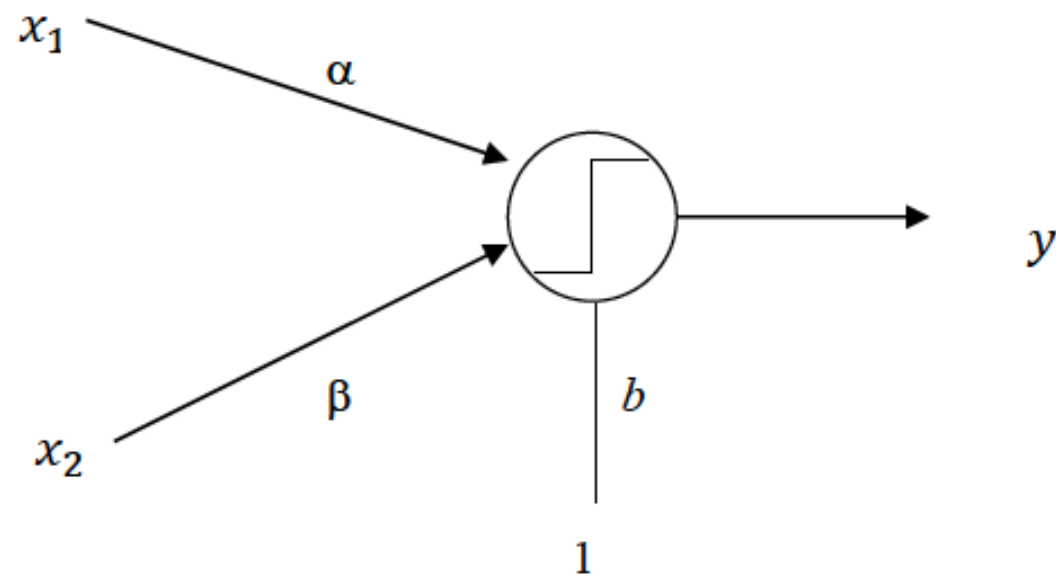
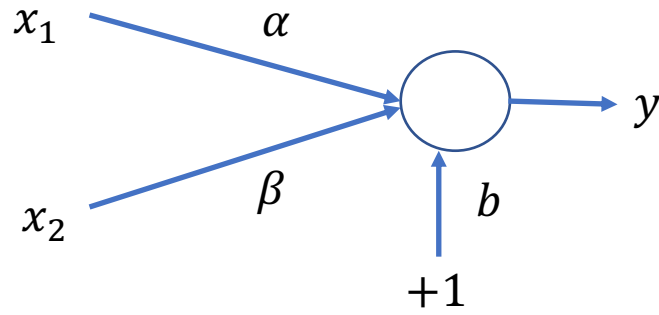


Figure 2

2. Two input binary neuron shown in figure 2 has a unit step activation function with bias $b = 0.5$ and receives two-dimensional input $(x_1, x_2) \in \mathbf{R}^2$.
 - (a) Find the space of possible values of weights (α, β) if the neuron is
 - (i) ON for input $(1.0, 1.0)$
 - (ii) ON for input $(0.5, -1.0)$
 - (iii) OFF for input $(2.0, -0.5)$.
 - (b) Indicate the weight space in 2-D α - β plot and show that $(-0.2, 0.2)$ is in this space.

Q2



Weight $\mathbf{w} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ and bias $b = 0.5$.

