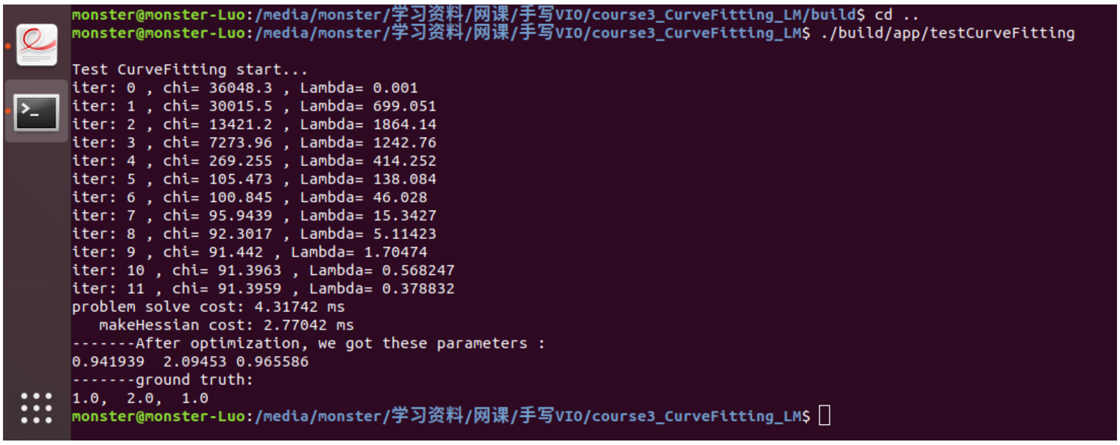
1 代码

1.1 阻尼因子变化曲线图

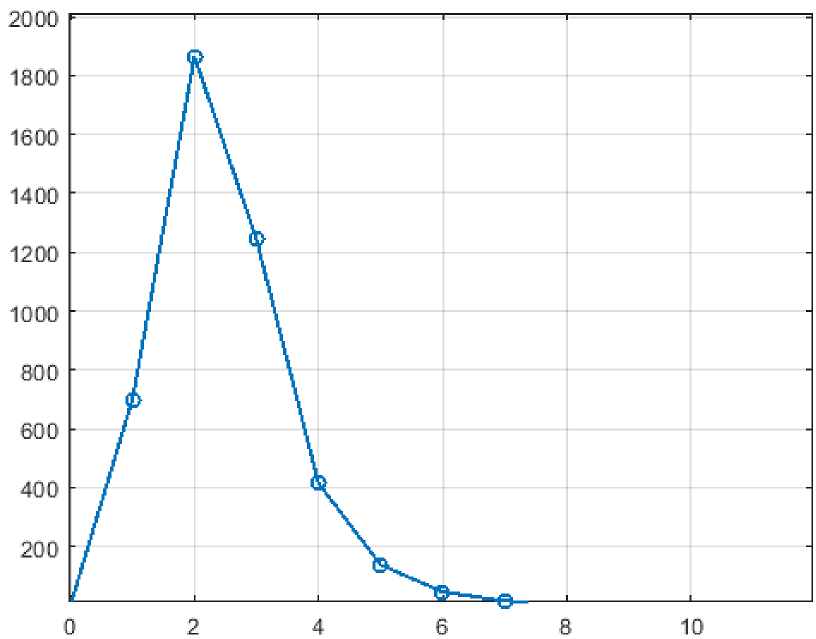
首先编译文件得到数值



```
monster@monster-Luo:/media/monster/学习资料/网课/手写VIO/course3_CurveFitting_LM/build$ cd ..
monster@monster-Luo:/media/monster/学习资料/网课/手写VIO/course3_CurveFitting_LM$ ./build/app/testCurveFitting

Test CurveFitting start...
iter: 0 , chi= 36048.3 , Lambda= 0.001
iter: 1 , chi= 30015.5 , Lambda= 699.051
iter: 2 , chi= 13421.2 , Lambda= 1864.14
iter: 3 , chi= 7273.96 , Lambda= 1242.76
iter: 4 , chi= 269.255 , Lambda= 414.252
iter: 5 , chi= 105.473 , Lambda= 138.084
iter: 6 , chi= 100.845 , Lambda= 46.028
iter: 7 , chi= 95.9439 , Lambda= 15.3427
iter: 8 , chi= 92.3017 , Lambda= 5.11423
iter: 9 , chi= 91.442 , Lambda= 1.70474
iter: 10 , chi= 91.3963 , Lambda= 0.568247
iter: 11 , chi= 91.3959 , Lambda= 0.378832
problem solve cost: 4.31742 ms
   makeHessian cost: 2.77042 ms
-------After optimization, we got these parameters :
0.941939  2.09453 0.965586
-------ground truth:
1.0,  2.0,  1.0
monster@monster-Luo:/media/monster/学习资料/网课/手写VIO/course3_CurveFitting_LM$
```

