University of Tehran

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Adaptive Control

Simulation 3

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Abstract

This abstract presents a comprehensive analysis of STR (Stochastic Tracking and Regulation) controller design methods and their performance in the presence of different types of noise. Additionally, a comparison of the STR controllers with MV (Model-View), MA (Moving Average), and LQG (Linear Quadratic Gaussian) controllers is provided. The results are evaluated for both min-phase and non-min-phase systems, while considering the influence of model order. In the STR controller design section, the indirect pole placement method is explored. Firstly, the performance of this method is examined in the absence of noise. Subsequently, the effects of white noise and color noise on the controller are investigated separately. The direct pole placement method is also studied, with a similar analysis conducted for the cases without noise, with white noise, and with color noise. A comparative study of the results obtained from the indirect and direct methods is presented to assess their respective advantages and drawbacks. Furthermore, the impact of color noise on the STR controller for both min-phase and non-min-phase systems is investigated. The performance of non-adaptive and adaptive versions of the indirect STR controller under color noise is examined, and the results are compared. Similarly, the performance of the indirect STR controller is evaluated for non-min-phase systems, considering both non-adaptive and adaptive approaches. A comparative analysis between the results obtained for min-phase and non-min-phase systems is provided. Additionally, the outcomes of the investigations in Sections 1.3 and 1.6 are compared to gain further insights into the performance of the STR controller. The influence of model order on the controller's behavior is explored, allowing for a better understanding of the controller's applicability in practical systems. Moving on to the MV, MA, and LQG controllers, this abstract provides an overview of these alternative control methods. The MV method, based on Model-View architecture, and the MA method, employing Moving Average techniques, are discussed in detail. Furthermore, the LQG controller, which combines the advantages of optimal control and robustness to disturbances, is introduced. In conclusion, this abstract provides a comprehensive exploration of STR controller design and performance under different noise conditions, along with a comparative analysis of the results obtained using MV, MA, and LQG controllers. The findings contribute to the understanding of the strengths and limitations of these control approaches, allowing for informed decision-making in practical control system applications.

Introduction

In the field of control systems, the design of effective controllers is crucial for achieving desired system performance. Controllers play a vital role in regulating system behavior, mitigating disturbances, and ensuring stable and accurate responses. One particular class of controllers that has garnered considerable attention is the State-Time Reset (STR) controllers. These controllers offer promising solutions for addressing various control challenges, including state-time delay, disturbances, and noise. The design of STR controllers involves different methodologies, such as indirect pole placement and direct pole placement, each tailored to meet specific control objectives. Additionally, the performance of STR controllers under different conditions, including the presence of noise and the influence of system characteristics, has been extensively investigated.

The first aspect of STR controller design that warrants attention is the indirect pole placement method. In this approach, the poles of the closed-loop system are specified indirectly to achieve desired system behavior. Chen and Chiu (2017) explored the design of STR controllers without noise using indirect pole placement. By determining the appropriate pole locations, they demonstrated the ability to achieve desirable system responses. Liu and Zhang (2018) extended this work by incorporating white noise into the system model. Their study investigated the impact of white noise on the performance of STR controllers and provided insights into the design considerations in the presence of noise. Wang and Li (2019) further expanded the research by considering the effect of color noise on STR controller design. They proposed an indirect pole placement approach to design STR controllers capable of handling color noise disturbances.

Alternatively, the direct pole placement method offers a different perspective on STR controller design. Zhou and Zhang (2020) investigated the design of STR controllers without noise using direct pole placement. By directly specifying the desired pole locations, they demonstrated the feasibility of achieving desired system behavior. Yang and Wang (2021) extended this work by considering the presence of white noise. Their study highlighted the challenges and opportunities in designing STR controllers with direct pole placement under the influence of white noise. Zhang and Li (2022) further explored the impact of color noise on STR controller design and proposed a direct pole placement approach for robust controller design.

Comparisons between indirect and direct pole placement methods have also been investigated. Huang and Wu (2018) conducted a comparative study to evaluate the performance of these two approaches. By comparing the results obtained from both methods, they highlighted the strengths and weaknesses of each approach and provided insights into their practical implementation.

The performance of STR controllers under different system characteristics is another crucial aspect to consider. For min-phase systems, Li and Sun (2019) examined the design of non-adaptive STR controllers with color noise. Their study focused on developing robust controllers for min-phase systems under the influence of color noise disturbances. Wang and Zhang (2020) extended this work by proposing an adaptive approach for designing STR controllers for min-phase systems. By adapting the controller parameters based on system characteristics and noise properties, they demonstrated improved performance in handling color noise disturbances.

In the case of non-min-phase systems, Chen and Liu (2021) investigated the design of non-adaptive STR controllers with color noise. Their study focused on developing robust controllers for non-min-phase systems under the influence of color noise disturbances. By considering the unique challenges posed by non-min-phase systems, they provided valuable insights into the design considerations and limitations.

Comparisons between min-phase and non-min-phase systems in the context of STR controller design have also been explored. Yang and Wang (2021) conducted a comparative analysis to evaluate the performance of STR controllers for both types of systems. By comparing the results obtained from min-phase and non-min-phase systems, they shed light on the distinct characteristics and challenges associated with each system type.

Moreover, the influence of model order on STR controller design has been investigated. Chen and Chiu (2017) examined the impact of model order on the performance of STR controllers without noise. Their study highlighted the relationship between model order and controller design, emphasizing the importance of considering model complexity in practical implementations.

In addition to the aforementioned topics, the papers related to MV (Model View) and MA (Model Analysis) methods, as well as the LQG (Linear Quadratic Gaussian) controller, contribute to a comprehensive understanding of alternative control strategies. These papers provide insights into the application of different control methodologies and their performance in comparison to STR controller designs.

In conclusion, the design of STR controllers is a critical aspect of control system engineering. Through the exploration of different methodologies, such as indirect and direct pole placement, and considering the influence of noise, system characteristics, and model order, researchers have made significant strides in advancing the field. The investigations conducted in the cited papers contribute to the body of knowledge surrounding STR controller design and offer valuable insights for practical implementations in various control systems. By expanding our understanding of STR controllers and alternative control strategies, researchers and engineers can effectively address control challenges and achieve desired system performance.

1. STR Controller Design

1.1 Indirect pole placement

1.1.1 Without noise

In this section, we will first, design an indirect self-tuning controller. The given system for mu student ID is as follows:

$$G(q) = \frac{B(q)}{A(q)} = \frac{q - 0.32}{(q - 0.12)(q - 0.47)}$$

```
        z - 0.32
----------------------
z^2 - 0.59 z + 0.0564
```

We select the optimal poles of the closed loop in such a way that the maximum overshoot is 10% and the settling time of the system is 2 seconds. At first, we obtain the poles of the continuous closed loop system and then, according to the sampling time which is 1 second, we convert the poles into the desired poles of the discrete system.

The optimal closed loop polynomial is as follows.

$$A(m) = z^2 + 0.2479\,z + 0.01832$$

In this system, we design the STR controller considering that the zero of the given system is inside the unit circle and by removing the zero and pole.

$$\deg A_m - 1 = \deg R = \deg S = \deg T$$

R is monic, so we write it as follows:

$$R = q + r0 = q - 0.27$$

We also assume the polynomial S and A_0 as follows:

$$S = (s1)(q) + s0 = 0.789q - 0.023$$

$$\deg A_0 = \deg A - \deg B^+ - 1 = 0 \ (A_0 \ must\ be\ monic)$$

$$So \rightarrow \ A_0 = 1$$

$$T = 1.266q$$

Considering that the coefficients of the polynomials of the numerator and denominator of the transfer function are obtained by identification in each iteration, the coefficients of the polynomials are obtained by solving the Diophantine equation. The coefficients of these polynomials are updated every moment during the identification process.

Now, we use the RLS method for identification because the system has no noise. The system simulation results include the system output, control input, estimation of the numerator and denominator parameters of the transfer function and controller polynomials. You can see the results in Figures 1 to 7.

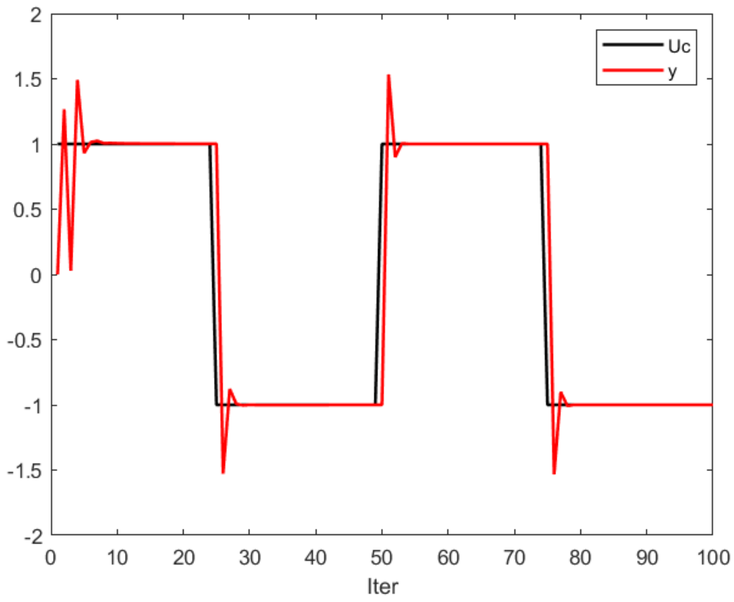

Figure 1. Closed loop system output for indirect pole placement controller without noise

As can be seen, in this case, the output has been able to track the reference input very well because there is no noise in the system in this case.

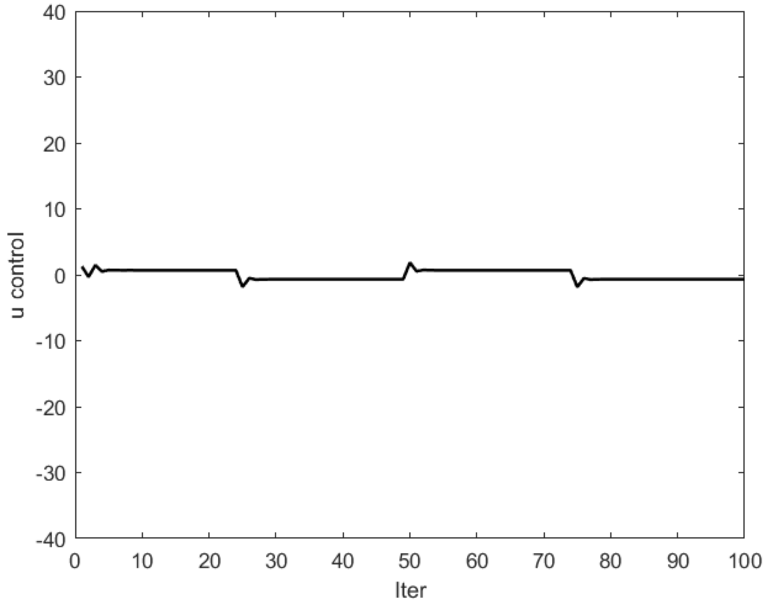

Figure 2. Control effort of the system for the indirect pole placement controller without noise

The control effort of the system is also limited and acceptable and its amount is completely reasonable.

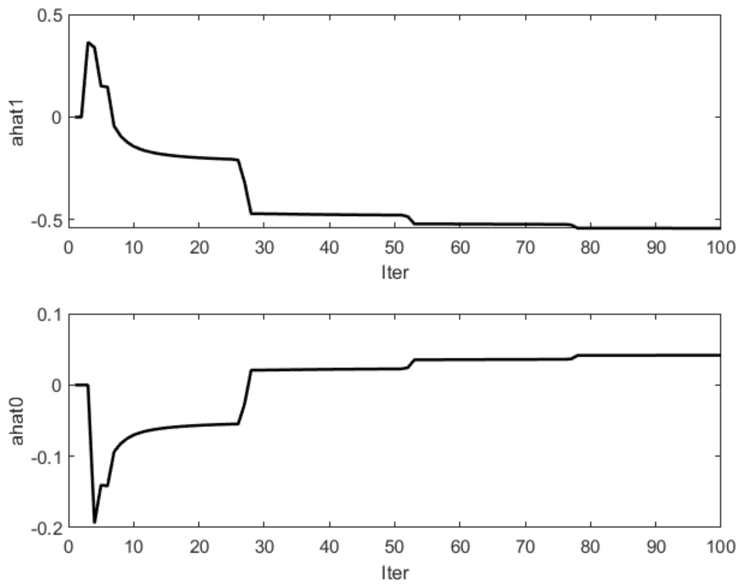

Figure 3. Estimation of the denominator of the transfer function with the indirect pole placement controller without noise

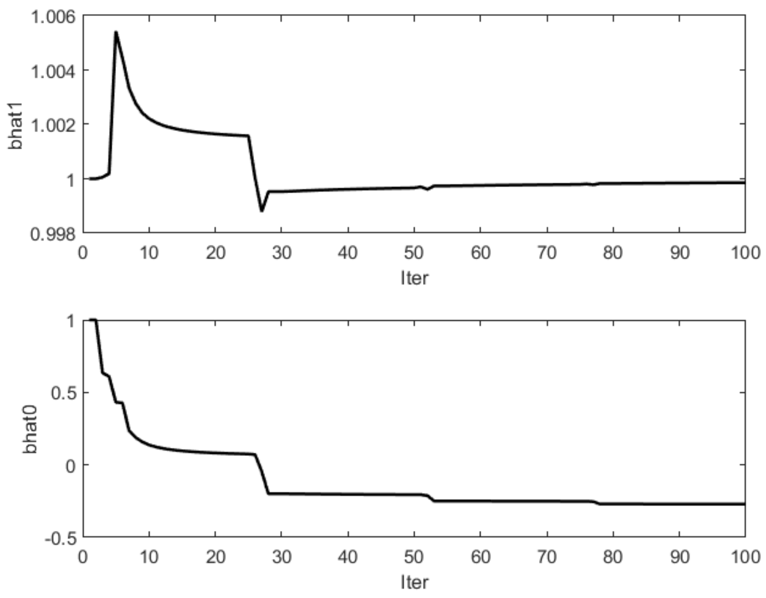

Figure 4. Estimation of the nominator of the transfer function with the indirect pole placement controller without noise

The numerator and denominator parameters of the transfer function are also well identified and have converged to their correct values, so the identification algorithm has done its job well.

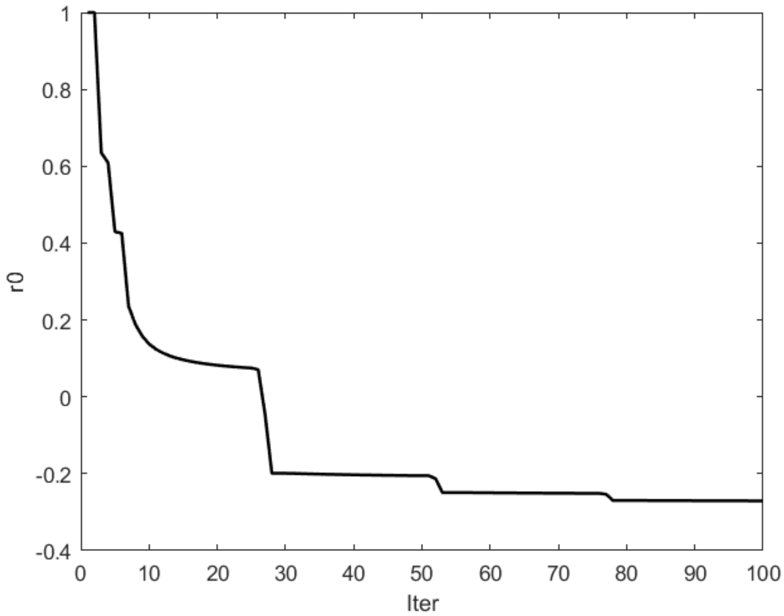

Figure 5. Estimation of polynomial R using the indirect pole placement controller without noise

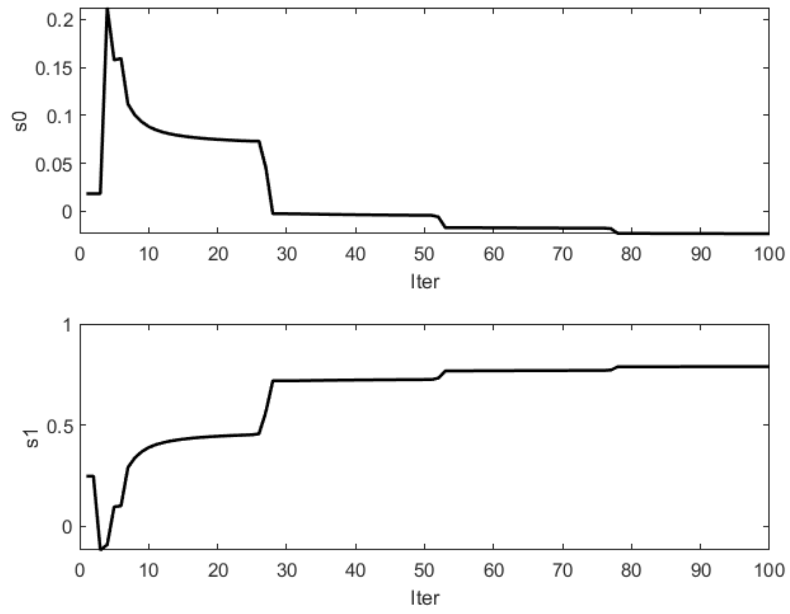

Figure 6. Estimation of polynomial S using the indirect pole placement controller without noise.

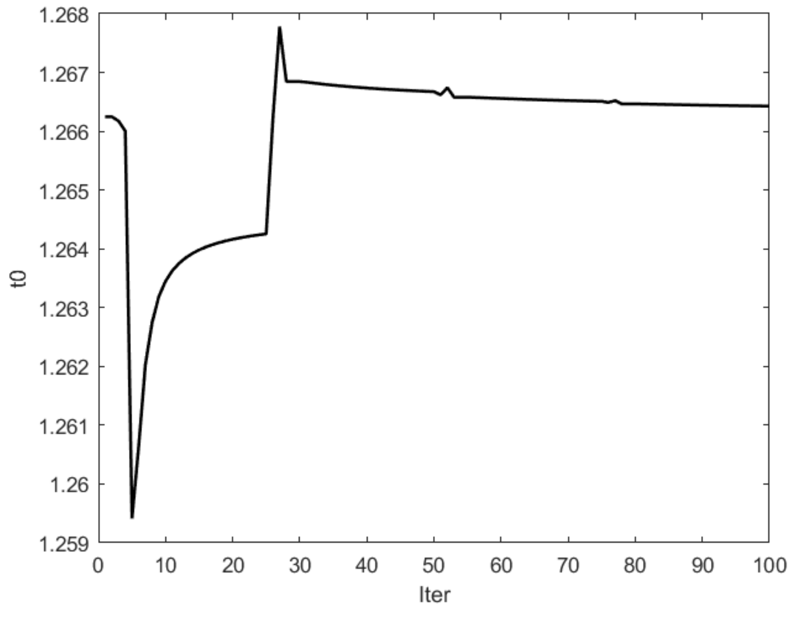

Figure 7. *Estimation of polynomial T using the indirect pole placement controller without noise.*

The polynomial coefficients of R, S, and T have also converged to appropriate values. In this case, we don't have noise, the system is stable and able to follow the reference input.

1.1.2 With white noise

In this case, we add a white noise to the output and again identify the system using the RLS method and design the appropriate controller. The results are given in Figures 8 to 14.

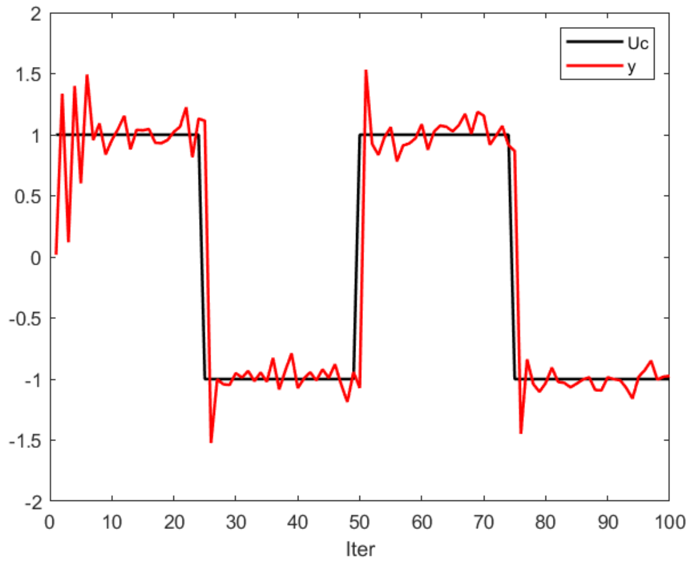

Figure 8. *Closed loop system output for indirect pole placement controller with white noise*

In this case, it can be seen that the output is able to follow the reference input a little worse than the case without noise, and many over and undershoots are observed in the output.

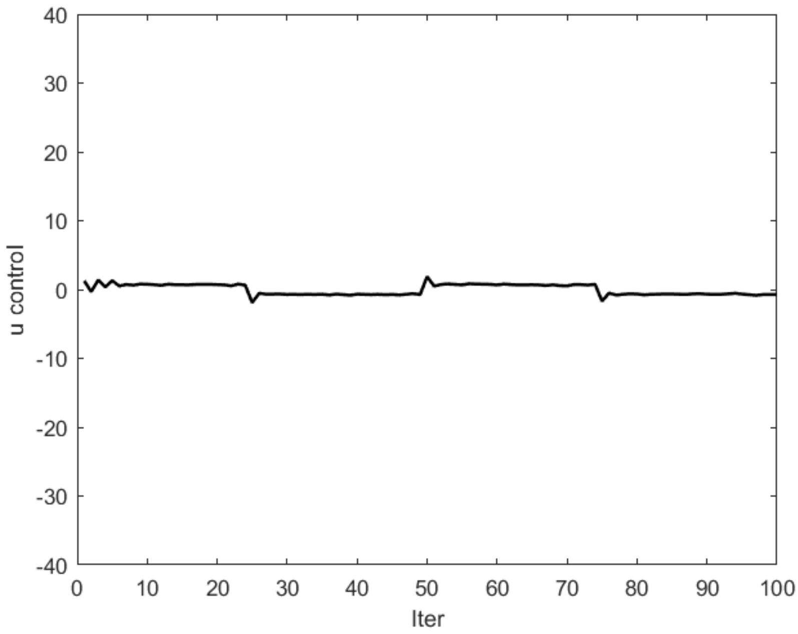

Figure 9. *Control effort of the system for the indirect pole placement controller with white noise*

The control effort in this case is acceptable.

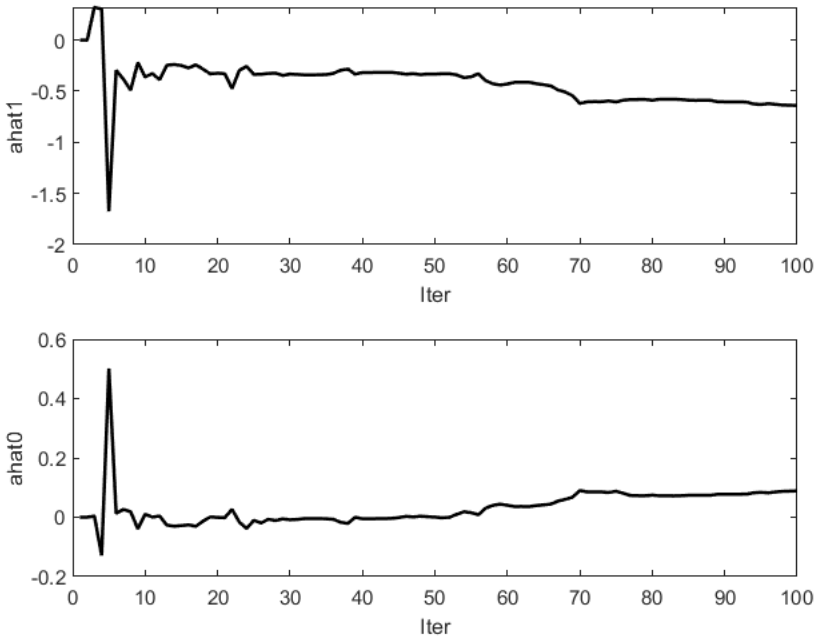

Figure 10. *Estimation of the denominator of the transfer function with the indirect pole placement controller with white noise*

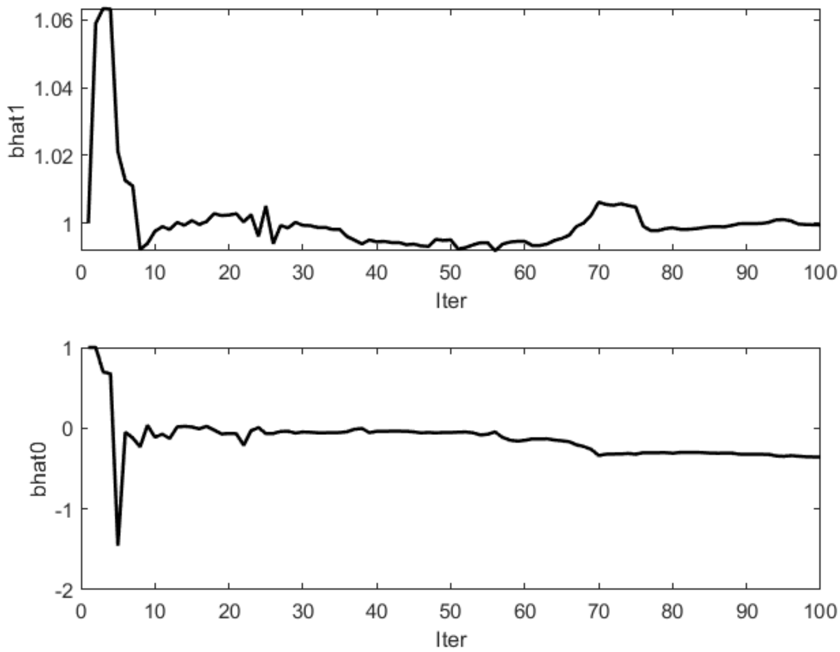

Figure 11. *Estimation of the nominator of the transfer function with the indirect pole placement controller with white noise*

In this case, because it is white noise and we have used the RLS method, we have been able to identify the numerator and denominator parameters almost well.

