Omdurman Islamic University

Faculty of Engineering

Electrical & Electronic Engineering

(4th year)

Signal Processing and Systems

Lecturer

FAWAZ FATHI

Module 2

Continuous-Time (CT) Signals and Systems

Course Description (Part 1)

- Module 1: Introduction to signals and systems.
- Module 2: Continuous-Time (CT) Signals and Systems
- Module 3: Continuous-Time Linear Time-Invariant (LTI) Systems
- Module 4: Continuous-Time Fourier Series (CTFS)
- Module 5: Continuous-Time Fourier Transform (CTFT)
- Module 6: Laplace Transform (LT)

Course Description (Part2)

- Module 1: Introduction to Digital signal Processing.
- Module 2: Analogue to Digital conversion, Sampling, Quantization
- Module 3-1: Digital signal and systems .
- Module 3-2: LTI systems described by difference equations.
- Module 4-1: Discrete Time Fourier Transform.
- Module 4-2: Fast Fourier Transforms (FFT).

Course Description (Part2)

- Module 5: Z Transform
- Module 6: Basic Filtering Types
- Module 7: FIR Filters design, implementation.
- Module 8: IIR Filters design, implementation.

Part 1

Continuous-Time (CT) Signals and Systems

Section
2.1

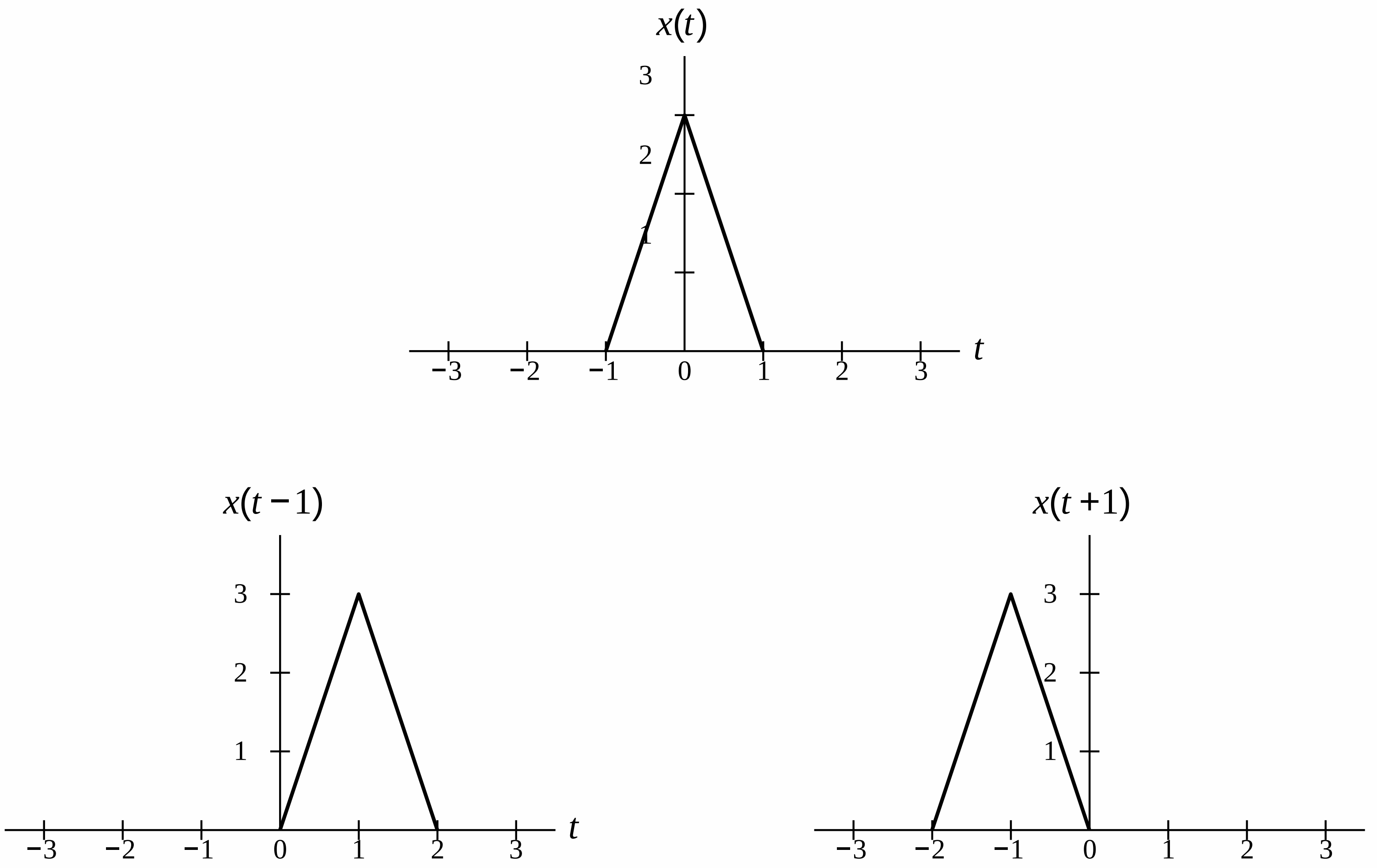
Independent- and Dependent-Variable Transformations

- Time shifting (also called translation) maps the input signal x to the output signal y as given by

$$y(t) = x(t-b),$$

where b is a real number.

- Such a transformation shifts the signal (to the left or right) along the time axis.
- If $b > 0$, y is *shifted to the right* by $|b|$, relative to x (i.e., delayed in time).
- If $b < 0$, y is *shifted to the left* by $|b|$, relative to x (i.e., advanced in time).

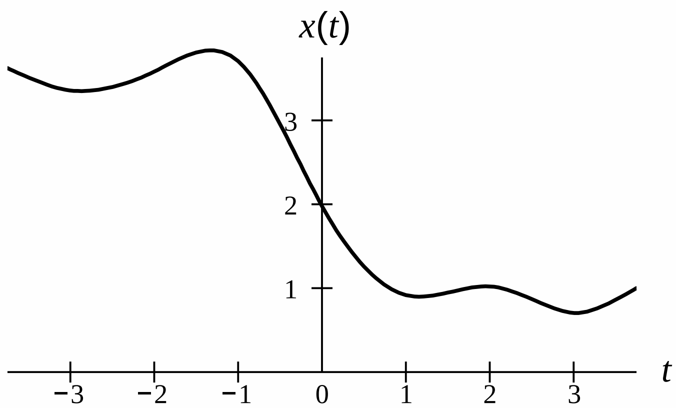

$x(t)$
3
2
1
t
−3
−2
−1
0
1
2
3
$x(t-1)$
3
2
1
t
−3
−2
−1
0
1
2
3
$x(t+1)$
3
2
1
t
−3
−2
−1
0
1
2
3

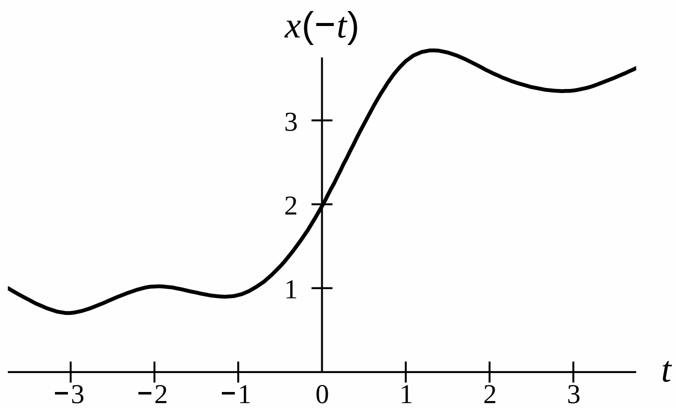
Time Reversal (Reflection)

- Time reversal (also known as reflection) maps the input signal x to the output signal y as given by

$$y(t) = x(-t).$$

- Geometrically, the output signal y is a reflection of the input signal x about the (vertical) line $t = 0$.





