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SLIDING MODE CONTROL DESIGN FOR THE BENCHMARK PROBLEM IN REAL-TIME HYBRID SIMULATION

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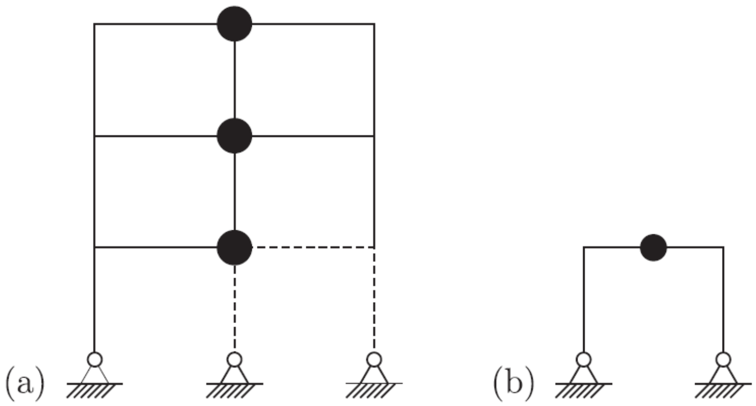
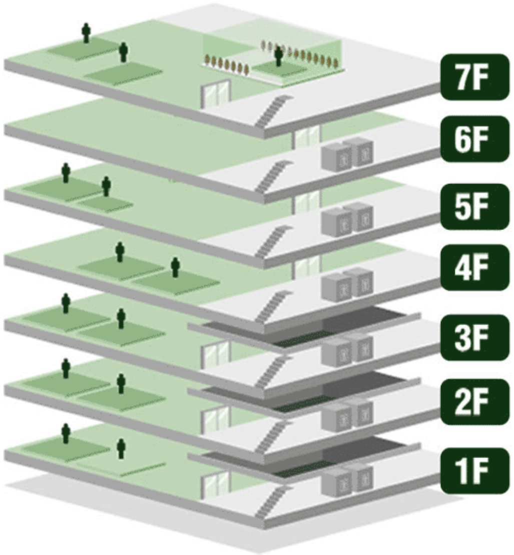
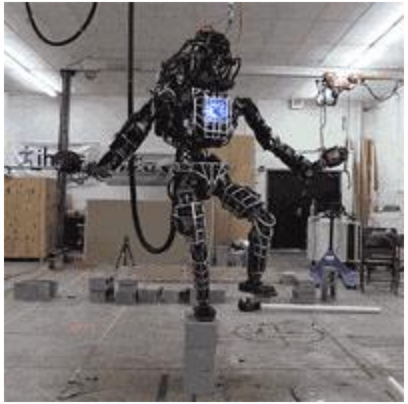
- **Real-time hybrid simulation**
- **Benchmark problem**