lambda 值整理后，在 matlab 下绘图



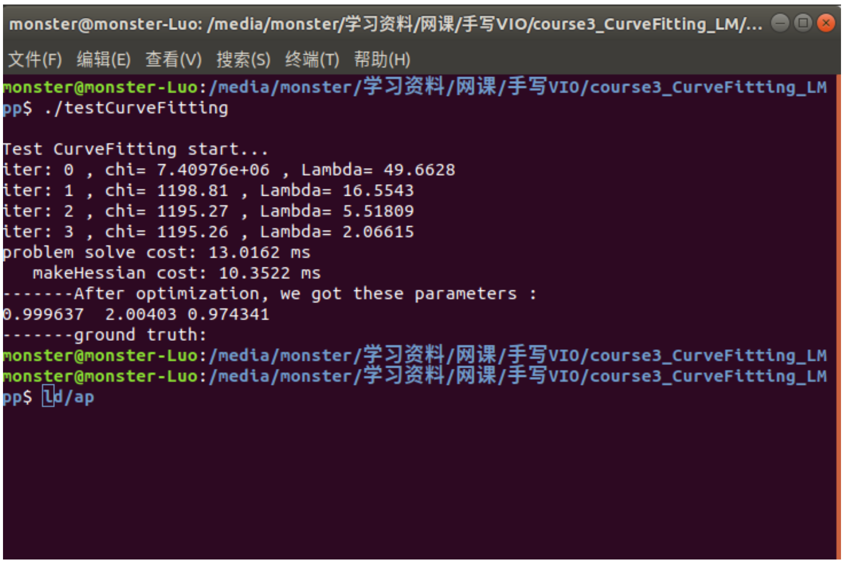
1.2 完成曲线 $y = ax^2 + bx + c$ 的参数估计

需要将代码中的 $y = \exp(ax^2 + bx + c)$改为 $y = ax^2 + bx + c$， 以及修改残差和对应雅可比

```
residual_(0) = abc(0)*x_*x_ + abc(1)*x_ + abc(2)  - y_;  // 构建残差
jaco_abc << x_ * x_ , x_  , 1 ;//雅可比
```

重新编译运行

参数估计误差较大，误差较大的原因可能是数据点数量不足。于是将数据点由 100 增加到 1200,再次编译，得到参数估计精度就足够高了



1.3 实现其他更优秀的阻尼因子策略

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2.关于 F 和 G 的公式推导

2.1

1) $f_{15} = \dfrac{\partial \alpha_{b_i b_{k+1}}}{\partial \delta b_k^g}$

$$a = \frac{1}{2}\left(q_{b_i b_k}(a^{b_k} - b_k^a) + q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a)\right)$$

$$\omega = \frac{1}{2}\left((\omega^{b_k} + n_k^g - b_k^g) + (\omega^{b_{k+1}} + n_{k+1}^g - b_k^g)\right) = \frac{1}{2}\left((\omega^{b_k} + n_k^g) + (\omega^{b_{k+1}} + n_{k+1}^g)\right) - b_k^g$$

$$\alpha_{b_i b_{k+1}} = \alpha_{b_i b_k} + \beta_{b_i b_k} + \frac{1}{2} a \delta t^2$$

b_k^g 与 ω 相关，误差传递，对 ω 右乘 $\delta b_k^g \delta t$ 求偏导

$$f_{15} = \frac{\partial \alpha_{b_i b_{k+1}}}{\partial \delta b_k^g} = \frac{\partial \frac{1}{2} a \delta t^2}{\partial \delta b_k^g}$$

$$= \frac{\partial \frac{1}{2} \cdot \frac{1}{2} q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a) \delta t^2}{\partial \delta b_k^g}$$

$$\Rightarrow \frac{1}{4} \frac{\partial q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -\frac{1}{2}\delta b_k^g \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a) \delta t^2}{\partial \delta b_k^g}$$

$$= \frac{1}{4} \frac{\partial R_{b_i b_{k+1}} \exp([-\frac{1}{2}\delta b_k^g \delta t]_\times)(a^{b_{k+1}} - b_k^a)\delta t^2}{\partial \delta b_k^g}$$

$$= \frac{1}{4} \partial R_{b_i b_{k+1}} (I + [-\frac{1}{2}\delta b_k^g \delta t]_\times)(a^{b_{k+1}} - b_k^a)\delta t^2 / \partial \delta b_k^g$$

$$= -\frac{1}{4} \frac{\partial R_{b_i b_{k+1}} [(a^{b_{k+1}} - b_k^a)]_\times \delta t^2 (-\delta b_k^g \delta t)}{\partial \delta b_k^g}$$

$$= -\frac{1}{4}\left(R_{b_i b_{k+1}} [(a^{b_{k+1}} - b_k^g)]_\times \delta t^2\right)(-\delta t)$$

