Analysis II

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Indice

1 The vector space $\mathbb{R}^n$

1.1 Operations

Addition and scalar multiplications are defined as follows:

$$\mathbf{x}+\mathbf{y}=\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}+\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}=\begin{pmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{pmatrix}$$

$$\lambda\mathbf{x}=\lambda\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} \lambda x_1 \\ \vdots \\ \lambda x_n \end{pmatrix}$$

$$\lambda\mathbf{x}+\mathbf{y}=\lambda\mathbf{x}+\lambda\mathbf{y}$$

Scalar product, $\langle\cdot,\cdot\rangle : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, in the vector space $\mathbb{R}^n$ is defined as:

$$\langle\mathbf{x},\mathbf{y}\rangle=\sum_{k=1}^{n} x_k y_k$$

The scalar product satisfies the following properties:
1 Positive definiteness: $\langle\mathbf{x},\mathbf{x}\rangle \geq 0 \forall x$ with $\langle\mathbf{x},\mathbf{x}\rangle = 0 \overset{\text{iff}}{\Longleftrightarrow} x = 0$

2 Allegati

2.1 Dimostrazione 1