PK

Signal Processing f. Computer Ing.
4th lecture, Suggested Solution.

1) Given $x_c(t) = \sin(2\pi(100)t)$ and $T = 1/400$ sec. Find $x[n]$.

$$x[n] = x_c(nT)$$

$$= \sin\left(2\pi(100)n\frac{1}{400}\right)$$

$$= \sin\left(\frac{\pi}{2}n\right)$$

2) Given $x[n] = \cos\left(\frac{\pi}{4}n\right) \quad -\infty < n < \infty$
This signal is derived by sampling of the continuous-time signal

$$x_c(t) = \cos(\Omega_0 t) \quad -\infty < t < \infty$$

using a sample-rate equal to 1000 sample/sec

Find two possible values of Ω_0

We have that $\omega = T \cdot \Omega$ and $T = \frac{1}{1000}$ sec.

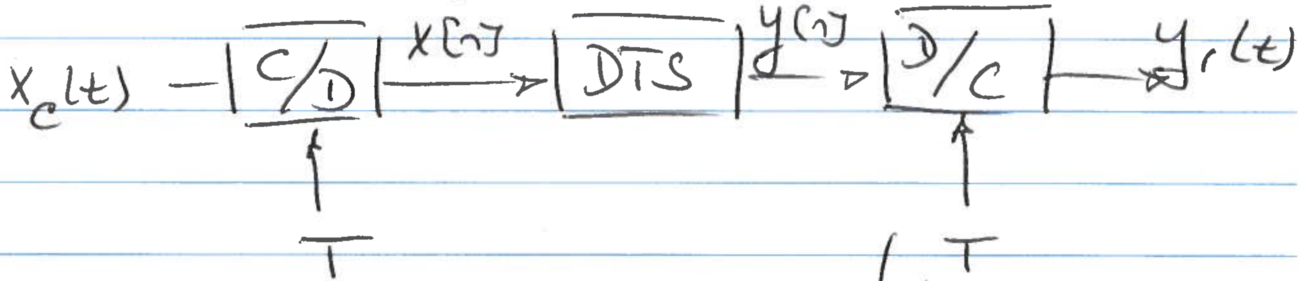
$\Downarrow$

$$\Omega_0 = \omega \cdot \frac{1}{T} = \frac{\pi}{4} \cdot 1000 = 250\pi$$

Since, in the discrete-time domain, it is not possible to distinguish a signal with frequency ω_0 from a signal with frequency $\omega_0 + 2\pi k$, then an alternative value of Ω_0 is;

$$\Omega_0 = \left(\frac{\pi}{4} + 2\pi\right) \cdot 1000 = 2250\pi$$

5) Given the system

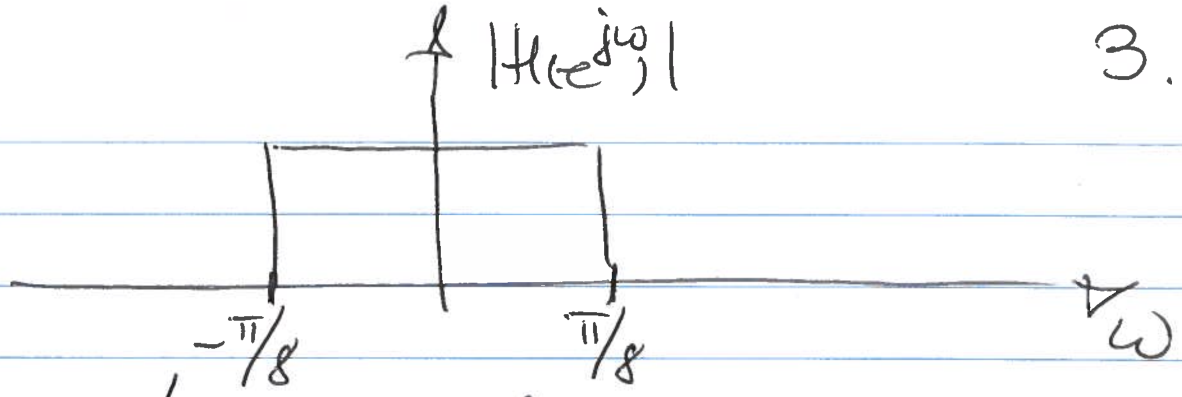


Where DTS is an ideal LP. filter with $f_{cut\text{-}off}$ equal to $\pi/8$ rad/sec.

a) Assume $x_c(t)$ is band-limited to 5kHz

Find the maximal value of T, such that there is no aliasing in the C/D.

