UNIVERZITET U SARAJEVU
ELEKTROTEHNIČKI FAKULTET SARAJEVO

ZADATAK 1 – LV5

OPERACIONA ISTRAŽIVANJA

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Potpisi:

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Zadatak 1

Brod od 10 tona treba natovariti s tri vrste robe. U tabeli je data jedinična težina u tonama za svaku vrstu robe, raspoloživa količina i cijena po utovarenom komadu robe u hiljadama KM. Kako treba natovariti brod da se dobije maksimalan profit od utovarene robe i koja je suma u pitanju? Ukoliko trgovac bude prinuđen da prebaci robu koristeći dva broda od 4 i 6 tona kako će tada izgledati optimalni utovar i hoće li doći do promjene u profitu? Koristiti dinamičko programiranje prilikom rješavanja ovog problema. Ukoliko ima više rješenja dovoljno je navesti samo jedno uz napomenu da nije jedinstveno.

Vrsta robe (i)	1	2	3
Težina (w_i) [t]	2	3	1
Količina (t_i)	4	3	5
Cijena (c_i) [hiljadu KM]	31	35	20

Rješenje:

Matematički model problema:

$$\arg\max Z = 31x_1 + 35x_2 + 20x_3$$

$$po. \quad 2x_1 + 3x_2 + x_3 \leq 10$$

$$0 \leq x_1 \leq 4$$

$$0 \leq x_2 \leq 3$$

$$0 \leq x_3 \leq 5$$

$$x_1 \in Z, x_2 \in Z, x_3 \in Z$$

Rekurzivna jednačina Bellmana (raspisana po iteracijama):

$$z(v,i) = \max\left\{c_i x_i + z(v - w_i x_i, i-1) \middle| x_i = 0, \dots, \min\left\{t_i, \left\lfloor \frac{v}{w_i} \right\rfloor\right\}\right\}, i = 1,2,3$$

$$z(v,0) = 0$$

$$z(v,1) = \max\left\{c_1 x_1 + z(v - w_1 x_1, 0) \middle| x_1 = 0, \dots, \min\left\{t_1, \left\lfloor \frac{v}{w_1} \right\rfloor\right\}\right\}$$

$$= \max\left\{31x_1 + z(v - 2x_1, 0) \middle| x_1 = 0, \dots, \min\left\{4, \left\lfloor \frac{v}{2} \right\rfloor\right\}\right\}$$

$$= \max\left\{31x_1 \middle| x_1 = 0, \dots, \min\left\{4, \left\lfloor \frac{v}{2} \right\rfloor\right\}\right\}$$

$$z(v,2) = \max\left\{c_2 x_2 + z(v - w_2 x_2, 1) \middle| x_2 = 0, \dots, \min\left\{t_2, \left\lfloor \frac{v}{w_2} \right\rfloor\right\}\right\}$$

$$= \max\left\{35x_2 + z(v - 3x_2, 1) \middle| x_2 = 0, \dots, \min\left\{3, \left\lfloor \frac{v}{3} \right\rfloor\right\}\right\}$$

$$z(v,2) = \max\left\{c_2x_2 + z(v - w_2x_2, 1) \middle| x_2 = 0, \dots, \min\left\{t_2, \left\lfloor\frac{v}{w_2}\right\rfloor\right\}\right\}$$

$$= \max\left\{35x_2 + z(v - 3x_2, 1) \middle| x_2 = 0, \dots, \min\left\{3, \left\lfloor\frac{v}{3}\right\rfloor\right\}\right\}$$

$$z(v,3) = \max\left\{c_3x_3 + z(v - w_3x_3, 2) \middle| x_3 = 0, \dots, \min\left\{t_3, \left\lfloor\frac{v}{w_3}\right\rfloor\right\}\right\}$$

$$= \max\left\{20x_3 + z(v - x_3, 2) \middle| x_3 = 0, \dots, \min\left\{5, \left\lfloor\frac{v}{1}\right\rfloor\right\}\right\}$$

