Control Systems (EE2227) Presentation

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Question

- A closed loop system has the characteristic equation given by $s^3 + Ks^2 + (K+2)s + 3 = 0$. For this system to be stable, which one of the following conditions should be satisfied?

  (Q.no:20, EE (Set-1), GATE-2017)

  (A) $0 < K < 0.5$
  (B) $0.5 < K < 1$
  (C) $0 < K < 1$
  (D) $K > 1$

Theory

The Routh array, for a characteristic function $H(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_n$ is given as-

$$\begin{array}{cccccc} s^n & a_0 & a_2 & a_4 & \ldots \\ s^{n-1} & a_1 & a_3 & a_5 & \ldots \\ s^{n-2} & b_0 & b_1 & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s^0 & \ldots & \ldots & \ldots & \ldots \end{array}$$

where $b_0 = \frac{a_1 a_2 - a_0 a_3}{a_1}$, $b_1 = \frac{a_1 a_4 - a_0 a_5}{a_1}$ $\ldots$

Continued...

Routh-Hurwitz criterion

- If there are any sign changes in the first column of the Routh array, then the system is unstable and the number of sign changes correspond to the number of poles in the right half of the s-plane.
- If all elements of a row are zero, then the system is marginally stable.
- Else, the system is stable.

Solution

Computing the Routh array for the given characteristic equation, we get-

s^3	1	$K+2$	0
s^2	K	3	0
s	$\frac{K^2+2K-3}{K}$	0	0
s^0	3	0	0

According to the Routh-Hurwitz stability criterion, for the system to be stable there should be no sign changes in the first column of the Routh array. That means-

$$K > 0 \text{ and } \frac{K^2+2K-3}{K} > 0$$

$$\Rightarrow K > 0 \text{ and } (K-1)(K+3) > 0$$

which gives us $K > 0$ and ($K > 1$ or $K < -3$).

Note that K cannot be negative.

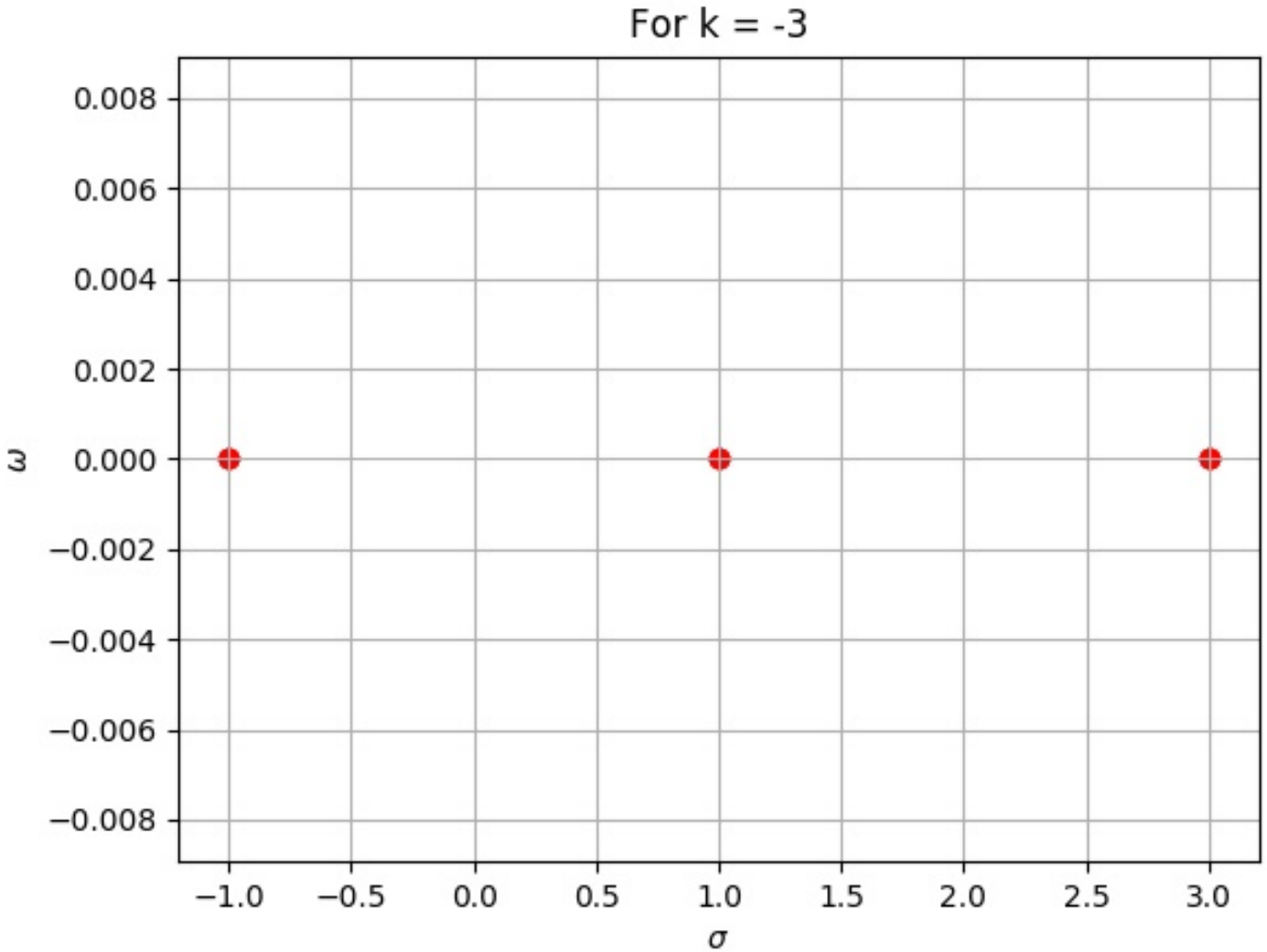
$\Rightarrow K > 1$, which is option (D).

