PK

8th LECTURE, SUGGESTED SOLUTION.

- Design a TYPE-I FIR filter with $\omega_c = \pi/4$ using the Window method. The filter should comply with the specifications for the LP-filter designed in Lecture 7.

$\Downarrow$

Desired frequency response;

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega M/2} & |\omega| \leq \pi/4 \\ 0 & \pi/4 < |\omega| \leq \pi \end{cases}$$

from which we can derive the ideal impulse response

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j\omega M/2} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} \left(\cos\{(n-M/2)\omega\} + j\sin\{(n-M/2)\omega\}\right) d\omega$$

$$= \frac{1}{2\pi}\left[\sin\{(n-M/2)\omega\}\right]_{-\pi/4}^{\pi/4} \cdot \frac{1}{n-M/2}$$

$$= \frac{1}{2\pi(n-M/2)} \cdot \left\{\sin\left(\pi/4(n-M/2)\right) - \sin\left(-\pi/4(n-M/2)\right)\right\}$$

$\Downarrow$

$$h_d[n] = \frac{\sin\left(\pi/4(n-M/2)\right)}{\pi(n-M/2)} \qquad -\infty < n < \infty$$

Now, $h_d[n]$ is → Symmetric around $M/2$
→ infinite
→ non-causal

Thus we truncate $h_d[n]$ with a symmetric window function;

$$h[n] = h_d[n] \cdot w[n], \quad w[n] \neq 0 \quad 0 \leq n \leq M$$

Using equ. 140a (p.343), we can now write the frequency response for the filter (TYPE-I)

$$H(e^{j\omega}) = e^{-j\omega M/2} \left(\sum_{k=0}^{M/2} a[k] \cdot \cos(\omega k) \right)$$

where

$$a[0] = h[M/2]$$
$$a[k] = 2 \cdot h[M/2 - k] \quad \text{for } k = 1, 2, \ldots, M/2$$

Due to the symmetric impulse response $h[n]$, the filter has linear phase response.

Thus we need to concern only about the amplitude response

$$|H(e^{j\omega})| = \left| \sum_{k=0}^{M/2} a[k] \cdot \cos(\omega k) \right|$$

In order to fulfill the 0dB DC Gain requirement we need to find G given as;

$$G \cdot |H(e^{j\omega})| = 1 \Big|_{\omega=0}$$

$$\Downarrow$$

$$G \cdot \left| \sum_{k=0}^{M/2} a[k] \right| = 1$$

$$\Downarrow$$

$$G = \frac{1}{\left| \sum_{k=0}^{M/2} a[k] \right|}$$

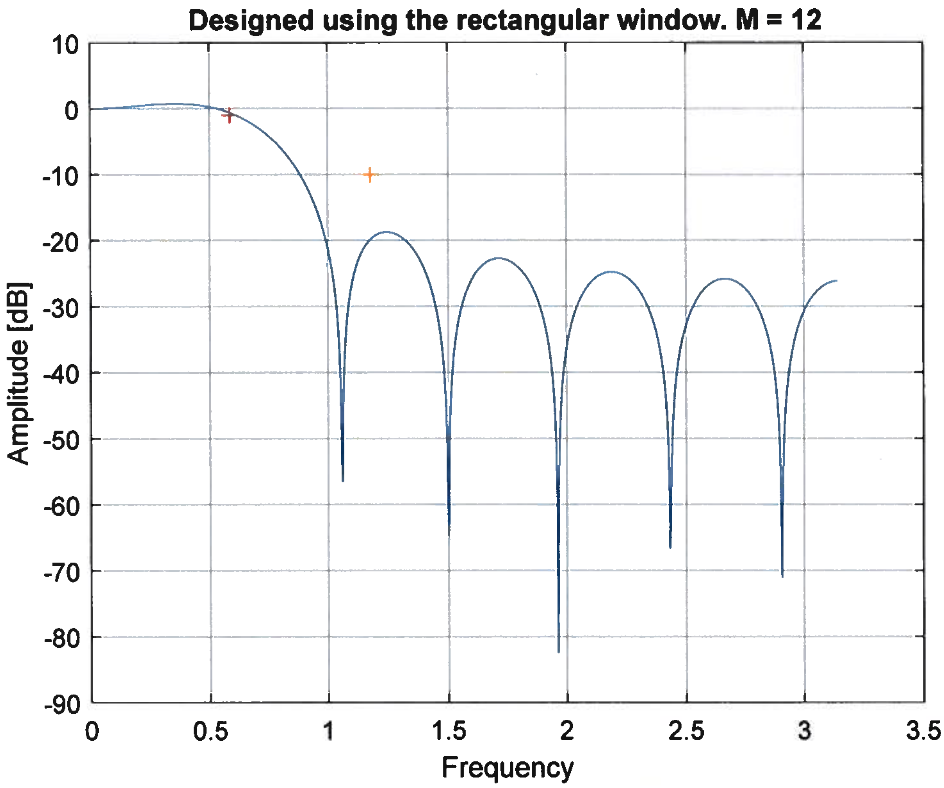
and thus the overall Amplitude response is