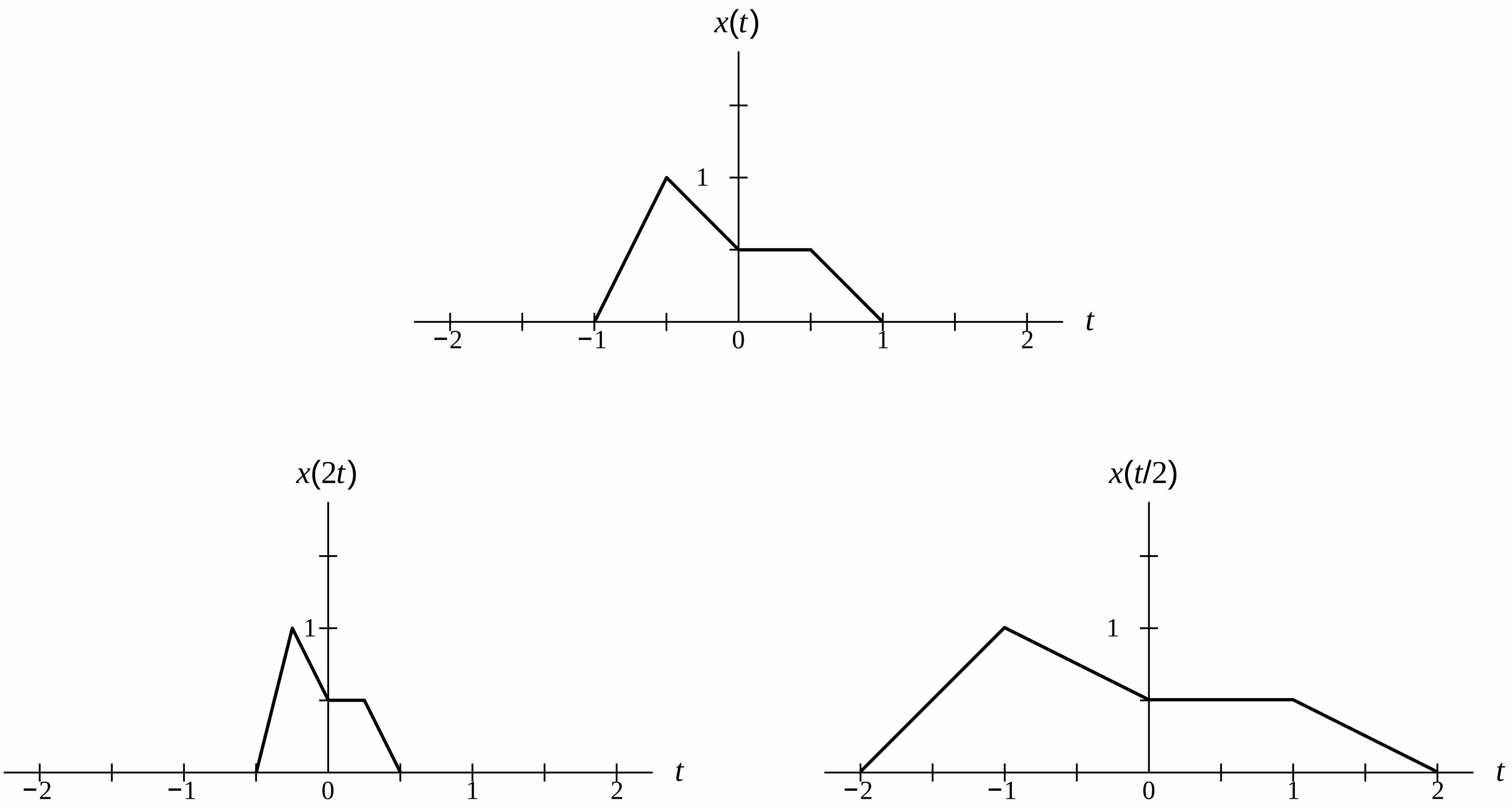
- Time compression/expansion (also called dilation) maps the input signal x to the output signal y as given by

$$y(t) = x(at),$$

where a is a *strictly positive* real number.

- Such a transformation is associated with a compression/expansion along the time axis.
- If $a > 1$, y is *compressed* along the horizontal axis by a factor of a, relative to x.
- If $a < 1$, y is *expanded* (i.e., stretched) along the horizontal axis by a factor of $\frac{1}{a}$, relative to x.

Time Compression/Expansion (Dilation): Example



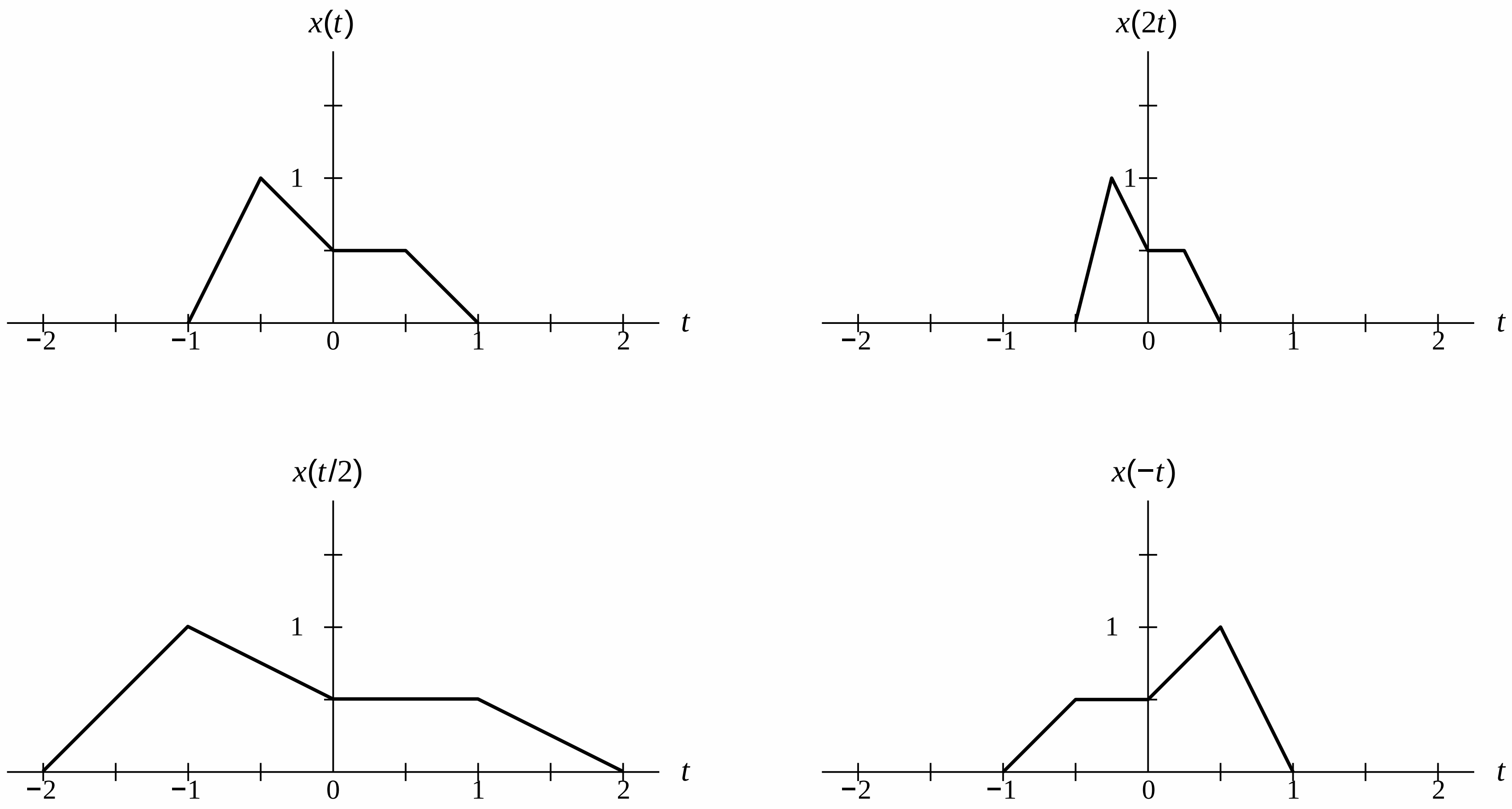
Time Scaling

- Time scaling maps the input signal x to the output signal y as given by

$$y(t) = x(at),$$

  where a is a *nonzero* real number.
- Such a transformation is associated with a dilation (i.e., compression/expansion along the time axis) and/or time reversal.
- If $|a| > 1$, the signal is *compressed* along the time axis by a factor of $|a|$. If
- $|a| < 1$, the signal is *expanded* (i.e., stretched) along the time axis by a factor of $\left|\frac{1}{a}\right|$.
- If $|a| = 1$, the signal is neither expanded nor compressed.
- If $a < 0$, the signal is also time reversed.
- Dilation (i.e., expansion/compression) and time reversal *commute*.
- Time reversal is a special case of time scaling with $a = -1$; and time compression/expansion is a special case of time scaling with $a > 0$.

