#### Foundations of Software

Martin Odersky, EPFL

Slides in part adapted from: University of Pennsylvania CIS 500: Software Foundations - Fall 2006 by Benjamin Pierce

1

# Course Overview

2

#### What is "software foundations"?

Software foundations (or "theory of programming languages") is the mathematical study of the  $\frac{1}{2}$  meaning of programs.

The goal is finding ways to describe program behaviors that are both precise and abstract.

- precise so that we can use mathematical tools to formalize and check interesting properties
- abstract so that properties of interest can be discussed clearly, without getting bogged down in low-level details

3

# Why study software foundations?

- ➤ To prove specific properties of particular programs (i.e., program verification)
  - Important in some domains (safety-critical systems, hardware design, security protocols, inner loops of key algorithms, ...), but still quite difficult and expensive
- ▶ To develop intuitions for *informal* reasoning about programs
- ➤ To prove general facts about all the programs in a given programming language (e.g., safety or isolation properties)
- ➤ To understand language features (and their interactions) deeply and develop principles for better language design (PL is the "materials science" of computer science...)

#### What you can expect to get out of the course

- ► A more sophisticated perspective on programs, programming languages, and the activity of programming
  - See programs and whole languages as formal, mathematical objects
  - ▶ Make and prove rigorous claims about them
  - Detailed knowledge of a variety of core language features
- ▶ Deep intuitions about key language properties such as type safety
- ▶ Powerful tools for language design, description, and analysis

Most software designers are language designers!

5

#### Greenspun's Tenth Rule Of Programming

Any sufficiently complicated C or Fortran program contains an ad-hoc, informally-specified, bug-ridden, slow implementation of half of Common Lisp.

- Philip Greenspun

 $\epsilon$ 

#### What this course is not

- ► An introduction to programming
- A course on functional programming (though we'll be doing some functional programming along the way)
- A course on compilers (you should already have basic concepts such as lexical analysis, parsing, abstract syntax, and scope under your belt)
- A comparative survey of many different programming languages and styles

# Approaches to Program Meaning

- Denotational semantics and domain theory view programs as simple mathematical objects, abstracting away their flow of control and concentrating on their input-output behavior.
- Program logics such as Hoare logic and dependent type theories focus on logical rules for reasoning about programs.
- Operational semantics describes program behaviors by means of abstract machines. This approach is somewhat lower-level than the others, but is extremely flexible.
- Process calculi focus on the communication and synchronization behaviors of complex concurrent systems.
- Type systems describe approximations of program behaviors, concentrating on the shapes of the values passed between different parts of the program.

8

# Overview

This course will concentrate on operational techniques and type systems.

- ▶ Part I: Modeling programming languages
  - ▶ Syntax and parsing
  - Operational semantics
  - ▶ Inductive proof techniques
  - ▶ The lambda-calculus
  - ▶ Syntactic sugar; fully abstract translations
- ▶ Part II: Type systems
  - ▶ Simple types
  - ▶ Type safety
  - References
  - Subtyping

### Overview

- ▶ Part III: Object-oriented features (case study)
  - A simple imperative object model
  - ▶ An analysis of core Java
  - ▶ An analysis of core Scala

10

# Organization of the Course

11

#### Staff

Instructor: Martin Odersky

martin.odersky@epfl.ch

Teaching Assistants: Denys Shabalin

Contact him about the mailing list! denys.shabalin@epfl.ch

Amir Shaikhha

 $\verb"amir.shaikhha@epfl.ch"$ 

Fengyun Liu

 ${\tt fengyun.liu@epfl.ch}$ 

# Information

Textbook: Types and Programming Languages,

Benjamin C. Pierce, MIT Press, 2002

Webpage: https://fos2017.github.io

13

#### Elements of the Course

- ▶ The Foundations of Software course consists of
  - ▶ lectures (Tue 10:15-12:00, INM 200)
  - ▶ exercises and project work (Wed 10:15-12:00, CO 020)
- ▶ The lecture will follow in large parts the textbook.
- ► For lack of time, we cannot treat all essential parts of the book in the lectures, that's why the textbook is required reading for participants of the course.