$$|H(e^{j\omega})| = \frac{\left| \sum_{k=0}^{M/2} a[k] \cdot \cos(\omega k) \right|}{\left| \sum_{k=0}^{M/2} a[k] \right|}$$

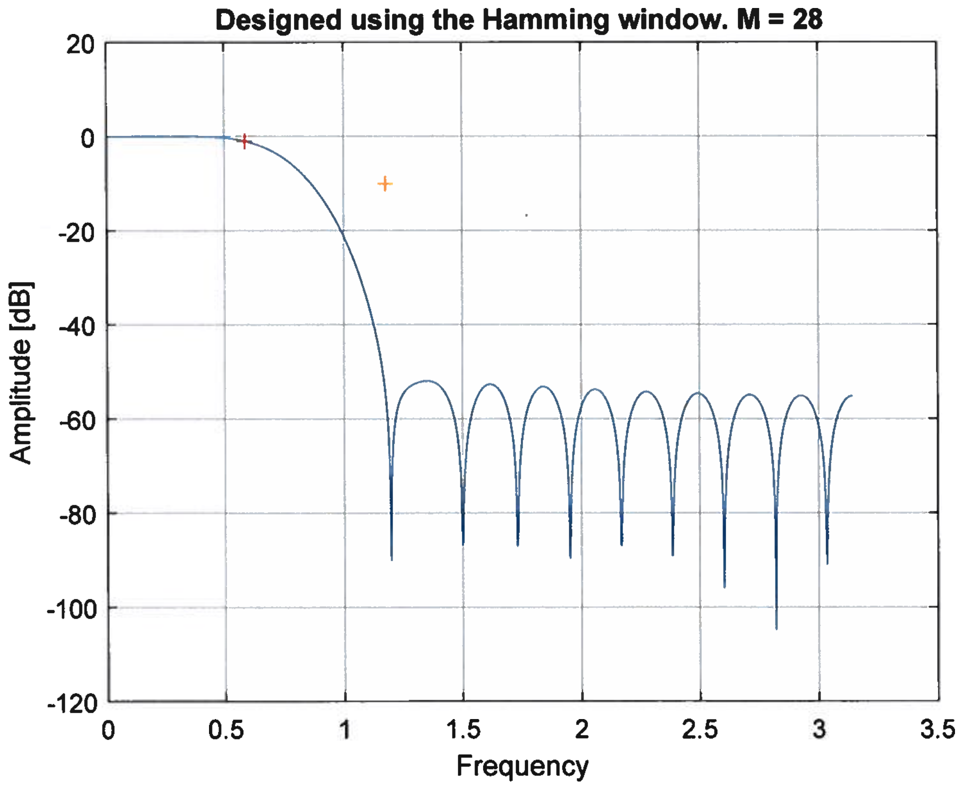
Now, for given Window function $w[n]$ our task is to find the smallest value of M which satisfies the design specifications

