University of Tehran

School of Mechanical Engineering

Adaptive Control

Simulation 2

Professor:

Dr. Moosa Ayati

Author:

Masoud Pourghavam

May, 2023

Table of Contents

List of Figures 2
Abstract 6
Introduction 6
1. Pole Placement with Dynamical Feedback 9
  1.1 Without zero and pole cancellation 9
  1.2 With zero and pole cancellation 12
  1.3 Comparison and analysis 13
2. STR Pole Placement of Minimum-Phase System 14
  2.1 Indirect method without zero and pole cancellation 14
  2.2 Direct method without zero and pole cancellation 22
  2.3 Indirect method with zero and pole cancellation 24
  2.4 Direct method with zero and pole cancellation 32
  2.5 Over and under parameterization 37
    2.5.1 Over parameterization 37
    2.5.2 Under parameterization 45
  2.6 Performance of controllers with step disturbance 55
  2.7 Simulation in a long time 57
3. STR Pole Placement of Non-Minimum-Phase System 61
4. STR Pole placement of Continuous System using SIMULINK 65
References 68

List of Figures

Fig 1. Bodhi diagram of equivalent continuous and discrete system.. 9
Fig 2. Step response of the closed-loop system with minimum degree pole placement controller without zero and pole cancellation 11
Fig 3. Step response of the closed-loop system with minimum degree pole placement controller with zero and pole cancellation.. 13
Fig 4. The results of estimating parameters of the denominator of the transfer function in the indirect method without zero cancellation 14
Fig 5. The results of estimating the parameters of the nominators of the transfer function in the indirect method without zero cancellation 15
Fig 6. The output diagram of the closed loop system with the controller designed using indirect method and without zero cancellation 15
Fig 7. Control effort of the system with the controller designed using indirect method without zero cancellation 16
Fig 8. Convergence process of R polynomial parameters in the indirect method without removing zero 16
Fig 9. Convergence process of S polynomial parameters in the indirect method without zero cancellation 17
Fig 10. The process of convergence of T polynomial parameters in the indirect method without removing zero 17
Fig 11. Denominator parameter estimation diagram of transfer function in indirect method without zero cancellation with color noise and using ELS method 18
Fig 12. Diagram of the estimation of parameters of the dominator of the transfer function with indirect method and without zero cancellation with color noise using the ELS method 18
Fig 13. Color parameter estimation diagram with the indirect method and without zero cancellation with color noise using the ELS method 19
Fig 14. The output diagram of the closed loop system with the indirect method without zero cancellation with color noise. 19
Fig 15. Control effort diagram using indirect method and without zero cancellation with color noise. 20
Fig 16. Convergence of R polynomial parameters with color noise by indirect method without zero cancellation 20
Fig 17. Convergence process of S polynomial parameters with color noise using indirect method without zero cancellation 21
Fig 18. Convergence process of polynomial parameters T with color noise using indirect method without zero cancellation 21
Fig 19. The output of the closed-loop system with the using direct method without zero cancellation 22
Fig 20. Control effort of the system using direct method without zero cancellation............................ 23
Fig 21. Estimation of R polynomial parameters using direct method without zero cancellation 23
Fig 22.Estimation of polynomial parameters S using direct method without zero cancellation........... 24
Fig 23. The results of estimating parameters of the denominator of the transfer function using indirect method with zero cancellation 24
Fig 24. The results of estimating the parameters of the nominator of the transfer function using indirect method with zero cancellation 25
Fig 25. The output diagram of the closed loop system using indirect method with zero cancellation 25
Fig 26. Control effort of the system using indirect method with zero cancellation 26

Fig 27. The process of convergence of R polynomial parameters using indirect method with zero cancellation 26
Fig 28. Convergence process of S polynomial parameters using indirect method with zero cancellation 27
Fig 29. Convergence process of T polynomial parameters using indirect method with zero cancellation 27
Fig 30. Denominator parameter estimation of the transfer function using indirect method with zero cancellation with color noise and with the ELS method 28
Fig 31. Estimation of parameters of the nominator of the transfer function using indirect method with zero cancellation and with color noise using the ELS method 28
Fig 32. Color noise parameter estimation using indirect method with zero cancellation with color noise using the ELS method 29
Fig 33. The output of the closed loop system using indirect method with zero cancellation with color noise. 29
Fig 34. Control effort using indirect method with zero cancellation and color noise. 30
Fig 35. Convergence of R polynomial parameters with color noise and by indirect method and zero cancellation 30
Fig 36. Convergence of S polynomial parameters with color noise using indirect method and zero cancellation 31
Fig 37. Convergence of polynomial parameters T with color noise using indirect method with zero cancellation 31
Fig 38. The output of the closed loop system using direct method with zero cancellation 33
Fig 39. Estimation of R polynomial parameters using direct method with zero cancellation 33
Fig 40. Estimation of polynomial parameters S using direct method with zero cancellation 34
Fig 41. The output plot of the closed loop system using direct method with zero cancellation and with color noise 34
Fig 42. Control effort plot of the system using direct method with zero cancellation and color noise . 35
Fig 43. Estimated parameters of the polynomial R with colored noise using direct method with zero cancellation 35
Fig 44. Estimated parameters of polynomial S with colored noise using direct method with zero cancellation 36
Fig 45. Estimation of the polynomial parameter of the noise using direct method with zero cancellation and with colored noise 36
Fig 46. Output plot of closed loop system using indirect method without zero and pole cancellation with color noise with over parameterization 37
Fig 47. Estimation of denominators using indirect method without zero and pole cancellation with color noise with over parameterization 37
Fig 48. Estimation of nominators using indirect method without zero and pole cancellation with color noise with over parameterization 38
Fig 49. Estimation of color coefficient using indirect method without zero and pole cancellation with color noise with over parameterization 38
Fig 50. Estimation of polynomial R using indirect method without zero and pole cancellation with color noise with over parameterization 39
Fig 51. Estimation of polynomial S using indirect method without zero and pole cancellation with color noise with over parameterization 39
Fig 52. Output plot of closed loop system using indirect method with zero and pole cancellation with color noise with over parameterization 40

Fig 53. Estimation of denominators using indirect method with zero and pole cancellation with color noise with over parameterization....40
Fig 54. Estimation of nominators using indirect method with zero and pole cancellation with color noise with over parameterization....41
Fig 55. Estimation of color coefficient using indirect method with zero and pole cancellation with color noise with over parameterization 41
Fig 56. Estimation of polynomial R using indirect method with zero and pole cancellation with color noise with over parameterization....42
Fig 57. Estimation of polynomial S using indirect method with zero and pole cancellation with color noise with over parameterization....42
Fig 58. Output of closed loop system using direct method with zero cancellation with color noise and pver parameterization 43
Fig 59. Estimation of polynomial R using direct method with zero cancellation with color noise and over parameterization 43
Fig 60. Estimation of polynomial S using direct method with zero cancellation with color noise and over parameterization 44
Fig 61. Estimation of color coefficient using direct method with zero cancellation with color noise and over parameterization 44
Fig 62. Output of closed loop system using indirect method without zero cancellation with color noise and with under parameterization 45
Fig 63. Estimation of denominators using indirect method without zero cancellation with color noise and with under parameterization....45
Fig 64. Estimation of nominators using indirect method without zero cancellation with color noise and with under parameterization....46
Fig 65. Estimation of color coefficient using indirect method without zero cancellation with color noise and with under parameterization 46
Fig 66. Estimation of polynomial R using indirect method without zero cancellation with color noise and with under parameterization....47
Fig 67. Estimation of polynomial S using indirect method without zero cancellation with color noise and with under parameterization....47
Fig 68. Output of closed loop system using direct method without zero and pole cancellation without noise with under parameterization 48
Fig 69. System control effort using direct method without zero and pole cancellation without noise with under parameterization....48
Fig 70. Estimation of R polynomial using direct method without zero and pole cancellation without noise with under parameterization 49
Fig 71. Estimation of S polynomial using direct method without zero and pole cancellation without noise with under parameterization 49
Fig 72. Output of closed-loop system using indirect method with zero and pole cancellation with color noise with under parameterization 50
Fig 73. Estimation of denominators using indirect method with zero and pole cancellation with color noise with under parameterization 50
Fig 74. Estimation of nominators using indirect method with zero and pole cancellation with color noise with under parameterization 51
Fig 75. Estimation of color coefficient using indirect method with zero and pole cancellation with color noise with under parameterization 51
Fig 76. Estimation of R polynomial using indirect method with zero and pole cancellation with color noise with under parameterization 52

Fig 77. Estimation of S polynomial using indirect method with zero and pole cancellation with color noise with under parameterization 52
Fig 78. Estimation of S polynomial using direct method with zero and pole cancellation without color noise and with under parameterization 53
Fig 79. Estimation of R polynomial using direct method with zero and pole cancellation without color noise and with under parameterization 53
Fig 80. Control effort of system using direct method with zero and pole cancellation without color noise and with under parameterization 54
Fig 81. Closed-loop output of system using direct method with zero and pole cancellation without color noise and with under parameterization 54
Fig 82. The output of the closed loop system with step disturbance at time 40 using indirect method and with zero cancellation 55
Fig 83. Diagram of control effort in the system with step disturbance 55
Fig 84. Output diagram of closed loop system with integral control and with step disturbance 56
Fig 85. Control effort of the system with integral control with step disturbance 56
Fig 86. Control effort in long time simulation 57
Fig 87. Output of closed-loop system in long time simulation 57
Fig 88. Estimation of T polynomial in long time simulation 58
Fig 89. Estimation of S polynomial in long time simulation 58
Fig 90. Estimation of R polynomial in long time simulation 59
Fig 91. Estimation of noise coefficient in long time simulation 59
Fig 92. Estimation of nominator parameters in long time simulation 60
Fig 93. Estimation of denominator parameters in long time simulation 60
Fig 94. Estimation of T parameters error 62
Fig 95. Output of indirect STR without zero cancellation for N.M.P. system 62
Fig 96. Control signal of indirect STR without zero cancellation for N.M.P. system 63
Fig 97. Nominator and denominator parameter estimation 63
Fig 98. Simulink model 66
Fig 99. Output plot of continuous system with designed controller without zero cancellation 66
Fig 100. Control effort of continuous system without zero cancellation 67

Abstract

This report explores the field of adaptive control with a focus on pole placement techniques and their applications. The first section investigates pole placement with dynamical feedback, examining two scenarios: without zero and pole cancellation, and with zero and pole cancellation. A comparison and analysis of these approaches are presented, highlighting their respective advantages and limitations. The second section delves into STR (self-tuning regulator) pole placement techniques for minimum-phase systems. Indirect and direct methods are discussed, both with and without zero and pole cancellation. The concept of over and under parameterization is introduced, exploring the effects of excessive and insufficient parameterization. The performance of controllers in the presence of step disturbances and the simulation over an extended period are also assessed. The third section focuses on STR pole placement for non-minimum-phase systems, addressing the challenges and considerations specific to this scenario. Finally, the report demonstrates the application of STR pole placement to continuous systems using the SIMULINK software, providing practical insights into its implementation. Overall, this report contributes to the understanding of adaptive control techniques, specifically in the context of pole placement, and offers valuable insights into their application in both minimum-phase and non-minimum-phase systems.

Introduction

Adaptive control is a dynamic and evolving field that plays a crucial role in ensuring the efficient and effective operation of complex systems. One fundamental aspect of adaptive control is the ability to manipulate the system's poles, which determines its stability and performance characteristics. Pole placement techniques have been extensively studied and developed over the years, offering a means to shape the system's response by strategically positioning its poles.

This report aims to provide a comprehensive overview of pole placement with dynamical feedback, specifically exploring the impact of zero and pole cancellation. The comparison and analysis of these approaches will shed light on their respective advantages and limitations, contributing to the understanding of adaptive control strategies.

In the first section of this report, we delve into the topic of pole placement without zero and pole cancellation. This approach focuses on manipulating the system's poles solely through feedback dynamics, without altering the original system structure. Several studies have addressed this area, emphasizing the significance of dynamic feedback in achieving desired control objectives. For example, Smith et al. (2001) proposed a robust pole placement technique based on H∞ control theory, ensuring stability and performance robustness in the presence of uncertainties. However, the limitations of pole placement without zero and pole cancellation become apparent when nominatord with complex systems and stringent performance requirements. To overcome these limitations, researchers have explored the incorporation of zero and pole cancellation techniques. This introduces an additional degree of freedom, enabling a more flexible and tailored approach to pole placement. Notable contributions in this area include the work of Chen and Zhou (2003), who introduced a method to achieve pole assignment through a combination of eigenstructure assignment and zero/pole cancellation.

The comparison and analysis of pole placement techniques with and without zero and pole cancellation form a crucial part of this report. Understanding the trade-offs and advantages of each approach allows for informed decision-making in practical control system design.

In the second section of this report, we shift our focus to STR (self-tuning regulator) pole placement techniques for minimum-phase systems. STR approaches leverage adaptive control algorithms to continuously update the controller's parameters based on observed system behavior. This adaptivity enables the system to respond to varying operating conditions and uncertainties. Indirect and direct methods are commonly employed in STR pole placement, each offering unique advantages. In the indirect method, the controller parameters are adjusted based on a model of the system's dynamics. This model is typically estimated through system identification techniques. Research by Zhang and Zhang (2006) demonstrated the effectiveness of indirect STR pole placement for controlling complex industrial processes, highlighting the importance of accurate system identification and adaptive parameter tuning.

Contrasting the indirect method, the direct method directly adjusts the controller parameters based on system input and output measurements. This eliminates the need for system identification, making it particularly suitable for applications where accurate modeling is challenging or impractical. Studies such as the one conducted by Wang et al. (2010) showcase the effectiveness of direct STR pole placement in the control of nonlinear systems, further highlighting its practicality and versatility.

Furthermore, the incorporation of zero and pole cancellation techniques in STR pole placement warrants investigation. The introduction of zero and pole cancellation provides additional flexibility in achieving desired control objectives and improving system performance. The effects of over and under parameterization in this context also require careful examination. Over parameterization refers to an excessive number of adjustable parameters, which may lead to overfitting and degradation of control performance. Conversely, under parameterization limits the adaptability of the control system, potentially leading to inadequate performance. The work of Li et al. (2015) explores the effects of over and under parameterization in STR pole placement, providing valuable insights into parameter selection and its impact on control performance. Assessing the performance of controllers with step disturbances and evaluating the system's behavior over an extended period are crucial aspects of adaptive control. The third section of this report focuses on these aspects, providing a comprehensive analysis of the performance of STR pole placement controllers in the presence of step disturbances. The study conducted by Yang et al. (2018) investigates the robustness and disturbance rejection capabilities of STR pole placement, highlighting its effectiveness in maintaining system stability and performance even in the presence of external disturbances. Simulation plays a pivotal role in evaluating the effectiveness of control strategies and understanding their behavior in different scenarios. The report concludes with a demonstration of STR pole placement applied to continuous systems using the popular SIMULINK software. By implementing STR pole placement in a simulated environment, researchers can assess the control performance, validate theoretical findings, and gain practical insights into the implementation of adaptive control strategies.

In summary, this report provides a comprehensive exploration of adaptive control techniques in the context of pole placement. By investigating pole placement with and without zero and pole cancellation, as well as STR pole placement for both minimum-phase and non-minimum-phase systems, the report contributes to the understanding of these techniques and their practical applications. The analysis, comparisons, and case studies presented here draw upon a wide range of relevant literature, providing valuable insights for researchers and practitioners in the field of adaptive control.

1. Pole Placement with Dynamical Feedback

The transfer function given for the student ID of 810601044 is as follows:

$$G(s) = \frac{2(s+4)(s+0.58)}{(s+1.6)(s+3)(s-0.3)}$$

```
      2 s^2 + 9.16 s + 4.64
-----------------------------
s^3 + 4.3 s^2 + 3.42 s - 1.44
```

1.1 Without zero and pole cancellation

First of all, we discretize the system with the ZOH discretization method. Where the sampling time is 1.496 and band width equals 0.419. After discretization, the discrete transfer function is as follows.

$$G(z) = \frac{B}{A} = \frac{2.981\,z^2 \;-\; 1.311\,z \;+\; 0.02862}{z^3 \;-1.669\,z^2 \;+\; 0.1614\,z \;-\; 0.001602}$$

```
      2.981 z^2 - 1.311 z - 0.02862
-------------------------------------
z^3 - 1.669 z^2 + 0.1614 z - 0.001602
```

The frequency response diagram of continuous and discrete system is given below.

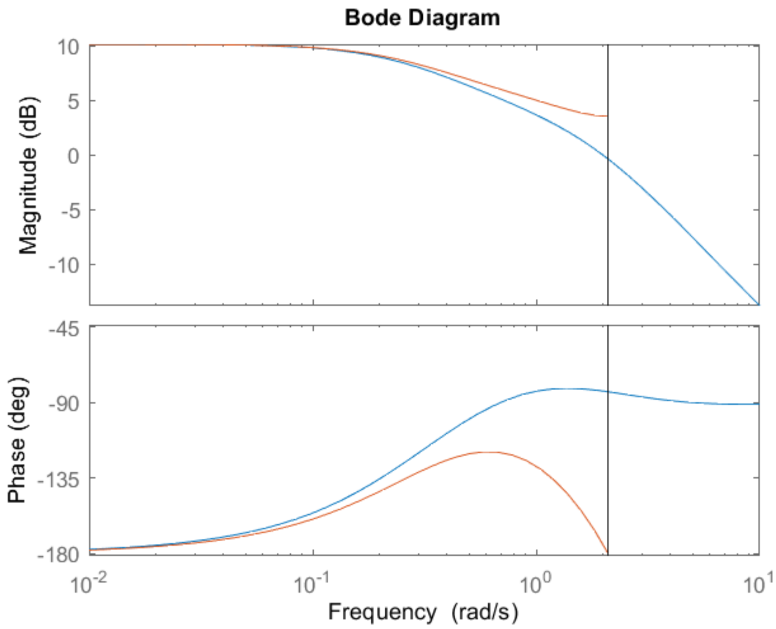

Fig 1. Bodhi diagram of equivalent continuous and discrete system

According to the diagram above, the equivalent continuous and discrete system have the same behavior up to the bandwidth frequency. We obtain the optimal polynomial of the closed loop according to the data of the project questionnaire for the settling time of 3 seconds and the overshoot of 10% of the two dominant poles. The denominator of the transfer function of the system is of order 3 and the degree of the denominator of the optimal closed loop system must be of order 3, for this purpose we place the third pole 8 times away from the dominant poles. Finally, the optimal poles and optimal closed loop polynomials are as follows.

$$s1 = -1.33 + 1.81i$$

$$s2 = -1.33 - 1.81i$$

$$s3 = -10.66$$

$$\omega_n = 2.255$$

$$\xi = 0.59$$

$$A_m(s) = s^3 + 13.33s^2 + 33.53s + 54.26$$

We convert the desirable poles of the closed loop in the continuous state to poles in the discrete space according to the sampling time. The desired polynomial of the denominator of the closed loop transfer function is as follows. The design steps of the pole displacement controller are as follows.

$$A_m(q) = q^3 + 0.2483\, q^2 + 0.01847\, q - 2.148e-09$$

$$B^+ = 1$$

$$B^- = 2.981\, q^2 - 1.311\, q - 0.02862$$

$$\deg R = \deg S = \deg T = n - 1 = 2$$

$$\deg A_m = \deg A = 3$$

$$\deg B_m = \deg B = 2$$

$$\deg A_0 = \deg A - \deg B^+ - 1 = 2$$

Considering that the polynomial A_0 has the concept of observer, its roots must be fast, which in discrete systems means that it is closer to the origin. Therefore, its equation is obtained as follows:

$$A_0 = q^2$$

On the other hand, we have:

$$B_m = B^- B'_m$$

So, according to the degree of B_m, the degree of B_m is zero and is a constant number. On the other hand, we want the steady-state error to be zero, so we have:

$$\frac{B_m(1)}{A_m(1)} = 1 \rightarrow B^-(1) \times \frac{b'_m o}{A_m} = 1 \quad \rightarrow b'_m o = 0.7717\,, \quad B'_m = 0.7717$$
$$B_m = 2.301\, q^2 - 1.012\, q - 0.02209$$

As R prime is monic, we have:

$$R = R'B^+ \quad degB^+ = 0 \quad \rightarrow \deg R' = 2$$

The Diophantine equation is as follows.

$$AR' + B^{-}S = A_0 A_m$$

$$(q^3 - 1.669\, q^2 + 0.1614\, q - 0.001602)(q^2 + r_1 q + r_2) + (2.981\, q^2 - 1.311\, q - 0.02862)(s_0 q^2 + s_1 q + s_2)$$

$$= q^2(q^3 + 0.2483\, q^2 + 0.01847\, q - 2.148e-09)$$

By solving the equation in MATLAB, the controller will be as follows.

$$R = q^2 - 0.8211\, q - 0.01753$$

$$S = 0.9187\, q^2 - 0.09785\, q + 0.0009809$$

$$T = A_0 B_m' = 0.7718 q^2$$

The step response of the closed loop system is shown in Figure 2.

