

Leonardo Fusser (1946995)

LABORATORY EXPERIMENT #4

Frequency Modulation

NOTE:

To be completed in one lab session of 3 hrs.

No lab report required. Just answer the questions and add your screen shots.
This exercise is to be done **individually** except where specified in the procedure.

OBJECTIVES:

After performing this experiment, the student will be able to:

1. Calculate the FM Modulation index and predict sideband amplitudes.
2. Generate an FM modulated signal and measure the frequency-domain characteristics.
3. Compare measured results with theoretical predictions.

DISCUSSION OF THEORY

As the name implies, Frequency Modulation is a technique to embed baseband information onto an RF carrier by modifying the frequency of the carrier according to the baseband signal. In FM, the amplitude of the carrier, and hence its power, remains constant.

The FM Modulation index m_f indicates the degree of modulation. Unlike the AM modulation index that is always less than or equal to one (or expressed as a percentage), m_f can be less than or greater than one.

$$m_f = \frac{\Delta f}{f_{MOD}}$$

Δf is the peak frequency change on either side
of the unmodulated (idle) carrier frequency.

f_{MOD} is the frequency of the baseband (modulating) signal.

When m_f is less than or approximately equal to 1, it is said to be Narrowband FM (NBFM). When m_f is greater than 1, it is said to be Wideband FM (WBFM). WBFM produces many more sidebands than NBFM, but its noise immunity performance is much better.

The total bandwidth of the modulated FM signal is the frequency difference between highest order significant sideband pair.

Carson's rule is a way to approximate the bandwidth, especially for non-sinusoidal baseband signals.

$$B_{FM} \approx 2(B_{MOD} + \Delta f)$$

Where: B_{MOD} is the baseband bandwidth (highest modulating frequency)

Δf is the deviation

PROCEDURE**Part 1:**

1. Check the following simulation tool for FM modulation. You can change modulation signal and carrier. Also, you can also change the modulation deviation to modulation voltage K_f .

<https://demonstrations.wolfram.com/PowerContentOfFrequencyModulationAndPhaseModulation/>

In order to change the parameters, you should download the wolfram player desktop version.

Take screenshot for you tests and save any Wolfram simulation to desktop version.

2. Write down the relation between K (frequency deviation) and modulation index m_f (it is called β in this tool).

$$m_f = \frac{f_d}{f_m}$$

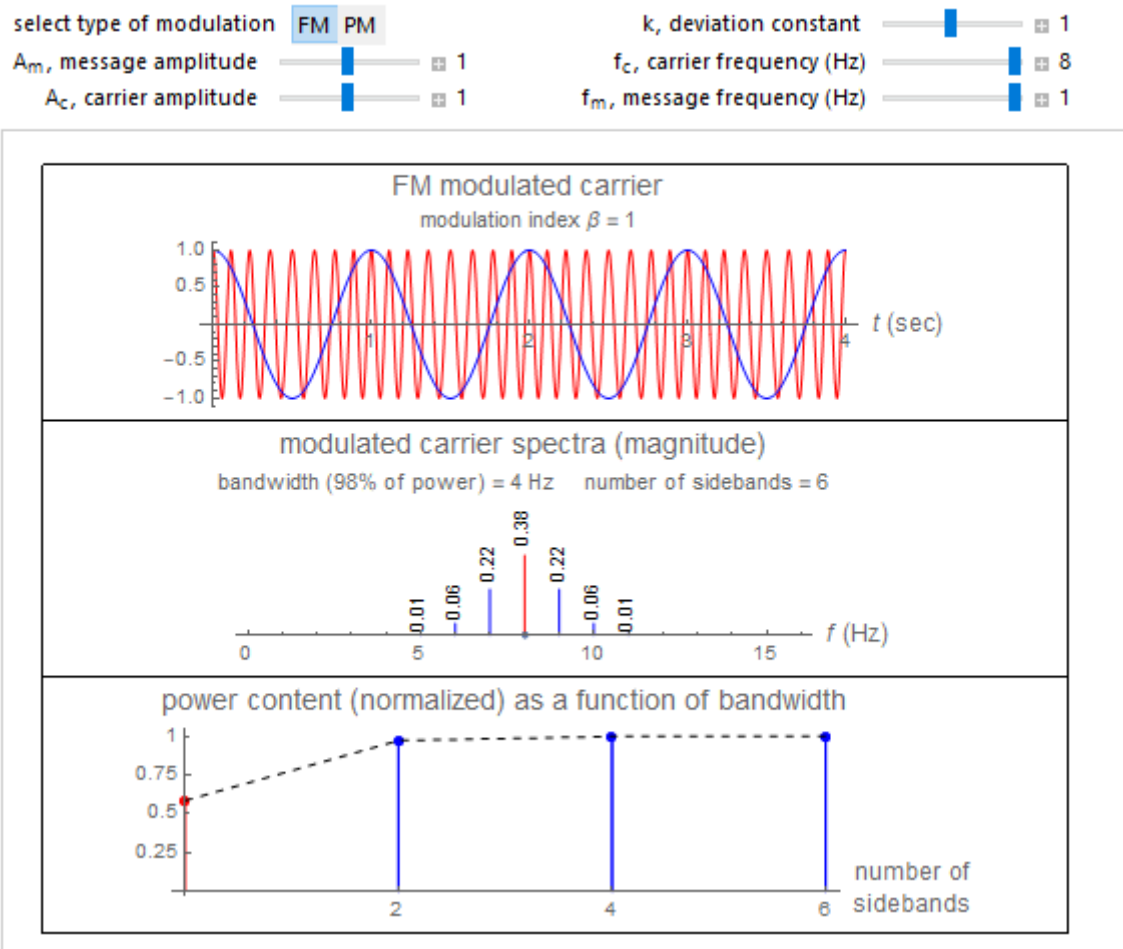
Where,

m_f = modulation index

f_d = frequency deviation (Hz)

f_m = modulating signal frequency (Hz)

3. Set carrier to 8Hz with amplitude of 1V. Modulation signal to 1Hz with amplitude of 1V. Set K=1.



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 1Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 1Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

