WWW.DAUPHINE.PSL.EU

13/11/2024

GAN improvements by rejection sampling methods

Authors :

Mohamed Amine Belkasmi, Emmarius DELAR, Rithy Sochet

Dauphine Université Paris | PSL

Rejection Sampling Methods

- **Intuition:**
  - Keep only good draw from the GAN's generator

- **Problem:**
  - How to determine what's a good draw ?
  - How to assure the diversity of the sample ?
  - What about the reject sampling time during generation ?

- **Approach:**
  - *Discriminator Rejection Sampling*
  - *Metropolis-Hastings GAN*
  - *Optimal Budgeted Rejection Sampling*

Rejection Sampling:

- We firstly train a VanillaGAN

- We assume our discriminator is optimal, i.e.:

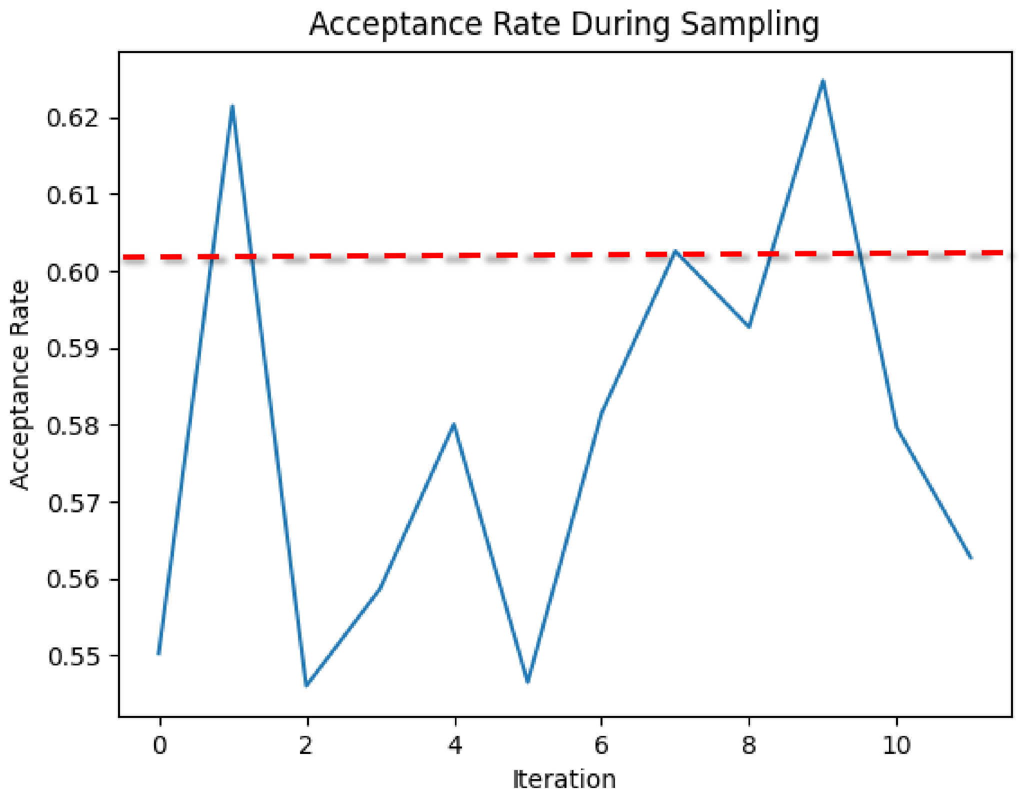
$$D(x) = \frac{p_d(x)}{p_d(x) + p_g(x)}$$

