UNIT 4 CONTROL CHARTS FOR DEFECTS

Structure

4.1 INTRODUCTION

In Unit 2, you have studied the $\overline{X}$, R and S control charts used for monitoring and controlling measurable quality characteristics such as diameter, length, width, weight, etc. In Unit 3, you have studied the p and np control charts applied for monitoring and controlling quality characteristics which are not measurable and the items or units are classified as defective or non-defective. These types of charts are broadly useful in statistical quality control.

Sometimes, an item may contain many defects, but it may not be classified as defective. For example, suppose a company manufactures, motorcycles. Suppose there are one or two minor scratches on the body of the motorcycle, which do not affect its functioning. So the motorcycle cannot be classified as defective. However, this is a defect. But if there are too many scratches, the motorcycle should be classified as defective because the scratches would be noticeable and affect its sale. So there are so many situations in industries in which we require to control the number of defects rather than the number of defective items. The c-chart and u-chart are used to control the number of defects.

In Secs. 4.2 and 4.3, we discuss the c-chart which is used to control the number of defects and its applications. We describe the u-chart in Sec 4.4. It is used to monitor and control the number of defects per unit when the sample size varies. In Sec 4.5, we compare the control charts for variables and attributes.

Objectives

After studying this unit, you should be able to:

- distinguish between defective and defect;
- describe the need of control charts for defects;
- select the appropriate control chart for defects;
- construct and interpret the control chart for number of defects (c-chart); and
- construct and interpret the control chart for number of defects per unit (u-chart).

4.2 CONTROL CHARTS FOR NUMBER OF DEFECTS (c-CHART)

You have learnt about the p-chart and np-chart in Unit 3. These control charts are used to control the fraction (proportion) defective and the number of defectives, respectively, on the basis of criterion: whether the item is defective or non-defective. But in many situations, it is advantageous economically to know the **number of defects** in an item instead of classifying it as defective or non-defective. For example, a glass bottle may have one small air bubble and the another glass bottle may have many small air bubbles. Even though both bottle sets are defective, the second one is far more serious as it has many defects. So it is also important to control the number of defects within the item. The c-chart is used for monitoring and controlling the number of defects in an item/unit. Traditionally, the number of defects was denoted by c. Therefore, the control chart for number of defects is known as the c-chart. It is used when the item/unit is largely assembled, e.g., TV, computer, mobile, laptop, aircraft, etc. In such cases, there are many opportunities for a defect to occur and the probability of the occurrence of a defect is very small. For example, in the case of commercial airplane there is a large number of rivets, but a small chance of any rivet having defects.

The sample/subgroup size for the c-chart is a single item/unit/article or constant size and we count the number of defects within the item/unit/article.

The underlying logic and procedure of constructing the c-chart are similar to the np-chart. The primary difference between the c-chart and the np-chart is that, instead of collecting data relating to the number of defective items, we collect data about the number of defects within the item and plot the number of defects against the sample number.

For obtaining the centre line and control limits of the c-chart, we require the sampling distribution of the number of defects. In this case, when there are many opportunities for a defect to occur and the probability of the occurrence of a defect is very small, the Poisson distribution is applied (recall Unit 10 of MST-003 entitled Poisson Distribution). So the number of defects follows the Poisson distribution. If λ is the average number of defects in the process, the mean and variance of the Poisson distribution are given as follows:

$$E(c) = \lambda \text{ and } Var(c) = \lambda \qquad \ldots (1)$$

We know the standard error of a random variable X is

$$SE(X) = \sqrt{Var(X)}$$

$$\therefore \quad SE(c) = \sqrt{Var(c)} = \sqrt{\lambda} \qquad \ldots (2)$$

In Unit 10 of MST-003, you have also learnt that the Poisson distribution is not symmetrical. Therefore, the upper and lower 3σ limits do not correspond to equal probabilities of a point on the control chart. But if the sample size is sufficiently large, then by the centre limit theorem, the sampling distribution of the number of defects is approximately normally distributed with mean λ and variance λ. Therefore, the centre line and control limits for the c-chart can be obtained as follows:

Centre line $(\text{CL}) = E(c) = \lambda$ … (3a)

Upper control limit $(\text{UCL}) = E(c) + 3SE(c) = \lambda + 3\sqrt{\lambda}$ … (3b)

Lower control limit $(\text{LCL}) = E(c) - 3SE(c) = \lambda - 3\sqrt{\lambda}$ … (3c)

Generally, λ is not known. So it is estimated from the sample information. Suppose we draw k samples of constant size or size 1 and $c_1, c_2, ..., c_k$ are the numbers of defects in the 1st, 2nd, …, kth sample, respectively. The average number of defects in the process is estimated by the average number of defects in the sample which is calculated by the formula given below:

$$\bar{c} = \frac{\text{Sum of defects}}{\text{Total number of samples inspected}} = \frac{1}{k}\sum_{i=1}^{k} c_i \quad \text{… (4)}$$

In this case, centre line and control limits of the c-chart are obtained by replacing λ by $\bar{c}$ in equations (3a to 3c) as follows:

Centre line $(\text{CL}) = \hat{\lambda} = \bar{c}$ … (5a)

Upper control limit (UCL) $= \hat{\lambda} + 3\sqrt{\hat{\lambda}} = \bar{c} + 3\sqrt{\bar{c}}$ … (5b)

Lower control limit (UCL) $= \hat{\lambda} - 3\sqrt{\hat{\lambda}} = \bar{c} - 3\sqrt{\bar{c}}$ … (5c)

We construct the c-chart by taking the sample number on the X-axis and the number of defects in the sample on the Y-axis. We draw the contre line as a solid line and control limits as dotted lines on the chart. We plot the number of defects in the sample against the sample number. The consecutive sample points are joined by line segments.