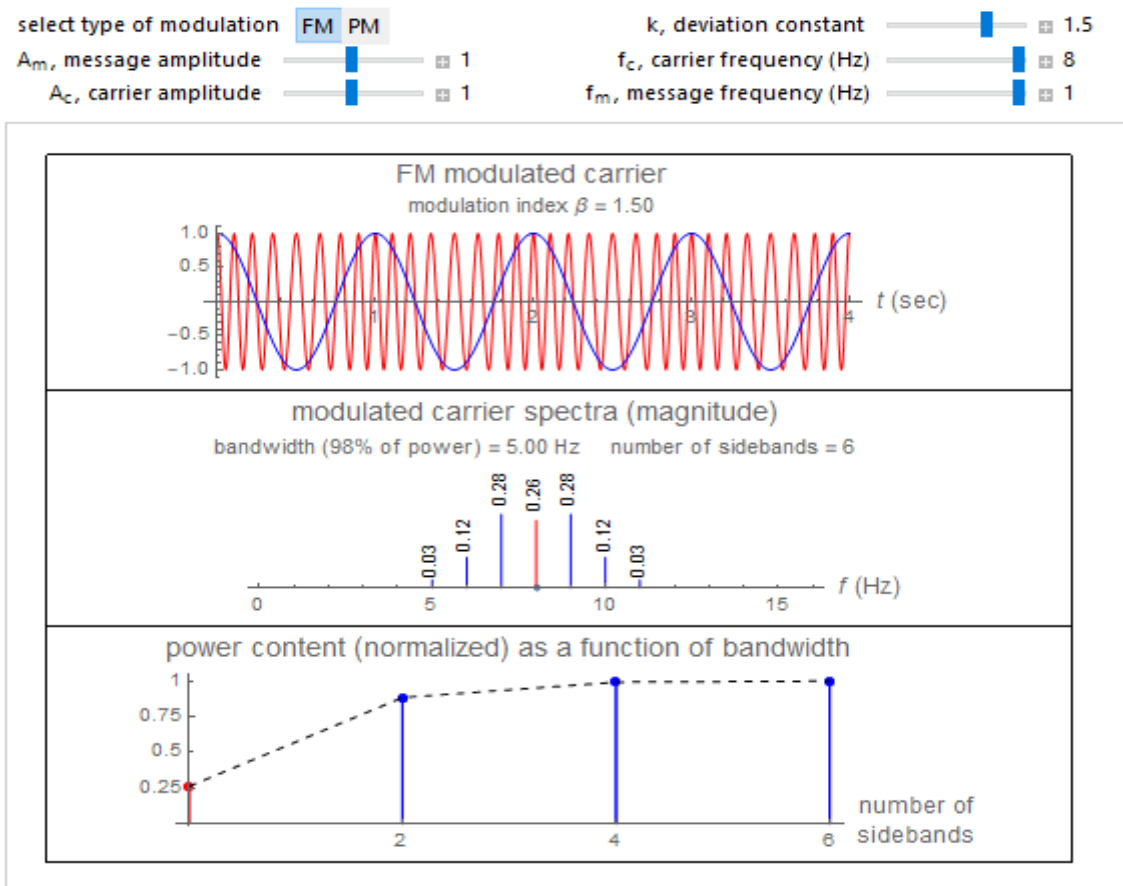
Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{1\text{Hz}}{1\text{Hz}}$$

$$m_f = 1$$

4. Check the effect of increasing K in
 - a. Modulation index
 - b. Signal in time domain and frequency domain
 - c. Explain your observation



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 1Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 1.5Hz Signal in both time domain and frequency is shown.

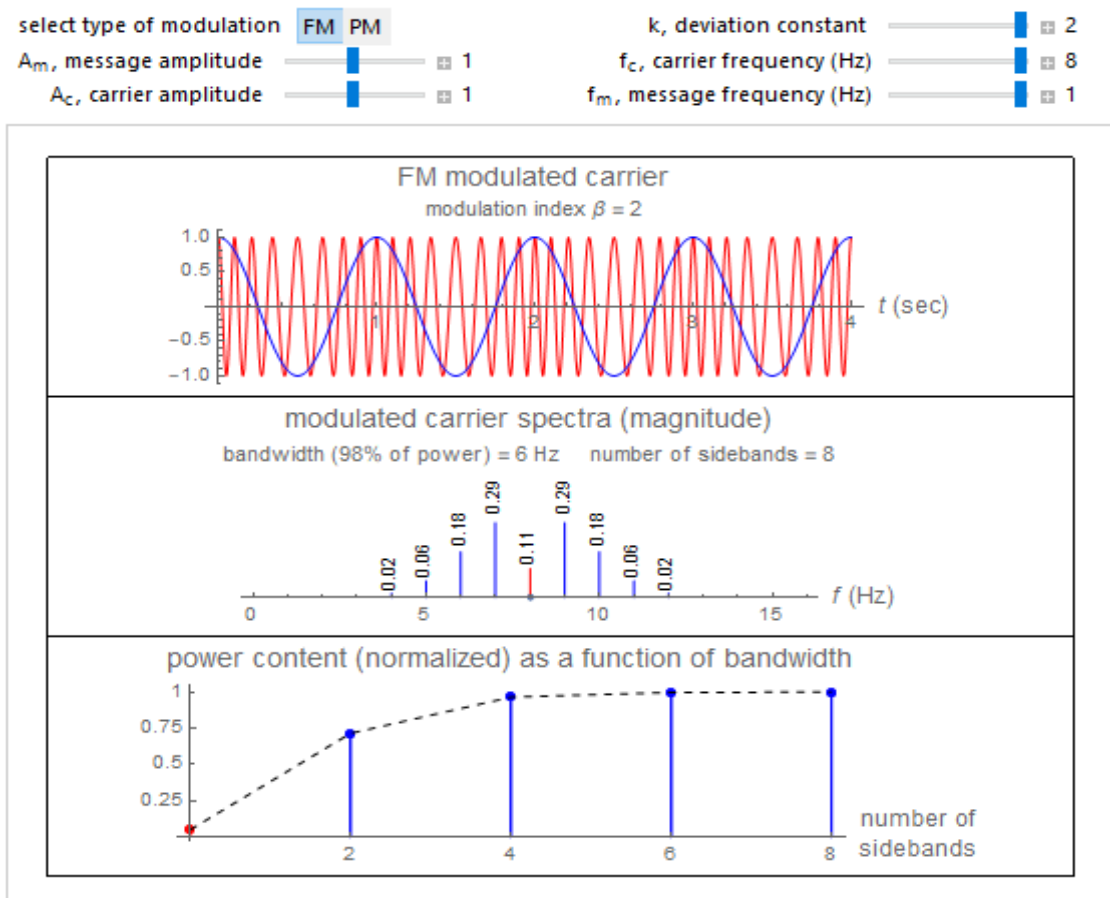
Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{1.5\text{Hz}}{1\text{Hz}}$$

$$m_f = 1.5$$



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 1Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 2Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{2\text{Hz}}{1\text{Hz}}$$

$$m_f = 2$$

Explanation:

From the two screenshots above, it is noticeable that with a higher frequency deviation (K), the number of significant sidebands increases in the frequency domain. In the time domain, the difference between the two screenshots and the impact of a higher frequency deviation is not noticeable. They look pretty much the same.