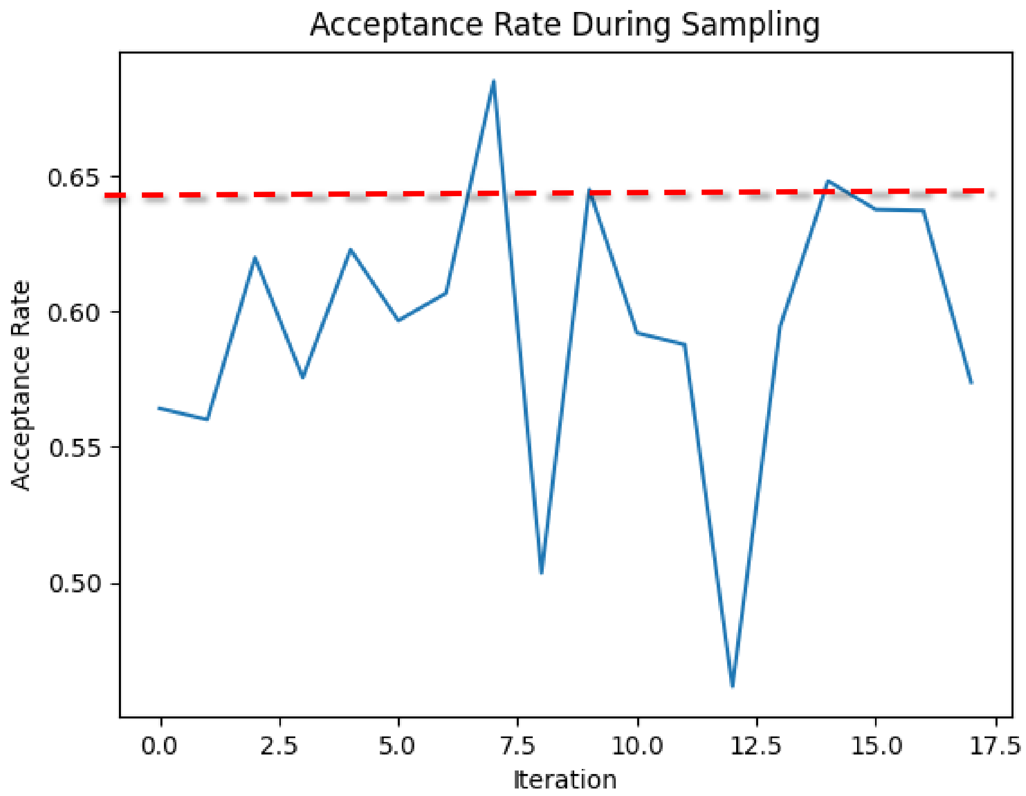
- We then obtain the acceptation rate:

$$\frac{p_d(x)}{p_g(x)} = \frac{D(x)}{1 - D(x)}$$

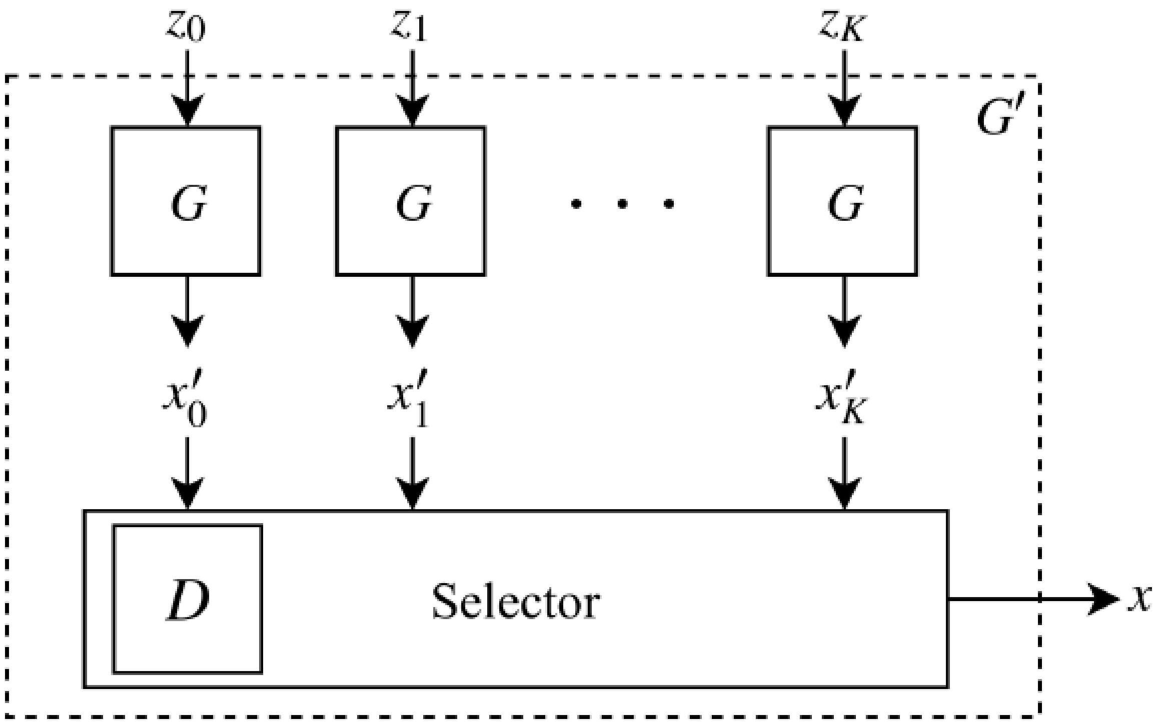
A figure of acceptation rate:

Precision: 0.52
Recall: 0.26





Metropolis-Hastings GAN



Algorithm 1 MH-GAN

Input: generator G, calibrated disc. D, real samples
Assign random real sample $\mathbf{x}_0$ to $\mathbf{x}$
for $k = 1$ **to** K **do**
  Draw $\mathbf{x}'$ from G
  Draw U from $\text{Uniform}(0, 1)$
  if $U \leq (D(\mathbf{x})^{-1} - 1)/(D(\mathbf{x}')^{-1} - 1)$ **then**
    $\mathbf{x} \leftarrow \mathbf{x}'$
  end if
end for
If $\mathbf{x}$ is still real sample $\mathbf{x}_0$ restart with draw from G as $\mathbf{x}_0$
Output: sample $\mathbf{x}$ from G'

Source : [2]

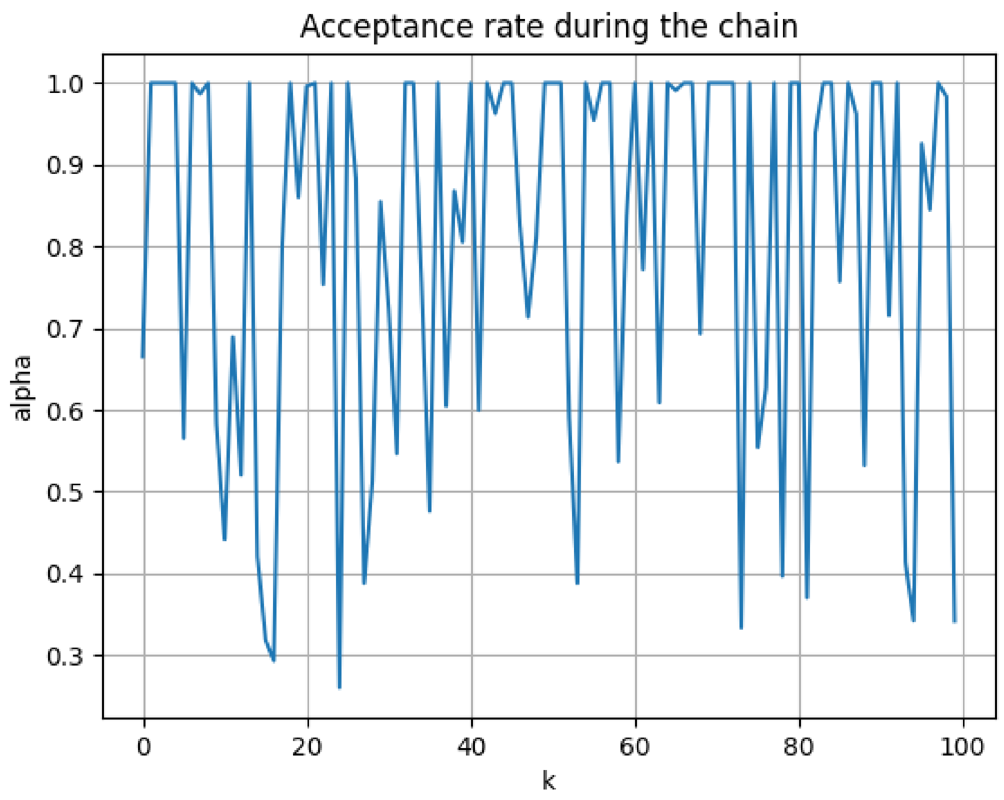
Metropolis-Hastings GAN

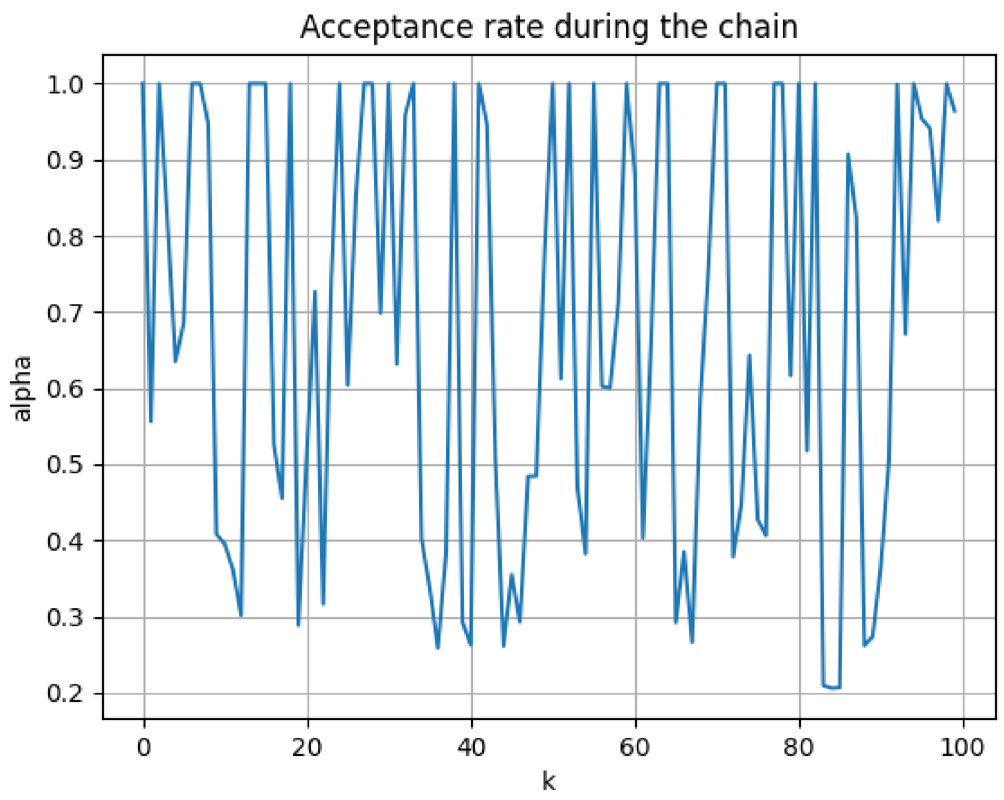
- Calibrator C, classifier where: $D(x_i) = C(\tilde{D}(x_i))$

- Acceptance rate : $\alpha(x', x_k) = \min(1, \frac{D(x_k)^{-1} - 1}{D(x')^{-1} - 1})$

- High computation time

A figure of acceptation rate:





Optimal Budgeted Rejection Sampling

- Optimal Acceptance function:

$$a_{\mathrm{O}}(\boldsymbol{x}) = \min\left(\frac{p(\boldsymbol{x})}{\widehat{p}(\boldsymbol{x})}\frac{c_K}{M}, 1\right)$$

Algorithm B.1 Dichotomy to compute c_K.

Input: N generated samples $\boldsymbol{x}_1^{\text{fake}}, \ldots, \boldsymbol{x}_N^{\text{fake}} \sim \widehat{P}$
Parameter: Budget K, Threshold ϵ
Output: Constant c_K

Let $c_{\min} = 1e^{-10}$ and $c_{\max} = 1e^{10}$.
$c_K = (c_{\max} + c_{\min})/2$
Define the loss $\mathcal{L}(c_K) = \sum_{i=1}^{N} a\left(\boldsymbol{x}_i^{\text{fake}}, c_K\right) - \frac{1}{K}$
while $|\mathcal{L}(c_K)| \geq \epsilon$ **do**
    if $\mathcal{L}(c_K) > \epsilon$ **then**
        $c_{\max} = c_K$
    else if $\mathcal{L}(c_K) < -\epsilon$ **then**
        $c_{\min} = c_K$
    end if
    Update: $c_K = (c_{\max} + c_{\min})/2$
    Update: $\mathcal{L}(c_K)$
end while

