Department of Electronic Systems

Written exam in
Signal processing, 3 ECTS

Wednesday, January 4, 2011
9.00 – 13.00

Remember to write your full name on every sheet you deliver!
The answers concerning analogue filters, digital filters and spectral estimation have to be written on separate sheets.
The problems are weighted as listed.
All ordinary tools may be used e.g. books, notes and laptop.
All electronic communication devices must be turned off at all times.
Communication with others is strictly prohibited.

Problem 1 (Weighted with 11 % - Digital filters)

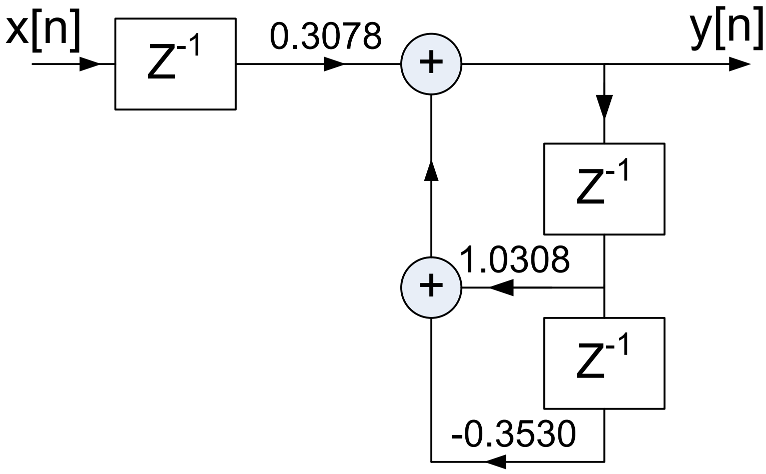


Figure 1. Block diagram for a digital filter.

A digital filter is described by the block diagram in figure 1. A sampling frequency of fs=1280 Hz is used.

Questions:

1) Write the difference equation for the filter.

   y[n] = 0.3078x[n-1] + 1.0308y[n-1] - 0.3530 y[n-2]

2) Determine the transfer function H(z).

$$H(z) = \frac{0.3078z^{-1}}{1-1.0308z^{-1}+0.3530z^{-2}}$$

3) Compute the first four samples of the step response

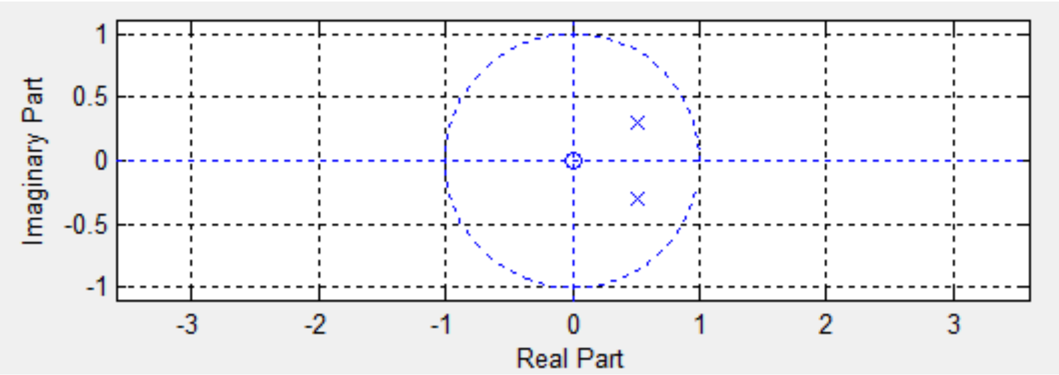
   y[0] = 0
   y[1] = 0.3078
   y[2] = 0.3078 + 1.0308 * 0.3078 =0.6251
   y[3] = 0.3078 + 1.0308 *0.6251 – 0.3530*0.3078 = 0.8435

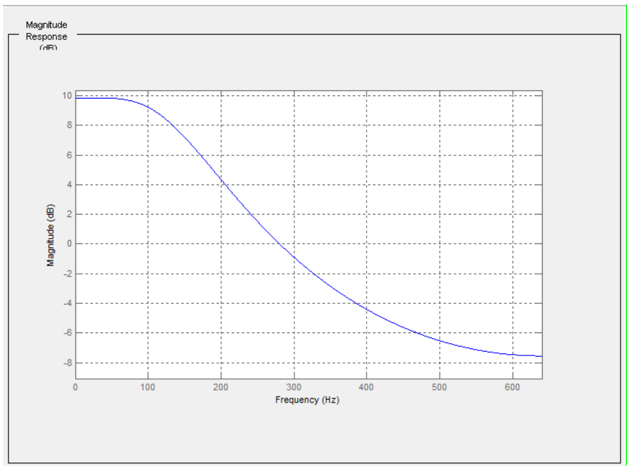
4) Determine the location of poles and zeros