In order to do that, write a program and conduct experiments with the rectangular as well as the Hamming window.

```
% The program calculates the amplitude response of an M'th order TYPE-I FIR
% filter, given the desired impulse response h_d[n]
%
% The Rectangular Window is used for truncation of h_d[n]
clear

% Filter order (has to be even for TYPE-I FIR filters)
M = 12;

% Frequency sweep from 0 til PI with 1000 points
for i=0:999,
 omega(i+1) = pi*i/999;
end;

% The amplitude response is now calculated for the 1000 discrete frequency
% values.
%
% The amplitude response has the form |H| = N(omega,k)/D(k)

for i=0:999
 % The numerator N is calculated using the summation variable sum_n
 sum_n = 0;
 % The denominator D is calculated using the summation variable sum_d
 sum_d = 0;
 % Each sum has a total of M/2+1 terms
 for k=1:(M/2)
   % First calculate the ideal impulse response sample. Note that the
   % symmetri point (k=0) is not included. This final term will be added
   % later.
   h(k) = (sin((pi/4)*k)/(pi*k));
   a(k) = 2*h(k);

   % Now both N and D are updated
   sum_n = sum_n + (a(k) * cos(k * omega(i+1)));
   sum_d = sum_d + a(k);
 end

 % Finally, add the contribution from the symmetri point, i.e., k=0
 % For the given impulse response, this value equals 1/4
 sum_n = sum_n + 0.25;
 sum_d = sum_d + 0.25;

 % Finally, the amplitude value at the actual frequency point is calculated
 amp(i+1) = abs(sum_n)/abs(sum_d);
end,

% Plot the amplitude response (in dB) together with the specifications.
plot(omega,20*log10(amp),omega(187),-1,'+',omega(375),-10,'+')
grid;
xlabel('Frequency')
ylabel('Amplitude [dB]')
title(sprintf('Designed using the rectangular window. M = %.0f', M))
```


Designed using the rectangular window. M = 12
Amplitude [dB]
Frequency
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
0
0.5
1
1.5
2
2.5
3
3.5

```
% The program calculates the amplitude response of an M'th order TYPE-I FIR
% filter, given the desired impulse response h_d[n]
%
% The Hamming Window is used for truncation of h_d[n]
clear

% Filter order (has to be even for TYPE-I FIR filters)
M = 28;

% Frequency sweep from 0 til PI with 1000 points
for i=0:999,
 omega(i+1) = pi*i/999;
end;

% The amplitude response is now calculated for the 1000 discrete frequency
% values.
%
% The amplitude response has the form |H| = N(omega,k)/D(k)

for i=0:999
 % The numerator N is calculated using the summation variable sum_n
 sum_n = 0;
 % The denominator D is calculated using the summation variable sum_d
 sum_d = 0;
 % Each sum has a total of M/2+1 terms
 for k=1:(M/2)
   % First calculate the ideal impulse response sample. Note that the
   % symmetri point (k=0) is not included. This final term will be added
   % later.
   h_d(k) = (sin((pi/4)*k)/(pi*k));
   % The impulse response is now multiplied with the Hamming window
   h(k) = h_d(k) * (0.54 - 0.46*cos((2*pi)*(k + M/2)/M));
   a(k) = 2*h(k);

   % Now both N and D are updated
   sum_n = sum_n + (a(k) * cos(k * omega(i+1)));
   sum_d = sum_d + a(k);
 end

 % Finally, add the contribution from the symmetri point, i.e., k=0
 % For the given impulse response, this value equals 1/4
 sum_n = sum_n + 0.25;
 sum_d = sum_d + 0.25;

 % Finally, the amplitude value at the actual frequency point is calculated
 amp(i+1) = abs(sum_n)/abs(sum_d);
end,

% Plot the amplitude response (in dB) together with the specifications.
plot(omega,20*log10(amp),omega(187),-1,'+',omega(375),-10,'+')
grid;
xlabel('Frequency')
ylabel('Amplitude [dB]')
title(sprintf('Designed using the Hamming window. M = %.0f', M))
```


Designed using the Hamming window. M = 28
Amplitude [dB]
Frequency
20
0
-20
-40
-60
-80
-100
-120
0
0.5
1
1.5
2
2.5
3
3.5