Statistic Stuffs

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I will drop here some statistic concepts I liked during the course I had at University of Milan.

1 Gini index

There are two types of Gini indexes. One has the purpose of telling how much a sample is heterogeneous. The other one is used for concentration. We will talk about the first one we have mentioned and is defined as follow:

$$I = 1 - \sum_{j=1}^{k} f_j^2$$

This index is always less than 1 and always more than 0.

$$0 \leq I < 1$$

So we are subtracting the squared relative frequencies from one. we know that:

$$\forall j f_j \geq 0 \longrightarrow \sum_{j=1}^{k} f_j = 1$$

.

This implies that $\exists \acute{j} : \acute{f_j} > 0 \longrightarrow f_{\acute{j}}^2 > 0$.
So:

$$\sum_{j=1}^{k} f_j^2 > 0 \longrightarrow I < 1$$

as said in before.

We can consider also that:

$$\sum_{j=1}^{k} f_j^2 \leq \sum_{j=1}^{k} f_j$$

.

This implies that i

$$1 - \sum_{j=1}^{k} f_j^2 \geq 0$$

.

We have just demonstrated what we said a few moments ago. $0 \leq I < 1$.

2 Probability calculus

Before we can talk about probability, we have to define a probability function. So we need to define as first Ω as the set of all possibles outcomes. Then we can say what is an event, which is just a subset of Ω. Now let's define what is an Event algebra:

$$\mathcal{A} = E_i \subseteq \Omega$$

E_i are all the subsets events in Ω. This algebra has to respect three rules:

1. $\Omega \in \mathcal{A}$
2. $\forall E \subseteq \Omega \longrightarrow E \in \Omega \rightarrow \bar{E} \in \Omega$
3. $\forall E, F \subseteq \Omega \longrightarrow E \in \mathcal{A}, F \in \mathcal{A} \rightarrow E \cup F \in \mathcal{A}$

The third rule can be extended to this:

$$\forall i = 1, 2, .., n E_i \subseteq \Omega, E_i \in \mathcal{A} \rightarrow \bigcup_{i=1}^{n} E_i \in \mathcal{A}$$

Finally we can define the probability function.

$P : \mathcal{A} \rightarrow R$

2.1 Kolmogorov axioms

Kolmogorov axioms are the fundamental rules of the probability calculus.

A1. $\forall E \in \mathcal{A} \ \ 0 \leq P(E) \leq 1$

A2. $P(\Omega) = 1$

A3. $\forall E_1 ... E_n \ \ \forall i \ E_i \in \mathcal{A}$ mutually exclusive $\rightarrow P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$