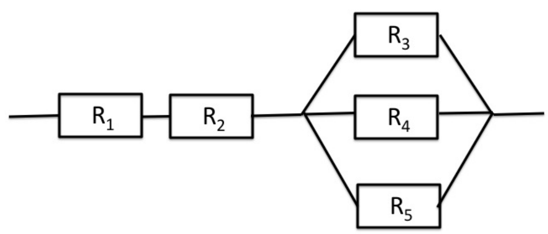
Introduction to Probability Theory and Statistics

Exam, June 2016

Probability Theory

Problem 1
Let a production line consist of 5 components (or robots $R_1, \ldots, R_5$), two robots in sequence, followed by 3 robots in parallel (see Figure). Each robot R_i functions independently from the others with a probability p_i, $i = 1, \ldots, 5$. Find the probability that the whole production line is functioning.



Problem 2
Suppose we have 3 different sensors measuring the same physical phenomenon. The first sensor S_1 is the most expensive and the most precise one: it provides the correct reading 99% of the time. The second sensor S_2 gives correct measurements 75% of the time and the third sensor S_3 only 50% of the time.

In one time unit we receive one reading from each of S_2 and S_3 and two readings from S_1. This gives us the probability that the received data originates from S_1 being equal to 50%, from S_2 25% and from S_3 25%.

(a) If a reading is received without knowing which sensors has produced this data, what is the probability that this reading is correct?

(b) If we have a reading and we know that it is a correct one, which of the sensors is most likely to have produced the reading?

Problem 3
Let X and Y be independent random variables with mean $E[X] = 0$ and $E[Y] = 1$ and variance $Var(X) = 1$ and $Var(Y) = 1$.

(a) Let $Z = 2X - 4Y + 10$. Find mean and variance of Z.

(b) Let $M = (X - 1)Y$. Find mean of M.

(c) Find covariance of X and Y.