Interpretation of the result

If all sample points lie on or between the upper and lower control limits, the control chart indicates that the process is under statistical control. Only chance causes are present in the process and no assignable cause is present. However, if one or more sample points lie outside the upper or lower control limit, the control chart alarms (indicates) that the process is not under statistical control and some assignable causes are present in the process.

To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once the assignable causes are eliminated, we delete the out-of-control points (samples) and calculate the revised centre line and control limits for the c-chart by using the remaining samples. These limits are known as the **revised control limits**. For the revised limits of the c-chart, we first calculate new $\bar{c}$ as follows:

$$\bar{c}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{k - d} \quad \text{… (6)}$$

where d – the number of discarded samples, and

$\sum_{j=1}^{d} c_j$ – the sum of the number of defects within the discarded samples.

After finding the $\overline{c}_{new}$, we reconstruct the centre line and control limits for the c-chart by replacing $\overline{c}$ by $\overline{c}_{new}$ in equations (5a to 5c) as follows:

Centre line (CL) $= \overline{c}_{new}$	… (7a)
Upper control limit (UCL) $= \overline{c}_{new} + 3\sqrt{\overline{c}_{new}}$	… (7b)
Lower control limit (UCL) $= \overline{c}_{new} - 3\sqrt{\overline{c}_{new}}$	… (7c)

Note: If the value of lower control limit is negative, we change the lower control limit to zero because a negative number of defects is not possible.

Let us take up some examples to illustrate how to construct the c-chart for practical situations.

Example 1: A control chart is to be formed for a process in which laptops are produced. The inspection unit is one laptop and control chart for the number of defects is to be used. Preliminary data are recorded and 45 defects are found in 30 laptops. Obtain the control limits for the chart.

Solution: Since we need to control the number of defects and the inspection unit is one laptop, we use the c-chart for number of defects.

The average number of defects in the process is not given. So, we use equations (5a to 5c) to calculate the centre line and control limits of the c-chart.

It is given that

the total number of defects in laptops = 45, and

the total number of laptops inspected (k) = 30.

$$\therefore \quad \overline{c} = \frac{1}{k}\sum_{i=1}^{k} c_i = \frac{1}{30} \times 45 = 1.5$$

Hence, using equations (5a to 5c), we calculate the centre line and control limits of the c-chart as follows:

$$CL = \overline{c} = 1.5$$

$$UCL = \overline{c} + 3\sqrt{\overline{c}} = 1.5 + 3 \times \sqrt{1.5} = 1.5 + 3 \times 1.225 = 5.175$$

$$LCL = \overline{c} - 3\sqrt{\overline{c}} = 1.5 - 3 \times \sqrt{1.5} = 1.5 - 3 \times 1.225 = -2.175 \approx 0$$

Example 2: The number of scratch marks on a particular piece of furniture is recorded. The data for 20 samples are given below:

Sample Number	1	2	3	4	5	6	7	8	9	10
Scratch Mark	6	3	14	7	2	5	12	4	7	3
Sample Number	11	12	13	14	15	16	17	18	19	20
Scratch Mark	2	7	6	8	4	10	5	4	13	9

Draw the appropriate control chart and write the comments about the state of the process when:

i) the management sets a goal of 5 scratch marks on an average per piece.

ii) the management does not set the average number of marks per piece.

Solution: Here the number of defects on a particular piece of furniture is given. So we use the c-chart.

i) The management sets a goal of 5 scratch marks on an average per piece. It means that the average number of defects in the process is known. Therefore, we use equations (3a to 3c) to calculate the centre line and control limits of the c-chart.

It is given that

the average number of defects per sample (λ) = 5, and

the number of items in the sample (n) = 1.

Therefore, we calculate the centre line and control limits of the c-chart using equations (3a to 3c) as follows:

$$\mathrm{CL} = \lambda = 5$$

$$\mathrm{UCL} = \lambda + 3\sqrt{\lambda} = 5 + 3 \times \sqrt{5} = 5 + 3 \times 2.236 = 11.708$$

$$\mathrm{LCL} = \lambda - 3\sqrt{\lambda} = 5 - 3 \times \sqrt{5} = 5 - 3 \times 2.236 = -1.708 \approx 0$$

For constructing the c-chart, we plot the points by taking the sample number on the X-axis and the number of defects (c) on the Y-axis. We draw the centre line as a solid line and control limits as dotted lines on the graph as shown in Fig. 4.1.

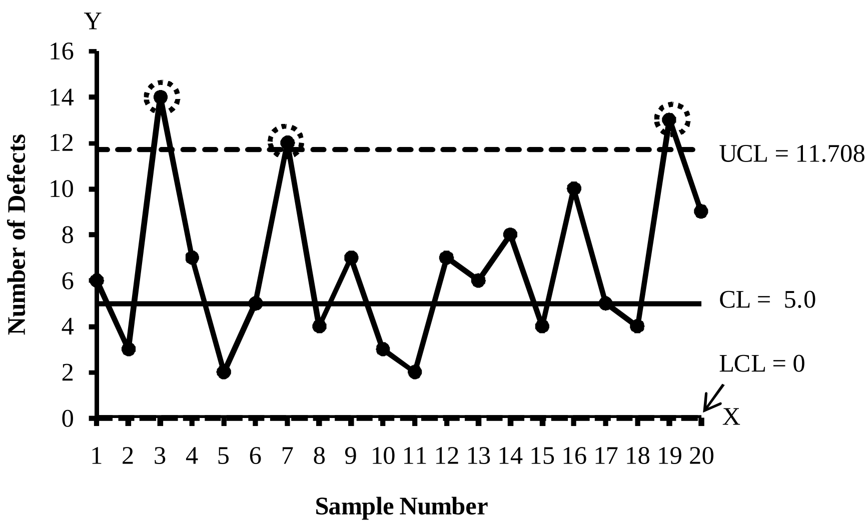

Fig. 4.1: The c-chart for number of defectes on an average per piece of the furniture, which is 5.