Source : [3]

Optimal Budgeted Rejection Sampling

Algorithm 2 GAN Tw/OBRS

---

repeat

Update T by ascending the gradient of

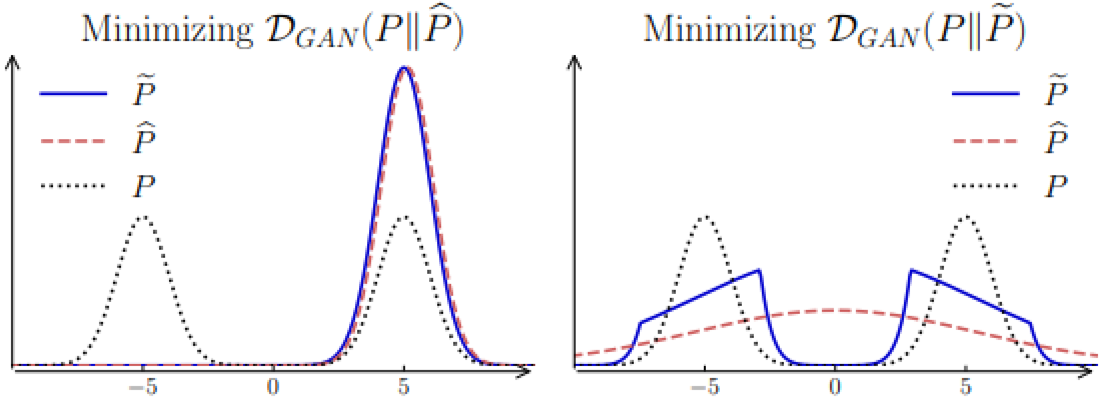
$$\mathbb{E}_{\boldsymbol{x}\sim P}\left[T(\boldsymbol{x})\right] - \mathbb{E}_{\boldsymbol{x}\sim \widehat{P}_G}\left[f^*(T(\boldsymbol{x}))\right].$$

Update c_K such that $\mathbb{E}_{\widehat{P}_G}\left[a_{\mathrm{O}}(\boldsymbol{x})\right] \leq 1/K$.
(See Alg B.1 in App B.2) for details.)

Update G by descending the gradient of

$$\mathbb{E}_{\boldsymbol{x}\sim \widehat{P}_G}\left[K a_{\mathrm{O}}(\boldsymbol{x}) f\left(\frac{r(\boldsymbol{x})}{K a_{\mathrm{O}}(\boldsymbol{x})}\right)\right].$$

until convergence.



Source : [3]

Optimal Budgeted Rejection Sampling

Algorithm 2 GAN Tw/OBRS

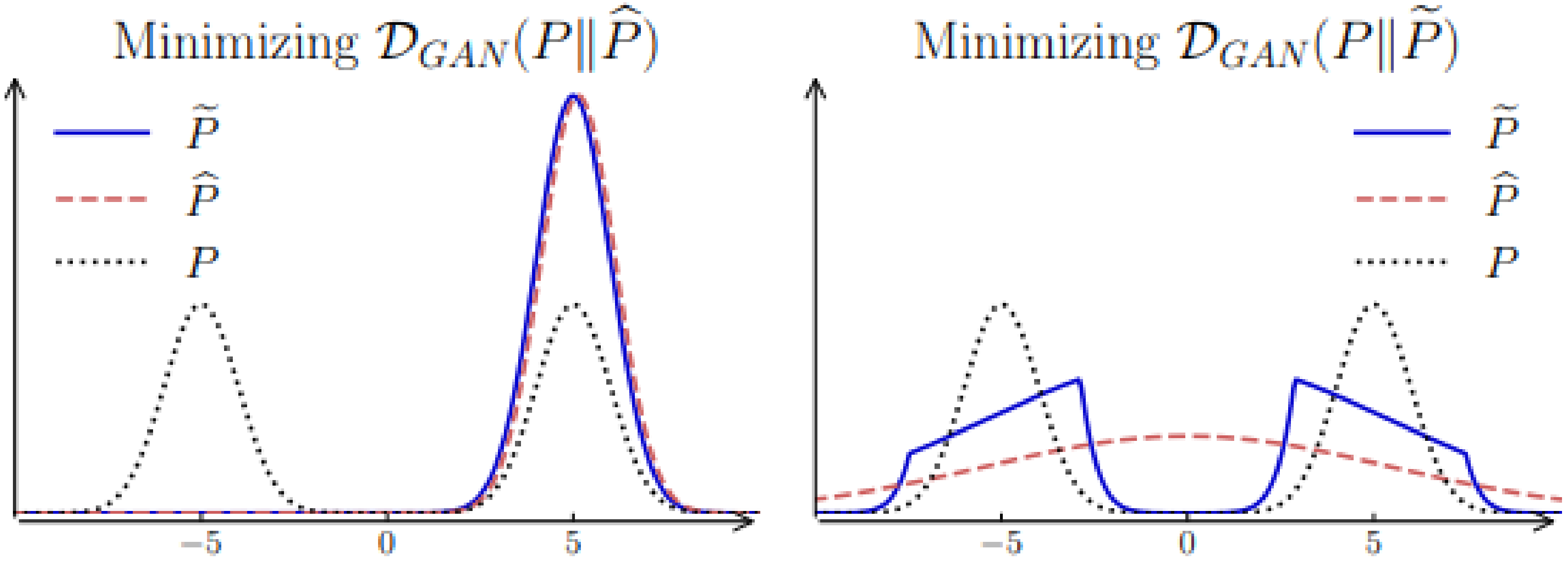
repeat
Update T by ascending the gradient of