$$y = f(u) = \begin{cases} 1.0, & u > 0.0 \\ 0.0, & u \leq 0.0 \end{cases}$$

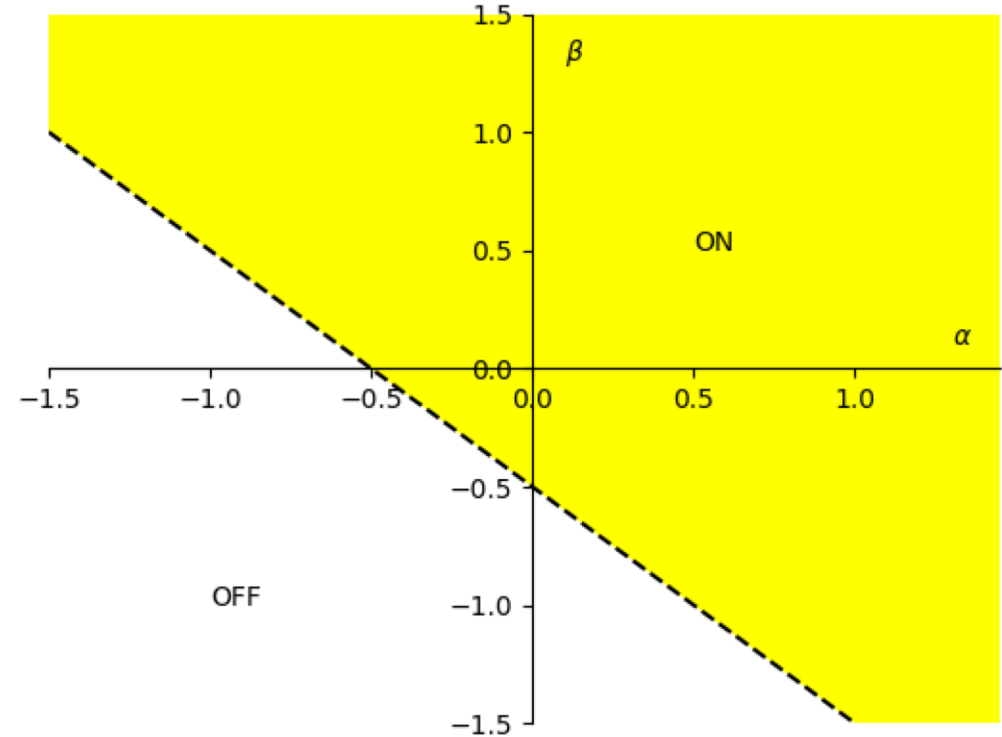
Input $\mathbf{x} = (1.0 \quad 1.0)^T$:

$$\begin{aligned} \text{Synaptic input } u &= \mathbf{x}^T \mathbf{w} + b \\ &= (1.0 \quad 1.0) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + 0.5 \\ &= \alpha + \beta + 0.5 \end{aligned}$$

Neuron is ON;

$$\text{So, } u = \alpha + \beta + 0.5 > 0.0$$

$$\beta > -\alpha - 0.5 \quad (1)$$



Q2

Input $\mathbf{x} = (0.5 \quad -1.0)^T$:

Synaptic input $u = \mathbf{x}^T \mathbf{w} + b$

$$= (0.5 \quad -1.0) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + 0.5$$

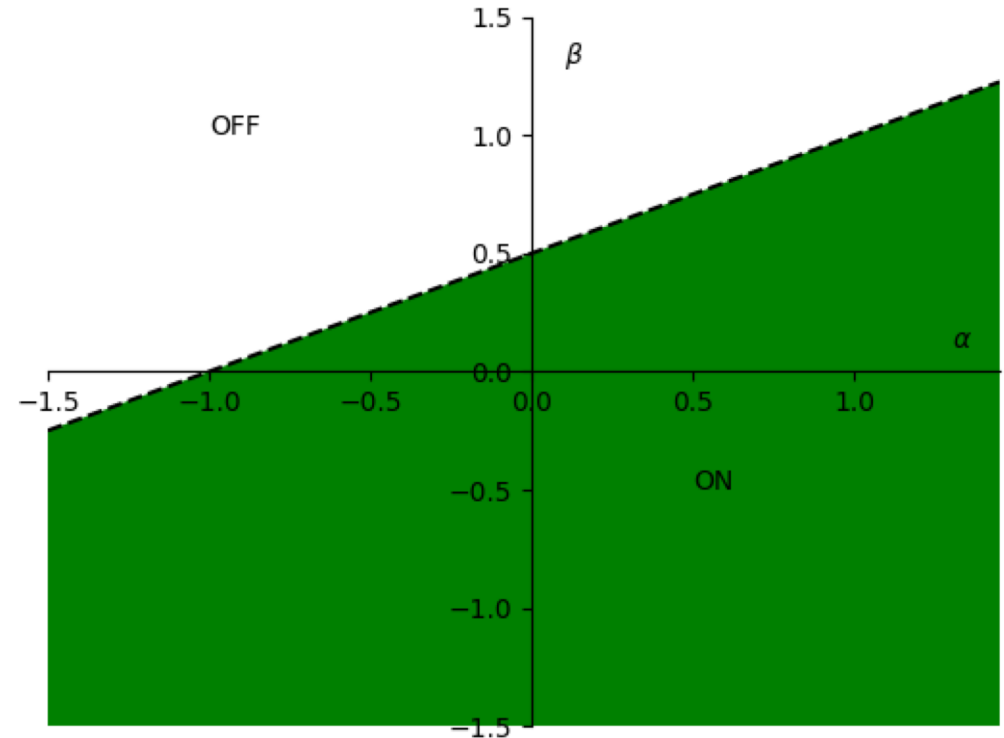
$$= 0.5\alpha - \beta + 0.5$$

Neuron is ON;

