$y'' + 2y' + y = 2x$

1)

$y^2 + 2y + 1 = 0$

$-2 \pm \sqrt{2^2 - 4\cdot1\cdot1}\ /2$

$-2 \pm \sqrt{4-4}\ /2$

$-2 \pm 0\ /2$

$-2/2$

-1 case 1

$y = c_1 e^{-1x} + c_2 x e^{-1x}$

$A = y' = 2$

$B = y'' = 0$

$1(0) + 2(A) + 1(Ax+B)$

$2x - 4 = y_p$

$2A + B = 0$

$4 + B = 0$

$B = -4$

$y_p = c_1 e^{-1x} + c_2 x e^{-1x} + 2x - 4$

2) $c_1 e^{-1x} + c_2 x e^{-1x}$

$y_1 = e^{-1x} \quad y_2 = x e^{-1x}$

$y_1' = -e^{-x} \quad y_2' = (1-x)\ e^{-x}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & xe^{-1x} \\ -e^{-x} & \cancel{x}e^{-x} \end{vmatrix} = e^{-x} \cdot (1-x) e^{-x} - -e^{-x} \quad xe^{-x}$

$(1-x) \cdot e^{-2x} - xe^{-2x} = e^{-2x}$

$v_2 = \dfrac{xe^{-x} \cdot 2x}{e^{-2x}} = 2x^2 e^x$

$v_1 = 2xe^x$

$\int 2x^2 e^x\ dx = 2(x^2 - 2x + 2)e^x \quad \int 2xe^x\ dx = 2(x-1)e^x$

$y_1' v_2 - y_2' v_1$

$-\underline{e^{-x}}\, 2(x^2 - 2x + 2)\underline{e^x} + x\underline{e^{-x}}\, 2(x-1)\underline{e^x}$

$-2(x^2 - 2x + 2) + 2x(x-1)$

$-2x^2 + 4x - 4 \quad + 2x^2 - 2x$

$y_p = 0 + 2x - 4$

$y(x) = c_1 e^{-x} + c_2 x e^{-x} + 2x - 4$

3)

$L(y) = y'' + 2y' + y$

$r^2 - r - 2$

$y_h = c_1 e^{-x} + c_2 x e^{-x}$

$$G(t,s) = \begin{cases} 0 & t < s \\ (t-s)e^{-(t-s)} & t \geq s \end{cases}$$

$$\int_0^t G(t,s)\, 2\, ds$$

$$2\int_0^t (t-s)e^{-(t-s)} s\, ds \quad = 2\left((t-2)e^{t} + t + 2\right)e^{-t}$$

$$2\left(te^{-t} - 2e^{-t} + t + 2\right)e^{-t}$$

$$2\left(t - 2 + te^{-t} + 2e^{-t}\right)$$

$$\left(2t - 4 + 2te^{-t} + 4e^{-t}\right)$$

②

$Y'' + Y = x^2 \; ; \; t \geq 0 \; ; \; Y(0) = Y'(0) = 0$

1) $r^2 + 0 + 1 = 0 \qquad 0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 1}/2 \qquad \sqrt{-4}/2 \qquad 2i/2 \quad i$

$Y = C_1 \cos(x) + C_2 \sin(x)$

$A = y^2 + 1 \qquad 2A + (At^2 + A_2 t + A_3)$

$A' = 2y \qquad A_1 t^2 + A_2 t + (2A_1 + A_3)$

$A'' = 2 \qquad A_1 = 0 \quad A_2 = 1 \quad A_3 = -2$

$Y_p = t^2 - 2$

$Y(t) = C_1 \cos(x) + C_2 \sin(x) + t^2 - 2$

2) $Y_1 = \cos(x) \qquad Y_2 = \sin(x)$

$Y_1' = -\sin(x) \qquad Y_2' = \cos(x)$

$\cos \cdot \cos - \sin \cdot -\sin$

$\cos^2(x) + \sin^2(x) = 1$

$v_1' = -\frac{Y_2 \, f x}{w} = -\sin(x)\, x^2 \qquad v_2' = \frac{Y_1 \, f(x)}{w} \qquad \cos(x)\, x^2$

$\int -\sin(x) x^2 \qquad \int \cos(x) x^2$

$v_1 = (x^2 - 2)\cos(x) - 2x\sin(x) \qquad v_2 = 2x\cos(x) + (x^2 - 2)\sin(x)$

$u_1 Y_1 + u_2 Y_2$

$((x^2 - 2)\cos(x) - 2x\sin(x))(\cos(x)) + ((2x\cos(x) + (x^2 - 2)\sin(x))(\sin(x))$

$x^2 - 2$

$C_1 \cos(t) + C_2 \sin(t) + x^2 - 2$

③ $G(t,s) = \sin(t - s) \qquad t \geq s$

$Y(t) = \int_0^t \sin(t - s)\, s^2 \, ds$

$t^2 - 2$

$C_1 \cos(t) + C_2 \sin(t) + t^2 - 2$