Interpretation of the result

From Fig. 4.1, we observe that the points corresponding to the sample numbers 3, 7 and 19 lie outside the upper control limit. Therefore, the process is not under statistical control with respect to the average number of defects per piece, which is 5.

ii) Here, the average number of defects in the process is not given. So we use equations (5a to 5c) to calculate the centre line and control limits of the c-chart.

Here total number of defects = 6 + 3 + … + 13 + 9 = 131.

$$\therefore \quad \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i = \frac{1}{20}\times 131 = 6.55$$

Therefore, using equations (5a to 5c), we calculate the centre line and control limits of the c-chart as follows:

$$CL = \bar{c} = 6.55$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 6.550 + 3\times\sqrt{6.55} = 6.550 + 3\times 2.559 = 14.227$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 6.550 - 3\times\sqrt{6.55} = 6.550 - 3\times 2.559 = -1.127 \approx 0$$

We construct the c-chart by taking the sample number on the X-axis and the number of defects (c) on the Y-axis as shown in Fig. 4.2.

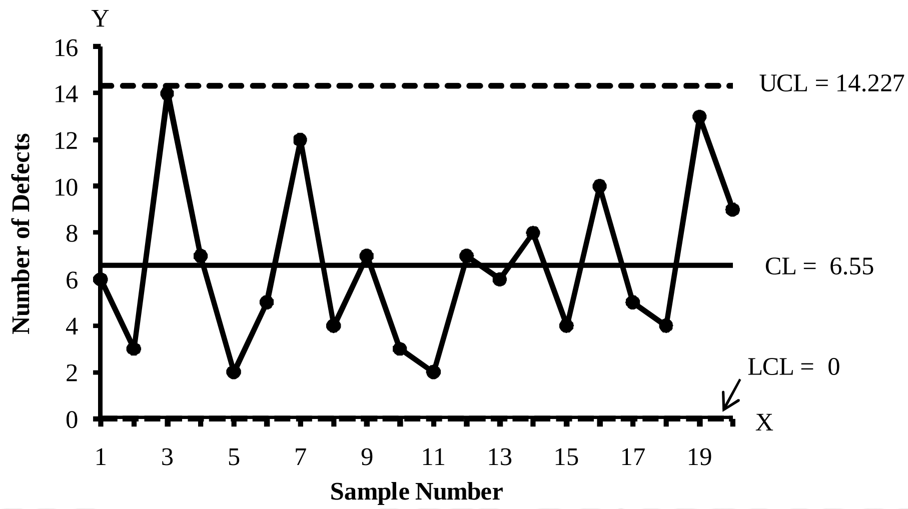

Fig. 4.2: The c-chart for number of defectes on an average per piece of the furniture, which is 6.55.

Interpretation of the result

From the c-chart (shown in Fig. 4.2), we observe that no point lies outside the control limits and there is no specific pattern of the sample points on the chart. So it indicates that the process is under control with respect to the average number of defects per piece, which is 6.55.

Example 3: As part of an overall quality improvement programme, a textile manufacturer decides to monitor the number of defects found in each inspected bolt (large bundle) of cloth. The data from 20 inspections are recorded in the table given below:

Bolt of Cloth	1	2	3	4	5	6	7	8	9	10
Number of Defects	10	19	5	9	2	8	7	13	3	2
Bolt of Cloth	11	12	13	14	15	16	17	18	19	20
Number of Defects	22	4	6	9	7	2	5	12	4	2

i) Which control chart should be used in this case? Calculate the control limits for this chart.

ii) Do these data from a controlled process? If not, calculate the revised control limits.

Solution:

i) Here the manufacturer monitors the number of defects found in each inspected bolt of cloth and only one bolt is inspected in each sample. So we can use the c-chart. The average number of defects in the bolt of cloth is not given in this case. So we use equations (5a to 5c) to calculate the centre line and control limits of the c-chart.

It is given that k = 20 and n = 1.

The total number of defects in bolts = 10 + 19 + … + 2 = 151

$$\therefore \quad \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i = \frac{1}{20} \times 151 = 7.550$$

Hence, using equations (5a to 5c), we calculate the centre line and control limits of the c-chart as follows:

$$CL = \bar{c} = 7.550$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 7.550 + 3 \times \sqrt{7.550} = 7.550 + 3 \times 2.748 = 15.794$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 7.550 - 3 \times \sqrt{7.550} = 7.550 - 3 \times 2.748 = -0.694 \approx 0$$

We construct the c-chart by taking the sample number on the X-axis and the number of defects (c) on the Y-axis as shown in Fig. 4.3.

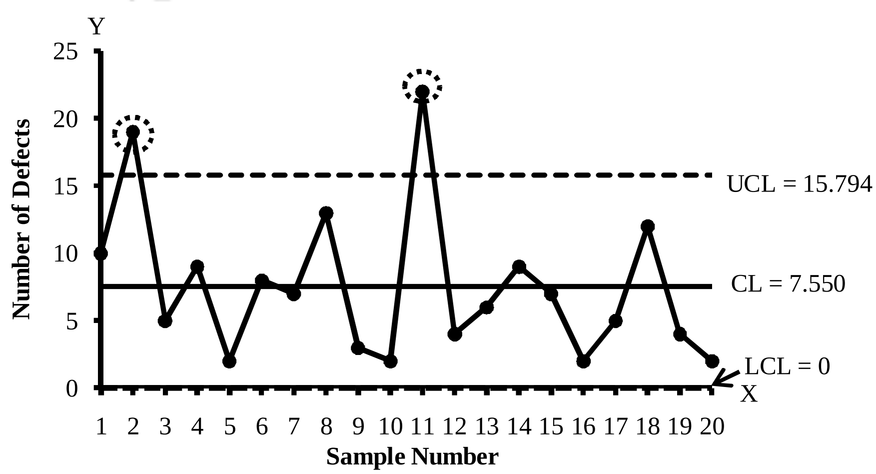

Fig. 4.3: The c-chart for number of defects in the bolt of cloth.

