**BMI 555 Homework Assignment 1**

Due Oct-10-2024

**Deliverables: Code, Plots, Observations.**

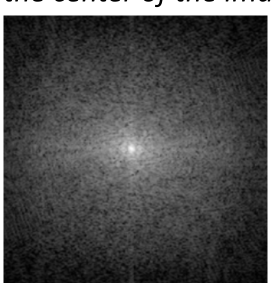
**Introduction.** Shepp-Logan phantom below is commonly used to test reconstruction algorithms. You can download the image from: http://bigwww.epfl.ch/thevenaz/shepplogan/phantom.gif

1. **MRI – Fourier Transform**

a. Simulate the acquisition of k-space data by applying a **2D** Fourier Transform to the Shepp-Logan image. Plot the 2D Fourier Domain **magnitude** image in log scale. (k-space data are complex, you will encounter error if you try to plot it directly).

*Hint1: use numpy.fft.fft2() in Python or equivalent in MATLAB.*

*Hint2: depending on the implementation of the 2D Fourier Transform, the DC point (the entry with the highest magnitude in the Fourier domain), might be at a corner. In that case, move it to the center of the image.* You should have something like this:



b. Perform inverse 2D Fourier Transform. Plot the magnitude image (i.e. the absolute value of the image matrix).

*Hint: using numpy.fft.ifft2() (or equivalent in MATLAB).*

c. Removing the 10x10 patch from the center of the k-space (setting all values within that patch to 0), perform inverse 2D Fourier Transform. Plot the magnitude image.

d. Perform inverse 2D Fourier Transform using only the center 10x10 patch (setting everything outside to 0).

e. What's your observations?

2. **CT and nuclear imaging – Radon Transform**

a. Apply the Radon Transform to the Shepp-Logan phantom image at multiple angles (0°, 45°, 90°). Plot the profiles using 2D line plot.

b. Generate the sinogram of the Shepp-Logan phantom with 1° increment from 0° – 360°.

*Hint: Use Python's skimage.transform.radon() or MATLAB's radon() function.*

c. Perform inverse radon transform (such as iradon() function) of the sinogram to reconstruct the image from the sinogram (**WITHOUT any filter**). This is inverse radon transfer, equivalent to back-projection.

d. Perform inverse radon transform  to reconstruct the image from the sinogram (**WITH Ram-Lak filter**).

e. Compare the two images you reconstructed with the original image. What's your observation?

3. **CT and nuclear imaging – Noisy data**

Now repeat problem 2 again **after** adding Gaussian noise to your data by doing the following:

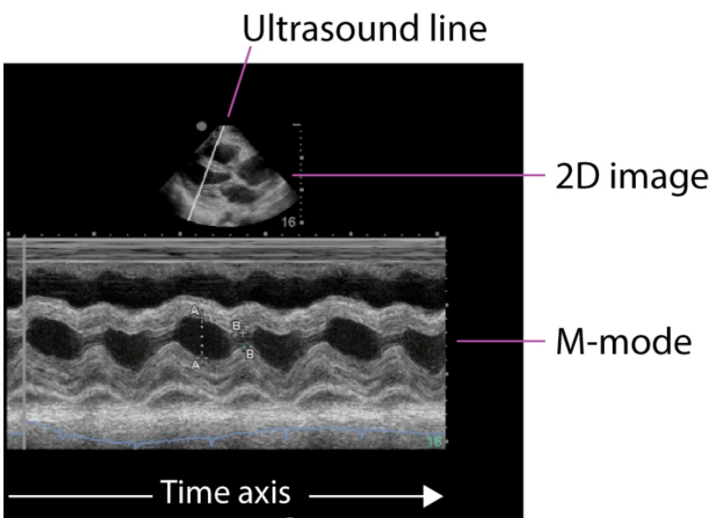
Step 1. Normalize your Shepp-Logan image such that black equals to 0, and the max is 1.

Step 2. Add Gaussian noise with the distribution of ~N(0, 0.1) to the image.

Step 3. repeat all subproblems in Problem 2.

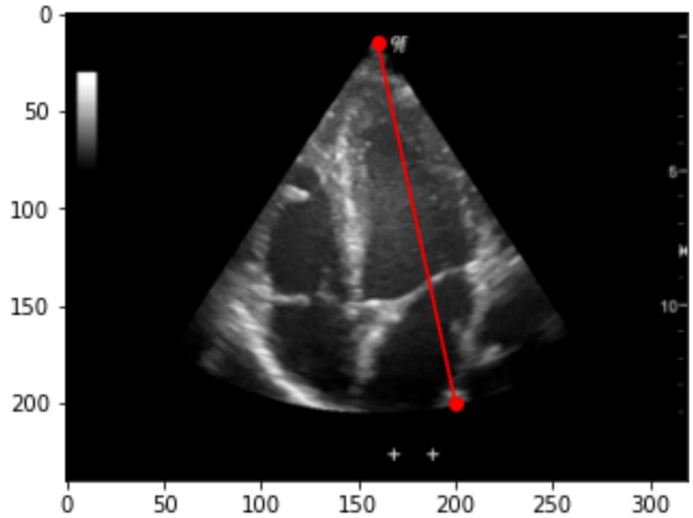
4. **Ultrasound imaging – M mode**

Simulate M-mode ultrasound from b-mode ultrasound image. One example is below.




You can download the cardiac ultrasound B-mode image from [https://commons.wikimedia.org/wiki/File:Ultrasound_of_human_heart_apical_4-cahmber_view.gif](https://commons.wikimedia.org/wiki/File:Ultrasound_of_human_heart_apical_4-cahmber_view.gif). This is not the one shown in the example above.

a. Simulate M-mode ultrasound by plotting data across the gif frame from the line defined by two point [160, 15] and [200, 200] as shown below.




**5. Monte Carlo method to Estimate $\frac{\pi}{4}$**

Generate N random points in a unit squre (with side length of 1) for N = 100, 1000, and 10000. Count how many fall inside the quarter circle defined by $x^2 + y^2 \leq 1$ as shown in the figure below. Repeat each experiment 10 times, calculate the mean and standard deviation of the estimate for $\frac{\pi}{4}$. For each N, pick one run (in the 10 runs) and plot the points generated, use different colors for points inside and outside the quarter circle.