   Using the matlab function roots with the coefficients [1 -1.0308 0.3530] we get:
   Zeros: z=0
   Poles: z= (0.5154 +/- 0.2956i)

5) What kind of filter does the block diagram in figure 1 represent (highpass, lowpass, etc)? Argue!

It’s a lowpass filter as the poles are located close to DC and there are no zeros outside origo.





6) Determine the gain at DC

$$H(e^{j\omega})\Big|_{\omega=0} = \frac{0.3078e^{-j\omega}}{1-1.0308e^{-j\omega}+0.3530e^{-2j\omega}}$$

$$= \frac{0.3078}{1-1.0308+0.3530} = 0.9553 = -0.3971dB$$

Problem 2 (Weighted with 11% - Digital filters)

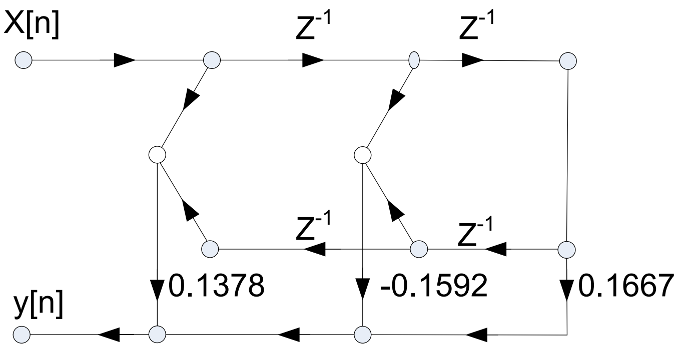


Figure 2. Signal flow graph for a digital highpass filter

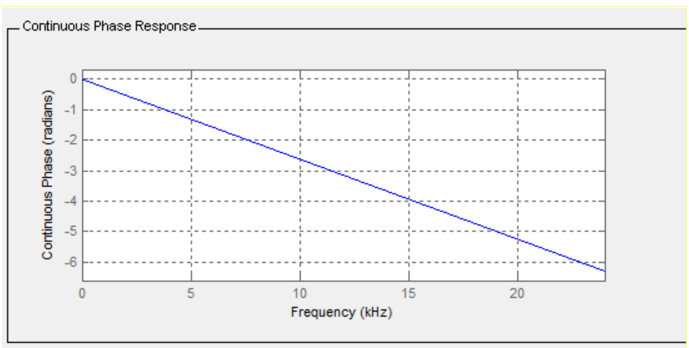
A digital highpass filter is defined by the flow graph shown in the above figure. All internal registers are reset before n=0. The sampling frequency is Fs=48 kHz and the -3 dB cutoff frequency is 20 kHz.

Questions:

1. Determine the initial six samples of the impulse response
   It's seen that the impulse response is given by the sequence:
   0.1378 -0.1592 0.1667 -0.1592 0.1378 0
2. Write the difference equation for the filter.
   $y[n] = 0.1378x[n] - 0.1592x[n-1] + 0.1667\,x[n-2] - 0.1592x[n-3] + 0.1378x[n-4]$
3. Determine the transfer function H(z).
   $H(z)= 0.1378 - 0.1592z^{-1} + 0.1667\,z^{-2} - 0.1592z^{-3} + 0.1378z^{-4}$
4. Draw the phase response in the interval [0Hz; 24kHz]
   It's a linear phase filter i.e. phase is given

$$\arg\{H(e^{j\omega})\} = -(M/2)\omega$$

   Where M=4

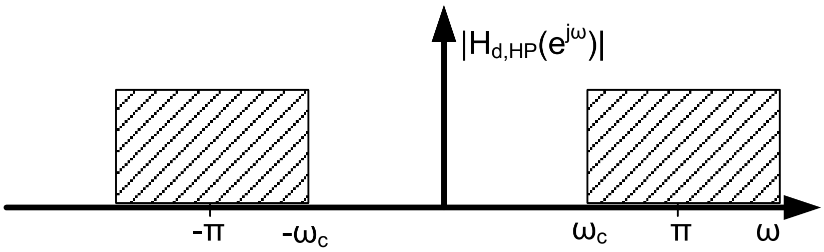


5. Verify that the filter coefficients can be obtained with the Windowing approach using a rectangular window.

The ideal frequency response for the highpass filter is:

$$H_{d,HP}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2} & \omega_c \le |\omega| \le \pi \\ 0 & 0 \le |\omega| \le \omega_c \end{cases}$$