Interpretation of the result

From Fig. 4.3, we observe that the points corresponding to the sample numbers (bolt) 2 and 11 lie outside the upper control limit. Therefore, the process is not under statistical control with respect to the number of defects. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

ii) For calculating the revised control limits for the c-chart, we first delete the out-of-control points (2 and 11) and then calculate the new $\bar{c}$ by using remaining samples.

In our case, $k = 20, d = 2, \sum_{j=1}^{d} c_j = 19 + 22 = 41$

$$\bar{c}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{k-d} = \frac{151-41}{20-2} = 6.111$$

After finding the $\bar{p}_{new}$, we calculate the revised centre line and control limits of the chart using equations (7a to 7c) as follows:

$$CL = \bar{c}_{new} = 6.111$$

$$UCL = \bar{c}_{new} + 3\sqrt{\bar{c}_{new}} = 6.111 + 3 \times \sqrt{6.111} = 6.111 + 3 \times 2.472 = 13.527$$

$$LCL = \bar{c}_{new} - 3\sqrt{\bar{c}_{new}} = 6.111 - 3 \times \sqrt{6.111} = 6.111 - 3 \times 2.472 = -1.305 \approx 0$$

So far you have studied the c-chart and learnt when it is used and how it is constructed. We now discuss the application of the c-chart.

Applications of the c-chart

Although the applications of the c-chart are limited compared to the $\bar{X}$, R, p and np-charts, yet a number of practical situations exist where we apply the c-chart. Some area where the c-chart is applied are listed below:

1. c is the number of defects observed in a television, computer, laptop, mobile, etc.
2. c is the number of defects observed in a woollen carpet, cloth, paper, etc.
3. c is the number of air bubbles in a glass, bottle, paper weight, etc.
4. c is the number of defects observed in aircraft engines.
5. c is the number of breakdowns at weak spots in insulation in a given length of insulated wire.
6. c is the number of imperfections observed in a bolt (large bundle) of cloth.
7. c is the number of surface defects observed in a galvanised sheet, furniture, automobile, etc.
8. The c-chart is also used in
   a) chemical laboratories,
   b) accident statistics both in highway accidents and industrial accidents.

You may like to check your understanding of the c-chart by answering the following exercises.

E1) Choose the correct option from the following:

i) The control chart for number of defects is called the

a) $\bar{X}$-chart b) p-chart c) np-chart d) c-chart

ii) The control limits for the c-chart are

a) $\lambda \pm 3\sqrt{\frac{\lambda}{n}}$ b) $\lambda \pm 3\sqrt{\lambda}$

c) $\lambda \pm \sqrt{\dfrac{\lambda}{n}}$ d) $n\lambda \pm 3\sqrt{n\lambda}$

iii) The control chart for number of defects per unit is called the

a) u-chart b) p-chart c) np-chart d) c-chart

E2) Write the control limits of the c-chart when the average number of defects of the process is not known.

E3) Twenty five samples of packets of match boxes containing 10 match boxes each, were selected at regular intervals. The match boxes in each sample were inspected and the number of defects with regard to the number of broken sticks, paste on the sticks, labeling, etc. were noted in the following table:

Sample Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of Defects	7	3	0	5	8	2	4	5	6	3	8	2	4
Sample Number	14	15	16	17	18	19	20	21	22	23	24	25	
Number of Defects	9	6	7	10	3	8	4	2	10	4	1	3	

Comment on the state of the process.

E4) A quality control technician has recorded the number of defects per 100 square meter on white papers. The results of 25 inspections are shown in the following table:

Sample Number	**Number of Defects**	**Sample Number**	**Number of Defects**	**Sample Number**	**Number of Defects**
1	6	10	6	18	7
2	8	11	10	19	6
3	12	12	9	20	10
4	5	13	6	21	5
5	10	14	8	22	8
6	18	15	20	23	10
7	5	16	5	24	8
8	8	17	9	25	7
9	8				

Construct a control chart for the number of defects. Revise the control limits, assuming special causes for the out-of-control.

4.3 CONTROL CHARTS FOR NUMBER OF DEFECTS PER UNIT (u-CHART)

In Sec. 4.2, you have learnt that the c-chart is used for monitoring and controlling the number of defects. But it is applied only when the sample (subgroup) size is one or constant, i.e., we have inspected the same number of units or items in each sample. To overcome this drawback, we use the u-chart.

When the sample size varies due to some reason, such as machinery, raw material, workers, etc., the u-chart is constructed to monitor the number of defects per unit. The u-chart can also be used when the **sample size is constant**.

The procedure for drawing the u-chart for variable sample size is similar to the c-chart with constant size. The primary difference between the c-chart and u-chart is that instead of plotting the number of defects per sample, we plot the number of defects per item/unit and monitor them.

Since the control limits are function of the sample size (n), these will vary with the sample size. So we should calculate the control limits separately for each sample. In such cases, the control limits are known as **variable control limits**.

Generally, two approaches are used for constructing variable control limits for the u-chart.

First Approach

In this approach, we calculate the control limits for each sample. Suppose $c_1, c_2, ..., c_k$ represent the number of defects in 1^{st}, 2^{nd}, …, k^{th} samples of size $n_1, n_2, ..., n_k$, respectively, we first calculate the number of defects per item for each sample as follows:

$$u_1 = \frac{c_1}{n_1},\ u_2 = \frac{c_2}{n_2}, ..., u_k = \frac{c_k}{n_k} \qquad \text{… (8)}$$

The average number of defects per item is given by

$$\bar{u} = \frac{1}{k}\sum_{i=1}^{n} u_i \quad \text{or} \quad \bar{u} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} n_i} \qquad \text{… (9)}$$

The centre line remains constant for each sample because it is not a function of sample size and is given by

$$CL = \bar{u} \qquad \text{… (10a)}$$

The control limits change due to variable sample size. The control limits for the i^{th} sample are given by

$$UCL_i = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}} \qquad \text{… (10b)}$$

$$LCL_i = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}} \qquad \text{… (10c)}$$

Second Approach

In this approach, we calculate the control limits by using the average sample size. This approach is used only when there is no large variation in the sample sizes and it is expected that the future sample sizes will not differ significantly from the average sample size. Using this approach, we get constant control limits just as we get in the case of constant sample size. We calculate the average sample size as follows:

$$\bar{n} = \frac{n_1 + n_2 + ... + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i \qquad \dots (11)$$

$$CL = \bar{u} \qquad \dots (12a)$$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}} \qquad \dots (12b)$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}} \qquad \dots (12c)$$

Therefore, the centre line and control limits of the u-chart are as follows:

The construction and interpretation of the u-chart are similar to that of the c-chart.