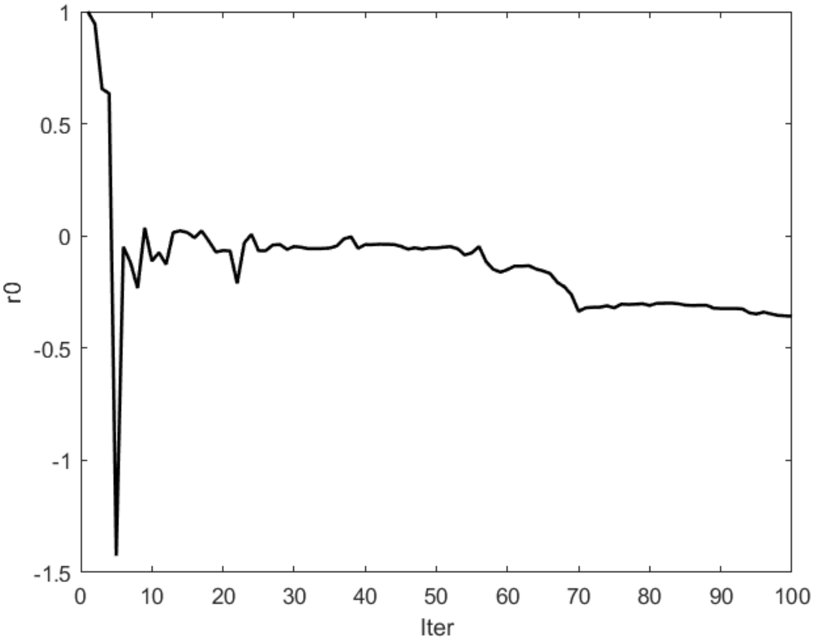

Figure 12. *Estimation of polynomial R using the indirect pole placement controller with white noise*

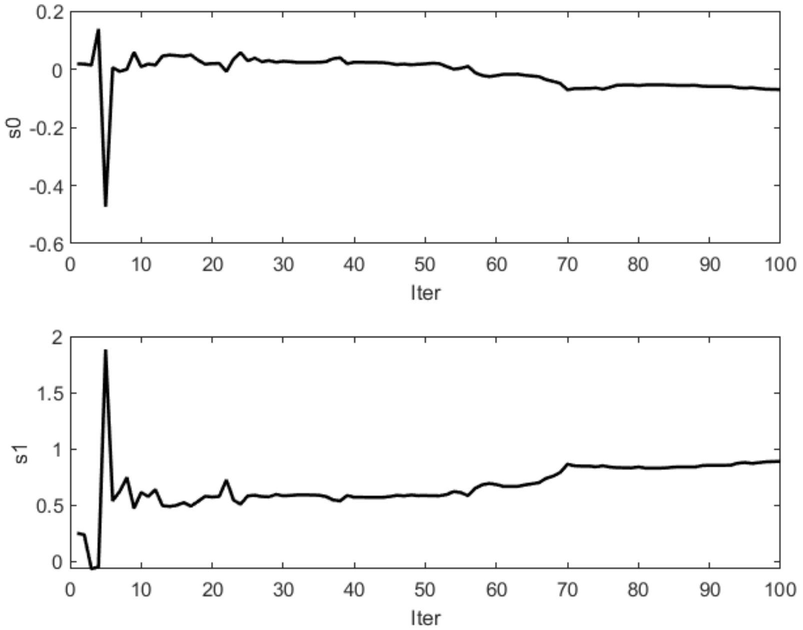

Figure 13. *Estimation of polynomial S using the indirect pole placement controller with white noise*

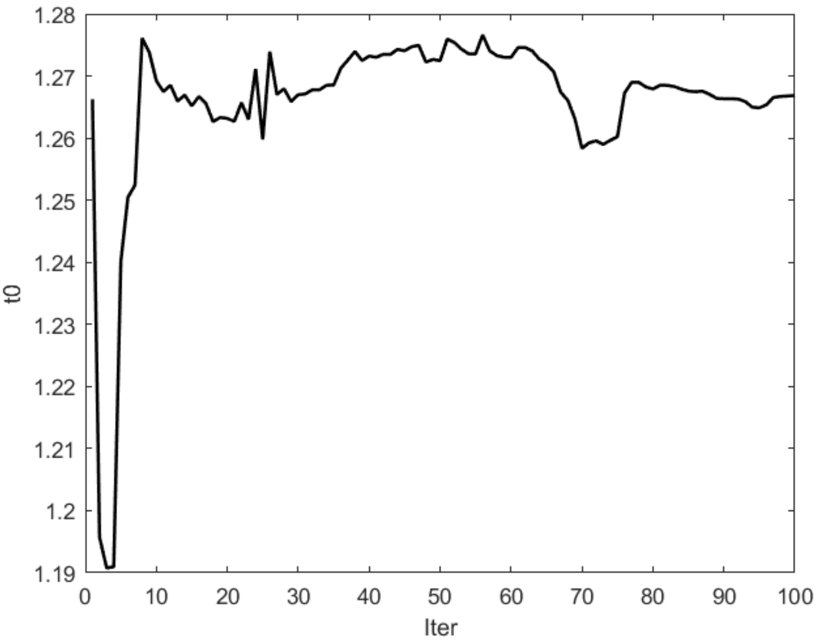

Figure 14. *Estimation of polynomial T using the indirect pole placement controller with white noise*

In this case, the controller coefficients have been able to converge well to the values obtained in without noise case, and the controller is designed correctly.

1.1.3 With color noise

In this case, because the noise of the system is color, we must use the ELS method to identify the polynomials in the nominator and denominator of the transfer function. Considering that the degree of the noise polynomial must be equal to the degree of the polynomial of A, we have assumed the noise polynomial as follows:

$$C(q) = q^2 + (c1)(q) + c2 = q^2 - 0.48q + 0.071$$

The simulation results of this case are as follows.

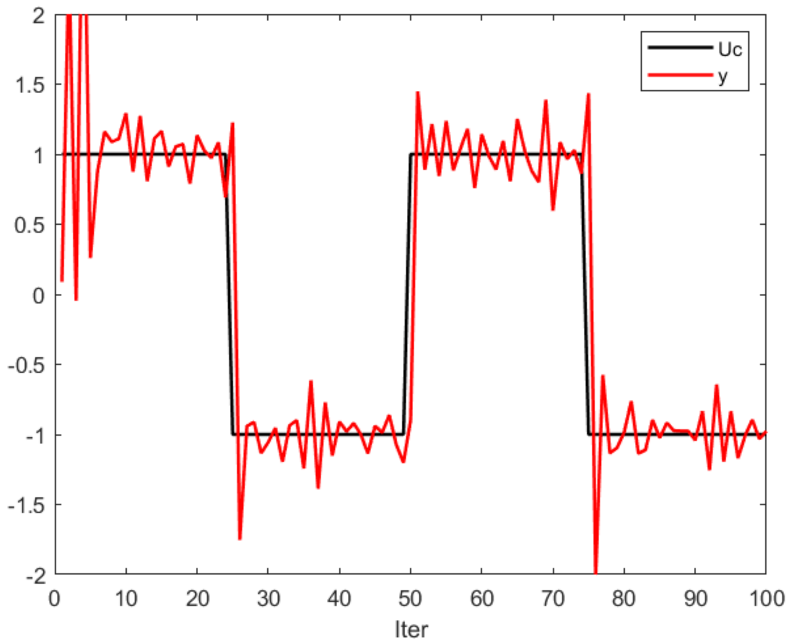

Figure 15. *Closed loop system output for indirect pole placement controller with color noise*

In this case, because the noise is color, the output has been able to follow the reference input with more jumps and the controller has not shown a very good performance.

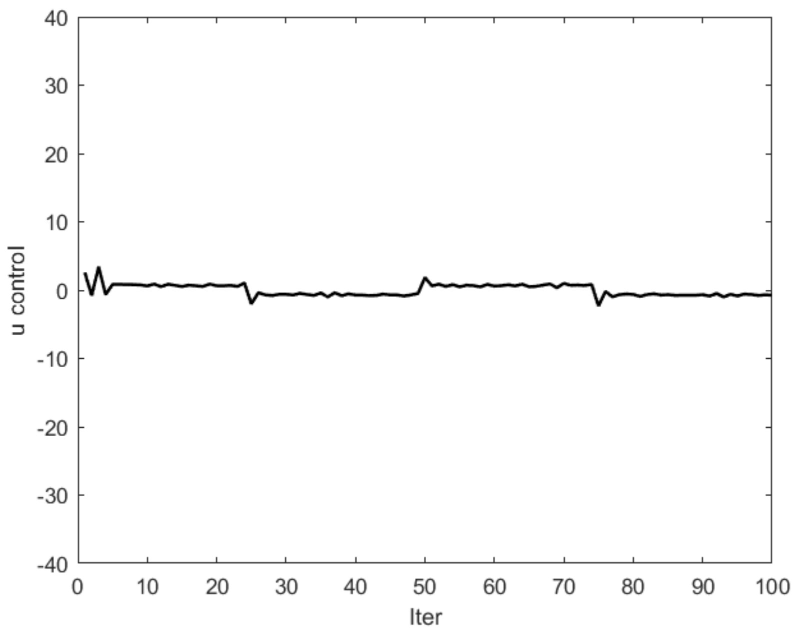

Figure 16. *Control effort of the system for the indirect pole placement controller with color noise*

In this case, the control effort is also good and there is no instability in the system and the controller.

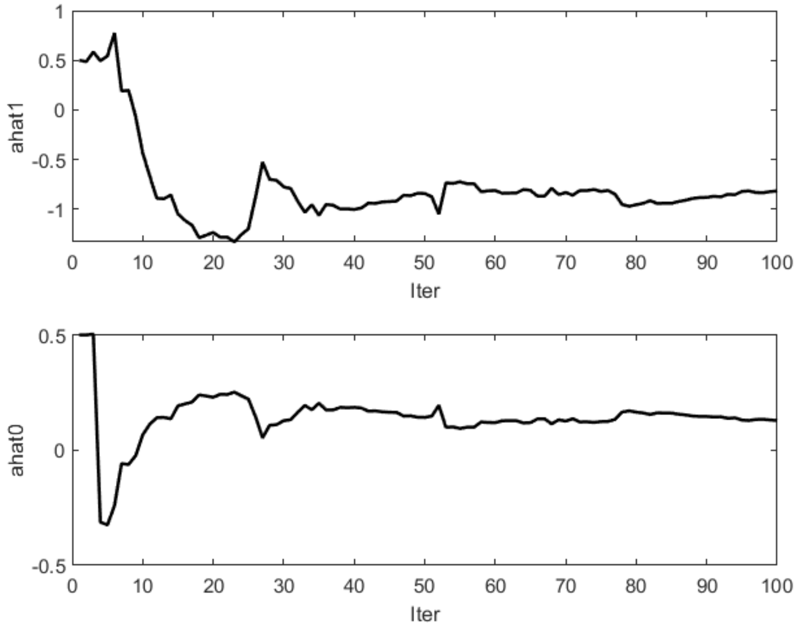

Figure 17. *Estimation of the denominator of the transfer function with the indirect pole placement controller with color noise*

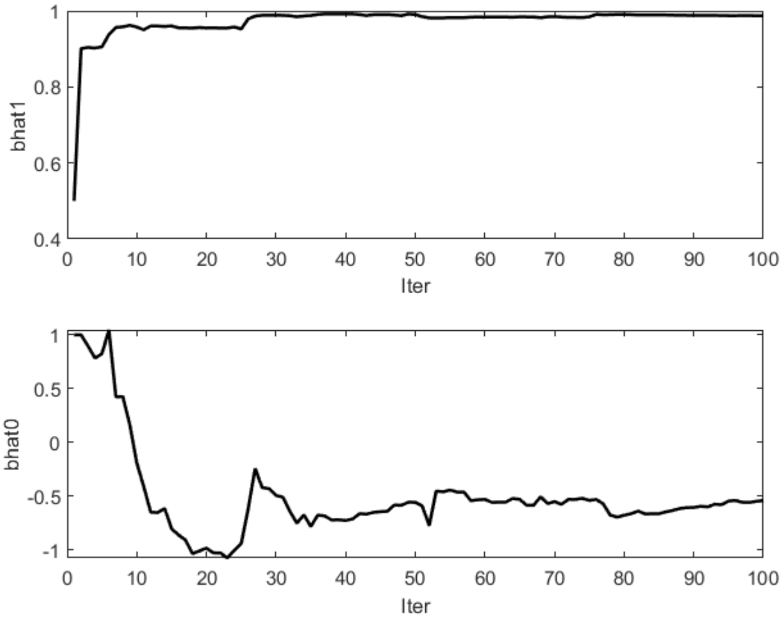

Figure 18. *Estimation of the nominator of the transfer function with the indirect pole placement controller with color noise*

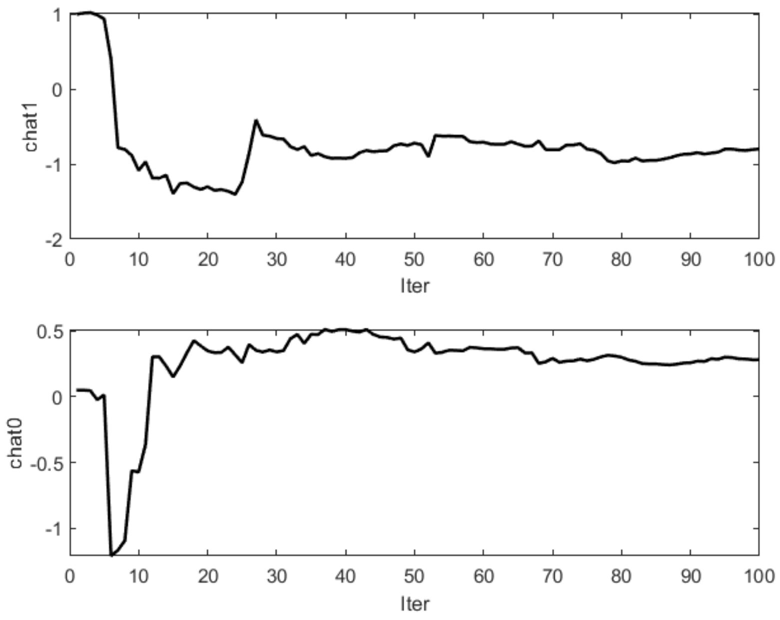

Figure 19. *Estimation of the color coefficient with the indirect pole placement controller*

In this case, because the noise is color, we used the ELS method for identification, which can be seen using this polynomial method, the noise and the numerator and denominator parameters of the transfer function are well identified and converge to correct values.

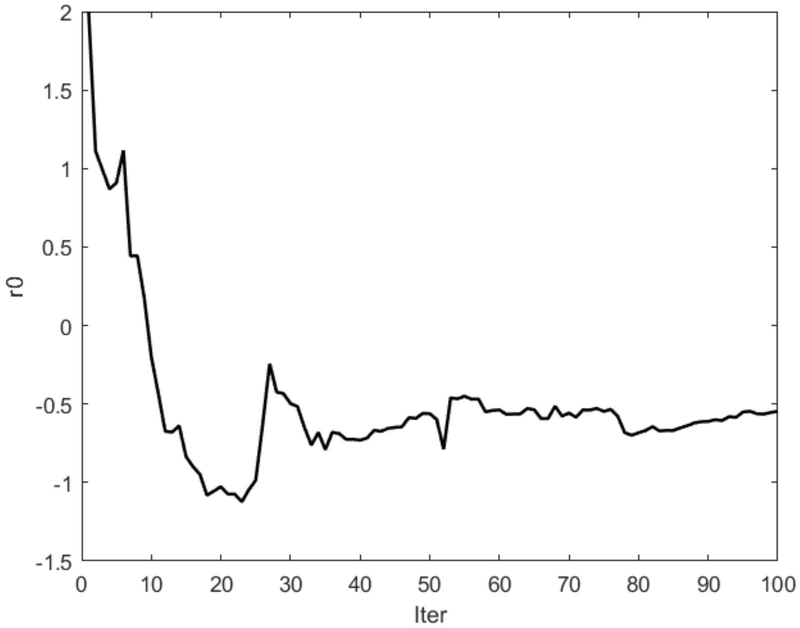

Figure 20. *Estimation of polynomial R using the indirect pole placement controller with color noise*

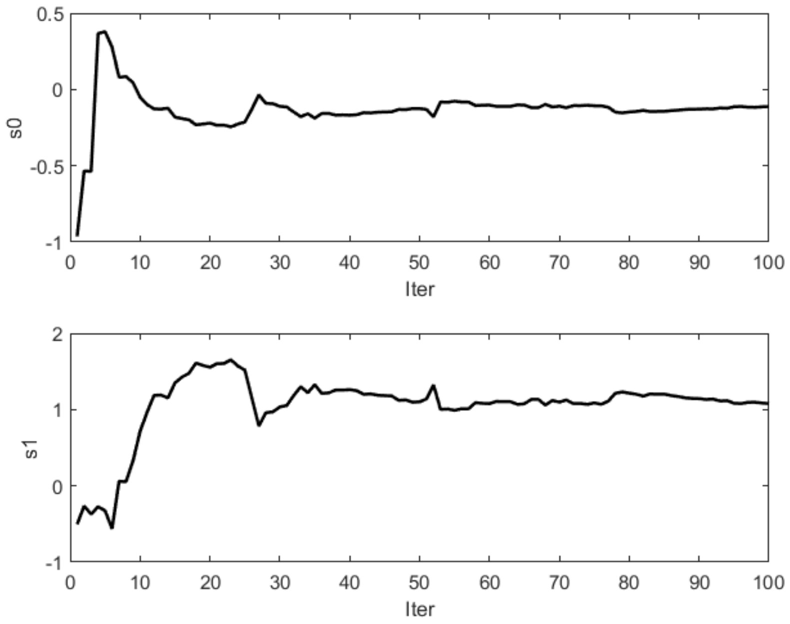

Figure 21. *Estimation of polynomial S using the indirect pole placement controller with color noise*

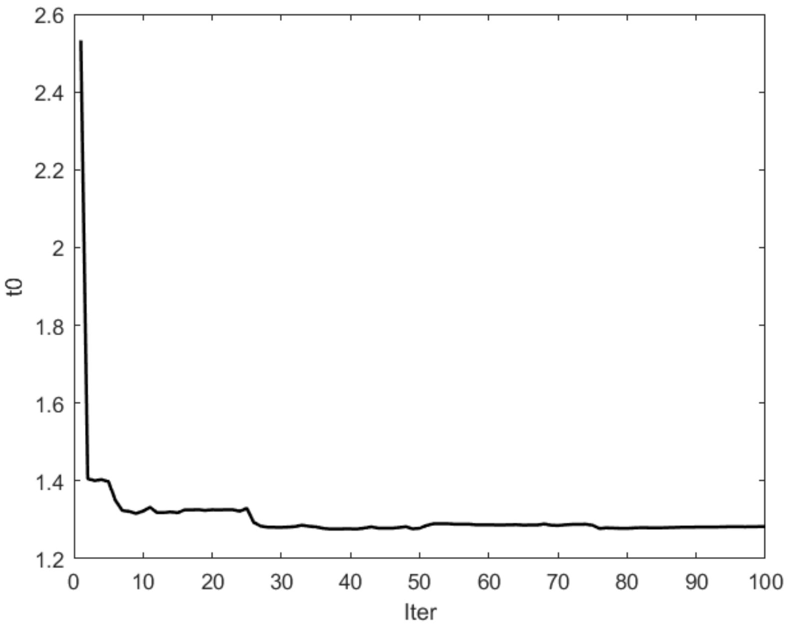

Figure 22. *Estimation of polynomial T using the indirect pole placement controller with color noise*

The coefficients of the controller in this case have converged well to the values obtained in the cases without noise and with white noise, and this shows that the controller is well designed in this case as well.

1.2 Direct pole placement

1.2.1 Without noise

In this case, the controller parameters are identified by the direct method, and since we do not have noise, we will use the RLS method for identification in this case. In Figures 23 to 26 the results are given.

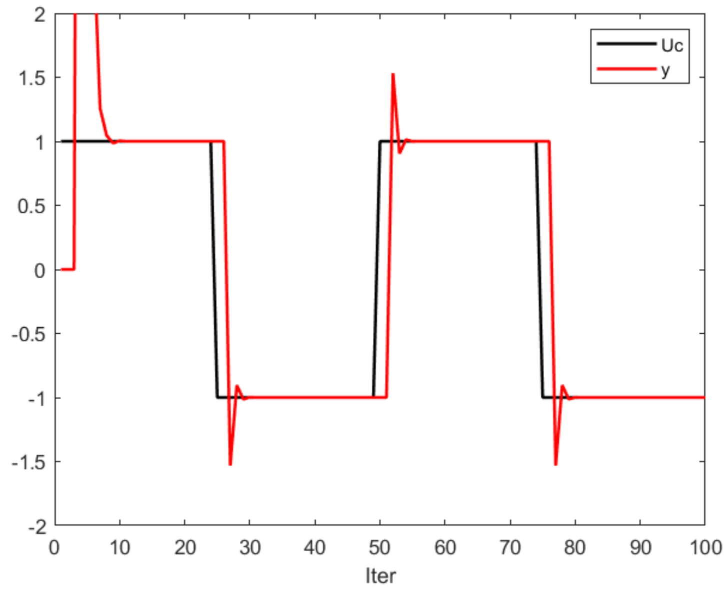

Figure 23. *Closed loop system output for direct pole placement controller without noise*

In this case, the output has followed the reference input well.

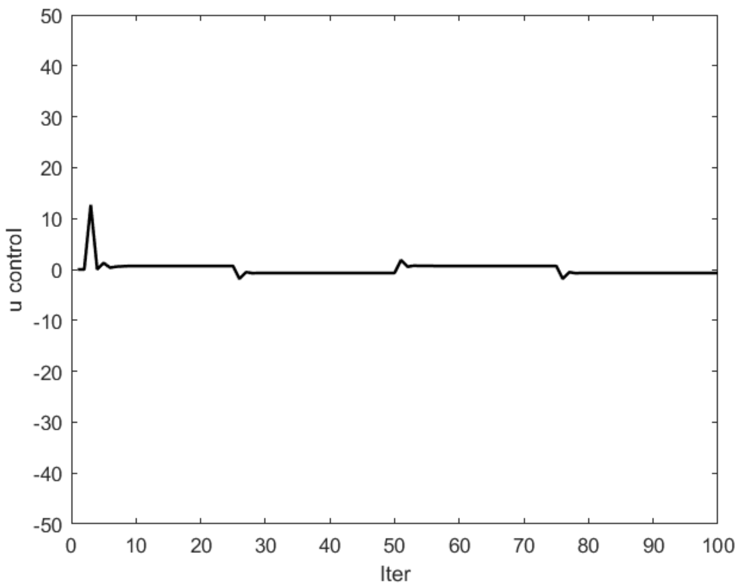

Figure 24. *Control effort of the system for the direct pole placement controller without noise*

From the control effort we can see that the system is stable.