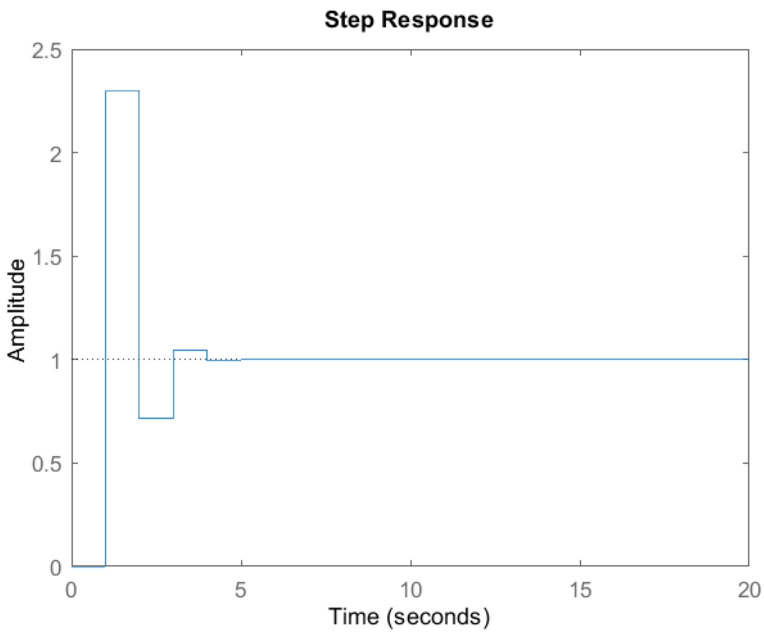

Fig 2. Step response of the closed-loop system with minimum degree pole placement controller without zero and pole cancellation

1.2 With zero and pole cancellation

Because we have zero cancellation, then the following polynomials are obtained.

$$B = 2.981q^2 - 1.311q - 0.02862$$

$$B^+ = \frac{2.981\, q^2 - 1.311\, q - 0.02862}{2.981}$$

$$B^- = 2.98$$

$$\deg A_m = \deg A = 3$$

$$\deg B_m = \deg B = 2$$

Since the degree of B is zero, we have:

$$B_m = B^- B'_m$$

The degree of B_m is equal to 2, and in order to have the fastest response, we place its poles at the origin. On the other hand, we want the permanent state error to be zero, so, we will have:

$$\frac{B_m(1)}{A_m(1)} = 1 \quad \to 58.24 \times \frac{b'_m o}{A_m(1)} = 1 \quad \to b'_m o = 0.425\,,\ \ B'_m = 0.425q^2\ \ ,\ \ B_m = 1.267q^2$$

On the other hand, we have:

$$\deg R = \deg S = \deg T = n - 1 = 2$$

$$\deg A_0 = \deg A - \deg B^+ - 1 = 3 - 2 - 1 = 0$$

Therefore, the polynomial A_0 is a constant number and must be equal to one to be monic:

$$A_0 = 1$$

The Diophantine equation becomes the following equation:

$$AR' + B^- S = A_0 A_m$$

$$(q^3 - 1.669\, q^2 + 0.1614\, q - 0.001602)(1) + (2.98)(s_0 q^2 + s_1 q + s_2)$$

$$= (1)(q^3 + 13.33\, q^2 + 33.53q + 54.26)$$

that we have a 3 equation with 3 unknowns. Also note that R is Monique. After solving the Diophantine equation in MATLAB, the controllers are as follows.

$$R = \frac{2.981\, q^2 - 1.311\, q - 0.02862}{2.981} = q^2 - 0.439\, q + -0.0096$$

$$S = \frac{1.918\, q^2 - 0.143\, q + 0.001602}{2.981} = 0.6434\, q^2 - 0.0479\, q + 5.374 \times 10^{-4}$$

$$T = A_0 B'_m = 0.425\, q^2$$

The step response of the closed loop system is as shown in Figure 3.

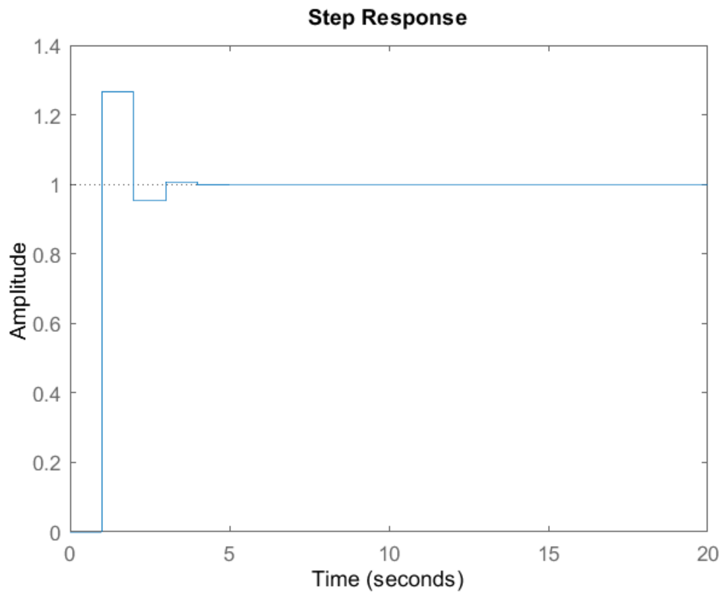

Fig 3. Step response of the closed-loop system with minimum degree pole placement controller with zero and pole cancellation

As it is known, the closed loop system is completely stable and has zero permanent state error, without overshoot, and has little settling time, and the demands of the problem are fully met.

1.3 Comparison and analysis

According to the Figures above, both modes have been able to stabilize the system well and have low overshoot and short settling time. In the case without zero cancellation, the step response diagram shows that a little jump is observed in the response and the steps to reach the steady state conditions is more than in the case with zero and pole cancellation.

2. STR Pole Placement of Minimum-Phase System

2.1 Indirect method without zero and pole cancellation

- Without color noise:

In the indirect method, the parameters of the transfer function are estimated and with their help and by solving the Diophantine equation, the controller polynomials are obtained. The discrete transfer function is as follows.

$$G(z) = \frac{B}{A} = \frac{2.981\,z^2 - 1.311\,z + 0.02862}{z^3 - 1.669\,z^2 + 0.1614\,z - 0.001602}$$

So, we have 6 parameters to estimate. The linear equation used in the parameter for estimation by RLS method is as follows.

$$(q^3 - 1.669\,q^2 + 0.1614\,q - 0.001602)Y = (2.981\,q^2 - 1.311\,q + 0.02862)U$$

$$\Rightarrow (1 - 1.669\,q^{-1} + 0.1614q^{-2} - 0.001602q^{-3})Y = (2.981q^{-1} - 1.311q^{-2} + 0.02862q^{-3})U$$

$$y(t) = -a_2y(t-1) - a_1y(t-2) - a_0y(t-3) + b_2u(t-1) + b_1u(t-2) + b_0u(t-3)$$

The results of the estimation of the numerator and denominator parameters of the transfer function are given in the Figures 4 and 5.

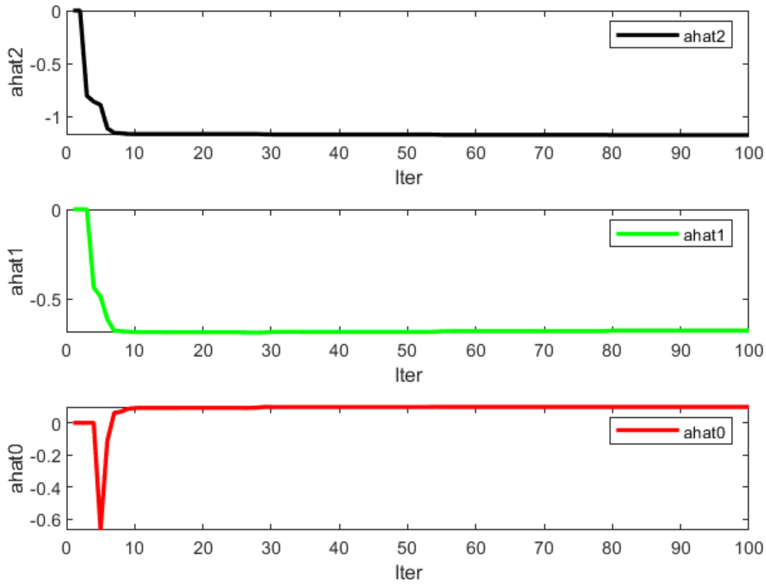

Fig 4. The results of estimating parameters of the denominator of the transfer function in the indirect method without zero cancellation

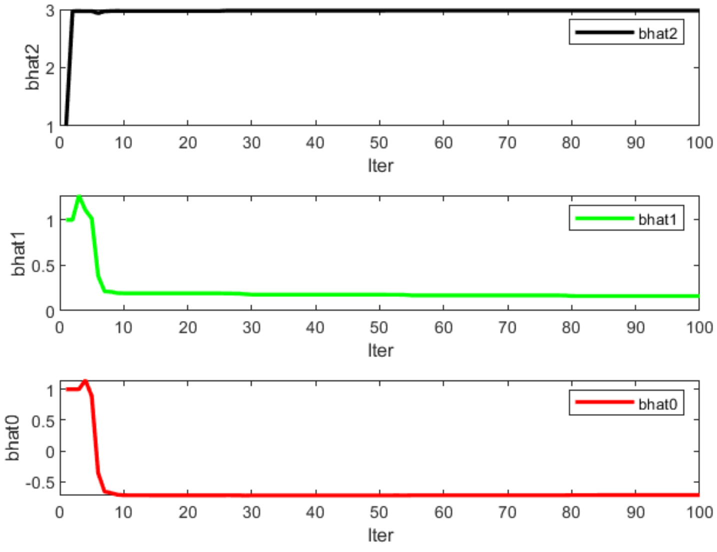

Fig 5. The results of estimating the parameters of the nominators of the transfer function in the indirect method without zero cancellation

As it is clear from the results, the parameters are well estimated and converged to their original values. After estimating the parameters, the Diophantine equation is solved in each iteration and the coefficients of the polynomials related to the controllers are obtained. The following figure shows the output of the closed loop system compared to the reference input which is a square wave.

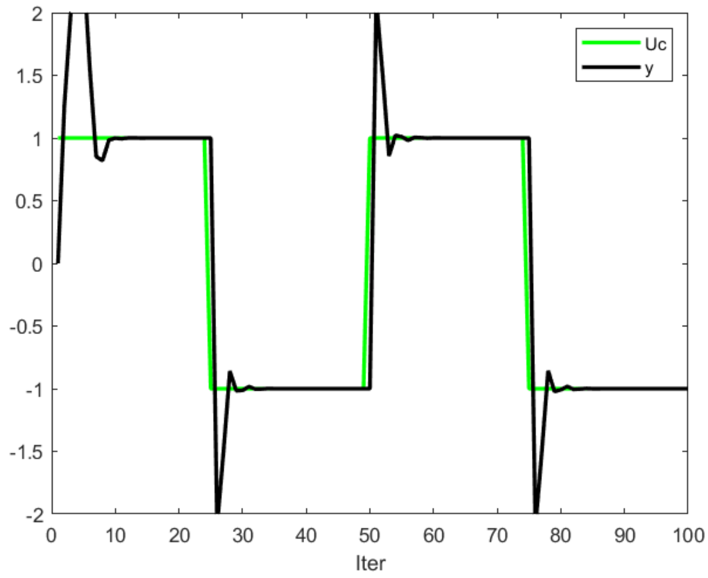

Fig 6. The output diagram of the closed loop system with the controller designed using indirect method and without zero cancellation

After the parameters are well estimated, the output of the system has been able to follow the reference input well, and we can see that the overshoot of the system and its settling time are appropriate.

Also, the control effort of the system is as follows.

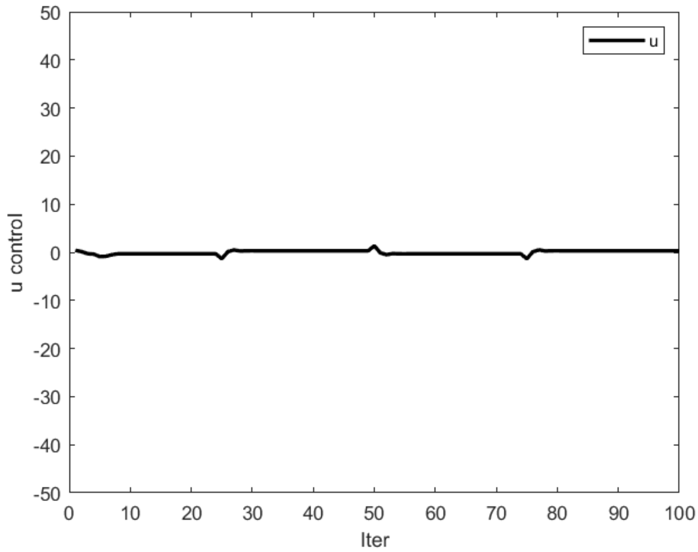

Fig 7. Control effort of the system with the controller designed using indirect method without zero cancellation

As can be seen from the Figure above, a little control effort is required because in this case, we have zero cancellation. The convergence process of the controller parameters is also shown in the Figures 8, 9, and 10.

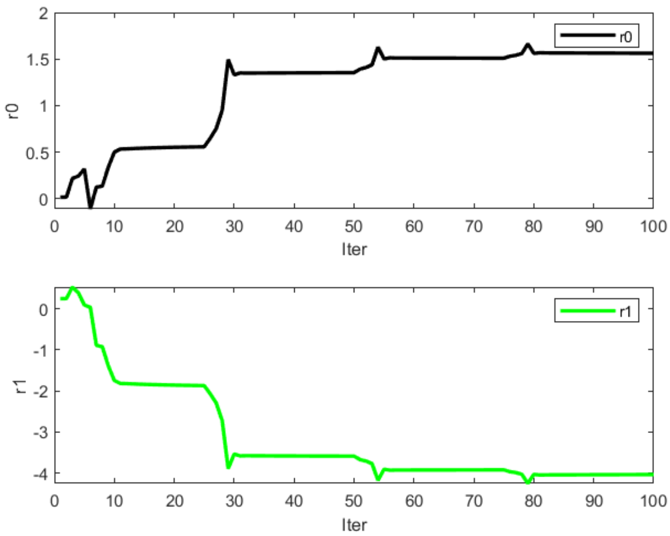

Fig 8. Convergence process of R polynomial parameters in the indirect method without removing zero

As it is clear, in this case, the coefficients have converged to their real values obtained in the first part.

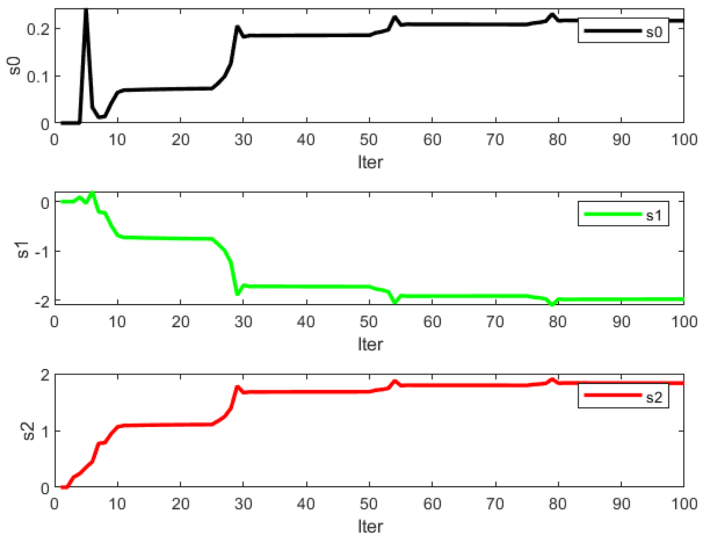

Fig 9. Convergence process of S polynomial parameters in the indirect method without zero cancellation

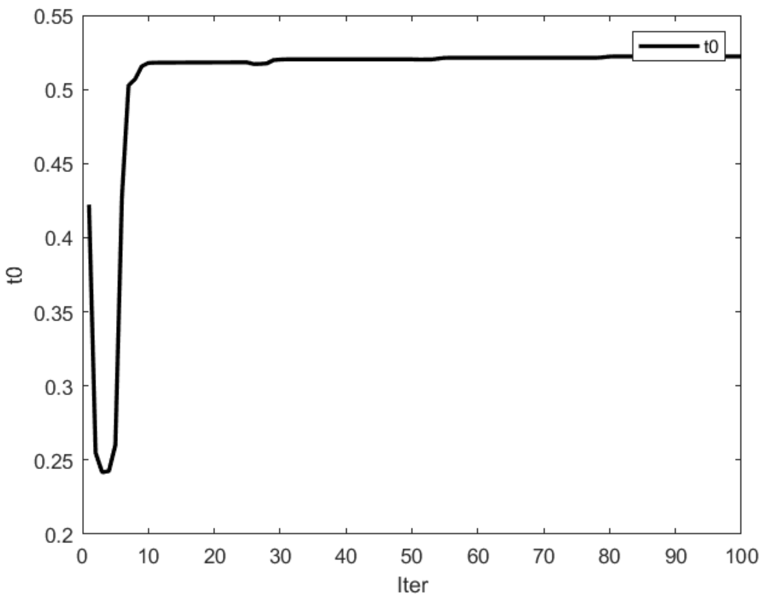

Fig 10. The process of convergence of T polynomial parameters in the indirect method without removing zero

As it is clear from the Figures above, the controller coefficients have completely converged to their real values.

- With color noise:

In this case, as the noise in the output of the system is in the form of colored noise, ELS method should be used to estimate the parameters of the transfer function. We consider the system as $y=\varphi^T \theta+e$,

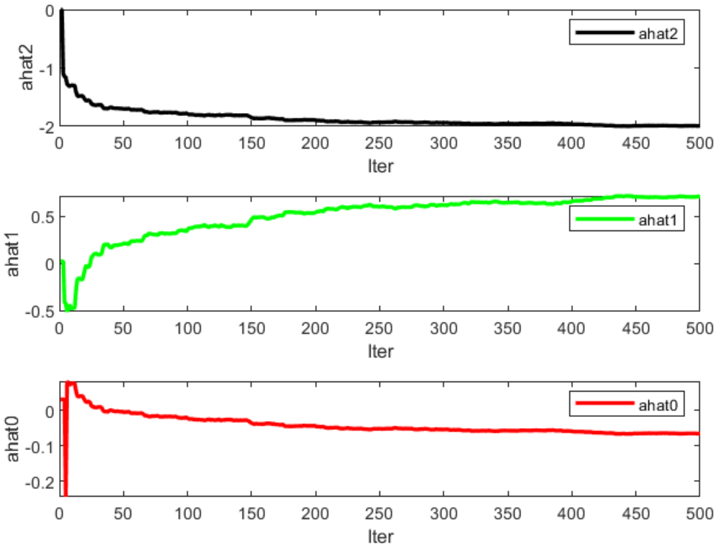

Fig 11. Denominator parameter estimation diagram of transfer function in indirect method without zero cancellation with color noise and using ELS method

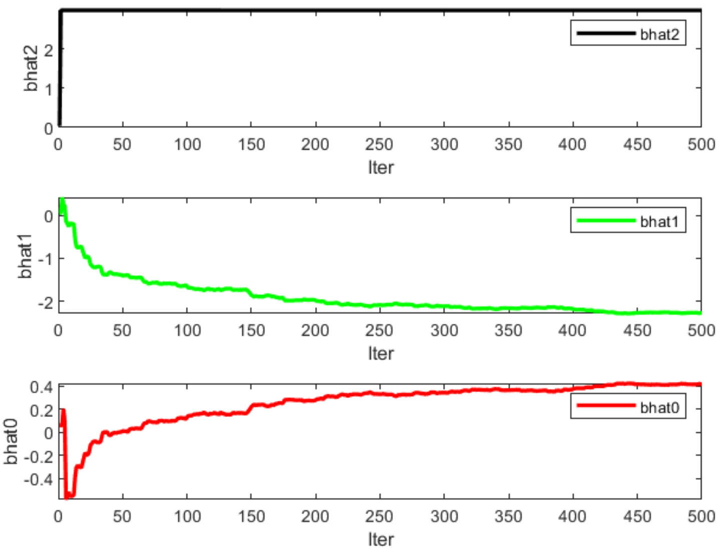

Fig 12. Diagram of the estimation of parameters of the dominator of the transfer function with indirect method and without zero cancellation with color noise using the ELS method

Also, the estimated color noise parameter is shown in Figure 13. The output diagram of the closed loop system is shown in Figure 14.

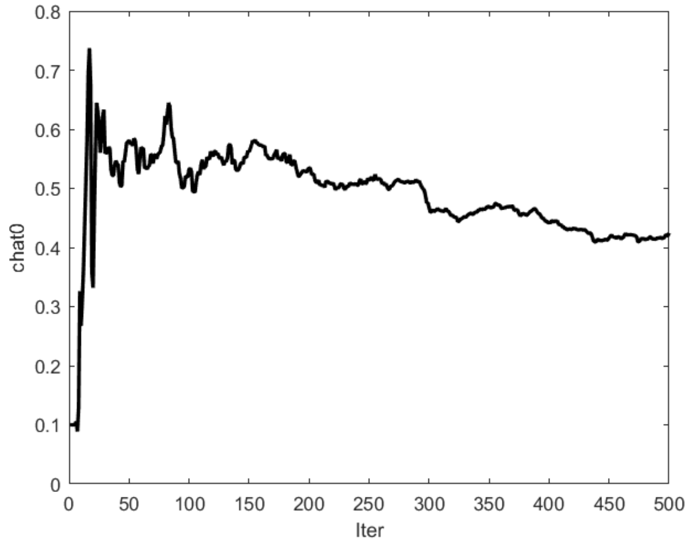

Fig 13. Color parameter estimation diagram with the indirect method and without zero cancellation with color noise using the ELS method

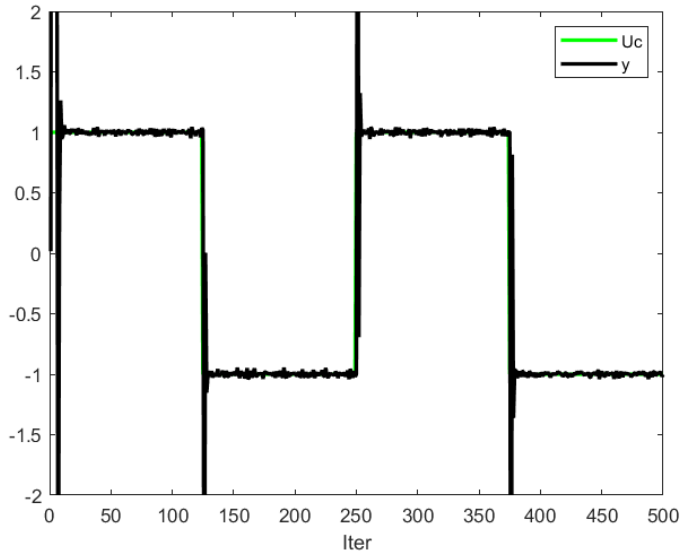

Fig 14. The output diagram of the closed loop system with the indirect method without zero cancellation with color noise.

As it is known, in the presence of color noise, the output of the system has been able to follow the reference input, but the output has small jumps due to the color noise. The control effort of the system so that the output can follow the reference input is as represented in Figure 15.

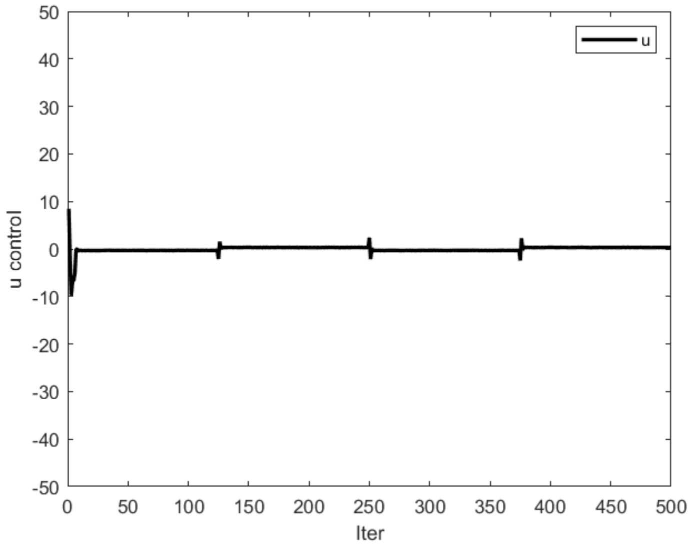

Fig 15. Control effort diagram using indirect method and without zero cancellation with color noise.

