UNIT 15 CONTINUOUS UNIFORM AND EXPONENTIAL DISTRIBUTIONS

Structure

15.1 INTRODUCTION

In Units 13 and 14, you have studied normal distribution with its various properties and applications. Continuing our study on continuous distributions, we, in this unit, discuss continuous uniform and exponential distributions. It may be seen that discrete uniform and geometric distributions studied in Unit 11 and Unit 12 are the discrete analogs of continuous uniform and exponential distributions. Like geometric distribution, exponential distribution also has the memoryless property. You have also studied that geometric distribution is the only discrete distribution which has the memoryless property. This feature is also there in exponential distribution and it is the only continuous distribution having the memoryless property.

The present unit discusses continuous uniform distribution in Sec. 15.2 and exponential distribution in Sec. 15.3.

Objectives

After studing the unit, you would be able to:

- define continuous uniform and exponential distributions;
- state the properties of these distributions;
- explain the memoryless property of exponential distribution; and
- solve various problems on the situations related to these distributions.

15.2 CONTINUOUS UNIFORM DISTRIBUTION

The uniform (or rectangular) distribution is a very simple distribution. It provides a useful model for a few random phenomena like having random number from the interval [0, 1], then one is thinking of the value of a uniformly distributed random variable over the interval [0, 1].

Definition: A random variable X is said to follow a continuous uniform (rectangular) distribution over an interval (a, b) if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a < x < b \\ 0, & \text{otherwise} \end{cases}$$

The distribution is called uniform distribution since it assumes a constant (uniform) value for all x in (a, b). If we draw the graph of y = f(x) over x-axis and between the ordinates x = a and x = b (say), it describes a rectangle as shown in Fig. 15.1

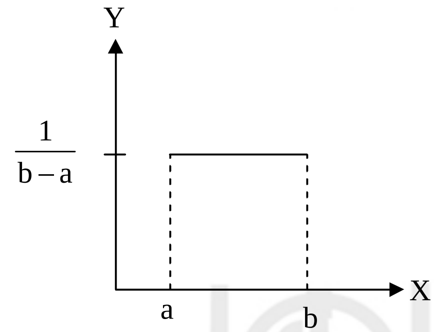

Fig. 15.1: Graph of uniform function

A uniform variate X on the interval (a, b) is written as X ~ U[a, b]

Cumulative Distribution Function

The cumulative distribution function of the uniform random variate over the interval (a, b) is given by:

For $x \le a$, $F(x) = P[X \le x] = \int_{-\infty}^{x} 0dx = 0$

For a < x < b,

$$F(x) = P[X \le x] = \int_{a}^{x} f(x)dx = \int_{a}^{x} \frac{1}{b-a}dx = \frac{1}{b-a}[x]_a^x = \frac{x-a}{b-a}.$$

For $x \ge b$

$$F(x) = P[X \le x] = \int_{-\infty}^{x} f(x)dx$$

$$= \int_{-\infty}^{a} f(x)dx + \int_{a}^{b} f(x)dx + \int_{b}^{\infty} f(x)dx$$

$$= \int_{-\infty}^{a} (0)dx + \int_{a}^{b} \frac{1}{b-a}dx + \int_{b}^{\infty} (0)dx$$

$$= 0 + \frac{1}{b-a}[x]_a^b + 0 = \frac{b-a}{b-a} = 1.$$

So,

$$F(x) = \begin{cases} 0 & \text{for } x \le a \\ \frac{x-a}{b-a} & \text{for } a < x < b \\ 1 & \text{for } x \ge b \end{cases}$$

On plotting its graph, we have

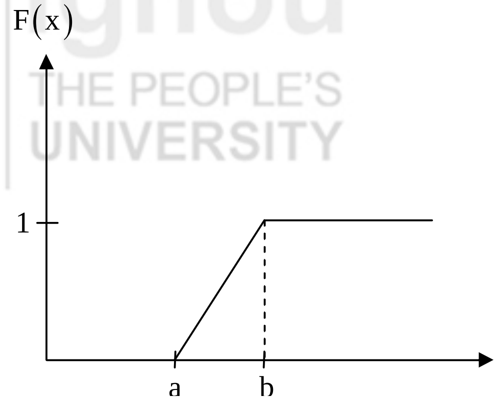

Fig. 15.2: Graph of distribution function

Mean and Variance of Uniform Distribution

Mean $= $ 1st order moment about origin $\left(\mu_1'\right)$

$$= \int_a^b x.f(x)dx = \int_a^b x.\frac{1}{b-a}dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{b-a}\left[\frac{b^2}{2}-\frac{a^2}{2}\right]$$