5. Set K again to 1
- a. What is modulation index?

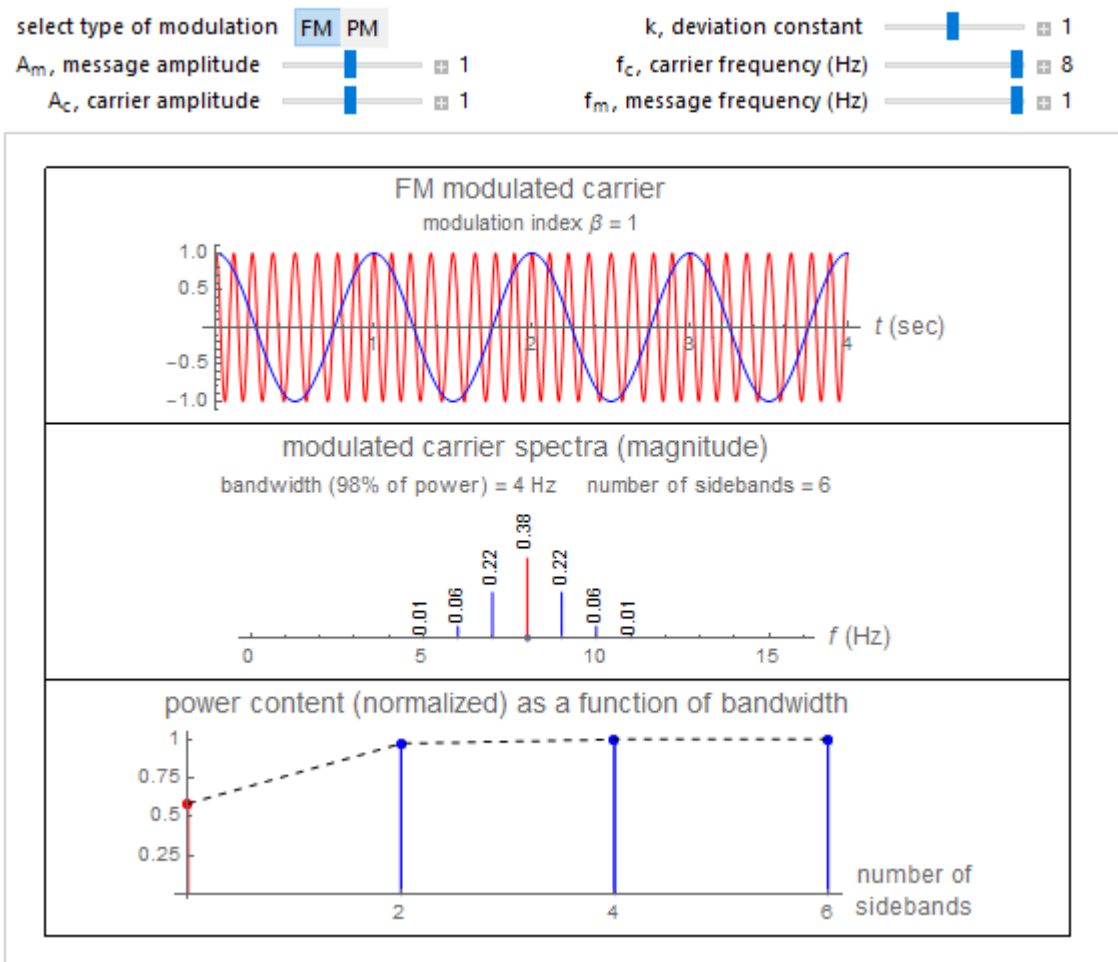
Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{1\text{Hz}}{1\text{Hz}}$$

$$m_f = 1$$

- b. Take a screenshot from signal in time domain and frequency domain.



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 1Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 1Hz. Signal in both time domain and frequency is shown.

- c. By looking to spectrum try to find the FM signal bandwidth and compare with the bandwidth given by simulation tool.

$$BW = 2 * f_m * N$$

$$BW = 2 * 1\text{Hz} * 3$$

$$BW_{(General)} = 6\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 1 (calculated above in 5a), there are three significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 4Hz) as shown in the above screenshot.

- d. Use the Carson Law to estimate the bandwidth and compare with result of section 5.c

Note: $f_d = k$

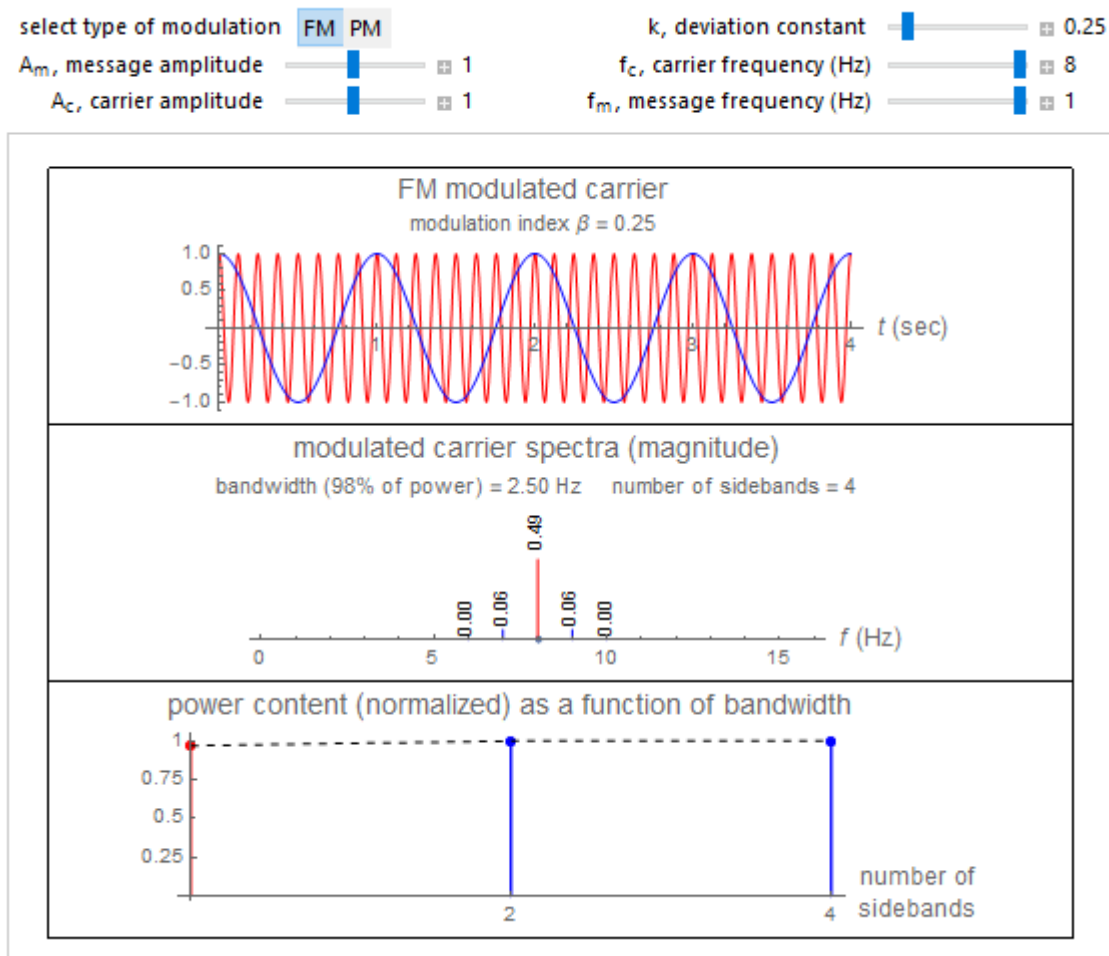
$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[1\text{Hz} + 1\text{Hz}]$$