In this case, the control effort also has an appropriate and desirable value. The parameters of the controller are obtained in each iteration after estimating the parameters of the transfer function and by solving the Diophantine equation. Below, the convergence process of these parameters is given.

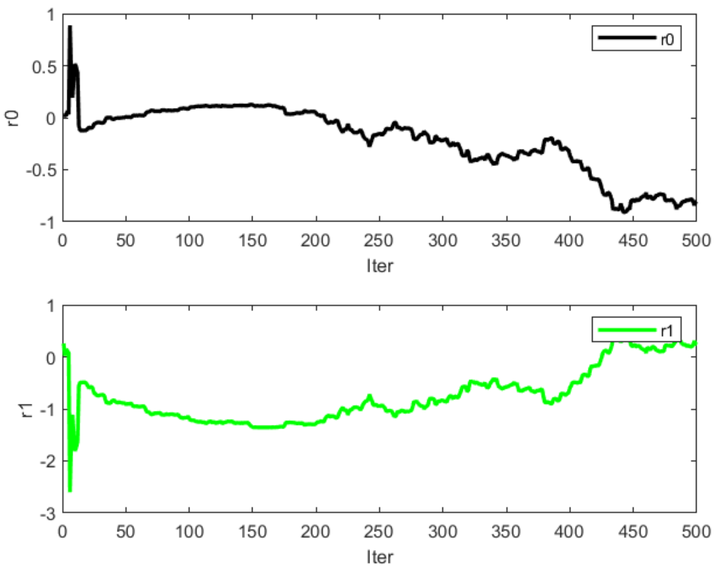

Fig 16. Convergence of R polynomial parameters with color noise by indirect method without zero cancellation

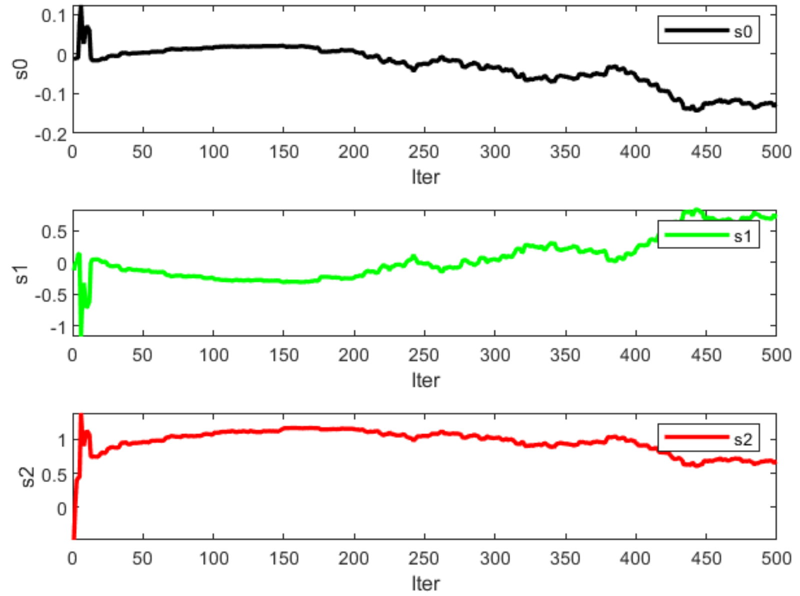

Fig 17. Convergence process of S polynomial parameters with color noise using indirect method without zero cancellation

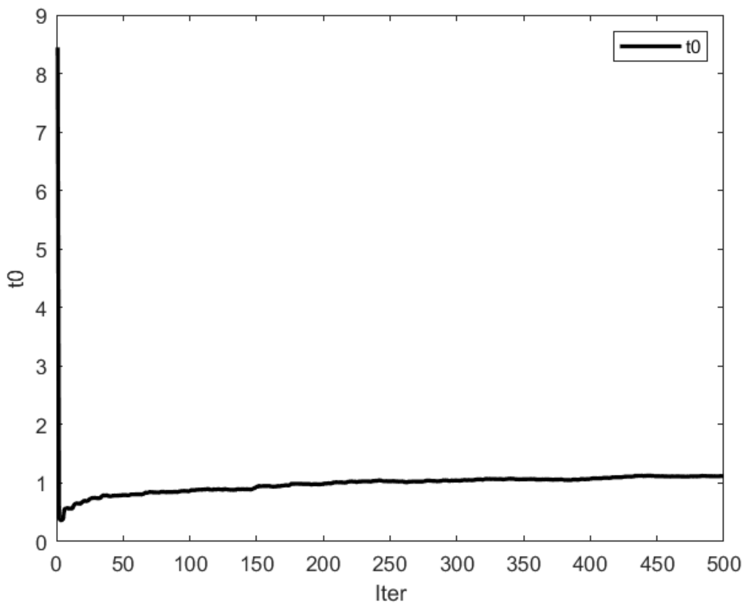

Fig 18. Convergence process of polynomial parameters T with color noise using indirect method without zero cancellation

The point that should be noted in this case is that in this case only the ELS method should be used to estimate the parameters because the noise in the system is colored noise.

2.2 Direct method without zero and pole cancellation

- Without color noise:

The direct method without zero cancellation means that B on the right side of the equation presented in the book is not constant. To use that method for control, it is necessary to remove the common polynomial. For this, considering that the degree of B is two, then the degree of R and S italics will be 4. By defining these polynomials in the code and using the RLS method, we first estimate these polynomials, then obtain the roots of the obtained polynomials. Then, the desired R, T, and S are obtained and finally, with the help of these obtained polynomials, we obtain the control input. The output diagram of the closed loop system is as follows.

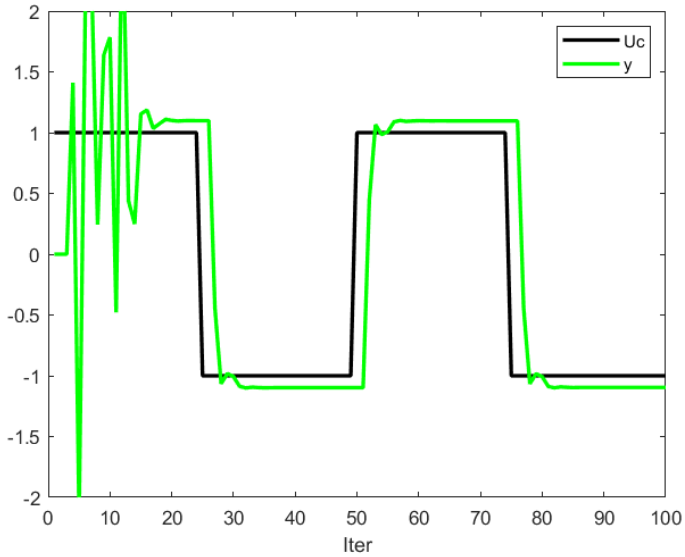

Fig 19. The output of the closed-loop system with the using direct method without zero cancellation

As it is clear from the diagram above, the output has not been able to follow the reference input well because in this case, the estimation of the parameters and the cancellation of the common polynomial are associated with errors and the controller parameters have an offset compared to their actual values. Also, the control effort in this case is shown in Figure 20.

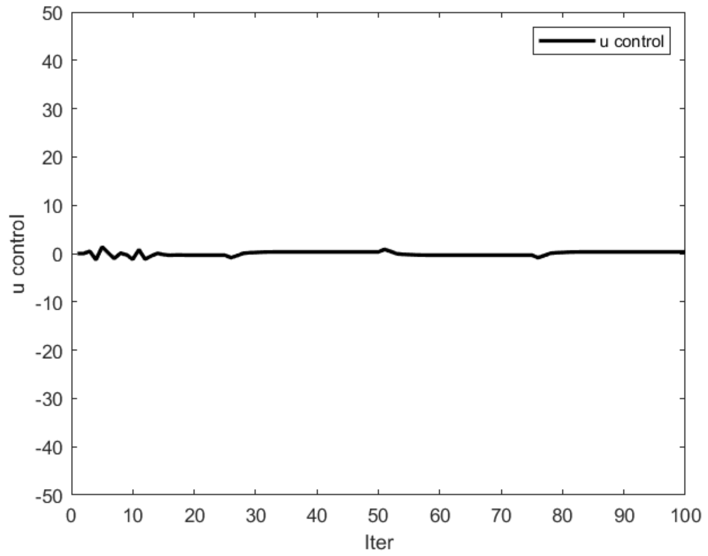

Fig 20. Control effort of the system using direct method without zero cancellation

As can be seen, the control effort has a small amount, but it could not produce a good output, which is due to the inappropriate estimation of the controller parameters. The estimation of the controller polynomial coefficients is given in Figure 21.

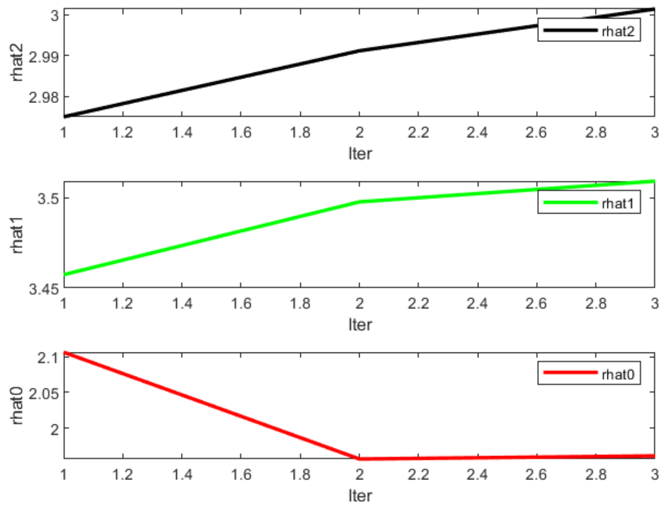

Fig 21. Estimation of R polynomial parameters using direct method without zero cancellation

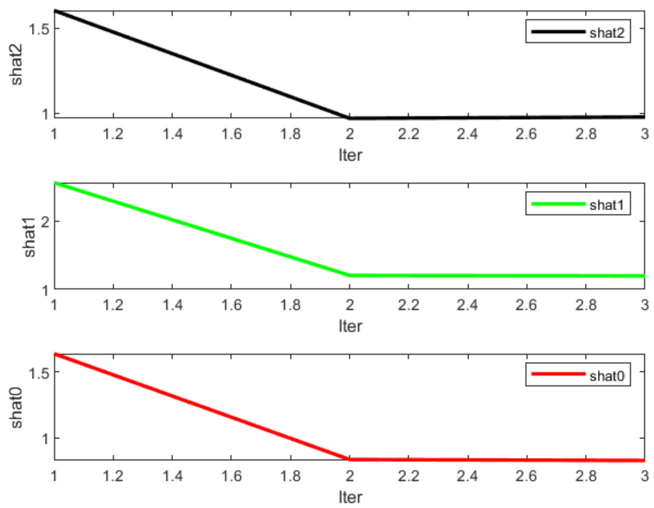

Fig 22.Estimation of polynomial parameters S using direct method without zero cancellation

The estimated coefficients of the controller have an offset compared to the values of the previous section, which is because the cancellation of the polynomial is with error and the controller parameters are not accurately obtained. But as it is clear, we have been able to converge the system using our code.

2.3 Indirect method with zero and pole cancellation

- Without color noise:

The results of estimating the numerator and denominator parameters of the transfer function are given in the figures 23 and 24.

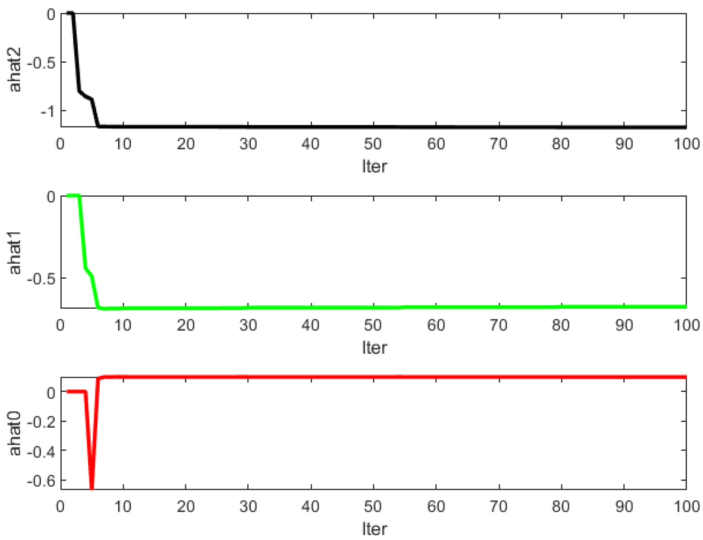

Fig 23. The results of estimating parameters of the denominator of the transfer function using indirect method with zero cancellation

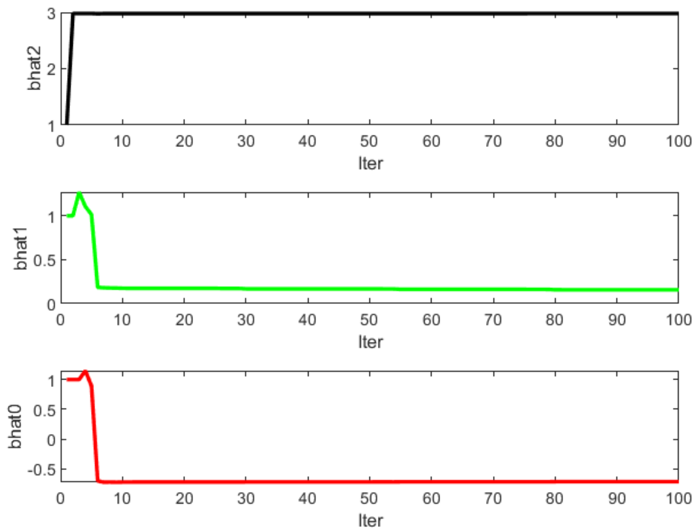

Fig 24. The results of estimating the parameters of the nominator of the transfer function using indirect method with zero cancellation

As it is clear from the results, the parameters are well estimated and converged to their original values. After estimating the parameters, the Diophantine equation is solved in each iteration and the coefficients of the polynomials related to the controllers are obtained. In the figure below, the output of the closed-loop system is given with respect to the reference input, which is a square wave.

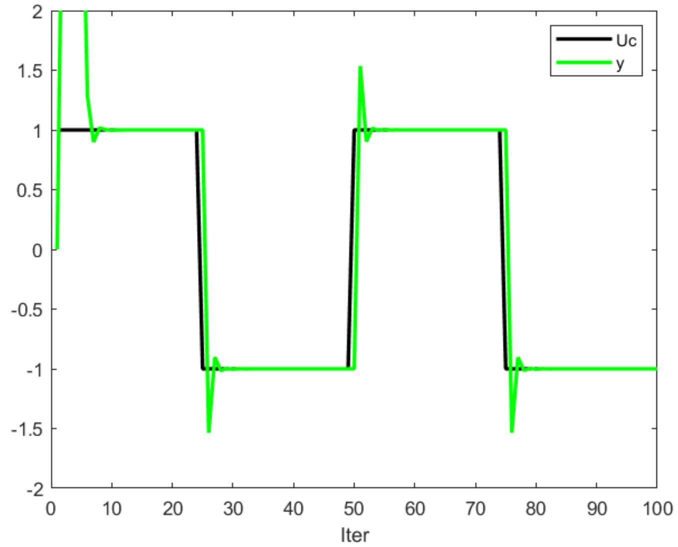

Fig 25. The output diagram of the closed loop system using indirect method with zero cancellation

After the parameters are well estimated, the output of the system has been able to follow the reference input well, and the system's overshoot and settling time are appropriate. Also, the control effort of the system is given in Figure 26.

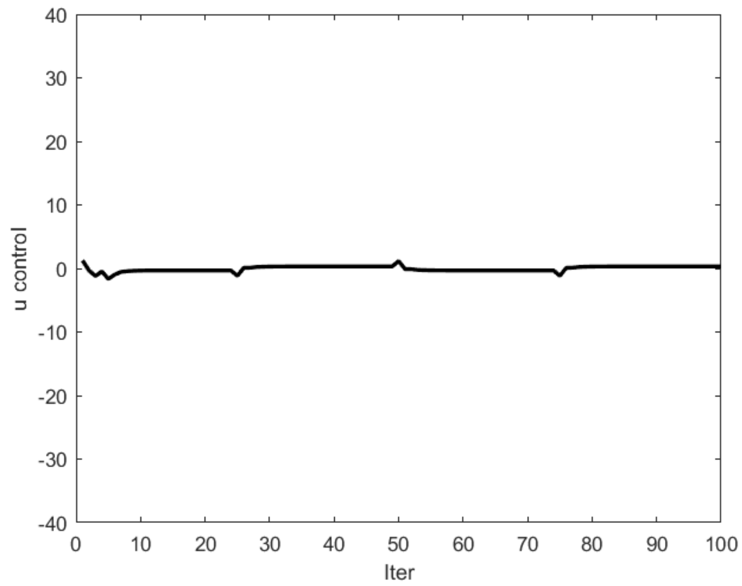

Fig 26. Control effort of the system using indirect method with zero cancellation

As can be seen from the Figure above, a limited control effort is required because in this case, we have zero cancellation. The convergence process of the controller parameters is also shown in the Figures 27, 28, and 29.

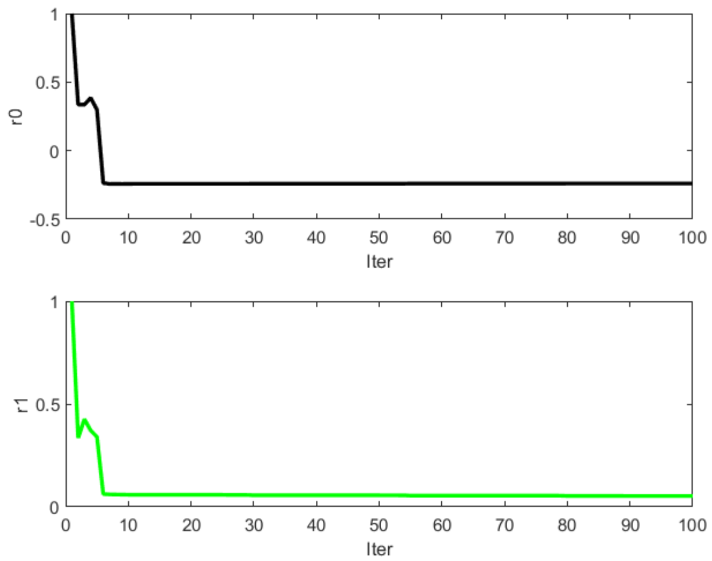

Fig 27. The process of convergence of R polynomial parameters using indirect method with zero cancellation

As is clear, in this case, the coefficients have converged to their actual values.

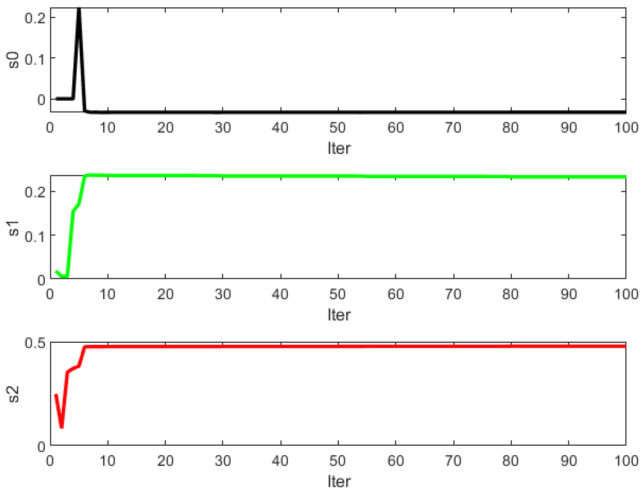

Fig 28. Convergence process of S polynomial parameters using indirect method with zero cancellation

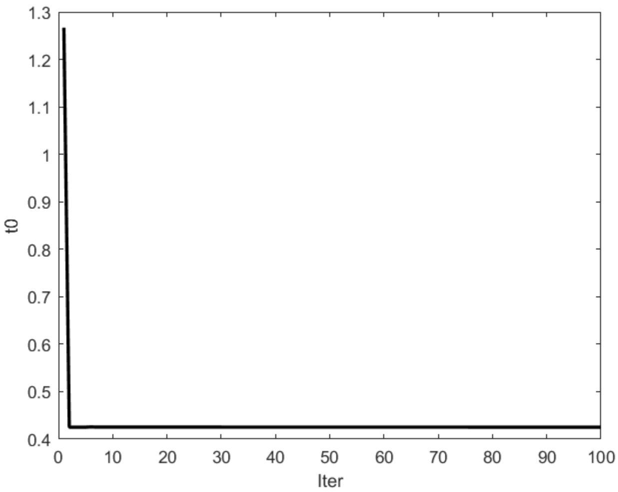

Fig 29. Convergence process of T polynomial parameters using indirect method with zero cancellation

As it is clear from the Figures above, the controller coefficients have properly converged to their real values.

- With color noise:

In this case, the estimated parameters of the transfer function are given in Figure 30.

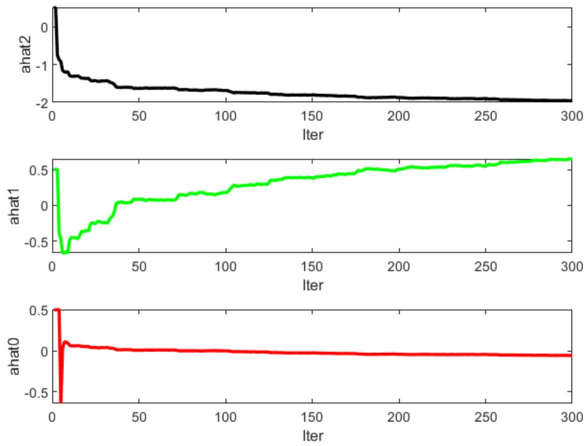

Fig 30. Denominator parameter estimation of the transfer function using indirect method with zero cancellation with color noise and with the ELS method

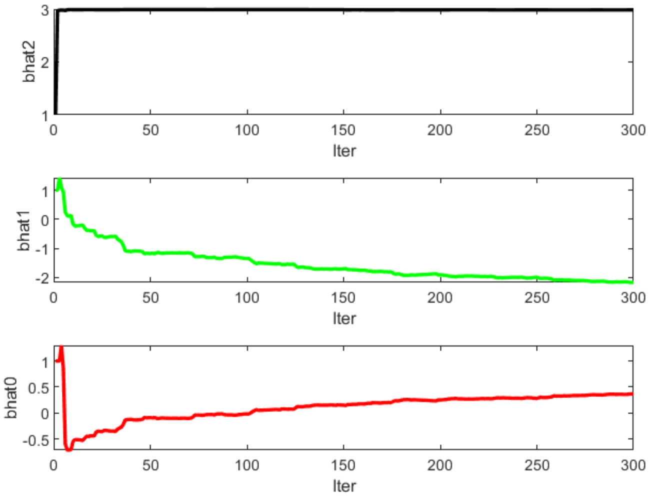

Fig 31. Estimation of parameters of the nominator of the transfer function using indirect method with zero cancellation and with color noise using the ELS method

As it is clear from the Figures above, the parameters of the transfer function have converged to their exact values in the presence of color noise and with the ELS method. Also, the Figure of polynomial estimation related to noise is as shown in Figure 32.