So, $u = 0.5\alpha - \beta + 0.5 > 0$

$$\beta < 0.5\alpha + 0.5$$

(2)



Q2

Input $\mathbf{x} = (2.0 \quad -0.5)^T$:

Synaptic input $u = \mathbf{x}^T \mathbf{w} + b$

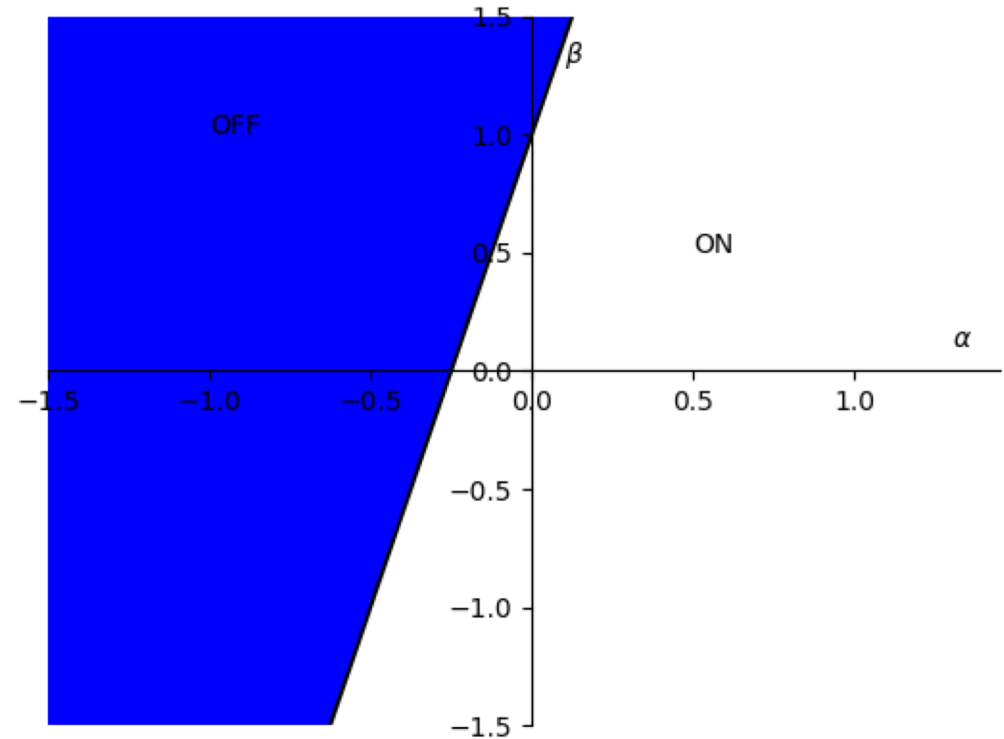
$$= (2.0 \quad -0.5) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + 0.5$$

$$= 2\alpha - 0.5\beta + 0.5$$

Neuron is OFF;

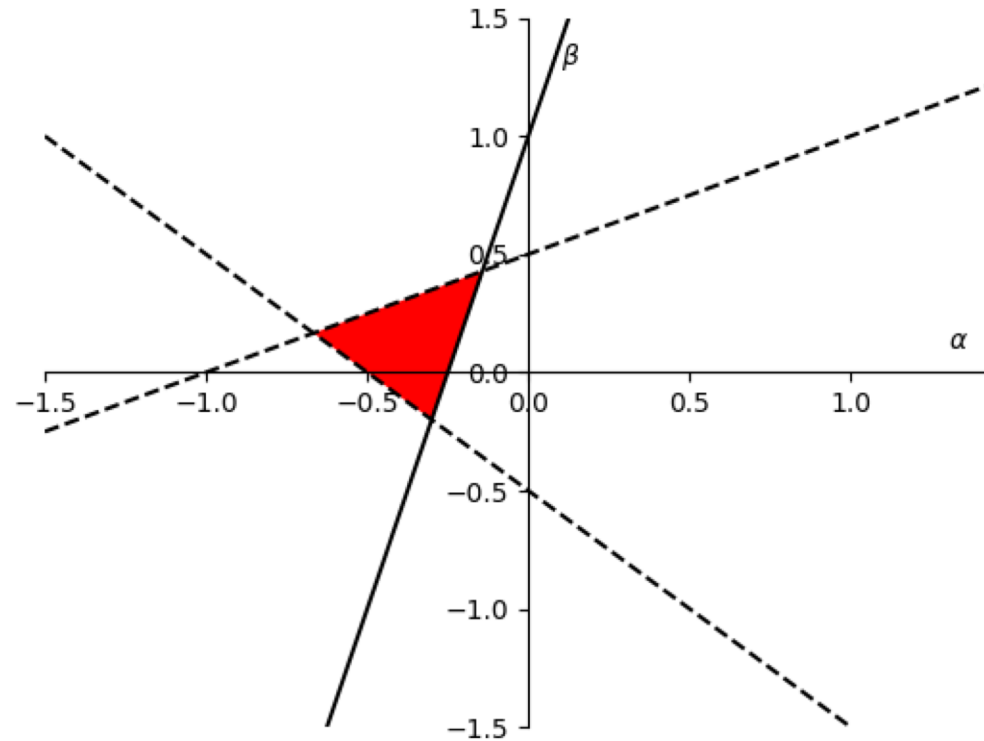
So, $u = 2\alpha - 0.5\beta + 0.5 \leq 0$