14

#### Homework and Projects

You will be asked to

- > solve and hand in some written exercise sheets,
- ▶ do a number of programming assignments, including
  - parsers,
  - $\,\,\vartriangleright\,\,$  interpreters and reduction engines,
  - ▶ type checkers

for a variety of small languages.

► The recommended implementation language for these assignments is Scala.

15

#### Scala

- Scala is a functional and object-oriented language that is closely interoperable with Java.
- It is very well suited as an implementation language for type-checkers, in particular because it supports:
  - ▶ pattern matching,
  - $\, \triangleright \, \, \mathsf{higher}\text{-}\mathsf{order} \,\, \mathsf{functions},$
  - ▶ inheritance and mixins.

#### Learning Scala

If you don't know Scala yet, there's help:

► The Scala web site:

www.scala-lang.org

- ➤ On this site, the documents:
  - $\begin{tabular}{ll} $ \land$ $A$ Brief Scala Tutorial an introduction to Scala for Java programmers. (short and basic). \end{tabular}$

  - $\,\,\vartriangleright\,\, An\,\, Overview\,\, of\,\, the\,\, Scala\,\, Programming\,\, Language$  (high-level).
  - $\,\,\vartriangleright\,\, Scala\,\, By\,\, Example$  (long, comprehensive, tutorial style).
- ► The assistants.

17

#### Grading and Exams

Final course grades will be computed as follows:

► Homework and project: 30%

▶ Mid-term exam: 30%

► Final exam: 40%

#### Exams:

- 1. Mid-term: first week of November (exact date TBA)
- 2. Final exam: to be announced

18

#### Collaboration

- ► Collaboration on homework is strongly encouraged.
- $\,\blacktriangleright\,$  Studying with other people is the best way to internalize the material
- ➤ Form pair programming and study groups!

  2-3 people is a good size. 4 is too many for all to have equal input.

"You never really misunderstand something until you try to teach it...
" - Anon.

– Anon.

19

#### Plagiarism

- ▶ A single group will of course share code.
- ▶ But plagiarizing code by other groups as part of a project is unethical and will not be tolerated, whatever the source.

#### Part I

# Modelling programming languages

21

#### Syntax and Parsing

- ► The first-level of modeling a programming language concerns its context-free syntax.
- ► Context free syntax determines a set of legal phrases and determines the (tree-)structure of each of them.
- ▶ It is often given on two levels:
  - ▷ concrete: determines the exact (character-by-character) set of legal phrases
  - □ abstract: concentrates on the tree-structure of legal phrases.
- ▶ We will be mostly concerned with abstract syntax in this course.
- But to be able to write complete programming tools, we need a convenient way to map character sequences to trees.

22

#### Approaches to Parsing

There are two ways to construct a parser:

- ▶ By hand Derive a parser program from a grammar.
- ► Automatic Submit a grammar to a tool which generates the parser

In the second approach, one uses a special grammar description language to describe the input grammar.

23

#### Domain-Specific Languages

- ► The grammar description language is an example of a domain-specific language (DSL).
- ➤ The parser generator acts as a processor ("compiler") for this language — that's why it's sometimes called grandly a "compiler-compiler".
- ▶ Example of a "program" in the grammar description DSL:

```
 \begin{array}{lll} \mathsf{Expr} & ::= \mathsf{Term} \ \{'+' \ \mathsf{Term} \ | \ '-' \ \mathsf{Term} \}. \\ \mathsf{Term} & ::= \mathsf{Factor} \ \{'*' \ \mathsf{Factor} \ | \ '/' \ \mathsf{Factor} \}. \\ \mathsf{Factor} & ::= \mathsf{Number} \ | \ '(' \ \mathsf{Expr} \ ')'. \end{array}
```

#### Embedded Domain Specific Languages

- ▶ An alternative to a stand-alone DSL is an Embedded DSL.
- Here, the DSL does not exist as a separate language but as an API in a host language.
- ► The host language is usually a general purpose programming language.

We will now develop this approach for grammar description languages.