Time Scaling (Dilation/Reflection): Example



Combined Time Scaling and Time Shifting

- Consider a transformation that maps the input signal x to the output signal y as given by

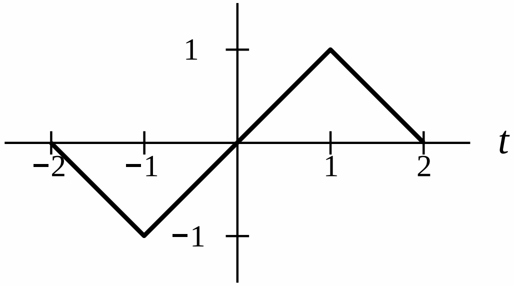
$$y(t) = x(at - b),$$

  where a and b are real numbers and $a \neq 0$.
- The above transformation can be shown to be the combination of a time-scaling operation and time-shifting operation.
- Since time scaling and time shifting *do not commute*, we must be particularly careful about the order in which these transformations are applied.
- The above transformation has two distinct but equivalent interpretations:
  1. first, time shifting x by b, and then time scaling the result by a;
  2. first, time scaling x by a, and then time shifting the result by b/a.
- Note that the time shift is not by the same amount in both cases.

Combined Time Scaling and Time Shifting: Example

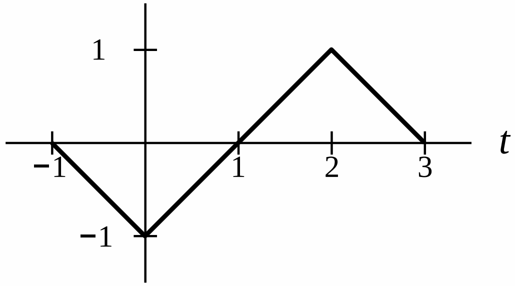
Given $x(t)$ as shown below, find $x(2t-1)$.

$x(t)$

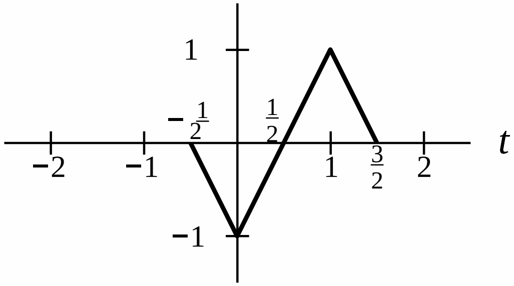


time shift by 1 and then time scale by 2

$p(t) = x(t-1)$

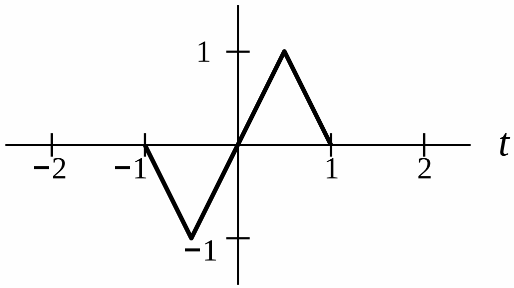


$p(2t) = x(2t-1)$

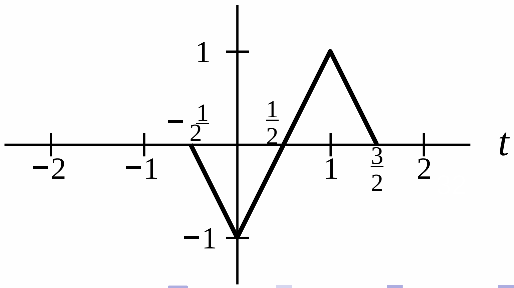


time scale by 2 and then time shift by $\frac{1}{2}$

$q(t) = x(2t)$



$q(t-1/2) = x(2(t-1/2)) = x(2t-1)$



Two Perspectives on Independent-Variable Transformations

- A transformation of the independent variable can be viewed in terms of
  1. the effect that the transformation has on the *signal*; or
  2. the effect that the transformation has on the *horizontal axis*.
- This distinction is important because such a transformation has *opposite* effects on the signal and horizontal axis.
- For example, the (time-shifting) transformation that replaces t by $t - b$ (where b is a real number) in $x(t)$ can be viewed as a transformation that
  1. shifts the signal x *right* by b units; or
  2. shifts the horizontal axis *left* by b units.
- In our treatment of independent-variable transformations, we are only interested in the effect that a transformation has on the *signal*.
- If one is not careful to consider that we are interested in the signal perspective (as opposed to the axis perspective), many aspects of independent-variable transformations will not make sense.

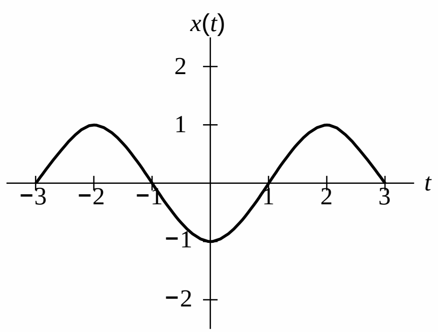
Amplitude Scaling

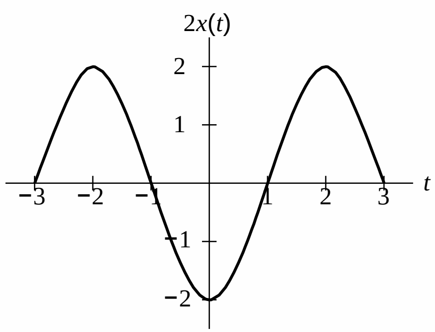
- Amplitude scaling maps the input signal x to the output signal y as given by

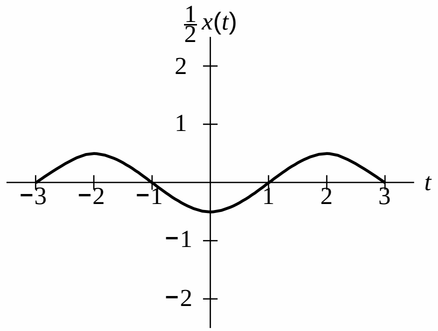
$$y(t) = ax(t),$$

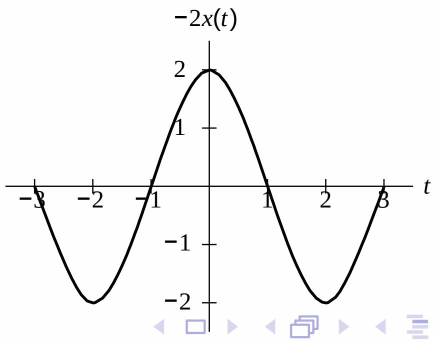
where a is a real number.

- Geometrically, the output signal y is *expanded/compressed* in amplitude and/or *reflected* about the horizontal axis.