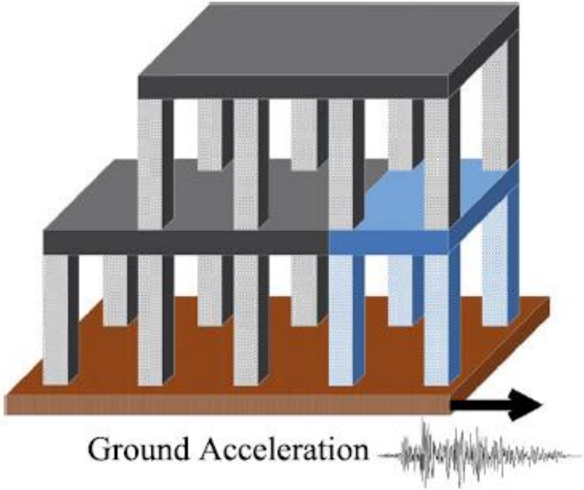


RTHS

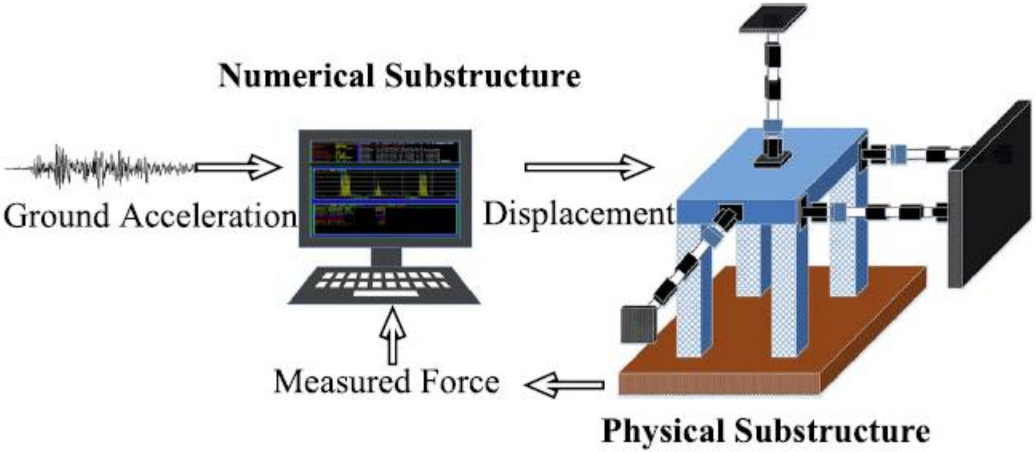




RTHS



(a) Reference structure

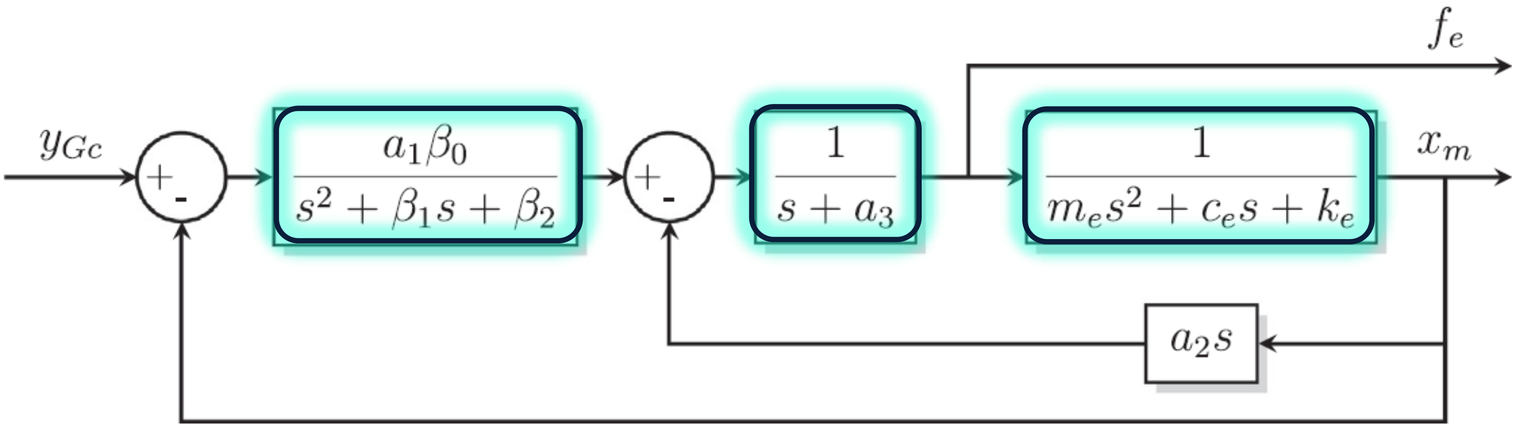


(b) Real-time hybrid simulation

OBJECTIVES

- **Utilizing a benchmark problem on RTHS control**
- **Designing a SMC strategy**
- **Demonstrate and validate its effectiveness**
- **Enhancing the performance and robustness of RTHS testing**

CONTROL PLANT



CONTROL PLANT

Table 1. Parametric values and uncertainties associated with the original model of the control plant

Parameter	Nominal value	Standard deviation	Units
$a_1\beta_0$	2.13×10^{13}	-	kg/s^5
a_2	4.23×10^{6}	-	kg/s^2
a_3	3.3	1.3	s^{-1}
β_1	425	3.3	s^{-1}
β_2	1×10^{5}	3.31×10^{3}	s^{-2}
m_e	29.1	-	kg
c_e	114.6	-	kg/s
k_e	1.19×10^{6}	5×10^{4}	kg/s^2

CONTROL PLANT

$$G_p(s) = \frac{B_0}{A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s^1 + A_0} \tag{1}$$

$$\begin{aligned}
B_0 &= a_1 \beta_0 \\
A_0 &= k_e a_3 \beta_2 + a_1 \beta_0 \\
A_1 &= k_e a_3 \beta_1 + (k_e + c_e a_3 + a_2)\beta_2 \\
A_2 &= k_e a_3 + (k_e + c_e a_3 + a_2)\beta_1 + (c_e + m_e a_3)\beta_2 \\
A_3 &= k_e + c_e a_3 + a_2 + (c_e + m_e a_3)\beta_1 + m_e \beta_2 \\
A_4 &= c_e + m_e a_3 + m_e \beta_1 \\
A_5 &= m_e
\end{aligned} \tag{2}$$

CONTROL PLANT

$$G_{pr}(s) = \frac{a}{s^2 + bs + c} \times \left(\frac{1 + ds}{1 + es} \right) \qquad (3)$$

$$u = \frac{a(1 + ds)}{1 + es} y_{Gc} \qquad (4)$$

$$G_r(s) = \frac{1}{s^2 + bs + c} \qquad (5)$$

CONTROL PLANT

Table 2. Parametric values and uncertainties associated with the reduced-model of the control plant

Parameter	Nominal value	Uncertainty	Units
a	53354	--	s^{-2}
b	221.64	[141.85,301.43]	s^{-1}
c	54290	[48861,59719]	s^{-2}
d	1.06×10^{-4}	--	s
e	2.11×10^{-2}	--	s

CONTROL PLANT

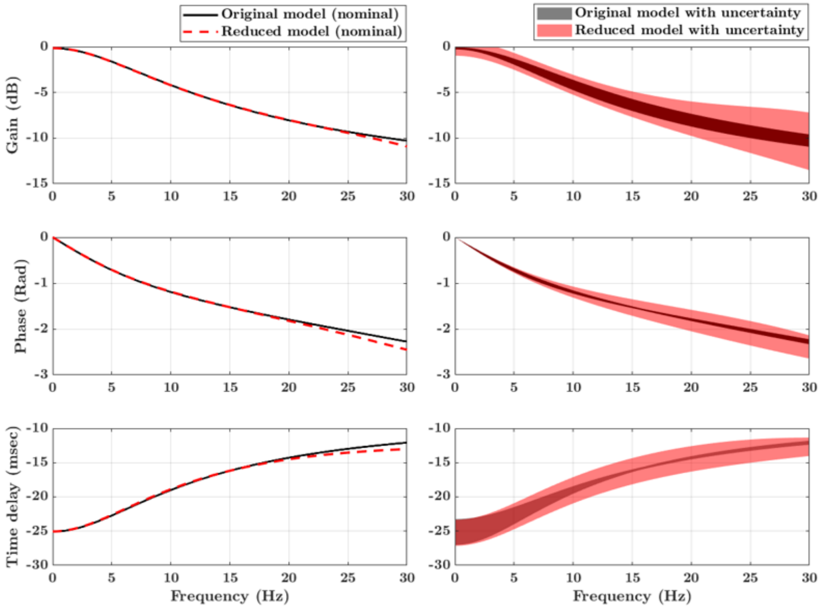


Fig 3. Frequency responses of original and reduced-order models of the control plant (left: nominal models; right: models with uncertainties)

KALMAN ESTIMATOR

$$\dot{X} = Ax + Bu$$
$$X_m = Cx + \nu \tag{6}$$

$$A = \begin{bmatrix} 0 & 1 \\ -\bar{c} & -\bar{b} \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = [10] \tag{7}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(x_m - C\hat{x})$$
$$\text{where } \hat{x} = \left[\hat{x}_m, \dot{\hat{x}}_m\right]^T \tag{8}$$

$$\bar{b} = 221.64 s^{-1} \text{ and } \bar{C} = 54290 s^{-2}$$

SLIDING MODE CONTROL

$$E = \dot{e} + \lambda e \qquad (15)$$

$$e = \hat{x}_m - x_d \qquad (14)$$

$$V = \frac{E^2}{2} \qquad (16)$$

$$\dot{V} = E\dot{E} \leq -\eta |E| \qquad (17)$$

$$\dot{E} = f(\hat{x}_m) + u - \ddot{x}_d + \lambda \dot{e} \qquad (18)$$

SLIDING MODE CONTROL

$$\bar{u} = -\bar{f}(\hat{x}_m) + \ddot{x}_d - \lambda\dot{e} \tag{19}$$

$$u' = \bar{u} - k\,\mathrm{sgn}(E) \tag{20}$$

$$k = F(\hat{x}_m) + \eta \tag{21}$$

$$\mathrm{sgn}(E) = \begin{cases} 1 & E > 0 \\ 0 & E = 0 \\ -1 & E < 0 \end{cases} \tag{22}$$