25

#### An EDSL for Parsing in Scala

```
\begin{array}{lll} \mathsf{def}\;\mathsf{expr}\; : \mathsf{Parser}[\mathsf{Any}] = \mathsf{term}\; \mathsf{"rep}("+"\; \mathsf{`term}\; |\; "-"\; \mathsf{`term}) \\ \mathsf{def}\;\mathsf{term}\; : \mathsf{Parser}[\mathsf{Any}] = \mathsf{factor}\; \mathsf{``rep}("*"\; \mathsf{``factor}\; |\; "/"\; \mathsf{``factor}) \\ \mathsf{def}\;\mathsf{factor}: \mathsf{Parser}[\mathsf{Any}] = "("\; \mathsf{``expr}\; \mathsf{``"})"\; |\; \mathsf{numericLit} \end{array}
```

A small Perl hack could derive the above code from the informal grammar definition above:

- ▶ Insert a def at the beginning of each production.
- ► The "::=" becomes ": Parser[Any] =".
- ➤ Sequential composition is now expressed by a ~.
- ▶ Repetition {...} is now expressed by rep(...).
- $\blacktriangleright$  Option [...] is now expressed by  $\mathsf{opt}(...).$

26

#### Parser Combinators

▶ The differences between Grammar A and Grammar B are fairly minor.

(Note in particular that existing DSLs for grammar descriptions also tend to add syntactic complications to the idealized Grammar A we have seen).

- ➤ The important difference is that Grammar B is a valid Scala program, when combined with an API that defines the necessary primitives.
- ▶ These primitives are called parser combinators.

27

#### The Basic Idea

For each language (identified by grammar symbol S), define a function  $f_S$  that, given an input stream i,

- ▶ if a prefix of i is in S, return Success(Pair(x, i')) where x is a result for S and i' is the rest of the input.
- ightharpoonup otherwise, return Failure(msg, i) where msg is an error message string.

The first behavior is called  ${\it success}$ , the second  ${\it failure}$ .

#### The Basic Idea in Code

#### Assume:

```
class StandardTokenParsers {
            type Parser = Input ⇒ ParseResult

where

type Input = Reader[Token] // a stream of Tokens with positions.

and we assume a class Token with subclasses

▷ case class Keyword(chars: String) for keywords,

▷ case class NumericLit(chars: String) for numbers,

▷ case class StringLit(chars: String) for strings,

▷ case class Identifier(chars: String) for identifiers.
```

Also assume a class ParseResult[T] with subclasses

30

#### Object-Oriented Parser Combinators

- ▶ In fact, we will also need to express | and ~ as methods of parsers.
- ▶ That's why we extend the function type of parsers as follows:

```
abstract class Parser[T] extends (Input \Rightarrow ParseResult[T]) { // An unspecified method that defines the parser function. def apply(in: Input): ParseResult // A parser combinator for sequential composition def ^{\prime} ... // A parser combinator for alternative composition def | ... }
```

It remains to define concrete combinators that implement this class (see below).  $% \begin{center} \end{center} \$ 

31

#### A Generic Single-Token Parser

- ► The following parser succeeds if the first token in the input satisfies a given predicate p.
- ▶ If it succeeds, it reads the token string and returns it as a result.

```
\label{eq:def_token} \begin{split} & \mathsf{def}\; \mathsf{token}(\mathsf{kind}:\mathsf{String})(\mathsf{p}\colon \mathsf{Token} \Rightarrow \mathsf{boolean}) = \mathsf{new}\; \mathsf{Parser}[\mathsf{String}] \; \{ \\ & \mathsf{def}\; \mathsf{apply}(\mathsf{in}\colon \mathsf{Input}) = \\ & \mathsf{if}\; (\mathsf{p}(\mathsf{in}.\mathsf{head})) \; \mathsf{Success}(\mathsf{in}.\mathsf{head}.\mathsf{chars}, \, \mathsf{in}.\mathsf{tail}) \\ & \mathsf{else}\; \mathsf{Failure}(\mathsf{kind}+"\; \mathsf{expected}", \, \mathsf{in}) \\ \} \end{split}
```