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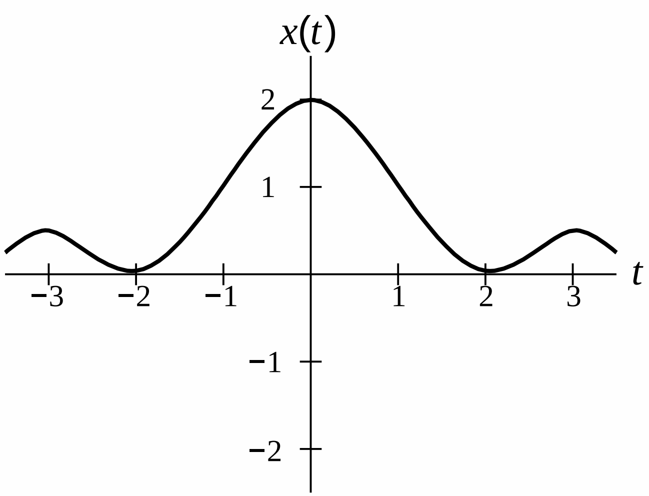
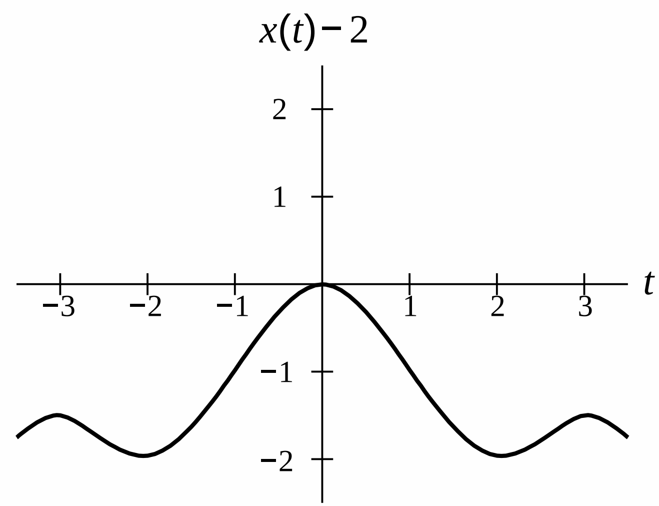
- Amplitude shifting maps the input signal x to the output signal y as given by

$$y(t) = x(t) + b,$$

where b is a real number.

- Geometrically, amplitude shifting adds a *vertical displacement* to x.





Combined Amplitude Scaling and Amplitude Shifting

- We can also combine amplitude scaling and amplitude shifting transformations.
- Consider a transformation that maps the input signal x to the output signal y, as given by

$$y(t) = ax(t) + b,$$

where a and b are real numbers.
- Equivalently, the above transformation can be expressed as

$$y(t) = a\left[x(t) + \tfrac{b}{a}\right].$$

- The above transformation is equivalent to:
  1. first amplitude scaling x by a, and then amplitude shifting the resulting signal by b; or
  2. first amplitude shifting x by b/a, and then amplitude scaling the resulting signal by a.

Section 2.2

Properties of Signals

Symmetry and Addition/Multiplication

- Sums involving even and odd functions have the following properties:
  - The sum of two even functions is even.
  - The sum of two odd functions is odd.
  - The sum of an even function and odd function is neither even nor odd, provided that neither of the functions is identically zero.
- That is, the *sum* of functions with the *same type of symmetry* also has the *same type of symmetry*.
- Products involving even and odd functions have the following properties:
  - The product of two even functions is even.
  - The product of two odd functions is even.
  - The product of an even function and an odd function is odd.
- That is, the *product* of functions with the *same type of symmetry* is *even*, while the *product* of functions with *opposite types of symmetry* is *odd*.

Decomposition of a Signal into Even and Odd Parts

- Every function x has a *unique* representation of the form

$$x(t) = x_\mathrm{e}(t) + x_\mathrm{o}(t),$$

  where the functions x_e and x_o are *even* and *odd*, respectively.
- In particular, the functions x_e and x_o are given by

$$x_\mathrm{e}(t) = \tfrac{1}{2}[x(t) + x(-t)] \quad \text{and} \quad x_\mathrm{o}(t) = \tfrac{1}{2}[x(t) - x(-t)].$$

- The functions x_e and x_o are called the even part and odd part of x, respectively.
- For convenience, the even and odd parts of x are often denoted as $\mathrm{Even}\{x\}$ and $\mathrm{Odd}\{x\}$, respectively.

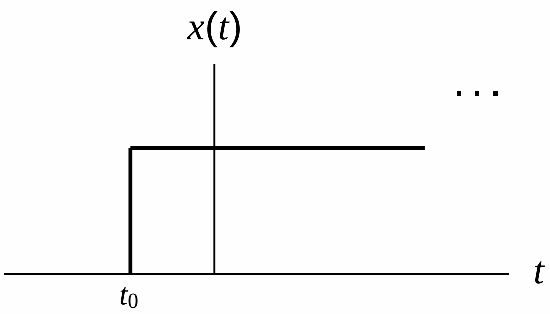
- **Sum of periodic functions.** Let x_1 and x_2 be periodic functions with fundamental periods T_1 and T_2, respectively. Then, the sum $y = x_1 + x_2$ is a periodic function if and only if the ratio *T_1/T_2 is a rational number* (i.e., the quotient of two integers). Suppose that $T_1/T_2 = q/r$ where q and r are integers and *coprime* (i.e., have no common factors), then the fundamental period of y is rT_1 (or equivalently, qT_2, since $rT_1 = qT_2$). (Note that rT_1 is simply the least common multiple of T_1 and T_2.)
- Although the above theorem only directly addresses the case of the sum of two functions, the case of N functions (where $N > 2$) can be handled by applying the theorem repeatedly $N - 1$ times.

Right-Sided Signals

- A signal x is said to be right sided if, for some (finite) real constant t_0, the following condition holds:

$$x(t) = 0 \quad \text{for all } t < t_0$$

  (i.e., x is *only potentially nonzero to the right of* t_0).
- An example of a right-sided signal is shown below.



- A signal x is said to be causal if

$$x(t) = 0 \quad \text{for all } t < 0.$$

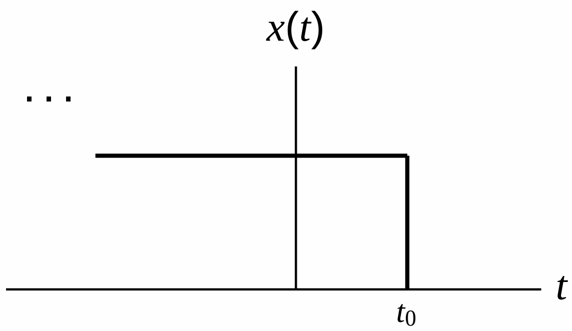
- A causal signal is a *special case* of a right-sided signal.
- A causal signal is not to be confused with a causal system. In these two contexts, the word "causal" has very different meanings.

Left-Sided Signals

- A signal x is said to be left sided if, for some (finite) real constant t_0, the following condition holds:

$$x(t) = 0 \qquad \text{for all } t > t_0$$

(i.e., x is *only potentially nonzero to the left of* t_0).
- An example of a left-sided signal is shown below.



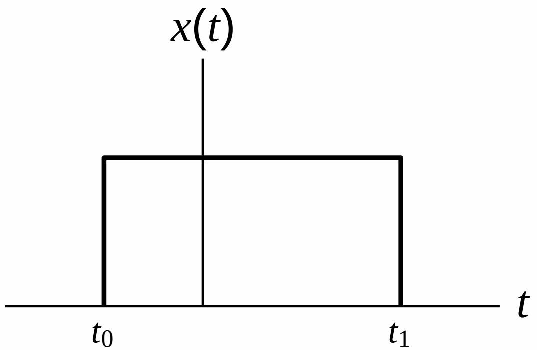
- Similarly, a signal x is said to be anticausal if

$$x(t) = 0 \qquad \text{for all } t > 0.$$

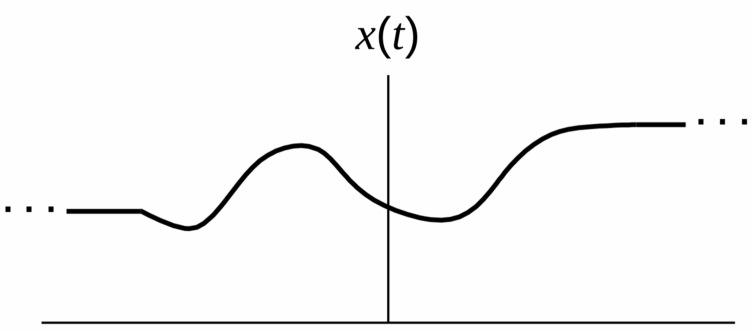
- An anticausal signal is a *special case* of a left-sided signal.
- An anticausal signal is not to be confused with an anticausal system. In these two contexts, the word "anticausal" has very different meanings.