Which may be illustrated as:



Hence the coefficients are:

$$h_{d,HP}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{d,HP}(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{\sin \pi(n-\frac{M}{2})}{\pi(n-\frac{M}{2})} - \frac{\sin \omega_c(n-\frac{M}{2})}{\pi(n-\frac{M}{2})}, \qquad -\infty < n < \infty$$

Using M=4 and $\omega_c = \left(\frac{20000}{48000}\right)2\pi = 2.6180\ radians\ /sample$ we find that the coefficients are:

0.1378 -0.1592 0.1667 -0.1592 0.1378

6. Determine the gain factor that will ensure a maximum gain of 0 dB.

The required gain can be ensured using:

$$H(e^{j\omega}) = \sum_{k=0}^{M} h[k]e^{-jk\omega} \Rightarrow$$

$$H(e^{j\omega}) = \sum_{k=0}^{k=4} h[k]e^{-jk\omega} \Rightarrow$$

$$G|H(e^{j\omega})| = 1|_{\omega=\pi} \Rightarrow$$

$$G = \frac{1}{h[0] - h[1] + h[2] - h[3] + h[4]}$$

Gain-faktoren G=1/(0.1378+0.1592+ 0.1667 + 0.1592 + 0.1378) =1/0.7607 = 1.3146

Problem 3 (Weighted with 12% - Digital filters)

An analog Butterworth filter is defined by the transfer function:

$$H_a(s) = \frac{\Omega_c}{s + \Omega_c}$$

Where $\Omega_c = 1000\ Hz$ is the cutoff frequency.

Questions:

1. Determine a digital filter with the same cutoff frequency using the bilinear transformation. The sampling frequency is 8kHz. The gain at DC should be 10 dB.

First we find $\omega_c = 2\pi\Omega_c / fs = \pi/4$

Pre-warping:

$$\Omega_c = \frac{2}{T_s}\tan\left(\frac{\omega_c}{2}\right) \Rightarrow$$

$$\Omega_c = \frac{2}{T_s}\tan\left(\frac{0.25\pi}{2}\right) = \frac{0.828}{T_s}$$

The system function for the analogue filter is:

$$H_a(s) = \frac{1}{1 + sT_s / 0.828}$$

Now we apply the bilinear transformation to the prewarped analogue filter

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T_S}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{1}{1 + 2/0.828((1-z^{-1})/(1+z^{-1}))}$$

$$= \frac{1+z^{-1}}{1+z^{-1} + 2.415(1-z^{-1})}$$

$$= \frac{1+z^{-1}}{3.415 - 1.415z^{-1}}$$

$$= 0.293\frac{1+z^{-1}}{1-0.414z^{-1}}$$

Der foretages en DC skalering:

$$|H(z)|_{\omega=0} = G * 0.293\frac{1+e^{-j\omega}}{1-0.414e^{-j\omega}}\Bigg|_{\omega=0} = 10dB = 10^{0.5} = 3.162$$

$$G * 0.293 \frac{1+1}{1-0.414} = 3.162$$

$$G = 3.162$$

Så det endelige filter er givet ved:

$$= 0.926 \frac{1+z^{-1}}{1-0.414z^{-1}}$$

2) Verify that the gain has dropped by 3dB at 1000 Hz as compared to DC.

$$H(e^{j\omega})\Big|_{\omega=0.25\pi} = 0.926 \frac{1+\cos(-0.25\pi)+j\sin(-0.25\pi)}{1-0.414(\cos(-0.25\pi)+j\sin(-0.25\pi))} =$$

$$= 0.926 \frac{\sqrt{(1+\cos(-0.25\pi))^2+(\sin(-0.25\pi))^2}}{\sqrt{(1-0.414\cos(-0.25\pi))^2+(0.414\sin(-0.25\pi))^2}}$$

$$= 0.926 \frac{1.8478}{0.7654} = 2.2355 \approx 7dB$$

i.e. the gain has dropped (10db – 7db =) 3dB at 1000 Hz as compared to DC