1. Training inputs for a dichotomizer are given as:

$$\mathbf{X}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}; \quad \mathbf{X}_2 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}; \quad \mathbf{X}_3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}; \quad \mathbf{X}_4 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}; \qquad \textit{Class } 1$$

$$\mathbf{X}_5 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \mathbf{X}_6 = \begin{bmatrix} -1 \\ -3 \end{bmatrix}; \quad \mathbf{X}_7 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}; \quad \mathbf{X}_8 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}; \qquad \textit{Class } 2$$

(a) Determine whether the two classes of patterns are linearly separable and find the center of gravity of patterns in each class.

(b) If the center points of two clusters of class 1 and class 2 are vectors χ_1 and χ_2, show that linear decision boundary that perpendicularly passes through the middle point of the line joining the two centroids and can be expressed in the form

$$(\chi_1 - \chi_2)^T \mathbf{x} + \frac{1}{2}(\|\chi_2\|^2 - \|\chi_1\|^2) = 0$$

(c) Design a dichotomizer using a perceptron having the decision boundary as in part (b) for the given classification and determine how it recognizes the following input patterns

$$\mathbf{x} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} \frac{36}{13} \\ 0 \end{bmatrix};$$

2. Design a dichotomizer that performs the following classification:

$$\boldsymbol{x}_1 = \begin{pmatrix} 0.8 \\ 0.5 \\ 0.0 \end{pmatrix}, x_2 = \begin{pmatrix} 0.9 \\ 0.7 \\ 0.3 \end{pmatrix}, \boldsymbol{x}_3 = \begin{pmatrix} 1.0 \\ 0.8 \\ 0.5 \end{pmatrix} \rightarrow Class\ A$$

$$\boldsymbol{x}_4 = \begin{pmatrix} 0.0 \\ 0.2 \\ 0.3 \end{pmatrix}, \boldsymbol{x_5} = \begin{pmatrix} 0.2 \\ 0.3 \\ 0.5 \end{pmatrix}, \boldsymbol{x}_6 = \begin{pmatrix} 0.4 \\ 0.7 \\ 0.8 \end{pmatrix} \rightarrow Class\ B$$

Demonstrate one iterations of learning of above two-class classification with

(a) a discrete perceptron using discrete perceptron learning algorithm.

(b) a logistic regression neuron using stochastic gradient descent learning.

Draw the decision boundary learned by the neurons.

Initialize weights randomly and biases to 0.0. Set the learning parameter $\alpha = 0.1$.

3. Use gradient descent learning on a logistic regression neuron to realize the following classification of 3-dimensional inputs.

Class 1	Class 2
$(-1.75 \quad 0.34 \quad 1.15)$	$(-0.25 \quad 0.98 \quad 0.51)$
$(0.22 \quad -1.07 \quad -0.19)$	$(-0.58 \quad 0.82 \quad 0.67)$
$(0.26 \quad -0.46 \quad 0.44)$	$(-0.1 \quad -0.53 \quad 1.03)$
$(-0.44 \quad -1.12 \quad 1.62)$	$(0.18 \quad 0.94 \quad 0.73)$
$(1.54 \quad -0.25 \quad -0.84)$	$(1.36 \quad -0.33 \quad 0.06)$

Initialize the weights randomly and bias to 0.0 and use a learning factor $\alpha = 0.4$

Show one iterations of learning of the neuron and plot classification error vs. iterations until convergence.

Find the probabilities that the trained network assigns to each input pattern belonging to its class label.