$$= \frac{(b-a)(b+a)}{2(b-a)} = \frac{a+b}{2}$$

Second order moment about origin $\left(\mu_2'\right)$

$$= \int_a^b x^2 f(x)dx$$

$$= \int_a^b x^2.\frac{1}{b-a}dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{1}{b-a}\left[\frac{b^3}{3}-\frac{a^3}{3}\right]$$

$$= \frac{b^3-a^3}{3(b-a)}$$

$$= \frac{(b-a)\left(b^2+ab+a^2\right)}{3(b-a)} \quad \left[\because x^3-y^3=(x-y)\left(x^2+xy+y^2\right)\right]$$

$$= \frac{a^2+ab+b^2}{3}$$

$\therefore$ Variance of X = E(X²) – [E(X)]² $= \dfrac{a^2+ab+b^2}{3}-\left(\dfrac{a+b}{2}\right)^2$

$$= \frac{4\left(a^2+ab+b^2\right)-3(a+b)^2}{12}$$

$$= \frac{4a^2 + 4ab + 4b^2 - 3a^2 - 3b^2 - 6ab}{12}$$

$$= \frac{b^2 + a^2 - 2ab}{12} = \frac{(b-a)^2}{12}.$$

So, Mean = $\frac{a+b}{2}$ and Variance = $\frac{(b-a)^2}{12}$.

Let us now take up some examples on continuous uniform distribution.

Example 1: If X is uniformly distributed with mean 2 and variance 12, find P[X < 3].

Solution: Let X ~ U [a, b]

$\therefore$ probability density function of X is

$f(x) = \frac{1}{b-a}, \ a < x < b.$

Now as Mean = 2

$\Rightarrow \ \frac{a+b}{2} = 2$

$\Rightarrow$ a + b = 4 … (1)

Variance = 12

$\Rightarrow \frac{(b-a)^2}{12} = 12$

$\Rightarrow (b-a)^2 = 144$

$\Rightarrow$ b – a = ± 12

$\Rightarrow$ b – a = 12 ... (2) $\left[\begin{array}{l}\because b-a=-12, \text{being negative is} \\ \text{rejected as b should be greater than a} \\ \Rightarrow b-a \ \text{should be positive}\end{array}\right]$

Adding (1) and (2), we have

2b = 16

$\Rightarrow$ b = 8 and hence a = 4 – 8 = – 4

$\therefore \ f(x) = \frac{1}{b-a} = \frac{1}{8-(-4)} = \frac{1}{12}$ for – 4 < x < 8.

Thus, the desired probability $= P[X < 3] = \int_{-4}^{3} \frac{1}{12} dx = \frac{1}{12} \int_{-4}^{3} 1 dx = \frac{1}{12} [x]_{-4}^{3}$

$$= \frac{1}{12}[3-(-4)] = \frac{7}{12}.$$

Example 2: Calculate the coefficient of variation for the rectangular distribution in (0, 12).

Solution: Here a = 0, b = 12.

$\therefore$ Mean = $\dfrac{a+b}{2} = \dfrac{0+12}{2} = 6,$

Variance = $\dfrac{(b-a)^2}{12} = \dfrac{(12-0)^2}{12} = \dfrac{144}{12} = 12.$

$\Rightarrow$ S.D. = $\sqrt{12}$

Thus, the coefficient of variation

$$= \frac{\text{S.D.}}{\text{Mean}} \times 100$$ [Also see Unit 2 of MST-002]

$$= \frac{\sqrt{12}}{6} \times 100 = 57.74\%$$

Example 3: Metro trains are scheduled every 5 minutes at a certain station. A person comes to the station at a random time. Let the random variable X count the number of minutes he/she has to wait for the next train. Assume X has a uniform distribution over the interval (0, 5). Find the probability that he/she has to wait at least 3 minutes for the train.

Solution: As X follows uniform distribution over the interval (0, 5),

$\therefore$ probability density function of X is

$$f(x) = \frac{1}{b-a} = \frac{1}{5-0} = \frac{1}{5},\ 0 < x < 5$$

Thus, the desired probability

$$P[X \geq 3] = \int_3^5 f(x)\,dx = \int_3^5 \frac{1}{5}\,dx = \frac{1}{5}\int_3^5 (1)\,dx$$

$$= \frac{1}{5}[x]_3^5 = \frac{1}{5}(5-3) = \frac{2}{5} = 0.4$$

Now, you can try the following exercises.