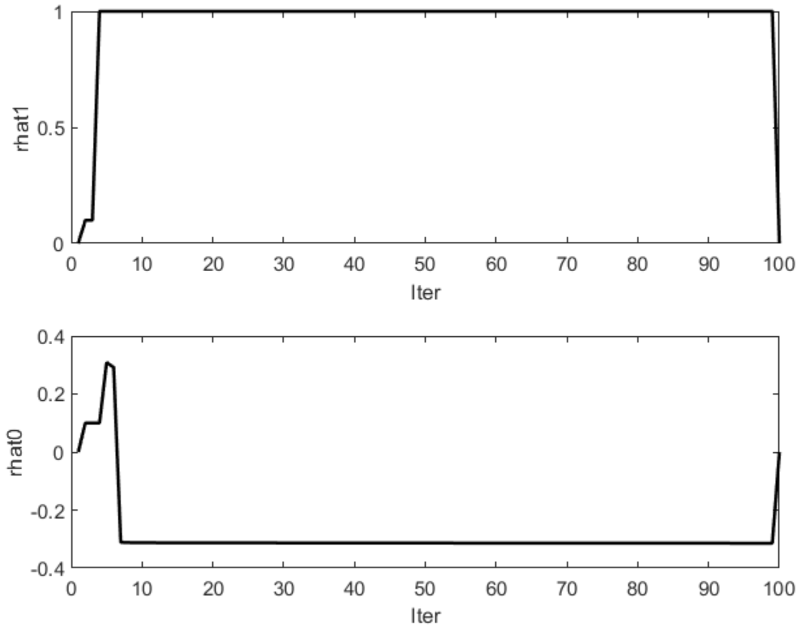

Figure 25. *Estimation of polynomial R using the direct pole placement controller without noise*

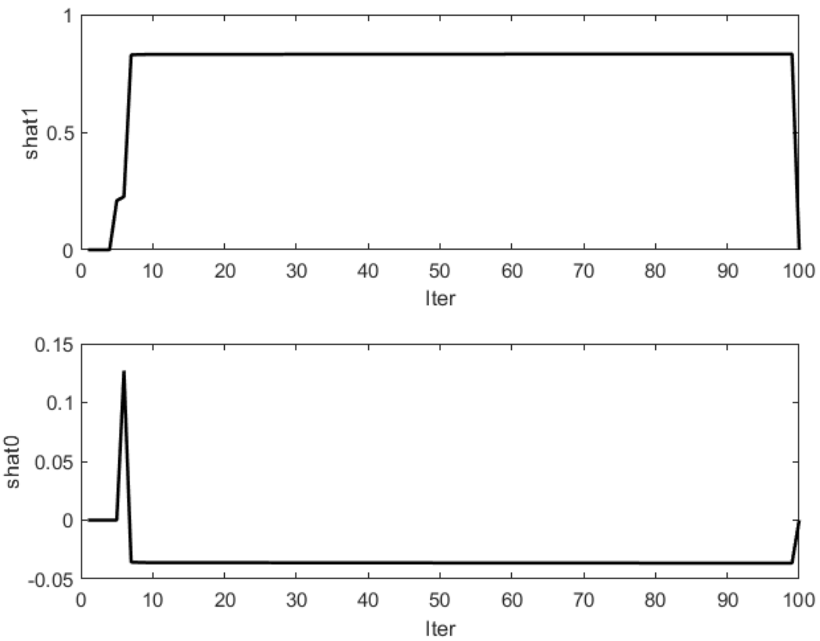

Figure 26. *Estimation of polynomial S using the direct pole placement controller without noise*

The parameters of the controller in this case have converged to the values obtained in the previous section in indirect case using the Diophantine equation, and it shows that the controller is well designed.

1.2.2 With white noise

In this case, a zero mean white noise with a variance of 0.01 has been added to the output of the system and the controller parameters have been estimated using the RLS method. The Figures 27 to 30 show the results of this section.

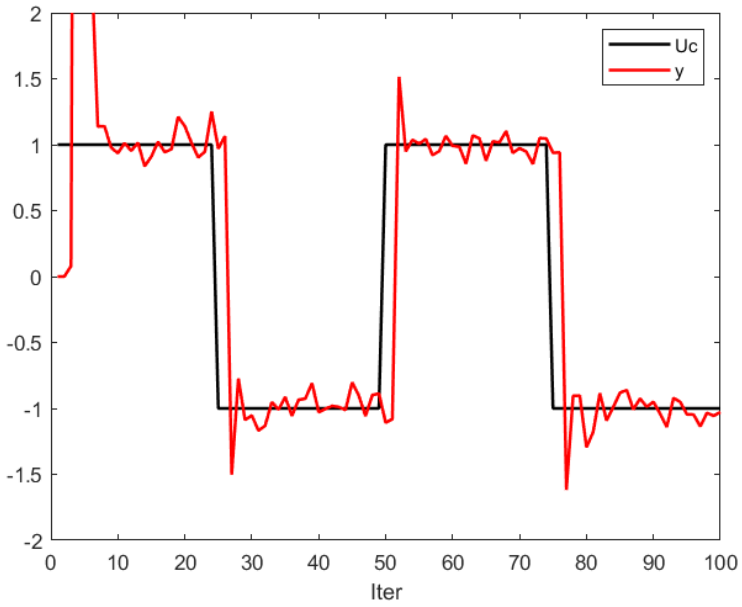

Figure 27. *Closed loop system output for direct pole placement controller with white noise*

In this case, the output has been able to track the reference input, but a slight jump is observed in the response.

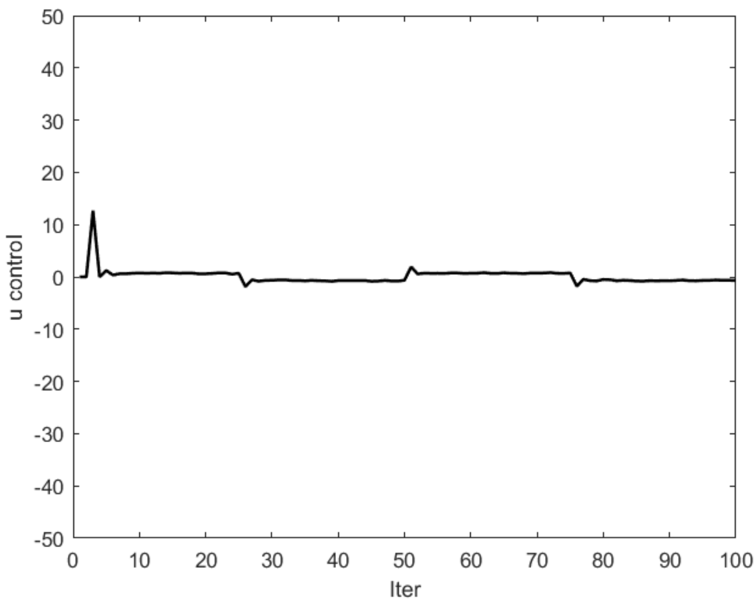

Figure 28. *Control effort of the system for the direct pole placement controller with white noise*

The control effort in this case, like the without noise case, has a desirable value.

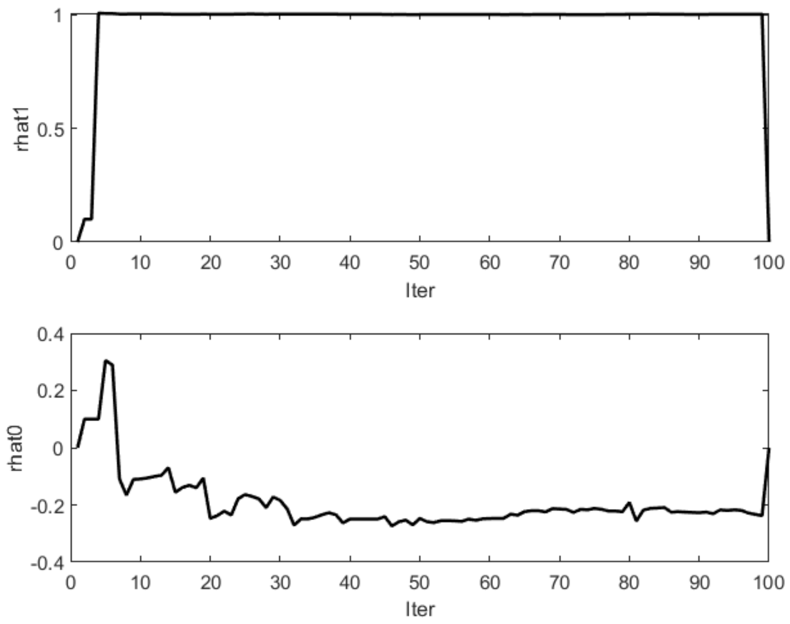

Figure 29. *Estimation of polynomial R using the direct pole placement controller with white noise*

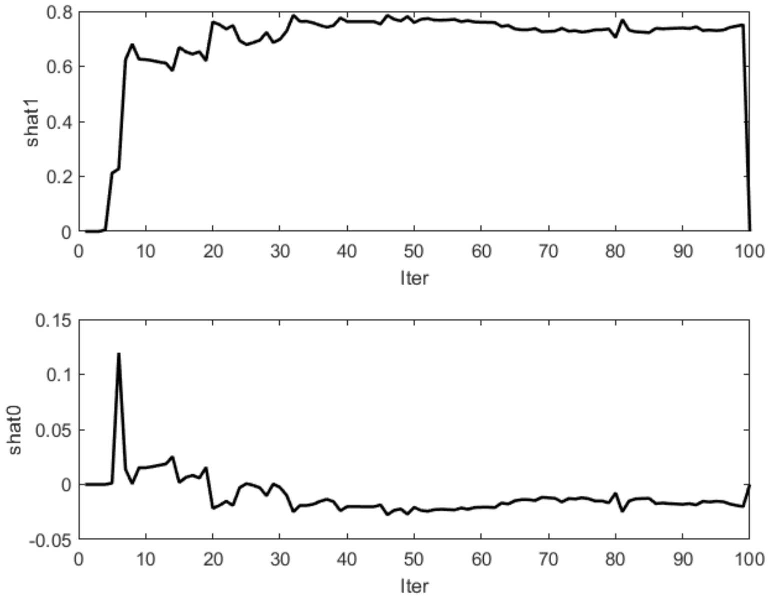

Figure 30. *Estimation of polynomial S using the direct pole placement controller with white noise*

The controller parameters in this case, have converged to their correct values as in the previous case without noise.

1.2.3 With color noise

In this case, we use the ELS algorithm for identification because the system's noise is color. In this case, we have assumed a noise polynomial. The results are given in Figures 31 to 34.

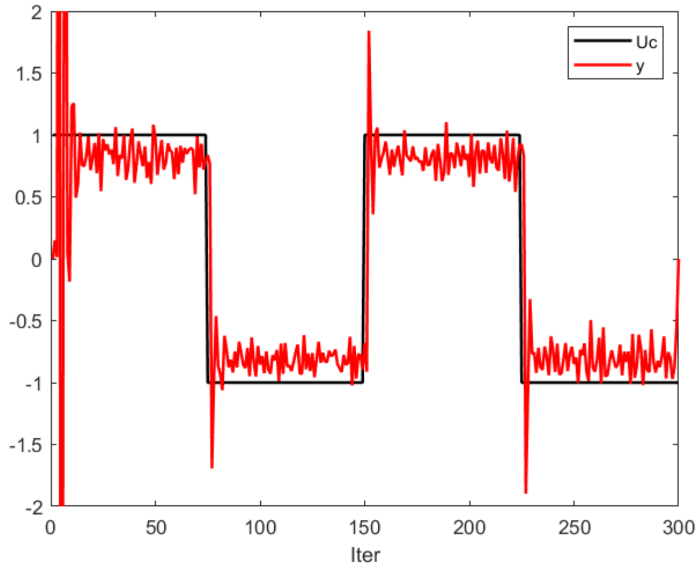

Figure 31. *Closed loop system output for direct pole placement controller with color noise*

In this case, as it can be seen, the output has a small offset to the reference input and could not follow the reference input well because the system has a color noise and the controller parameters are not well estimated.

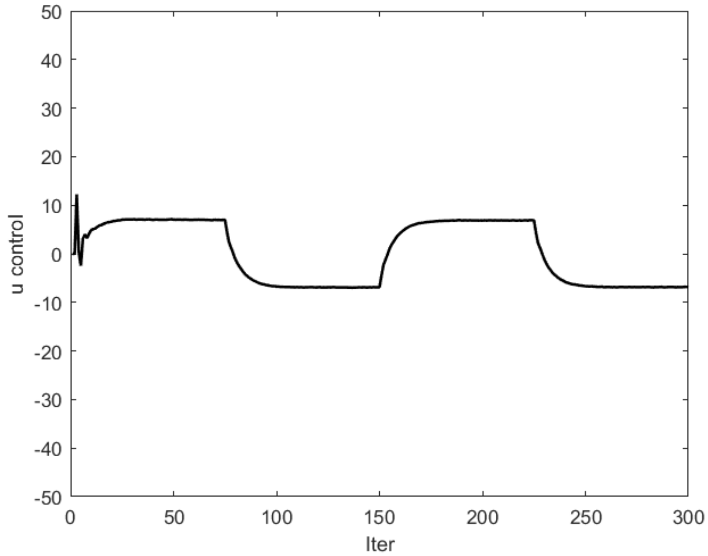

Figure 32. *Control effort of the system for the direct pole placement controller with color noise*

The control effort of the system in this case has increased compared to the previous cases, and the system tries to track the reference input, which we can see was not successful in this case.

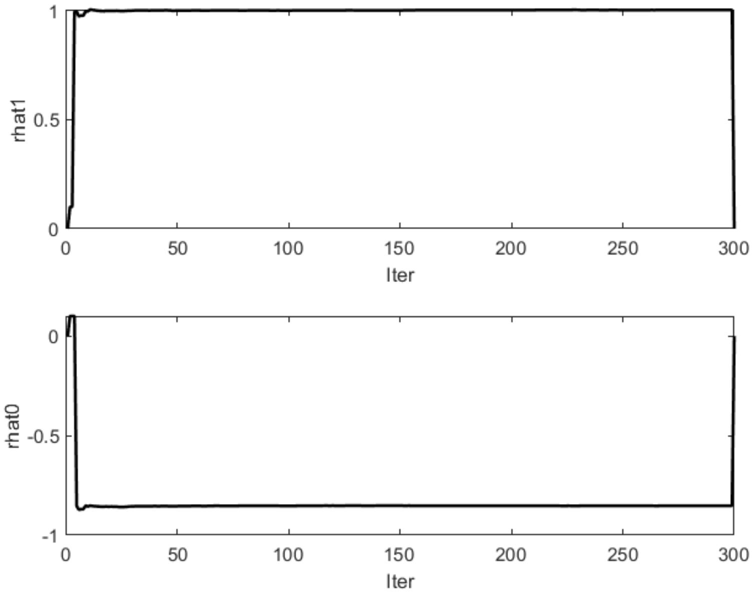

Figure 33. *Estimation of polynomial R using the direct pole placement controller with color noise*

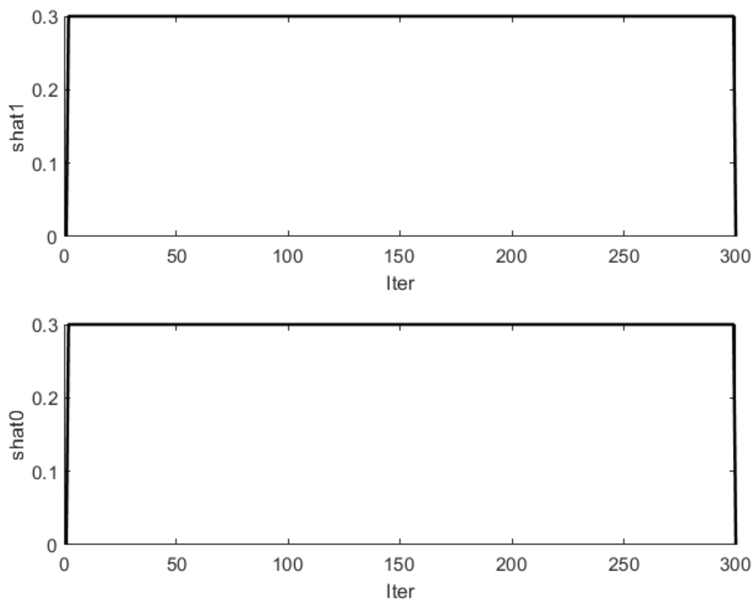

Figure 34. *Estimation of polynomial S using the direct pole placement controller with color noise*

The parameters of the R controller have converged to their correct values, but the parameters of the S controller have not been able to converge to the desired values because of the color noise, which can be caused by the polynomial of the noise. So, in this case, the controller could not work well.

1.3 Comparison of results for indirect and direct methods

In the Table below, the controllers designed by direct and indirect methods and in the presence of white and color noise are compared. The comparison results are given in Table 1.

Table 1. Comparison of direct and indirect methods

Controller	Output Variance	Output Mean	Input Variance	Input Mean
Indirect with white noise	1.031855	-1.307264e-02	5.232295e-01	-1.939262e-02
Indirect with color noise	1.167418	1.044827e-02	7.169496e-01	-3.881307e-03
Direct with white noise	2.832689	1.482932e-01	2.148741	8.927768e-02
Direct with color noise	2.084380	-4.437278e-03	3.994534e+01	5.182482e-02

As it is clear from the Table above, the control signal in direct method, has a high variance. Also, the variance of the output and the controller is better for the indirect case and a better controller is designed in this case. The reason is that the equations used in the book are specific to the without noise case, so polynomial of color noise or white noise reduces the system performance.

1.4 Indirect STR with color noise for min-phase system

1.4.1 Non-adaptive

Minimum variance controller:

This controller tries to minimize the variance of the output, but it has no control over the variance of the control signal. Considering that the desired system has a zero in the numerator and two poles in the denominator, its relative degree is equal to:

$$d_0 = \deg A - \deg B = 2 - 1 = 1$$

According to the Diophantine equation, we have:

$$q^{d_0-1}C(q) = A(q)F(q) + G(q) \Rightarrow C(q) = A(q)F(q) + G(q)$$

$$\deg F = d_0 - 1 = 1 - 1 = 0 \rightarrow F = 1\ (monic)$$

$$C(q) = A(q) + G(q) \Rightarrow G(q) = C(q) - A(q)$$

So, the control input is as follows:

$$u(t) = -\frac{G(q)}{B(q)F(q)}y(t) = -\frac{S(q)}{R(q)} = \frac{C(q) - A(q)}{B(q)}y(t)$$

The simulation results of this case are given in Figures 35 to 38.

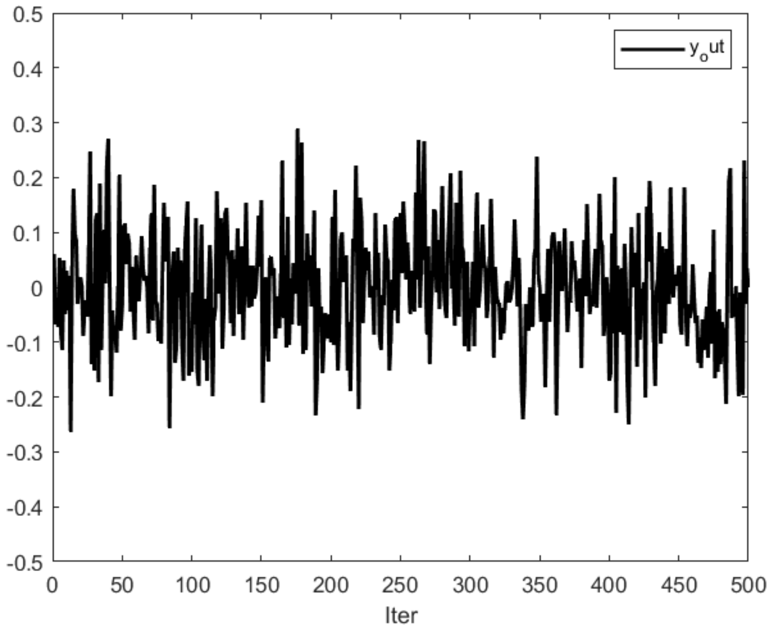

Figure 35. *System output of non-adaptive minimum variance controller for min phase system*

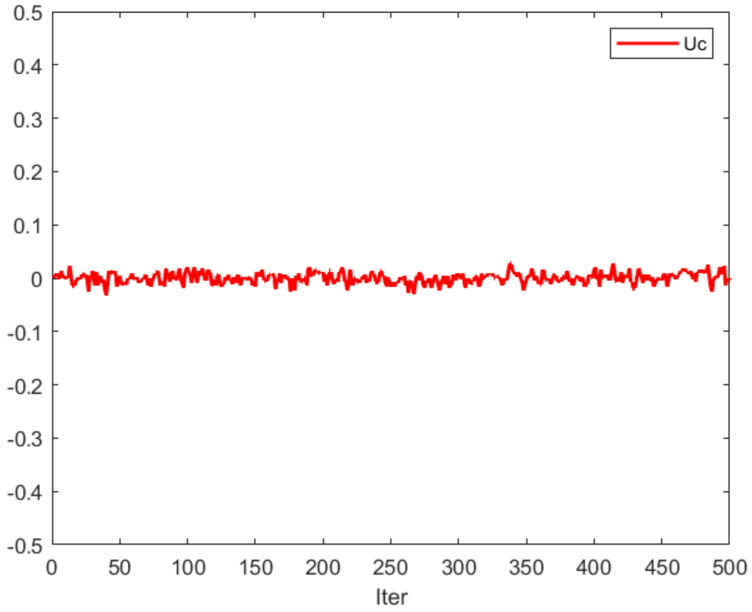

Figure 36. *Control signal of non-adaptive minimum variance controller for min phase system*

As it is clear from the figures above, the variance of the control signal and the output, in the case with color noise, is low, and the controller in this case has been able to show good performance and reduce its variance to its lowest value. Also, in this case, the variance of the control signal is even less than the variance of the output signal, which is caused by the dynamics of the system and the controller.

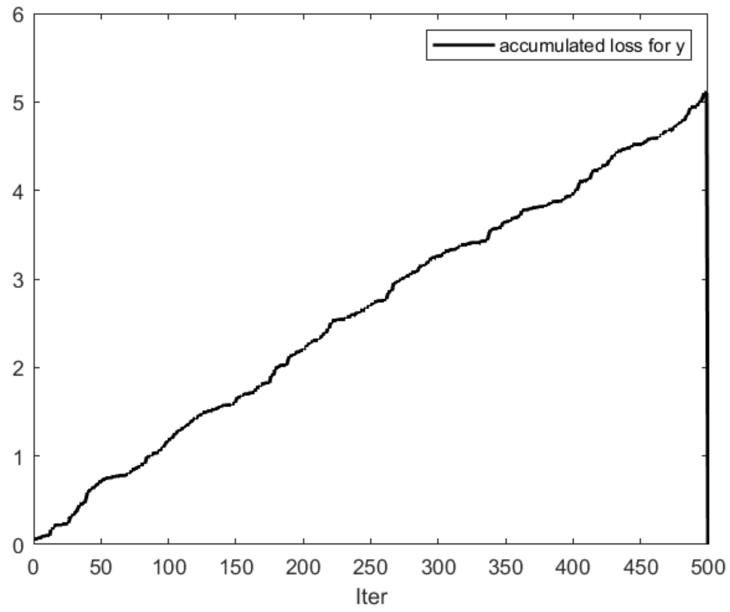

Figure 37. *Accumulated loss of y for non-adaptive minimum variance controller for min phase system*

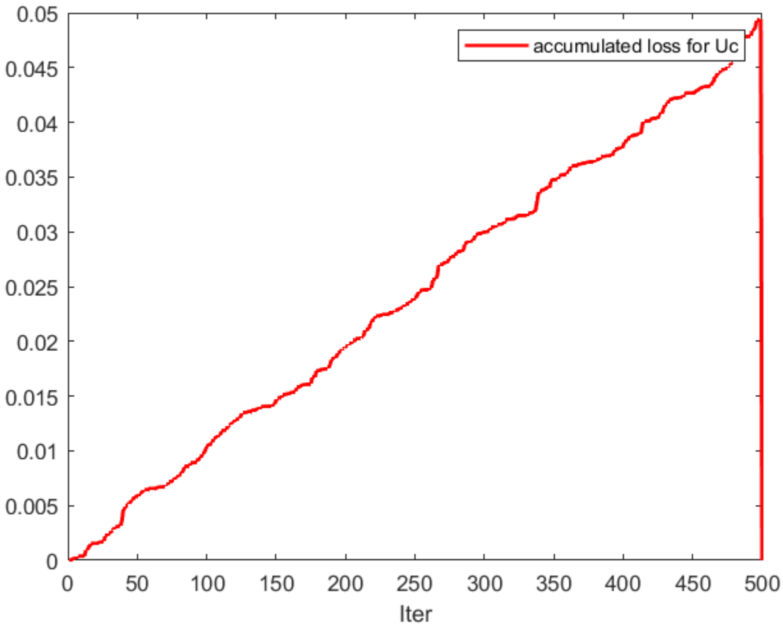

Figure 38. *Accumulated loss of Uc for non-adaptive minimum variance controller for min phase system*

The Figures above, show the accumulated losses of output and control signals. As it is clear from the figures above, the accumulated loss of the output signal is increasing with a slope. Also, the accumulated loss in this case is the lowest value for the output because the controller used in this section is the minimum variance controller.

In these cases, color noise parameters are considered as follows:

$$C(q) = (q - 0.6)(q - 0.17) = q^2 - 0.77q + 0.102$$

Moving average controller:

In this case, in the loss function, in addition to the output variance, the variance of the control signal is considered so that we can obtain more desirable control signals. but instead, the variance of the output signal will become worse. The Figures 39 to 42, show the simulation results.