Finite-Duration and Two-Sided Signals

- A signal that is both left sided and right sided is said to be finite duration (or time limited).
- An example of a finite duration signal is shown below.



- A signal that is neither left sided nor right sided is said to be two sided.
- An example of a two-sided signal is shown below.



Bounded Signals

- A signal x is said to be bounded if there exists some (*finite*) positive real constant A such that

$$|x(t)| \le A \quad \text{for all } t$$

  (i.e., $x(t)$ is *finite* for all t).
- Examples of bounded signals include the sine and cosine functions.
- Examples of unbounded signals include the $\tan$ function and any nonconstant polynomial function.

Signal Energy and Power

- The energy E contained in the signal x is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

- A signal with finite energy is said to be an energy signal.
- The average power P contained in the signal x is given by

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt.$$

- A signal with (nonzero) finite average power is said to be a power signal.

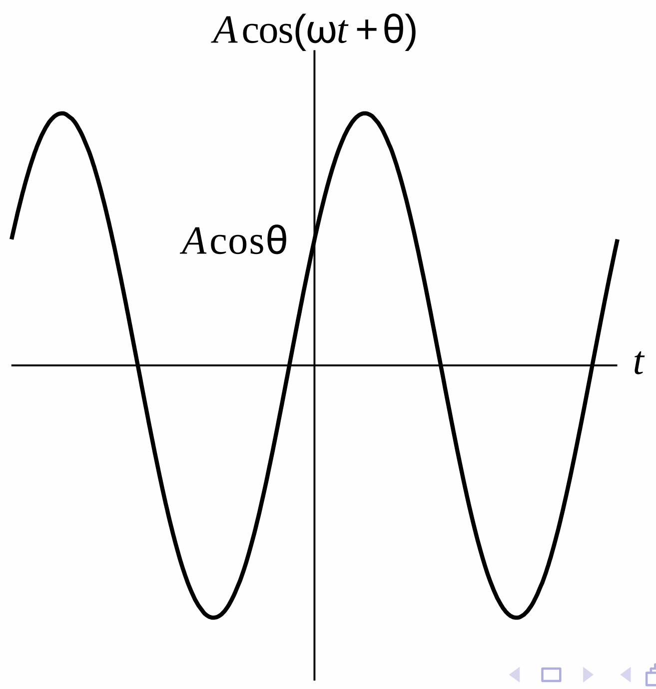
Section 2.3

Elementary Signals

- A (CT) real sinusoid is a function of the form

$$x(t) = A\cos(\omega t + \theta),$$

where A, ω, and θ are *real* constants.
- Such a function is periodic with *fundamental period* $T = \frac{2\pi}{|\omega|}$ and *fundamental frequency* $|\omega|$.
- A real sinusoid has a plot resembling that shown below.



Complex Exponentials

- A (CT) complex exponential is a function of the form

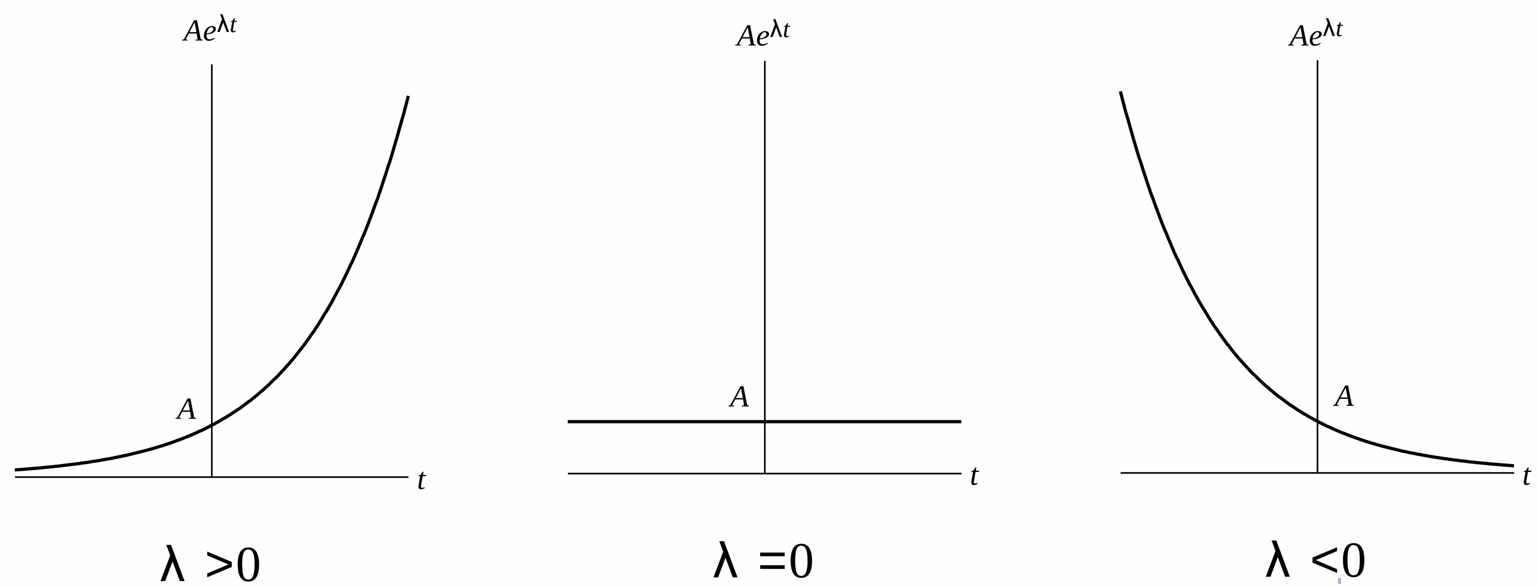
$$x(t) = Ae^{\lambda t},$$

where A and λ are *complex* constants.

- A complex exponential can exhibit one of a number of *distinct modes of behavior*, depending on the values of its parameters A and λ.
- For example, as special cases, complex exponentials include real exponentials and complex sinusoids.

Real Exponentials

- A real exponential is a special case of a complex exponential $x(t) = Ae^{\lambda t}$, where A and λ are restricted to be *real* numbers.
- A real exponential can exhibit one of *three distinct modes* of behavior, depending on the value of λ, as illustrated below.
- If $\lambda > 0$, $x(t)$ *increases* exponentially as t increases (i.e., a growing exponential).
- If $\lambda < 0$, $x(t)$ *decreases* exponentially as t increases (i.e., a decaying exponential).
- If $\lambda = 0$, $x(t)$ simply equals the *constant* A.



- A complex sinusoid is a special case of a complex exponential $x(t) = Ae^{\lambda t}$, where A is *complex* and λ is *purely imaginary* (i.e., $\operatorname{Re}\{\lambda\} = 0$).
- That is, a (CT) complex sinusoid is a function of the form