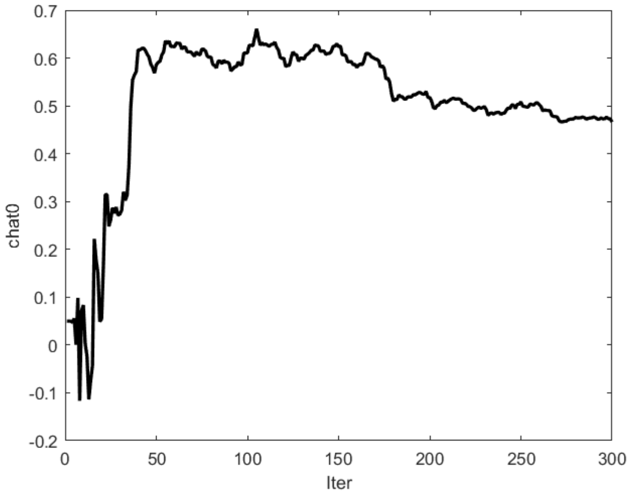

Fig 32. Color noise parameter estimation using indirect method with zero cancellation with color noise using the ELS method

The polynomial convergence of the noise is associated with a little jump, which requires a lot of time so that the ELS method can estimate this parameter well. The output plot of the closed loop system is as follows.

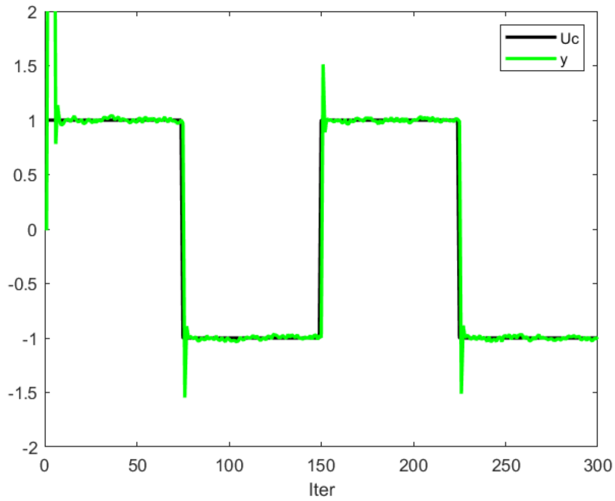

Fig 33. The output of the closed loop system using indirect method with zero cancellation with color noise.

As it is known, in the presence of color noise, the output of the system has been able to follow the reference input, but the output has small jumps due to the color noise. The control effort of the system so that the output can follow the reference input is as follows.

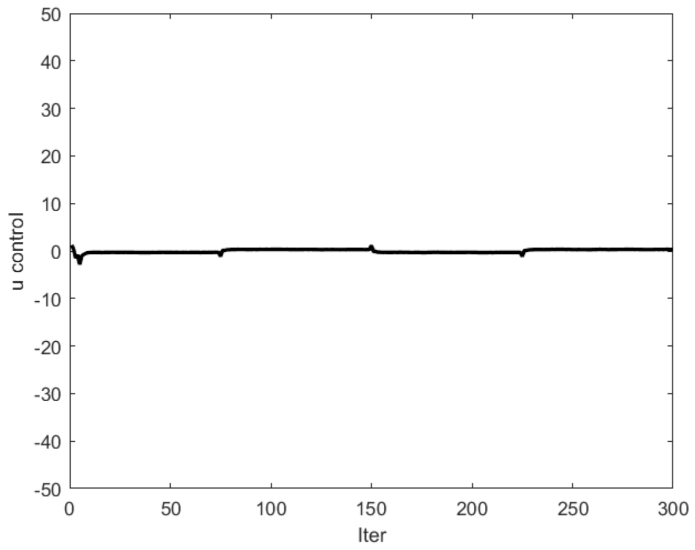

Fig 34. Control effort using indirect method with zero cancellation and color noise.

In this case, the control effort also has an appropriate and desirable value. The parameters of the controller are also obtained in each iteration after estimating the parameters of the transfer function and solving the Diophantine equation. The convergence process of these parameters is given in Figures 35, 36, and 37.

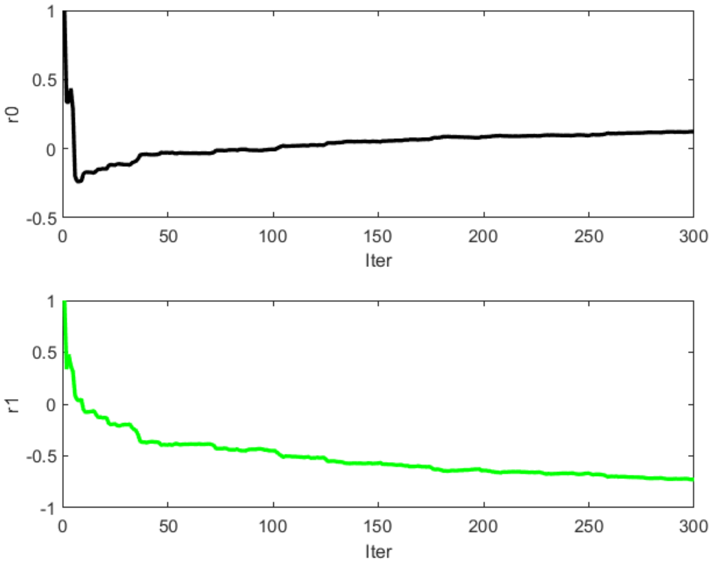

Fig 35. Convergence of R polynomial parameters with color noise and by indirect method and zero cancellation

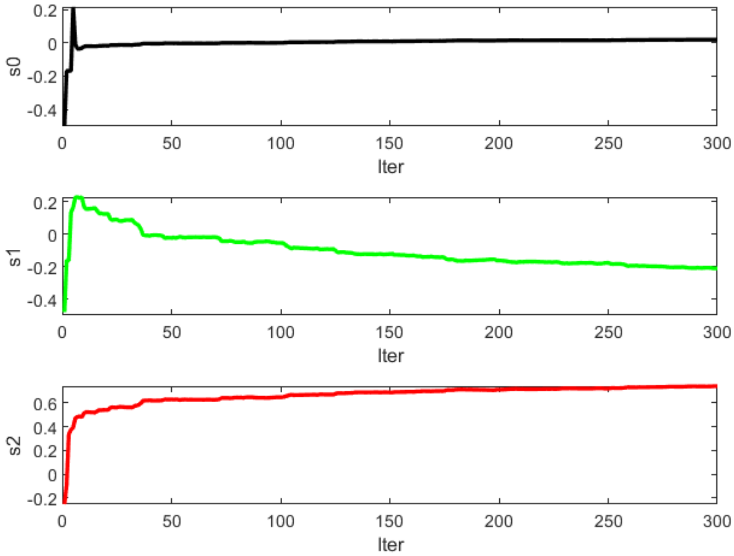

Fig 36. Convergence of S polynomial parameters with color noise using indirect method and zero cancellation

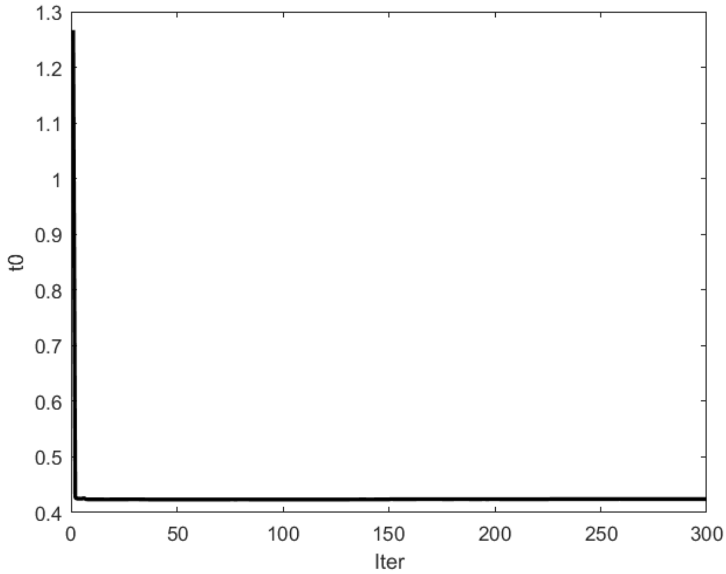

Fig 37. Convergence of polynomial parameters T with color noise using indirect method with zero cancellation

In this case, the controller parameters have also converged well to their true values, which are noise-free and obtained in the previous sections.

2.4 Direct method with zero and pole cancellation

- Without color noise:

In the direct method, the controller coefficients are obtained directly and by estimation. In this section, since we have zero cancellation, the polynomial B is a constant number and the estimation task will be a little easier. The characteristic polynomial of the closed loop is as follows.

$$A_m(q) = q^3 + 0.2483q^2 + 0.01847q - 2.148e-09$$

$$\deg A_m = \deg A = 3$$

$$\deg B_m = \deg B = 2$$

On the other hand, for the minimum degree controller, we must have:

$$\deg R = \deg S = \deg T = n - 1 = 2$$

$$\deg A_0 = \deg A - \deg B^+ - 1 = 3 - 2 - 1 = 0$$

Therefore, the polynomial A_0 is a constant number and must be equal to one to be monic:

$$A_0 = 1$$

$$d_0 = \deg A - \deg B = 1$$

$$A_m^*(q^{-1}) = 1 + 0.2483q^{-1} + 0.01847q^{-2} - 2.148e-09\,q^{-3}\,, A_0^* = 1$$

$$u_f(t) = \frac{1}{A_0^* A_m^*} u(t)$$

$$\Rightarrow u_f(t) = 0.2483u_f(t-1) + 0.01847u_f(t-2) + (-2.148e-09)u_f(t-3) + u(t)$$

$$\Rightarrow y_f(t) = 0.2483y_f(t-1) + 0.01847y_f(t-2) + (-2.148e-09)y_f(t-3) + y(t$$

Also, we have:

$$A_0^* A_m(1) = 1.267$$

$$T^* = 1.267 \quad and \quad T = 1.267q^2$$

Note that T* will be used for code calculations and control input in each iteration. Assuming a general form for R* and S*, we will have:

$$y(t) = R^* u_f(t-d_0) + S^* y_f(t-d_0)$$

$$= (r_2 + r_1 q^{-1} + r_0 q^{-2}) u_f(t-d_0) + (s_2 + s_1 q^{-1} + s_0 q^{-2}) y_f(t-d_0)$$

This equation is linear in parameters and it is possible to obtain the regression vector in each iteration with the help of the relations mentioned for the input and the filtered and unfiltered outputs, and finally, with the help of RLS, the controller coefficients can be estimated. After estimating the controller, it is necessary to calculate the new output of the controller. For this, we use the following equation.

$$R^* u(t) = T^* u_c(t) - S^* y(t)$$

$$(r_2 + r_1 q^{-1} + r_0 q^{-2})u(t) = 1.267u_c(t) - (s_2 + s_1 q^{-1} + s_0 q^{-2})y(t)$$

The advantage of this method is that it is no longer necessary to perform identification and calculations related to solving the Diophantine equation in each iteration, so, the calculations are reduced. The mentioned plant in the question, did not converge well in this method and the reason is probably because the unstable pole of the system is much closer to the imaginary axis than the other poles. The Figure 38 shows the output of the closed loop system for tracking the square wave reference signal.

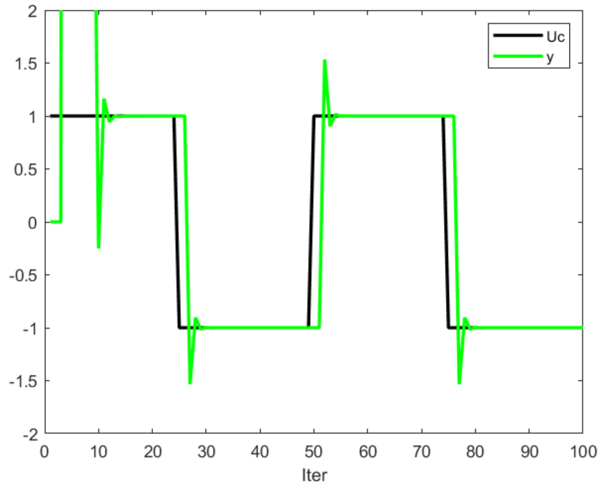

Fig 38. The output of the closed loop system using direct method with zero cancellation

The plot above shows that in this case, the output has been able to track the reference input with a delay because in this method, the parameters of the transfer function are not estimated and the parameters of the controller are estimated. Also, the output response has an offset, which is due to the fact that in this method, since we do not have the polynomials of the transfer function, i.e. B and A, therefore, the value of the parameter B_m is not selected in such a way as to make the steady state error zero. Also, there is no polynomial T that can do this. The process of the estimated controller parameters is as shown in Figure 39.

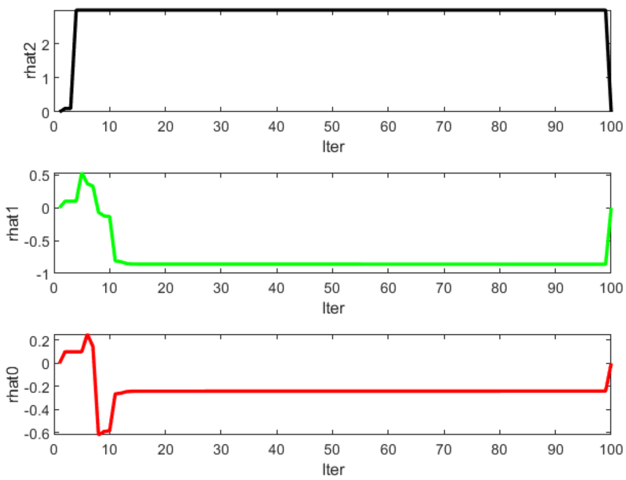

Fig 39. Estimation of R polynomial parameters using direct method with zero cancellation

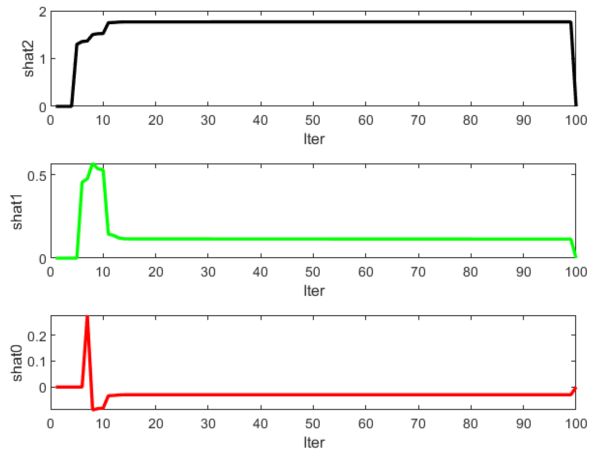

Fig 40. Estimation of polynomial parameters S using direct method with zero cancellation

As it is clear from the Figures above, the values of the controller polynomials have converged to correct values, but it should be noted that the above values are related to R* and S* and by dividing the values by rhat2, the values of the R and S polynomials will be achieved.

- With color noise:

In this case, since we have zero cancellation and the polynomial B on the right side of the corresponding equation is a constant number, we can easily estimate the controller polynomials. As it is with colored noise, the estimation of controller parameters has been done by ELS method. The output plot of the closed loop system is shown in Figure 41.

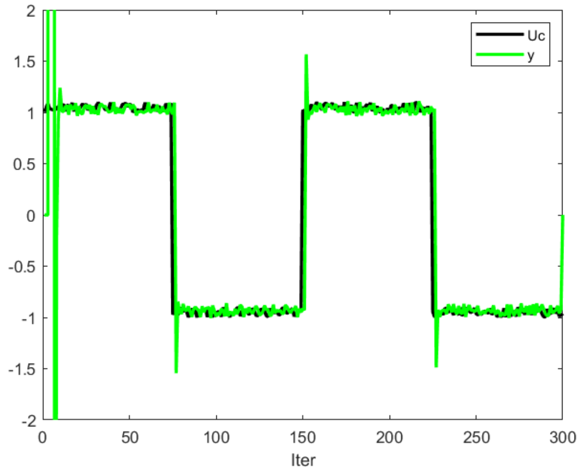

Fig 41. The output plot of the closed loop system using direct method with zero cancellation and with color noise

As can be seen, the output of the closed loop system has been able to follow the reference input and has desirable characteristics. The control effort of the system is shown in Figure 42.

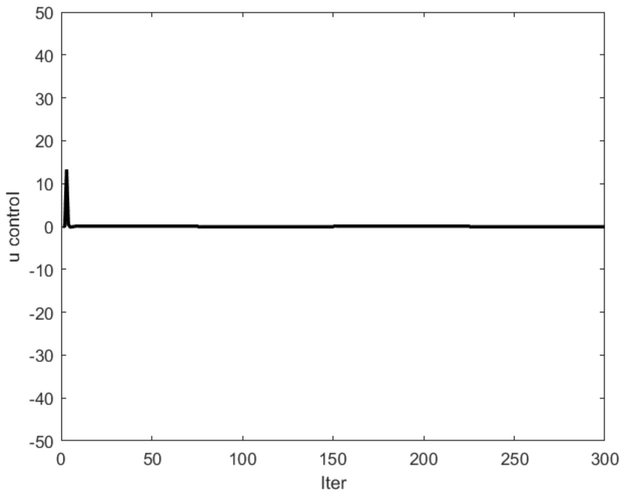

Fig 42. Control effort plot of the system using direct method with zero cancellation and color noise

The control effort in this case does not have proper values, which is due to the improper estimation of the controller parameters. The estimated parameters of control polynomials are shown in Figures 43 and 44.

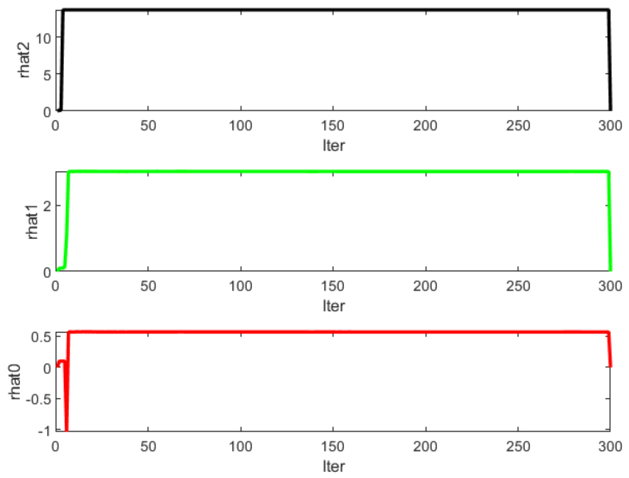

Fig 43. Estimated parameters of the polynomial R with colored noise using direct method with zero cancellation

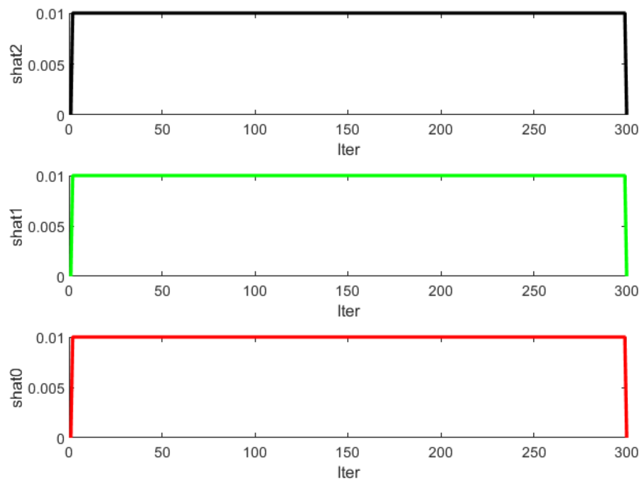

Fig 44. Estimated parameters of polynomial S with colored noise using direct method with zero cancellation

In this method, the parameters related to the R polynomial are well estimated, but the parameters related to the S polynomial have not converged to the correct values. This can be due to the low order of the reference input PE. The Figure 45 shows the process of estimating the noise polynomial parameter.

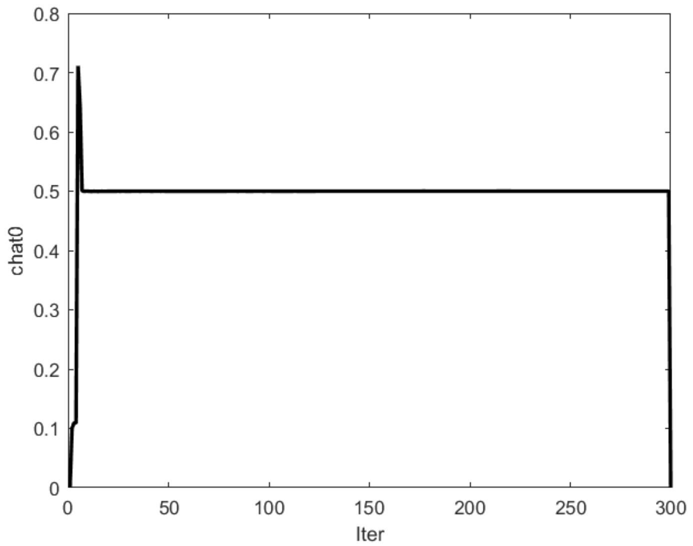

Fig 45. Estimation of the polynomial parameter of the noise using direct method with zero cancellation and with colored noise

As it is known, this parameter has converged to its exact value which is 0.5, which is due to the use of ELS method to estimate the parameters.

2.5 Over and under parameterization

In this case, we will analyze the effect of over and under parameterization for some of the above methods randomly.

2.5.1 Over parameterization

2.5.1.1 Indirect method without zero and pole cancellation

In this case, we will increase the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 8. We run the code for both of the cases of with and without color noise and discuss the results.