E1) Suppose that X is uniformly distributed over (–a, a). Determine ‘a’ so that

i) $P[X > 4] = \dfrac{1}{3}$

ii) $P[X < 1] = \dfrac{3}{4}$

iii) $P[|X| < 2] = P[|X| > 2]$

E2) A random variable X has a uniform distribution over (–2, 2). Find k for which P[X > k] = $\dfrac{1}{2}$.

Now, let us discuss exponential distribution in the next section.

15.3 EXPONENTIAL DISTRIBUTION

The exponential distribution finds applications in the situations related to lifetime of an equipment or service time at the counter in a queue. So, the exponential distribution serves as a good model whenever there is a waiting time involved for a specific event to occur e.g. waiting time for a failure to occur in a machine. The exponential distribution is defined as follows:

Definition: A random variable X is said to follow exponential distribution with parameter $\lambda > 0$, if it takes any non-negative real value and its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

Its cumulative distribution function (c.d.f.) is thus given by

$$F(x) = P[X \leq x] = \int_0^x f(x)\,dx = \int_0^x \lambda e^{-\lambda x} dx$$

$$= \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^x = -1\left[e^{-\lambda x}\right]_0^x = -\left(e^{-\lambda x} - e^0\right)$$

$$= -\left(e^{-\lambda x} - 1\right) = 1 - e^{-\lambda x}.$$

$$\text{So, } F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{for } x \geq 0 \\ 0, & \text{elsewhere} \end{cases}.$$

Mean and Variance of Exponential Distribution

$$\text{Mean} = E(X) = \int_0^\infty x\,f(x)\,dx = \int_0^\infty x\,\lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^\infty x\,e^{-\lambda x} dx$$

$$= \lambda\left[\left[(x)\frac{e^{-\lambda x}}{-\lambda}\right]_0^\infty - \int_0^\infty (1)\frac{e^{-\lambda x}}{-\lambda}dx\right] \quad \text{[Integrating by parts]}$$

In case of integration of product of two different types of functions, we do integration by parts i.e. the following formula is applied:

$$\int(\text{First function})(\text{Second fuction})\,dx$$

$$= (\text{First function as it is})\ (\text{Integral of second})$$

$$-\int(\text{Differentiation of first})(\text{Integral of second})\,dx$$

$$\therefore \text{ Mean } = \lambda\left[(0-0)+\frac{1}{\lambda}\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^{\infty}\right]$$

$$= \lambda\left[-\frac{1}{\lambda^2}(0-1)\right] = \lambda\left(\frac{1}{\lambda^2}\right) = \frac{1}{\lambda}.$$

Now, $E(X^2) = \int_0^{\infty} x^2 f(x)\,dx = \int_0^{\infty} x^2\left(\lambda e^{-\lambda x}\right)dx$

$$= \lambda\int_0^{\infty} x^2 e^{-\lambda x}\,dx$$

$$= \lambda\left[\left(x^2\frac{e^{-\lambda x}}{-\lambda}\right)_0^{\infty} - \int_0^{\infty}(2x)\frac{e^{-\lambda x}}{-\lambda}\,dx\right] \qquad \text{[Integrating by parts]}$$

$$= \lambda\left[(0-0)+\frac{2}{\lambda}\int_0^{\infty} x\,e^{-\lambda x}\,dx\right]$$

$$= \frac{2}{\lambda}\lambda\int_0^{\infty} x\ e^{-\lambda x}\,dx = \frac{2}{\lambda}\int_0^{\infty} x\left(\lambda e^{-\lambda x}\right)dx$$

$$= \frac{2}{\lambda}E(X)$$

$$= \frac{2}{\lambda}\frac{1}{\lambda} \qquad \text{[E(X) is mean and has already been obtained]}$$

$$= \frac{2}{\lambda^2}$$

Thus, Variance = $E(X^2) - [E(X)]^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

So, Mean = $\frac{1}{\lambda}$ and Variance = $\frac{1}{\lambda^2}$.

Remark 1: Variance = $\frac{1}{\lambda^2} = \frac{1}{\lambda.\lambda} = \frac{\text{Mean}}{\lambda} \Rightarrow$ Mean = $\lambda \times$ Variance

So,

Value of λ	Implies
$\lambda < 1$	Mean < Variance
$\lambda = 1$	Mean = Variance
$\lambda > 1$	Mean > Variance

Hence, for exponential distribution,

Mean > or = or < Variance according to whether λ > or = or < 1.