#### Specific Single-Token Parsers

- ➤ The following parser succeeds if the first token in the input is the given keyword chars:
- ▶ If it succeeds, it returns a keyword token as a result.

```
\label{eq:continuous} \begin{split} & \mathsf{implicit} \ \mathsf{def} \ \mathsf{keyword}(\mathsf{chars}: \mathsf{String}) = \mathsf{token}("""+\mathsf{chars}+""") \ \{ \\ & \mathsf{case} \ \mathsf{Keyword}(\mathsf{chars1}) \Rightarrow \mathsf{chars} == \mathsf{chars1} \\ & \mathsf{case} \ \_ \Rightarrow \mathsf{false} \ \} \end{split}
```

 Note that keyword is marked implicit, so that you can usually write "if" instead of keyword("if").

33

The following parsers succeed if, respectively, the first token in the input is a numeric or string literal, or an identifier.

```
\label{eq:defnumericLit} \begin{split} & \mathsf{def}\;\mathsf{numericLit} = \mathsf{token}("\mathsf{number"})(\_\mathsf{isInstanceOf}[\mathsf{NumericLit}]) \\ & \mathsf{def}\;\mathsf{stringLit} = \mathsf{token}("\mathsf{string}\;\mathsf{literal"})(\_\mathsf{isInstanceOf}[\mathsf{StringLit}]) \\ & \mathsf{def}\;\mathsf{ident} = \mathsf{token}("\mathsf{identifier"})(\_\mathsf{isInstanceOf}[\mathsf{Identifier}]) \end{split}
```

34

#### The Sequence Combinator

- ➤ The sequence combinator P ~ Q succeeds if P and Q succeed sequentially, and returns the concatenation of their results.
- ▶ " is implemented as a method of class Parser.

```
abstract class Parser[T] {  \begin{tabular}{ll} def & [U](q\colon Parser[U]) = new \ Parser[T & U] \ def \ apply(in\colon lnput) = Parser.this(in) \ match \ \{ & case Success(x, in1) \Rightarrow & q(in1) \ match \ \{ & case Success(x, in2) \Rightarrow Success(new & (x, y), in2) \ case \ failure \Rightarrow failure \ \} \\ & case \ failure \Rightarrow failure \ \} \\ \} \\ \end{tabular}
```

35

Concatenating Results

Normally, the  $\tilde{\ }$  combinator returns a the results of the two parsers that are run sequentially wrapped in a "-object.

The case class  $\tilde{\ }$  is similar to Pair (but  $\tilde{\ }$  turns out to be nicer – see below).

```
case class ~[T, U](_1: T, _2: U) { override def toString = "(" + _1 +" ~ " + _2 +")" }
```

There are also two variants of  $\tilde{\ }$  which return only the left or only the right operand:

```
P < ^{\sim} Q // returns only P's result P \sim Q // returns only Q's result
```

### The Alternative Combinator

- $\blacktriangleright$  The alternative combinator  $P \mid Q$  succeeds if either P or Q succeeds
- ▶ It returns the result of P if P succeeds, or the result of Q, if Q succeeds.
- The alternative combinator is implemented as a method of class Parser.

```
 \begin{array}{l} \mathsf{def} \mid (\mathsf{q} : \Rightarrow \mathsf{Parser}[T]) = \mathsf{new} \, \mathsf{Parser}[T] \; \{ \\ \mathsf{def} \, \mathsf{apply}(\mathsf{in} \colon \mathsf{Input}) = \mathsf{Parser.this}(\mathsf{in}) \, \, \mathsf{match} \; \{ \\ \mathsf{case} \, \mathsf{s1} \; \emptyset \; \mathsf{Success}( \neg \, \neg \, ) \Rightarrow \mathsf{s1} \\ \mathsf{case} \; \mathsf{failure} \Rightarrow \mathsf{q}(\mathsf{in}) \\ \} \\ \} \end{array}
```