SLIDING MODE CONTROL

$$sat(E/\Phi) = \begin{cases} E/\Phi & |E/\Phi| \le 1 \\ \mathrm{sgn}(E) & |E/\Phi| > 1 \end{cases} \qquad (23)$$

$$u = \bar{u} - \bar{k}sat(E/\Phi) \qquad (24)$$

$$\bar{k} = k - \dot{\Phi} \qquad (25)$$

$$\dot{\Phi} = F(x_d) + \eta - \lambda\Phi \qquad (26)$$

SLIDING MODE CONTROL

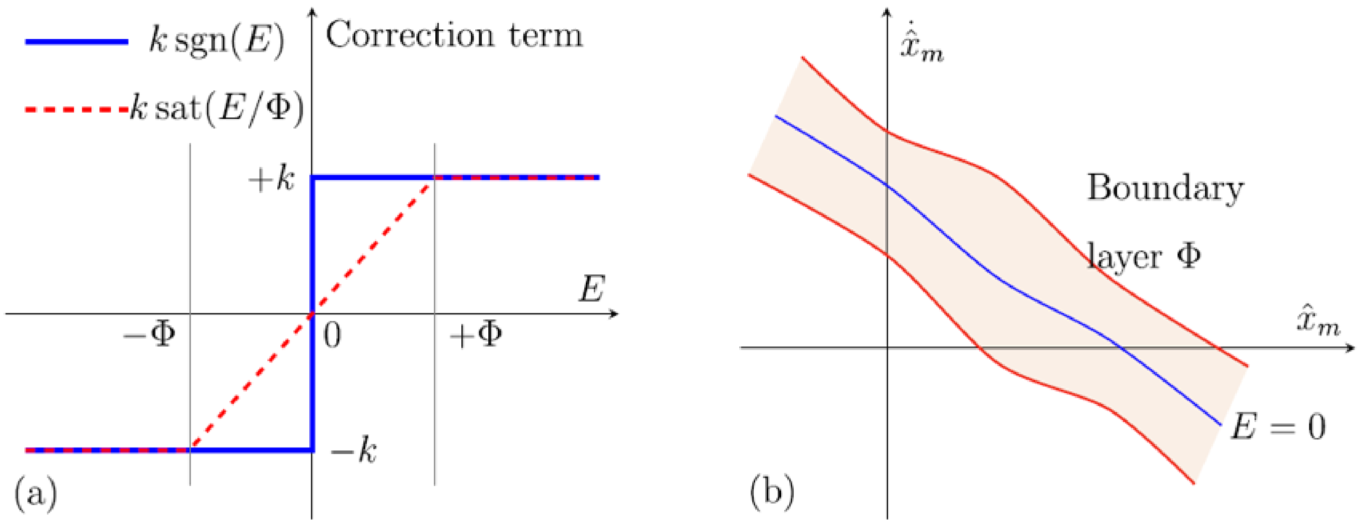


Fig 5. (a) Correction term of the control command with and without boundary layer: (b) Sketch of the sliding surface and time-varying boundary layer

PHASE-LEAD COMPENSATOR

$$e^{\tau s} = \frac{e^{2\tau s}}{e^{\tau s}} \approx \frac{1+2\tau s}{1+\tau s} = G_{pl}(s) \tag{27}$$

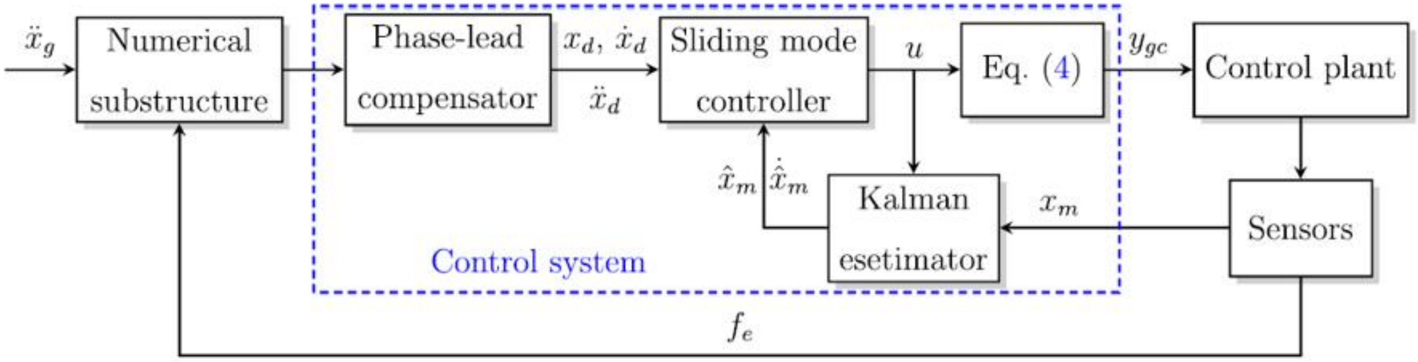
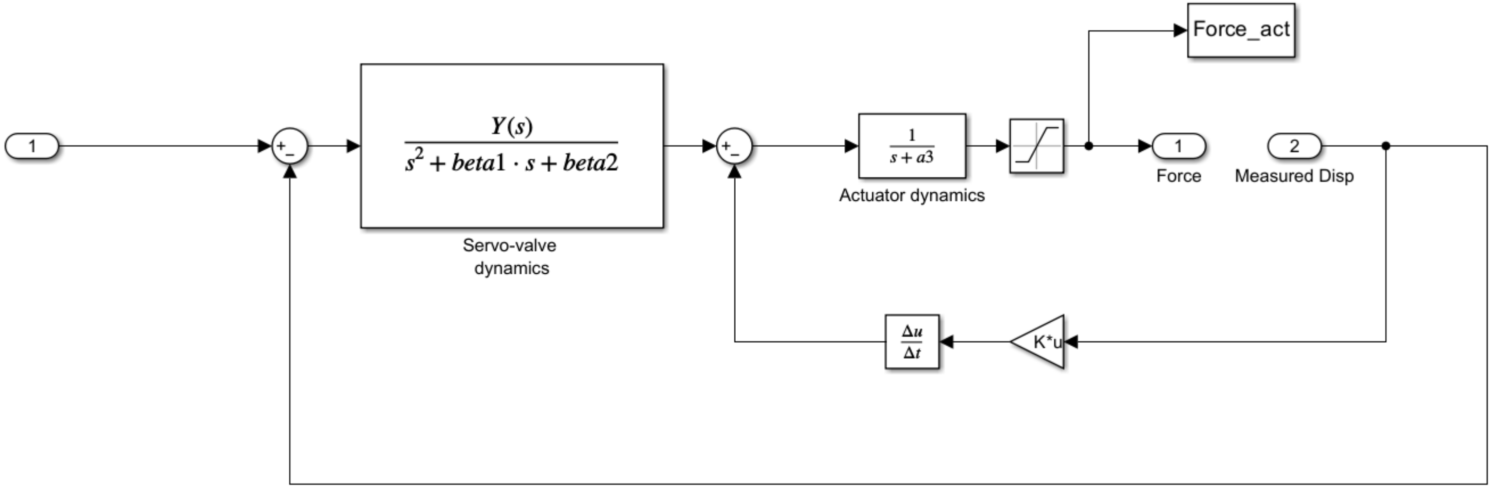


