Portfolio 2

1. Write a method that solves the selection problem using a priority queue and conduct a series of experiments that indicate that the time complexity of your method is O(N log N).
2. Implement the quickSelect method and conduct a series of experiments that indicate that the time complexity of the method is O(N).
3. Solve the average case recurrence equation for quickSelect.

1: Priority Queue

The experiment is done on a random generate list from 0 to 10 000 with a step size of 100. Each iteration is done 100 times. The average of each iteration is plotted in figure 1.

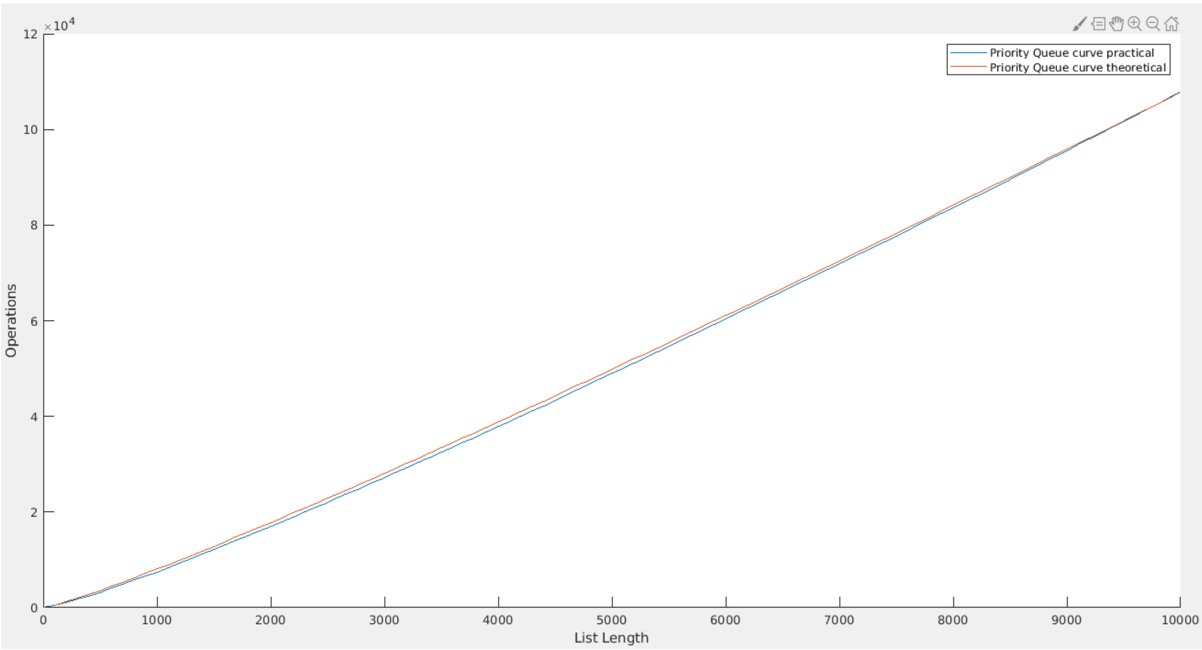

Figure 1: Priority Queue: Operation versus list length.

Theoretical line is plotted using function: $N \cdot Log_2(N) \cdot factor$ where factor is to get the two graphs intersect each other in endpoints. The practical data is a variable counted one up each time perculate_down is called after buildheap is done. It can be seen from the plot that the data follow $N \cdot Log_2(N)$ pretty good

2: Quick Select

The experiment is done on a random generate list from 0 to 10 000 with a step size of 100. Each iteration is done 100 times. The average of each iteration is plotted in figure 2.

