Statistical Distributions

Produced by David Diez

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx,\ VarX = EX^2 - (EX)^2$$

Bernoulli: $f(x|p) = p^x(1-p)^{1-x}, x \in \{0,1\}$

$$EX = p,\ VarX = p(1-p),\ \Psi(t) = pe^t + (1-p)$$

Exponential family. $p(x,\theta) = \exp\{x\text{logit}(p) - (-\log(1-p))\}$

Beta(α, β): $f(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, x \in (0,1).$

$$EX = \frac{\alpha}{\alpha+\beta},\ VarX = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Exp. family. $p(x,\theta) = \frac{1}{x(1-x)}\exp\left\{\alpha\log x + \beta\log(1-x) + \log\left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]\right\}$

Binomial: $f(x|n,p) = \left(\begin{array}{c} n \\ x \end{array}\right)p^x(1-p)^{n-x}, x \in \{1,...,n\}$

$$EX = np,\ VarX = np(1-p),\ \Psi(t) = (1-p+pe^t)^n$$

Exp. family. $p(x,\theta) = \left(\begin{array}{c} n \\ x \end{array}\right)\exp\{x\text{logit}(p) - (-n\log(1-p))\}$

Cauchy: $f(x) = \frac{1}{\pi(1+x^2)}, x \in \Re$. The mean and variance do not exist.

Chi-Square: $f(x|\nu) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}x^{\nu/2-1}e^{-x/2}$, for $x > 0$.

$$EX = \nu,\ VarX = 2\nu$$

Dirichlet: $f(\mathbf{x}|\alpha) = \frac{\Gamma(\alpha.)}{\prod^d \Gamma(\alpha_i)}x_1^{\alpha_1} * ... * x_d^{\alpha_d}$.

$$EX_i = \frac{\alpha_i}{\alpha.},\ VarX_i = \frac{\alpha_i(\alpha. - \alpha_i)}{\alpha.^2(\alpha. + 1)},\ Cov(X_i,X_j) = -\frac{\alpha_i\alpha_j}{\alpha.^2(\alpha. + 1)}$$

Exponential: $f(x|\lambda) = \lambda e^{-\lambda x}, x > 0$. Exp. family. $p(x,\theta) = \exp\{-\lambda x - (-\log\lambda)\}$

$$EX = \frac{1}{\lambda},\ VarX = \frac{1}{\lambda^2},\ \Psi(t) = \left(1 - \frac{t}{\lambda}\right)^{-1},\ MLE(\lambda) = \frac{1}{\bar{x}}$$

Gamma: $f(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x > 0.$

$$EX = \frac{\alpha}{\beta},\ VarX = \frac{\alpha}{\beta^2},\ \Psi(t) = \left(\frac{\beta}{\beta - t}\right)^\alpha$$

Exp. family. $p(x,\theta) = \exp\left\{(\alpha-1)\log x - \beta x - \left(-\log\left[\frac{\beta^\alpha}{\Gamma(\alpha)}\right]\right)\right\}$

Geometric: $f(x|p) = (1-p)^{x-1}p, x \in \{1,2,...\}$

$$EX = 1/p,\ VarX = \frac{1-p}{p^2},\ \Psi(t) = \frac{pe^t}{1-(1-p)e^t},\ , MLE(p) = \left(\frac{1}{n}\sum x_i\right)^{-1}$$

Exp. family. $p(x,\theta) = \exp\{(x-1)\log(1-p) - (-\log p)\}$

Inverse-Gamma: $f(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{-(\alpha+1)}e^{-\beta/x}, x > 0.$

$$EX = \frac{\beta}{\alpha-1} \text{ for },\ VarX = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)},$$

Lognormal: $f(x|\mu,\sigma) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-(1/2)\left[\frac{\log(x)-\mu}{\sigma}\right]^2\right\}, x \in [0,\infty)$

$$\begin{aligned} EX &= e^{\mu+(1/2)\sigma^2},\ VarX = \left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}, \\ MLE(\mu) &= \frac{\sum \log x_i}{n},\ MLE(\sigma^2) = \frac{\sum(\log x_i - MLE(\mu))^2}{n} \end{aligned}$$

Negative Binomial: $f(x|r,p) = \begin{pmatrix} r+x-1 \\ x \end{pmatrix} p^r(1-p)^x, x \in \{0,1,...\}$

$$EX = r\frac{1-p}{p},\ VarX = r\frac{1-p}{p^2},\ \Psi(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r$$

Exp. family. $p(x,\theta) = \begin{pmatrix} r+x-1 \\ x \end{pmatrix} \exp\{x\log(1-p) - (-r\log p)\}$

Normal: $f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, x \in \Re.$

$$EX = \mu,\ VarX = \sigma^2,\ \Psi(t) = \exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$$

Exp. family. $p(x,\theta) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2\sigma^2} + \frac{x\mu}{\sigma^2} - \left(\frac{\mu^2}{2\sigma^2} + \log\sigma\right)\right\}$

Pareto: $f(x|\theta,a) = \theta a^\theta / x^{(\theta+1)}, x > a > 0, \theta > 0$

$$EX = \frac{\theta a}{\theta-1},\ VarX = \frac{\theta a^2}{(\theta-1)^2(\theta-2)}$$

Exp. family. $p(x,\theta) = \frac{1}{x}\exp\{-\theta\log x - (-\log\theta - \theta\log a)\}$

Poisson: $f(x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x \in \{0,1,...\}$

$$EX = \lambda,\ VarX = \lambda,\ \Psi(t) = e^{\lambda(e^t-1)},\ MLE(\lambda) = \frac{1}{n}\sum x_i$$

Exp. family. $p(x,\theta) = \frac{\exp\{x\log(\lambda)-\lambda\}}{x!}$

Rayleigh: $p(x,\theta) = \frac{x}{\theta^2} exp\left[-\frac{x^2}{2\theta^2}\right], x \geq 0, \theta > 0.$

$$EX = \theta\sqrt{\pi/2},\ VarX = \frac{4-\pi}{2}\theta^2,\ MLE(\theta) = \sqrt{\frac{1}{2n}\sum x_i^2}$$

Exp. family. $p(x,\theta) = x\exp\left\{-\frac{x^2}{2\theta^2} - 2\log\theta\right\}$

t: $f(x|\nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)(1+x^2/\nu)^{(\nu+1)/2}}, \nu > 0$(degrees of freedom)

$$EX = 0 \text{ for } \nu > 1,\ VarX = \frac{\nu}{\nu-2} \text{ for } \nu > 2$$

Uniform: $f(x|a,b) = \frac{1}{b-a}, x \in (a,b)$

$$EX = (a+b)/2,\ VarX = (b-a)^2/12,\ \Psi(t) = \frac{e^{tb}-e^{ta}}{t(b-a)}$$

Weibull: $f(x|\theta,a) = \theta a x^{a-1}\exp(-\theta x^a), x > 0, \theta > 0, a > 0$

Wishart: $P(A|m,\Lambda) \propto |A|^{\frac{m-p-1}{2}} exp\left(-\frac{1}{2}tr(\Lambda^{-1}A)\right)$ for positive definite matrices A.

$$EA = m\Lambda$$