We now consider an example to illustrate the use of the u-chart.

Example 4: Twenty samples of carpets are inspected for varying sample size and the number of defects in each sample is noted in the following table:

Sample Number	Number of Carpet Inspected	Number of Defects	Sample Number	Number of Carpet Inspected	Number of Defects
1	25	12	11	20	9
2	20	5	12	20	12
3	25	7	13	15	14
4	15	7	14	25	6
5	25	10	15	20	7
6	15	4	16	25	12
7	20	6	17	15	5
8	15	2	18	25	6
9	15	4	19	15	8
10	25	10	20	25	4

Construct a suitable control chart for the number of defects per carpet.

Solution: Since the number of defects is given for varying sample size, we use the u-chart.

We solve this example using both approaches.

<u>First Approach</u>

We use equations (10a to 10c) to calculate the centre line and control limits of the i^{th} sample.

To calculate the control limits and draw the u-chart, we first calculate the number of defects per carpet using equation (8) and then $\bar{u}$.

Sample Number	Number of Carpet Inspected (n)	Number of Defects (c)	u = c / n	CL	UCL	LCL
1	25	12	0.480	0.370	0.736	0.004
2	20	5	0.250	0.370	0.778	– 0.038 □ 0
3	25	7	0.280	0.370	0.736	0.004
4	15	7	0.467	0.370	0.841	– 0.101 □ 0
5	25	10	0.400	0.370	0.736	0.004
6	15	4	0.267	0.370	0.841	– 0.101 □ 0
7	20	6	0.300	0.370	0.778	– 0.038 □ 0
8	15	2	0.133	0.370	0.841	– 0.101 □ 0
9	15	4	0.267	0.370	0.841	– 0.101 □ 0
10	25	10	0.400	0.370	0.736	0.004
11	20	9	0.450	0.370	0.778	– 0.038 □ 0
12	20	12	0.600	0.370	0.778	– 0.038 □ 0
13	15	14	0.933	0.370	0.841	– 0.101 □ 0
14	25	6	0.240	0.370	0.736	0.004
15	20	7	0.350	0.370	0.778	– 0.038 □ 0
16	25	12	0.480	0.370	0.736	0.004
17	15	5	0.333	0.370	0.841	– 0.101 □ 0
18	25	6	0.240	0.370	0.736	0.004
19	15	8	0.533	0.370	0.841	– 0.101 □ 0
20	25	4	0.160	0.370	0.736	0.004
Total	405	150				

From the above table and equation (9), we have

$$\bar{u} = \frac{\sum_{i=1}^{k} c_i}{\sum_{i=1}^{k} n_i} = \frac{150}{405} = 0.370$$

Therefore, using equations (10a to 10c), we calculate the centre line and control limits which are shown in the table as follows:

$$CL = \bar{u} = 0.370$$

$$UCL_1 = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_1}} = 0.370 + 3\sqrt{\frac{0.370}{25}}$$

$$= 0.370 + 3 \times 0.122 = 0.736, \text{ and so on}$$

$$LCL_1 = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_1}} = 0.370 - 3\sqrt{\frac{0.370}{25}}$$

$$= 0.370 - 3 \times 0.122 = 0.004, \text{ and so on}$$

We construct the u-chart by taking the sample number on the X-axis and the number of defects per carpet (u) on the Y-axis. We draw the centre line as a solid line and control limits as dotted lines on the graph and plot the points as shown in Fig. 4.4.

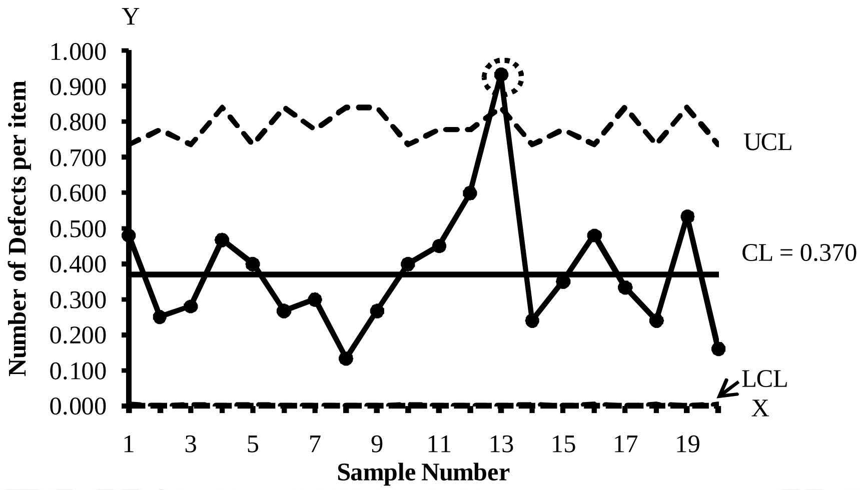

Fig. 4.4: The u-chart for number of defects per carpet.

Interpretation of the result

From the u-chart shown in Fig. 4.4, we observe that the point corresponding to sample 13 lies outside the upper control limit. Therefore, the process is not under statistical control with respect to the number of defects per carpet.