Za i = 1:

$$z(v,1) = \max\left\{31x_1 \middle| x_1 = 0, \dots, \min\left\{4, \left\lfloor\frac{v}{2}\right\rfloor\right\}\right\} = 31 \min\left\{4, \left\lfloor\frac{v}{2}\right\rfloor\right\}$$

v	x	Z(v,1)
0	0	0
1	0	0
2	1	31
3	1	31
4	2	62
5	2	62
6	3	93
7	3	93
8	4	124
9	4	124
10	4	124

Za i = 2:

$$z(v,2) = \max\left\{35x_2 + z(v - 3x_2, 1) \middle| x_3 = 0, \dots, \min\left\{3, \left\lfloor\frac{v}{3}\right\rfloor\right\}\right\}$$

v	$(x_2, v - 3x_2)$	$35x_2 + z(v - 3x_2, 1)$	Z(v,2)
0	(0,0)	0	0
1	(0,1)	0 + z(1,1)=0	0
2	(0,2)	0+z(2,1)=31	31
3	(0,3), (1,0)	31, 35	35
4	(0,4), (1,1)	62, 35	62
5	(0,5), (1,2)	62, 66	66
6	(0,6), (1,3), (2,0)	93, 66, 70	93
7	(0,7), (1,4), (2,1)	93, 97, 70	97
8	(0,8), (1,5), (2,2)	124, 97, 101	124
9	(0,9), (1,6), (2,3), (3,0)	124, 97, 101, 105	124
10	(0,10), (1,7), (2,4), (3,1)	124, 128, 70+62, 105	132

Za i = 3:

$$z(v,3) = \max\left\{20x_3 + z(v - x_3, 2)\middle|x_3 = 0, \dots, \min\left\{5, \left\lfloor\frac{v}{1}\right\rfloor\right\}\right\}$$

v	$(x_3, v - x_3)$	$20x_3 + z(v - x_3, 2)$	Z(v,2)
0	(0,0)	0	0
1	(0,1), (1,0)	0, 20	20
2	(0,2), (1,1), (2,0)	31, 20, 40	40
3	(0,3), (1,2), (2,1), (3,0)	35, 51, 40, 60	60
4	(0,4), (1,3), (2,2), (3,1), (4,0)	62, 55, 71, 60, 80	80
5	(0,5), (1,4), (2,3), (3,2), (4,1), (5,0)	66, 82, 75, 91, 80, 100	100
6	(0,6), (1,5), (2,4), (3,3), (4,2), (5,1)	93, 86, 102, 95, 111, 100	111
7	(0,7), (1,6), (2,5), (3,4), (4,3), (5,2)	97, 113, 106, 122, 115, 131	131
8	(0,8), (1,7), (2,6), (3,5), (4,4), (5,3)	124, 117, 133, 126, 142, 135	142
9	(0,9), (1,8), (2,7), (3,6), (4,5), (5,4)	124, 144, 137, 153, 146, 162	162
10	(0,10), (1,9), (2,8), (3,7), (4,6), (5,5)	132, 144, 164, 157, 173, 166	173

Optimalna vrijednost funkcije cilja Z = 173. Optimalna strategija:

- Za i=3 i v = 10 maksimum za $x_3 = 4$, preostaje kapacitet 10-4 = 6.
- Za i=2 i v = 6 maksimum za $x_2 = 0$ što ostavlja ranac neizmjenjenog kapaciteta.
- Za i = 1 i v = 6 maksimum za $x_1 = 3$ preostaje kapacitet ranca v=0 (u potpunosti popunjen ranac).

Optimalno rješenje:

$$x_1 = 3, x_2 = 0, x_3 = 4, Z = 173$$

Za dva broda od 4 i 6 tona imamo sljedeće. Za brod od 4 tone:

Za i = 1:

$$z(v,1) = \max\left\{31x_1\middle|x_1 = 0, \dots, \min\left\{4, \left\lfloor\frac{v}{2}\right\rfloor\right\}\right\} = 31 \min\left\{4, \left\lfloor\frac{v}{2}\right\rfloor\right\}$$

v	x	Z(v,1)
0	0	0
1	0	0
2	1	31
3	1	31
4	2	62

Za i = 2:

$$z(v,2) = \max\left\{35x_2 + z(v-3x_2,1)\middle| x_3 = 0, \dots, \min\left\{3, \left\lfloor\frac{v}{3}\right\rfloor\right\}\right\}$$

v	$(x_2, v-3x_2)$	$35x_2 + z(v-3x_2,1)$	Z(v,2)
0	(0,0)	0	0
1	(0,1)	0 + z(1,1)=0	0
2	(0,2)	0+z(2,1)=31	31
3	(0,3), (1,0)	31, 35	35
4	(0,4), (1,1)	62, 35	62