- With noise

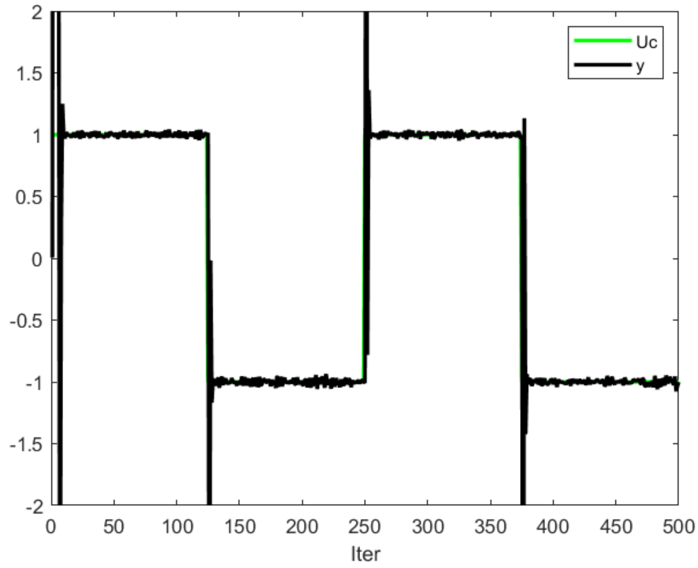

Fig 46. Output plot of closed loop system using indirect method without zero and pole cancellation with color noise with over parameterization

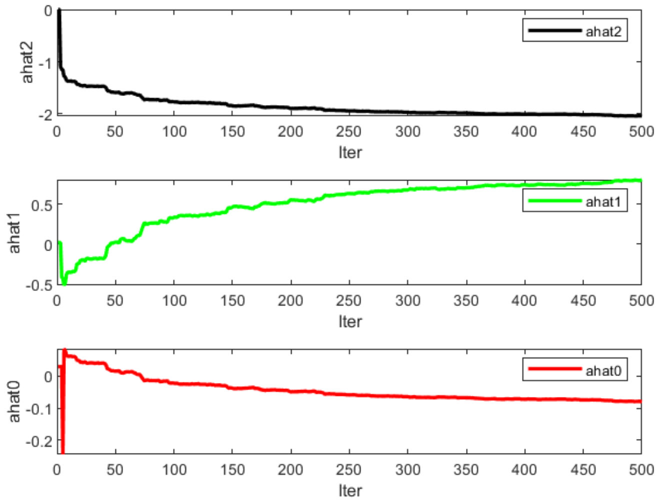

Fig 47. Estimation of denominators using indirect method without zero and pole cancellation with color noise with over parameterization

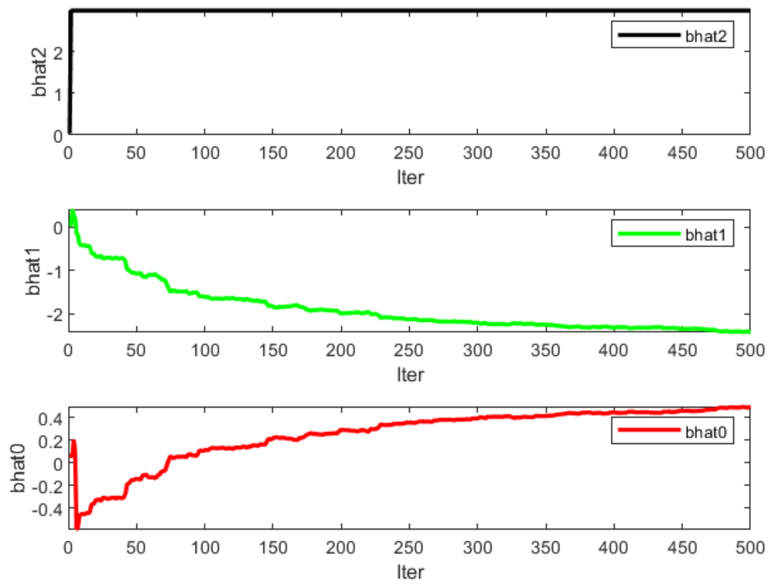

Fig 48. Estimation of nominators using indirect method without zero and pole cancellation with color noise with over parameterization

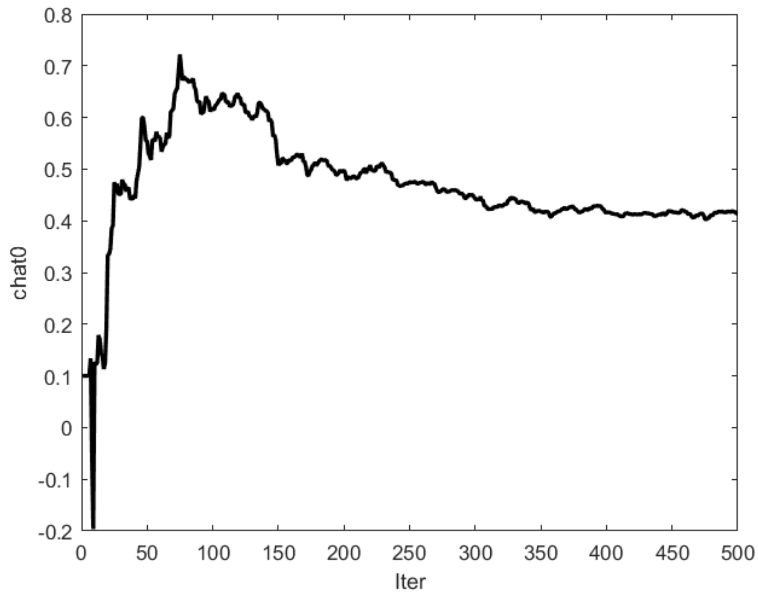

Fig 49. Estimation of color coefficient using indirect method without zero and pole cancellation with color noise with over parameterization

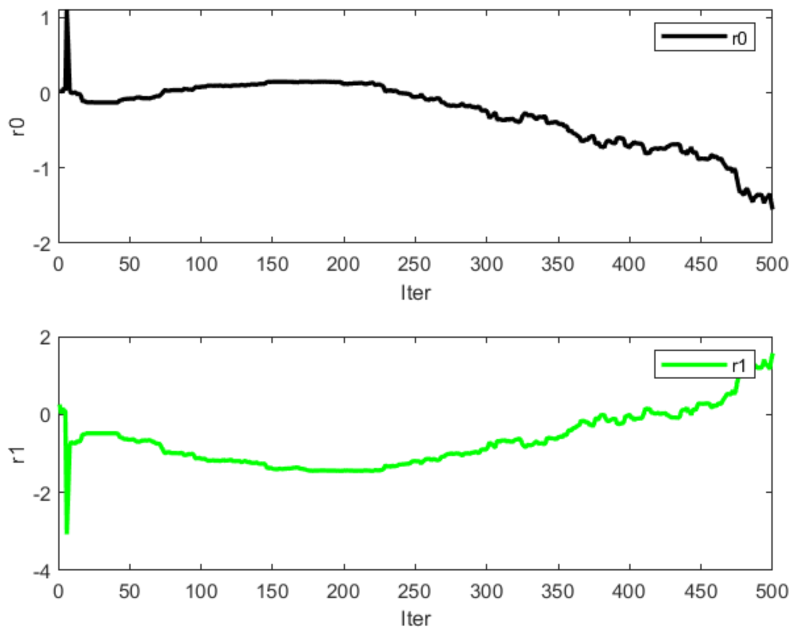

Fig 50. Estimation of polynomial R using indirect method without zero and pole cancellation with color noise with over parameterization

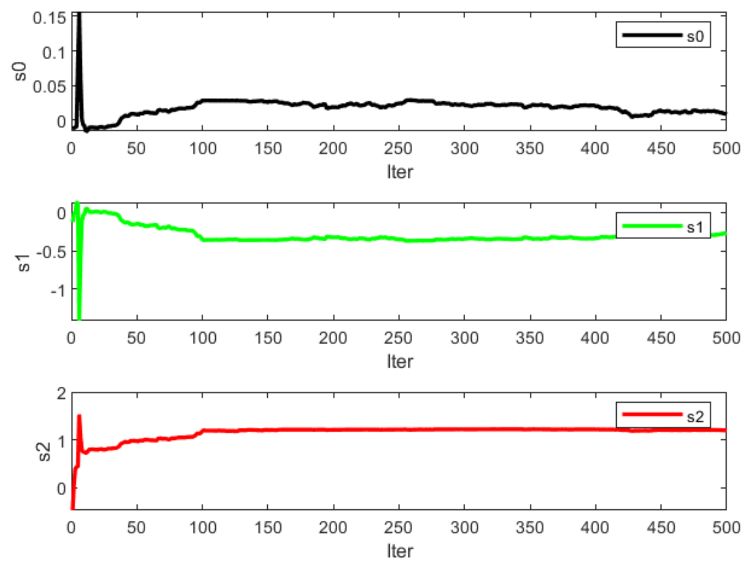

Fig 51. Estimation of polynomial S using indirect method without zero and pole cancellation with color noise with over parameterization

We can see from Figures 46 to 51 that almost all the parameters are converged properly but polynomial R and color coefficient are converged with delay and many disturbances.

2.5.1.3 Indirect method with zero and pole cancellation

In this case, we will increase the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 8. We run the code for both of the cases of with and without color noise and discuss the results.

- With noise

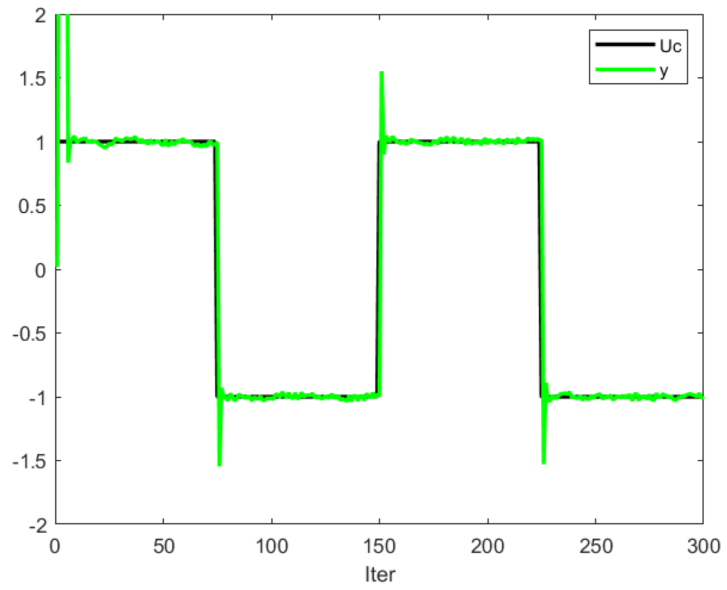

Fig 52. Output plot of closed loop system using indirect method with zero and pole cancellation with color noise with over parameterization

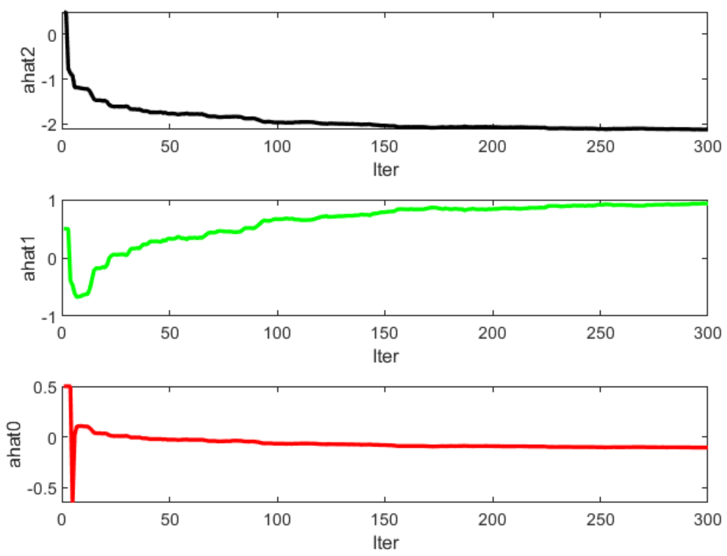

Fig 53. Estimation of denominators using indirect method with zero and pole cancellation with color noise with over parameterization

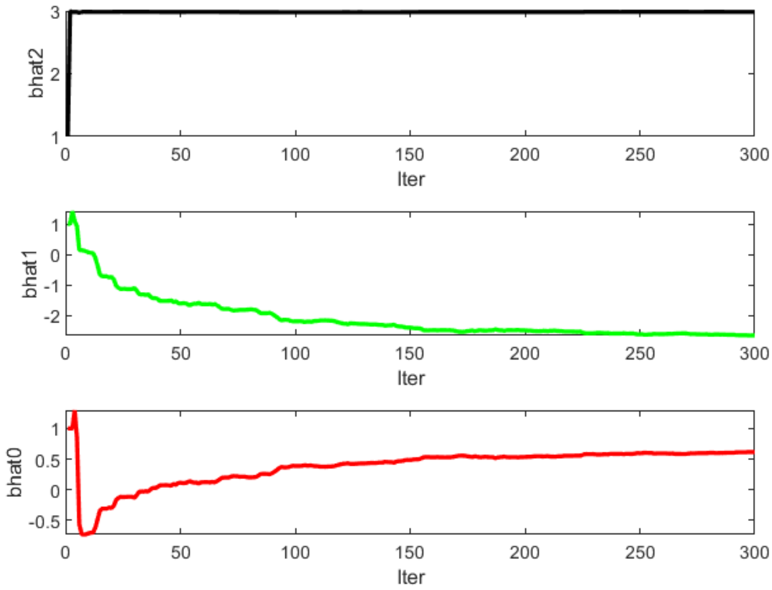

Fig 54. Estimation of nominators using indirect method with zero and pole cancellation with color noise with over parameterization

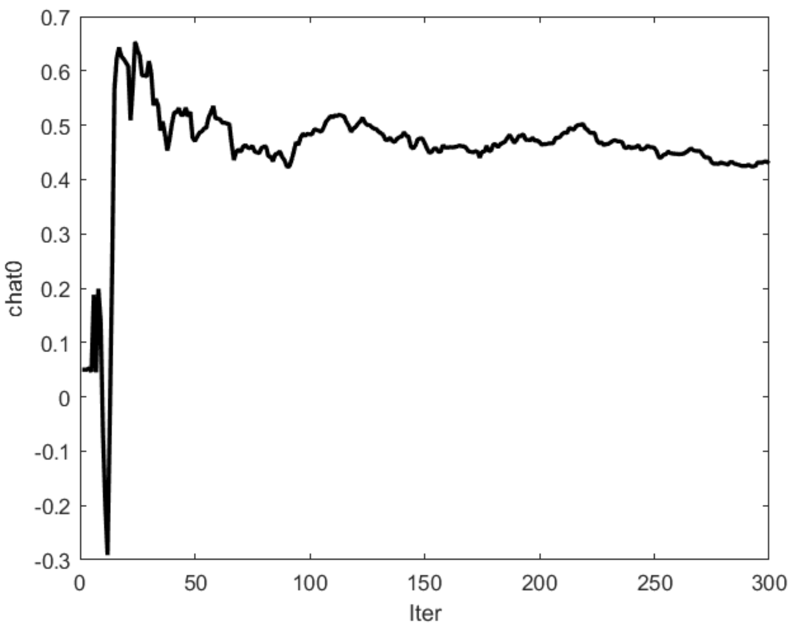

Fig 55. Estimation of color coefficient using indirect method with zero and pole cancellation with color noise with over parameterization

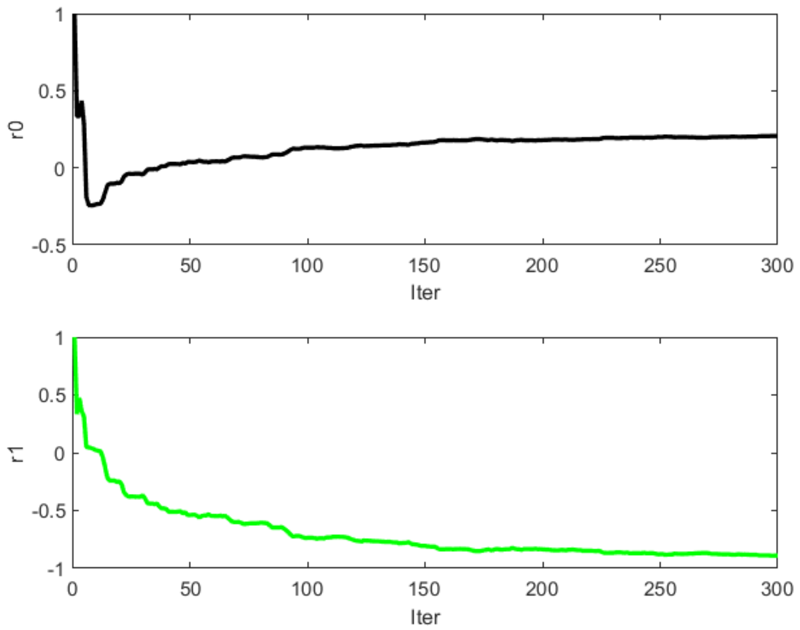

Fig 56. Estimation of polynomial R using indirect method with zero and pole cancellation with color noise with over parameterization

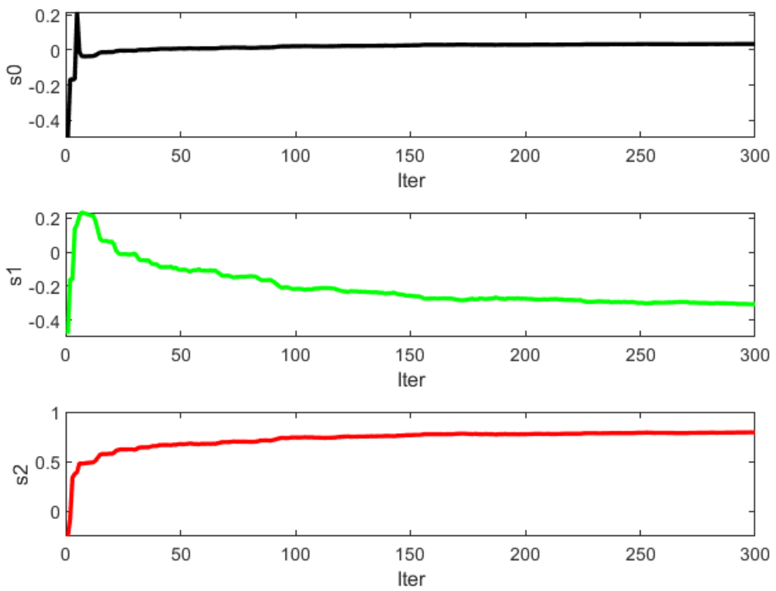

Fig 57. Estimation of polynomial S using indirect method with zero and pole cancellation with color noise with over parameterization

2.5.1.4 Direct method with zero and pole cancellation

In this case, we will increase the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 8. We run the code for both of the cases of with and without color noise and discuss the results.

- With noise

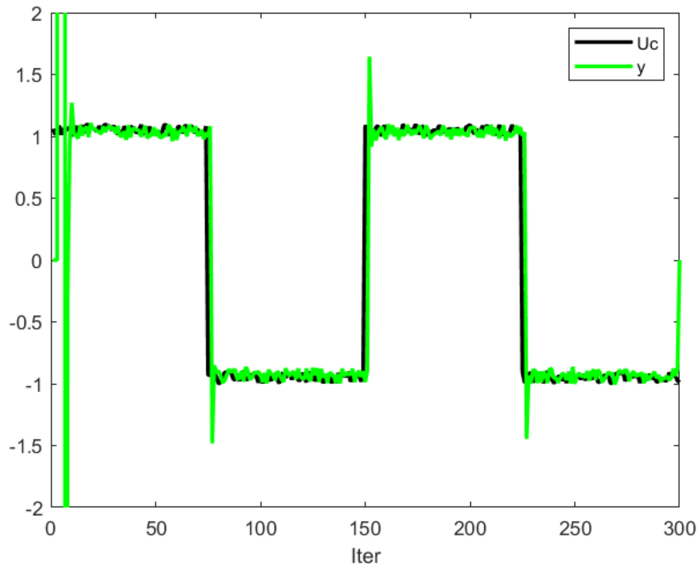

Fig 58. Output of closed loop system using direct method with zero cancellation with color noise and pver parameterization

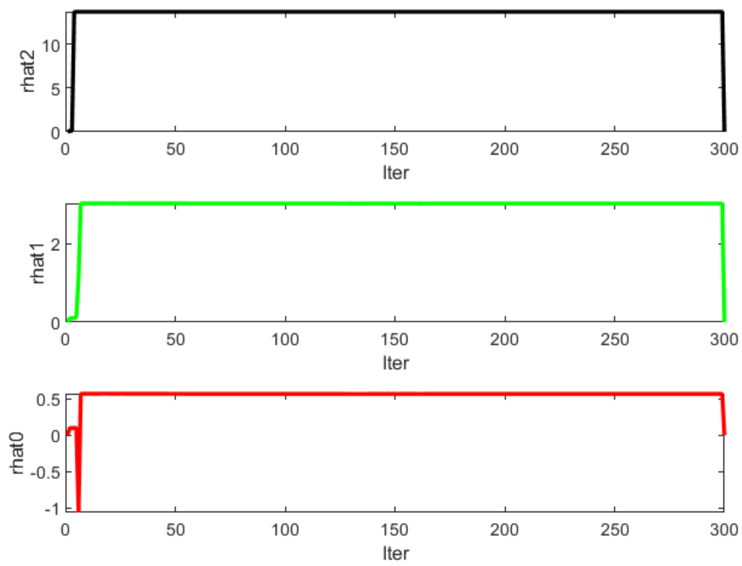

Fig 59. Estimation of polynomial R using direct method with zero cancellation with color noise and over parameterization

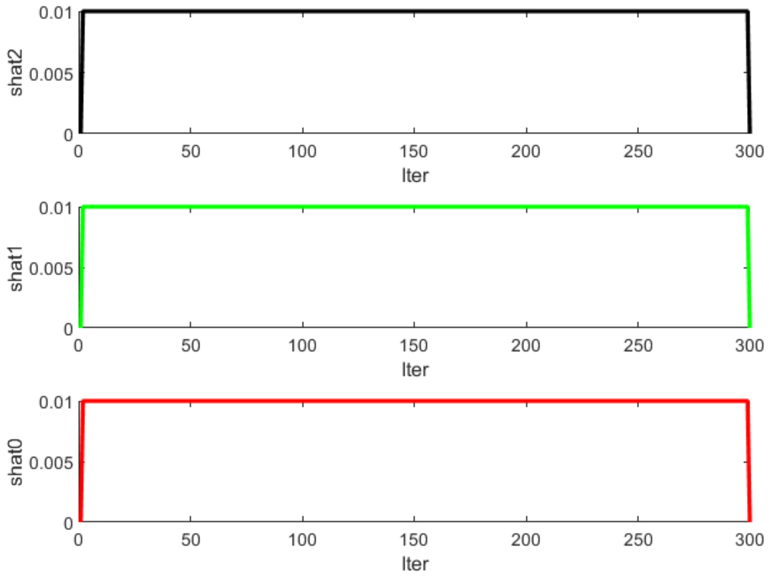

Fig 60. Estimation of polynomial S using direct method with zero cancellation with color noise and over parameterization

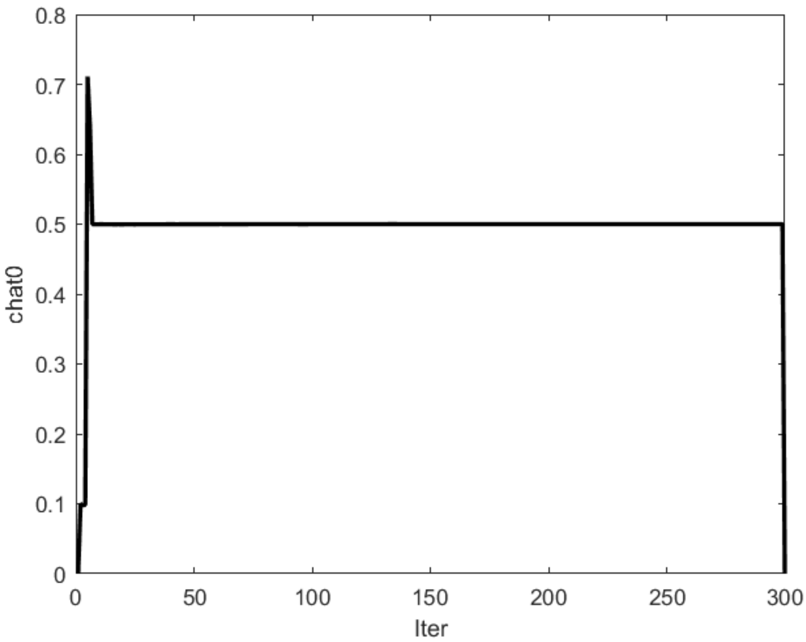

Fig 61. Estimation of color coefficient using direct method with zero cancellation with color noise and over parameterization

2.5.2 Under parameterization

2.5.2.1 Indirect method without zero and pole cancellation

In this case, we will decrease the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 4. We run the code for both of the cases of with and without color noise and discuss the results.