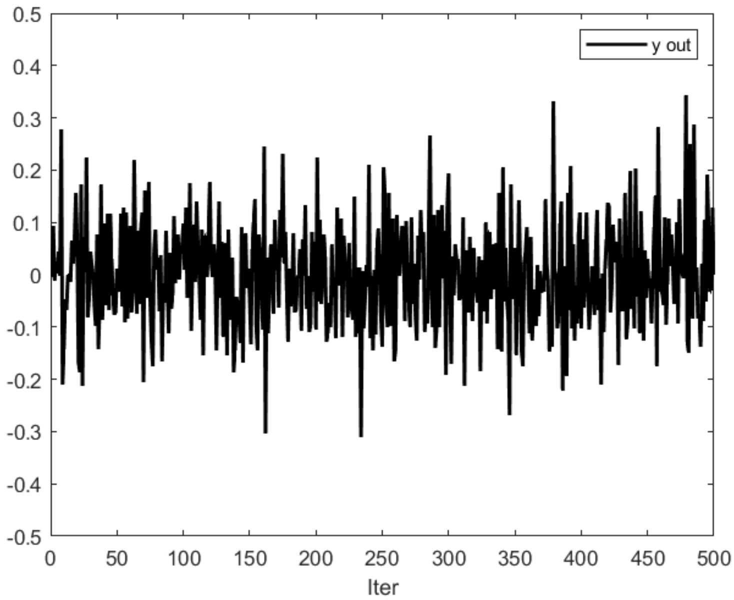

Figure 39. *System output of non-adaptive moving average controller for min phase system*

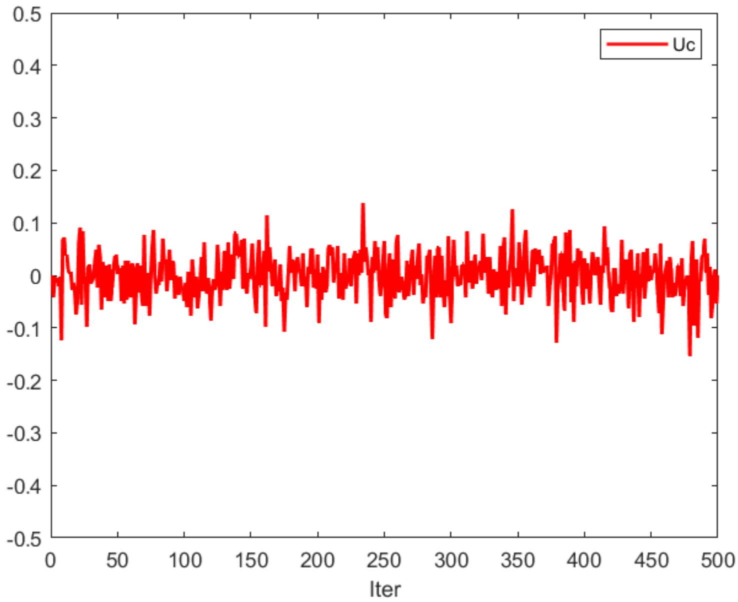

Figure 40. Control signal of non-adaptive moving average controller for min phase system

In this case, the output variance is higher than the minimum variance controller, because in addition to the output error, the control signal is also considered.

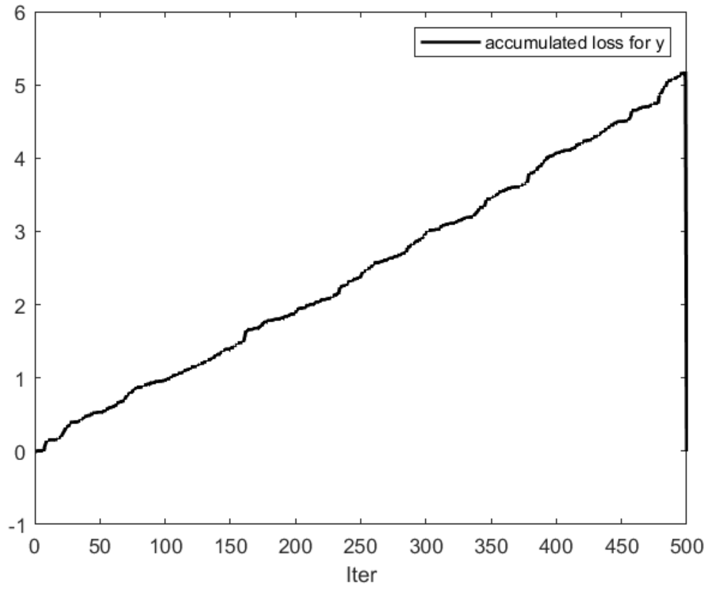

Figure 41. Accumulated loss of y for non-adaptive moving average control for min phase system

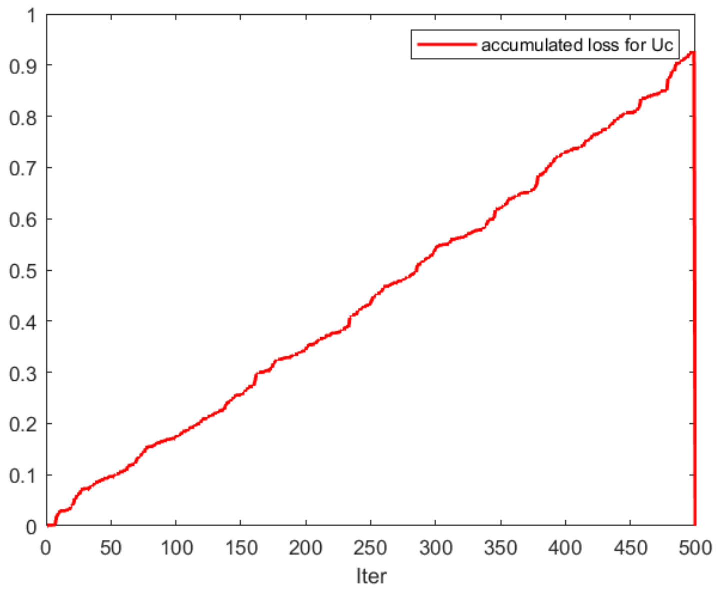

Figure 42. Accumulated loss of Uc for non-adaptive moving average control for min phase system

In this case, the output accumulated losses have increased compared to the previous case. Also, the accumulated loss has increased for the control signal. In general, in the case that the system is non-adaptive, both for the moving average controller and for the minimum variance controller, the variance of the output and control signal is small.

1.4.2 Adaptive

Minimum variance controller:

In this case, the parameters of the transfer function are estimated first, and in each iteration, by solving the Diophantine equation and using the indirect method, the minimum variance controller parameters are obtained. Also, because we have color noise in the system, we have performed the identification process with the ELS method. The Figures 43 to 51 show the simulation results which are estimated by ELS algorithm in each iteration.

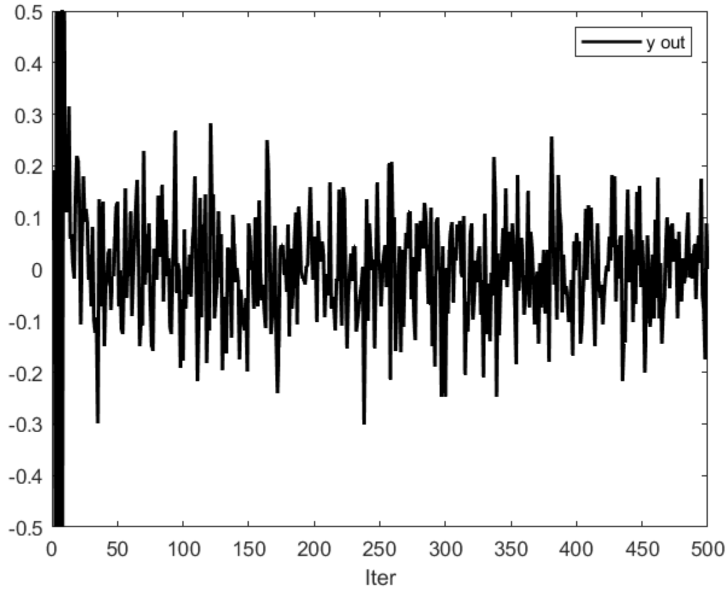

Figure 43. System output of adaptive minimum variance controller for min phase system

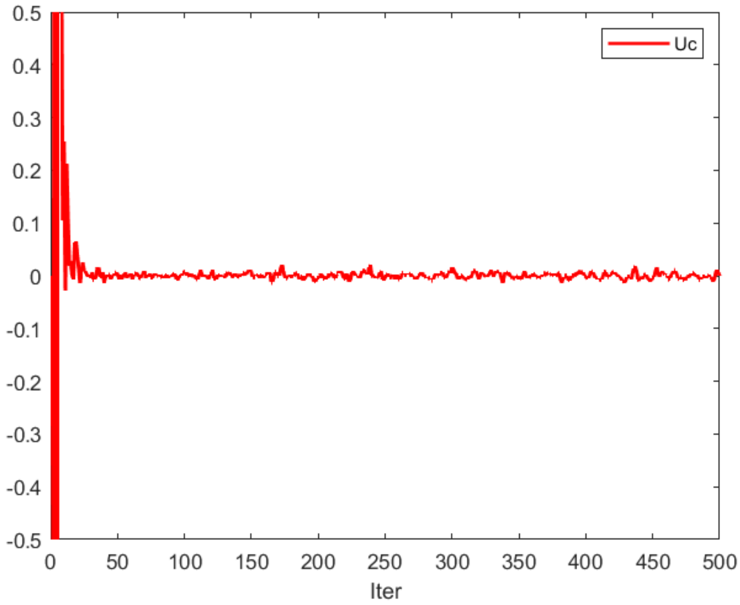

Figure 44. Control signal of adaptive minimum variance controller for min phase system

As can be seen from the Figures above, in the adaptive case, because the system parameters are initially unknown, the output variance and control effort are very large. This case has performed worse than the non-adaptive case, which is due to the incorrect identification of color noise parameters. Therefore, the presence of color noise in the system also reduces the performance of the system.

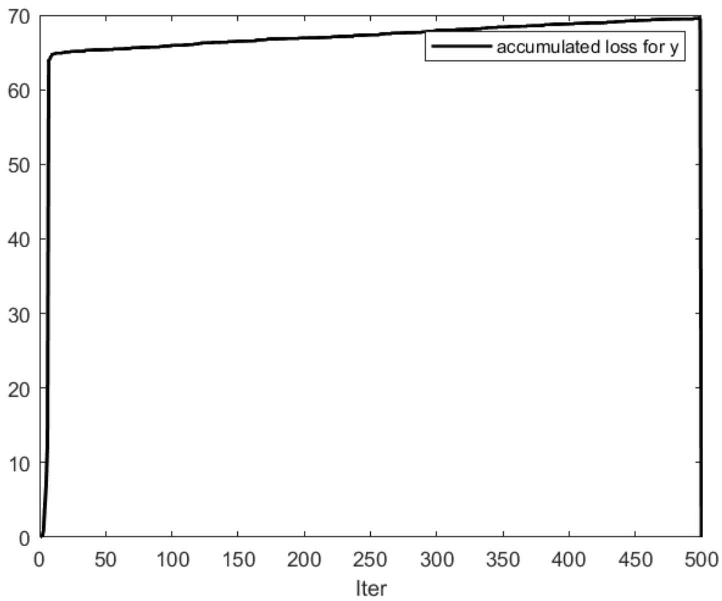

Figure 45. Accumulated loss of y for adaptive minimum variance controller for min phase system

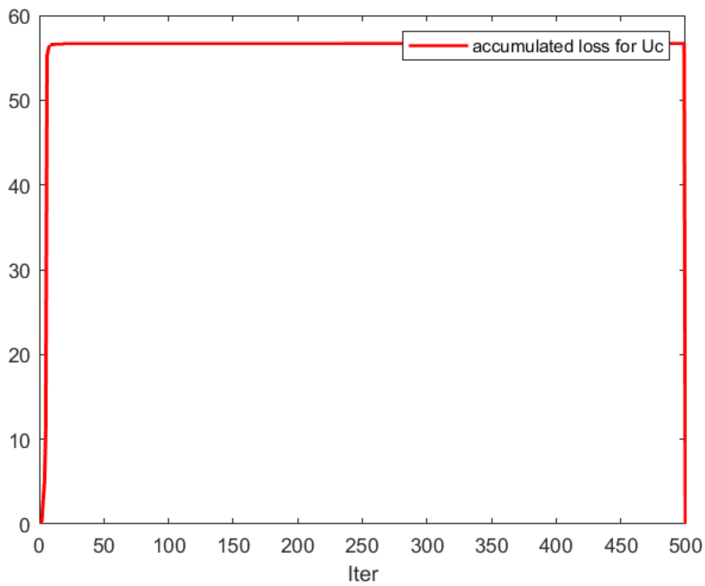

Figure 46. Accumulated loss of Uc for adaptive minimum variance controller for min phase system

Accumulated loss of output and control effort in this case is much more than the non-adaptive case, which is due to incorrect identification of color noise parameters that affect the output and the control signal.

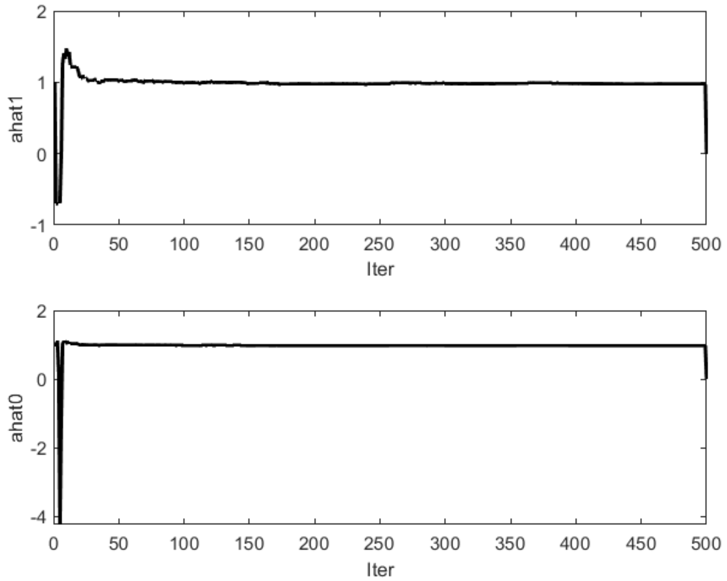

Figure 47. Denominator estimation for adaptive minimum variance controller for min phase system

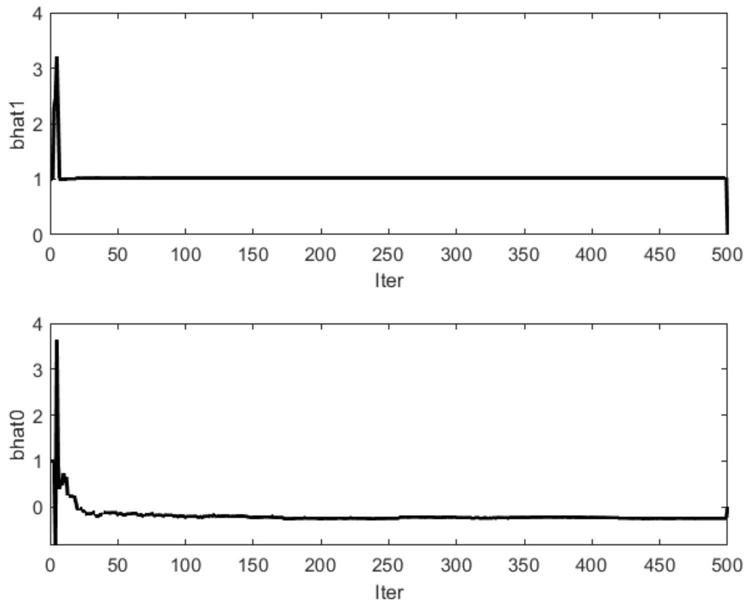

Figure 48. Nominator estimation for adaptive minimum variance controller for min phase system

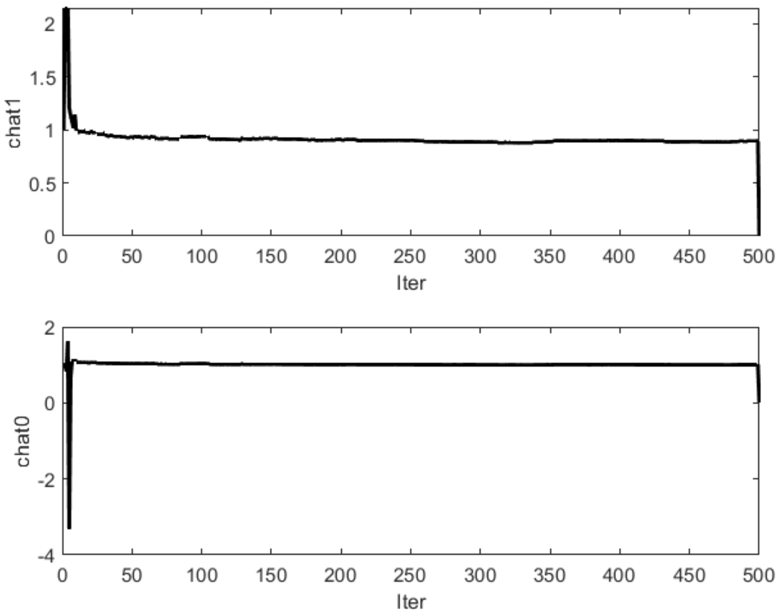

Figure 49. Noise coefficient estimation for adaptive minimum variance controller for min phase system

In the Figures above, polynomials A and B are not well estimated. For example, in polynomial A, the parameters have been wrongly estimated, and this has caused an increase in output variance and accumulated loss of output.

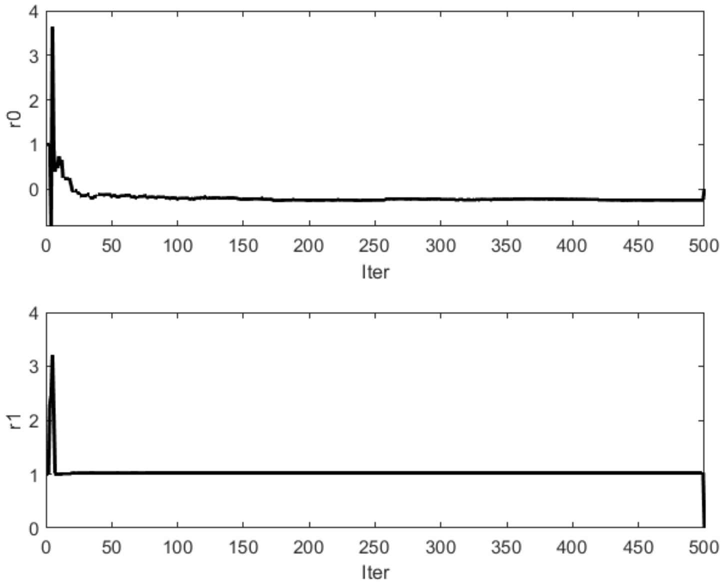

Figure 50. Estimation of polynomial R parameters for adaptive minimum variance controller for min phase system

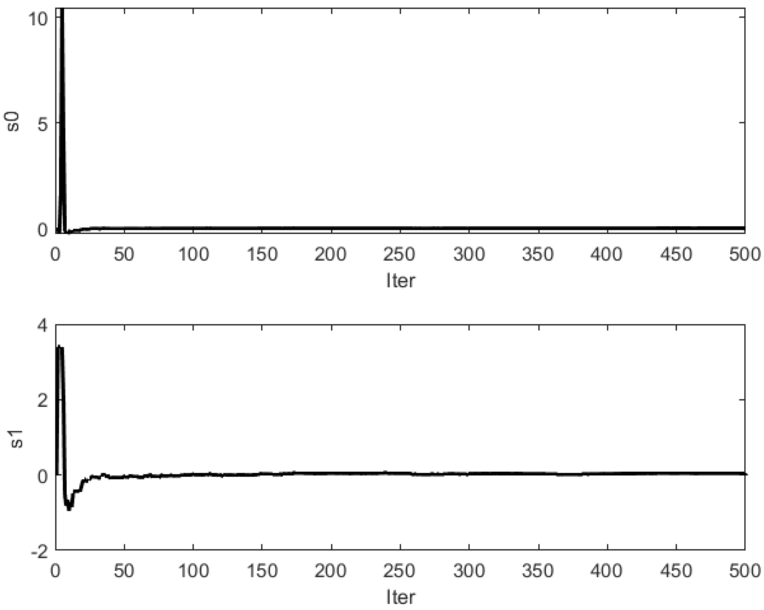

Figure 51. Estimation of polynomial S parameters for adaptive minimum variance controller for min phase system

In this case, the controller parameters have converged to wrong values because the main parameters of the system are not well estimated.

Moving average controller:

In this case, we expect that the output variance will increase from its correct value, which is equal to the noise variance value, but instead, the variance of the control signal will decrease slightly. In the figures below, the simulation results of this case are given.

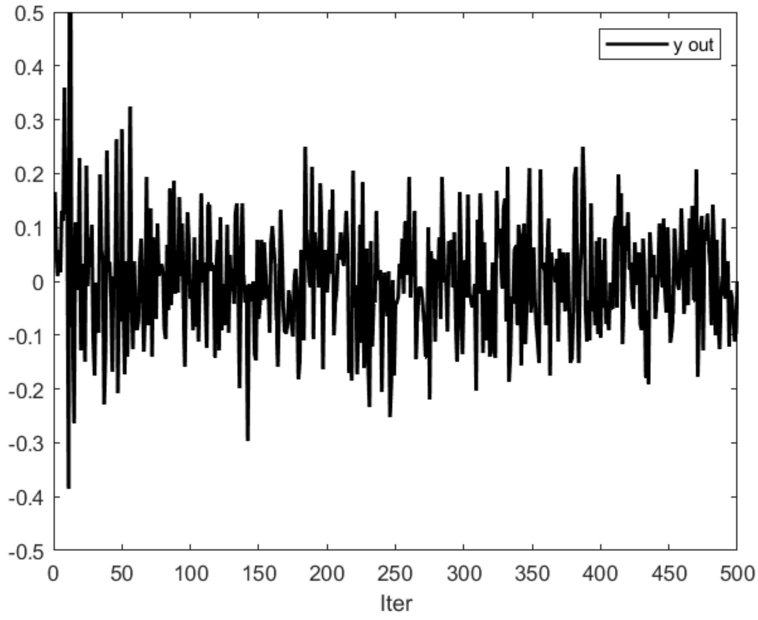

Figure 52. System output of adaptive moving average controller for min phase system

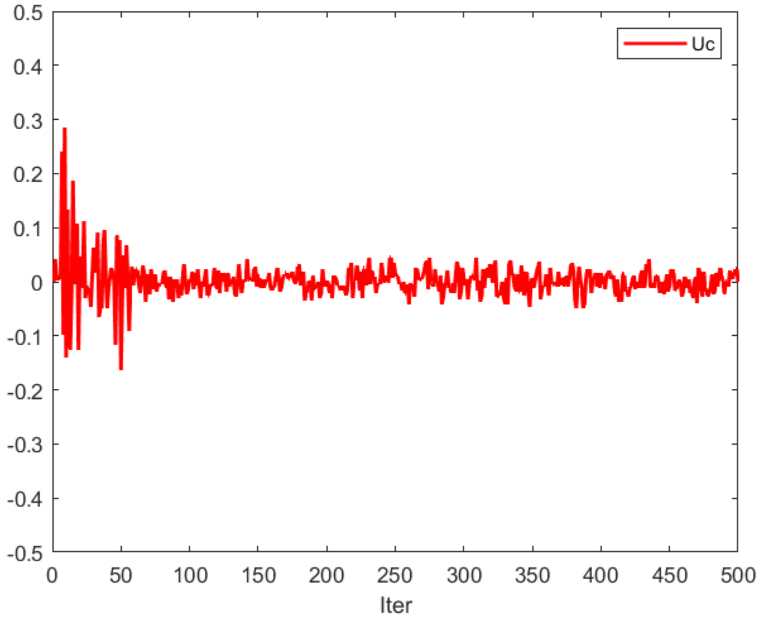

Figure 53. Control signal of adaptive moving average controller for min phase system

As it is clear from the Figures above, the output variance in this case is much higher than the minimum variance controller case. In this case, the variance of the control effort has also been slightly reduced and the polynomial parameters of the noise have been estimated better, and this has caused the variance of the control signal to be better than the minimum variance controller.

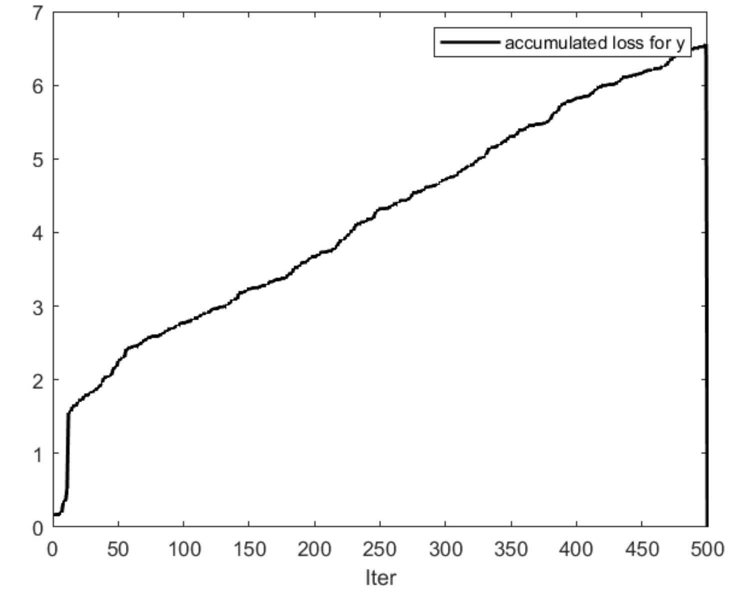

Figure 54. Accumulated loss of y for adaptive moving average controller for min phase system

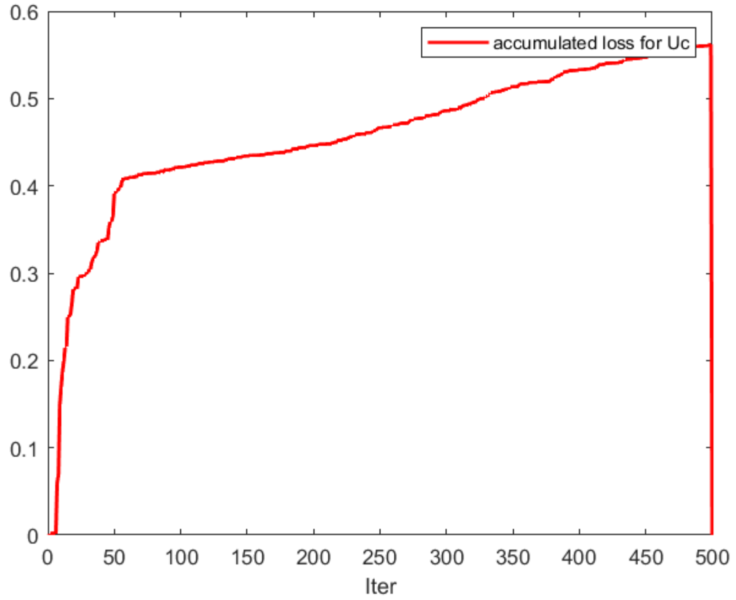

Figure 55. Accumulated loss of Uc for adaptive moving average controller for min phase system

The accumulated loss of output and control effort in this case is much less than the minimum variance controller case because the noise polynomial is better estimated in the presence of color noise.