$$BW_{(Carson)} = 4\text{Hz}$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated in 5c (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.

6. Try Number 5 (above) for $K = 0.25$ and 2



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 1Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 0.25Hz Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{0.25\text{Hz}}{1\text{Hz}}$$

$$m_f = 0.25$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 1\text{Hz} * 1$$

$$BW_{(General)} = 2\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 0.25 (calculated above), there is only one significant sideband (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly lower than what is shown in the simulation tool (BW in simulation tool = 2.5Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

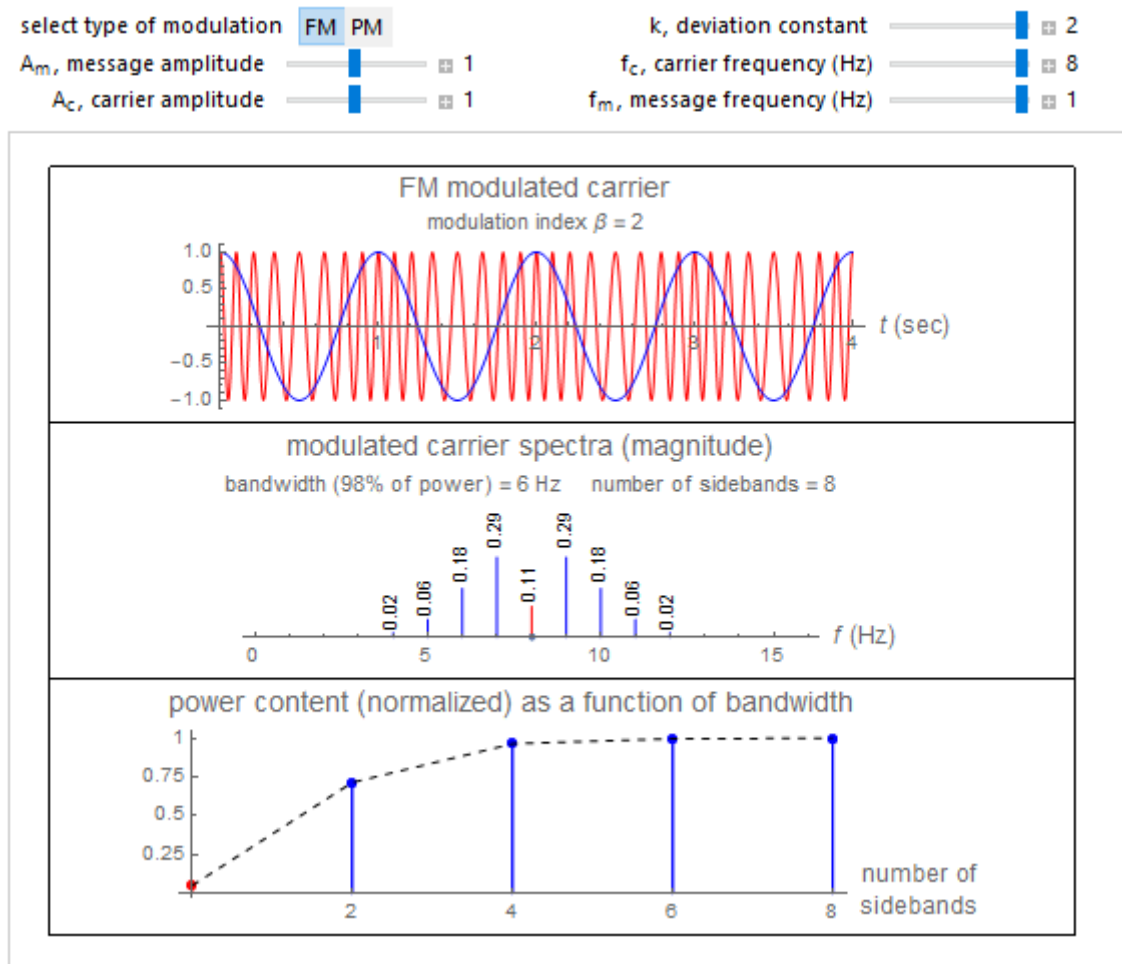
Note: $f_d = k$

$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[0.25\text{Hz} + 1\text{Hz}]$$

$$BW_{(Carson)} = 2.5\text{Hz}$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 1Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 2Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{2\text{Hz}}{1\text{Hz}}$$

$$m_f = 2$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 1\text{Hz} * 4$$

$$BW_{(General)} = 8\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 2 (calculated above), there are four significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 6Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

Note: $f_d = k$

$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[2\text{Hz} + 1\text{Hz}]$$