- With noise

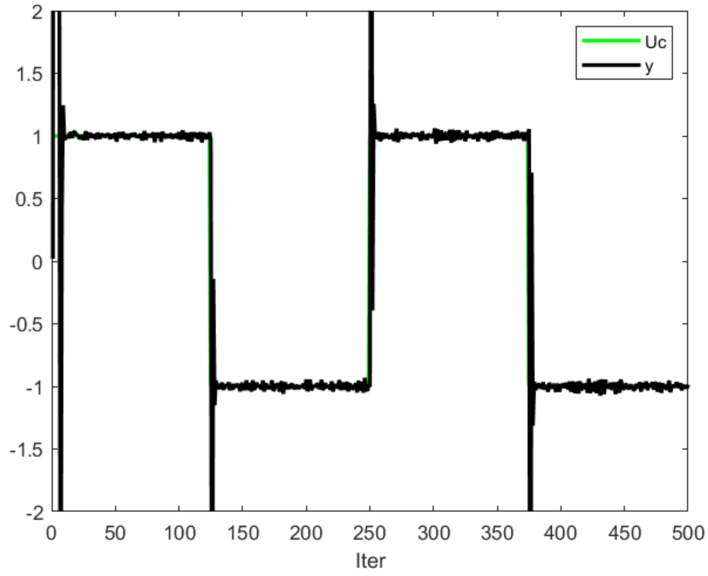

Fig 62. Output of closed loop system using indirect method without zero cancellation with color noise and with under parameterization

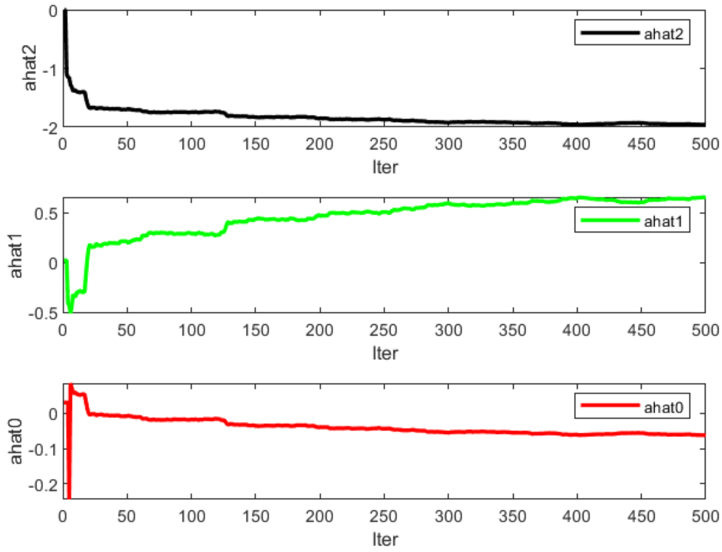

Fig 63. Estimation of denominators using indirect method without zero cancellation with color noise and with under parameterization

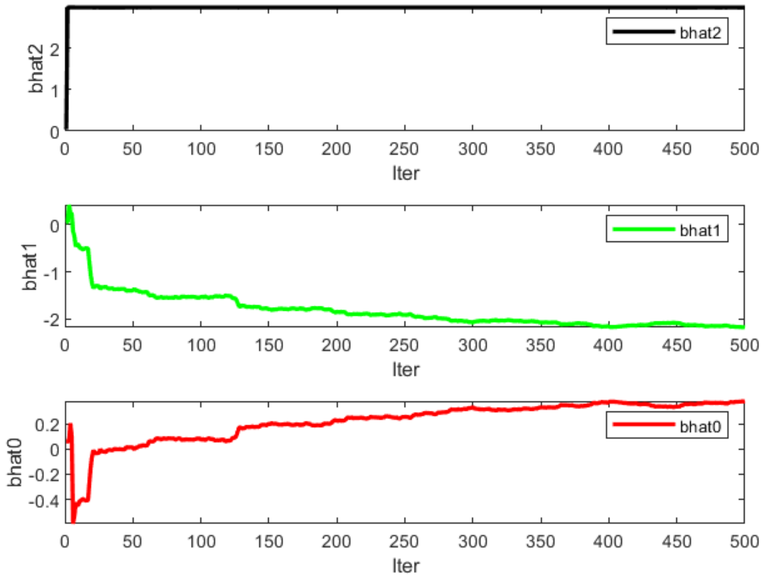

Fig 64. Estimation of nominators using indirect method without zero cancellation with color noise and with under parameterization

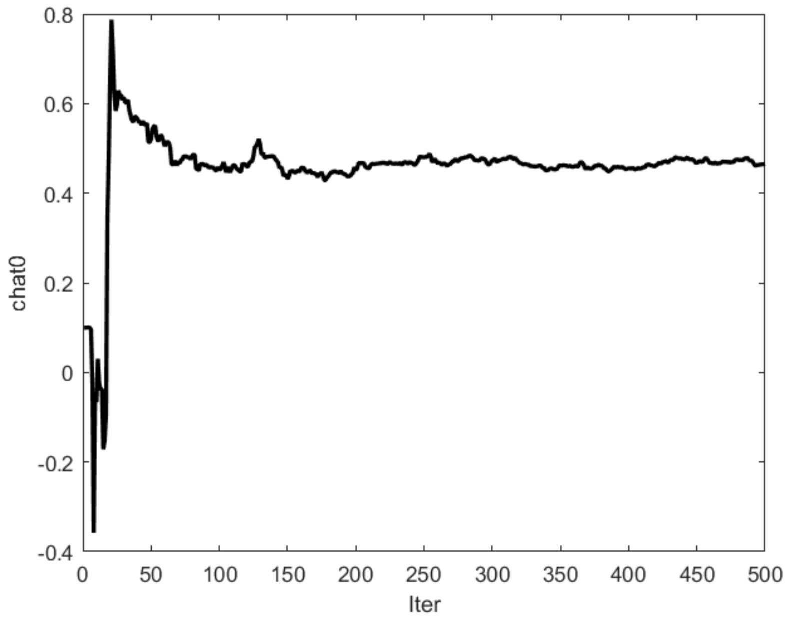

Fig 65. Estimation of color coefficient using indirect method without zero cancellation with color noise and with under parameterization

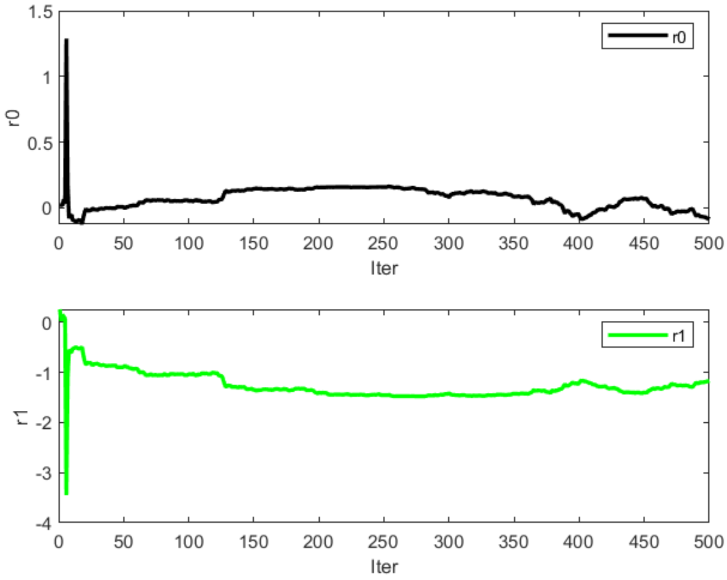

Fig 66. Estimation of polynomial R using indirect method without zero cancellation with color noise and with under parameterization

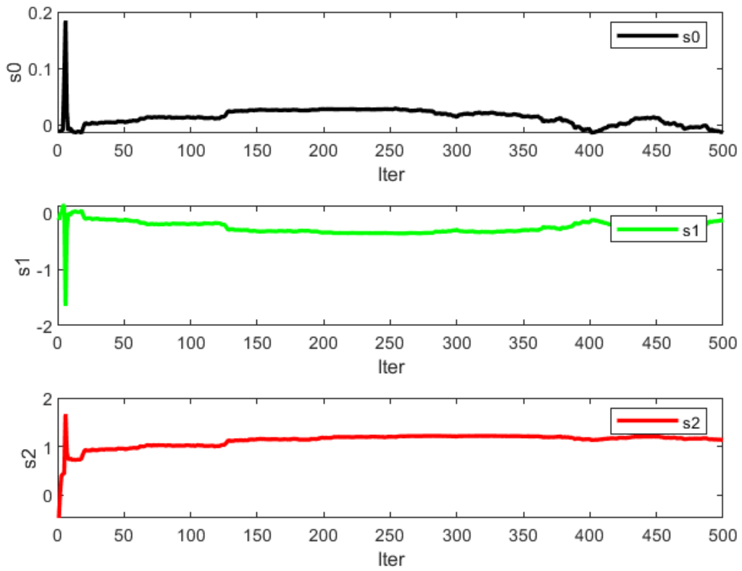

Fig 67. Estimation of polynomial S using indirect method without zero cancellation with color noise and with under parameterization

2.5.2.2 Direct method without zero and pole cancellation

In this case, we will decrease the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 4. We run the code for both of the cases of with and without color noise and discuss the results.

- Without noise

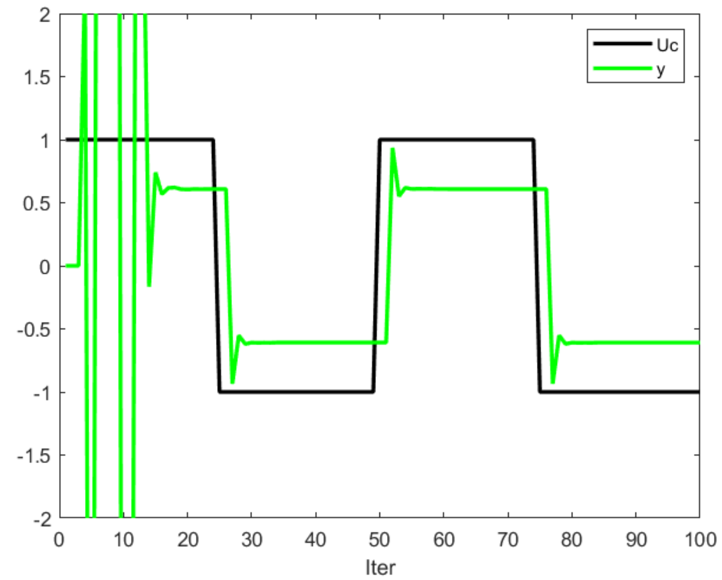

Fig 68. Output of closed loop system using direct method without zero and pole cancellation without noise with under parameterization

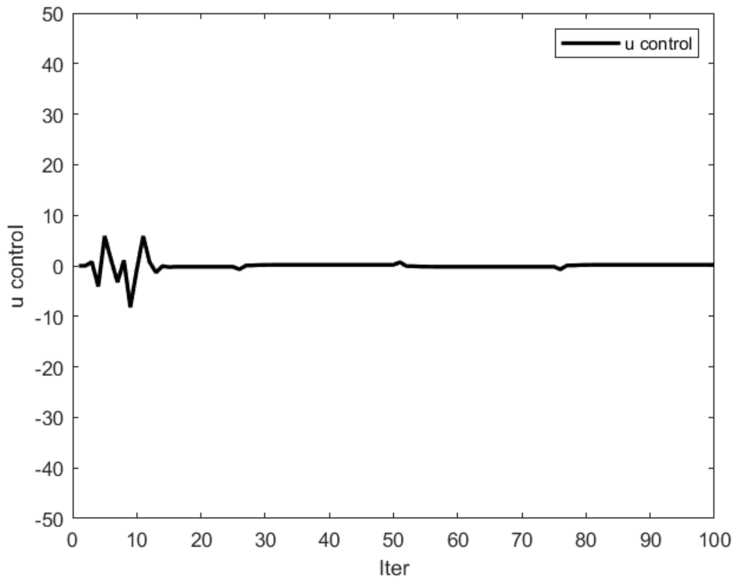

Fig 69. System control effort using direct method without zero and pole cancellation without noise with under parameterization

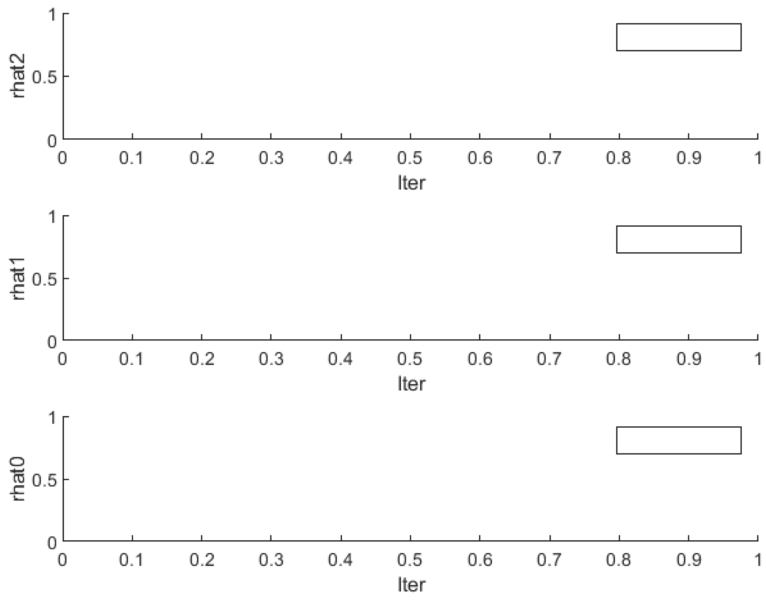

Fig 70. Estimation of R polynomial using direct method without zero and pole cancellation without noise with under parameterization

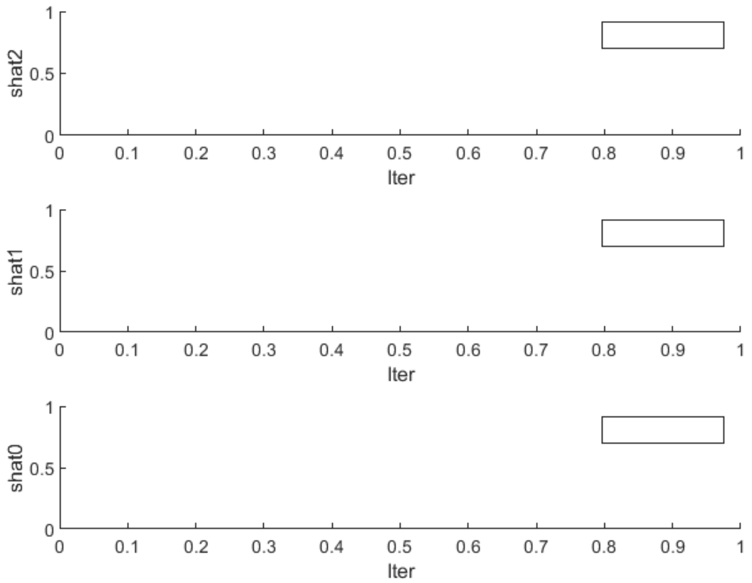

Fig 71. Estimation of S polynomial using direct method without zero and pole cancellation without noise with under parameterization

2.5.2.3 Indirect method with zero and pole cancellation

In this case, we will decrease the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 4. We run the code for both of the cases of with and without color noise and discuss the results.

- With noise

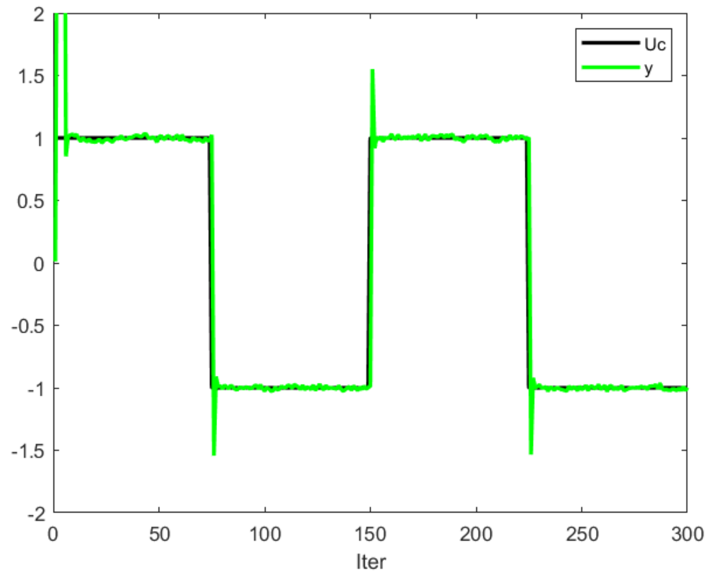

Fig 72. Output of closed-loop system using indirect method with zero and pole cancellation with color noise with under parameterization

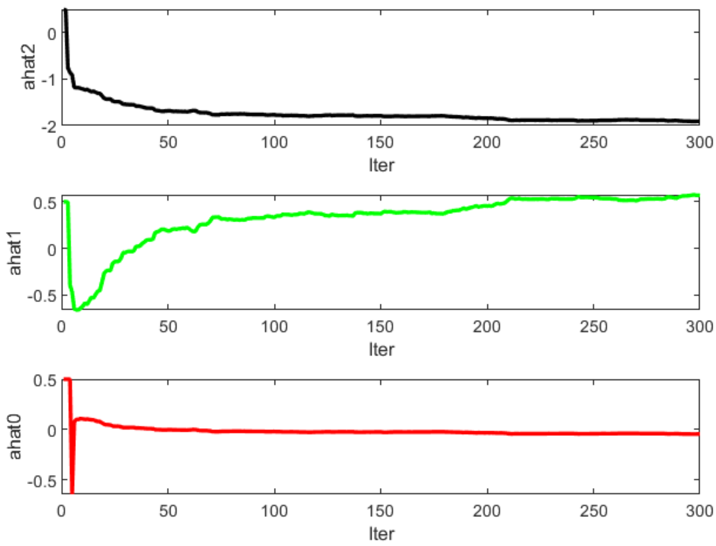

Fig 73. Estimation of denominators using indirect method with zero and pole cancellation with color noise with under parameterization

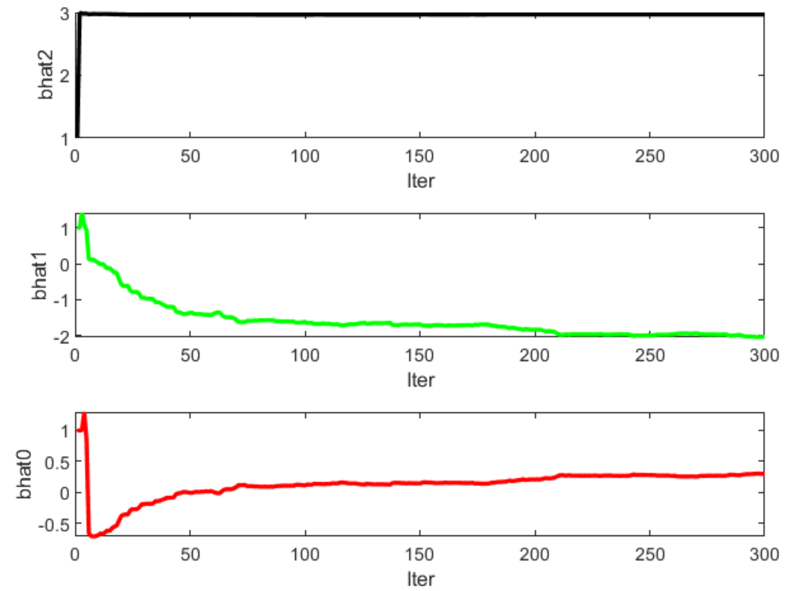

Fig 74. Estimation of nominators using indirect method with zero and pole cancellation with color noise with under parameterization

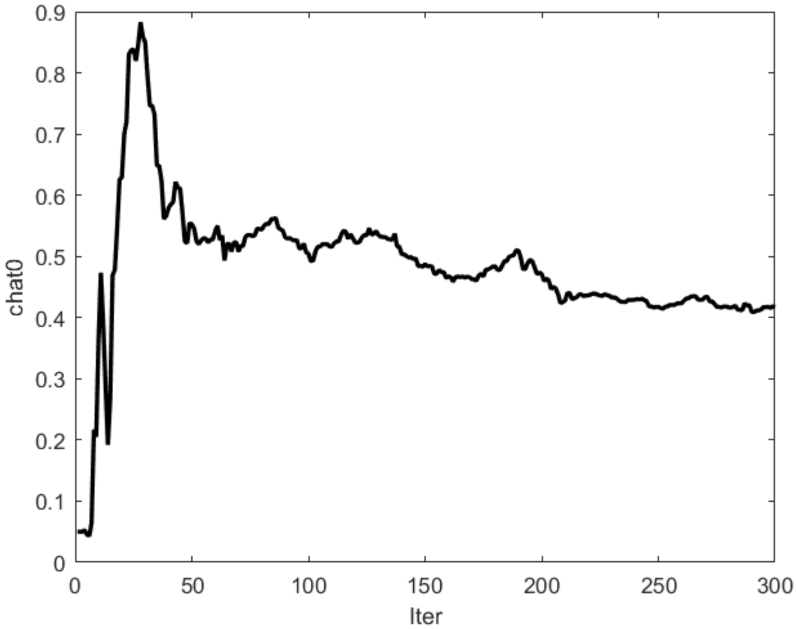

Fig 75. Estimation of color coefficient using indirect method with zero and pole cancellation with color noise with under parameterization