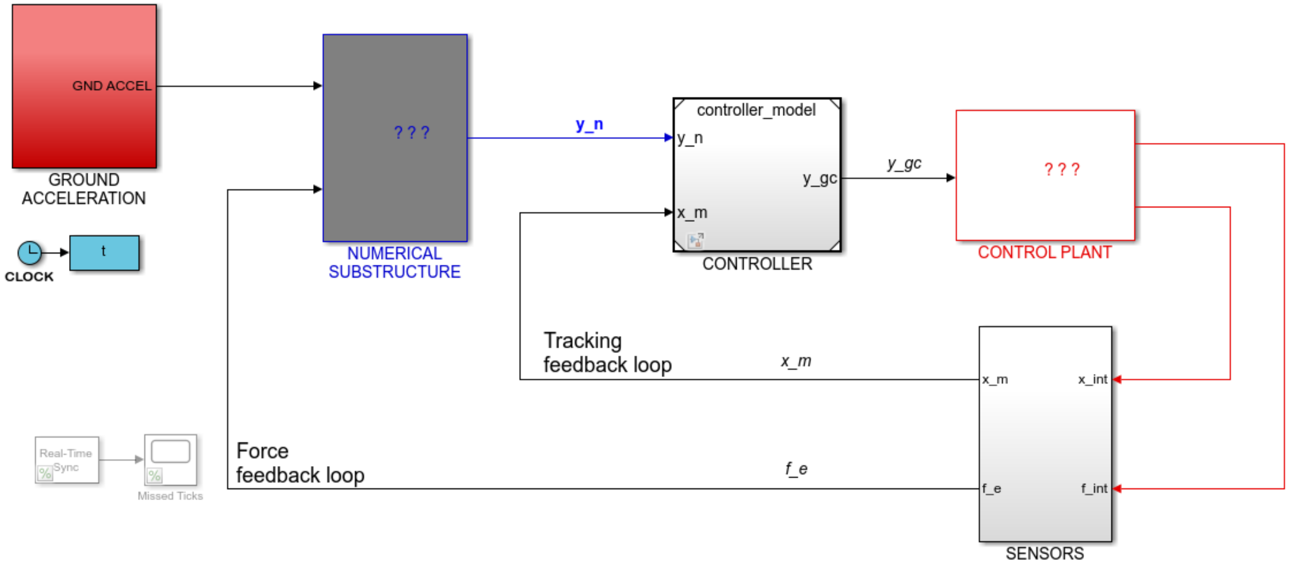
Fig 6. RTHS block diagram

SIMULATION



1
+
_
$\frac{Y(s)}{s^2 + beta1 \cdot s + beta2}$
Servo-valve dynamics
$\frac{1}{s+a3}$
Actuator dynamics
Force_act
1
Force
2
Measured Disp
$\frac{\Delta u}{\Delta t}$
K*u

SIMULATION


GND ACCEL
GROUND ACCELERATION
CLOCK
t
? ? ?
NUMERICAL SUBSTRUCTURE
y_n
controller_model
y_n
y_gc
x_m
CONTROLLER
y_gc
? ? ?
CONTROL PLANT
Tracking feedback loop
x_m
x_m
x_int
f_e
f_int
SENSORS
Real-Time Sync
Missed Ticks
Force feedback loop
f_e

SIMULATION

Table 3. RTHS partitioning cases of the benchmark problem

Partitioning Case (#)	Reference floor (kg)	Reference modal damping (%)
Case 1	1000	5
Case 2	1100	4
Case 3	1300	3
Case 4	1000	3

