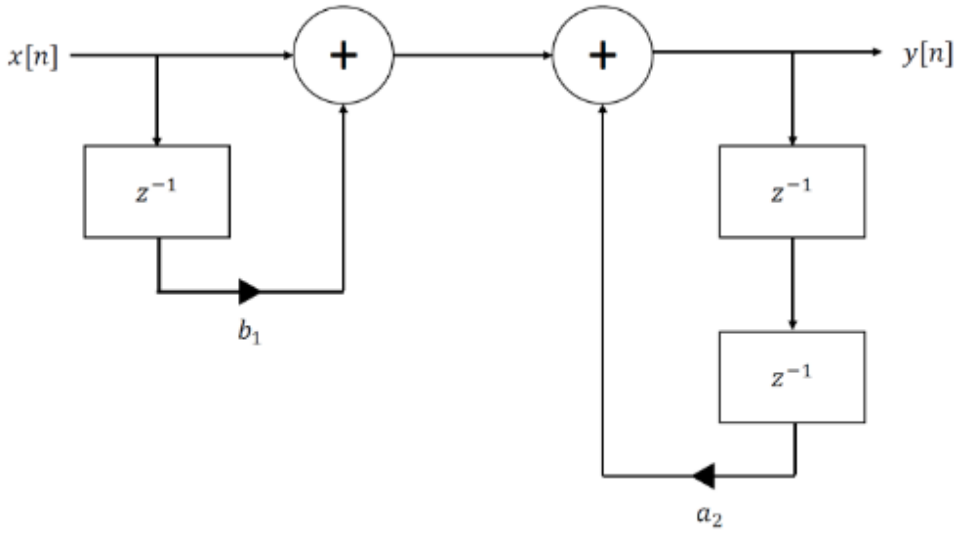
Exam set 2021

A.1 (15%, Digital Filters) A discrete time filter is given in terms of its Direct Form I structure,



A. Derive the difference equation for the filter.

The difference equation is defined as follows:

$$y[n] = x[n] + b_1 x[n-1] + a_2 y[n-2]$$

B. Write an expression for the filter transfer function, H(z)

First lets z-transform the expression above:

$$Y(z) = X(z) + b_1 X(z) z^{-1} + a_2 Y(z) z^{-2}$$

Now we will move the part of the right side of the expression with $Y(z)$ over to the left side:

$$Y(z) - a_2 Y(z) z^{-2} = X(z) + b_1 X(z) z^{-1}$$

now we will remove $Y(z)$ from the right side expression such that we can divide the expression:

$$Y(z)(1 - a_2 z^{-2}) = X(z) + b_1 X(z) z^{-1}$$

and now we move the left side back to the right side by division:

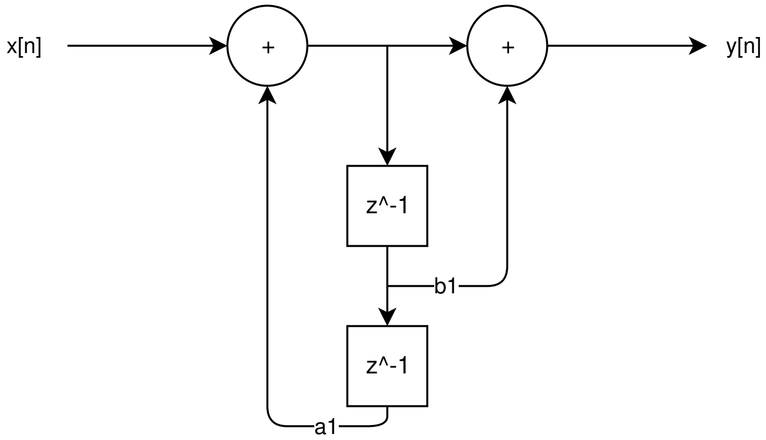
$$Y(z) = \frac{X(z) + b_1 X(z) z^{-1}}{1 - a_2 z^{-2}}$$

Since the following expression is true: $H(z) = \frac{Y(z)}{X(z)}$ then the following can be expressed:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{X(z)+b_1 X(z) z^{-1}}{1-a_2 z^{-2}}}{X(z)} = \frac{1 + b_1 z^{-1}}{1 - a_2 z^{-2}}$$

as the second divison will remove the term $X(z)$ from the top of the fraction, giving the final expression for $H(z)$. This also means that $Y(z) = 1 + b_1 z^{-1}$ and $X(z) = 1 + a_2 z^{-2}$

C. Draw the Direct Form II structure of the filter.



D. Assume that $H(z)$ should represent a stable filter. In such a case, determine the requirements on the filter coefficients b_1 and a_2.