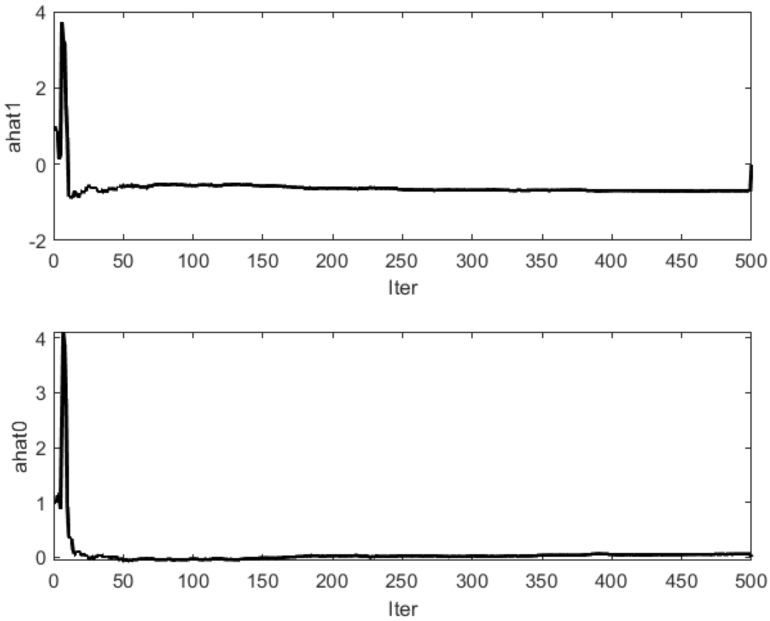

Figure 56. Denominator estimation of adaptive moving average controller for min phase system

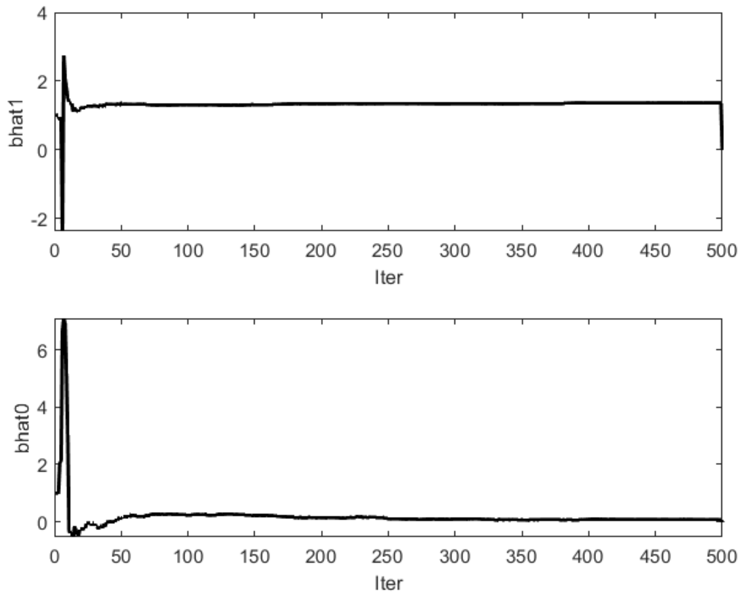

Figure 57. Nominator estimation of adaptive moving average controller for min phase system

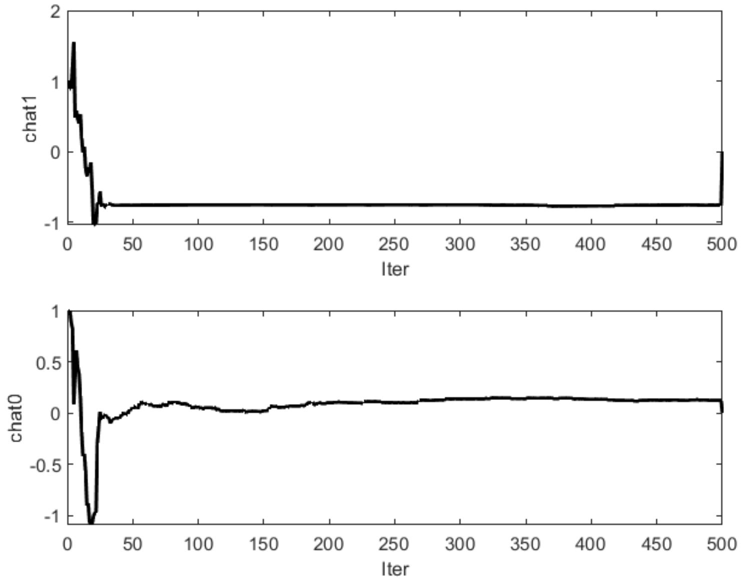

Figure 58. Estimation of noise coefficient for adaptive moving average controller for min phase system

The polynomial parameters of the system in this case are estimated much better than the minimum variance controller, because this controller shows more resistance to color noise and has better performance.

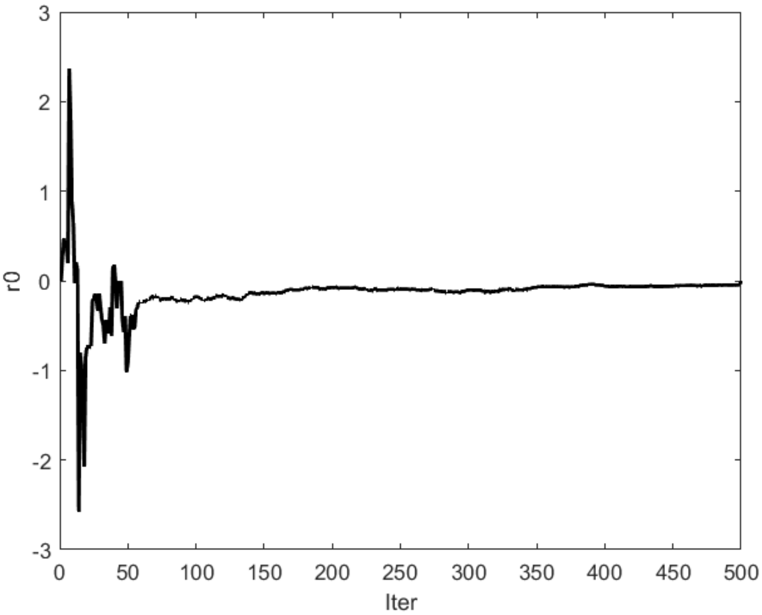

Figure 59. Estimation of R polynomial parameters for adaptive moving average controller for min phase system

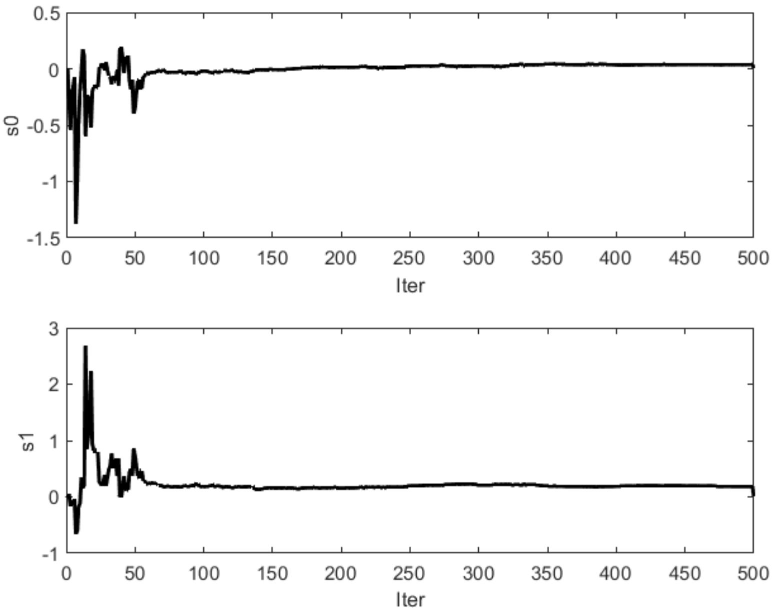

Figure 60. Estimation of S polynomial parameters for adaptive moving average controller for min phase system

The parameters of the controller are also well converged to the desired values, and this makes the variances and accumulated losses of the output and the control signal better.

1.5 Indirect STR with color noise for non-min-phase system

1.5.1 Non-adaptive

Minimum variance controller:

In this case, we have used non-minimum phase system. For this purpose, we have reversed the zero of the system so that it falls outside the unit circle. For the time when the system becomes non-minimum phase, it should be noted that the Diophantine equation must be changed and the B polynomial must be converted into a star polynomial whose zeros are reversed. In this case, the results of the simulation are shown in Figures 61 to 64.

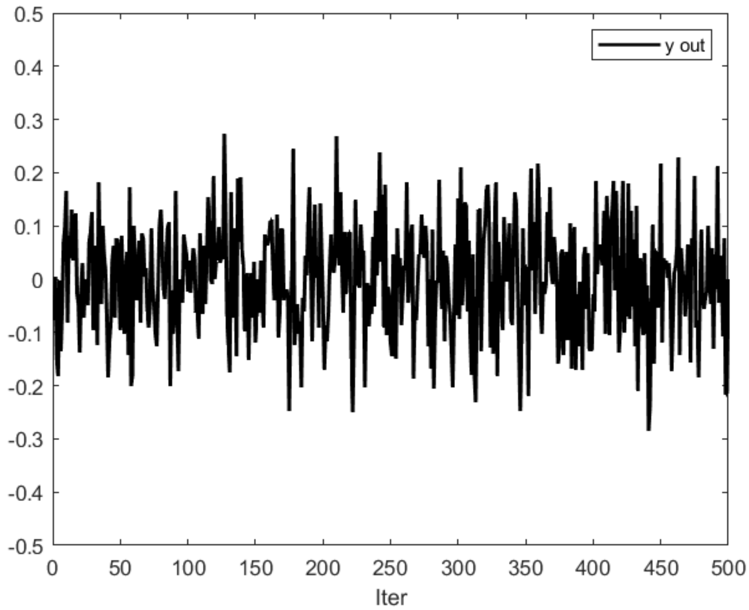

Figure 61. System output of minimum variance non-adaptive controller for non-min phase system

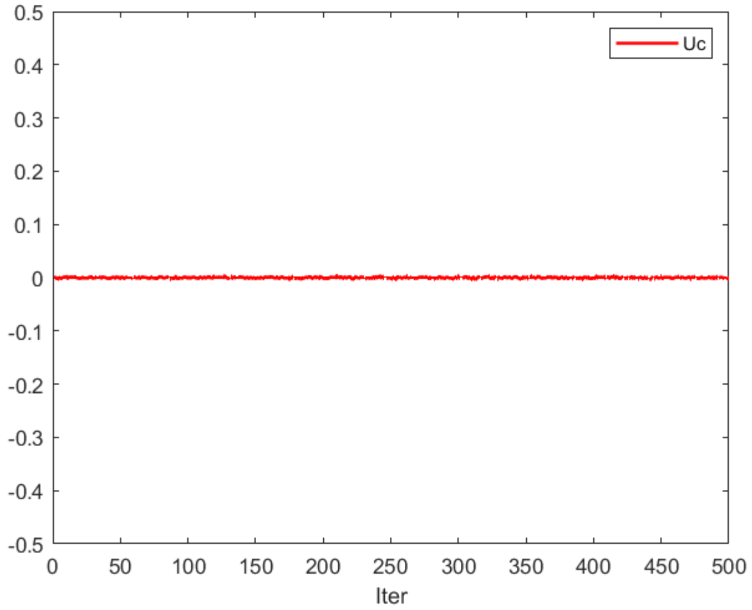

Figure 62. Control signal of minimum variance non-adaptive controller for non-min phase system

In this case, the output of the system has a higher variance than when the system is minimum phase. However, the control signal has an appropriate variance and because we have non-adaptive controllers, it shows better results. In the case where the system is non-minimum phase, we have instability and the necessary changes have been applied in solving the Diophantine equation.

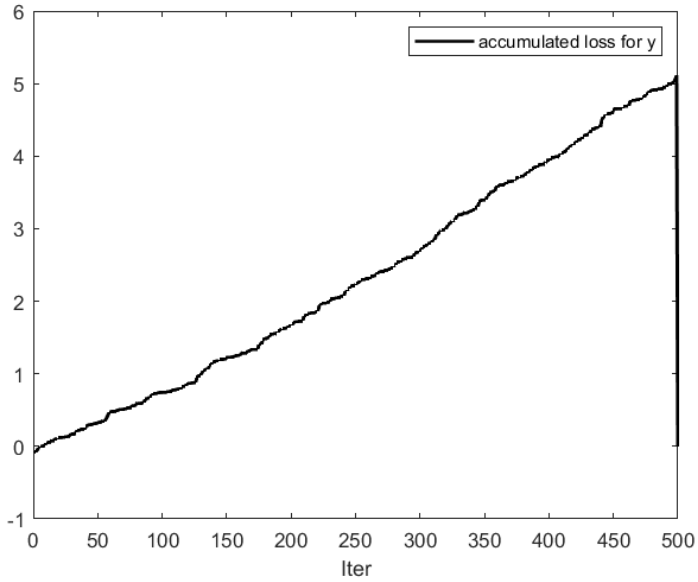

Figure 63. Accumulated loss of y for minimum variance non-adaptive controller for non-min phase system

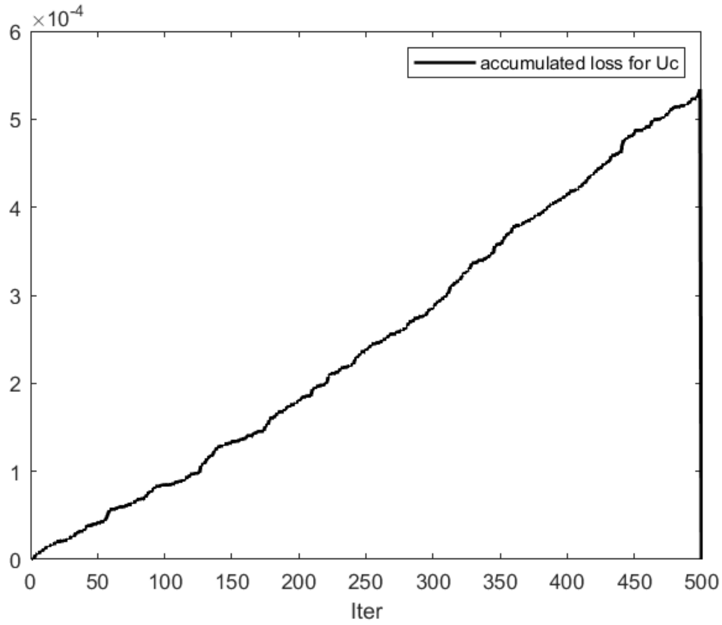

Figure 64. Accumulated loss of Uc for minimum variance non-adaptive controller for non-min phase system

In this case, the accumulated loss of the non-minimum phase system is the same as the minimum phase system, and a limited value has been obtained for the accumulated loss of the output and the accumulated loss of the control effort, which is due to the non-adaptive controller.

Moving average controller:

In this case, the simulation results are given in Figures 65 to 68.

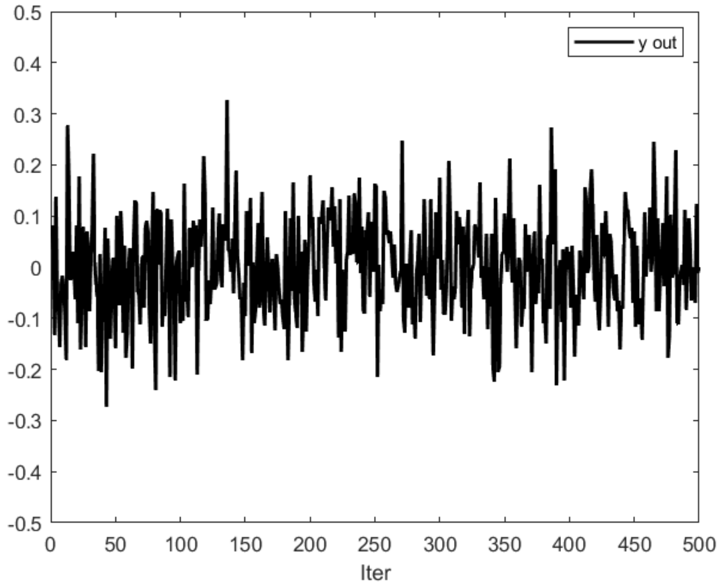

Figure 65. System output of moving average non-adaptive controller for non-min phase system

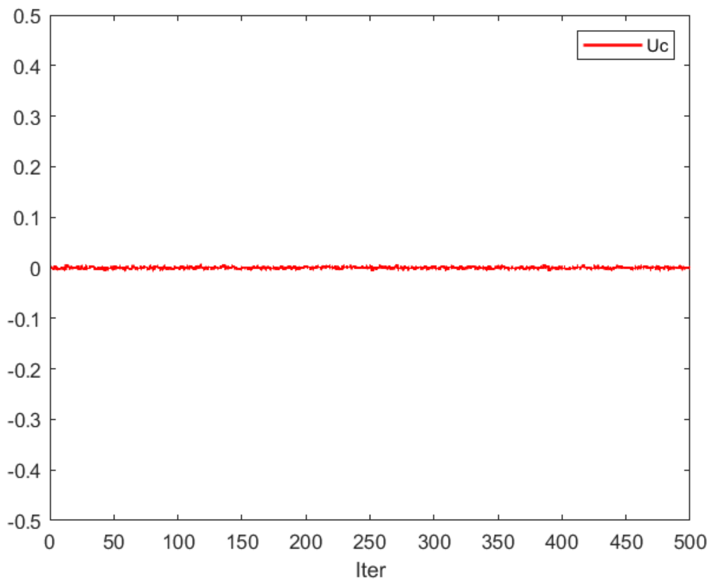

Figure 66. Control signal of moving average non-adaptive controller for non-min phase system

As it is clear from the Figures above, in this case, because we have used moving average controller, the output variance has increased. Also, the output signal variance has also increased slightly.

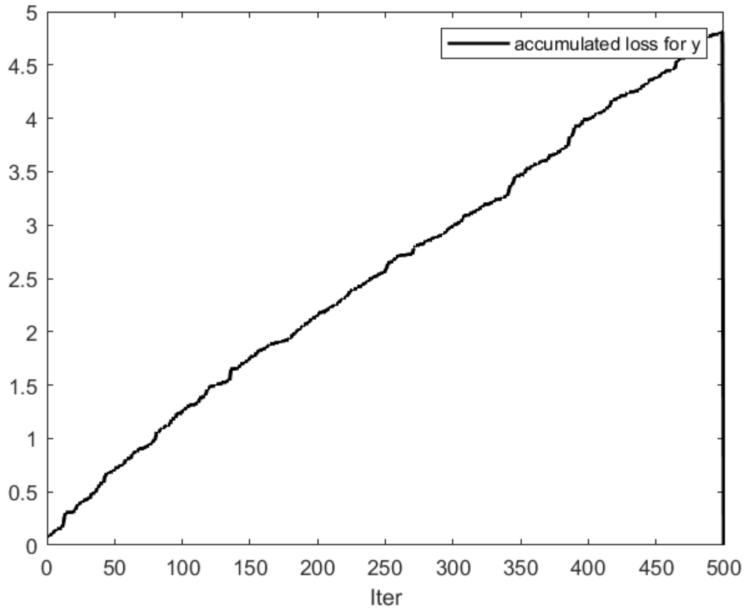

Figure 67. Accumulated loss of y for moving average non-adaptive controller for non-min phase system

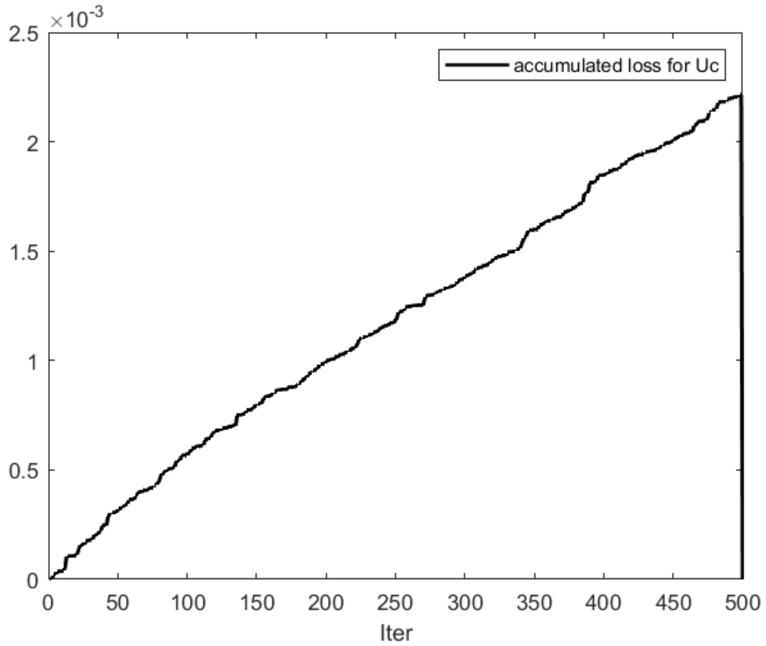

Figure 68. Accumulated loss of Uc for moving average non-adaptive controller for non-min phase system

Accumulated loss for the output and control signal in this case is slightly higher than the case where we used the minimum variance controller.

1.5.2 Adaptive

Minimum variance controller:

In this case, in the presence of color noise, first the system parameters should be identified by the ELS method, and then, by using the Diophantine equation, the controller parameters are obtained and finally, the control signal is calculated. In the figures below, the simulation results are given for this non-minimum phase system.

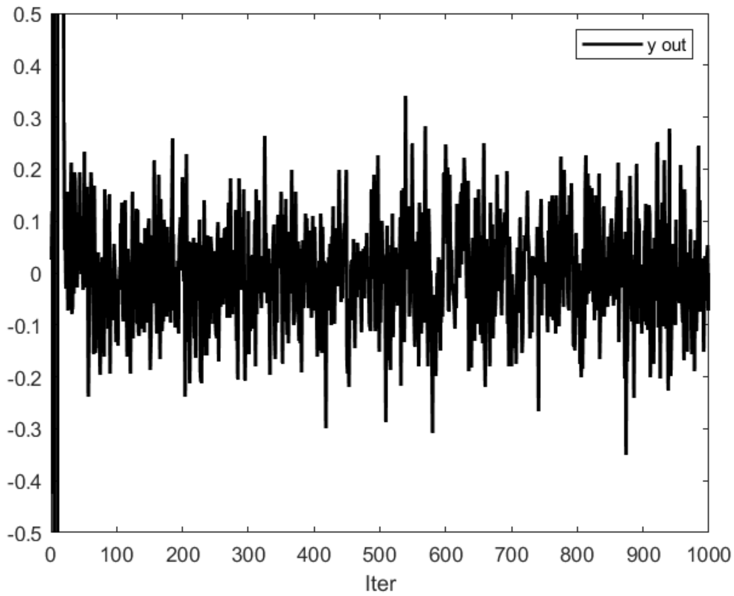

Figure 69. System output of minimum variance adaptive controller for non-min phase system

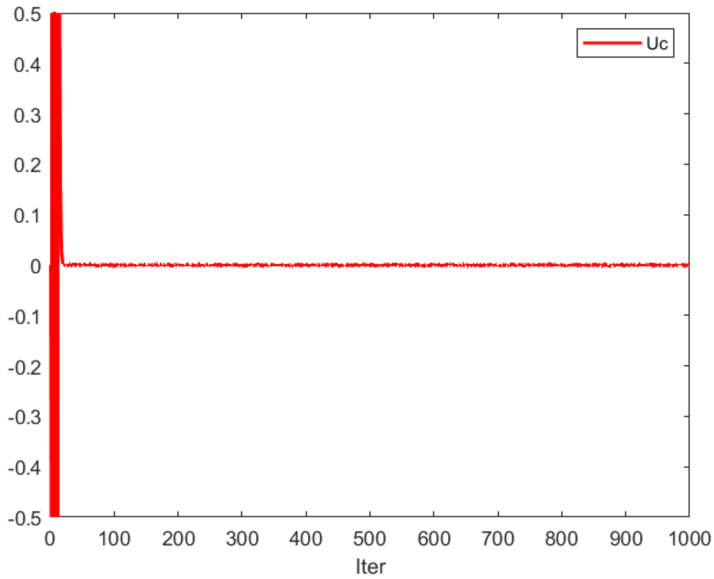

Figure 70. Control signal of minimum variance adaptive controller for non-min phase system

As it is clear from the Figures above, in the adaptive case, the variance of the system output and the control signal has increased compared to the non-adaptive case, which is due to the inaccurate estimation of the polynomial parameters of noise and polynomial A.