RESULTS

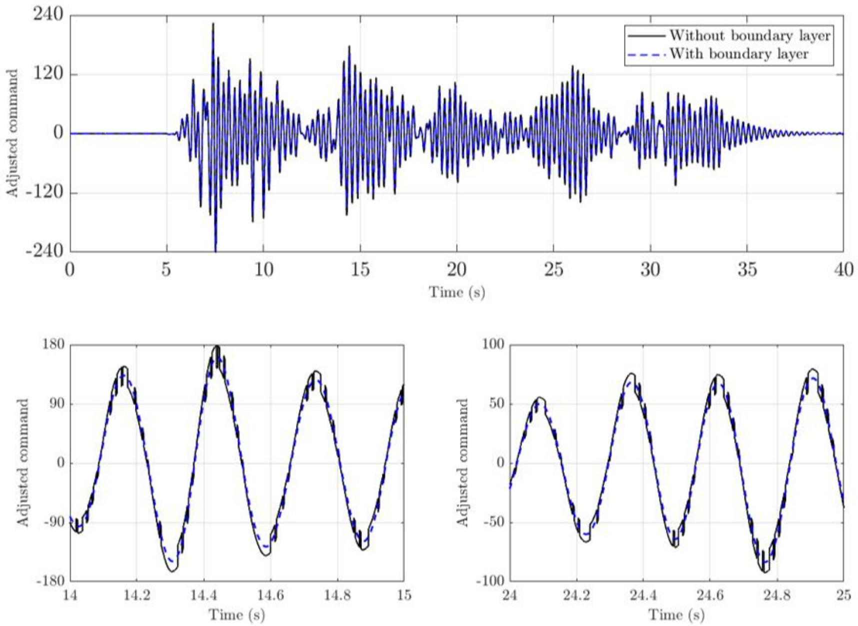


Fig 9. Adjusted command: EL Centro earthquake, case 4, nominal control plant

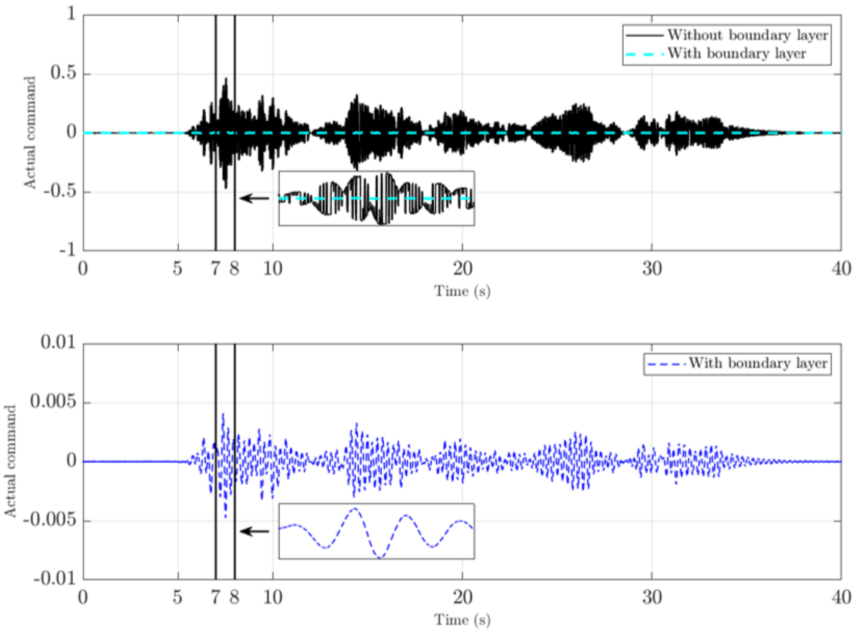


Fig 10. Actual command: El Centro earthquake, case 4, nominal control plant

RESULTS

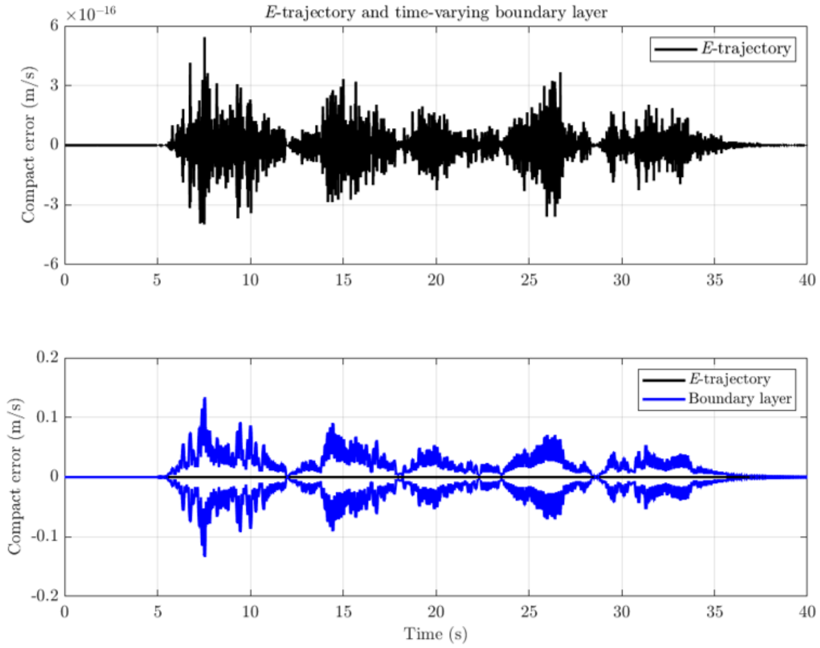


Fig 11. Compact error trajectory and time-varying boundary layer: El Centro earthquake, Case 4, nominal control plant

