AALBORG UNIVERSITY

Written exam in
Signal Processing for Computer Engineers, 5 ECTS
Wednesday January 4, 2023
9.00 – 12.00

Read carefully:

- Remember to write your **full name on every sheet** that you scan and submit electronically.
- Problems are weighted according to the listed percentages.
- Results **without sufficient explanations will not give full credits**.
- Grading is dependent on the number of correct answers but also on the depth as well as the width of the answers.
- You should demonstrate knowledge in both main subjects: Digital Filters, and Spectral Estimation.
- All ordinary tools may be used i.e., books, lecture notes, slides, calculators, and laptops.
- **Communication with others is strictly prohibited..**.!!

A.1 (20%, Digital Filters)

A continuous-time filter is given by the transfer function $H_c(s)$;

$$H_c(s) = \frac{s+1}{s^2+5s+6}$$

We want to design the corresponding discrete-time filter $H(z)$ using the Impulse Invariant Method.

A. Find a closed-form expression for the continuous-time impulse response $h_c(t)$. Assume that the continuous-time system is causal. *Hint: You may want to first decompose $H_c(s)$ into two 1st order sections using Partial Fraction Expansion.*
B. Using $h_c(t)$, derive an expression for the discrete-time impulse response $h[n]$. The sample frequency f_s is 10 Hz.
C. Using $h[n]$, establish an expression for the transfer function $H(z)$. The filter coefficients should be calculated with 4 decimals.

A.2 (15%, Digital Filters)

A discrete-time filter $H(z)$ is specified by two zeros in $z = 0$ and $z = 0.8966$, respectively, and two poles in $z = 0.8197$ and $z = 0.7399$, respectively.

A. Draw a figure showing the pole-zero diagram.
B. Decide whether the filter is of type IIR of FIR. Argue your answer.
C. Find the filter's I/O-relation in terms of the difference equation. Give the result with 4 decimals. *Hint: First find $H(z)$ and then conduct inverse z-transform on $Y(z)$.*
D. Derive an expression for the filter's frequency response $H(e^{j\omega})$.
E. Use $H(e^{j\omega})$ to calculate the DC-gain. Give the result in dB.

A.3 (15%, Digital Filters)

An M^{th}order Type I FIR filter should be designed using the Window Method. The desired (i.e., the ideal) impulse response of the filter is

$$h_d[n] = \frac{\sin\{(n-\frac{M}{2})\cdot\frac{\pi}{5}\}}{(n-\frac{M}{2})\cdot\pi} \qquad -\infty < n < \infty$$

A. Calculate the impulse response $h[n]$ for $M = 6$ using the Hamming window. Give the result with 4 decimals. *Hint: Use L'Hopital's rule for $n = 3$.*
B. Considering $h[n]$, decide whether the filter is causal or non-causal. Argue your answer.
C. Decide whether the filter has a linear phase response. Argue your answer.

B.1 (10%, Spectral Estimation)

A continuous-time signal $x_c(t)$ has the following spectral representation,

$$|X_c(\Omega)| = \begin{cases} 1 & |\Omega| \leq 2\pi \cdot 600 \quad rad/\sec \\ 1 & 2\pi \cdot 1200 \;\; rad/\sec \leq |\Omega| \leq 2\pi \cdot 1600 \, rad/sec \\ 0 & otherwise \end{cases}$$

A. We want to sample $x_c(t)$ uniformly without aliasing. What is the lowest possible sample frequency f_s. Argue your answer.
B. Now, assume that prior to sampling, $x_c(t)$ is passed through an ideal High-Pass filter (i.e., passband gain is 1, and stopband gain is 0) with cut-off frequency $f_c = 1\ kHz$. What is then the lowest possible sample frequency f_s if we want to avoid aliasing. Argue your answer.

B.2 (5%, Spectral Estimation)

A continuous-time signal $x_c(t)$ is sampled into $x[n]$ using a sample frequency $f_s = 8\ kHz$. To conduct a DFT-based spectral estimation, a finite-length segment of the signal is next generated by multiplying $x[n]$ by a window of length equal to 20 ms. Calculate the spectral resolution (in Hz) of the resulting DFT-spectrum.

B.3 (15%, Spectral Estimation)

One period of a discrete-time signal is given as $x[n] = (-0.25)^n \cdot u[n],\ 0 \leq n \leq N-1,\ \ N = 100$

A. Derive a closed-form expression for the Discrete Time Fourier Transform $X(e^{j\omega})$ of $x[n]$.
B. By sampling the Discrete Time Fourier Transform, now calculate the Discrete Fourier Transform $X[k_1]$, where the integer value k_1 corresponds to $\omega = \frac{\pi}{5}\ rad$. Find k_1 and give $X[k_1]$ with 4 decimals.

B.4 (15%, Spectral Estimation)

Given two real finite-length sequences $x_1[n] = \{1,0,1,-1\}$ and $x_2[n] = \{-1,0,0,1\}$.

A. Calculate the Discrete Fourier Transform, $X[k]$ for $0 \leq k \leq N-1,\ \ N = 4,$ given that $x[n] = 2x_1[n] + 4x_2[n]$. Argue your answer.
B. Calculate $|X[k]|$ for $k = 3$. Give the result with 4 decimals.
C. Calculate $arg\{X[k]\}$ for $k = 2$. Give the result with 4 decimals.

B.5 (5%, Spectral Estimation)

Assume that the computation of a 256-point DFT-based spectral estimation can be done in about 10 μs on a given computer. What is the approximate computation time on the same computer if the spectral estimation instead is calculated using the FFT algorithm.