$$\beta \geq 4\alpha + 1.0 \quad (3)$$



Q2

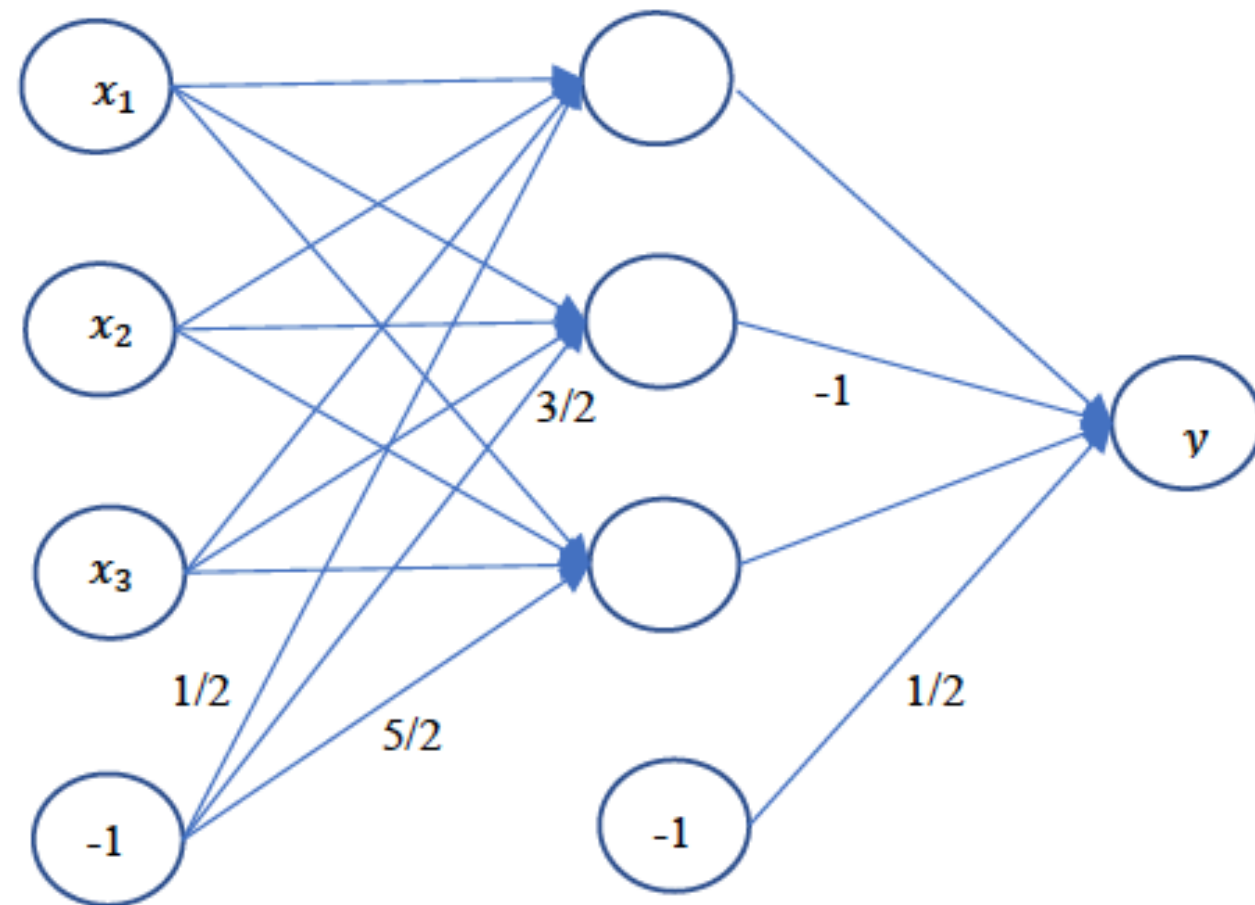
The neuron should satisfy conditions (1), (2) and (3).
Intersection of the regions gives the space of (α, β) .



