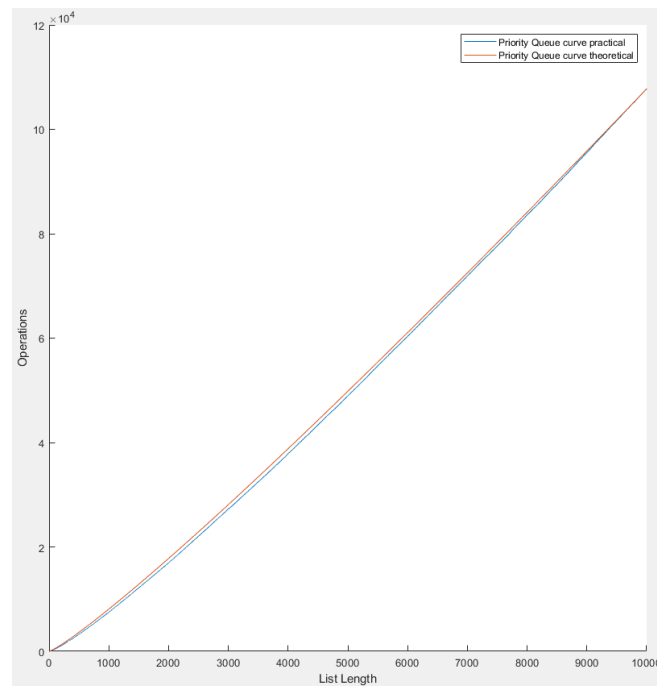


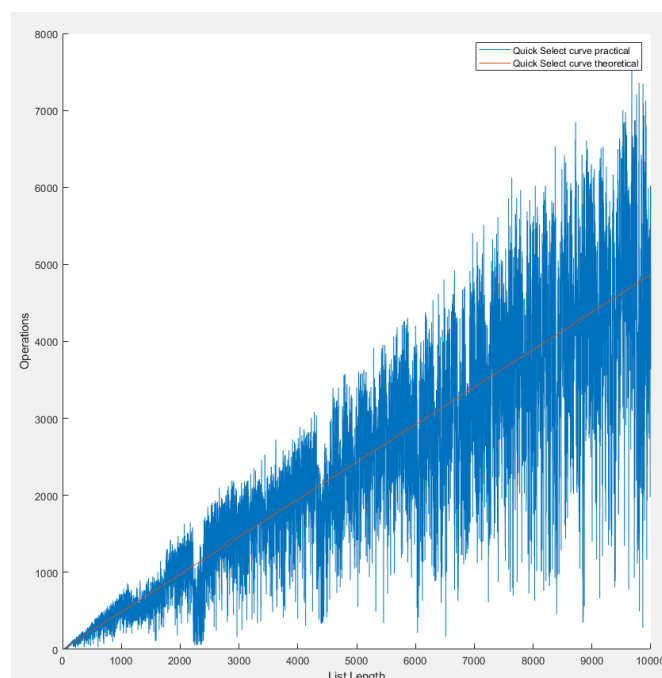
Portfolio 2

1:



Theoretical line is plotted using function: $N \cdot \log_2(N) \cdot factor$ where factor is to get the two graphs intersect each other in endpoints. The practical data is a variable counted one up each time `percolate_down` is called after `buildheap` is done. It can be seen from the plot that the data follow $N \cdot \log_2(N)$ pretty good

2:



Theoretical line is plotted using function: $N \cdot factor$ where factor is to get the two graphs intersect each other in endpoints. The practical data is a variable counted one up each time it swaps in the while(true) inside quick_select. This means we do not count one up if it is using insertions_sort (only for small list sizes). It can be seen from the plot that the operations vary a lot. This is due to the list generated is random which mean if the list is nearly sorted, the operations used is few. The tendency of the graph is linear.

3: Bevis average case quick select

Average case:

$$T(N) = \frac{1}{N} \left[\sum_{j=0}^{N-1} T(j) \right] + c \cdot N \quad (1)$$

Multiple by N:

$$N \cdot T(N) = 1 \cdot \left[\sum_{j=0}^{N-1} T(j) \right] + c \cdot N^2 \quad (2)$$

Substitute N with N-1:

$$(N-1) \cdot T(N-1) = \left[\sum_{j=0}^{N-2} T(j) \right] + c \cdot (N-1)^2 \quad (3)$$

Subtract (3) from (2):

$$N \cdot T(N) - (N-1) \cdot T(N-1) = T(N-1) + c \cdot N^2 - c \cdot (N-1)^2$$

$$N \cdot T(N) - (N-1) \cdot T(N-1) = T(N-1) + c \cdot N^2 - c(N^2 - 2N + 1)$$

$$N \cdot T(N) - ((N-1) \cdot T(N-1)) = T(N-1) + 2cN - c$$

Rearrange and drop insignificant c:

$$N \cdot T(N) = T(N-1) + (N-1) \cdot T(N-1) + 2cN \quad (4)$$

$$N \cdot T(N) = T(N-1) + NT(N-1) - T(N-1) + 2cN$$

$$N \cdot T(N) = N \cdot T(N-1) + 2cN \quad (5)$$

Divide equation (5) by N:

$$T(N) = T(N-1) + 2c \quad (6)$$

Telescope:

$$T(N-1) = T(N-2) + 2c$$

$$T(N-2) = T(N-3) + 2c$$

\vdots

$$T(2) = T(1) + 2c$$

Adding all equations:

$$T(N) = T(1) + \sum_{i=1}^N 2c = O(N) \quad (7)$$