AALBORG UNIVERSITY

Written re-exam in
Signal Processing for Computer Engineers, 5 ECTS
Friday February 10, 2023
9.00 – 12.00

Read carefully:

- Remember to write your **full name on every sheet** that you scan and submit electronically.
- Problems are weighted according to the listed percentages.
- Results **without sufficient explanations will not give full credits**.
- Grading is dependent on the number of correct answers but also on the depth as well as the width of the answers.
- You should demonstrate knowledge in both main subjects: Digital Filters and Spectral Estimation.
- All ordinary tools may be used i.e., books, lecture notes, slides, calculators, and laptops.
- **Communication with others is strictly prohibited..**.!!

A.1 (50%, Digital Filters)

An FIR filter has the impulse response; $h[0] = h[4] = \frac{1}{4},\ h[1] = h[3] = \frac{1}{2},\ h[2] = 1.$

A. Find the difference equation for the filter.

B. Using the difference equation found in question A, calculate the first 3 samples of the step response.

C. A finite length sequence $x[n] = \{1, 2, 3\},\ 0 \leq n \leq 2,$ is used as input to the filter. Calculate the response $y[n],\ 0 \leq n \leq 2$, using the convolution sum. *Hint: You may want to make a drawing illustrating the convolution process*.

D. Find the pole location of the filter. *Hint: It might be useful to first establish the transfer function $H(z)$.*

E. Derive the frequency response for the filter.

F. You are informed that the filter has a linear phase response. Draw the phase response in the interval $0 \leq \omega \leq \frac{\pi}{2}$. Argue your answer.

G. The filter is executed using a sample frequency $f_s = 1kHz$. How often is a new sample $y[n]$ available on the filter output? Argue your answer.

H. How many arithmetic operations are needed to calculate one sample $y[n]$?

I. Is it possible to somehow reduce the number of arithmetic operations found in question H? Argue your answer. *Hint: You may take advantage of the symmetric impulse response.*

J. You are informed that the filter has two complex conjugated zero pairs in $-0.7429\ \pm j1.5291$ and $-0.2571 \pm j0.5291$, respectively. Draw the pole/zero diagram.

K. Discuss whether the filter is stable. Argue your answer.

L. Assume that the output of the filter, $y[n]$, is next passed through an LTI-system with the transfer function $G(z) = z^{-M}$ to generate a modified output $y'[n]$. Derive an expression of $y'[n]$ as a function of $y[n]$.

B.1 (5%, Spectral Estimation)

A continuous-time signal $x_c(t)$ is sampled into $x[n]$ using a sample frequency $f_s = 5\ kHz$. To conduct a DFT-based spectral estimation, a finite-length segment of the signal is next generated by multiplying $x[n]$ by a window function. Calculate the necessary length of the window if the spectral resolution of the resulting DFT-spectrum should be $10Hz$. Give the result in seconds.

B.2 (15%, Spectral Estimation)

A continuous-time signal is generated by adding two sinusoids, $x_c(t) = \sin(\Omega_1 t) + \sin(\Omega_2 t)$, where $\Omega_1 = 6283.2\ rad/sec$ and $\Omega_2 = 9424.8\ rad/sec$.

- **A.** Calculate the lowest possible sample frequency needed if we want to sample $x_c(t)$ into $x[n]$ without aliasing. Give the result in Hz.
- **B.** Assume now that prior to the sampling, $x_c(t)$ is passed through an ideal band pass filter with the center frequency at $950Hz$ and a bandwidth equal to $200Hz$. What is then the lowest possible sample frequency needed if we want to sample the filtered signal into $x[n]$ without aliasing. Give the result in Hz.

B.3 (25%, Spectral Estimation)

One period of a discrete-time square-wave signal is given as

$$x[n] = \begin{cases} 1 & 0 \le n \le 1 \\ 0 & 2 \le n \le 7 \end{cases}$$

- **A.** Use the Discrete Fourier Transform to calculate $X[k],\ 0 \le k \le N-1,\ N = 8$.
- **B.** Explain how we should interpret $\arg\{X[4]\}$.
- **C.** You are informed that $|X[7]| = |X[1]|$, $|X[6]| = |X[2]|$, and $|X[5]| = |X[3]|$. Explain why we see this symmetry in the amplitude spectrum.

B.4 (5%, Spectral Estimation)

We want to reduce the computation time of a 1024-point DFT with a factor of 2. Is this possible to do by using the FFT algorithm instead of the DFT algorithm? Argue your answer.