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Multivariate Analysis and Statistical Learning

Backfitting process in Generalized Additive Model

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Generalized Additive Model

Generalized additive models or simply GAMs extend the standard linear model by allowing non-linear functions of each of the variables, while maintaining additivity.

The GAM is in the form

$$y_i = \beta_0 + \sum_{j=1}^{p} f_j(x_{ij}) + \varepsilon_i$$

It is called an additive model because we calculate a separate f_j for each X_j , and then add together all of their contributions

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Backfitting

Standard software such as the `gam()` function in R can be used to fit GAMs using smoothing splines, via an approach known as *backfitting*.

This method fits a model involving multiple predictors by *backfitting* repeatedly updating the fit for each predictor in turn, holding the others fixed.

To understand the backfitting process, we can simulate it on a traditional linear model.

Backfitting

Let's pretend we don't have R functions able to fit more than one parameter.

To estimate parameters we keep fix one estimated (hat) parameter at time e repeat the process until results converge.

For each parameter let's estimate the following formula

$$Y - \widehat{\beta_1} X_1 - \widehat{\beta_2} X_2 - \ldots - \widehat{\beta_{k-1}} X_{k-1} - \widehat{\beta_{k+1}} X_{k+1} - \ldots - \widehat{\beta_p} X_p = \beta_0 + \beta_k X_k + \varepsilon$$

until it converges

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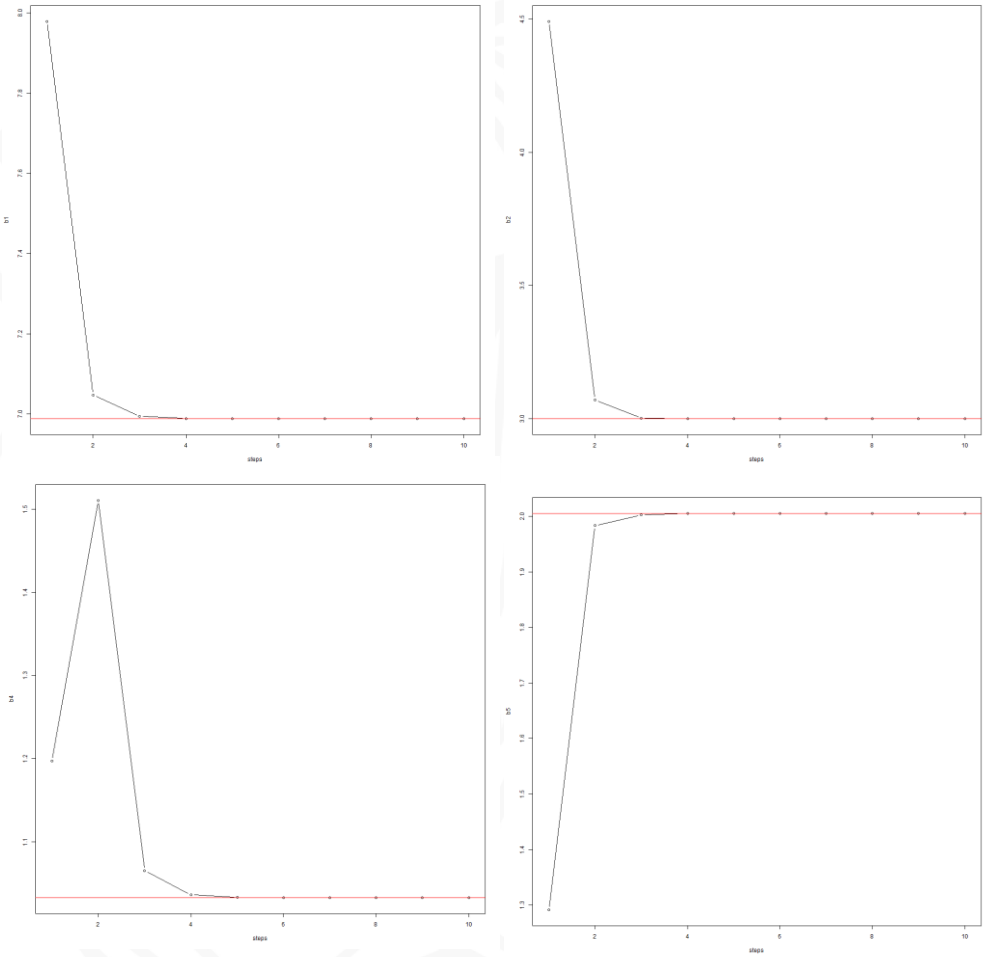
Results

In few steps all five parameters are found even if we start from random values.

In the same way, parameters in GAM are found one by one, using backfitting



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References

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