37

#### Failure And Success Parsers

➤ The parser failure(msg) always fails with the given error message. It is implemented as follows:

```
\begin{array}{l} \mathsf{def}\;\mathsf{failure}(\mathsf{msg}\colon\mathsf{String}) = \mathsf{new}\;\mathsf{Parser}[\mathsf{Nothing}]\;\{\\ \mathsf{def}\;\mathsf{apply}(\mathsf{in}\colon\mathsf{Input}) = \mathsf{Failure}(\mathsf{msg},\;\mathsf{in})\\ \} \end{array}
```

► The parser success(result) always succeeds with the given result. It does not consume any input. It is implemented as follows:

```
\begin{array}{l} \mathsf{def} \ \mathsf{success}[\mathsf{T}](\mathsf{result} \colon \mathsf{T}) = \mathsf{new} \ \mathsf{Parser}[\mathsf{T}] \ \{ \\ \ \ \mathsf{def} \ \mathsf{apply}(\mathsf{in} \colon \mathsf{Input}) = \mathsf{Success}(\mathsf{result}, \ \mathsf{in}) \\ \} \end{array}
```

38

# Result Conversion

The parser  $P \ \hat{} \ f$  succeeds iff P succeeds. In that case it returns the result of applying f to the result of P.

```
 \begin{array}{l} \mathsf{def} \, {}^{\smallfrown} \, [\mathsf{U}](\mathsf{f}\colon\mathsf{T} \Rightarrow \mathsf{U}) = \mathsf{new} \, \mathsf{Parser}[\mathsf{U}] \, \{ \\ \mathsf{def} \, \mathsf{apply}(\mathsf{in}\colon\mathsf{Input}) = \mathsf{Parser.this}(\mathsf{in}) \, \, \mathsf{match} \, \, \{ \\ \mathsf{case} \, \mathsf{Success}(\mathsf{x}, \, \mathsf{in1}) \Rightarrow \mathsf{Success}(\mathsf{f}(\mathsf{x}), \, \mathsf{in1}) \\ \mathsf{case} \, \mathsf{f} \Rightarrow \mathsf{f} \\ \} \end{array}
```

A variant  $\hat{\ \ }\hat{\ \ }$  takes a value V as right hand side argument.

It returns  $\boldsymbol{V}$  if the left hand parser succeeds:

```
\mathsf{def} \; \widehat{} \; \widehat{} \; [U](r \colon U) \colon \mathsf{Parser}[U] = \; \widehat{} \; \widehat{} \; (x \Rightarrow r)
```

39

#### Option and Repetition Combinators

 $\blacktriangleright$  The  $\mathsf{opt}(P)$  combinator always succeeds and returns an Option result.

It returns  $\mathsf{Some}(R)$  iff P succeeds with R. In case of failure, the result is None.

 $\begin{tabular}{ll} \hline \begin{tabular}{ll} \hline \end{tabular} \hline \end{tabular} \end{$ 

The two combinators are implemented as follows:

```
\begin{array}{ll} \mathsf{def}\;\mathsf{opt}[T](p\colon\mathsf{Parser}[T])\colon\mathsf{Parser}[\mathsf{Option}[T]] &= \\ p \; \widehat{}^{\mathsf{Nome}} \; |\;\mathsf{success}(\mathsf{None}) \\ \mathsf{def}\;\mathsf{rep}[T](p\colon\mathsf{Parser}[T])\colon\mathsf{Parser}[\mathsf{List}[T]] &= \\ p \; \widehat{}^{\mathsf{nep}}(p) \; \widehat{}^{\mathsf{Nome}} \; |\; \mathsf{case}\; x \; \widehat{}^{\mathsf{nep}}(x) \; |\; \mathsf{success}(\mathsf{List}(x)) \end{array}
```

Note that neither of these combinators can fail!

#### The Interleaved Repetition Combinator

The  $\operatorname{repsep}(P,\,Q)$  parser parses a (possibly empty) sequence

```
P\ Q\ P\ ...\ Q\ P
```

It returns a list of all results returned by P.

