Vectors

- `Complex Vector dot product:` $\vec{v} \cdot \vec{w} = \sum_{i=1}^{n} \bar{v}_i w_i$
- `length of a vector:` $||\vec{v}|| = \sqrt{\vec{v} \cdot \vec{v}}$ or $||\vec{v}|| = \sqrt{\sum_{i=1}^{n} v_i^2}$
- `orthogonal vectors:` $\vec{v} \cdot \vec{w} = 0$
- `parallel vectors:` $\vec{v} = c\vec{w}$
- `angles between vectors:` $\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{||\vec{v}||||\vec{w}||}$
- `cauchy-bunyakovsky-schwarz (CBS) inequality:` $|\vec{v} \cdot \vec{w}| \leq ||\vec{v}||||\vec{w}||$
- `triangle inequality:` $||\vec{v} + \vec{w}|| \leq ||\vec{v}|| + ||\vec{w}||$
- `backwards triangle inequality:` $||\vec{v} - \vec{w}|| \geq |||\vec{v}|| - ||\vec{w}|||$
- `unit vector:` $\hat{v} = \frac{\vec{v}}{||\vec{v}||}$
- `find middle of line:` $\frac{\vec{v} + \vec{w}}{2}$

Matrices

- `symmetric:` $A^T = A$
- `skew-symmetric:` $A^T = -A$
- `hermitian:` $A^H = A \rightarrow (a_{ij} = \bar{a_{ji}})$
- `skew-hermitian:` $A^H = -A \rightarrow (a_{ij} = -\bar{a_{ji}})$

Matrix Multiplication

- $A_{m \times n}$, `rows:` m , `columns:` n

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

- reverse order law of transposition: $(AB)^T = B^T A^T$

liniar systems:

$A\vec{x} = \vec{b}$ A is invertible $\iff$ $A\vec{x} = \vec{0}$ has only the trivial solution $\vec{x} = \vec{0}$, or A is non singular

linear transformations

$cT(\vec{v}) = T(c\vec{v})$ and $T(\vec{v}) + T(\vec{w}) = T(\vec{v} + \vec{w})$

range and kernel of T

- `range:` $R(T) = \{T(\vec{v}) | \vec{v} \in V\}$
- `kernel:` $N(T) = \{\vec{v} \in V | T(\vec{v}) = \vec{0}\}$

rotations

- `2D:` $R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
- `3D:` $R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- `opposite rotation:` $R_\theta^T = R_{-\theta}$

projections

`orthogonal projection:` get rid of the z component

lines to lines

equally spaced lines stay equally spaced

matrix of a linear transformation

$T : V \rightarrow W$, $dim(V) = n$, $dim(W) = m$ then A is a $m \times n$ matrix