$$\mathbb{E}_{\boldsymbol{x}\sim P}\left[T(\boldsymbol{x})\right] - \mathbb{E}_{\boldsymbol{x}\sim \widehat{P}_G}\left[f^*(T(\boldsymbol{x}))\right].$$

Update c_K such that $\mathbb{E}_{\widehat{P}_G}\left[a_O(\boldsymbol{x})\right] \le 1/K$.
(See Alg B.1 in App B.2) for details.)
Update G by descending the gradient of

$$\mathbb{E}_{\boldsymbol{x}\sim \widehat{P}_G}\left[K a_O(\boldsymbol{x}) f\left(\frac{r(\boldsymbol{x})}{K a_O(\boldsymbol{x})}\right)\right].$$

until convergence.



Source : [3]

EXPERIMENTAL RESULTS

Model	FID	Precision	Recall	Time (in s)
Vanilla GAN (100 epochs)	26.86	**0.53**	0.24	90
DRS	30.04	0.52	**0.26**	106.27
Vanilla GAN + MH-GAN (K=10)	**25.36**	**0.53**	0.22	1044
Vanilla GAN Tw/ ORBS	27.69	0.51	0.2	101

Table 1: Results on testing platform

Model	FID	Precision	Recall	Time (in s)
Vanilla GAN (100 epochs)	27.62	0.328	0.176	33
Vanilla GAN + MH-GAN (K=10)	25.42	0.324	0.177	942
Vanilla GAN + MH-GAN (K=100)	25.43	0.322	0.196	6201
Vanilla GAN + MH-GAN (K=100)	**24.77**	0.332	0.190	8243
Vanilla GAN Tw/ ORBS	27.4	**0.352**	**0.21**	**33.52**

Table 2: Results with our own testing

DISCUSSION

- Computing M, the maximum of the quotient
- The assumption on the optimal discriminator
- The two distributions don't necessarily have the same support
- Computation time may be long (especially for MH-GAN)

THANKS FOR LISTENING

Time for your questions

UNIVERSITÉ PARIS DAUPHINE - PSL
Place du Maréchal de Lattre de Tassigny – 75775 Paris cedex 16

References

[1] Samaneh Azadi, Catherine Olsson, Trevor Darrell, Ian Goodfellow, and Augustus Odena. ***Discriminator rejection sampling***. arXiv preprint arXiv:1810.06758, (2018)

[2] Ryan Turner, Jane Hung, Eric Frank, Yunus Saatchi, and Jason Yosinski. ***Metropolis-hastings generative adversarial networks.*** (2019)

[3] Alexandre Verine, Muni Sreenivas Pydi, Benjamin Negrevergne, and Yann Chevaleyre. ***Optimal budgeted rejection sampling for generative models***. (2024)