

P5.2 Statistics for Medicine

Massimo Borelli

Master of Advanced Studies in Medical Physics



Commonly used random variables

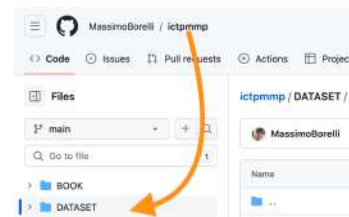


the Normal distribution

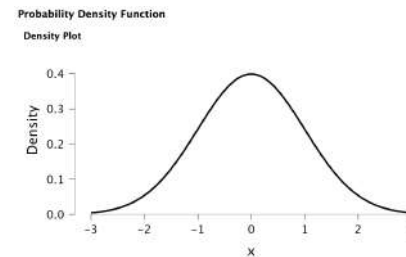
Example (B. Rosner, example 5.22)

The cerebral blood flow (CBF) in the general population is, approximately, normally distributed with mean $\mu = 75$ and standard deviation $\sigma = 17$ ($\sigma^2 = 289$). Which could be the percentage of persons having a CBF < 40 ?

Brief recalls on random variables



the Normal distribution



the Normal distribution

Example (B. Rosner, example 5.23)

Glaucoma is characterized by intraocular pressure greater than 20 mmHg, while in normal population intraocular pressure X has mean $\mu = 16$ and standard deviation $\sigma = 3$. How much it could be $P(12 \leq X \leq 20)$?

Finite random variables

The otitis dataset
(number of episodes of otitis media in the first two years of life)

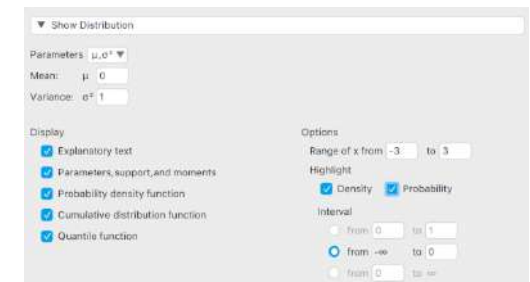
$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0.129 & 0.264 & 0.271 & 0.185 & 0.095 & 0.039 & 0.017 \end{pmatrix}$$

- probability mass / discrete density function

Example (graph)

Draw the barplot with JASP

the Normal distribution



the Normal distribution

Example (again intraocular pressure)

Can you find the upper and the lower fifth percentile of the intraocular pressure, as above defined ($\mu = 16, \sigma = 3$)?

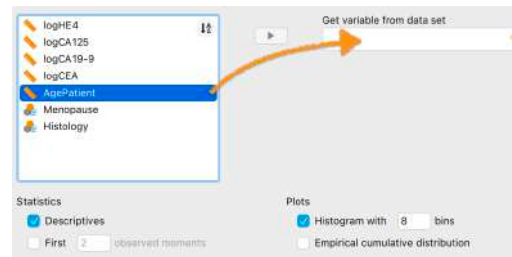
the Normal distribution: the QQ plot



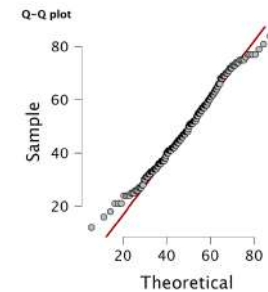
S. Najaf Zadeh
the roma dataset

- Histology
- AgePatient
- Menopause status
- four biomarkers (log transformed):
 - logHE4,
 - logCA125
 - logCA19.9
 - logCEA

the Normal distribution



the Normal distribution



caveat: the Normal distribution

Do two dromedaries make a camel?



caveat: the Normal distribution

Do two dromedaries make a camel?

Bernard Rosner

... linear combination of normal random variables are often of specific concern. It can be shown that any linear combination of normal random variables is itself normally distributed.

Martin Bland:

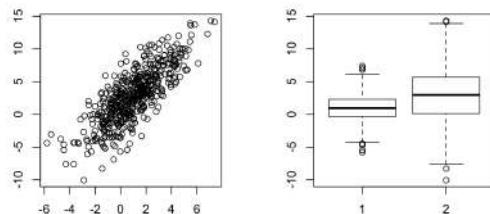
... If we add two variables from Normal distributions together, even with different means and variances, the sum follows a Normal distribution.

caveat: the Normal distribution

Do two dromedaries make a camel?

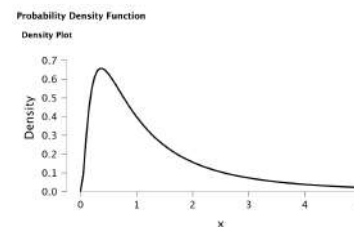


caveat: the Normal distribution



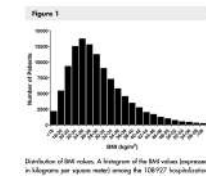
Edward L. Melnick, Aaron Tenenbein. 2012
Misspecifications of the Normal Distribution
<https://www.tandfonline.com/doi/pdf/10.1080/00031305.1992.10483052>

the log-Normal distribution



Eckhard Limpert, et al.
Log-normal Distributions across the Sciences: Keys and Clues
<https://academic.oup.com/bioscience/article/51/5/341/243981>

the logNormal distribution



Example (Gregg Fonarow et al. - summarizing body mass index)

Suppose that you are required to lead a pilot study concerning radiation dosimetry in 25 obese patients. How do you think you are going to describe the data? Using the mean and the standard deviation, or the median and the quartiles? What are here the difficulties?