<u>Second Approach</u>

According to this approach, the control limits are calculated using the average sample size and we get constant control limits. We first calculate the average sample size using equation (11) as follows:

$$\bar{n} = \frac{1}{k}\sum_{i=1}^{k} n_i = \frac{1}{20} \times 405 = 20.25$$

Using equations (12a to 12c), we calculate the centre line and control limits as follows:

$$CL = \bar{u} = 0.370$$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}} = 0.370 + 3\sqrt{\frac{0.370}{20.25}} = 0.370 + 3 \times 0.135 = 0.775$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}} = 0.370 - 3\sqrt{\frac{0.370}{20.25}} = 0.370 - 3 \times 0.135 = -0.035 \approx 0$$

We draw the u-chart in the same way as in the first approach (see Fig. 4.5).

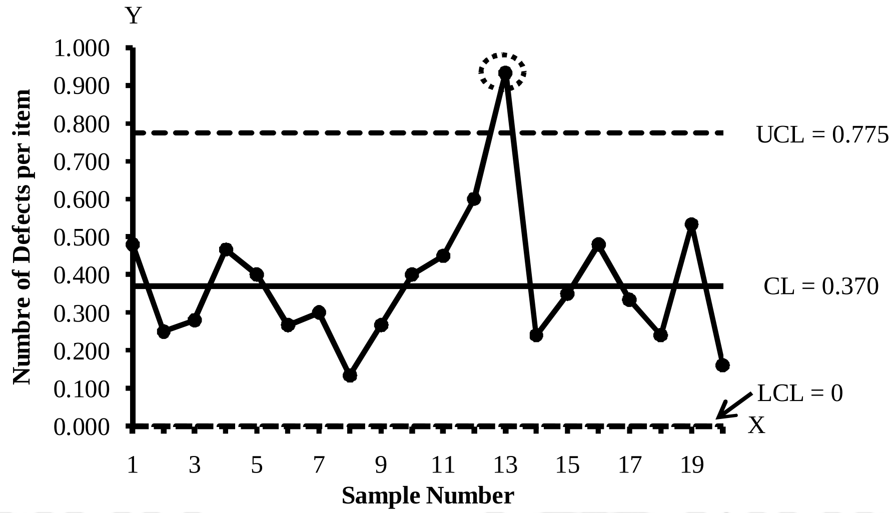

Fig. 4.5: The u-chart for number of defects per carpet.

Interpretation of the result

From the u-chart shown in Fig. 4.5, we observe that the point corresponding to sample 13 lies outside the upper control limit. Therefore, the process is not under statistical control with respect to the number of defects per carpet.

You can check your understanding of the u-chart by answering the following exercises.

E6) A quality control technician notes the number of defects per 100 square meter on white paper, but the amount of paper inspected for each sample varies. The results of 25 inspections are shown in the following table:

Sample Number	Amount of the Paper Inspected (in square meter)	Number of Defects	Sample Number	Amount of the Paper Inspected (in square meter)	Number of Defects
1	300	7	14	250	8
2	200	8	15	300	6
3	250	5	16	250	5
4	150	5	17	150	9
5	250	10	18	200	7
6	100	4	19	150	6
7	200	5	20	300	10
8	150	8	21	200	5
9	150	8	22	250	8
10	250	6	23	200	5
11	200	5	24	100	8
12	250	9	25	150	5
13	100	6			

Construct a control chart for the number of defects per 100 square meter.

4.4 COMPARISON BETWEEN CONTROL CHARTS FOR VARIABLES AND ATTRIBUTES

Now that you have studied the control charts for variables and attributes, you know which type of control chart to use in a given situation. In the table below, we present a comparison of the control charts for variables and attributes.

S. No.	Control Charts for Variables	Control Charts for Attributes
1	These charts are used for measurable characteristics.	These charts are used for non-measurable characteristics and the situations where the control charts for variables are not applied due to time and cost factors.
2	The types of charts are the $\bar{X}$-chart, R-chart and S-chart.	The types of charts are the p-chart, np-chart, c-chart and u-chart.
3	A separate control chart is needed for each quality characteristic.	A single chart is enough for a number of quality characteristics.
4	Small samples serve the purpose very well.	Large samples are required for correct conclusions.
5	The cost of inspection of units is large.	The cost of inspection of units is small since no measurements are made.
6	These charts provide the better quality control.	These charts are not very effective in quality control.
7	These charts are not easily understood.	These charts are easily understood.

With this we end the unit and summarise our discussion.

4.5 SUMMARY

1. The c-chart is used when we want to control the number of defects and sample size is one unit or constant.
2. The u-chart is used when we want to control the number of defects per unit and sample size varies.
3. The centre line and control limits of the c-chart are given as

   Centre line (CL) $= \bar{c}$

   Upper control limit (UCL) $= \bar{c} + 3\sqrt{\bar{c}}$

   Lower control limit (UCL) $= \bar{c} - 3\sqrt{\bar{c}}$

   where $\bar{c} = \dfrac{\text{Sum of defects}}{\text{Total number of samples inspected}} = \dfrac{1}{k}\sum_{i=1}^{k} c_i.$

4. The centre line and control limits of the u-chart are given as

   Centre line (CL) $= \bar{u}$

$$\text{Upper control limit (UCL)} = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$\text{Lower control limit (UCL)} = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$\text{where } \bar{u} = \frac{1}{k}\sum_{i=1}^{n} u_i \text{ or } \bar{u} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} n_i} \text{ and } \bar{n} = \frac{n_1 + n_2 + \ldots + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i$$

4.6 SOLUTIONS / ANSWERS

E1) i) Option (d) is the correct option because we know that the c-chart and u-chart are the control charts for number of defects.

ii) Option (b) is the correct option because we know that the control limits for the c-chart are $= \lambda \pm 3\sqrt{\lambda}$.

iii) Option (a) is the correct option because we know that the u-chart is the control chart for number of defects per unit whereas the c-chart is the control chart for number of defects.

