第八次作业：

证明教材9.10式：

$$z(x_{jk}) \approx (1+\varepsilon_k^\wedge + \frac{1}{2}\varepsilon_k^\wedge \varepsilon_k^\wedge) T_{op,k}(p_{op,j} + D\xi_j)$$

$$\approx T_{op,k} p_{op,j} + \varepsilon_k^\wedge T_{op,k} p_{op,j} + T_{op,k} D\xi_j + \frac{1}{2}\varepsilon_k^\wedge \varepsilon_k^\wedge T_{op,k} p_{op,j} + \varepsilon_k^\wedge T_{op,k} D\xi_j$$

$$= z(x_{op,jk}) + Z_{jk}\delta x_{jk} + \frac{1}{2}\sum_i 1_i \underbrace{\delta x_{jk}^T Z_{ijk} \delta x_{jk}}_{\text{标量}}$$

其中 $z(x_{op,jk}) = T_{op,k} p_{op,j}$

$$Z_{jk} = [(T_{op,k} p_{op,j})^\odot \quad T_{op,k} D]$$

$$Z_{i,jk} = \begin{bmatrix} 1_i^\circledcirc (T_{op,k} p_{op,j})^\odot & 1_i^\circledcirc T_{op,k} D \\ (1_i^\circledcirc T_{op,k} D)^T & 0 \end{bmatrix}$$

自定义：

$$z(x_{op,jk}) = T_{op,k} p_{op,j}$$

对一次项和二次型分别证明

即 $Z_{jk}\delta x_{jk} = \varepsilon_k^\wedge T_{op,k} p_{op,j} + T_{op,k} D\xi_j$

$$\frac{1}{2}\sum_i 1_i \delta x_{jk}^T Z_{ijk} \delta x_{jk} = \varepsilon_k^\wedge T_{op,k} D\xi_j + \frac{1}{2}\varepsilon_k^\wedge \varepsilon_k^\wedge T_{op,k} p_{op,j}$$

一次型：$Z_{jk}\delta x_{jk} = [(T_{op,k} p_{op,j})^\odot \quad T_{op,k} D]\begin{bmatrix}\varepsilon_k \\ \xi_j\end{bmatrix}$

$$= (T_{op,k} p_{op,j})^\odot \varepsilon_k + T_{op,k} D\xi_j = \varepsilon_k^\wedge T_{op,k} p_{op,j} + T_{op,k} D\xi_j$$

$$x^\wedge p = p^\odot x$$

二次项：$\frac{1}{2}\sum_i 1_i \delta x_{jk}^T Z_{ijk} \delta x_{jk} = \frac{1}{2}\sum_i 1_i [\varepsilon_k^T \quad \xi_j^T]\begin{bmatrix} 1_i^\circledcirc (T_{op,k} p_{op,j})^\odot & 1_i^\circledcirc T_{op,k} D \\ (1_i^\circledcirc T_{op,k} D)^T & 0 \end{bmatrix}\begin{bmatrix}\varepsilon_k \\ \xi_j\end{bmatrix}$

$$= \frac{1}{2}\sum_i 1_i (\varepsilon_k^T 1_i^\circledcirc (T_{op,k} p_{op,j})^\odot \varepsilon_k + \xi_j^T (1_i^\circledcirc T_{op,k} D)^T \varepsilon_k + \varepsilon_k^T 1_i^\circledcirc T_{op,k} D\xi_j)$$

$$\Rightarrow \frac{1}{2}\sum_i 1_i \varepsilon_k^T 1_i^\circledcirc (T_{op,k} p_{op,j})^\odot \varepsilon_k$$

$$= \frac{1}{2}\sum_i 1_i 1_i^T \varepsilon_k^\wedge \varepsilon_k^\wedge T_{op,k} p_{op,j} = \frac{1}{2}\varepsilon_k^\wedge \varepsilon_k^\wedge T_{op,k} p_{op,j}$$

$$\frac{1}{2}\sum_i 1_i (\xi_j^T (1_i^\circledcirc T_{op,k} D)^T \varepsilon_k + \varepsilon_k^T 1_i^\circledcirc T_{op,k} D\xi_j) = \sum_i 1_i (\varepsilon_k^T 1_i^\circledcirc T_{op,k} D\xi_j).$$

$$= \sum_i 1_i (1_i^T \varepsilon_k^\wedge T_{op,k} D\xi_j) = \varepsilon_k^\wedge T_{op,k} D\xi_j \quad (p^T x^\wedge \equiv x^T p^\circledcirc;\ x^\wedge p \equiv p^\odot x)$$

$$\xi_j^T (1_i^\circledcirc T_{op,k} D)^T \varepsilon_k = (T_{op,k} D\xi_j)^T 1_i^\circledcirc \varepsilon_k = 1_i (T_{op,k} D\xi_j)^\wedge \varepsilon_k \ ???$$