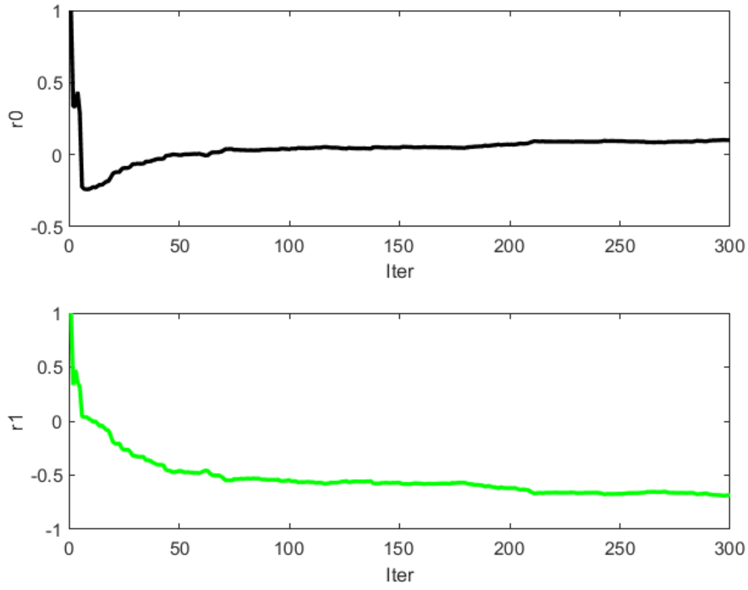

Fig 76. Estimation of R polynomial using indirect method with zero and pole cancellation with color noise with under parameterization

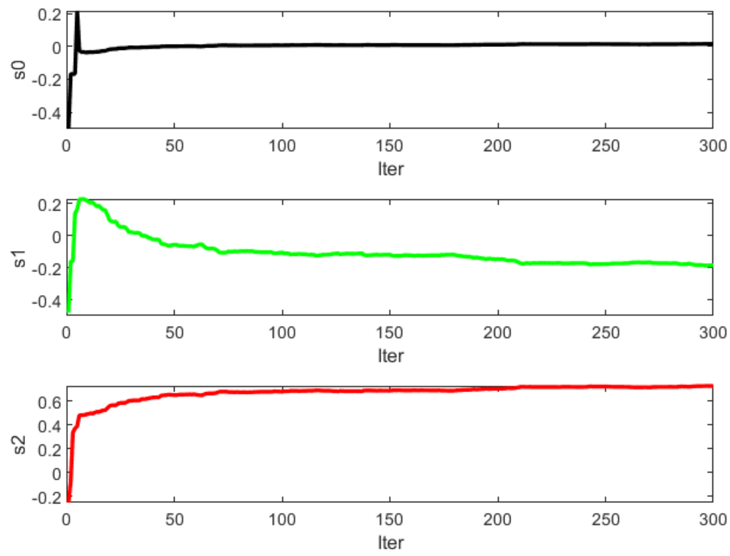

Fig 77. Estimation of S polynomial using indirect method with zero and pole cancellation with color noise with under parameterization

2.5.2.4 Direct method with zero and pole cancellation

In this case, we will decrease the nominator and denominator parameters, so, the parameters of transfer function should change from 6 to 4. We run the code for both of the cases of with and without color noise and discuss the results.

- Without noise

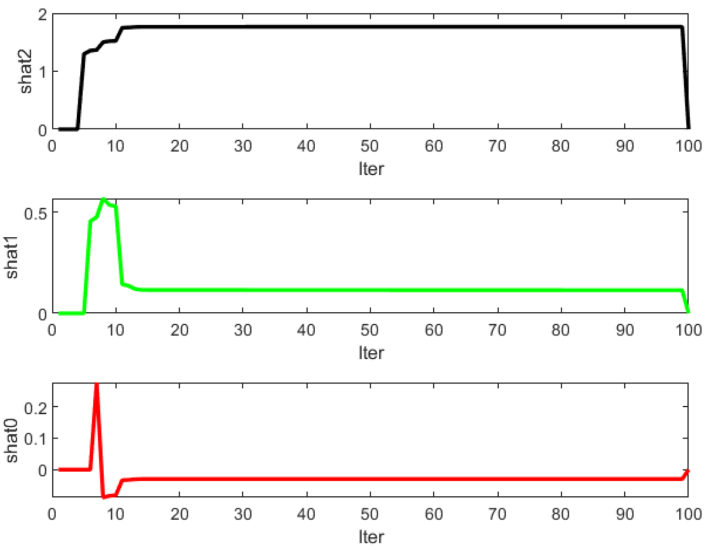

Fig 78. Estimation of S polynomial using direct method with zero and pole cancellation without color noise and with under parameterization

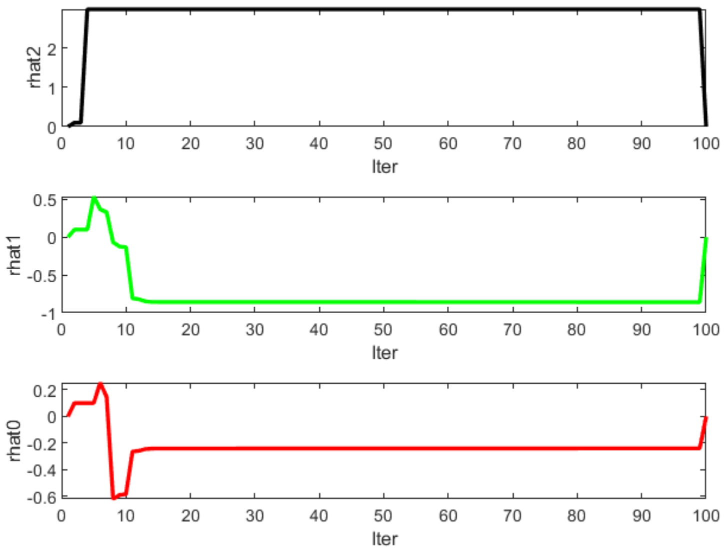

Fig 79. Estimation of R polynomial using direct method with zero and pole cancellation without color noise and with under parameterization

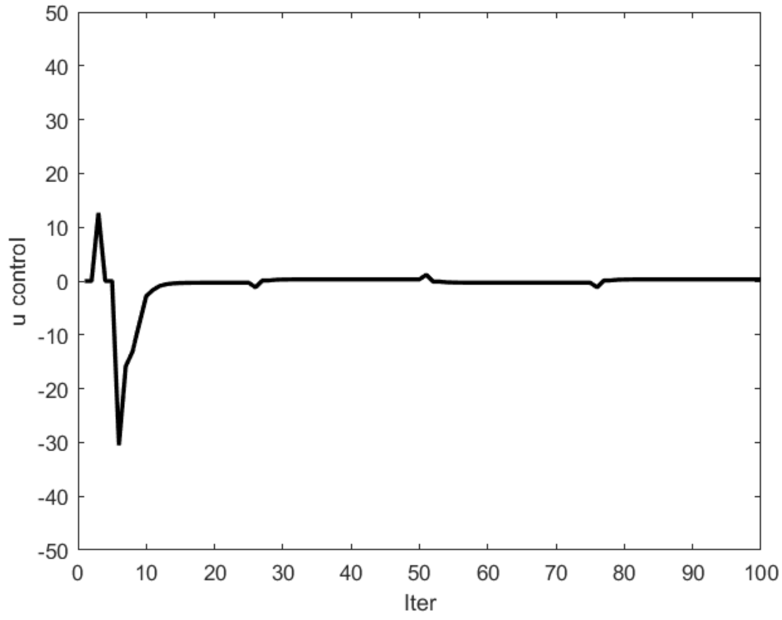

Fig 80. Control effort of system using direct method with zero and pole cancellation without color noise and with under parameterization

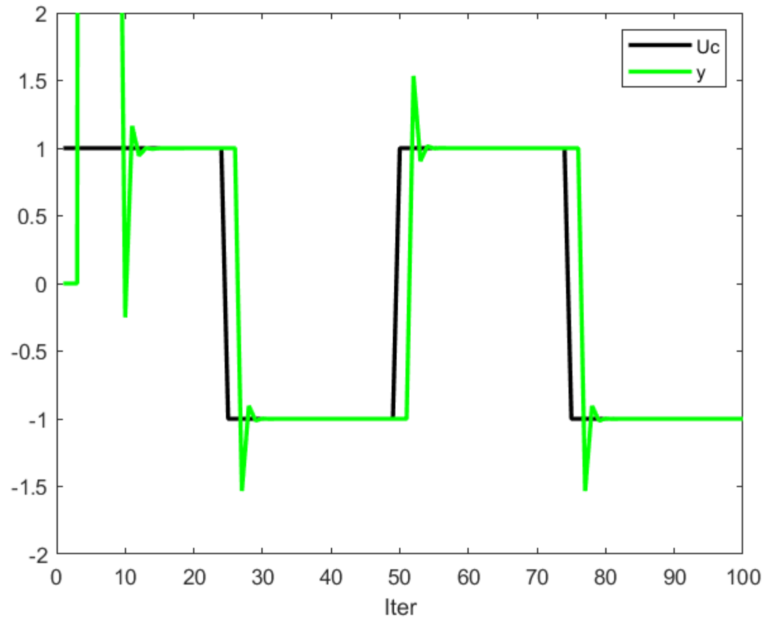

Fig 81. Closed-loop output of system using direct method with zero and pole cancellation without color noise and with under parameterization

We can see from the result that almost in all cases the parameters are estimated properly except direct method without zero and pole cancellation and without color noise which in that, the parameters couldn't estimate at all. Also, in the case of direct method with zero and pole cancellation and without color noise we didn't have an accurate estimation for output of closed loop control system.

2.6 Performance of controllers with step disturbance

In this method, at time 40, a step disturbance is exerted into the system. Also, the design of the controller is indirect and with zero cancellation, that is, the parameters of the transfer function are estimated first, and then the polynomials of the controller are obtained with the help of the Diophantine equation. The following Figure shows the output of the closed-loop transfer function.

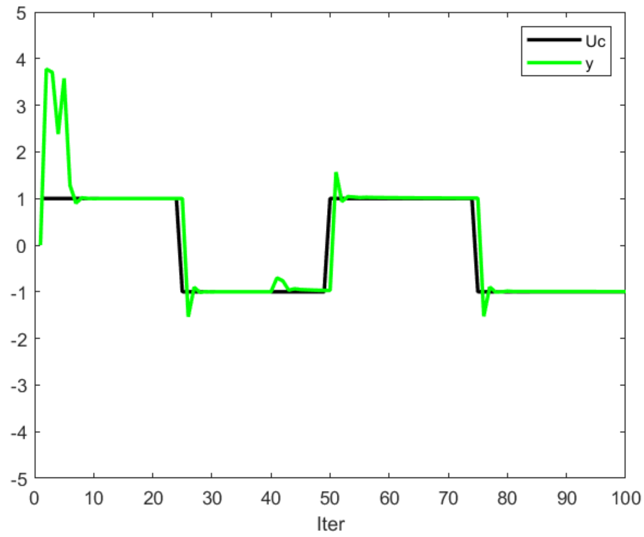

Fig 82. The output of the closed loop system with step disturbance at time 40 using indirect method and with zero cancellation

As it is known, the presence of disturbance has a bad effect on the system and has caused the system output to be completely distorted. This is due to the fact that in the designed controller, there is no integration that can cancel the disturbance effect. The control effort in the presence of the step disturbance is given in Figure 83.

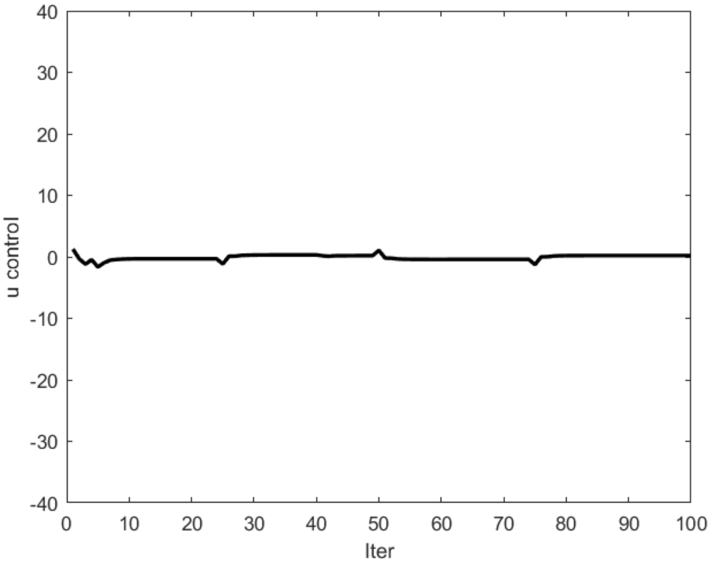

Fig 83. Diagram of control effort in the system with step disturbance

Now, to solve this problem, by modifying the control algorithm and identification mechanism and add an integral control to this system and simulate the system again. In this case, the output of the closed loop system is as follows.

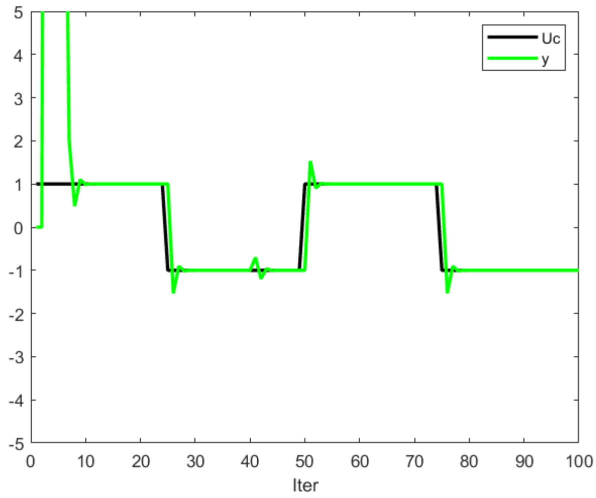

Fig 84. Output diagram of closed loop system with integral control and with step disturbance

As it is clear from the Figure 84, by modifying the control system and adding an integral control, the disturbance effect has been reduced and the output has been able to follow the reference input.

The control effort in the presence of integral control is also as follows.

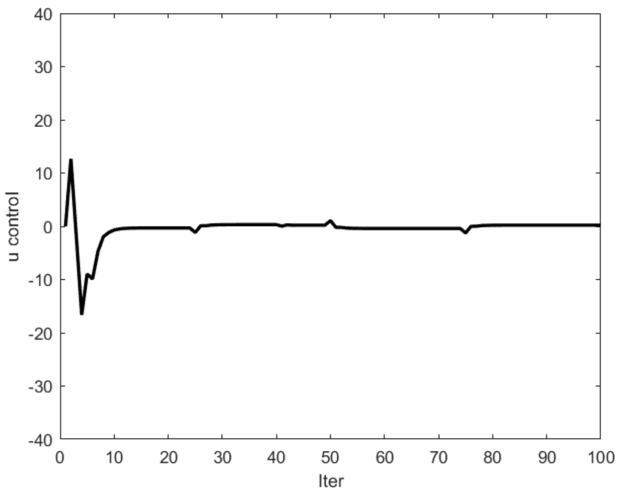

Fig 85. Control effort of the system with integral control with step disturbance

As it is clear from the plot, the control effort of the system in this case is improved by integral control, but as the controller used is integral, it will lead to saturation of the controller.

2.7 Simulation in a long time

In this case, we design a controller using indirect method without zero and pole cancellation for a minimum-phase System. We simulate the system in long time to see how the parameters will be estimated.