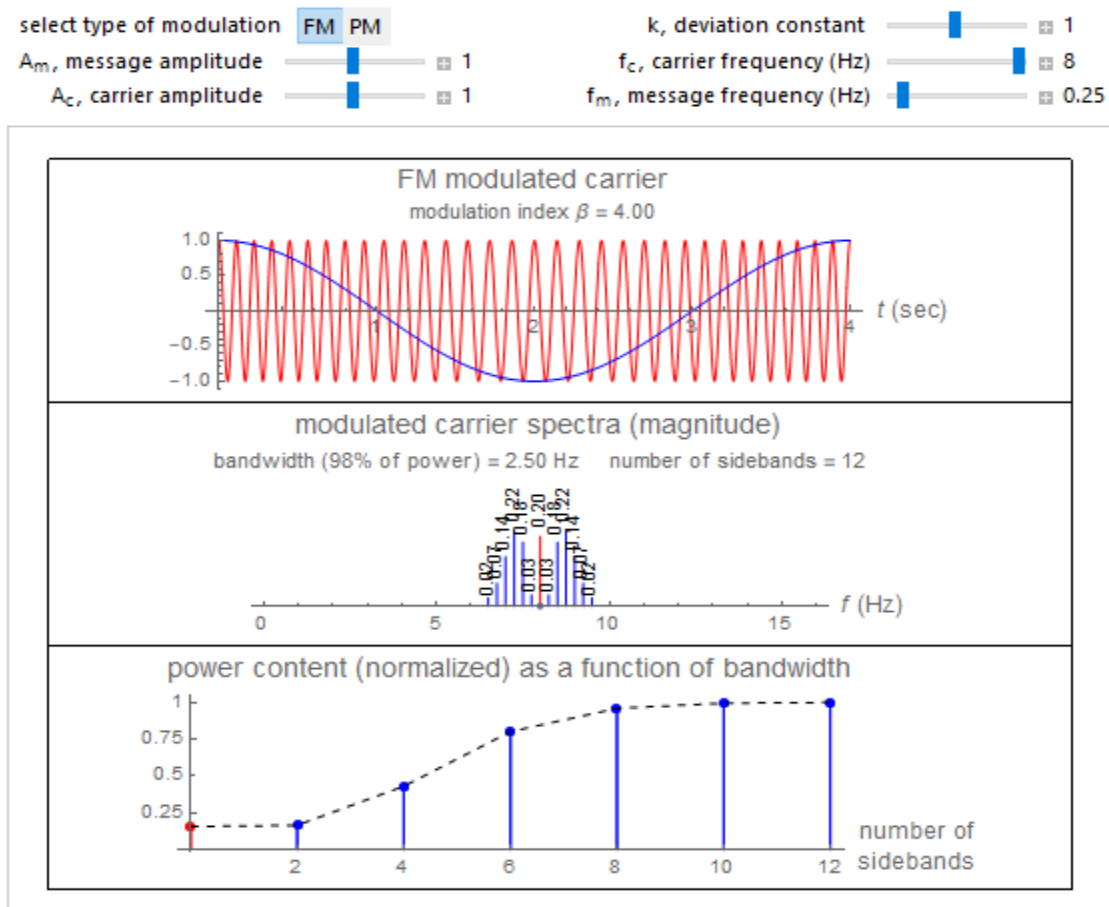
$$BW_{(Carson)} = 6\text{Hz}$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.

7. Try Number 5 and 6 (above) for $f_m = 0.25\text{Hz}$ and 0.5Hz and explain the effect of frequency of modulating signal in time and frequency domain.

the following three screenshots are for $f_m = 0.25\text{Hz}$

$f_m = 0.25\text{Hz}$ (I)



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 0.25Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 1Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{1\text{Hz}}{0.25\text{Hz}}$$

$$m_f = 4$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 0.25\text{Hz} * 7$$

$$BW_{(General)} = 3.5\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 4 (calculated above), there are seven significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 2.5Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

Note: $f_d = k$

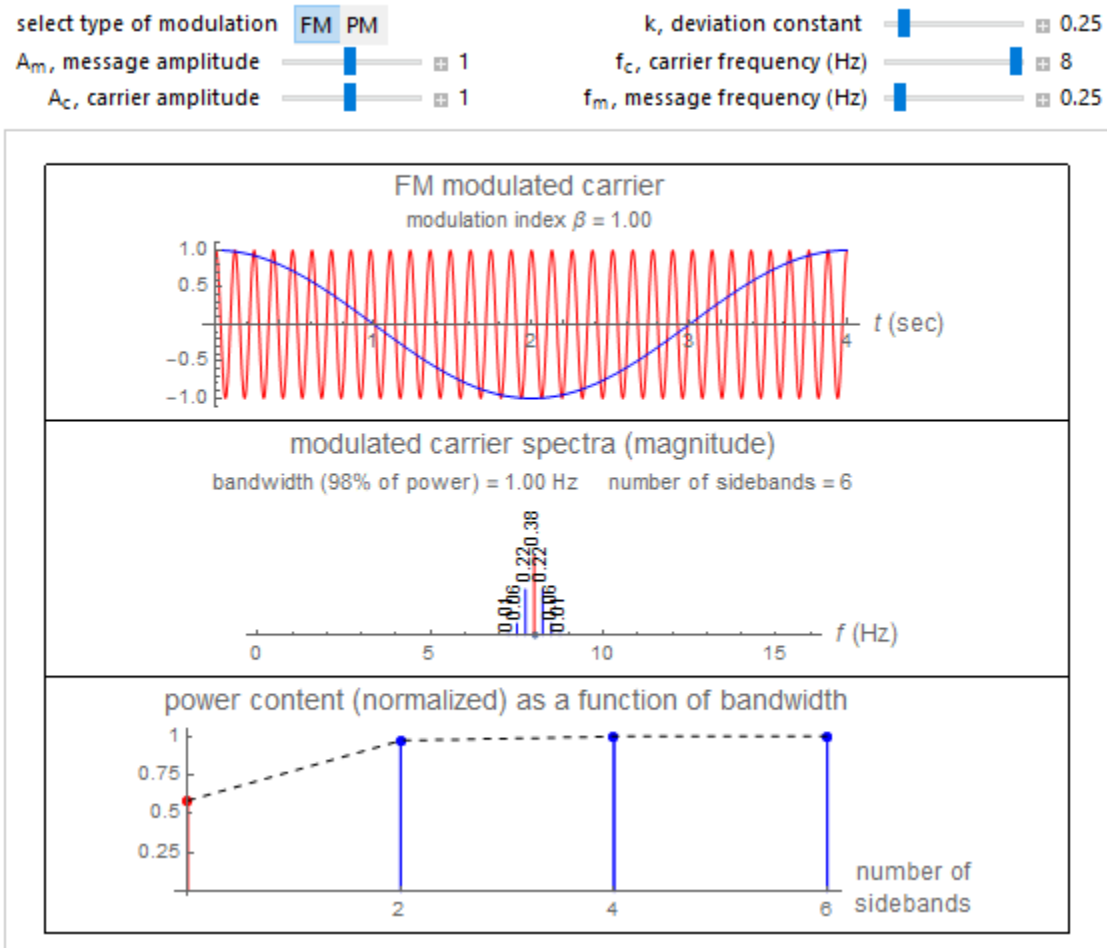
$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[1\text{Hz} + 0.25\text{Hz}]$$

$$BW_{(Carson)} = 2.5\text{Hz}$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.