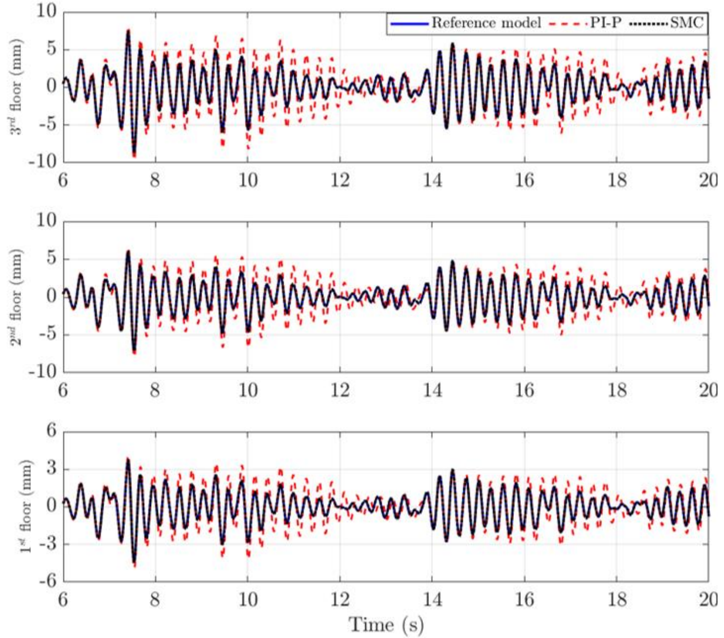
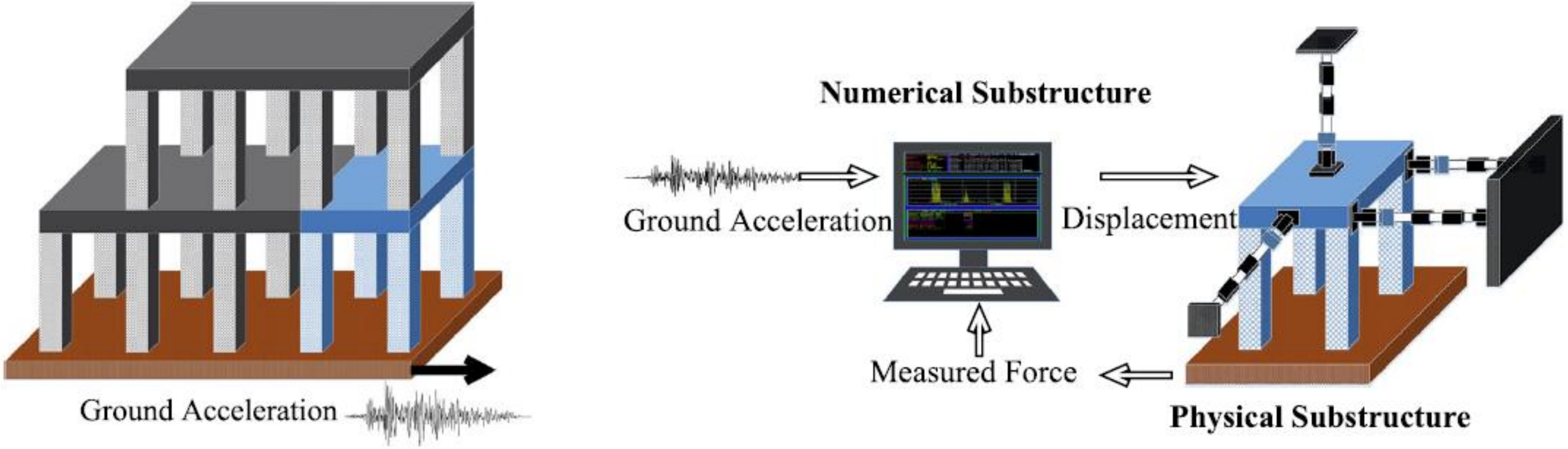


Fig 12. Floor displacement in the reference model and virtual RTHS: El Centro earthquake, case 4, nominal control plant

CONCLUSION

- The sliding mode control system exhibits significantly improved robustness and accuracy compared to the demo controller provided in the benchmark problem.
- The proposed control strategy is expected to be highly effective for complex control plants.

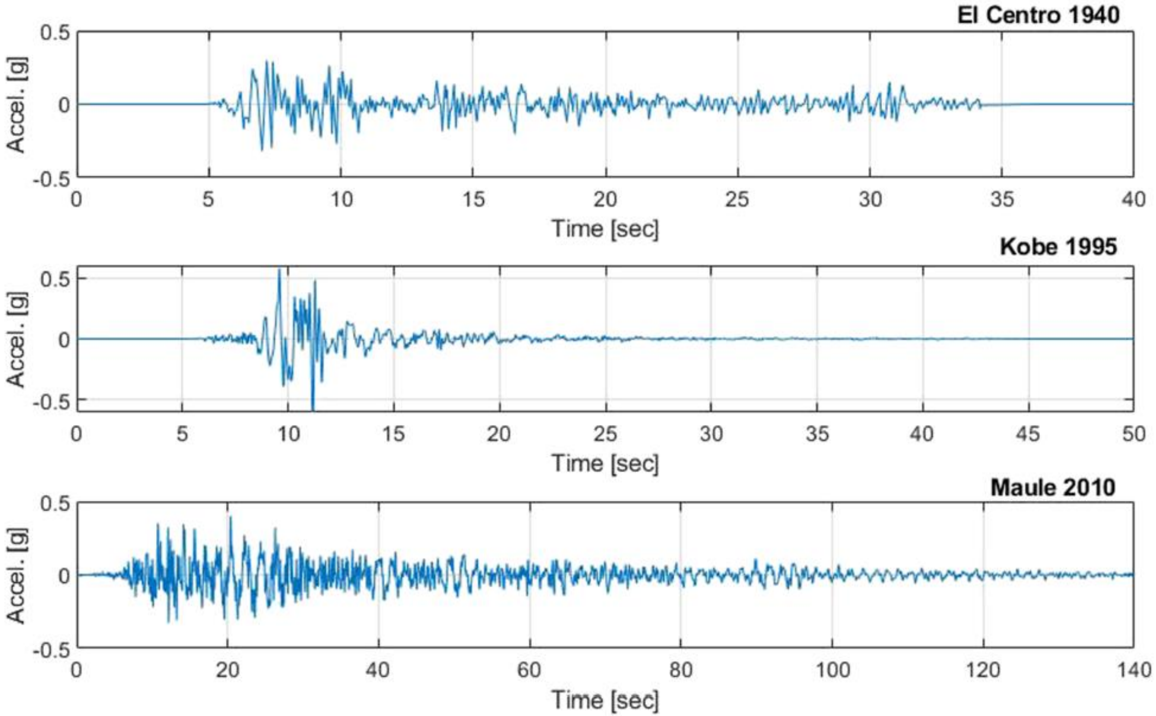
NOVELTY



(a) Reference structure

(b) Real-time hybrid simulation

NOVELTY



El Centro 1940
Accel. [g]
0.5
0
-0.5
0
5
10
15
20
25
30
35
40
Time [sec]
Kobe 1995
Accel. [g]
0.5
0
-0.5
0
5
10
15
20
25
30
35
40
45
50
Time [sec]
Maule 2010
Accel. [g]
0.5
0
-0.5
0
20
40
60
80
100
120
140
Time [sec]

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ALL downloaded records are UNSCALED and as-recorded (UNROTATED). The scaling tool available on this site is to be used to determine the scale factors to be used in the simulation platform. These scale factors can be found with the record metadata in the download (Scaling the traces within this tool would only cause confusion with file versioning).

Please note that, due to copyright issues, a strict limit has been imposed on the number of records that can be downloaded within a unique time window. The current limit is set at approximately 200 records every two weeks, 400 every month. Abusive downloads will result in further restrictions.