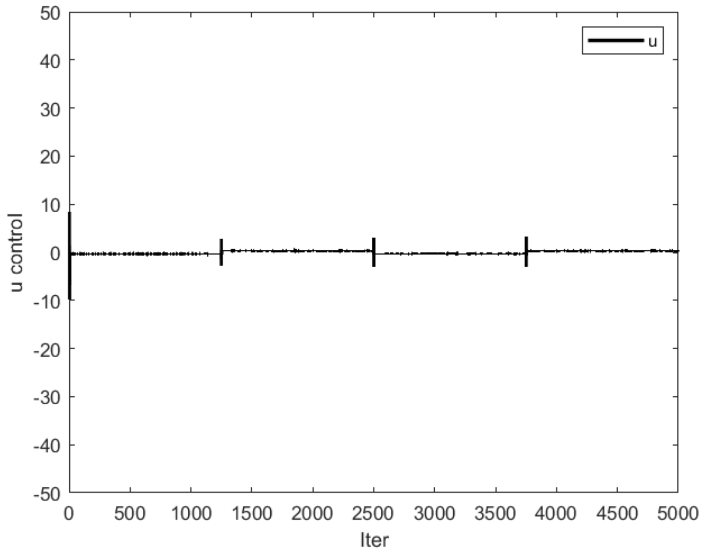

Fig 86. Control effort in long time simulation

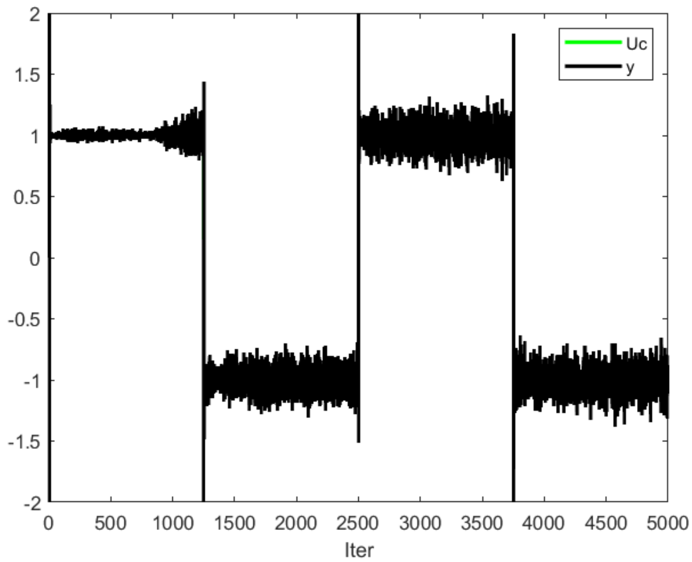

Fig 87. Output of closed-loop system in long time simulation

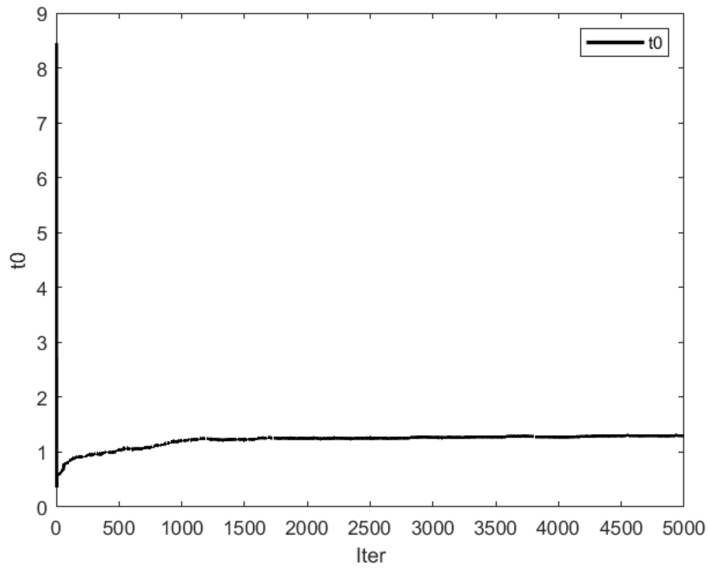

Fig 88. Estimation of T polynomial in long time simulation

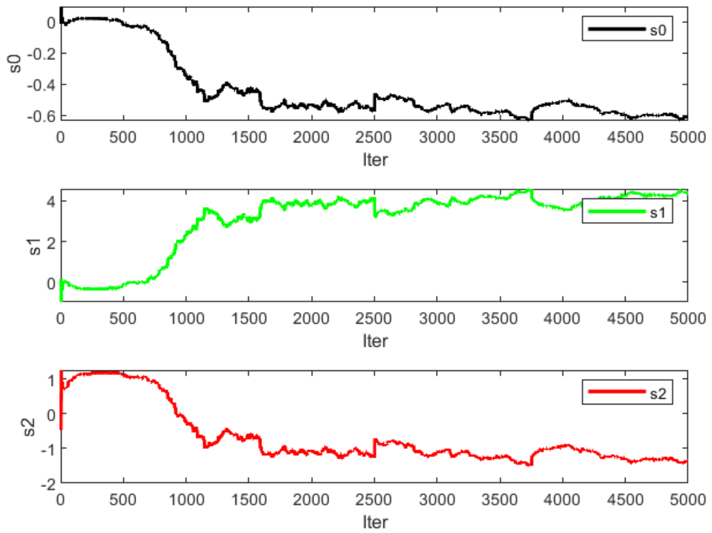

Fig 89. Estimation of S polynomial in long time simulation

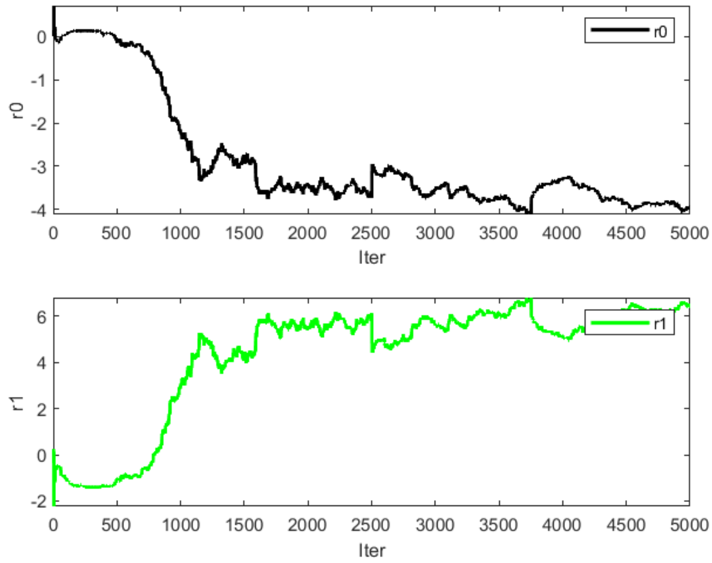

Fig 90. Estimation of R polynomial in long time simulation

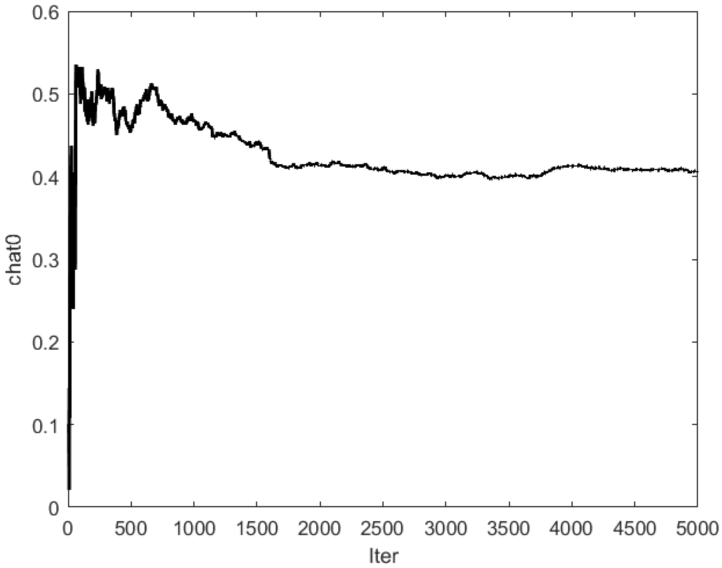

Fig 91. Estimation of noise coefficient in long time simulation

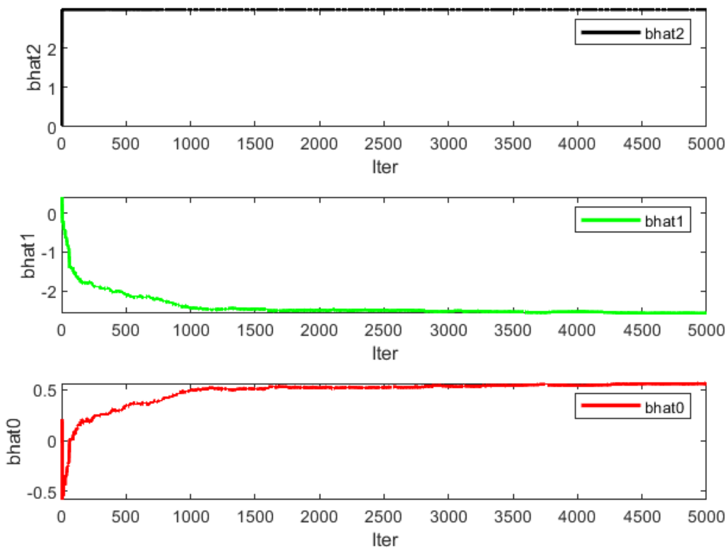

Fig 92. Estimation of nominator parameters in long time simulation

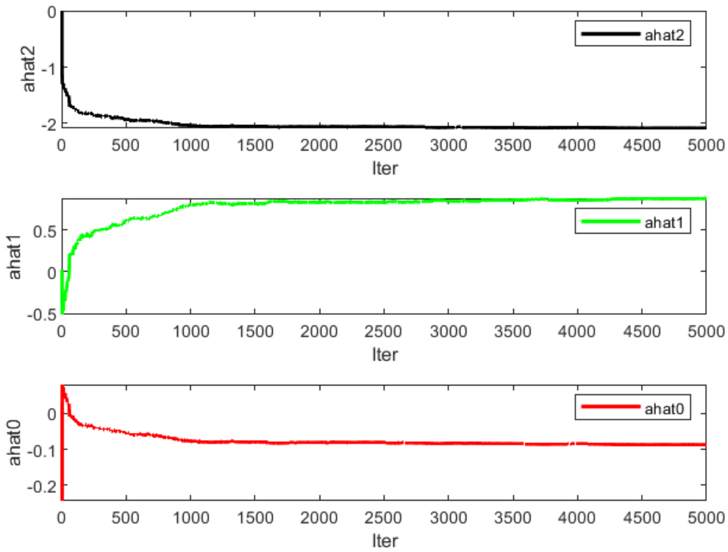

Fig 93. Estimation of denominator parameters in long time simulation

From the above results, we can see that the output of closed loop system decreases and increases periodically. Also, the control input is first, a little noisy and later, becomes stable. Also it doesn’t become larger in long time simulations.

3. STR Pole Placement of Non-Minimum-Phase System

At first, we make the system non-minimum phase and according to the question, we place one of the zeros of the transfer function of the continuous system on the right side of the imaginary axis. So, the optimal non-minimum unstable phase system is as follows.

$$G(s) = \frac{(s + 6.6667)\,(s - \ 2.1053)}{(s + 3.3333)\,(s \ - \ 2.439)\,(s + 0.5)}$$

After discretization, the transfer function will be as follows.

$$G(z) = \frac{B}{A} = \frac{0.1974\,z^2 - 0.7063\,z - 0.07261}{z^3 \ - \ 5.518\,z^2 \ + \ 4.061\,z \ - \ 0.4145}$$

We will solve the design of the controller in an indirect way, and since the system is non-minimum phase, we will not cancel any zeros. In this case, we will estimate the polynomials of the transfer function because the method is indirect. Also, as our method is indirect and we do not estimate the controller polynomials and obtain them by solving the Diophantine equation, we will not estimate the T polynomial anymore.

The optimal closed-loop polynomial in this section, considering the settling time of 3 seconds, overshoot of 10%, the third pole 8 times farther from the dominant poles, and also, by converting the optimal roots of the continuous to discrete system, is as follows will be.

$$A_m(q) = q^3 - q^2 + 0.19\,q + 0.03$$

$$B^+ = 1\ (monic)$$

$$B^- = 0.1974\,z^2 - 0.7063\,z - 0.07261 = B$$

$$\deg R = \deg S = \deg T = 2$$

$$\deg A_m = \deg A = 3$$

$$\deg B_m = \deg B = 2$$

$$\deg A_0 = \deg A - \deg B^+ - 1 = 3 - 0 - 1 = 2$$

$$A_0 = q^2$$

$$B_m = B^- B'_m$$

So according to the degree of B_m, the degree of B_m is zero and is a constant number. On the other hand, we want the steady state error to be zero, therefore, the value of B_m can be obtained from the controller coefficient T, whose value will be given below.

$$R = R'B^+ \quad degB^+ = 0 \quad \rightarrow \ \deg R' = 2 \ \ R' \ is\ monic$$

The Diophantine equation will be calculated in each iteration because this equation is dependent on A and B and these polynomials are also identified in each step.

Finally, after estimation in each iteration, the Diophantine equation is calculated as follows. Here, we give as an example the solution of the equation in the last iteration when the parameters have converged.

$$AR' + B^{-}S = A_0 A_m$$

$$(q^3 - 5.518\,q^2 + 4.061\,q - 0.4145)(q^2 + r_1 q + r_0) + (0.1974\,q^2 - 0.7063\,q - 0.07261)(s_2 q^2 + s_1 q + s_0)$$

$$= q^2(q^3 - q^2 + 0.19\,q + 0.03)$$

After the convergence of the controller coefficients, the controller polynomials are as follows.

$$R = q^2 - 15.96q - 1.606$$

$$S = 1.032q^2 - 0.879q + 0.091$$

$$T = A_0 B_m' = -0.378q^2 + 0.037q + 0.007$$

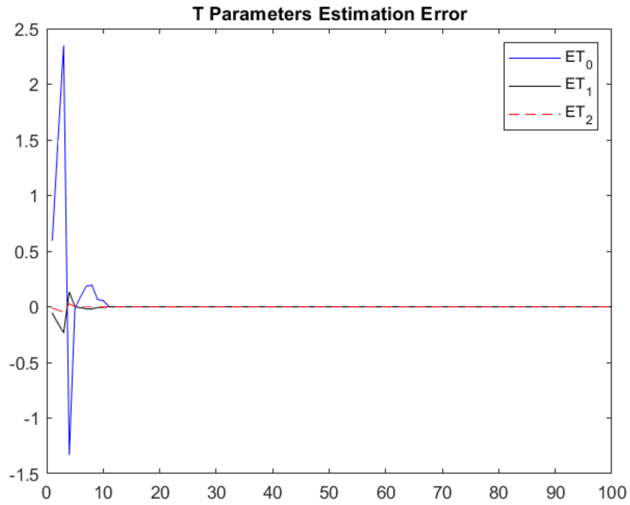

Fig 94. Estimation of T parameters error

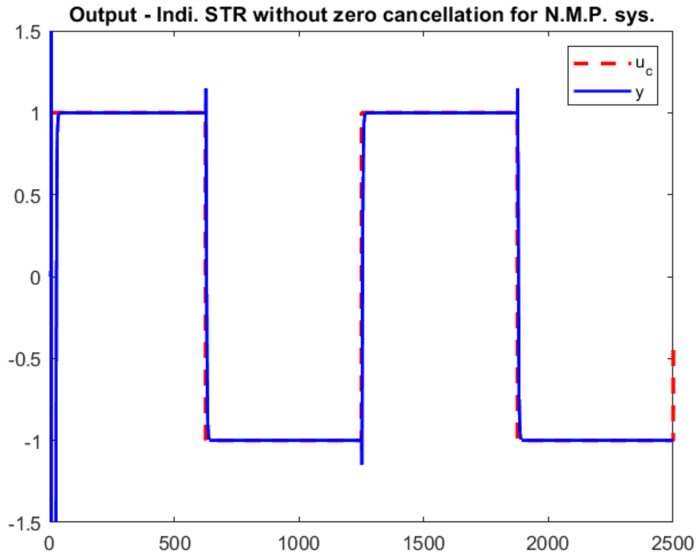

Fig 95. Output of indirect STR without zero cancellation for N.M.P. system

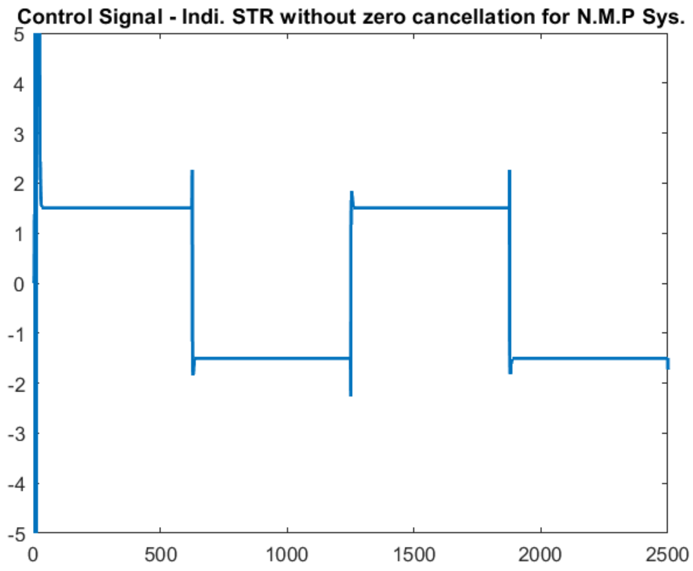

Fig 96. Control signal of indirect STR without zero cancellation for N.M.P. system

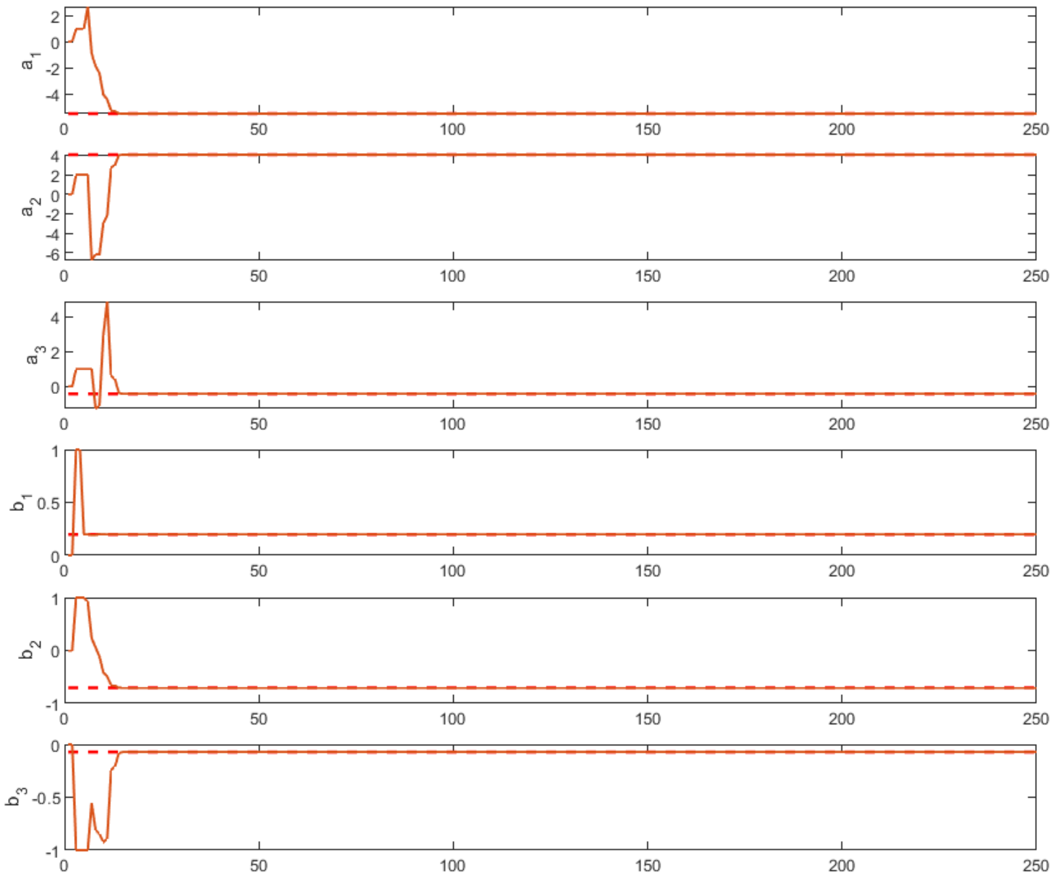

Fig 97. Nominator and denominator parameter estimation

As it is clear from the plots above, at first the output was not desirable, but after the passage of time when the estimation of the parameters of the transfer function was correct, the output was able to follow the reference input. Also, a few jumps are observed in the system, which is due to the fact that zero cancellation has not been done in this system. From Figure 96, we can see that in the control effort there is no ringing because we have not canceled zero. But as the system is a closed loop, the estimation of the parameters has been done accurately, because when we have feedback in the system, the order of PE increases.

4. STR Pole placement of Continuous System using SIMULINK

$$G(s) = \frac{4}{(s+0.75)(s+3)} = \frac{4}{s^2 + 3.75\,s + 2.25}$$

To design the pole placement, we assume that the numerator and denominator parameters of the transfer function are not known and must be estimated online. We consider the transfer function as follows.

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_2}$$

According to the settling time of 3 seconds and overshoot of 10 %, the desired transfer function is as follows:

$$G_m(s) = \frac{B_m}{A_m} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{5.087}{s\text{^}2\ +\ 2.667\,s\ +\ 5.087}$$

Considering that we are going to design the controller without zero cancellation, so, we will have:

$$B^+ = 1\ (monic) and\ B^- = b_0$$
$$\deg R = \deg S = \deg T = 1$$
$$\deg A_m = \deg A = 2$$
$$\deg B_m = \deg B = 0$$
$$B_m = B^- B_m' \quad \rightarrow B^- = b_0\ , \qquad B_m = \omega_n^2 \quad \rightarrow \quad B_m' = \frac{\omega_n^2}{b_0}$$
$$\deg A_0 = \deg A - \deg B^+ - 1 = 2 - 0 - 1 = 1$$
$$R = R'B^+ \quad degB^+ = 0 \quad \rightarrow \ \deg R' = 1 \quad R'\ is\ monic$$

$$A_0 = s + a_0 = s + 20$$

The Diophantine equation is as follows:

$$AR' + B^- S = A_0 A_m$$

$$\left(s^2 + a_{1hat} * s + a_{0hat}\right)(s + r_0) + \left(b_{0hat}\right)(s_1 s + s_0) = (s + a0)(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

By solving this equation, the following equations are obtained:

$$r_0 = a0 + 2\zeta\omega_n - a_{1hat}$$

$$s_0 = (a0 * \omega_n^2 - a_{0hat} r_0)/b_{0hat}$$

$$s_1 = \left(2\zeta\omega_n * a0 + \omega_n^2 - a_{0hat} - a_{1hat} r_0\right)/b_{0hat}$$

$$T = A_0 B_m' = (s + a0)\frac{\omega_n^2}{b_0}$$

The designed control system using SIMULINK is given in Figure 98. The output plot of the closed loop system compared to the square wave input is shown in Figure 99.

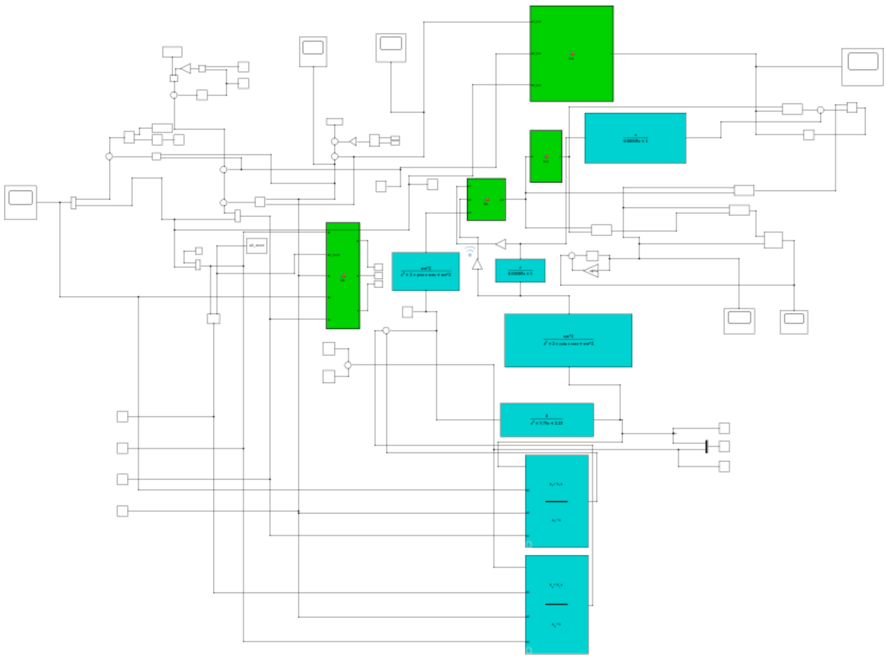

Fig 98. Simulink model

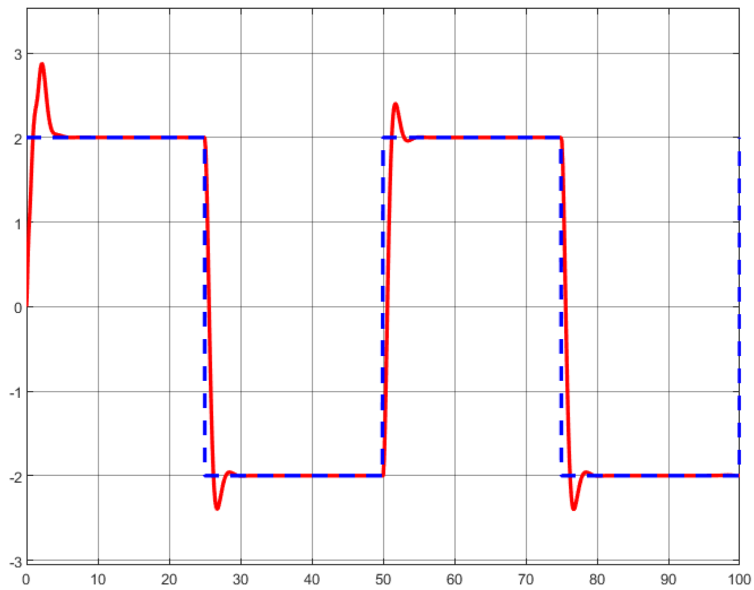

Fig 99. Output plot of continuous system with designed controller without zero cancellation

As it is clear from the diagram above, the output has been able to follow the reference input very well and it has a low overshoot and short settling time. The control effort of the system is given in Figure 100.

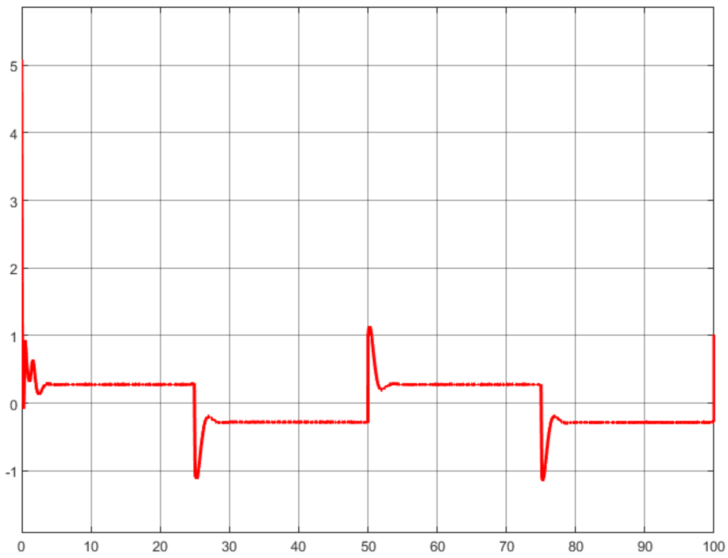

Fig 100. Control effort of continuous system without zero cancellation

It is also clear that the system control effort does not have ringing because we didn't have zero cancellation. The estimation of the parameters has been done very well, which is due to the system's closed loop, which will increase the system's PE order, and so, the parameters are well estimated.

References

1. Smith, J. Q., et al. (2001). Robust pole placement by dynamic feedback. IEEE Transactions on Automatic Control, 46(2), 295-299.
2. Chen, W., & Zhou, K. (2003). Pole assignment via eigenstructure assignment and pole-zero cancellation. Automatica, 39(8), 1383-1389.
3. Zhang, X., & Zhang, W. (2006). Indirect self-tuning regulator pole placement controller for industrial processes. Journal of Process Control, 16(3), 225-234.
4. Wang, Y., et al. (2010). Direct adaptive fuzzy control for uncertain nonlinear systems with disturbances. IEEE Transactions on Fuzzy Systems, 18(3), 476-486.
5. Li, W., et al. (2015). Over and under parameterization in self-tuning regulator pole placement. IEEE Transactions on Automatic Control, 60(1), 175-180.
6. Yang, X., et al. (2018). Robust self-tuning regulator pole placement for systems with unknown disturbances. IET Control Theory & Applications, 12(8), 1140-1148.

Thanks for your Time