$$f_m = 0.25\text{Hz (11)}$$



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 0.25Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 0.25Hz Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{0.25\text{Hz}}{0.25\text{Hz}}$$

$$m_f = 1$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 0.25\text{Hz} * 3$$

$$BW_{(General)} = 1.5\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 1 (calculated above), there are three significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 1Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

Note: $f_d = k$

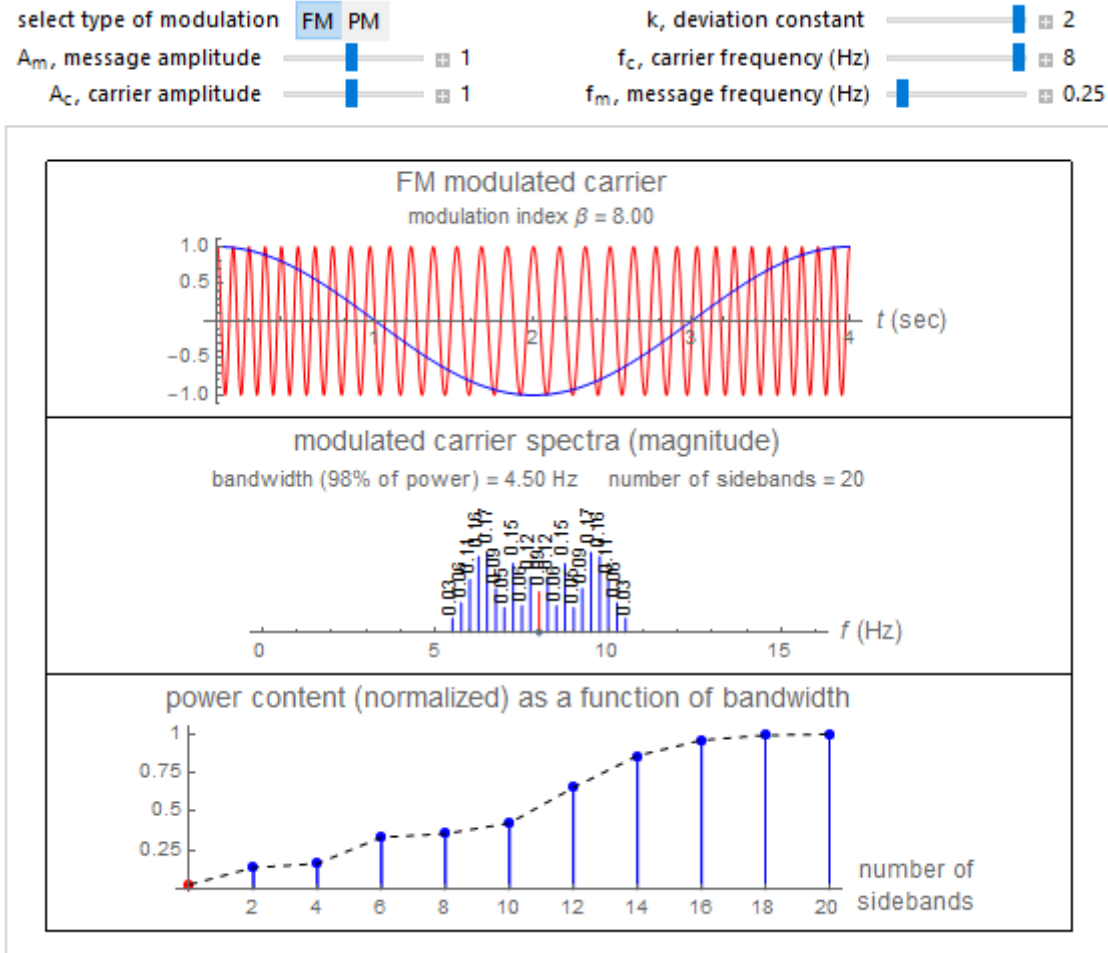
$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[0.25\text{Hz} + 0.25\text{Hz}]$$

$$BW_{(Carson)} = 1\text{Hz}$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.

$$f_m = 0.25\text{Hz (III)}$$



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 0.25Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 2Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{2\text{Hz}}{0.25\text{Hz}}$$

$$m_f = 8$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 0.25\text{Hz} * 11$$

$$BW_{(General)} = 5.5\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 8 (calculated above), there are eleven significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 4.5Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

Note: $f_d = k$

$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[2\text{Hz} + 0.25\text{Hz}]$$

$$BW_{(Carson)} = 4.5\text{Hz}$$

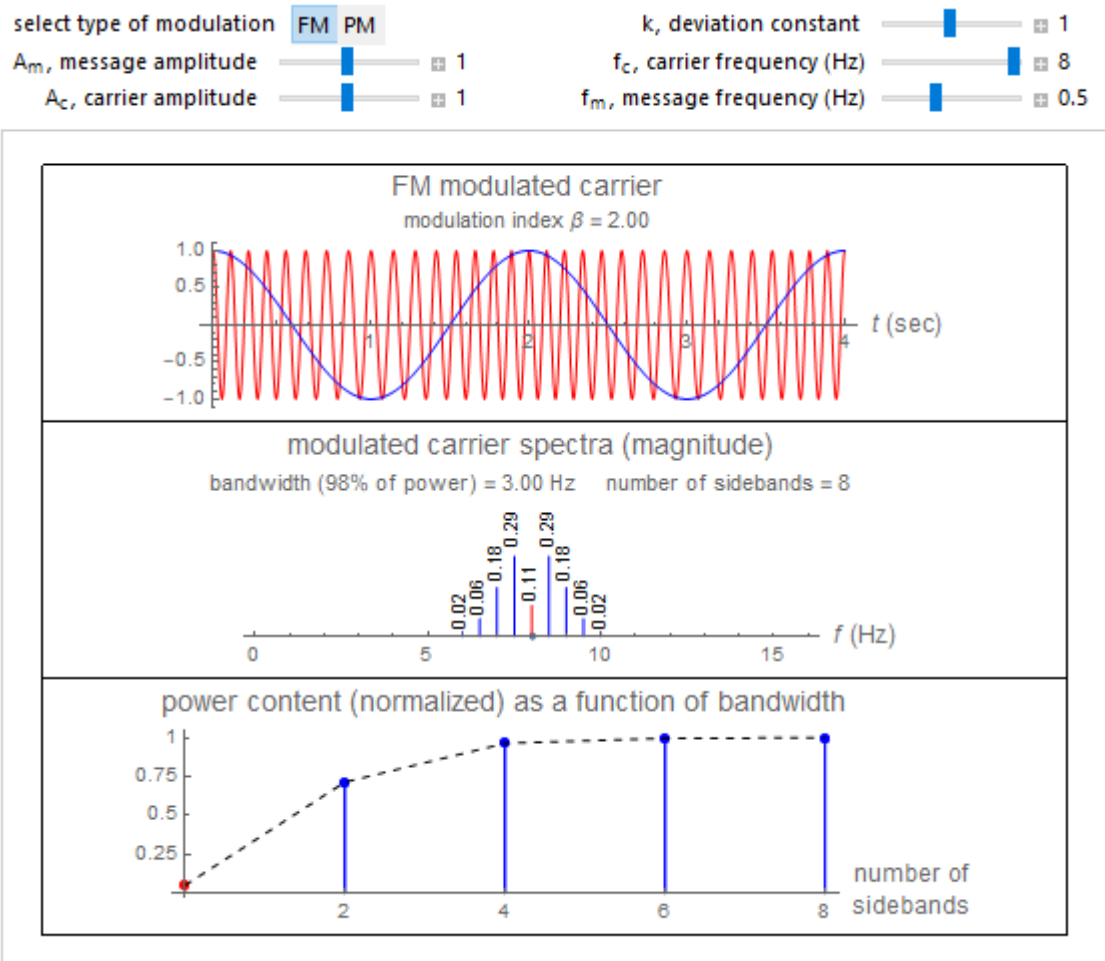
This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.