Memoryless Property of Exponential Distribution

Now, let us discuss a very important property of exponential distribution and that is the memoryless (or forgetfulness) property. Like geometric distribution in the family of discrete distributions, exponential distribution is the only distribution in the family of continuous distributions which has memoryless property. The memorless property of exponential distribution is stated as:

If X has an exponential distribution, then for every constant a ≥ 0, one has

$P[X \le x + a \mid X \ge a] = P\left[X \le x\right]$ for all x i.e. the conditional probability of waiting up to the time $'x+a'$ given that it exceeds 'a' is same as the probability of waiting up to the time 'x'. To make you understand the above concept clearly let us take the following example: Suppose you purchase a TV set, assuming that its life time follows exponential distribution, for which the expected life time has been told to you 10 years (say). Now, if you use this TV set for say 4 years and then you ask a TV mechanic, without informing him/her that you had purchased it 4 years ago, regarding its expected life time. He/she, if finds the TV set as good as new, will say that its expected life time is 10 years.

So, here, in the above example, 4 years period has been forgotten, in a way, and for this example:

P[life time up to 10 years]

= P[life time up to 14 years | life time exceeds 4 years]

i.e. $P[X \le 10] = P\,[X \le 14 \mid X \ge 4]$

or $P[X \le 10] = P[X \le 10 + 4 \mid X \ge 4]$

Here a = 4 and $x = 10$.

Let us now prove the memoryless property of exponential distribution.

Proof: $P\left[X \le x + a \mid X \ge a\right] = \dfrac{\left[\left(X \le x+a\right)\cap\left(X \ge a\right)\right]}{P\left[X \ge a\right]}$ [By conditional probability]

where

$$P\left[\left(X \le x+a\right)\cap\left(X \ge a\right)\right] = P\left[a \le X \le x+a\right]$$

$$= \int_a^{x+a} f\left(x\right)dx \;= \lambda \int_a^{x+a} e^{-\lambda x}dx$$

$$= \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_a^{x+a} = \lambda\left[\frac{e^{-\lambda(x+a)}}{-\lambda} - \frac{e^{-\lambda a}}{-\lambda}\right]$$

$$= \left[-e^{-\lambda(x+a)} + e^{-\lambda a}\right] = \left[-e^{-\lambda x}.e^{-\lambda a} + e^{-\lambda a}\right]$$

$$= e^{-\lambda a}\left[1 - e^{-\lambda x}\right], \text{ and}$$

$$P[X \ge a] = \int_a^{\infty} f\left(x\right)dx = \int_a^{\infty} \lambda e^{-\lambda x}dx = \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_a^{\infty} = -\left[0 - e^{-a\lambda}\right] = e^{-\lambda a}$$

$\therefore P[X \le x + a \mid X \ge a] = \dfrac{e^{-\lambda a}\left[1-e^{-\lambda x}\right]}{e^{-\lambda a}} = 1 - e^{-\lambda x}$

Also, $P[X \le x] = \int_0^x \lambda e^{-\lambda x} dx$

$= 1 - e^{-\lambda x}$ [On simplification]

Thus,

P[X ≤ x + a | X ≥ a] = P[X ≤ x].

Hence proved

Example 4: Show that for the exponential distribution:
$f(x) = Ae^{-x}$, $0 \le x < \infty$, mean and variance are equal.

Solution: As $f(x)$ is probability function,

$\therefore \int_0^{\infty} f(x) dx = 1$

$\Rightarrow \int_0^{\infty} Ae^{-x} dx = 1 \Rightarrow A\left[\dfrac{e^{-x}}{(-1)}\right]_0^{\infty} = 1$

$\Rightarrow -A[0-1] = 1 \Rightarrow A = 1$

$\therefore f(x) = e^{-x}$

Now, comparing it with the exponential distribution

$f(x) = \lambda e^{-\lambda x}$, we have

$\lambda = 1$

Hence, mean = $\dfrac{1}{\lambda} = \dfrac{1}{1} = 1$,

and variance = $\dfrac{1}{\lambda^2} = \dfrac{1}{1} = 1$.

So, the mean and variance are equal for the given exponential distribution.

Example 5: Telephone calls arrive at a switchboard following an exponential distribution with parameter λ = 12 per hour. If we are at the switchboard, what is the probability that the waiting time for a call is

i) at least 15 minutes

ii) not more than 10 minutes.

Solution: Let X be the waiting time (in hours) for a call.