Basically, this is the same as finding the lowest possible f_{sample}.



and $X_c(\Omega) = 0$ for $|\Omega| \geq 2\pi \cdot 5000$

According to Nyquist Sampling Theorem, the sample frequency should be at least twice the bandwidth of the input signal, i.e.,

$$T = \frac{1}{2 \cdot 5000} \text{ sec} = 0{,}1 \text{ ms}$$

b) Now, assuming that $T = 0{,}1$ ms, find f_{cut_off} of the EFFECTIVE CONTINUOUS-TIME FILTER. (ECTF)

The ECTF is the filter seen between the terminals $x_c(t)$ and $y_r(t)$

$$\begin{cases} f_s = \dfrac{1}{T} = 10 \text{ kHz} \\ \omega = T \cdot \Omega \\ \dfrac{\pi}{8} = \dfrac{1}{10 \cdot 10^3} \cdot \Omega_{cut\_off} \\ \Omega_{cut\_off} = 2 \cdot \pi \cdot f_{cut\_off} \end{cases}$$

$\Downarrow$

$$f_{cut\_off} = \frac{1}{2\pi} \cdot \left(10 \cdot 10^3 \cdot \frac{\pi}{8}\right) = 625\,Hz$$

c) Redo b) but now assuming $T = (20 \cdot 10^3\,Hz)^{-1}$

$$\begin{cases} \frac{1}{T} = 20 \cdot 10^3\,Hz \\ \omega = T \cdot \Omega \\ \frac{\pi}{8} = \frac{1}{20 \cdot 10^3} \cdot \Omega_c \\ \Omega_c = 2\pi \cdot f_{cut\_off} \end{cases}$$

$\Downarrow$

$$f_{cut\_off} = \frac{1}{2\pi}\left(20 \cdot 10^3 \cdot \frac{\pi}{8}\right) = 1250\,Hz$$

What can we learn from this result?:

By changing the sample frequency, we also change the DTS

In other words: It doesn't make sense to just increase the sample frequency because all frequencies in the discrete-time system are related to the sample frequency, and thus they change if the sample frequency is changed.

IMPORTANT

6) $h_c(t)$ is the impulse response of an LTI continuous-time filter, and $h[n]$ is the impulse response of a discrete-time filter.

a) For, $h_c(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$, $a > 0$ and real)

find the frequency response of the continuous-time filter, and plot the amplitude-response.

The frequency response $H_c(j\Omega)$ is found by Fourier transforming $h_c(t)$

$$H_c(j\Omega) = \int_{-\infty}^{\infty} h_c(t) \cdot e^{-j\Omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\Omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\Omega)t} \, dt$$

$$= \frac{1}{a + j\Omega}$$

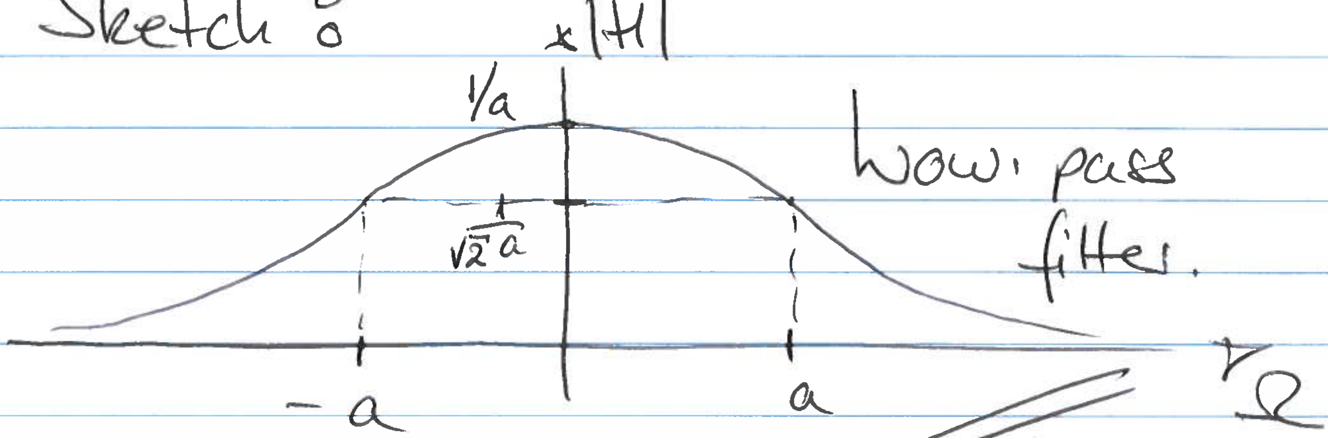
Can be found in any Fourier transform table