$$x(t) = Ae^{j\omega t},$$

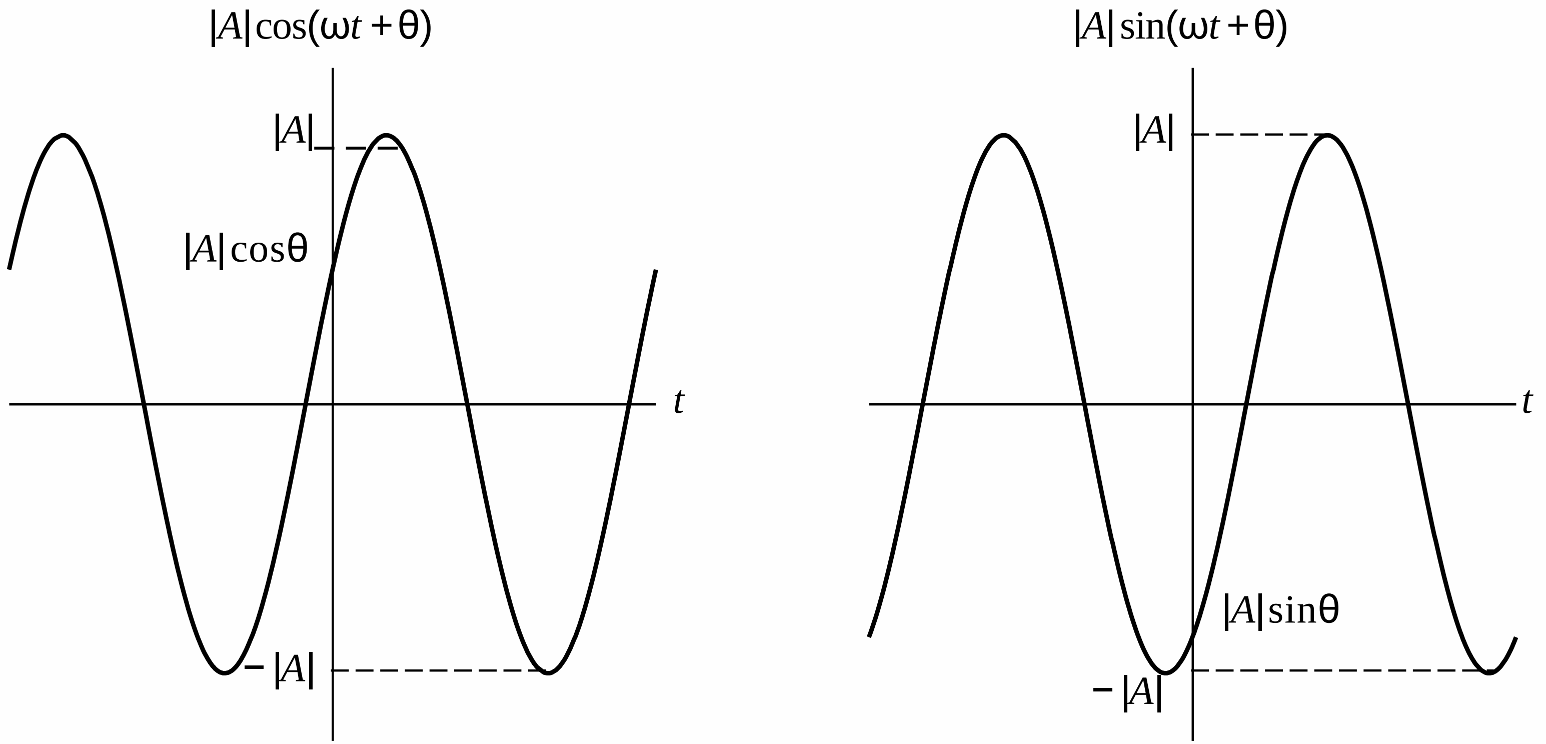
where A is *complex* and ω is *real*.

- By expressing A in polar form as $A = |A|\, e^{j\theta}$ (where θ is real) and using Euler's relation, we can rewrite $x(t)$ as

$$x(t) = \underbrace{|A|\cos(\omega t + \theta)}_{\operatorname{Re}\{x(t)\}} + j\underbrace{|A|\sin(\omega t + \theta)}_{\operatorname{Im}\{x(t)\}}.$$

- Thus, $\operatorname{Re}\{x\}$ and $\operatorname{Im}\{x\}$ are the same except for a time shift.
- Also, x is periodic with *fundamental period* $T = \frac{2\pi}{|\omega|}$ and *fundamental frequency* $|\omega|$.

- The graphs of $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ have the forms shown below.



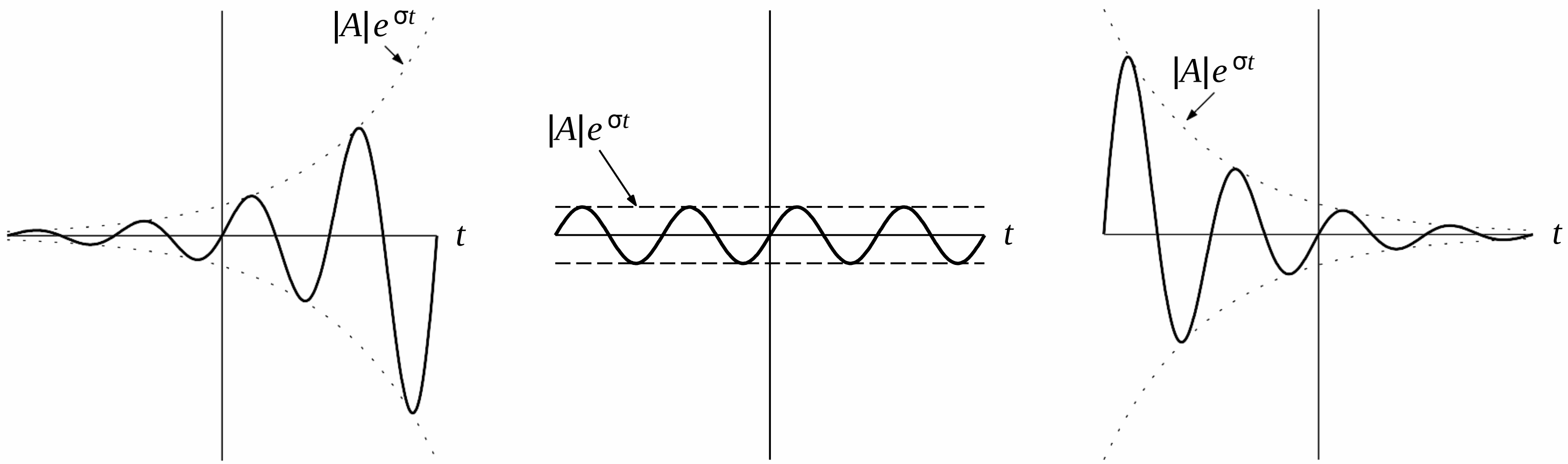
General Complex Exponentials

- In the most general case of a complex exponential $x(t) = Ae^{\lambda t}$, A and λ are both *complex*.
- Letting $A = |A|e^{j\theta}$ and $\lambda = \sigma + j\omega$ (where θ, σ, and ω are real), and using Euler's relation, we can rewrite $x(t)$ as

$$x(t) = \underbrace{|A|e^{\sigma t}\cos(\omega t + \theta)}_{\mathrm{Re}\{x(t)\}} + j\underbrace{|A|e^{\sigma t}\sin(\omega t + \theta)}_{\mathrm{Im}\{x(t)\}}.$$

- Thus, $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ are each the product of a real exponential and real sinusoid.
- One of *three distinct modes* of behavior is exhibited by $x(t)$, depending on the value of σ.
- If $\sigma = 0$, $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ are *real sinusoids*.
- If $\sigma > 0$, $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ are each the *product of a real sinusoid and a growing real exponential*.
- If $\sigma < 0$, $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ are each the *product of a real sinusoid and a decaying real exponential*.

- The *three modes of behavior* for $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ are illustrated below.



$\sigma > 0$ $\sigma = 0$ $\sigma < 0$

Relationship Between Complex Exponentials and Real Sinusoids

- From Euler's relation, a complex sinusoid can be expressed as the sum of two real sinusoids as

$$Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t.$$

- Moreover, a real sinusoid can be expressed as the sum of two complex sinusoids using the identities

$$A\cos(\omega t+\theta) = \frac{A}{2}\left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right] \quad \text{and}$$

$$A\sin(\omega t+\theta) = \frac{A}{2j}\left[e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}\right].$$

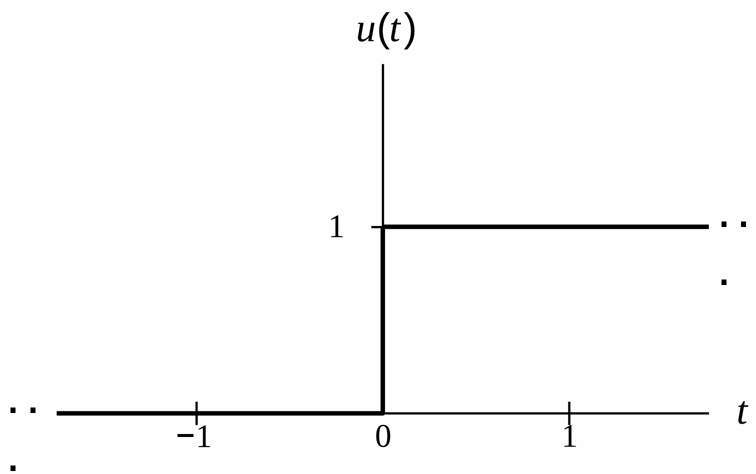
- Note that, above, we are simply *restating results* from the (appendix) material on complex analysis.