The repsep combinator is implemented as follows:

```
\begin{array}{ll} \mathsf{def}\;\mathsf{repsep}[\mathsf{T},\;\mathsf{U}](\mathsf{p}\colon\mathsf{Parser}[\mathsf{T}],\;\mathsf{q}\colon\mathsf{Parser}[\mathsf{U}])\colon\mathsf{Parser}[\mathsf{List}[\mathsf{T}]] \;=\; \\ \mathsf{p}\;\;\tilde{\;}\;\mathsf{rep}(\mathsf{q}\;\;\tilde{\;}>\;\mathsf{p})\;\; ^{\smallfrown}\; \{\;\mathsf{case}\;\mathsf{r}\;\;\tilde{\;}\;\mathsf{rs}\Rightarrow\mathsf{r}\;\colon\colon\mathsf{rs}\;\} \quad | \quad \mathsf{success}(\mathsf{List}()) \end{array}
```

41

#### Other Combinators

More combinators can be defined if necessary.

**Exercise:** Implement the rep1(P) parser combinator, which applies P one or more times.

Exercise: Define opt and rep directly, without making use of  $\tilde{\ }$ ,  $\ |\$ , and empty.

42

### An Example: JSON

 $\ensuremath{\mathsf{JSON}},$  or  $\ensuremath{\mathsf{JavaScript}}$  Object Notation, is a popular data interchange format.

JSON data essentially consists of objects  $\{...\},$  arrays [...], numbers, and strings.

Here is an example of a JSON value:

43

#### A JSON parser

### Testing the JSON Parser

Add a method main that can be used to test the parser.

45

```
> java examples.parsing. JSON " { "x": true "y": [1, 2] } " { x: true y: [1, 2] } " [1.13] failure: unexpected token string literal y { "x": true "y": [1, 2] } ^{\circ}
```

# Getting Better Output

- ➤ The result of the previous JSON parser was a tree containing all input tokens (slightly chaotically).
- ► We transform the productions using the ^^ combinator to get more structured output:

```
\begin{array}{lll} \mathsf{def} \; \mathsf{obj} & : \mathsf{Parser}[\mathsf{Any}] = \ // \; \mathsf{return} \; \mathsf{a} \; \mathsf{Map} \\ \text{$"\{"\ ^-> \mathsf{rep}(\mathsf{member}) < ^-"\}"\ ^- (\mathsf{ms} \Rightarrow \mathsf{Map}() \ ++ \ \mathsf{ms})$ \\ \mathsf{def} \; \mathsf{arr} \; : \mathsf{Parser}[\mathsf{Any}] = \ // \; \mathsf{return} \; \mathsf{a} \; \mathsf{List} \\ \text{$"["\ ^-> \mathsf{rep}\{\mathsf{value}\} < ^-"]"$} \\ \mathsf{def} \; \mathsf{member} : \mathsf{Parser}[\mathsf{Any}] = \ // \; \mathsf{return} \; \mathsf{a} \; \mathsf{name}/\mathsf{value} \; \mathsf{pair} \\ \mathsf{stringlit} \; \texttt{$"":"\ ^-} \; \mathsf{value} \; ^- \\ \mathsf{\{} \; \mathsf{case} \; \mathsf{name} \; "":"\ ^- \; \mathsf{value} \Rightarrow (\mathsf{name}, \; \mathsf{value}) \; \mathsf{\}} \end{array}
```

Question: What are more precise types for obj, arr, and member?

```
 \begin{array}{c} \text{def value: Parser}[Any] = (\\ \text{obj}\\ | \text{ arr}\\ | \text{ stringLit}\\ | \text{ numericLit } ^ (\_.\text{toInt}) \text{ // return an Int}\\ | \text{ "null" } ^ \wedge \text{ null} \text{ // return 'null'}\\ | \text{ "true" } ^ \wedge \text{ fulle} \text{ // return 'true'}\\ | \text{ "false" } ^ \wedge \text{ false} \text{ // return 'false'} \end{array}  If we run the test now, we get:  \begin{array}{c} \text{ > java examples.parsing. JSON1 "{ "x": true, "y": [1, 2, 3] }" \\ \{ \text{ x: true, y: } [1, 2, 3] \} \\ [1.30] \text{ parsed: Map}(\text{x} \rightarrow \text{true, y} \rightarrow \text{List}(1, 2, 3)) } \end{array}
```

