Probability and Statistics Notes

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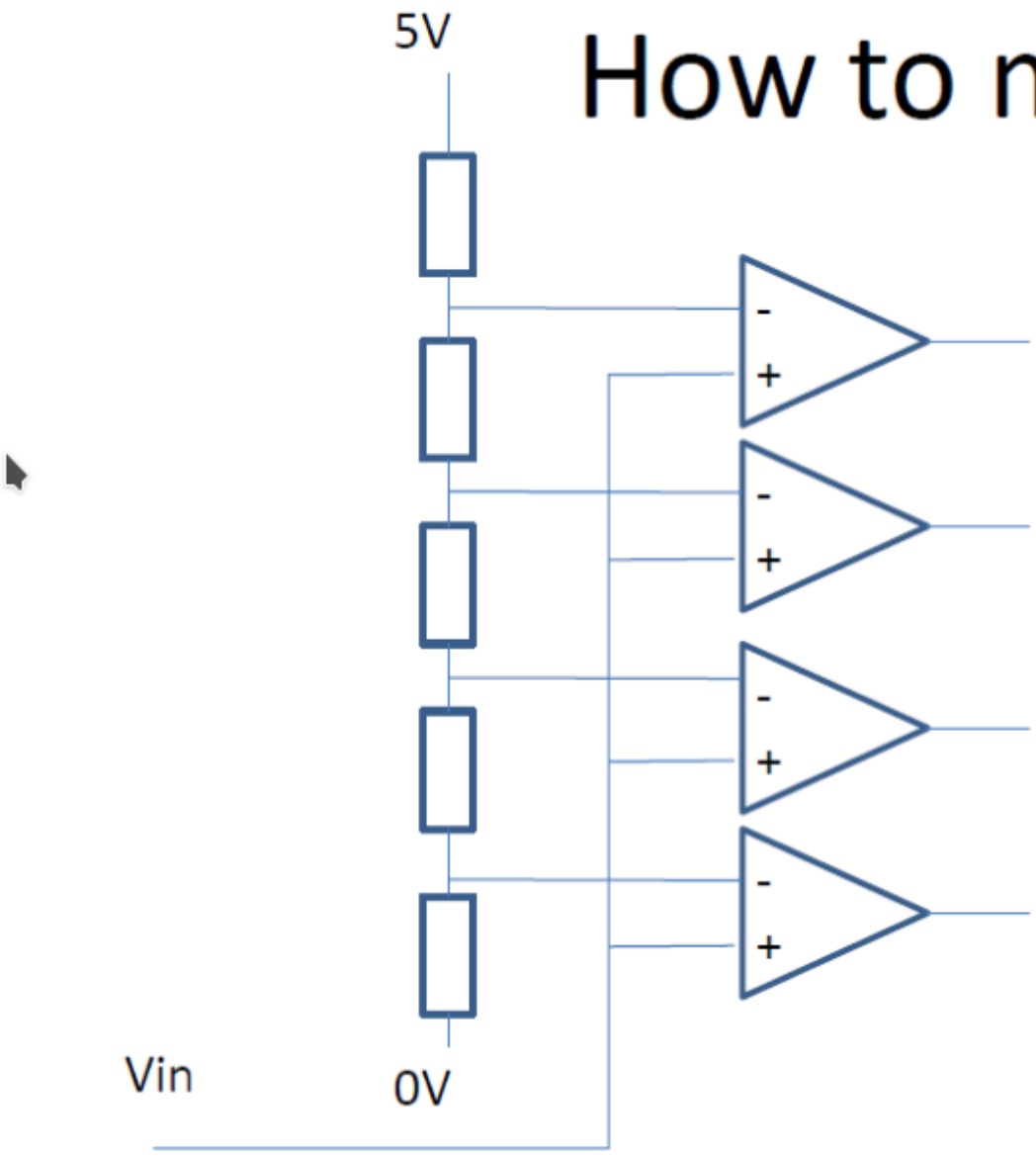
Lecture 1: Elementary digital circuits

The flash converter, an analog signal can be converted from analog to digital using this converter. it will divide a voltage into different levels, shown as the resistors on the figure below. This will produce a truth table of sorts, if we put in a signal that is between 2 and 3 volts in the example on the blackboard, it will correspond to the 2nd row in the table below:



Exercises

1. Show by perfect induction the following relations:

1. $(A + B) * (A + C) = A + (B * C)$

A	B	C	$A+B$	$A+C$	$(A+B)*(A+C)$	$B*C$	$A+(B*C)$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

2. $A*(A+B)=A$

A	B	$A+B$	$A*(A+B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

3. $A+\overline{A}=1$

A	$\overline{A}$	$A+\overline{A}$
0	1	1
1	0	1

4. $\overline{A+B+C}=\overline{A}*\overline{B}*\overline{C}$

A	B	C	$A+B+C$	$\overline{A+B+C}$	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A}*\overline{B}*\overline{C}$
0	0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	1	0	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0	0

2. Show that the following expression is equivalent to the exclusive or function This is denoted by $\oplus$

A	B	$A*\overline{B}$	$\overline{A}*B$	$\overline{(A*\overline{B})}$	$\overline{(\overline{A}*B)}$	$\overline{(A*\overline{B})}*\overline{(\overline{A}*B)}$	$\overline{\overline{(A*\overline{B})}*\overline{(\overline{A}*B)}}$
0	0	0	0	1	1	1	0
0	1	0	1	1	0	0	1
1	0	1	0	0	1	0	1
1	1	0	0	1	1	1	0

3. Reduce the following expressions:

1. $A * \overline{B} * \overline{C} + A * B * \overline{C} + \overline{A} * \overline{C}$

$$\overline{C}(A * \overline{B} + A * B + \overline{A})$$

2. $M * \overline{N} * P + \overline{L} * M * P + \overline{L} * M * \overline{N} + \overline{L} * M * N * \overline{P} + \overline{L} * \overline{N} * \overline{P}$

$$M(\overline{N} * P + \overline{L} * P + \overline{L} * \overline{N} + \overline{L} * N * \overline{P}) + \overline{L} * \overline{N} * \overline{P}$$

$$M(\overline{N} * P + \overline{L}(P + \overline{N} + N * \overline{P})) + \overline{L} * \overline{N} * \overline{P}$$

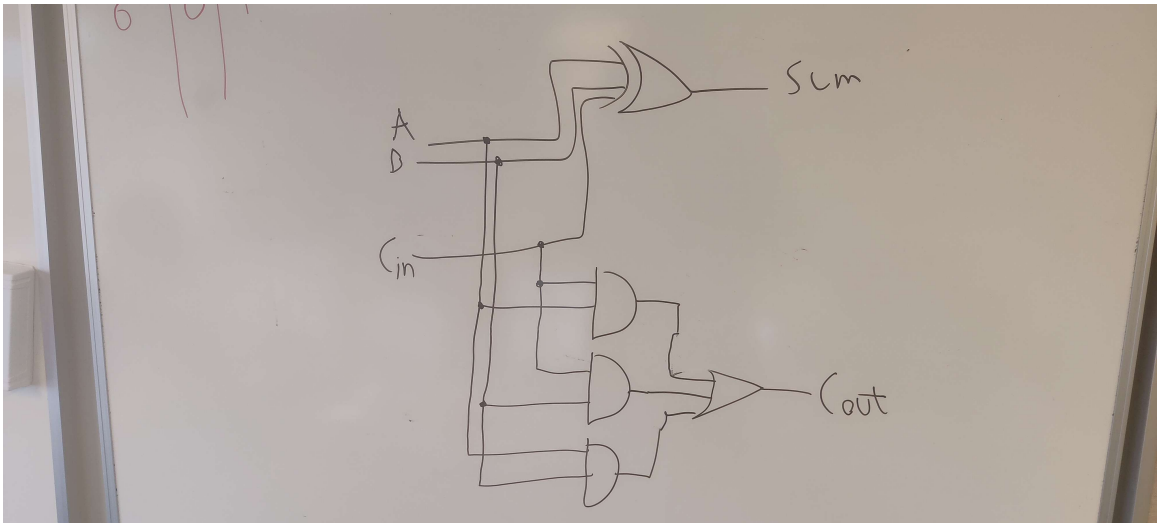
4. Find expressions for D and C cricuit below. What is the purpose of the circuit?

$$C = \overline{(\overline{A(\overline{AB})})(\overline{B(\overline{AB})})}$$

$$D = \overline{\overline{A * B}}$$

This circuit is a 1-bit adder, with C being the sum, and D being the carry.

5. Find the Gate Input Count (GIC) of the cirquit below.



6. Extend the half adder from slide 9 to take a carry as an additional input.