2 图像去畸变

4.矩阵运算

1.

答：x是$n \times 1$列向量

令矩阵$A = [a_1, a_2, ..., a_n]$, $A = [a_1'; a_2'; ...; a_n']$。

$$\frac{\partial Ax}{\partial x} = \begin{bmatrix} \frac{\partial Ax_1}{\partial x_1} & \frac{\partial Ax_2}{\partial x_1} & \dots & \frac{\partial Ax_n}{\partial x_1} \\ \frac{\partial Ax_1}{\partial x_2} & \frac{\partial Ax_2}{\partial x_2} & \dots & \frac{\partial Ax_n}{\partial x_2} \\ \dots & \dots & \dots & \dots \\ \frac{\partial Ax_1}{\partial x_n} & \frac{\partial Ax_2}{\partial x_n} & \dots & \frac{\partial Ax_n}{\partial x_n} \end{bmatrix}$$

先对x的第i个分量求导：

$$\frac{\partial Ax_i}{\partial x_k} = \frac{\partial a_i x}{\partial x_k} = a_{ik}$$

导入前式有：

$$\frac{\partial Ax}{\partial x} = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{bmatrix} = A^T$$

2.

$$\frac{\partial x^T Ax}{\partial x} = \begin{bmatrix} \frac{\partial x^T Ax}{\partial x_1} & \frac{\partial x^T Ax}{\partial x_2} & \cdots & \frac{\partial x^T Ax}{\partial x_n} \end{bmatrix}$$

先对x的第k个分量求导，结果如下：

$$\begin{aligned} \frac{\partial x^T Ax}{\partial x_k} &= \frac{\partial \sum_{i=1}^n \sum_{j=1}^n x_i A_{ij} x_j}{\partial x_k} \\ &= \sum_{i=1}^n A_{ik} x_i + \sum_{j=1}^n A_{kj} x_j \\ &= a_k^T x + a_k' x \end{aligned}$$

可以看出第一部分是矩阵A的第k列转置后和x相乘得到，第二部分是矩阵A的第k行和x相乘得到，排列好就是:
$\frac{\partial x^T Ax}{\partial x} = A^T x + Ax$

3.

设a,b都是n维列向量，显然有

$$ab^T = \begin{bmatrix} a_1 b_1 & a_1 b_2 & \dots & a_1 b_n \\ a_2 b_1 & a_2 b_2 & \dots & a_2 b_n \\ \dots & \dots & \dots & \dots \\ a_n b_1 & a_n b_2 & \dots & a_n b_n \end{bmatrix}$$

$$b^T a = \sum_{i=1}^n a_i b_i$$

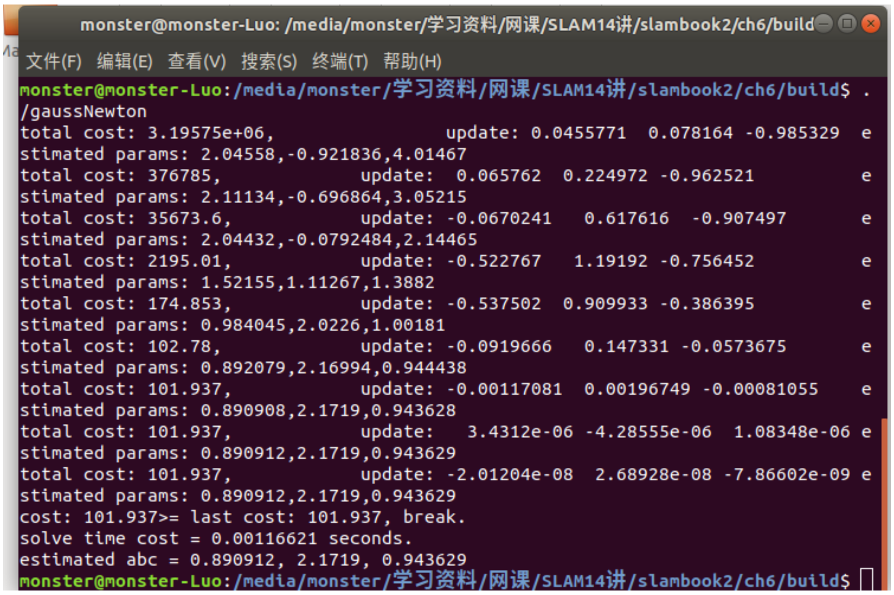
显然，可以得到：

$$tr(ab^T) = b^T a$$

令 $a = Ax$, $b = x$ 可得

$$tr(Axx^T) = tr((Ax)x^T) = x^T Ax$$

5 高斯牛顿法的曲线拟合实验



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monster@monster-Luo: /media/monster/学习资料/网课/SLAM14讲/slambook2/ch6/build
文件(F) 编辑(E) 查看(V) 搜索(S) 终端(T) 帮助(H)
monster@monster-Luo:/media/monster/学习资料/网课/SLAM14讲/slambook2/ch6/build$ ./gaussNewton
total cost: 3.19575e+06,            update: 0.0455771  0.078164 -0.985329  e
stimated params: 2.04558,-0.921836,4.01467
total cost: 376785,         update:  0.065762  0.224972 -0.962521       e
stimated params: 2.11134,-0.696864,3.05215
total cost: 35673.6,        update: -0.0670241   0.617616  -0.907497       e
stimated params: 2.04432,-0.0792484,2.14465
total cost: 2195.01,        update: -0.522767   1.19192 -0.756452       e
stimated params: 1.52155,1.11267,1.3882
total cost: 174.853,        update: -0.537502  0.909933 -0.386395       e
stimated params: 0.984045,2.0226,1.00181
total cost: 102.78,         update: -0.0919666   0.147331 -0.0573675       e
stimated params: 0.892079,2.16994,0.944438
total cost: 101.937,        update: -0.00117081  0.00196749 -0.00081055    e
stimated params: 0.890908,2.1719,0.943628
total cost: 101.937,        update:   3.4312e-06 -4.28555e-06  1.08348e-06 e
stimated params: 0.890912,2.1719,0.943629
total cost: 101.937,        update: -2.01204e-08  2.68928e-08 -7.86602e-09 e
stimated params: 0.890912,2.1719,0.943629
cost: 101.937>= last cost: 101.937, break.
solve time cost = 0.00116621 seconds.
estimated abc = 0.890912, 2.1719, 0.943629
monster@monster-Luo:/media/monster/学习资料/网课/SLAM14讲/slambook2/ch6/build$
```