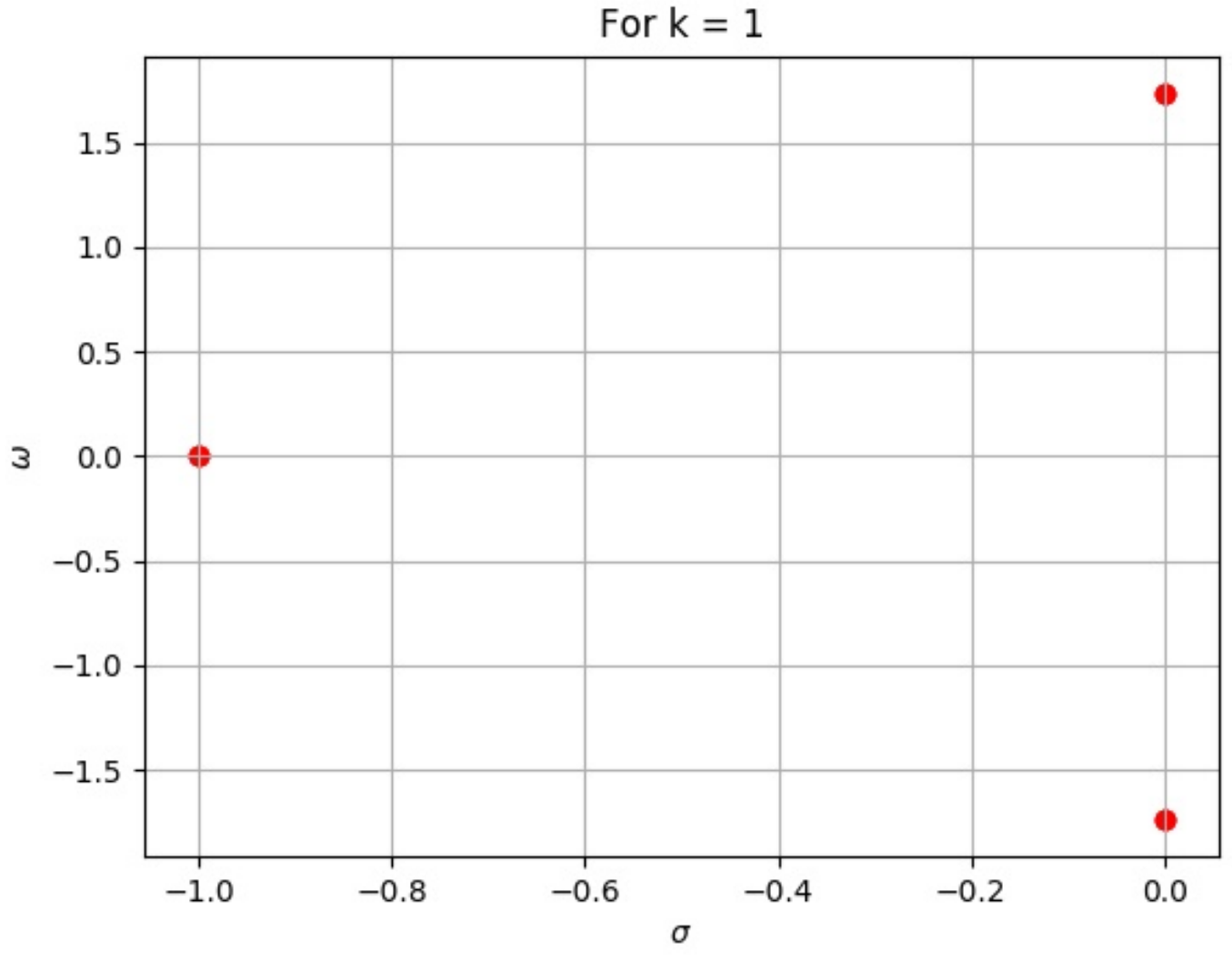
Result of the program

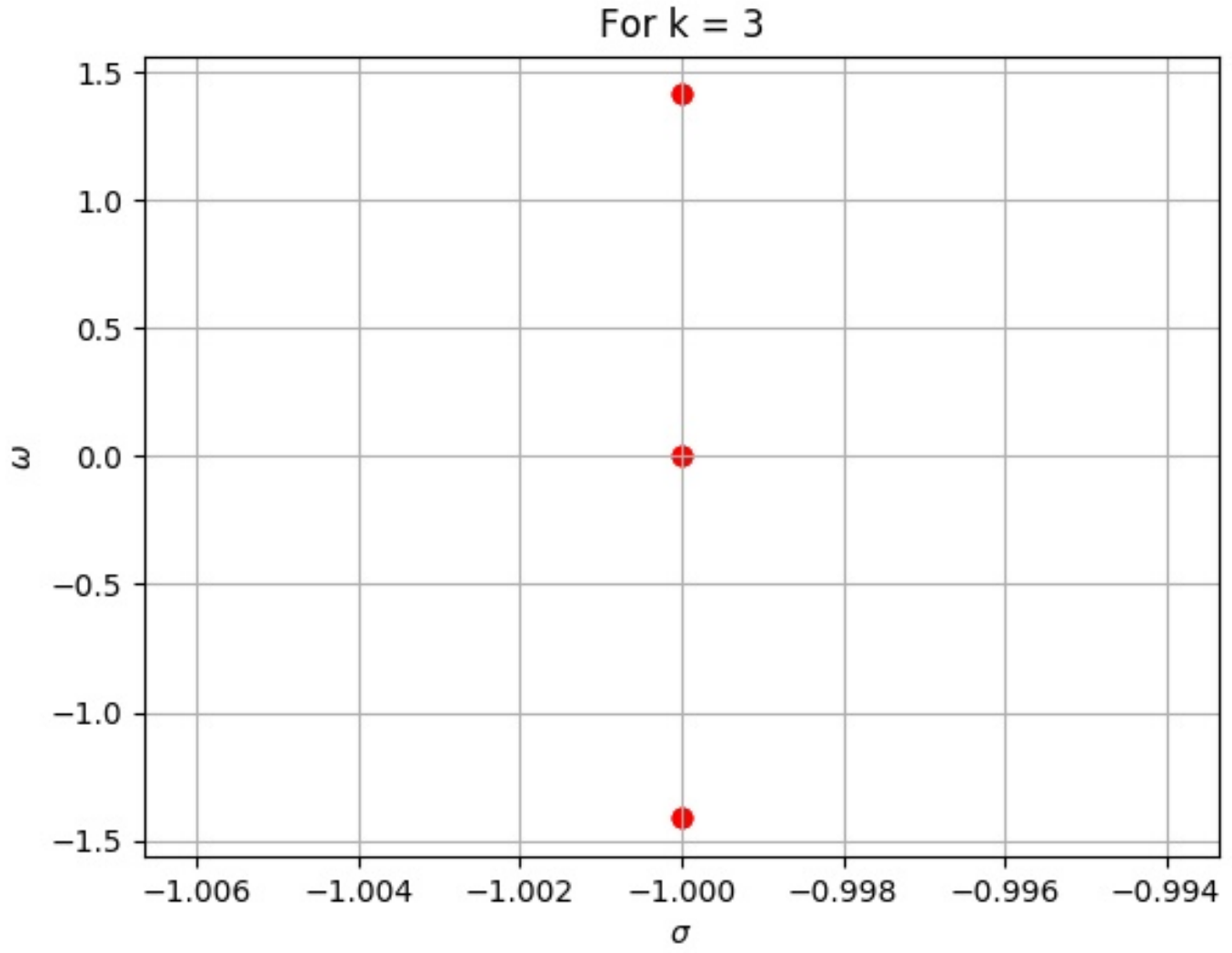
```
shaik-mastan@shaik-mastan-HP-Laptop-15-da1xxx:~/Control_Systems$ python -u "/home/shaik-mastan/Control_Systems/ppt.py"
For k = -3
Unstable system
Routh array:
 [[ 1. -1.  0.]
 [-3.  3.  0.]
 [-6.  0. -0.]
 [ 3.  0.  0.]]

For k = 1
Marginally Stable system
Routh array:
 [[ 1.  3.  0.]
 [ 1.  3.  0.]
 [ 2.  0. -0.]
 [ 3.  0.  0.]]

For k = 2
Stable system
Routh array:
 [[1.  4.  0. ]
 [2.  3.  0. ]
 [2.5 0.  0. ]
 [3.  0.  0. ]]

For k = 3
Stable system
Routh array:
 [[1. 5. 0.]
 [3. 3. 0.]
 [4. 0. 0.]
 [3. 0. 0.]]
```

Verification of the solution

The poles have been plotted for three different values of k.




For k = 1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
ω
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
σ


For k = 3
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
ω
−1.006
−1.004
−1.002
−1.000
−0.998
−0.996
−0.994
σ