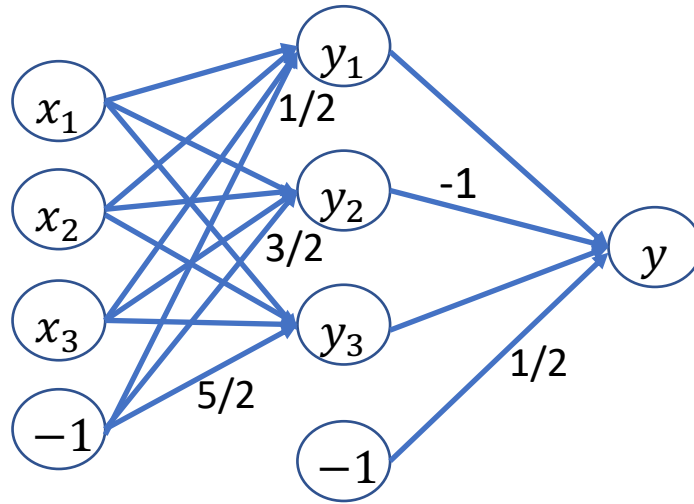
From the figure, $(-0.2, 0.2)$ is within the space of (α, β) .

Q3

The network shown in figure 3 has neurons having threshold activation functions and receives three-bit binary patterns $(x_1, x_2, x_3) \in \{0,1\}^3$. By analyzing for the outputs for all possible three-bit input patterns, determine the logic function that the network implements. All unlabeled weights shown in figure 3 are of unity weight.



Q3



Input

$$\mathbf{x} = (x_1, x_2, x_3) \in \{0, 1\}^3$$

Synaptic inputs to hidden neurons:

$$u_1 = x_1 + x_2 + x_3 - 0.5$$

$$u_2 = x_1 + x_2 + x_3 - 1.5$$

$$u_3 = x_1 + x_2 + x_3 - 2.5$$

Activation function:

$$f(u) = \begin{cases} 1.0, & u > 0.0 \\ 0.0, & u \leq 0.0 \end{cases}$$

Outputs of hidden neurons:

$$y_1 = f(u_1)$$

$$y_2 = f(u_2)$$

$$y_3 = f(u_3)$$

Synaptic input to output neuron:

$$u = y_1 - y_2 + y_3 - 0.5$$

Output of the network:

$$y = f(u)$$

Q3

$$u_1 = x_1 + x_2 + x_3 - 0.5$$

$$u_2 = x_1 + x_2 + x_3 - 1.5$$

$$u_3 = x_1 + x_2 + x_3 - 2.5$$

$$u = y_1 - y_2 + y_3 - 0.5$$

$$y = f(u) = \begin{cases} 1.0, & u > 0.0 \\ 0.0, & u \leq 0.0 \end{cases}$$

Note that $\mathbf{x} = (x_1, x_2, x_3) \in \{0, 1\}^3$

x1	x2	x3	u1	y1	u2	y2	u3	y3	u	y
0	0	0	-1/2	0	-3/2	0	-5/2	0	-1/2	0
0	0	1	1/2	1	-1/2	0	-3/2	0	1/2	1
0	1	0	1/2	1	-1/2	0	-3/2	0	1/2	1
0	1	1	3/2	1	1/2	1	-1/2	0	-1/2	0
1	0	0	1/2	1	-1/2	0	-3/2	0	1/2	1
1	0	1	3/2	1	1/2	1	-1/2	0	-1/2	0
1	1	0	3/2	1	1/2	1	-1/2	0	-1/2	0
1	1	1	5/2	1	3/2	1	1/2	1	1/2	1

$$y = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$$

Q3

$$\begin{aligned}O &= \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3 \\&= \bar{x}_1 (\bar{x}_2 x_3 + x_2 \bar{x}_3) + x_1 (\bar{x}_2 \bar{x}_3 + x_2 x_3) \\&= \bar{x}_1 (x_2 \oplus x_3) + x_1 (\overline{x_2 \oplus x_3}) \\&= x_1 \oplus (x_2 \oplus x_3) \\&= x_1 \oplus x_2 \oplus x_3\end{aligned}$$

$\oplus \rightarrow$ Exclusive OR

The function implemented by the network is 'Three input Exclusive OR'