Introduction to Probability Theory and Statistics

Exam, June 2015

Probability Theory

Problem 1
Suppose that X and Y are two discrete random variables whose joint probability mass function is given as following:

$$P(X=-1,Y=-1)=1/4;\ \ P(X=0,Y=-1)=1/4;\ \ P(X=1,Y=-1)=0$$

$$P(X=-1,Y=1)=0;\ \ P(X=0,Y=1)=1/4;\ \ P(X=1,Y=1)=1/4$$

(a) Find marginal distributions for X and Y.

(b) Are these random variables independent? In case they are not independent, suggest new random variables X_1 and Y_1 such that the new random variables have the same marginal distributions as the old ones and are independent.

(c) Calculate the expected values for X and Y.

(d) Calculate the covariance matrix. Are these random variables uncorrelated?

(e) Find $E[2X-4Y]$.

(f) Find $E[Z]$ where $Z=X\cdot Y^2$.

Problem 2
Suppose there are two classes in a school: 2.A and 2.B. In class 2.A there are 12 girls and 18 boys. In class 2.B there are 20 girls and 15 boys. We select a child at random by first selecting at random a class and then select at random a child from that class.

(a) What is the probability that the selected child is a girl?

(b) Compute the conditional probability that the child is from class 2.B given that it is a girl.