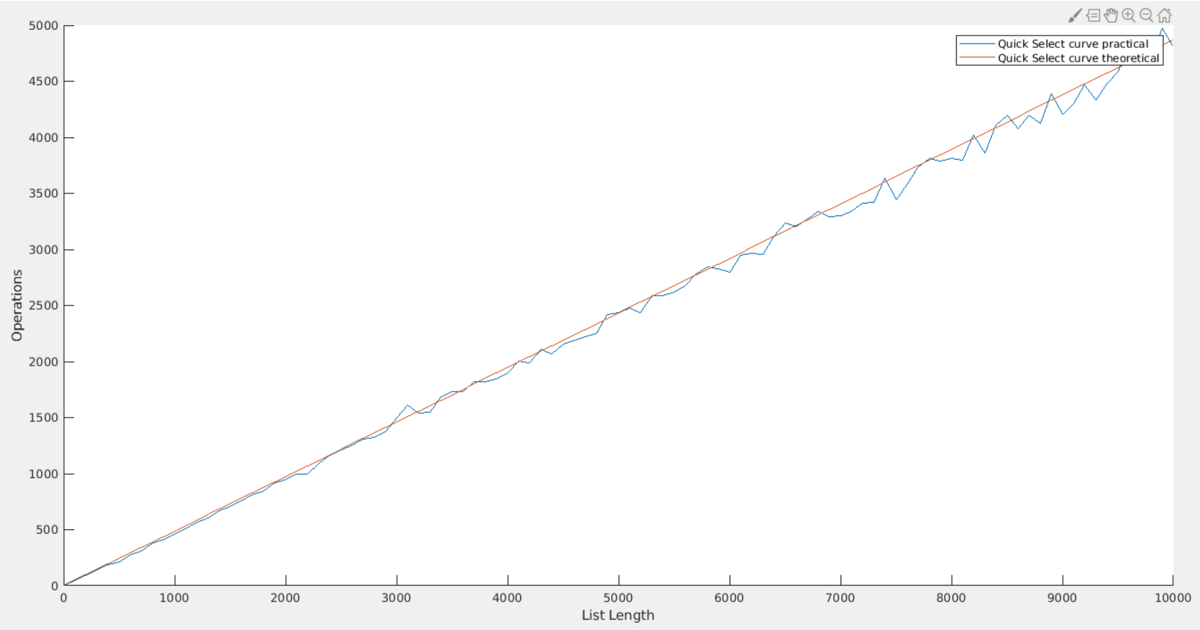

Figure 2: Quick Select: Operation versus list length

Theoretical line is plotted using function: $N \cdot factor$ where factor is to get the two graphs intersect each other in endpoints. The practical data is a variable counted one up each time it swaps in the while(true) inside quick_select. This means we do not count one up if it is using insertions_sort (only for small list sizes).

3: Average case Quick Select

Average case:

$$T(N) = \frac{1}{N}\left[\sum_{j=0}^{N-1} T(j)\right] + c \cdot N \quad (1)$$

Multiple by N:

$$N \cdot T(N) = 1 \cdot \left[\sum_{j=0}^{N-1} T(j)\right] + c \cdot N^2 \qquad (2)$$

Substitute N with N-1:

$$(N-1) \cdot T(N-1) = \left[\sum_{j=0}^{N-2} T(j)\right] + c \cdot (N-1)^2 \, (3)$$

Subtract (3) from (2):

$$N \cdot T(N) - (N-1) \cdot T(N-1) = T(N-1) + c \cdot N^2 - c \cdot (N-1)^2$$

$$N \cdot T(N) - (N-1) \cdot T(N-1) = T(N-1) + c \cdot N^2 - c\left(N^2 - 2N + 1\right)$$

$$N \cdot T(N) - ((N-1) \cdot T(N-1)) = T(N-1) + 2cN - c$$

Rearrange and drop insignificant c:

$$N \cdot T(N) = T(N-1) + (N-1) \cdot T(N-1) + 2cN \qquad (4)$$

$$N \cdot T(N) = T(N-1) + NT(N-1) - T(N-1) + 2cN$$

$$N \cdot T(N) = N \cdot T(N-1) + 2cN \qquad (5)$$

Divide equation (5) by N:

$$T(N) = T(N-1) + 2c \qquad (6)$$

Telescope:

$$T(N-1) = T(N-2) + 2c$$

$$T(N-2) = T(N-3) + 2c$$

$$\vdots$$

$$T(2) = T(1) + 2c$$

Adding all equations:

$$T(N) = T(1) + \sum_{i=1}^{N} 2c = O(N) \qquad (7)$$