Za i = 3:

$$z(v,3) = \max\left\{20x_3 + z(v-x_3,2)\middle| x_3 = 0, \dots, \min\left\{5, \left\lfloor\frac{v}{1}\right\rfloor\right\}\right\}$$

v	$(x_3, v-x_3)$	$20x_3 + z(v-x_3,2)$	Z(v,2)
0	(0,0)	0	0
1	(0,1), (1,0)	0, 20	20
2	(0,2), (1,1), (2,0)	31, 20, 40	40
3	(0,3), (1,2), (2,1), (3,0)	35, 51, 40, 60	60
4	(0,4), (1,3), (2,2), (3,1), (4,0)	62, 55, 71, 60, 80	80

Optimalna vrijednost funkcije cilja Z = 80. Optimalna strategija:

- Za i=3 i v = 4 maksimum je postignut za $x_3 = 4$, kapacitet ranca v=0 (u potpunosti popunjen ranac).

Optimalno rješenje za brod kapaciteta 4 glasi:

$$x_1 = 0, x_2 = 0, x_3 = 4, Z = 80$$

Za brod od 6 tona imamo:

Za i = 1:

$$z(v,1) = \max\left\{31x_1 \middle| x_1 = 0, \dots, \min\left\{4, \left\lfloor\frac{v}{2}\right\rfloor\right\}\right\} = 31 \min\left\{4, \left\lfloor\frac{v}{2}\right\rfloor\right\}$$

v	x	Z(v,1)
0	0	0
1	0	0
2	1	31
3	1	31
4	2	62
5	2	62
6	3	93

Za i = 2:

$$z(v,2) = \max\left\{35x_2 + z(v - 3x_2, 1)\middle| x_3 = 0, \dots, \min\left\{3, \left\lfloor\frac{v}{3}\right\rfloor\right\}\right\}$$

v	$(x_2, v - 3x_2)$	$35x_2 + z(v - 3x_2, 1)$	Z(v,2)
0	(0,0)	0	0
1	(0,1)	0 + z(1,1)=0	0
2	(0,2)	0+z(2,1)=31	31
3	(0,3), (1,0)	31, 35	35
4	(0,4), (1,1)	62, 35	62
5	(0,5), (1,2)	62, 66	66
6	(0,6), (1,3), (2,0)	93, 66, 70	93

Za i = 3:

$$z(v,3) = \max\left\{20x_3 + z(v - x_3, 2)\middle| x_3 = 0, \dots, \min\left\{1, \left\lfloor\frac{v}{1}\right\rfloor\right\}\right\}$$

v	$(x_3, v - x_3)$	$20x_3 + z(v - x_3, 2)$	Z(v,2)
0	(0,0)	0	0
1	(0,1), (1,0)	0, 20	20
2	(0,2), (1,1)	31, 20	31
3	(0,3), (1,2)	35, 51	51
4	(0,4), (1,3)	62, 55	62
5	(0,5), (1,4)	66, 82	82
6	(0,6), (1,5)	93, 86	93

Optimalna vrijednost funkcije cilja Z = 93.

Optimalna strategija:

- Za i=3 i v =6 maksimum je postignut za $x_3 = 0$ te kapacitet ostaje nepromjenjen
- Za i=2 i v= 6 maksimum je postignut za $x_2 = 0$ te kapacitet ostaje nepromjenjen
- Za i=1 v=6 maksimum je postignut za $x_1 = 3$ nakon čega je kapacitet ranca v=0 (u potpunosti popunjen ranac).

Optimalno rješenje za brod kapaciteta 6 glasi:

$$x_1 = 3, x_2 = 0, x_3 = 0, Z = 93$$

Vidimo da smo opet postigli isto rješenje kao i u slučaju kada imamo samo jedan brod kapaciteta 10 jer je 93+80=173 te smo uzimali iste predmete. Rješenje nije jedinstveno.