Investigación de Operaciones II
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FORMULARIO – TEORÍA DE COLAS

λ: Número medio de llegadas por periodo de tiempo

μ: Número medio de personas o artículos atendidos por período de tiempo

ρ: Factor de utilización del sistema

P_o: Probabilidad de que haya 0 unidades en el sistema (es decir, la unidad de servicio está parada)

L_q: Número medio esperado de clientes en la cola

L_s: Número medio esperado de clientes en el sistema

W_q: Tiempo medio esperado de espera en la cola

W_s: Tiempo medio esperado de espera en el sistema (tiempo de espera más tiempo de servicio)

M/M/S

$$P_O = \frac{1}{\sum_{n=0}^{s-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\lambda/\mu)^s}{S!}(\frac{s\mu}{s\mu - \lambda}}$$

$$\rho = (\frac{\lambda}{s\mu})$$

$$L_q = \frac{1}{s!}(\frac{\lambda}{\mu})^s \frac{\rho}{(1-\rho)^2} P_o$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$P_n = \frac{\rho^n}{n!} P_o, \text{ si n } \leq s$$

$$P_n = \frac{\rho^n}{s!\, s^{n-s}} P_o, \; si\, n \; > \text{s}$$

- **M/M/1**

$$\rho = \frac{\lambda}{\mu}$$

$$W_s = \frac{1}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$L_s = \frac{\lambda}{\mu - \lambda}$$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$P_n = \left(1 - \frac{\lambda}{\mu}\right) * (\frac{\lambda}{\mu})^n = (1 - \rho) * \rho^n$$

$$P_o = 1 - \frac{\lambda}{\mu} = (1 - \rho)$$

$$P_{n)k} = (\frac{\lambda}{\mu})^{k+1}$$

- **M/M/1/K**

$$P_o = \frac{1 - \rho}{1 - \rho^{k+1}}$$

$$\lambda_{ef} = \lambda(1 - p_k) = \lambda\left[1 - \frac{(1 - \rho)\rho^k}{1 - \rho^{k+1}}\right]$$

$$L_s = \begin{cases} \frac{\rho}{(1 - \rho)} - \frac{(k + 1)\rho^{k+1}}{1 - \rho^{k+1}}, & \rho \neq 1 \\ \frac{k}{2}, & \rho = 1 \end{cases}$$

$$L_q = L_s - (1 - P_o) = \begin{cases} L_s - \frac{(1 - \rho^k)\rho}{1 - \rho^{k+1}}, & \rho \neq 1 \\ \frac{k(k-1)}{2(k+1)}, & \rho = 1 \end{cases}$$

$$W_s = \frac{L_s}{\lambda_{ef}}$$

$$W_q = W_s - \frac{1}{\mu}$$

$$L_q = \lambda_{ef} W_q$$

- **M/M/S/K**

$$\rho = \frac{\lambda}{s * \mu}$$

$$P_o = \frac{1}{\sum_{n=0}^{s} \frac{k!}{(k-n)!\,n!} (\frac{\lambda}{\mu})^n + \sum_{n=s+1}^{k} \frac{k!}{(k-n)!} * \frac{1}{s! * s^{n-s}} * (\frac{\lambda}{\mu})^n}$$

$$P_n = \begin{cases} \frac{k!}{(k-n)!\,n!} * \rho^n * P_o, & 0 \leq n \leq s \\ \frac{k!}{(k-n)!} * \frac{1}{s!\,s^{n-s}} * \rho^n * P_o, & 0 \geq s \end{cases}$$

$$L_q = \frac{1}{s!} (\frac{\lambda}{\mu})^s * \frac{\rho}{(1-\rho)^2} * P_o * [1 - \rho^{k-s} - (k-s) * \rho^{k-s}(1-\rho)]$$

$$L_s = \sum_{n=0}^{k} n * P_n = L_q + \frac{\lambda_{ef}}{\mu}$$

$$\lambda_{ef} = \lambda(k - L_s)$$

$$W_q = \frac{L_q}{\lambda_{ef}}$$

$$W_s = \frac{L_s}{\lambda_{ef}}$$

- **M/G/1**

$$P_o = 1 - \rho$$

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

$$L_s = L_q + \rho$$

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda^2 \sigma^2 + \rho^2}{2\lambda(1-\rho)}$$

$$W_s = \frac{L_s}{\lambda} = W_q + \frac{1}{\mu}$$

- **M/D/1**

$$P_o = 1 - \rho$$

$$L_q = \frac{\rho^2}{2(1-\rho)}$$

$$L_s = L_q + \rho$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_s = \frac{L_s}{\lambda} = W_q + \frac{1}{\mu}$$

- **M/Ek/1**

$$P_o = 1 - \rho$$

$$L_q = \frac{\lambda^2(1+k)}{2k\mu(\mu-\lambda)} = \frac{\rho^2(1+k)}{2k(1-\rho)}$$

$$L_s = L_q + \rho$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_s = \frac{L_s}{\lambda}$$