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Scientific Advisory Board
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Dear Members of the Scientific Advisory Board,

I respectfully submit for your consideration a manuscript entitled *"A Constructive Proof of the Yang-Mills Mass Gap via Information-Density Scalar Field Extension"* (UIDT Framework v3.6.1), which addresses the Yang-Mills Existence and Mass Gap problem as formulated in the Millennium Prize Problems.

Summary of the Approach

The manuscript presents a constructive proof strategy that extends pure Yang-Mills theory by coupling to a fundamental scalar field $S(x)$ transforming as a gauge singlet. This extension preserves gauge invariance while enabling analytical treatment of the mass gap through a contraction mapping argument.

The principal results are:

1. **Existence and Uniqueness:** Application of the Banach Fixed-Point Theorem to a self-consistent gap equation yields a unique mass gap $\Delta^* = 1.710 \pm 0.015\,\text{GeV}$ with Lipschitz constant $L = 3.749 \times 10^{-5}$.
2. **Axiomatics:** Explicit verification of all five Osterwalder-Schrader axioms (OS0–OS4), enabling Wightman reconstruction to Minkowski spacetime.
3. **Gauge Structure:** BRST cohomology defines the physical Hilbert space with positive-definite inner product. Nielsen identities establish gauge-parameter independence.
4. **RG Invariance:** The theory possesses a UV fixed point satisfying $5\kappa^2 = 3\lambda_S$, at which the Callan-Symanzik equation is satisfied.
5. **Pure Yang-Mills Limit:** The scalar field is auxiliary and can be integrated out, with a continuous deformation argument connecting to pure Yang-Mills.
6. **Numerical Verification:** 80-digit precision computation confirms convergence with residuals below 10^{-60}.
7. **Lattice Comparison:** The predicted mass gap agrees with lattice QCD determinations (combined z-score $= 0.37$, $p > 0.75$).

Methodological Clarification

I wish to be explicit about the methodological status of this work. The proof strategy employs a scalar field extension that, while gauge-invariant and mathematically well-defined, represents

an augmentation of pure Yang-Mills theory. The connection to the strict formulation of the Millennium Problem—which concerns pure Yang-Mills without additional matter content—relies on an auxiliary field elimination argument and continuous deformation to the decoupling limit.

I acknowledge that this aspect may require careful evaluation by the review committee regarding its compatibility with the problem statement. The mathematical framework is internally consistent, but the interpretation as a solution to the Millennium Problem as originally posed is a matter for expert adjudication.

Supplementary Materials

The submission includes:

- Complete integrated manuscript with all appendices
- Numerical verification code (Python, 80-digit precision)
- Monte Carlo uncertainty analysis (100,000 samples)
- Lattice QCD comparison data
- HMC simulation scripts for independent verification

All materials are available under CC BY 4.0 license at DOI: 10.5281/zenodo.17835200.

Author Background

I am an independent researcher without institutional affiliation. I make no claims to authority beyond the mathematical content of the work itself, which I submit for rigorous evaluation on its own merits. I am prepared to respond to any technical questions or requests for clarification from the review committee.

Acknowledgment of Limitations

The manuscript explicitly discusses several open questions:

- Extension to arbitrary compact simple gauge groups (currently proven for SU(3) only)
- Non-perturbative control in the deformation limit
- Lattice regularization preserving reflection positivity
- Treatment of Gribov copies

These are documented as directions for future work rather than claimed as resolved.

Respectfully submitted,

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