Unit-Step Function

- The unit-step function (also known as the Heaviside function), denoted u, is defined as

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

- Due to the manner in which u is used in practice, the actual *value of $u(0)$* is unimportant. Sometimes values of 0 and $\frac{1}{2}$ are also used for $u(0)$.
- A plot of this function is shown below.

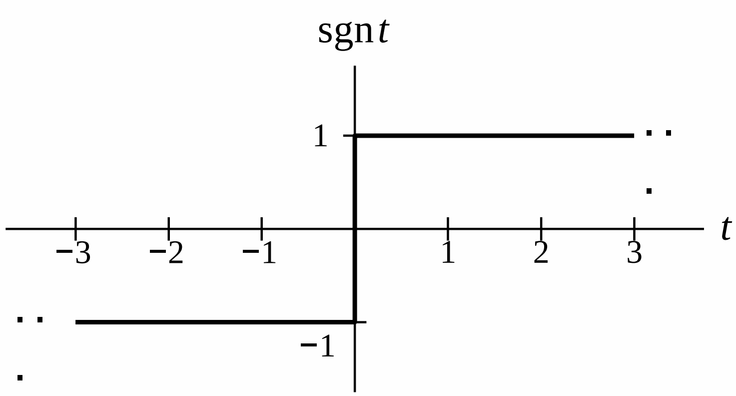


Signum Function

- The signum function, denoted sgn, is defined as

$$\operatorname{sgn} t = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t = 0 \\ -1 & \text{if } t < 0. \end{cases}$$

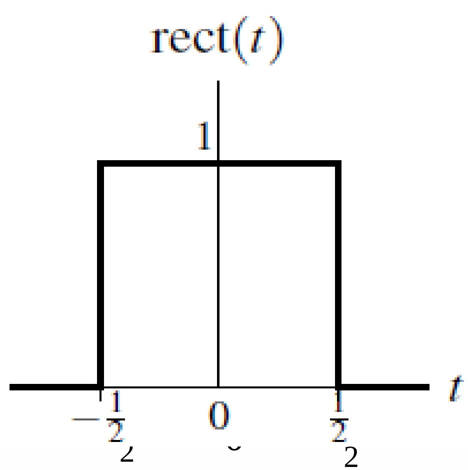
- From its definition, one can see that the signum function simply computes the *sign* of a number.
- A plot of this function is shown below.



- The rectangular function (also called the unit-rectangular pulse function), denoted rect, is given by

$$\operatorname{rect}(t) = \begin{cases} 1 & \text{if } -\frac{1}{2} \leq t < \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

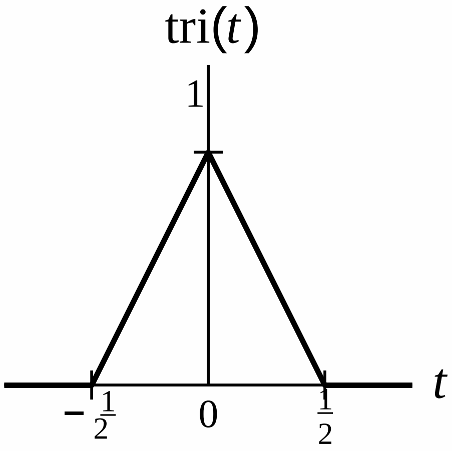
- Due to the manner in which the rect function is used in practice, the actual *value of* $\operatorname{rect}(t)$ *at* $t = \pm\frac{1}{2}$ is unimportant. Sometimes different values are used from those specified above.
- A plot of this function is shown below.



- The triangular function (also called the unit-triangular pulse function), denoted tri, is defined as

$$\operatorname{tri}(t) = \begin{cases} 1 - 2|t| & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

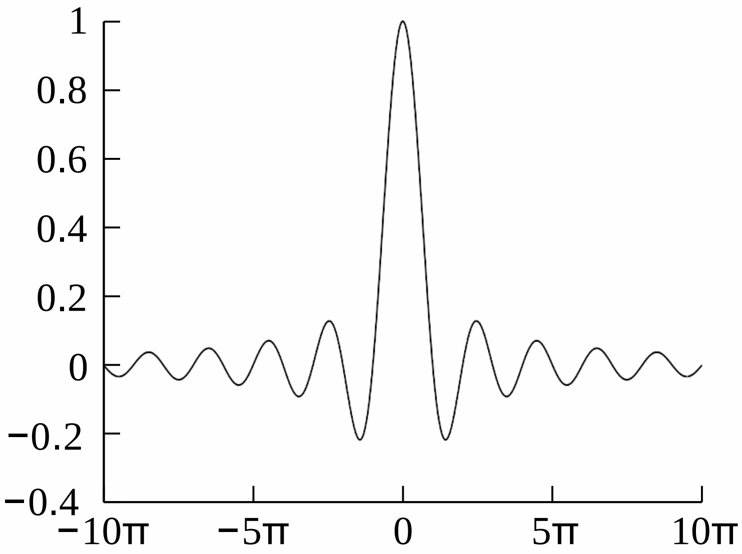
- A plot of this function is shown below.



- The cardinal sine function, denoted sinc, is given by

$$\operatorname{sinc}(t) = \frac{\sin t}{t}.$$

- By l'Hopital's rule, $\operatorname{sinc} 0 = 1$.
- A plot of this function for part of the real line is shown below. [Note that the oscillations in $\operatorname{sinc}(t)$ do not die out for finite t.]

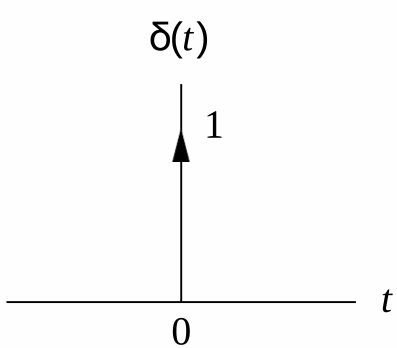


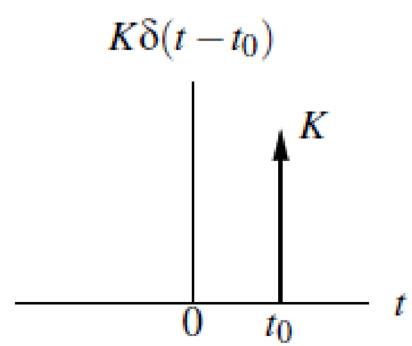
- The unit-impulse function (also known as the Dirac delta function or delta function), denoted δ, is defined by the following two properties:

$$\delta(t) = 0 \quad \text{for } t \neq 0 \quad \text{and}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1.$$

- Technically, δ is not a function in the ordinary sense. Rather, it is what is known as a *generalized function*. Consequently, the δ function sometimes behaves in unusual ways.
- Graphically, the delta function is represented as shown below.



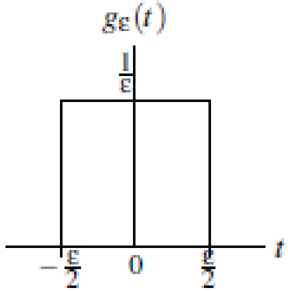


Unit-Impulse Function as a Limit

- Define

$$g_\varepsilon(t) = \begin{cases} 1/\varepsilon & \text{for } |t| < \varepsilon/2 \\ 0 & \text{otherwise.} \end{cases}$$

- The function g_ε has a plot of the form shown below.



- Clearly, for any choice of ε, $\int_{-\infty}^{\infty} g_\varepsilon(t)dt = 1$.
- The function δ can be obtained as the following limit:

$$\delta(t) = \lim_{\varepsilon \to 0} g_\varepsilon(t).$$

- That is, δ can be viewed as a *limiting case of a rectangular pulse* where the pulse width becomes infinitesimally small and the pulse height becomes infinitely large in such a way that the integral of the resulting function remains unity.