2) $g_{12} = \dfrac{\partial \alpha_{b_i b_{k+1}}}{\partial \delta n_k^g}$

$$a = \frac{1}{2}\left(q_{b_i b_k}(a^{b_k} - b_k^a) + q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a)\right)$$

$$\omega = \frac{1}{2}\left((\omega^{b_k} + n_k^g - b_k^g) + (\omega^{b_{k+1}} + n_{k+1}^g - b_k^g)\right) = \frac{1}{2}\left((\omega^{b_k} + n_k^g) + (\omega^{b_{k+1}} + n_{k+1}^g)\right) - b_k^g$$

$$\alpha_{b_i b_{k+1}} = \alpha_{b_i b_k} + \beta_{b_i b_k} + \frac{1}{2} a \delta t^2$$

对 ω 右乘个微小扰动 $\frac{1}{2}\delta n_k^g \delta t$

$$g_{12} = \frac{\partial \alpha_{b_i b_{k+1}}}{\partial \delta n_k^g} = \frac{\partial \frac{1}{2} a \delta t^2}{\partial \delta n_k^g} = \frac{1}{4} \cdot \frac{\partial q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a) \delta t^2}{\partial \delta n_k^g}$$

$$\Rightarrow \frac{1}{4} \partial q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -\frac{1}{2}\delta n_k^g \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a) \delta t^2$$

$$= \frac{1}{4} \frac{\partial R_{b_i b_{k+1}} \operatorname{Exp}([-\frac{1}{2}\delta n_k^g \delta t]_\times)(a^{b_{k+1}} - b_k^a)\delta t^2}{\partial \delta n_k^g}$$

$$= \frac{1}{4} \frac{\partial R_{b_i b_{k+1}} (I - [\frac{1}{2}\delta n_k^g \delta t]_\times)(a^{b_{k+1}} - b_k^a)\delta t^2}{\partial \delta n_k^g}$$

$$= -\frac{1}{4} \frac{\partial R_{b_i b_{k+1}} [(a^{b_{k+1}} - b_k^a)]_\times \delta t^2 (\frac{1}{2}\delta n_k^g \delta t)}{\partial \delta n_k^g}$$

$$= -\frac{1}{4}\left(R_{b_i} R_{b_{k+1}} [(a^{b_{k+1}} - b_k^a)]_\times \delta t^2\right)\left(\frac{1}{2}\delta t\right)$$

3.证明式(9)

题 3

$$\Delta x_{lm} = -\sum_{j=1}^{n} \frac{v_j^T F'^T}{\lambda_j + \mu} v_j ,$$

根据矩阵运算推导：

$$(J^T J + \mu I)\Delta x_{lm} = -J^T f \quad , \mu \geq 0$$

$J^T J$ 为半正定信息矩阵，特征值 $\{\lambda_j\}$。满足 $J^T J = V \Lambda V^T$，其中 V 为 $J^T J$ 特征向量组成的特征矩阵，向量之间相互正交。

Λ 为特征值组成的对角阵。 $F'(\underline{x}) = (J^T f)^T$

且因为 V 为正交阵，具有性质 $V^T = V^{-1}$

$$(J^T J + \mu I)\Delta x_{lm} = -J^T f$$

$$\begin{aligned}
\Delta x_{lm} &= (J^T J + \mu I)^{-1}(-J^T f) \\
&= (V \Lambda V^T + \mu V V^T)^{-1}(-J^T f) \\
&= (V \Lambda V^T + V \mu I V^T)^{-1}(-J^T f) = [V(\Lambda + \mu I)V^T]^{-1}(-F'^T)
\end{aligned}$$

展开：

$$[v_1 \ v_2 \ \cdots \ v_n]\begin{bmatrix} \frac{1}{\lambda_1+\mu} & 0 & \cdots & 0 \\ 0 & \frac{1}{\lambda_2+\mu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\lambda_n+\mu} \end{bmatrix}\begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_n^T \end{bmatrix}(-F'^T)$$

$$= \begin{bmatrix} \frac{v_1}{\lambda_1+\mu} & \frac{v_2}{\lambda_2+\mu} & \cdots & \frac{v_n}{\lambda_n+\mu} \end{bmatrix}\begin{bmatrix} -v_1^T F'^T \\ -v_2^T F'^T \\ \vdots \\ -v_n^T F'^T \end{bmatrix} = -\sum_{j=1}^{n} \frac{v_j v_j^T F}{\lambda_j + \mu}$$