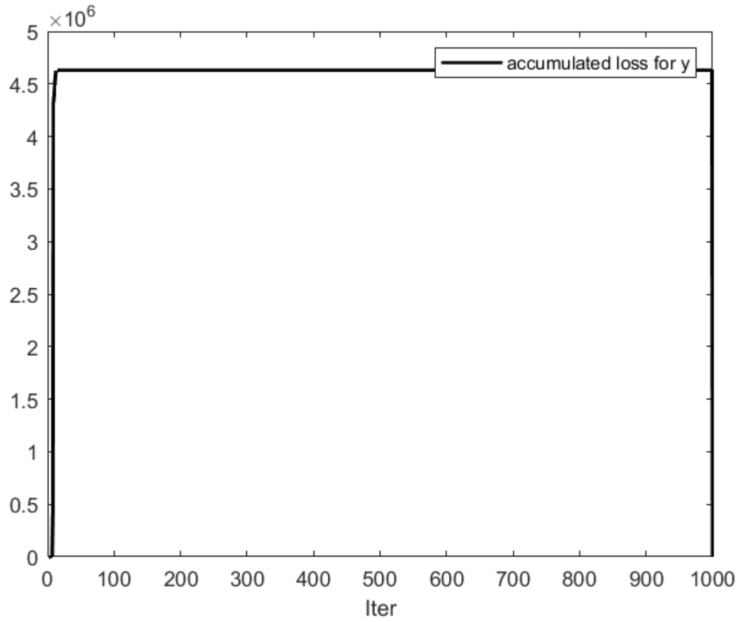

Figure 71. Accumulated loss of y for minimum variance adaptive controller for non-min phase system

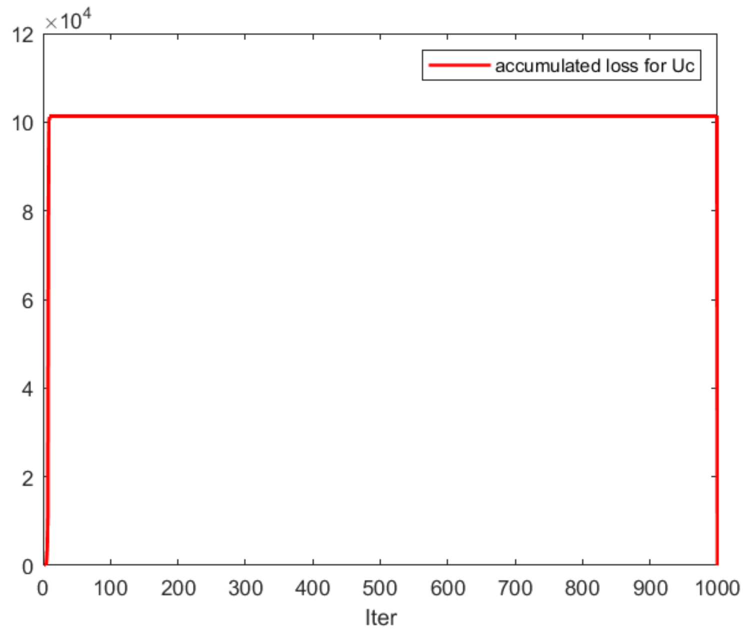

Figure 72. Accumulated loss of Uc for minimum variance adaptive controller for non-min phase system

The accumulated loss of the output and the output signal, in this case, is much higher than the non-adaptive case, which is due to the incorrect estimation of the polynomial parameters of the noise in the presence of color noise.

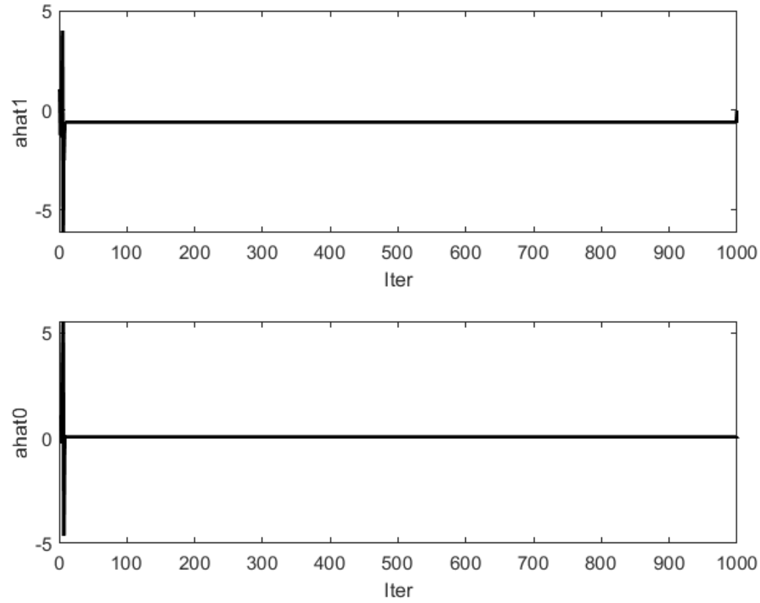

Figure 73. Estimation of denominator parameters of minimum variance adaptive controller for non-min phase system

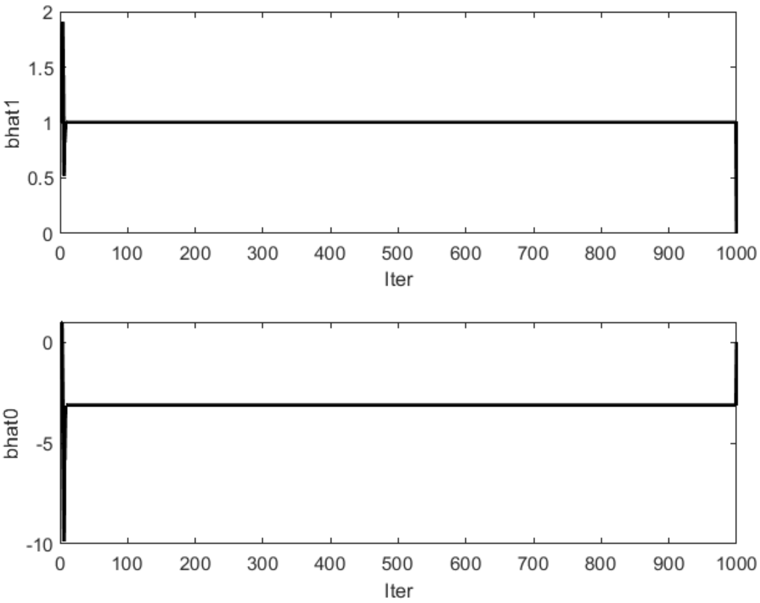

Figure 74. Estimation of nominator parameters of minimum variance adaptive controller for non-min phase system

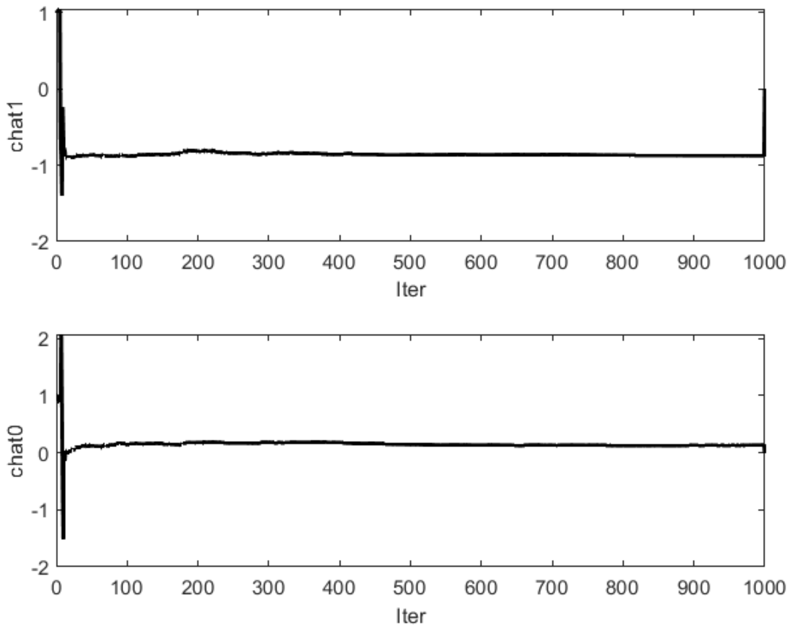

Figure 75. Estimation of color coefficient of minimum variance adaptive controller for non-min phase system

In the above Figures, it is clear that the parameters are not well estimated due to the presence of color noise in the system.

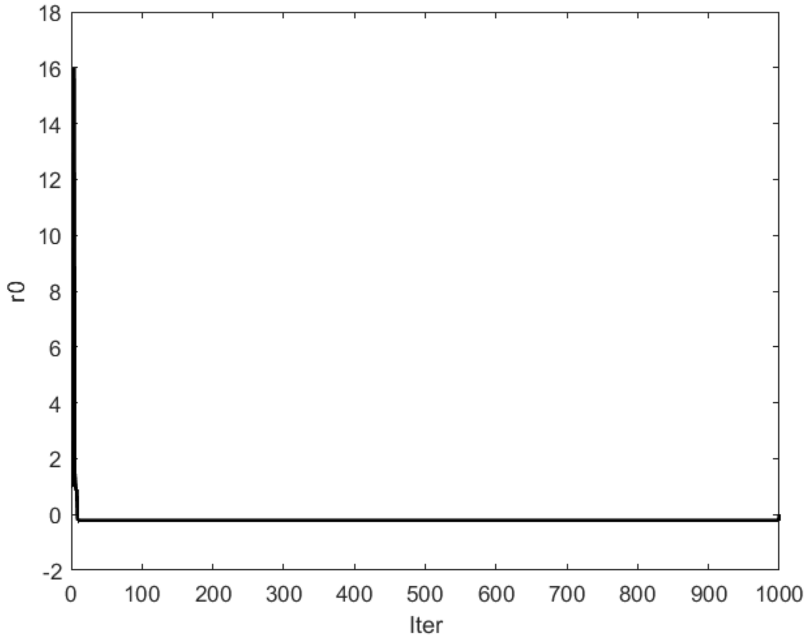

Figure 76. Estimation of R polynomial parameters of minimum variance adaptive controller for non-min phase system

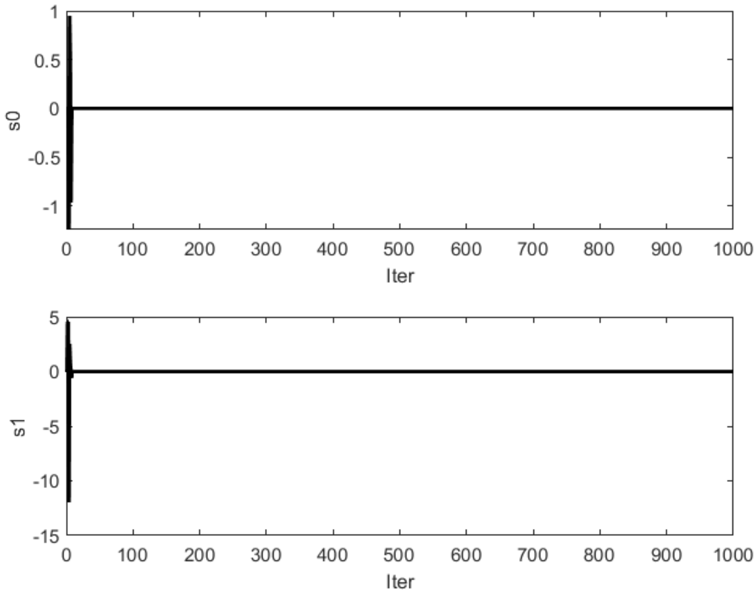

Figure 77. Estimation of S polynomial parameters of minimum variance adaptive controller for non-min phase system

The controller parameters are also not converged to the correct values, because the estimation of the parameters of the noise polynomial and the denominator polynomial of the transfer function has not been done well.

Moving average controller:

In this case, the system simulation results are given in Figures 78 to 86.

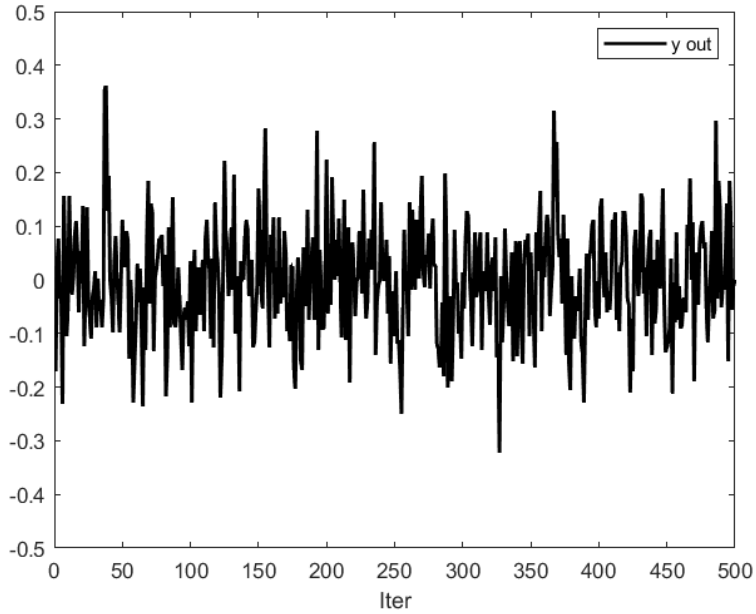

Figure 78. System output of adaptive moving average controller for non-minimum phase system

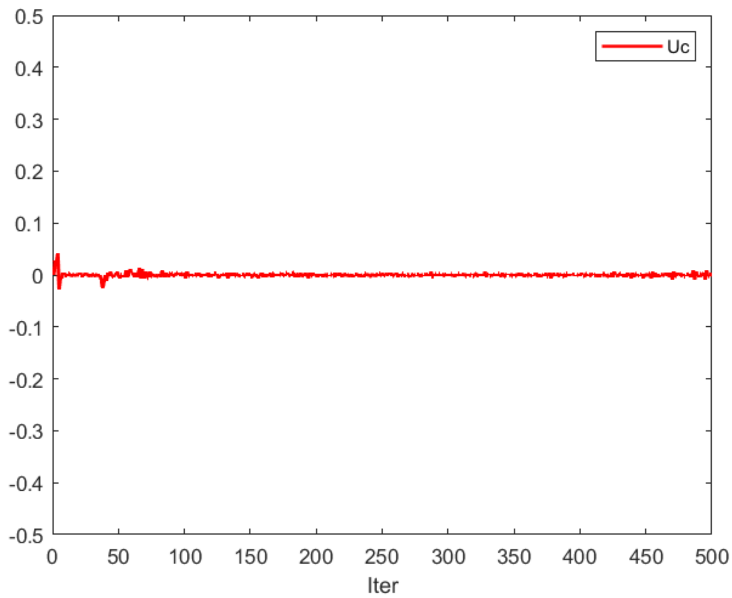

Figure 79. Control signal of adaptive moving average controller for non-minimum phase system

In this case, the variance of the output has increased compared to the minimum variance controller, because in the loss function of this controller, in addition to the output, there is also a control input. At the beginning of identifying the parameters, it is clear that the output values and the control signal are large values, which is because the identification has not been done well in the early stages.

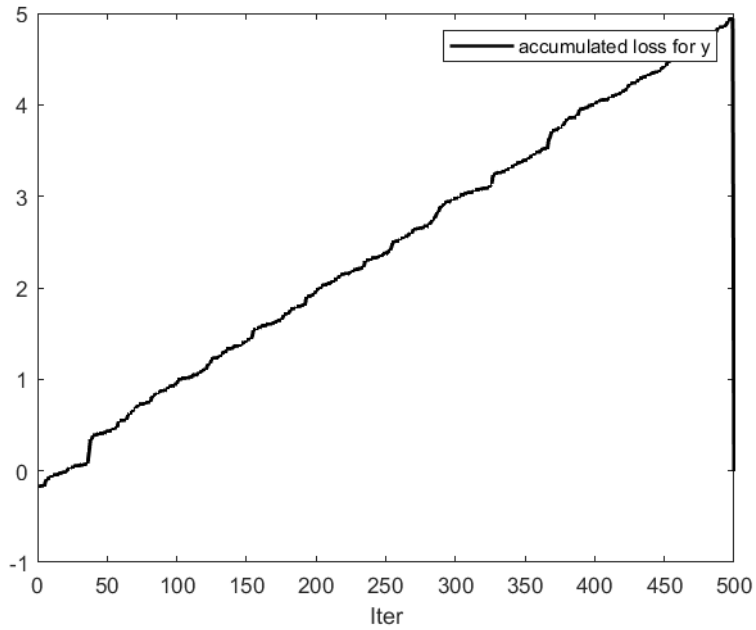

Figure 80. Accumulated loss of y for adaptive moving average controller for non-minimum phase system

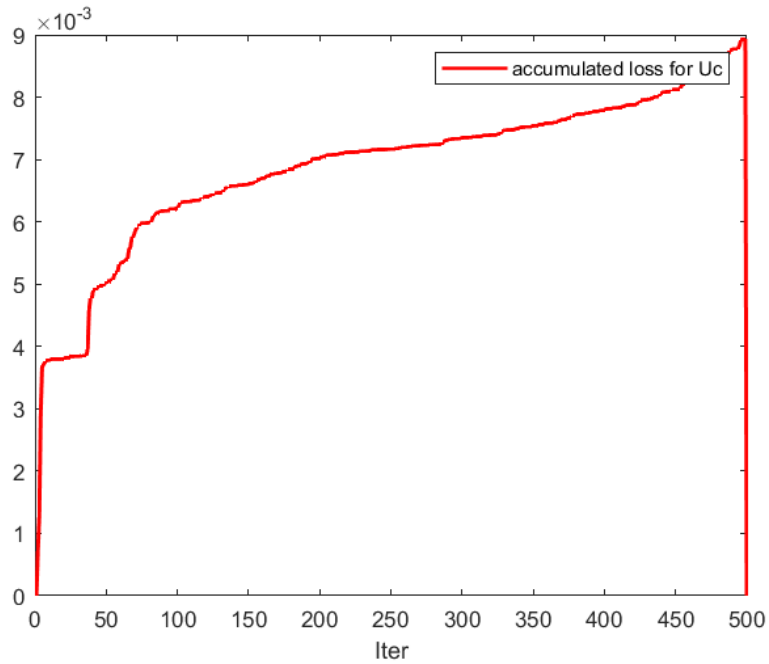

Figure 81. Accumulated loss of Uc for adaptive moving average controller for non-minimum phase system

In this case, the accumulated losses of the control input and output of the system also have greater values than the minimum variance controller, which is a very high initial value, due to the fact that the parameters are not well identified at the initial times, and as a result, the output and the control input have had large values and this has caused an increase in accumulated losses at the beginning of time. But, after the correct convergence of the parameters, it can be seen that these variables have changed and the system became stable.

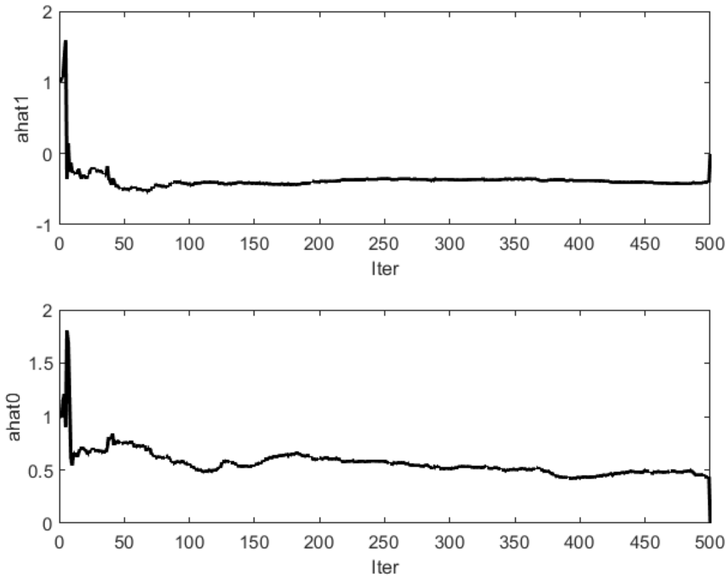

Figure 82. Denominator parameters estimation for adaptive moving average controller for non-minimum phase system

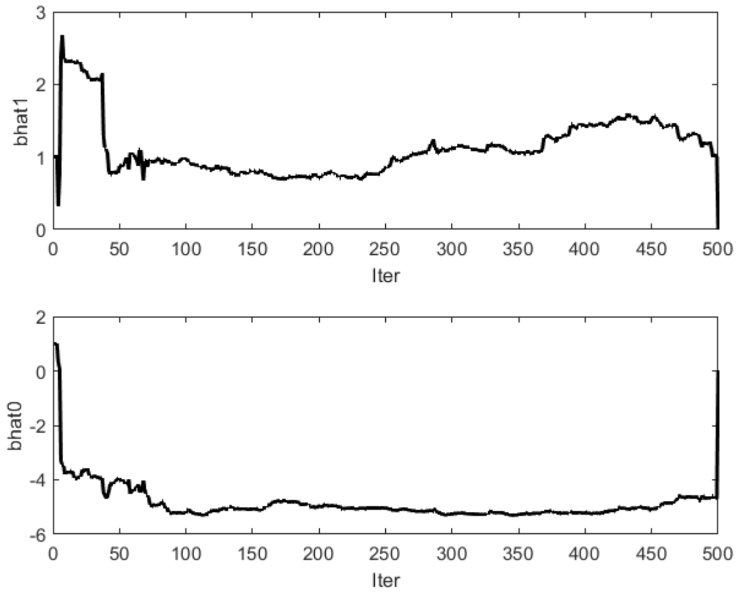

Figure 83. Nominator parameters estimation for adaptive moving average controller for non-minimum phase system

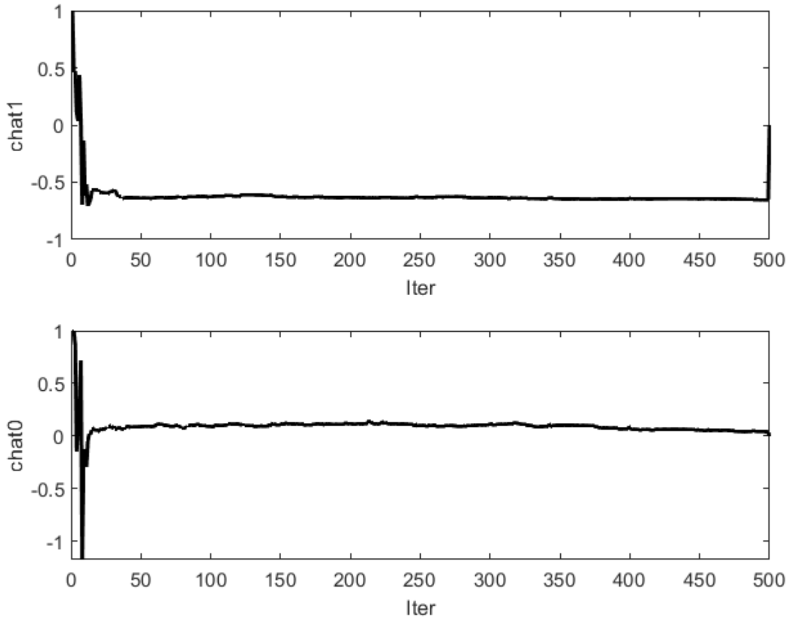

Figure 84. Color coefficient estimation for adaptive moving average controller for non-minimum phase system

In this case, compared to the case where we used the minimum variance controller, the parameters are well estimated and have converged to their correct values, because this controller has a single formulation for the minimum and non-minimum phase systems and changing the system, will not affect the performance of the controller.

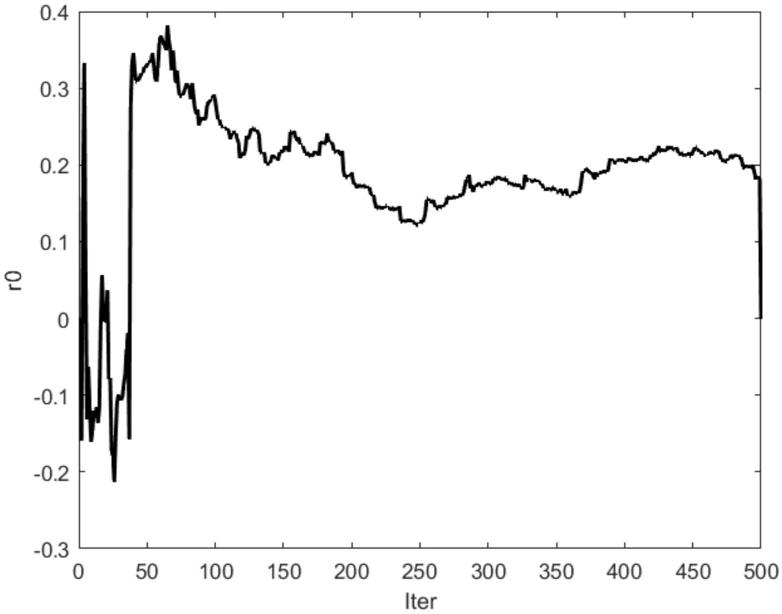

Figure 85. R polynomial parameter estimation for adaptive moving average controller for non-minimum phase system

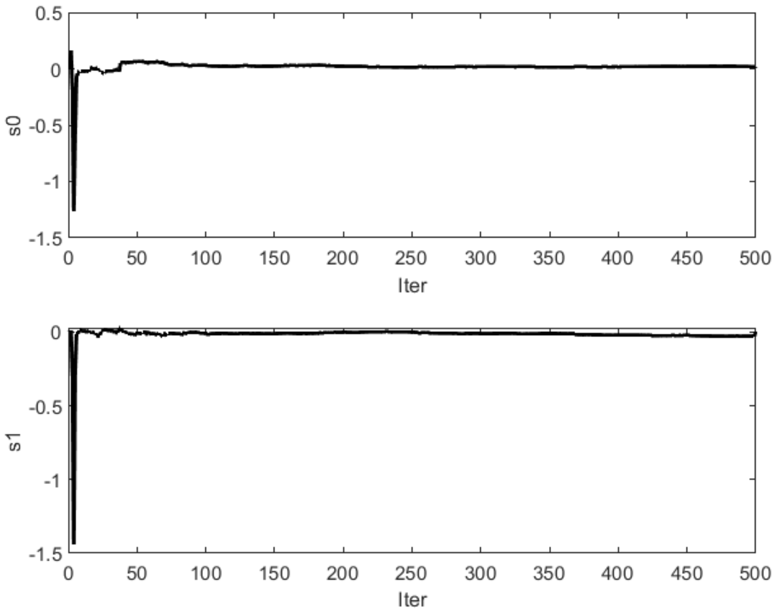

Figure 86. S polynomial parameter estimation for adaptive moving average controller for non-minimum phase system

Considering that the system parameters have converged to correct values, the controller parameters have also converged accurately.