Explanation:

From the three screenshots above, the effect of the value of the frequency deviation on the modulating frequency is clearly shown in the frequency domain. In the frequency domain, as the frequency deviation increases, there are more significant sidebands shown. In the time domain, there isn't any noticeable changes between the three screenshots as the value of the frequency deviation changes and the modulation frequency stays the same throughout.

the following three screenshots are for $f_m = 0.5\text{Hz}$

$$f_m = 0.5\text{Hz (I)}$$



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 0.5Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 1Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{1\text{Hz}}{0.5\text{Hz}}$$

$$m_f = 2$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 0.5Hz * 4$$

$$BW_{(General)} = 4Hz$$

**N is the number of significant sidebands, and for a modulation index of 2 (calculated above), there are four significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 3Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

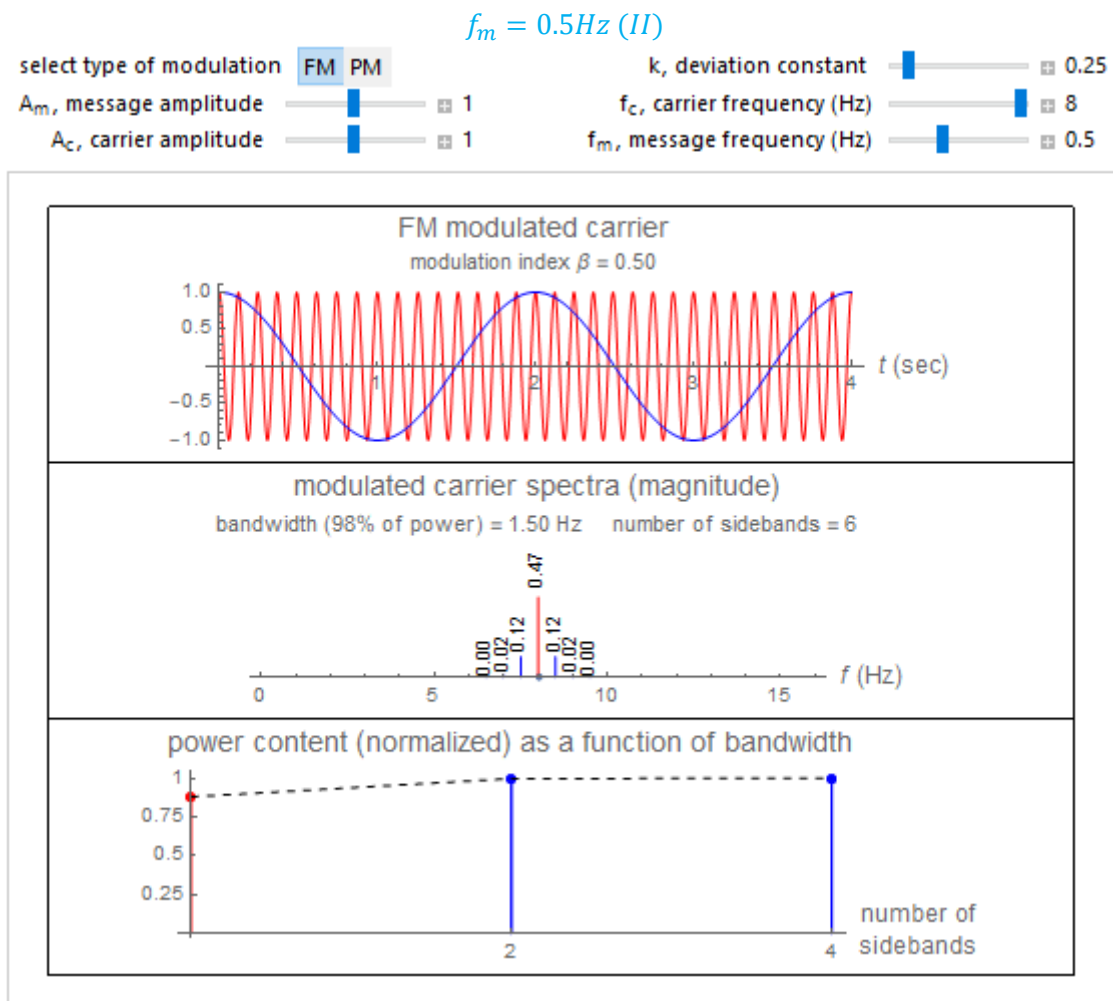
Note: $f_d = k$

$$BW = 2[f_{d(max)} + f_{m(max)}]$$

$$BW = 2[1Hz + 0.5Hz]$$

$$BW_{(Carson)} = 3Hz$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 0.5Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 0.25Hz Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{0.25\text{Hz}}{0.5\text{Hz}}$$

$$m_f = 0.5$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 0.5Hz * 2$$

$$BW_{(General)} = 2Hz$$

**N is the number of significant sidebands, and for a modulation index of 0.5 (calculated above), there are two significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 1.5Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