Amplitude response:

$$|H_c(j\Omega)| = \left|\frac{1}{a+j\Omega}\right| = \frac{1}{|a+j\Omega|}$$

$$= \frac{1}{\sqrt{a^2+\Omega^2}} = \begin{cases} 1/a & \text{for } \Omega = 0 \\ 1/\sqrt{2}\cdot a & \text{for } |\Omega| = a \\ \to 0 & \text{for } |\Omega| \to \infty \end{cases}$$

Sketch:



Low pass filter.

b) Assume that $h_d[n] = T\cdot h_c(nT)$.

Find the frequency response of the discrete-time filter.

First we determine the discrete-time impulse response;

$$h_d[n] = T\cdot e^{-anT}\cdot u[n]$$

Based on the impulse response, we now calculate the frequency response; (using the Discrete Time Fourier Transform)

$$H_d(e^{j\omega}) = DTFT\{h_d[n]\} = \sum_{n=-\infty}^{\infty} h_d[n] \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} T \cdot e^{-anT} \cdot e^{-j\omega n}$$

$$= T \cdot \sum_{n=0}^{\infty} e^{-(aT+j\omega)n}$$

$$= T \cdot \frac{1-e^{-\infty}}{1-e^{-aT} \cdot e^{-j\omega}} = \frac{T}{1-e^{-aT} \cdot e^{-j\omega}}$$

In order to find the amplitude response, we now find modulus of $H_d(e^{j\omega})$

$$\left|H_d(e^{j\omega})\right| = \left|\frac{T}{1-e^{-aT}(\cos\omega - j\sin\omega)}\right|$$

$$= \frac{T}{\sqrt{(1-e^{-aT}\cos\omega)^2 + (e^{-aT}\sin\omega)^2}}$$

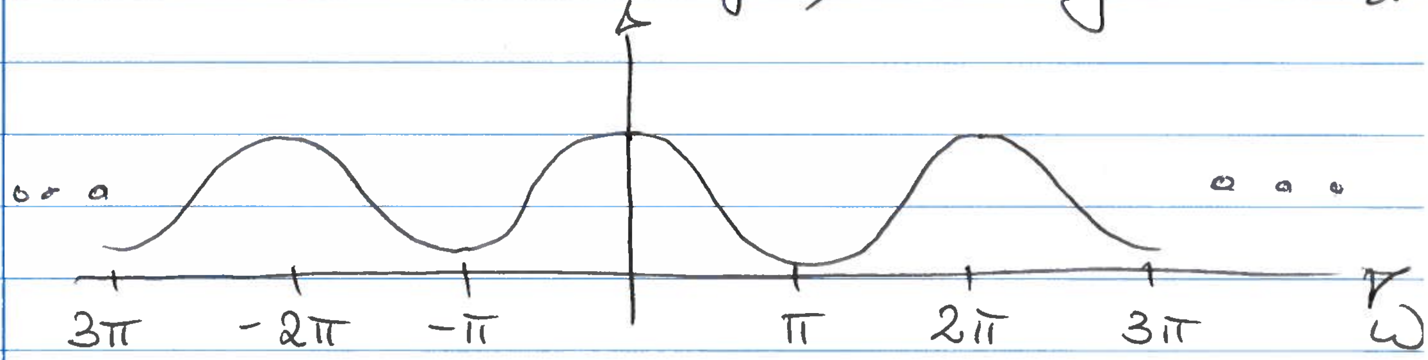
$$= \frac{T}{\sqrt{\left(1-2e^{-aT}\cdot\cos\omega + e^{-2aT}\right)}}$$

Let's have a closer look at this function:

Due to the $\cos\omega$ function, $|H_d(e^{j\omega})|$ is <u>PERIODIC</u>.

When $\omega = 0$ and $|\omega| = 2\pi k$, $|H_d(e^{j\omega})|$ has max value, and it has min value for $\omega = k\cdot\pi$, $k = \pm1, \pm3, \pm5 \ldots$

Based on this analysis, we may sketch $|H_d|$



Well, yes it looks like an HP-filter.

Lesson learned:

Sampling in the time domain leads to periodic frequency domain — not only for signals, but also for systems.