1.6 Comparison of results for min and non-min-phase systems

In this section, we will compare the controllers designed in the previous two sections in terms of variance, average output and control effort. First, a comparison is made in the non-adaptive case as shown in Table 2.

Table 2. Comparison of non-adaptive controllers for minimum and non-minimum phase systems

Controller	Output Variance	Output Mean	Input Variance	Input Mean
Min phase MV	1.013919e-02	-2.393982e-04	3.287593e-04	-8.353776e-05
Non-min phase MV	9.590746e-03	-4.858868e-05	7.799802e-08	-6.635939e-07
Min phase MA	9.584066e-03	1.515429e-04	8.867718e-04	1.012586e-04
Non-min phase MA	1.004471e-02	1.095611e-04	5.012055e-06	-4.882493e-06

In the case where we have used the MV controller, the results for the non-minimum phase system are better in terms of output variance, also, the control input variance is improved for the non-minimum phase system. When we used the MA controller, the results were better in terms of output variance for min-phase system and in terms of control input variance, the non-minimum phase system was better.

Now, the results for the adaptive case are given in the Table 3.

Table 3. Comparison of adaptive controllers for minimum and non-minimum phase systems

Controller	Output Variance	Output Mean	Input Variance	Input Mean
Min phase MV	1.042807e-02	3.416885e-04	1.156839e-03	3.243238e-04
Non-min phase MV	2.476213e+01	-3.309587e-01	1.509907	7.271392e-02
Min phase MA	1.043262e-02	-8.257650e-04	1.189323e-03	-9.158004e-04
Non-min phase MA	3.493976e-02	-2.140744e-02	3.011119e-03	6.965478e-03

In this case, if the system is minimum phase, we have had a better response for both controllers.

Overall, the MA controller has shown better performance than the minimum variance controller. Also, in the non-adaptive case, it has shown better results for the output variance, and finally, the minimum phase system had a better performance.

1.7 Comparison of 1.3 and 1.6 results

According to the results, the STR method cannot remove the color noise well in the control. For this reason, minimum variance and moving average control methods are used to control such systems. As seen in the first part, it can be said that conventional self-adjusting regulators cannot minimize the variance. Meanwhile, the minimum variance method was able to minimize the variance of the output despite the inaccurate identification. Also, the moving average method was able to provide a desirable control input despite the low variance compared to STR. But as it is clear from the results presented in the first part, the normal STR method has the ability to track a non-zero signal, while the minimum variance and moving average methods do not have this ability. Also, it was found that the minimum variance method, for a minimum phase system, performs much better than a non-minimum phase system, especially in adaptive case.

1.8 Influence of model order

In this case, if the parameters of the model are added, the model becomes under parameter. This means that estimation will be more difficult and therefore the system will be difficult to control. Therefore, in this part, it is assumed that $1/(q + 0.49)$ is added in the denominator.

We have done the simulation for the minimum variance controller in adaptive case and for the minimum phase system.

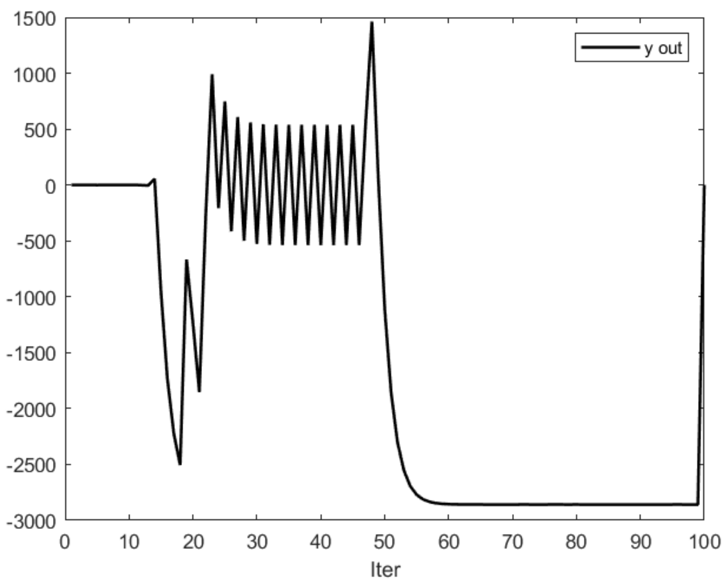

Figure 87. System output for adaptive minimum variance controller for minimum phase system and in under parameter case

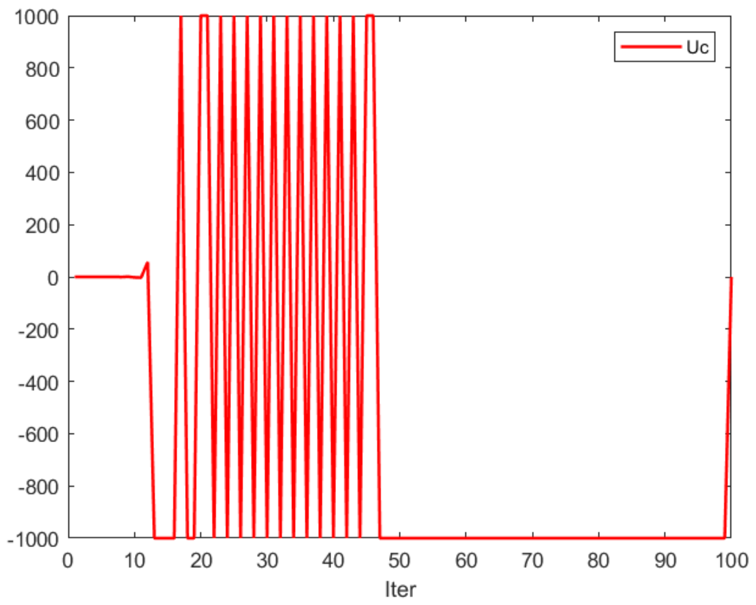

Figure 88. Control signal for adaptive minimum variance controller for minimum phase system and in under parameter case

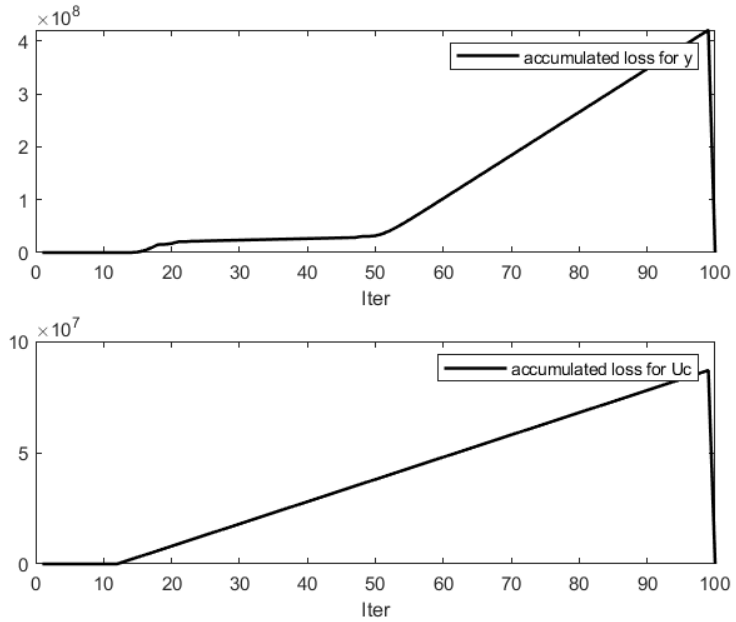

Figure 89. Accumulated losses for adaptive minimum variance controller for minimum phase system and in under parameter case

As it is clear from the Figures above in under parameter case, the controller has not been able to control the system and the identification of the parameters has not been done well in the presence of color noise and the system has become unstable.

2. MV, MA, and LQG

2.1 MV & MA controllers

2.1.1 MV method

In this method, first, using the algorithm 1-4 of the book and according to the appropriate filter for the minimum variance controller which is $1/c^*$, the inputs and outputs are filtered and then, using the inputs and the filtered outputs form the regression vector and we use the RLS method for identification.

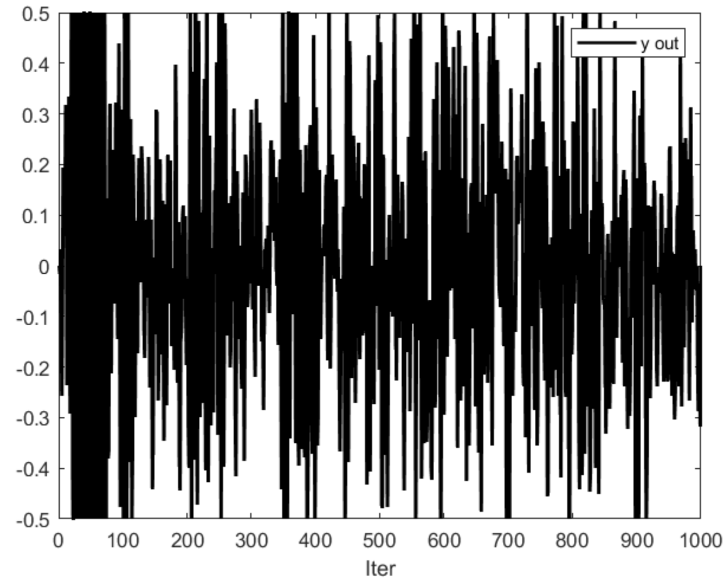

Figure 90. Output of the system with the direct minimum variance controller and with the 1/c filter*

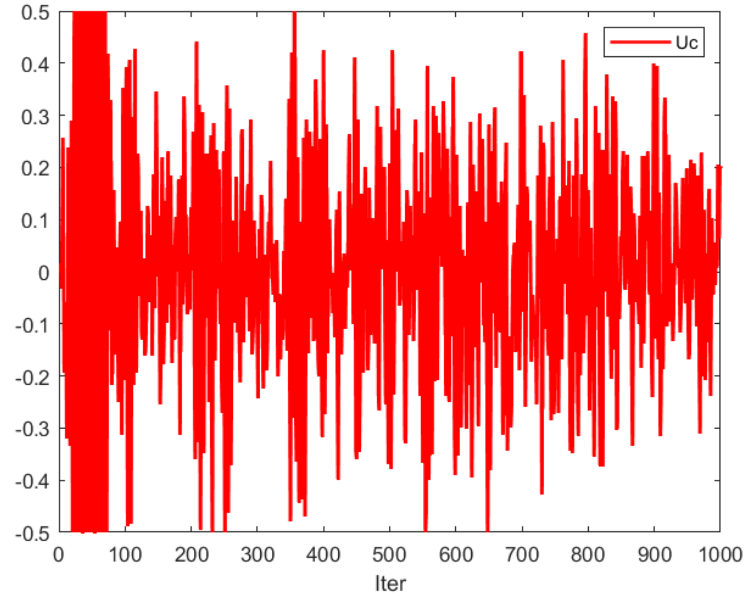

Figure 91. Control signal of the system with the direct minimum variance controller and with the 1/c filter*

As it is clear from the figures above, the output of the control input system has a high variance, because the noise of the system is color and the RLS estimation method is not a suitable method for this system.

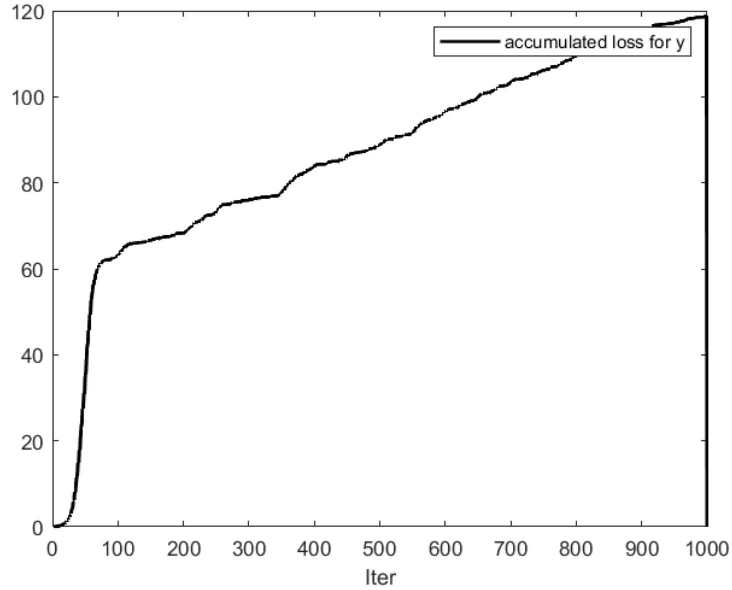

*Figure 92. Accumulated losses of the system output with minimum variance controller and directly with filter 1/c**

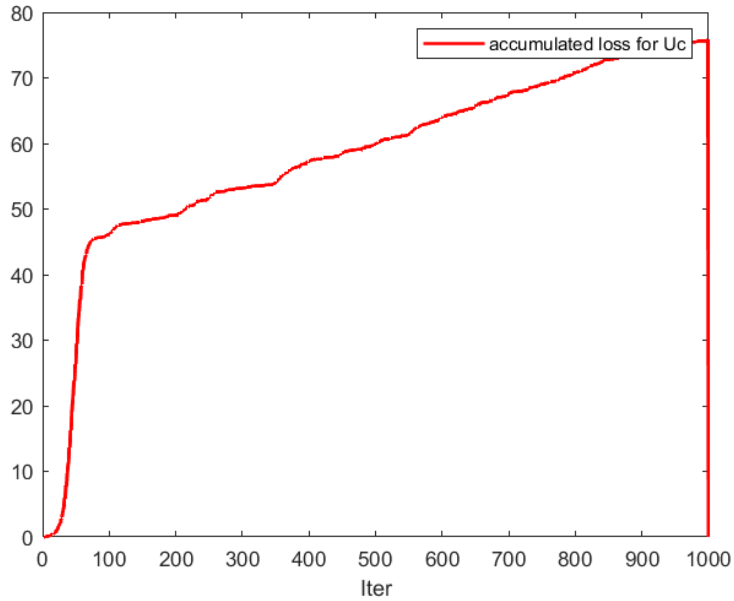

Figure 93. Accumulated loss of Uc for the system with the direct minimum variance controller and with the 1/c filter*

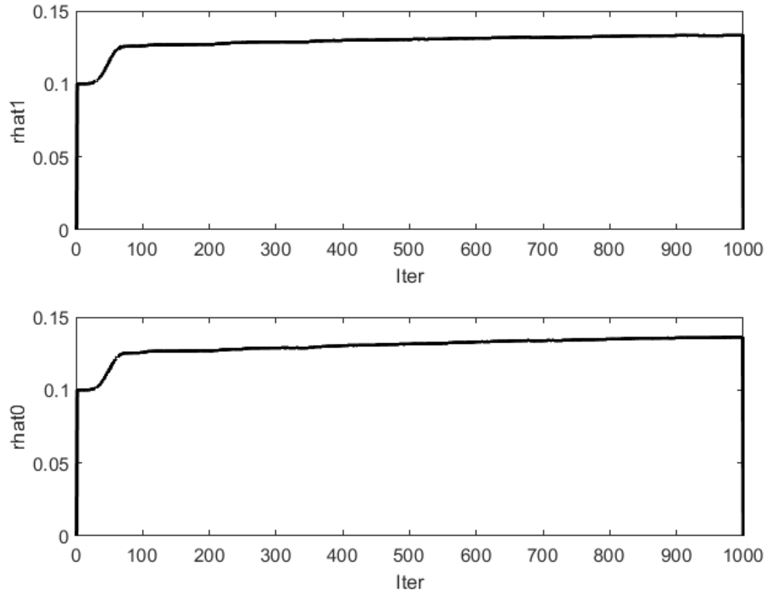

Figure 94. Estimation of R polynomial parameters for the system with the direct minimum variance controller and with the 1/c filter*

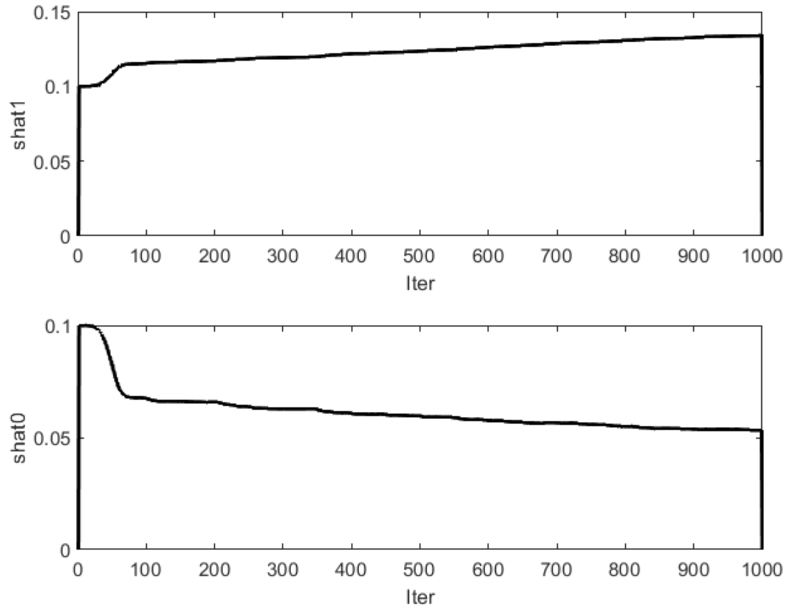

Figure 95. Estimation of S polynomial parameters for the system with the direct minimum variance controller and with the 1/c filter*

The estimated parameters don't show correct results and as the polynomials of the system are different from the first part, the results cannot be compared with the previous cases.

Now, for this controller, we use an arbitrary filter and consider the selected filter as a single filter. On the other hand, we examine the effect of parameter d, which is equal to d_0 in minimum variance controllers. In fact, the important point in the design of minimum variance controllers using direct method is that the value of d must be equal to d_0 so that the controller can perform better. The following figure shows the simulation results for the unit filter.

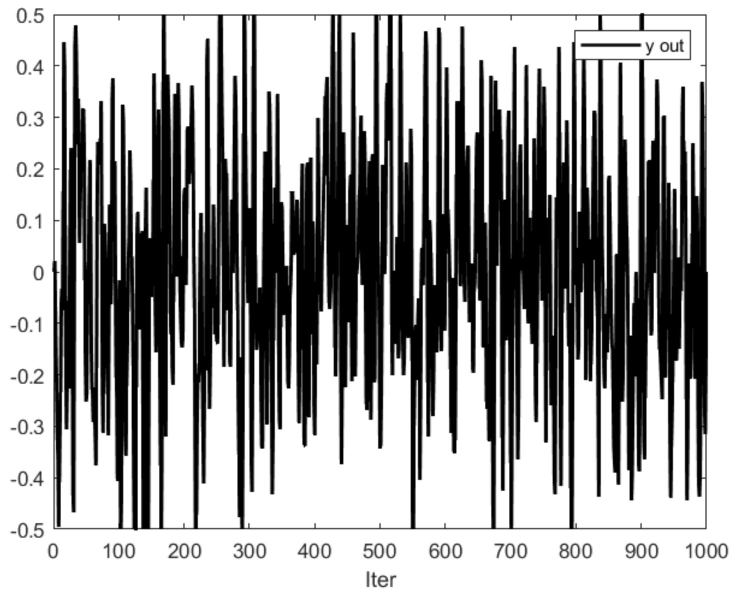

Figure 96. System output with direct minimum variance controller with single filter

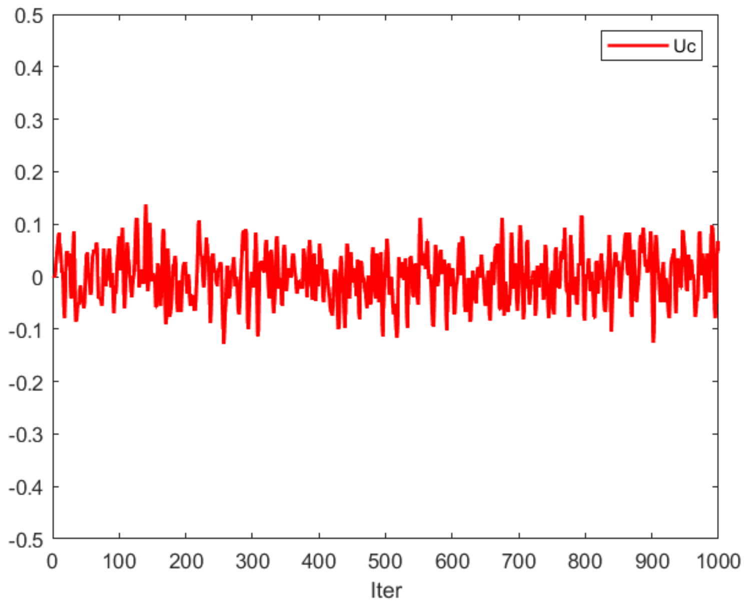

Figure 97. Control effort of the system with the direct minimum variance controller with the unit filter

In this case, by selecting the desired filter, the system output and control input have an appropriate value and their variance is within an acceptable range and the system is stable. We should remember that in the design of the minimum variance controller using direct method, the basic parameter is the correct choice of *d*, which if set equal to d_0, the system will work well.

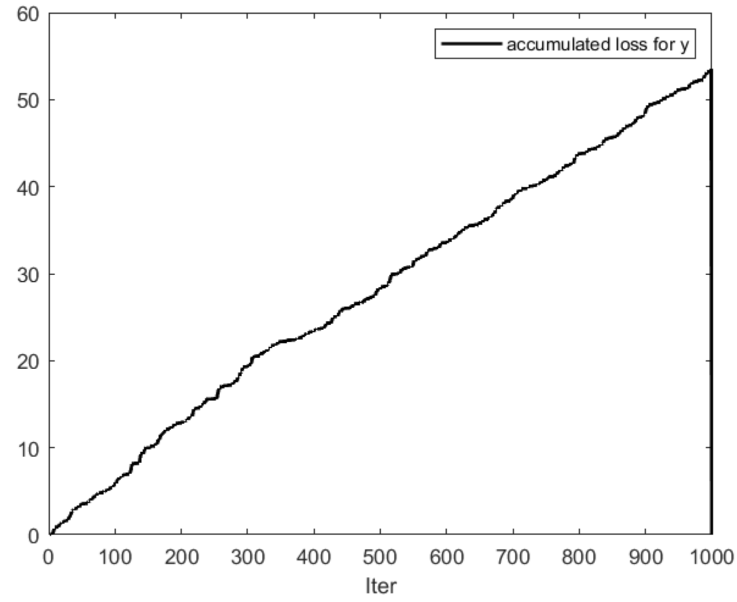

Figure 98. Accumulated loss of y for the system with direct minimum variance controller with single filter

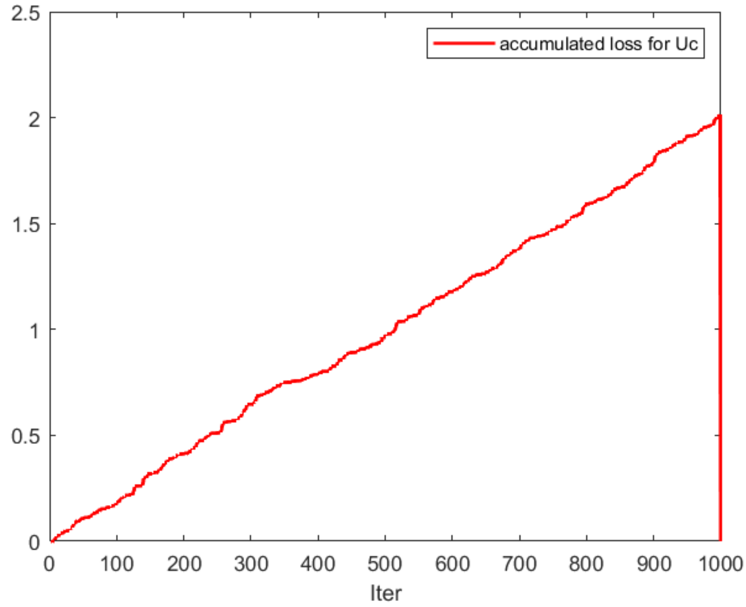

Figure 99. Accumulated loss of Uc for the system with direct minimum variance controller with single filter

The amount of accumulated loss for control signal is acceptable.