E2) The control limits of the c-chart when the average number of defects of the process is not known are

$$\text{Centre line (CL)} = \bar{c}$$

$$\text{Upper control limit (UCL)} = \bar{c} + 3\sqrt{\bar{c}}$$

$$\text{Lower control limit (UCL)} = \bar{c} - 3\sqrt{\bar{c}}$$

$$\text{where } \bar{c} = \frac{\text{Sum of defects}}{\text{Total number of samples inspected}} = \frac{1}{k}\sum_{i=1}^{k} c_i.$$

E3) Since the sample size is constant, we can use the c-chart. Also, the average number of defects in the process is not given. So we use equations (5a to 5c) to calculate the centre line and control limits of the c-chart.

It is given that k = 25 and n = 10.

The total number of defects in all match boxes = 7 + 1 + … + 3 = 124

$$\therefore \quad \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i = \frac{1}{25} \times 124 = 4.960$$

Hence, using equations (5a to 5c), we calculate the centre line and control limits of the c-chart as follows:

$$CL = \bar{c} = 4.960$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 4.960 + 3 \times \sqrt{4.960} = 4.960 + 3 \times 2.227 = 11.641$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 4.960 - 3 \times \sqrt{4.960} = 4.960 - 3 \times 2.227 = -1.721 \approx 0$$

We construct the c-chart by taking the sample number on the X-axis and the number of defects (c) per sample on the Y-axis as shown in Fig. 4.6.

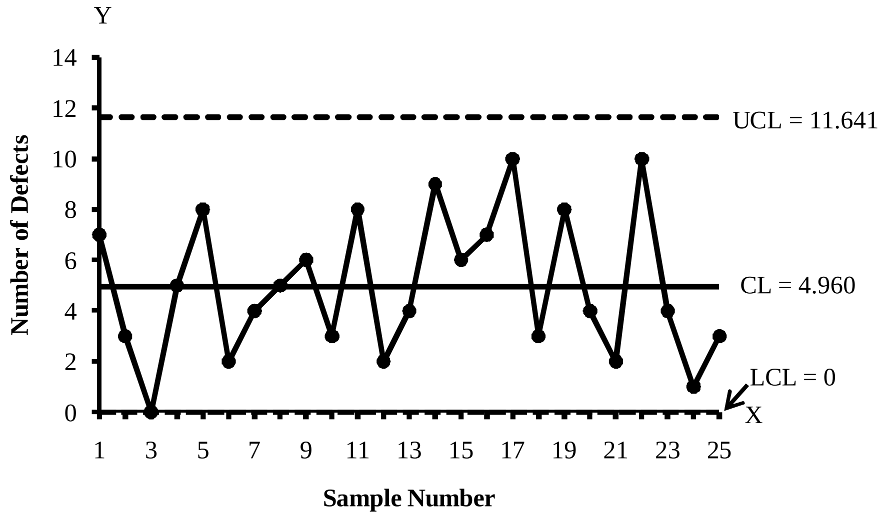

Fig. 4.6: The c-chart for the number of defects in packets of match box.

Interpretation of the result

From the c-chart shown in Fig. 4.6, we observe that no point lies outside the control limits and there is no specific pattern of the sample points on the chart. It indicates that the process is under statistical control.

E4) Since the number of defects on the white paper per 100 square meter is given. It means that the sample size is constant. So we use the c-chart.

Since the average number of defects of the process is not given, So we use equations (5a to 5c) to calculate the centre line and control limits of the c-chart.

It is given that k = 25 and n = 1.

The total number of defects = 6 + 8 + … + 7 = 214

$$\therefore \quad \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i = \frac{1}{25} \times 214 = 8.560$$

Hence, using equations (5a to 5c), we calculate the centre line and control limits of the c-chart as follows:

$$CL = \bar{c} = 8.560$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 8.560 + 3 \times \sqrt{8.560} = 8.560 + 3 \times 2.926 = 17.338$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 8.560 - 3 \times \sqrt{8.560} = 8.560 - 3 \times 2.926 = -0.218 \approx 0$$

We construct the c-chart by taking the sample number on the X-axis and the number of defects (c) on the white paper on the Y-axis as shown in Fig. 4.7.

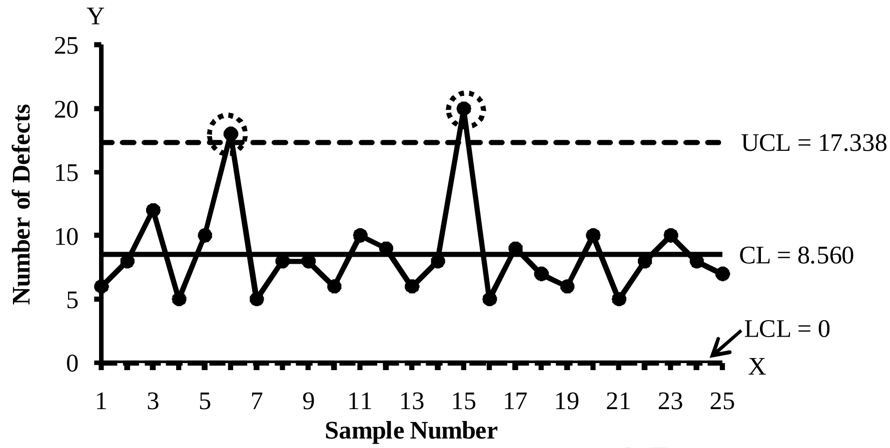

Fig. 4.7: The c-chart for the number of defects on the white paper.

Interpretation of the result

From Fig. 4.7, we observe that the points corresponding to the sample numbers 6 and 15 lie outside the upper control limit. Therefore, the process is not under statistical control with respect to the number of defects. To bring the process under statistical control it is necessary to investigate the assignable causes and take corrective action to eliminate them.