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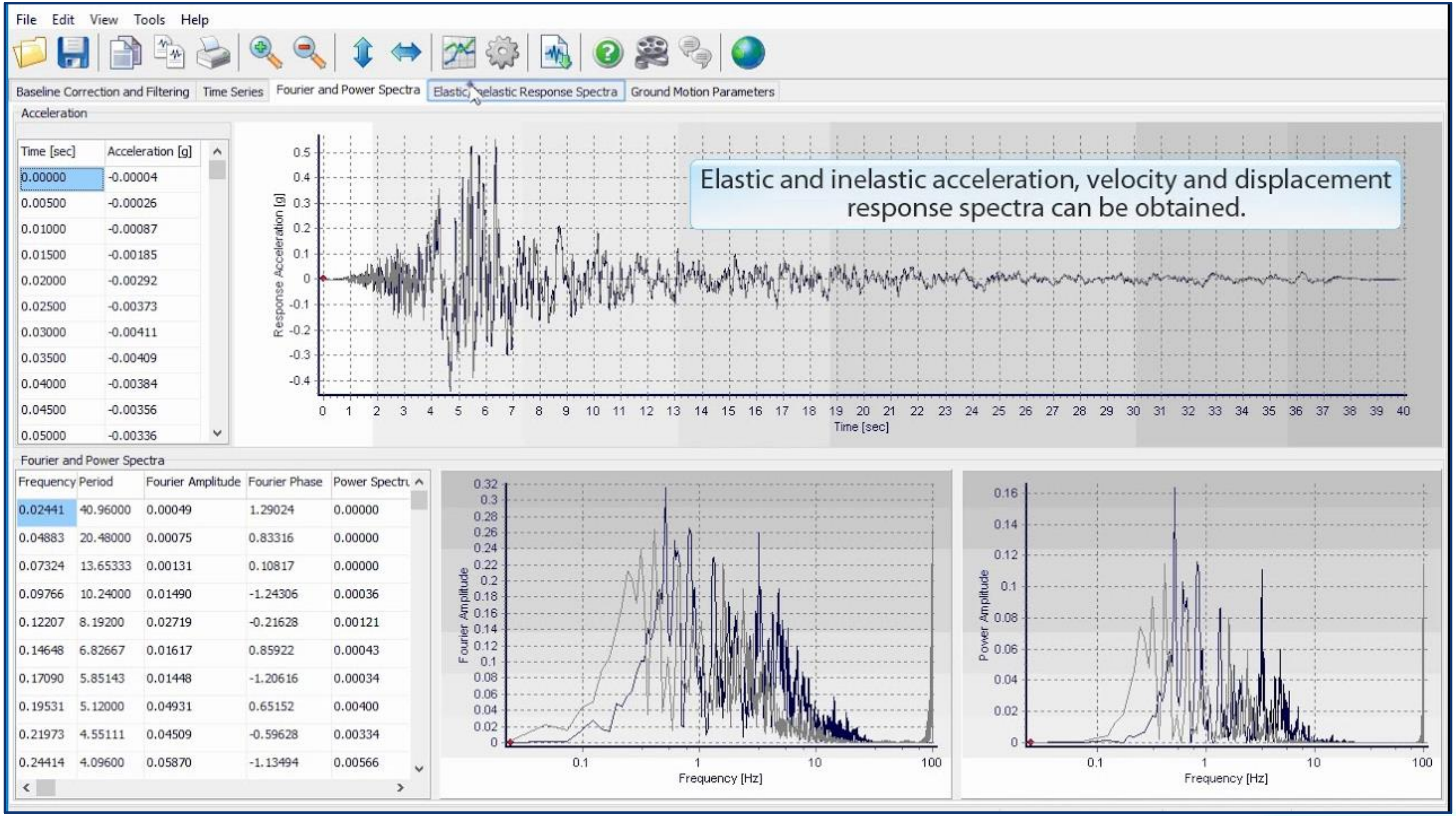
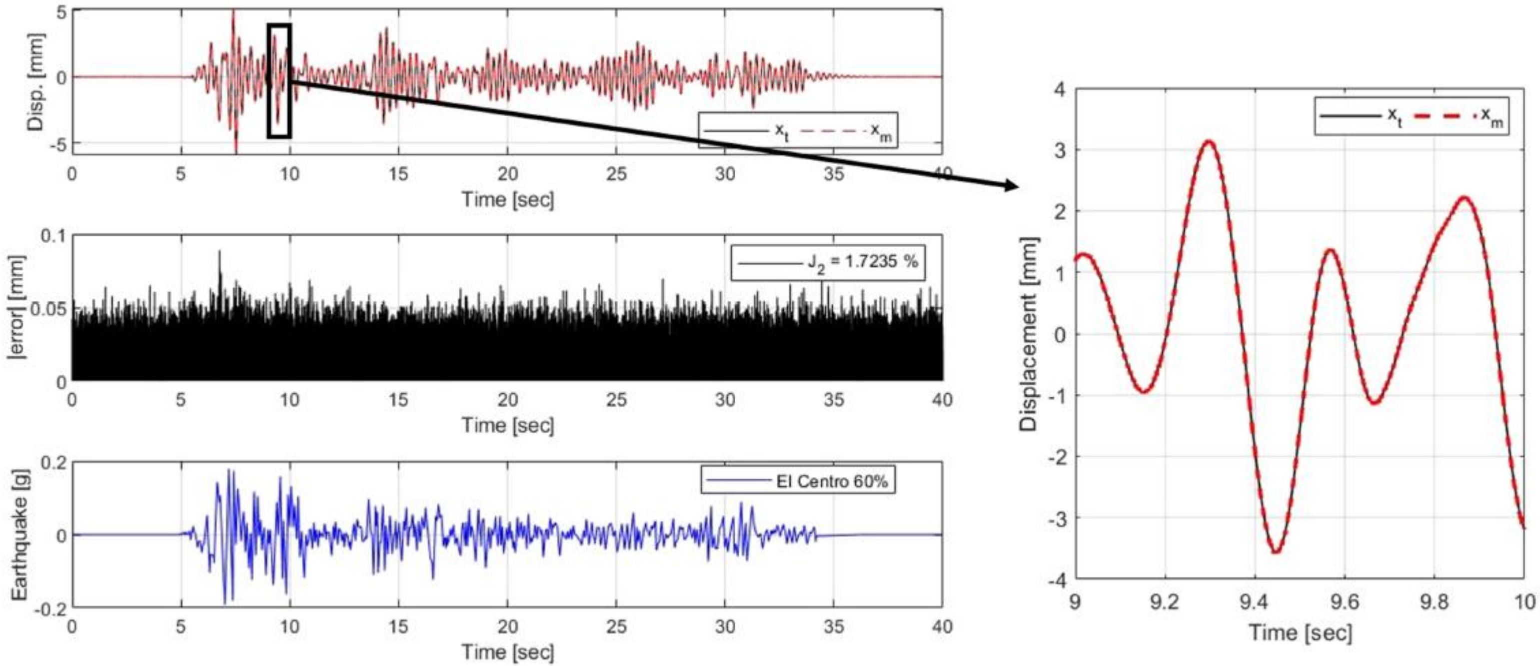

