Mission Analysis — Atmosphere Model

This note explains the mathematical foundations and implementation details of the Python code used for atmosphere model analysis. Each section aligns with the structure of the source code and references the International Standard Atmosphere (ISA) model.

ISA Atmospheric Model

1. Temperature Variation with Altitude

The International Standard Atmosphere (ISA) divides the atmosphere into layers with different temperature gradients. The base implementation considers the **troposphere**, **isothermal lower stratosphere**, and **polytropic upper stratosphere**.

1.1 Troposphere (0–11 km)

In this layer, temperature decreases linearly with altitude:

$$T(h) = T_0 + a \cdot h$$

- $T_0 = 288.15\ \mathrm{K}$: Sea-level standard temperature
- $a = -0.0065\ \mathrm{K/m}$: Lapse rate
- h: Geopotential altitude in meters

The pressure and density are computed using polytropic relations:

$$p(h) = p_0 \left(1 + \frac{ah}{T_0}\right)^{\frac{n}{n-1}}, \quad \rho(h) = \rho_0 \left(1 + \frac{ah}{T_0}\right)^{\frac{1}{n-1}}$$

- $n = 1.235$: Effective polytropic index for the troposphere

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1.2 Lower Stratosphere (11–20 km)

This is an **isothermal layer**, meaning temperature remains constant:

$$T(h) = T_{11} = 216.65 \text{ K}$$

Pressure and density decay exponentially due to hydrostatic equilibrium:

$$p(h) = p_{11} \cdot e^{-\frac{g_0(h-h_{11})}{RT_{11}}}, \quad \rho(h) = \rho_{11} \cdot e^{-\frac{g_0(h-h_{11})}{RT_{11}}}$$

1.3 Upper Stratosphere (>20 km)

Temperature increases slightly with height, modeled using a **modified polytropic** relation:

$$T(h) = T_{20} \left(1 + \frac{\gamma(h - h_{20})}{T_{20}}\right)$$

$$p(h) = p_{20} \left(1 + \frac{\gamma(h - h_{20})}{T_{20}}\right)^{\frac{n}{n-1}}, \quad \rho(h) = \rho_{20} \left[1 - \frac{(n-1)}{n} \cdot \frac{g_0}{RT_{20}} \cdot (h - h_{20})\right]^{\frac{1}{n-1}}$$

- $\gamma = 0.001 \text{ K/m}$: Positive lapse rate in upper stratosphere
- $n = 0.001$: Effective index for upper stratosphere

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2. Speed of Sound in the Atmosphere

The speed of sound varies with temperature and is computed using:

$$a(h) = \sqrt{\gamma \cdot R \cdot T(h)}$$

Where:

- $\gamma = 1.4$: Ratio of specific heats for air
- $R = 287.05 \text{ J/(kg} \cdot \text{K)}$: Specific gas constant for air
- $T(h)$: Temperature at altitude h

Or nondimensionally:

$$a(h) = a_0 \cdot \sqrt{\Theta}, \quad \text{where} \quad \Theta = \frac{T(h)}{T_0}, \quad a_0 = \sqrt{\gamma R T_0} \approx 340.3 \text{ m/s}$$

This is implemented in the `get_speed_of_sound()` method.

Sources / References

[1] Anderson, J. D. (2016). *Introduction to Flight*.

8th Edition, McGraw-Hill Education, New York.
Ch. 4, pp. 177–178. Key Equations:

- $a = \sqrt{\gamma R T}$
- $M = \frac{V}{a}$

[2] U.S. Standard Atmosphere, 1976.

NOAA, NASA, USAF. Washington, D.C.: U.S. GPO.
Provides full profiles for $T(h), p(h), \rho(h)$ and validation data for standard layers.

[2.1] Institute of Flight System Dynamics, TUM.

Centralized Definition Notes, Version 3.0, p. 101.
Provides layer-specific modeling parameters for polytropic and isothermal atmospheric sections.