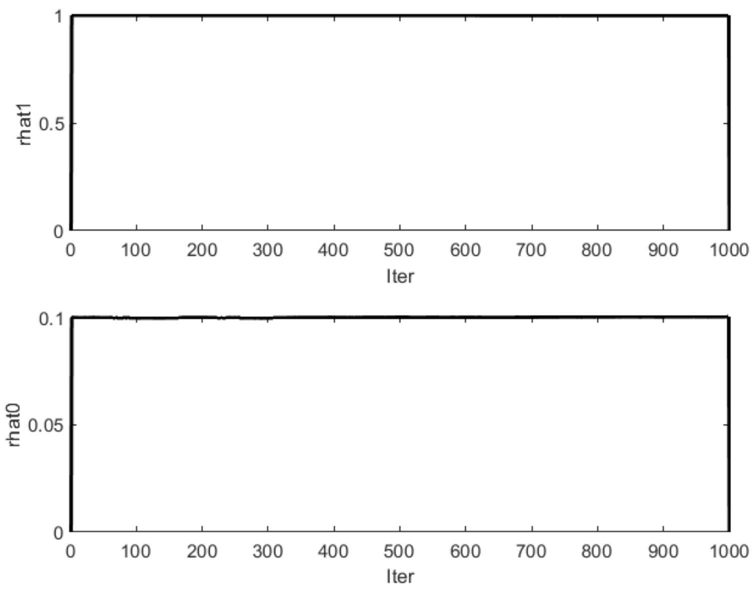

Figure 100. Estimated parameters of R for direct minimum variance controller with unit filter

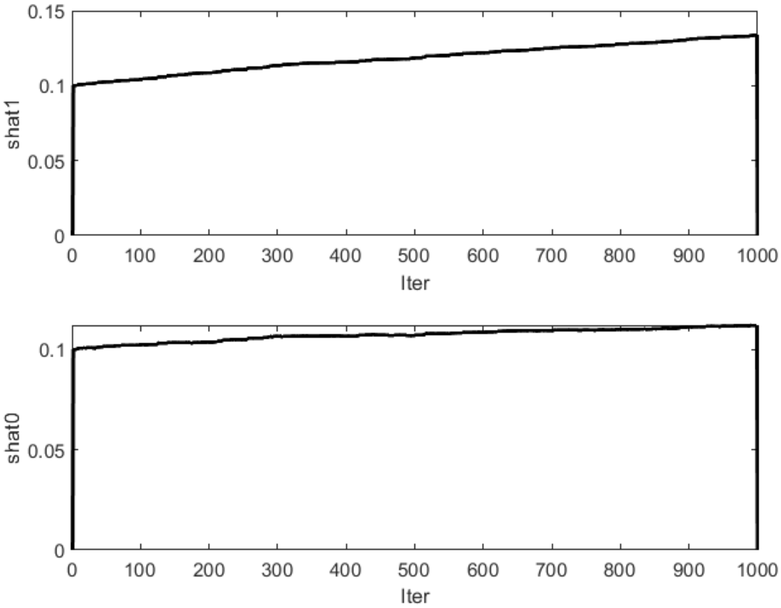

Figure 101. Estimated parameters of S for direct minimum variance controller with unit filter

2.1.2 MA method

In this case, the appropriate filter is $\frac{B^{-*}}{C^*}$, where B^- includes roots of B that are outside the unit circle. In this example, because the polynomial B itself, its root is outside the unit circle. So, as a result, B^- is the same as B and B^{-*} is also the inverse polynomial of B^- whose roots we have moved inside the unit circle. In this case, the value of d should be greater than the value of d_0 and should be less than the value of n, which is the degree of polynomial A of the system. In this case, we have considered the value of d to be equal to 2 in the simulation of this section, both with the desired filter mentioned above and with the desired filter which we have set as equal to one, the system identification has problems and becomes unstable. We have used RLS and ELS methods for identification, but in both cases, the results diverged. For this reason, we have placed a saturation on the control effort signal. The simulation results of this section are given in Figures 102 to 107.

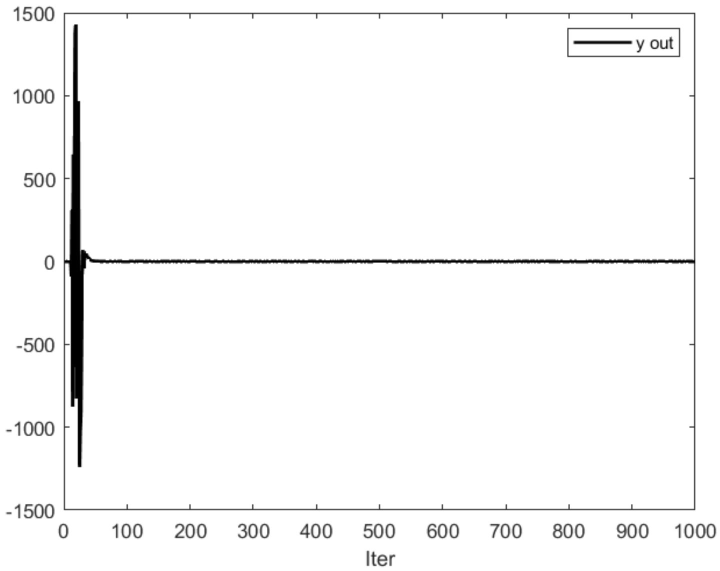

Figure 102. System output for moving average controller with unit arbitrary filter and non-minimum phase system

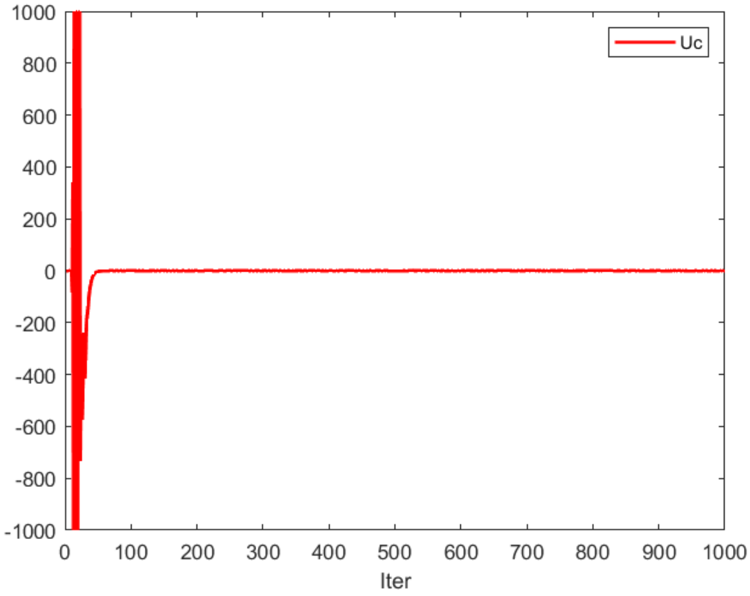

Figure 103. System control effort for moving average controller with unit arbitrary filter and non-minimum phase system

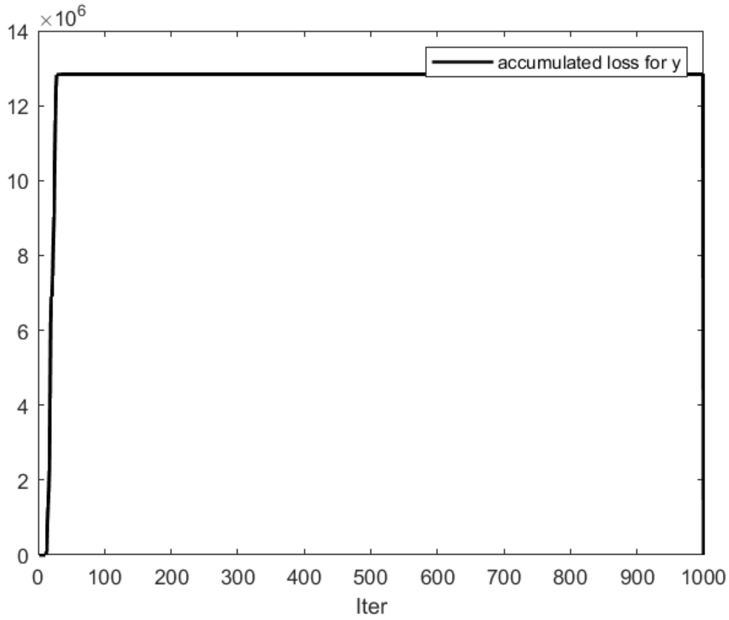

Figure 104. Accumulated loss of y for system with moving average controller with unit arbitrary filter and non-minimum phase system

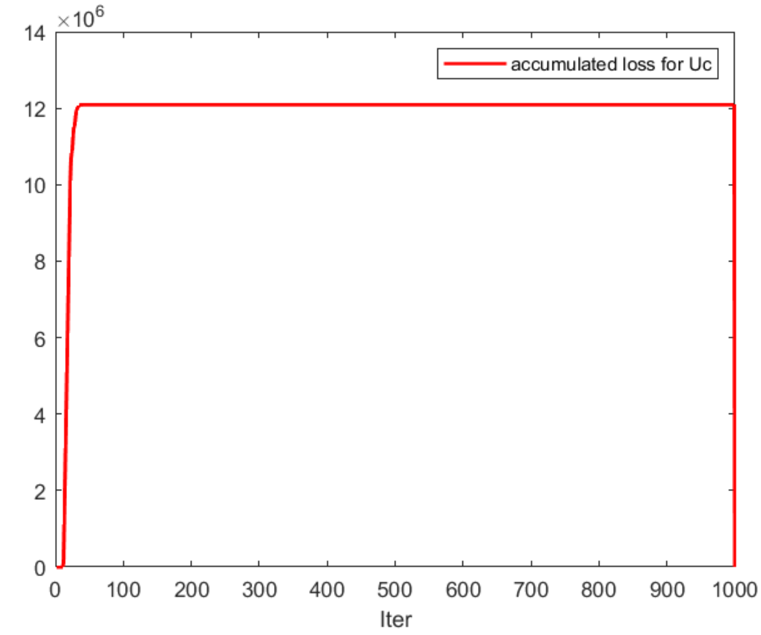

Figure 105. Accumulated loss of Uc for system with moving average controller with unit arbitrary filter and non-minimum phase system

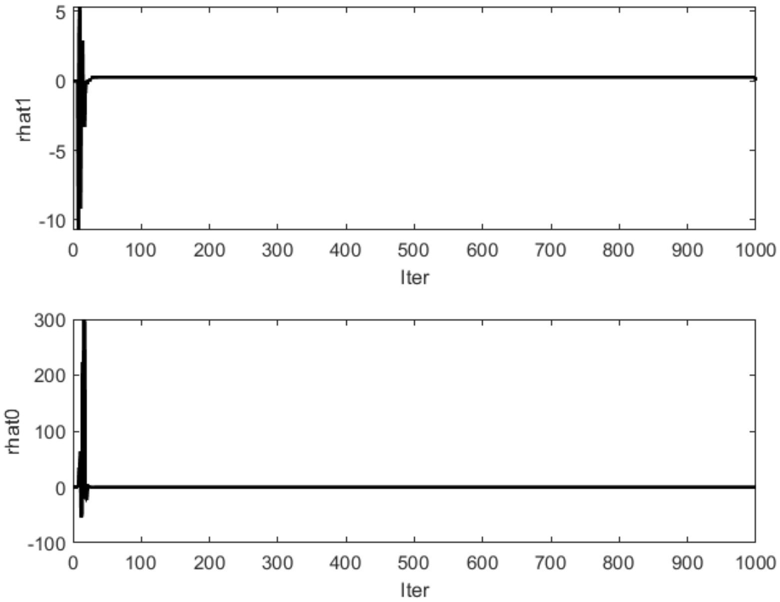

Figure 106. Polynomial R estimated parameters for moving average controller with unit arbitrary filter and non-minimum phase system

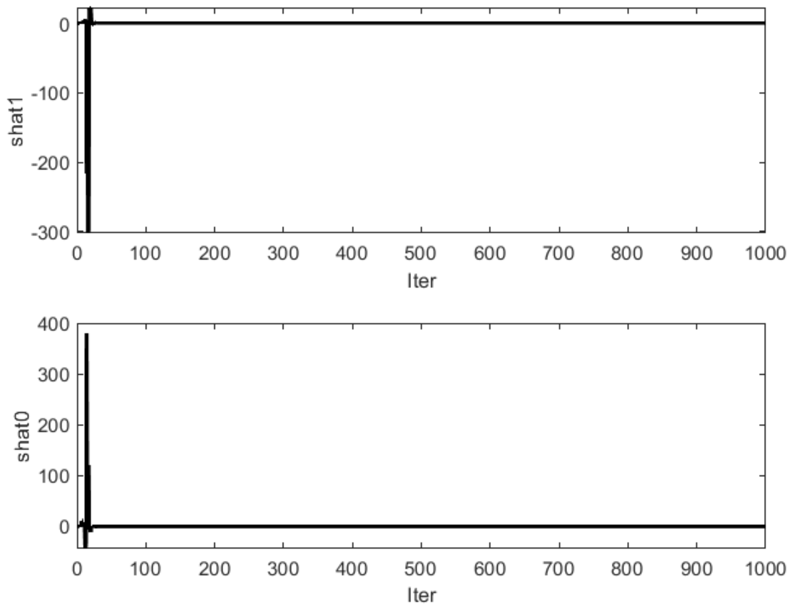

Figure 107. estimated parameters of polynomial S for moving average controller with unit arbitrary filter and non-minimum phase system

As it is clear from the Figures above, the system is unstable in this case and results are not good. By placing a saturation on the control input, we have only prevented the output values to become infinite, but the results are not acceptable. Also, we should note that the amount of noise in this system, produces different results in different runs.

2.2 LQG controller

The goal in this section is to trace the square signal in the presence of color noise, the equation of the system is as follows.

$$y(t) = \frac{q-0.5}{(q-0.3)(q-0.45)}u(t) + \frac{(q-0.7)}{q-0.9}e(t)$$

The polynomials of the system are as follows.

$$B1 = (q-0.5)$$
$$A1 = (q-0.3)(q-0.45)$$
$$C1 = (q-0.7)$$
$$A2 = (q-0.9)$$
$$A = (A1)(A2) = q^3 - 1.65q^2 + 0.81q - 0.1215 \qquad n = 3$$
$$B = (B1)(A2) = q^2 - 1.4q + 0.45$$
$$C = (C1)(A1) = q^3 - 1.45q^2 + 0.66q - 0.0945$$

We write the polynomial A_2 as follows.

$$A2 = A2^+ A2^-$$

that A_2 all the roots are inside the unit circle and for A_2, the roots are on or outside the circle, as a result, in this case we have for this problem:

$$A2^+ = A2 = q - 0.9 \ \ , \ \ degA2^+ = L = 1$$
$$A2^- = 1 \ \ , \ \ degA2^- = m = 0$$

First, we must obtain a positive r and a stable P_1 by solving the following equation. The equation to be solved is as follows:

$$rP1(q)P1(q^{-1}) = \rho A1(q)A2^-(q)A1(q^{-1})A2^{-1}(q^{-1}) + B1(q)B1(q^{-1})$$

In this equation, ρ is a coefficient that expresses the effect of the input on the loss function of the controller. In this equation, the degree of P_1 is as follows.

$$degP1 = degA1 + degA2^{-1} = 2 + 0 = 2$$

And since P_1 is monic, we consider it as follows:

$$P1 = q^2 + (p1)q + p0$$

By solving the given equation, *P₁* and *r* are as follows. Also, it should be kept in mind that in this case, the value of $\rho = 0.5$ is assumed for simulation.

$$r = 3.0423$$
$$P1 = q^2 - 0.6649q + 0.0887$$

The obtained P_1 coefficients make the P_1 polynomial stable in this system. Now, we need to obtain the controller polynomials from the following Diophantine equation.

$$A1(q)A2^{-}(q)\tilde{R}(q) + q^m B_1(q)\tilde{S}(q) = P_1(q)C(q) \qquad (1)$$

After solving this equation, we need to get *R* and *S* polynomials. which is calculated as follows:

$$R(q) = A_2^{-}(q) * \tilde{R}(q) = \tilde{R}(q)$$
$$S(q) = q^m\tilde{S}(q) = \tilde{S}(q)$$

Because $m = 0$ and $A2^{-} = 1$. The degree of these polynomials is as follows.

$$degR = degS = degT = n + m = 3 + 0 = 3$$

And we know that *R* is Monique.

So, by solving the equation 1 simultaneously, the controller polynomials are obtained. It is worth noting that considering that we have:

$$\tilde{S}(0) = 0$$

Polynomial *S* does not have a constant coefficient, there are 5 equations on the right side of equation 1, so there must be 5 unknowns on the left side of the equation, which are the coefficients of polynomials *R* and *S*. Polynomial *R* which has 3 unknown coefficients, then, polynomial *S* must also have 2 unknown coefficients. The condition of the answer is that *S* is monic and is as follows:

$$S(q) = q^3 + s_2q^2 + s_1q$$

By solving equation 1, *R* and *S* polynomials are obtained as follows.

$$R(q) = q^3 - 2.282q^2 + 0.9523q - 0.03106$$
$$S(q) = q^3 - 0.07499q^2 + 0.1349q$$

The polynomial *T* is also obtained as follows.

$$T = t_0 q^m C(q) = 0.8476q^3 - 1.229q^2 + 0.5594q - 0.0801$$

$$t_0 = \frac{P_1(1)}{B_1(1)}$$

By simulation, the output result is as follows.

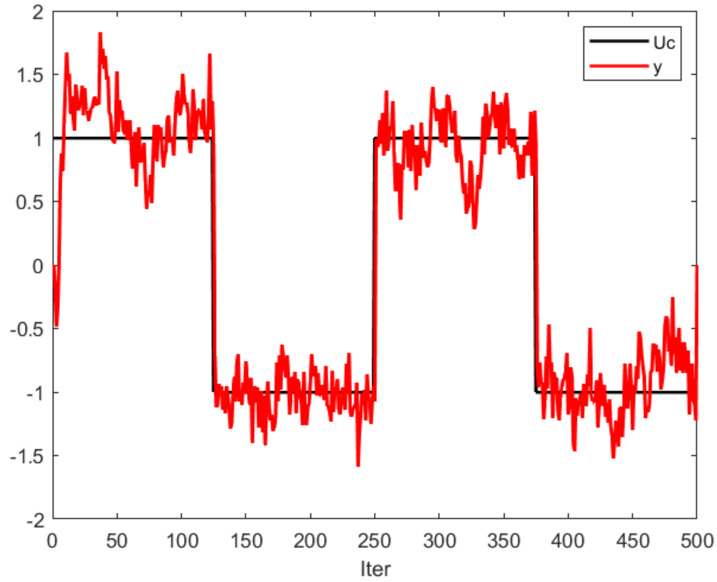

Figure 108. System output of LQG controller with $\rho = 0.5$

As it is clear from the diagram above, the output has been able to track the reference input, but it has many jumps and fluctuations due to the choice of $\rho = 0.5$. Also, the control effort is as follows:

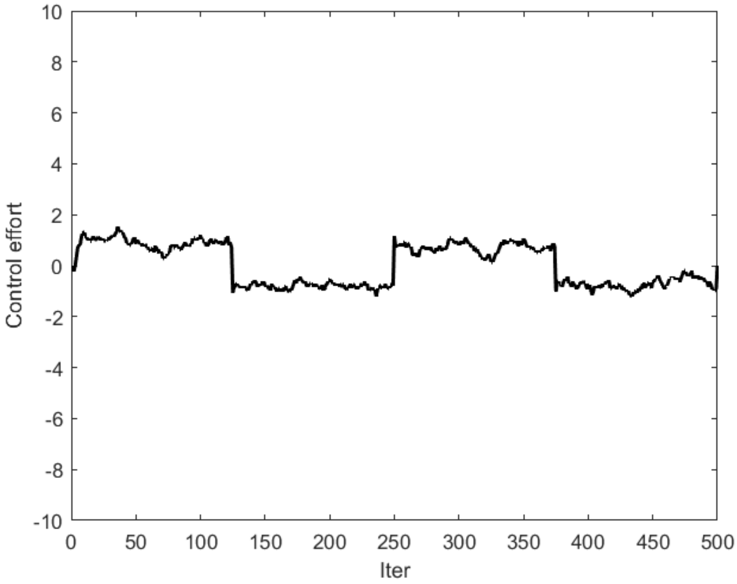

Figure 109. Control signal of LQG controller with $\rho = 0.5$

The control effort of the system also has an optimal value. In this case, the mean and variance of the control effort are equal to:

$$Mean\ of\ Uc = -0.0501$$

$$Variance\ of\ Uc = 0.6411$$

Now the value of $\rho = 2$ is set and the system is simulated again. In this case, the results are as follows.

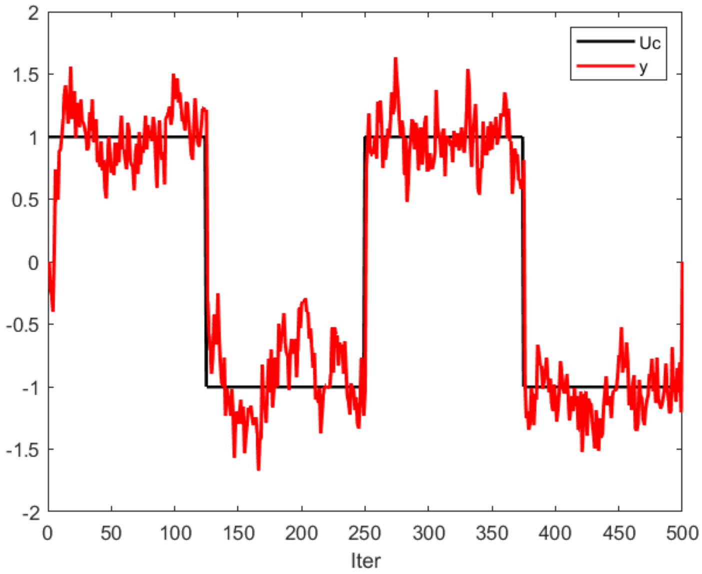

Figure 110. System output of LQG controller with value $\rho = 2$

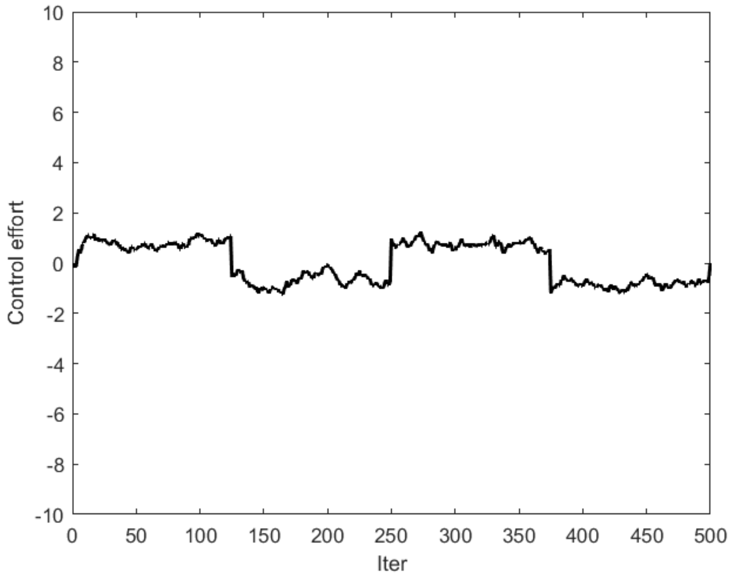

Figure 111. Control signal of LQG controller with value $\rho = 2$

In this case, the mean and variance of the control effort equal to

$$Mean\ of\ Uc = -0.0179$$

$$Variance\ of\ Uc = 0.5868$$

As can be seen, the changes in the value of ρ have reduced the variance of the control effort, because, in the controller loss function, we have increased the weight corresponding to this term and we expected to get the same result.

References

1. Chen, T. H., & Chiu, C. Y. (2017). Indirect pole placement design for STR controllers without noise. Journal of Control Engineering and Applied Informatics, 19(1), 25-34.
2. Liu, S., & Zhang, Y. (2018). Design of STR controllers with white noise using indirect pole placement. IEEE Transactions on Control Systems Technology, 26(3), 924-932.
3. Wang, J., & Li, Q. (2019). STR controller design with color noise: An indirect pole placement approach. Journal of Process Control, 75, 10-18.
4. Zhou, H., & Zhang, X. (2020). Direct pole placement design of STR controllers without noise. Automatica, 112, 108633.
5. Yang, C., & Wang, L. (2021). Designing STR controllers with white noise using direct pole placement. Journal of Dynamic Systems, Measurement, and Control, 143(8), 081003.
6. Zhang, Q., & Li, M. (2022). STR controller design with color noise: A direct pole placement approach. International Journal of Control, 95(9), 1876-1886.
7. Huang, W., & Wu, Z. (2018). A comparative study of indirect and direct pole placement methods for STR controller design. Control Engineering Practice, 71, 131-141.
8. Li, H., & Sun, Y. (2019). Non-adaptive indirect STR design with color noise for min-phase systems. IFAC-PapersOnLine, 52(5), 187-192.
9. Wang, X., & Zhang, S. (2020). Adaptive indirect STR design with color noise for min-phase systems. Asian Journal of Control, 22(6), 2471-2480.
10. Chen, G., & Liu, X. (2021). Non-adaptive indirect STR design with color noise for non-min-phase systems. Journal of Systems Science and Complexity, 34(3), 895-909.

Thanks for your Time