AALBORG UNIVERSITY

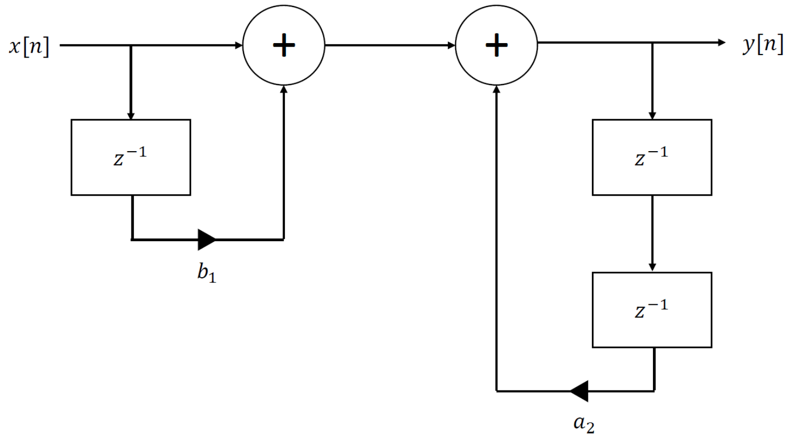
Written exam in
Signalbehandling for Computer Ingeniører, 5 ECTS
Monday January 3, 2022
9.00 – 13.00

Read carefully:

• Remember to write your **full name on every sheet** you return!
• Write **legible** with a pen that allows your answers to be scanned electronically.
• Problems are weighted as listed.
• Results **without sufficient explanations will not give full credits**!
• Grading is dependent on the number of correct answers but also on the depth as well as the width of the answers. You should demonstrate knowledge in each of the main subjects: digital filters and spectral estimation.
• All ordinary tools may be used, e.g., books, notes, programmable calculators, and laptops.
• All electronic communication devices **must be switched off** at all times.
• **Communication with others is strictly prohibited..**!!

A.1 (15%, Digital Filters)

A discrete time filter is given in terms of its Direct Form I structure,



A. Derive the difference equation for the filter.
B. Write an expression for the filter transfer function, $H(z)$.
C. Draw the Direct Form II structure of the filter.
D. Assume that $H(z)$ should represent a stable filter. In such a case, determine the requirements on the filter coefficients b_1 and a_2.
E. Given that $b_1 = 1$ and $a_2 = 0.9$, calculate the first 8 samples of the filter impulse response (assume that the filter is causal).
F. Based on the result found in E., and given that the sample frequency is $1\ kHz$, determine the time it takes before the impulse response has decreased to 5% of its initial value, i.e., the value at time $n = 0$.

A.2 (20%, Digital Filters)

A continuous time filter is given in terms of the transfer function,

$$H_c(s) = \frac{1}{s + \Omega_c}$$

where $\Omega_c = 2\pi \cdot 600\ rad/sec$.

A. Derive an expression for the discrete time transfer function $H(z)$ using the impulse invariance method. The sample frequency is $f_s = 3.6\ kHz$, and the DC-gain of the discrete time filter should equal $0\ dB$. The filter coefficients are calculated with 4 decimals.
B. Using $H(z)$ found in A., calculate the $3\ dB$ cut-off frequency for the discrete time filter. The result should be stated in radians.
C. Using $H(z)$ found in A., calculate the phase response at the frequency $\omega = \frac{\pi}{2}$. The result is calculated with 4 decimals.

A.3 (15%, Digital Filters)

A 4^{th} order Type III Linear Phase FIR filter is given by the impulse response,

$$h[0] = -h[4] = \frac{1}{8}$$

$$h[1] = -h[3] = \frac{1}{4}$$

$$h[2] = 0$$

A. Sketch the impulse response for $0 \leq n \leq 10$.
B. Determine the amplitude response of the filter at $\omega = \frac{\pi}{2}$.
   Hint: You may want to use equations 143, p. 344 in our Textbook; Oppenheim & Schafer, Discrete-Time Signal Processing, 3rd ed. (Type III FIR Linear-Phase Systems).
C. Derive the transfer function $H(z)$ for the filter.
D. You are now informed that the zeros of the filter are located as follows; one zero in $z = 1$ and three zeros in $z = -1$. Draw the pole-zero diagram for the filter.
E. Based on the information given in D., and the result calculated in B., indicate which filter function $H(z)$ represents, i.e., is it Low Pass, High Pass, etc…? Argue your answer.

B.1 (20%, Spectral Estimation)

A continuous time signal $x_c(t)$ is assumed to have the following spectral representation,

$$|X_c(\Omega)| = \begin{cases} 1 & |\Omega| \leq 2\pi \cdot 1000 \quad rad/\sec \\ 1 & 2\pi \cdot 1500 \;\; rad/\sec \leq |\Omega| \leq 2\pi \cdot 2500 \; rad/sec \\ 0 & otherwise \end{cases}$$

The signal $x_c(t)$ is now sampled uniformly with $f_s = 6\ kHz$.

A. Draw a sketch of the amplitude spectrum of the discrete time signal (you may limit your figure to the positive frequencies only).
B. In order to conduct a DFT-based spectral analysis, a segment with length 20 ms of the discrete time signal is selected using a window function. What is the frequency resolution of the resulting spectrum?
C. What can be done in order to increase the spectral resolution in the frequency domain, and what is the related consequence in the time domain? Explain and argue your answer.
D. The spectral analysis is conducted using either 1) the Rectangular window, or 2) the Hamming window. Which one of the two window functions would you use, if it is required that the calculated spectrum should match as good as possible the discontinuities at $1000Hz$, $1500Hz$, and $2500Hz$? Argue your answer.

B.2 (10%, Spectral Estimation)

Given the real sequence $x[n] = \{1,1,0,0\}$, where $N = 4$.

A. Calculate the Discrete Fourier Transform, $X[k]$ for $k = 0, \ldots, N-1$
B. Draw a plot of $|X[k]|$ and $arg\{X[k]\}$ for $k = 0, \ldots, N-1$

B.3 (12%, Spectral Estimation)

One period of a discrete time signal is given as $x[n] = 0.5^n$, $n = 0..N-1$, $N = 100$. Derive a closed form expression for the Discrete Time Fourier Transformation $X(e^{j\omega})$, and next use this expression to calculate the value of the Discrete Fourier Transform $X[k]$ for the value k which corresponds to $\omega = \frac{3\pi}{2}$.

B.4 (8%, Spectral Estimation)

Given the twiddle factor W_N.

A. Show that $W_N^{k(N-n)} = W_N^{-kn} = (W_N^{kn})^*$.
B. Express W_8^1 in polar coordinates.