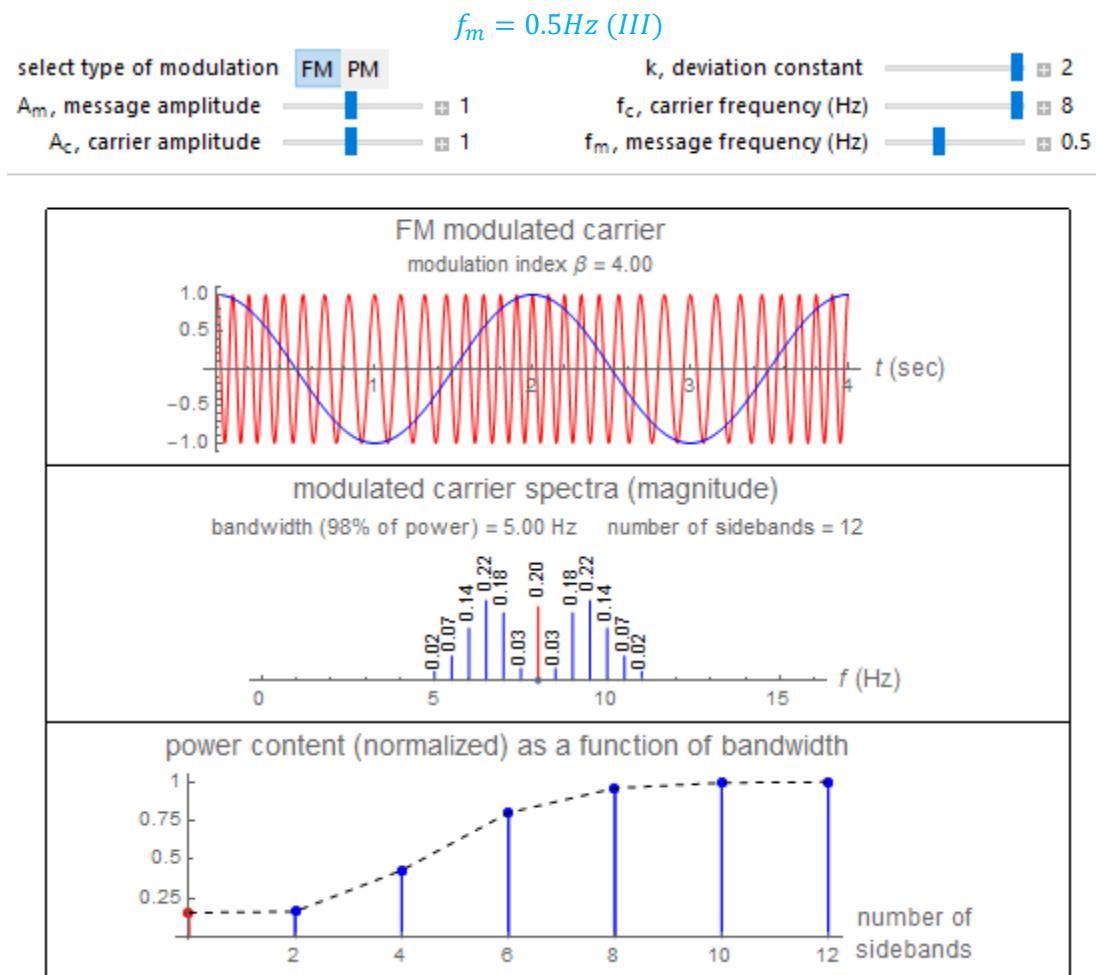
Note: $f_d = k$

$$BW = 2[f_{d(max)} + f_{m(max)}]$$

$$BW = 2[0.25Hz + 0.5Hz]$$

$$BW_{(Carson)} = 1.5Hz$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.



Above screenshot for when the carrier frequency is 8Hz and modulation frequency is 0.5Hz. Carrier amplitude and modulation amplitude is 1V. K is set to 2Hz. Signal in both time domain and frequency is shown.

Calculated modulation index:

Note: $f_d = k$

$$m_f = \frac{f_d}{f_m}$$

$$m_f = \frac{2\text{Hz}}{0.5\text{Hz}}$$

$$m_f = 4$$

Calculating bandwidth of signal using general rule:

$$BW = 2 * f_m * N$$

$$BW = 2 * 0.5\text{Hz} * 7$$

$$BW_{(General)} = 7\text{Hz}$$

**N is the number of significant sidebands, and for a modulation index of 4 (calculated above), there are seven significant sidebands (according to the Bessel function table).*

This calculated bandwidth does not match exactly to what the simulation tool gave as a result. This calculated bandwidth is slightly higher than what is shown in the simulation tool (BW in simulation tool = 5Hz) as shown in the above screenshot.

Calculating bandwidth of signal using Carson's rule:

Note: $f_d = k$

$$BW = 2[f_{d(\max)} + f_{m(\max)}]$$

$$BW = 2[2\text{Hz} + 0.5\text{Hz}]$$

$$BW_{(Carson)} = 5\text{Hz}$$

This calculated bandwidth matches exactly to what the simulation tool gave as a result and does not quite match to what was calculated above (using the general rule to calculate the bandwidth of the signal). It seems that the simulation tool is using Carson's rule to determine the bandwidth of the signal.

Explanation:

From the three screenshots above, the effect of the value of the frequency deviation on the modulating frequency is clearly shown in the frequency domain. In the frequency domain, as the frequency deviation increases, there are more significant sidebands shown. In the time domain, there isn't any noticeable changes between the three screenshots as the value of the frequency deviation changes and the modulation frequency stays the same throughout.

Part 2

1. Round the modulation indexes that you found in part 1 to the closest one in the Bessel function table. Calculate the Bandwidth for each section of your test.
2. Compare the result with simulation result.

Question	f_d (Hz)	f_m (Hz)	Calculated modulation index (m_f)	Number of significant sidebands (N)	Calculated bandwidth (BW in Hz - General rule)	Calculated bandwidth (BW in Hz - Carson's rule)	Simulation result (BW in Hz)
3	1	1	1	3	6	4	4
4 (I)	1.5	1	1.5	4	8	5	5
4 (II)	2	1	2	4	8	6	6
5	1	1	1	3	6	4	4
6 (I)	0.25	1	0.25	1	2	2.5	2.5
6 (II)	2	1	2	4	8	6	6
7: $f_m = 0.25\text{Hz}$ (I)	1	0.25	4	7	3.5	2.5	2.5
7: $f_m = 0.25\text{Hz}$ (II)	0.25	0.25	1	3	1.5	1	1
7: $f_m = 0.25\text{Hz}$ (III)	2	0.25	8	11	5.5	4.5	4.5
7: $f_m = 0.5\text{Hz}$ (I)	1	0.5	2	4	4	3	3
7: $f_m = 0.5\text{Hz}$ (II)	0.25	0.5	0.5	2	2	1.5	1.5
7: $f_m = 0.5\text{Hz}$ (III)	2	0.5	4	7	7	5	5

Table for question 1 and 2 shown above. Calculated bandwidth (using Carson's rule) is exactly the same bandwidth shown in the simulation tool. Attached to this report submission is the MS Excel spreadsheet for this table.