- Equivalence property. For any continuous function x and any real constant t_0,

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0).$$

- Sifting property. For any continuous function x and any real constant t_0,

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0).$$

- The δ function also has the following properties:

$$\delta(t) = \delta(-t) \quad \text{and}$$
$$\delta(at) = \frac{1}{|a|}\delta(t),$$

where a is a nonzero real constant.

Representing a Rectangular Pulse Using Unit-Step Functions

- For real constants a and b where $a \leq b$, consider a function x of the form

$$x(t) = \begin{cases} 1 & \text{if } a \leq t < b \\ 0 & \text{otherwise} \end{cases}$$

  (i.e., $x(t)$ is a *rectangular pulse* of height one, with a *rising edge at a* and *falling edge at b*).
- The function x can be equivalently written as

$$x(t) = u(t-a) - u(t-b)$$

  (i.e., the difference of two time-shifted unit-step functions).
- Unlike the original expression for x, this latter expression for x *does not involve multiple cases*.
- In effect, by using unit-step functions, we have collapsed a formula involving multiple cases into a single expression.

- The idea from the previous slide can be extended to handle any function that is defined in a *piecewise manner* (i.e., via an expression involving multiple cases).
- That is, by using unit-step functions, we can always collapse a formula involving multiple cases into a single expression.
- Often, simplifying a formula in this way can be quite beneficial.

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Section 2.4

Continuous-Time (CT) Systems

- A system with input x and output y can be described by the equation

$$y = H\{x\},$$

where H denotes an operator (i.e., transformation).

- Note that the operator H *maps a function to a function* (not a number to a number).
- Alternatively, we can express the above relationship using the notation

$$x \xrightarrow{\mathcal{H}} y.$$

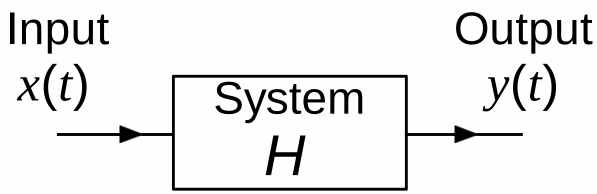
- If clear from the context, the operator H is often omitted, yielding the abbreviated notation

$$x \longrightarrow y.$$

- Note that the symbols "$\longrightarrow$" and "=" have *very different*
- meanings. The symbol "$\longrightarrow$" should be read as "*produces*" (not as "equals").

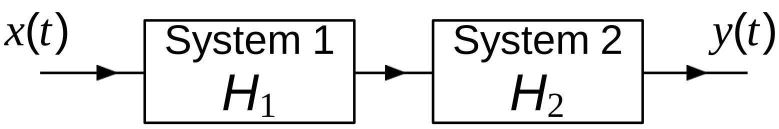
Block Diagram Representations

- Often, a system defined by the operator $\mathcal{H}$ and having the input x and output y is represented in the form of a *block diagram* as shown below.

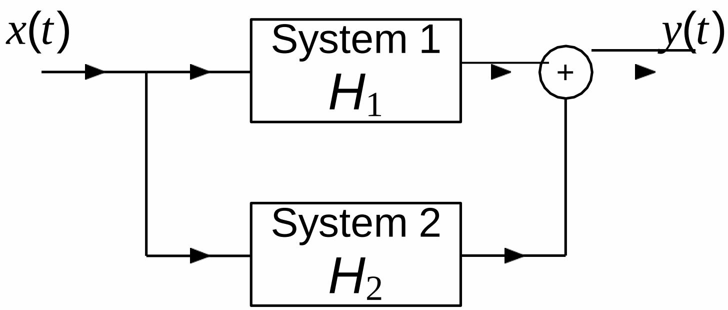


Interconnection of Systems

- *Two basic ways* in which systems can be interconnected are shown below.



Series



Parallel

- A series (or cascade) connection ties the output of one system to the input of the other.
- The overall series-connected system is described by the equation

$$y = H_2 \circ H_1\{x\}.$$

- A parallel connection ties the inputs of both systems together and sums their outputs.
- The overall parallel-connected system is described by the equation

$$y = H_1\{x\} + H_2\{x\}.$$

Section 2.5

Properties of (CT) Systems

- A system with input x and output y is said to have memory if, for any real t_0, $y(t_0)$ depends on $x(t)$ for some $t \neq t_0$.
- A system that does not have memory is said to be memoryless.
- Although simple, a memoryless system is *not very flexible*, since its current output value cannot rely on past or future values of the input.
- A system with input x and output y is said to be causal if, for every real t_0, $y(t_0)$ does not depend on $x(t)$ for some $t > t_0$.
- If the independent variable t represents time, a system must be causal in order to be *physically realizable*.
- Noncausal systems can sometimes be useful in practice, however, since the independent variable *need not always represent time*. For example, in some situations, the independent variable might represent position.

Invertibility

- The inverse of a system H is another system H^{-1} such that the combined effect of H cascaded with H^{-1} is a system where the input and output are equal.
- A system is said to be invertible if it has a corresponding inverse system (i.e., its inverse exists).
- Equivalently, a system is invertible if its input x can always be *uniquely* determined from its output y.
- Note that the invertibility of a system (which involves mappings between *functions*) and the invertibility of a function (which involves mappings between *numbers*) are *fundamentally different* things.
- An invertible system will always produce *distinct outputs* from any two *distinct inputs*.
- To show that a system is *invertible*, we simply find the *inverse system*.
- To show that a system is *not invertible*, we find *two distinct inputs* that result in *identical outputs*.
- In practical terms, invertible systems are "nice" in the sense that their *effects can be undone*.

Bounded-Input Bounded-Output (BIBO) Stability

- A system with input x and output y is BIBO stable if, for every bounded x, y is bounded (i.e., $|x(t)| < \infty$ for all t implies that $|y(t)| < \infty$ for all t).
- To show that a system is *BIBO stable*, we must show that *every bounded input* leads to a *bounded output*.
- To show that a system is *not BIBO stable*, we only need to find a single
- *bounded input* that leads to an *unbounded output*.

  In practical terms, a BIBO stable system is *well behaved* in the sense that, as long as the system input remains finite for all time,
- the output will also remain finite for all time.

  Usually, a system that is not BIBO stable will have *serious safety issues*. For example, an iPod with a battery input of 3.7 volts and headset output of ∞ volts would result in one vaporized Apple customer and one big lawsuit.

Time Invariance (TI)

- A system $\mathcal{H}$ is said to be time invariant (TI) if, for every function x and every real number t_0, the following condition holds:

$$y(t-t_0) = \mathcal{H}x'(t) \quad \text{where} \quad y = \mathcal{H}x \quad \text{and} \quad x'(t) = x(t-t_0)$$

  (i.e., $\mathcal{H}$ *commutes with time shifts*).
- In other words, a system is time invariant if a time shift (i.e., advance or delay) in the input always results only in an *identical time shift* in the output.
- A system that is not time invariant is said to be time varying.
- In simple terms, a time invariant system is a system whose behavior *does not change* with respect to time.
- Practically speaking, compared to time-varying systems, time-invariant systems are much *easier to design and analyze*, since their behavior does not change with respect to time.

Additivity, Homogeneity, and Linearity

- A system $\mathcal{H}$ is said to be additive if, for all functions x_1 and x_2, the following condition holds:

$$\mathcal{H}(x_1 + x_2) = \mathcal{H}x_1 + \mathcal{H}x_2$$

  (i.e., $\mathcal{H}$ *commutes with sums*).
- A system $\mathcal{H}$ is said to be homogeneous if, for every function x and every complex constant a, the following condition holds:

$$\mathcal{H}(ax) = a\mathcal{H}x$$

  (i.e., $\mathcal{H}$ *commutes with multiplication by a constant*).
- A system that is both additive and homogeneous is said to be linear.
- In other words, a system $\mathcal{H}$ is *linear*, if for all functions x_1 and x_2 and all complex constants a_1 and a_2, the following condition holds:

$$\mathcal{H}(a_1x_1 + a_2x_2) = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$$

  (i.e., $\mathcal{H}$ *commutes with linear combinations*).
- The linearity property is also referred to as the superposition property.
- Practically speaking, linear systems are much *easier to design and analyze* than nonlinear systems.