#### Table of Parser Combinators

ident	identifier
keyword()	keyword or special symbol (implicit)
numericLit	integer number
stringLit	string literal
P~Q	sequential composition
P < ~ Q, P ~> Q	sequential composition; keep left/right only
P   Q	alternative
opt(P)	option
rep(P)	repetition
repsep(P, Q)	interleaved repetition
P ^^ f	result conversion
P ^^^ v	constant result

49

#### Arithmetic Expressions Again

Here is the full parser for arithmetic expressions:

```
\begin{split} & \text{object Arithmetic extends StandardTokenParsers } \{ \\ & \text{lexical.delimiters } ++= \text{List}("(",")","+","-","*","/") \\ & \text{def expr: Parser[Any]} = \text{term $^{\circ}$ rep}("+" $^{\circ}$ term $|"-" $^{\circ}$ term) \\ & \text{def term } = \text{factor $^{\circ}$ rep}("*" $^{\circ}$ factor $|"/" $^{\circ}$ factor) \\ & \text{def factor: Parser[Any]} = "(" $^{\circ}$ expr $^{\circ}")" $| numericLit \\ \end{split}
```

Question: How can we make it evaluate the parsed expression?

50

### A Problem with Top-Down Parsing

As parser combinators work top-down, they don't allow left-recursion. A production like

```
\mathsf{def}\;\mathsf{expr}=\mathsf{expr}\;\tilde{\ }"-"\;\tilde{\ }\mathsf{term}
```

would go into an infinite recursion when executed.

The alternative:

```
\label{eq:defeq} \mbox{def expr} = \mbox{term "rep("-" " term)} \mbox{produces a "right-leaning" tree: } X-Y-Z \mbox{ parses}
```

X  $\tilde{\ }$  List("-"  $\tilde{\ }$  Y, "-"  $\tilde{\ }$  Z)

But the correct reduction/evaluation of +,-,\*,/ is left-leaning!

51

#### Evaluate by FoldLeft

We can solve this problem by delaying reduction until all elements of a repetition have been parsed and then performing a fold left on the list:

Here, reduceList is defined in terms of the fold-left operation  $/: \dots$ 

```
 \begin{array}{l} \mathsf{val} \ \mathsf{reduceList} \colon \mathsf{Expr} \ \tilde{\ } \ \mathsf{List}[\mathsf{String} \ \tilde{\ } \ \mathsf{Expr}] \Rightarrow \mathsf{Expr} = \{ \\ \mathsf{case} \ \mathsf{i} \ \tilde{\ } \ \mathsf{ps} \Rightarrow (\mathsf{i} \ / ; \ \mathsf{ps})(\mathsf{reduce}) \\ \} \end{array}
```

... and reduce is defined as follows:

```
\label{eq:continuous_state} \begin{array}{l} \text{def reduce}(x\colon Int, \, r\colon String \ ^{\circ}\ Int) = r\ \text{match}\ \{\\ \text{case}\ ^{\circ}+^{\circ}\ ^{\circ}\ y \Rightarrow x + y\\ \text{case}\ ^{\circ}-^{\circ}\ ^{\circ}\ y \Rightarrow x - y\\ \text{case}\ ^{\circ}-^{\circ}\ ^{\circ}\ y \Rightarrow x / y\\ \text{def}\ ^{\circ}\ \text{throw new MatchError}(\text{"illegal case: "+r})\\ \end{array} With this, we get:
```

java examples.parsing.ArithmeticParsers1 " 2\*(3+7) " 2\*(3+7) [1.12] parsed : 20

(Note: the chainl combinator in the standard library does this for you.)

53

#### Conclusion

- Combinator parsers give a provide a concise, flexible, and high-level way to construct parsers.
- ► The token classes of a context free grammar are modeled as primitive parsers.
- ▶ The combination forms are modeled as higher-order parsers.
- ▶ Combinator parsers are an example of an embedded DSL.
- ▶ By contrast, classical parser generators are stand-alone DSLs.
- Advantage of an embedded DSL over a parser generator: It's easier to connect the results of combinator parsers with the environment.
- ▶ Disadvantage: lower efficiency but this can be overcome (using packrat parsing)