For calculating the revised limits of the c-chart, we first delete the out-of-control points (6 and 15) and calculate the new $\bar{c}$ using remaining samples.

In our case, $k = 25, d = 2, \sum_{j=1}^{d} c_j = 18 + 20 = 38$

$$\therefore \quad \bar{c}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{k - d} = \frac{214 - 38}{25 - 2} = \frac{176}{23} = 7.652$$

After finding the $\bar{p}_{new}$, we calculate the revised centre line and control limits using equations (7a to 7c) as follows:

$$CL = \bar{c}_{new} = 7.652$$

$$UCL = \bar{c}_{new} + 3\sqrt{\bar{c}_{new}} = 7.652 + 3 \times \sqrt{7.652} = 7.652 + 3 \times 2.766 = 15.950$$

$$LCL = \bar{c}_{new} - 3\sqrt{\bar{c}_{new}} = 7.652 - 3 \times \sqrt{7.652}$$

$$= 7.652 - 3 \times 2.766 = -0.646 \Box 0$$

E5) Since the number of defects is given, we may use either c-chart or u-chart. Since the sample size varies, we use the u-chart.

For calculating the centre line and control limits of the u-chart, we first calculate the defects per unit (u) and then $\bar{u}$.

Sample Number	Amount of the Carpet Inspected	Number of Unit Inspected (n)	Number of Defects (c)	u = c/n	CL	UCL	LCL
1	300	3.0	7	2.333	3.301	6.486	0.168
2	200	2.0	8	4.000	3.301	7.196	–0.542 □ 0
3	250	2.5	5	2.000	3.301	6.788	–0.134 □ 0
4	150	1.5	5	3.333	3.301	7.795	–1.141 □ 0
5	250	2.5	10	4.000	3.301	6.788	–0.134 □ 0
6	100	1.0	4	4.000	3.301	8.799	–2.145 □ 0
7	200	2.0	5	2.500	3.301	7.196	–0.542 □ 0
8	150	1.5	8	5.333	3.301	7.795	–1.141 □ 0
9	150	1.5	8	5.333	3.301	7.795	–1.141 □ 0
10	250	2.5	6	2.400	3.301	6.788	–0.134 □ 0
11	200	2.0	5	2.500	3.301	7.196	–0.542 □ 0
12	250	2.5	9	3.600	3.301	6.788	–0.134 □ 0
13	100	1.0	6	6.000	3.301	8.799	–2.145 □ 0
14	250	2.5	8	3.200	3.301	6.788	–0.134 □ 0
15	300	3.0	6	2.000	3.301	6.486	0.168
16	250	2.5	5	2.000	3.301	6.788	–0.134 □ 0
17	150	1.5	9	6.000	3.301	7.795	–1.141 □ 0
18	200	2.0	7	3.500	3.301	7.196	–0.542 □ 0
19	150	1.5	6	4.000	3.301	7.795	–1.141 □ 0
20	300	3.0	10	3.333	3.301	6.486	0.168
21	200	2.0	5	2.500	3.301	7.196	–0.542 □ 0
22	250	2.5	8	3.200	3.301	6.788	–0.134 □ 0
23	200	2.0	5	2.500	3.301	7.196	–0.542 □ 0
24	100	1.0	8	8.000	3.301	8.799	–2.145 □ 0
25	150	1.5	5	3.333	3.301	7.795	–1.141 □ 0
Total	5050	50.5	168				

From the above table and equation (9), we have

$$\bar{u} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} n_i} = \frac{168}{50.5} = 3.327$$

Therefore, using equation (10a to 10c), we calculate the centre line and control limits which are shown in the table as follows:

$$CL = \bar{u} = 3.327$$

$$UCL_1 = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_1}} = 3.327 + 3\sqrt{\frac{3.327}{3}}$$

$$= 3.327 + 3 \times 1.053 = 6.486,$$ and so on.

$$LCL_1 = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_1}} = 3.327 - 3\sqrt{\frac{3.327}{3}}$$

$$= 3.327 - 3 \times 1.053 = 0.168, \text{ and so on.}$$

We construct the u-chart by taking the sample number on the X-axis and the number of defects per 100 square meter paper (u) on the Y-axis as shown in Fig. 4.8.

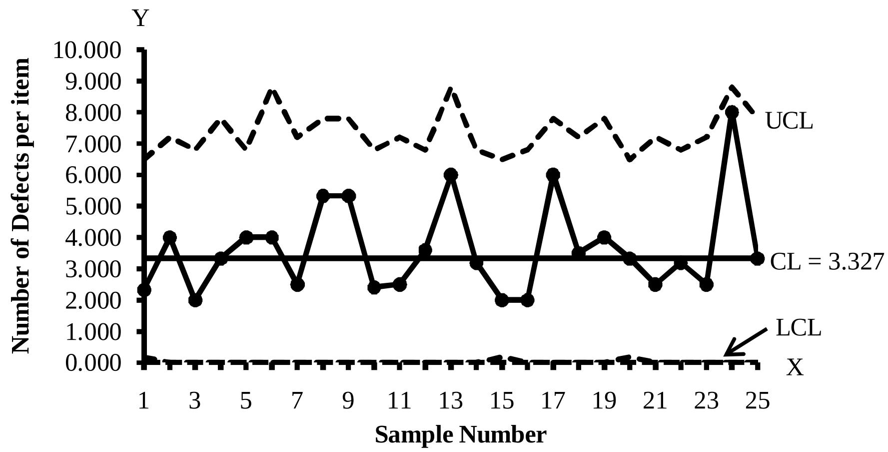

Fig. 4.8: The u-chart for the number of defects per 100 square meter paper.

Interpretation of the result

From Fig. 4.8, we observe that no point lies outside the control limits of the u-chart and there is no specific pattern of the sample points on the chart. So it indicates that the process is under statistical control with respect to the number of defects per 100 square meter paper.