File Edit View Tools Help
Baseline Correction and Filtering
Time Series
Fourier and Power Spectra
Elastic/Inelastic Response Spectra
Ground Motion Parameters
Acceleration
Time [sec] Acceleration [g]
0.00000 -0.00004
0.00500 -0.00026
0.01000 -0.00087
0.01500 -0.00185
0.02000 -0.00292
0.02500 -0.00373
0.03000 -0.00411
0.03500 -0.00409
0.04000 -0.00384
0.04500 -0.00356
0.05000 -0.00336
Elastic and inelastic acceleration, velocity and displacement response spectra can be obtained.
Response Acceleration [g]
Time [sec]
Fourier and Power Spectra
Frequency Period Fourier Amplitude Fourier Phase Power Spectru
0.02441 40.96000 0.00049 1.29024 0.00000
0.04883 20.48000 0.00075 0.83316 0.00000
0.07324 13.65333 0.00131 0.10817 0.00000
0.09766 10.24000 0.01490 -1.24306 0.00036
0.12207 8.19200 0.02719 -0.21628 0.00121
0.14648 6.82667 0.01617 0.85922 0.00043
0.17090 5.85143 0.01448 -1.20616 0.00034
0.19531 5.12000 0.04931 0.65152 0.00400
0.21973 4.55111 0.04509 -0.59628 0.00334
0.24414 4.09600 0.05870 -1.13494 0.00566
Fourier Amplitude
Frequency [Hz]
Power Amplitude
Frequency [Hz]

NOVELTY

$$x_m(s) = G_p(s)x_c(s) = \left(\frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}\right)x_c(s),$$

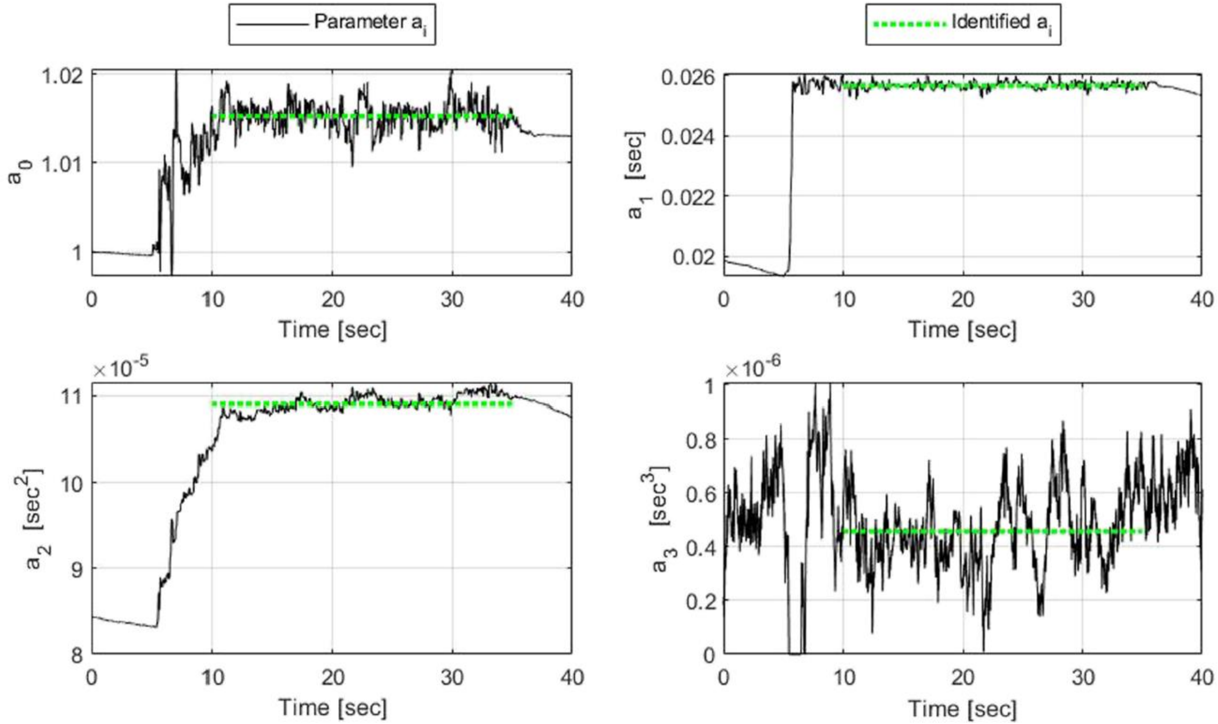
$$\hat{x}_c = AX_m = [a_0 \; a_1 \; a_2 \; a_3][x_m \; \dot{x}_m \; \ddot{x}_m \; \dddot{x}_m]^T,$$

NOVELTY



Disp. [mm]
x_t
x_m
Time [sec]
|error| [mm]
J_2 = 1.7235 %
Time [sec]
Earthquake [g]
El Centro 60%
Time [sec]
Displacement [mm]
Time [sec]

NOVELTY



Parameter a_i
Identified a_i
a_0
1.02
1.01
1
a_1 [sec]
0.026
0.024
0.022
0.02
$\times 10^{-5}$
a_2 [sec^2]
11
10
9
8
$\times 10^{-6}$
a_3 [sec^3]
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
Time [sec]

THANK YOU!