Brief recalls on random variables

the binomial distribution

Free parameter	Fixed parameter
Probability of success: p 0.186	Number of trials: n 210
Display	Options
<input type="checkbox"/> Explanatory text	Range of x from 20 to 60
<input type="checkbox"/> Parameters, support, and moments	Highlight
<input checked="" type="checkbox"/> Probability mass function	<input type="checkbox"/> Mass <input checked="" type="checkbox"/> Cumulative Probability
<input type="checkbox"/> Cumulative distribution function	Interval 30 $\leq X \leq$ 50

Example (probability)

Suppose that you collect a new sample of 210 women with the same symptoms of those enrolled in roma. Obviously, only by chance you will observe exactly '39' malignancies. Can you compute the probability to observe a number of malignancy between 30 and 50?

Brief recalls on random variables

the Poisson distribution

1				1	2
2	3	1	2	1	
	1		2		1
1	1	2		4	1
1		1		3	1
	2	1	1		

Example (probability)

Use JASP to discover in a $\lambda = 0.37$ Poisson distribution how many, in probability, cells could have a value greater or equal than 2.

Brief recalls on random variables

the Poisson distribution

The screenshot shows the JASP R interface. At the top, there are tabs for 'Factor', 'Distributions', and 'R (Beta)'. The 'R (Beta)' tab is selected. Below the tabs, there is a text area with the following text:

```
Welcome to R in JASP!  
> rpois(n = 36, lambda = 0.37)  
[1] 0 0 0 1 0 0 0 1 0 1 0 0 0 1 0 0 1 1 0 1 0 1 1 0 0 1 0 0 2  
0 0 0 0 0 0
```

Below the text area, there is a button labeled 'Run Code' and a button labeled 'Clear Output'. An orange arrow points from the 'Run Code' button to the 'R (Beta)' tab. Another orange arrow points from the 'Run Code' button to the code editor area.

Statistics for Medicine

Massimo Borelli

Master of Advanced Studies in Medical Physics



Massimo Borelli

Statistics for Medicine

Frequencies

Histology	Menopause		Total
	ante	post	
benign	106	65	171
malignant	12	27	39
Total	118	92	210

Table: Menopausal status and malignancy in ovarian cancer

Massimo Borelli

Statistics for Medicine

- prevalence
- sensitivity and specificity
- predictive values
- ROC curves
- ...

<https://github.com/MassimoBorelli/ictpmmp/tree/main/BOOK>

professor Luigi Rigon

Massimo Borelli

Statistics for Medicine

Evaluating odds and risks

Odds Ratio

Histology	Menopause		Total
	ante	post	
benign	106	65	171
malignant	12	27	39
Total	118	92	210

Example (Odds Ratio)

Explore the output of the Odds Ratio (2 × 2 only) checkbox in the Statistics menu of the contingency table of Histology (Rows) versus Menopause (Columns).

Massimo Borelli

Statistics for Medicine

the Bayes factor: JASP core business!

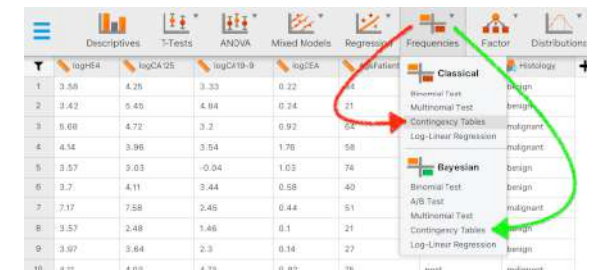
- Alice has a balanced urn with 5 winning black balls and 5 white balls ($p = 0.5$)
- Bob has a tricky urn with 6 winning black balls and 4 white balls ($p = 0.6$).

(binomial scheme, extractions with replacement)
we observe 115 successes over 200 draws, but without knowing if they are generated from Alice's or Bob's urn.

Massimo Borelli

Statistics for Medicine

Frequencies



Massimo Borelli

Statistics for Medicine

Bayes Theorem

$$P(\text{malignant}|\text{ante}) = \frac{P(\text{ante}|\text{malignant}) \cdot P(\text{malignant})}{P(\text{ante})}$$

Histology	Menopause		Total
	ante	post	
benign	106	65	171
malignant	12	27	39
Total	118	92	210

Massimo Borelli

Statistics for Medicine

the Bayes factor: JASP core business!

$$P(X = 115 | \text{Alice}) = \binom{200}{115} \cdot 0.5^{115} \cdot 0.5^{200-115} \approx 0.006$$

$$P(X = 115 | \text{Bob}) = \binom{200}{115} \cdot 0.6^{115} \cdot 0.4^{200-115} \approx 0.044$$



Massimo Borelli

Statistics for Medicine

the Bayes factor: JASP core business!

$$\frac{P(X = 115 | \text{Bob})}{P(X = 115 | \text{Alice})} \approx \frac{.044}{.006} \approx 7.4$$

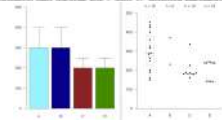
it is much more likely that the balls have been drawn by Bob's urn: about seven times higher

the Bayes factor:

$$\frac{P(D|M_1)}{P(D|M_2)} = \frac{P(M_1|D)}{P(M_2|D)} \cdot \frac{P(M_2)}{P(M_1)}$$

Mismatching variability with reliability

transfected endothelial cells. A shows migration assay for control cells and cells transfected with siRNA against AP-5. Error bars in A (n = 3), and B (n = 3) represent SD while in C represent SEM (n = 4). D shows gelatin zymography of MMP-9 levels in AP-5 and AP-5 siRNA (25 ng/ml).



Tatsuki Koyama.

Beware of Dynamite

<https://biostat.app.vumc.org/wiki/pub/Main/TatsukiRcode/Poster3.pdf>