$\therefore f(x) = \lambda e^{-\lambda x}, x \ge 0$

$\Rightarrow F(x) = P[X \le x] = 1 - e^{-\lambda x}$ [c.d.f. of exponential distribution]

$= 1 - e^{-12x}$ … (1) [$\because \lambda = 12$]

Now,

i) P[waiting time is at least 15 minutes] = P[waiting time is at least $\frac{1}{4}$ hours]

$$= P\left[X \geq \frac{1}{4}\right] = 1 - P\left[X < \frac{1}{4}\right]$$

$$= 1 - \left[1 - e^{-12 \times \frac{1}{4}}\right] \quad \text{[Using (1) above]}$$

$$= e^{-3}$$

$$= 0.0498 \quad \left[\begin{array}{l}\text{See table given at the}\\ \text{end of Unit 10}\end{array}\right]$$

ii) P[waiting time not more than 10 minutes]

= P[waiting time not more than $\frac{1}{6}$ hrs]

$$= P\left[X \leq \frac{1}{6}\right] = 1 - e^{-12 \times \frac{1}{6}}$$

$$= 1 - e^{-2} = 1 - (0.1353) = 0.8647$$

Now, we are sure that you can try the following exercises.

E3) What are the mean and variance of the exponential distribution given by:

$$f(x) = 3e^{-3x}, x \geq 0$$

E4) Obtain the value of k > 0 for which the function given by

$$f(x) = 2e^{-kx}, x \geq 0$$

follows an exponential distribution.

E5) Suppose that accidents occur in a factory at a rate of $\lambda = \frac{1}{20}$ per working day. Suppose in the factory six days (from Monday to Saturday) are working. Suppose we begin observing the occurrence of accidents at the starting of work on Monday. Let X be the number of days until the first accident occurs. Find the probability that

i) first week is accident free

ii) first accident occurs any time from starting of working day on Tuesday in second week till end of working day on Wednesday in the same week.

We now conclude this unit by giving a summary of what we have covered in it.

15.4 SUMMARY

Following main points have been covered in this unit.

1) A random variable X is said to follow a **continuous uniform (rectangular)** distribution over an interval (a, b) if its probability density function is given by

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a < x < b \\ 0, & \text{otherwise} \end{cases}$$

2) For **continuous uniform distribution**, **Mean** $= \frac{a+b}{2}$ and **variance** $= \frac{(b-a)^2}{12}$.

3) A random variable X is said to follow **exponential distribution** with parameter $\lambda > 0$, if it takes any non-negative real value and its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \ge 0 \\ 0 & \text{, elsewhere} \end{cases}$$

4) For **exponential distribution**, **Mean** $= \frac{1}{\lambda}$ and **Variance** $= \frac{1}{\lambda^2}$.

5) Mean > or = or < Variance according to whether λ > or = or < 1.

6) **Exponential distribution** is the **only continuous distribution** which has the **memoryless property** given by:

   $P[X \le x + a \mid X \ge a] = P[X \le x]$.

15.5 SOLUTIONS/ANSWERS

E1) As $X \sim U[-a, a]$,

$\therefore$ probability density function of X is

$$f(x) = \frac{1}{a-(-a)} = \frac{1}{a+a} = \frac{1}{2a}, \quad -a < x < a.$$

i) Given that $P[X > 4] = \frac{1}{3}$

$$\Rightarrow \int_4^a \frac{1}{2a} dx = \frac{1}{3}$$

$$\Rightarrow \frac{1}{2a}[x]_4^a = \frac{1}{3}$$

$$\Rightarrow \frac{a-4}{2a} = \frac{1}{3}$$

$$\Rightarrow 3a - 12 = 2a$$

$$\Rightarrow a = 12.$$

ii) $P[X < 1] = \frac{3}{4}$

$$\Rightarrow \int_{-a}^{1} \frac{1}{2a} dx = \frac{3}{4}$$

$$\Rightarrow \frac{1}{2a}[x]_{-a}^{1} = \frac{3}{4}$$

$$\Rightarrow \frac{1}{2a}[1+a] = \frac{3}{4}$$

$$\Rightarrow 1 + a = \frac{3}{2}a$$

$$\Rightarrow 2 + 2a = 3a$$

$$\Rightarrow a = 2$$

iii) $P[|X| < 2] = P[|X| > 2]$

$$\Rightarrow P[-2 < X < 2] = P[X < -2 \text{ or } X > 2] \quad \begin{bmatrix} \because |X| < 2 \Rightarrow \pm X < 2 \\ \Rightarrow X < 2 \text{ or } -X < 2 \\ \Rightarrow -2 < X < 2 \text{ and} \\ |X| > 2 \Rightarrow \pm X > 2 \\ \Rightarrow X > 2 \text{ or } -X > 2 \\ \Rightarrow X > 2 \text{ or } X < -2 \end{bmatrix}$$

$$\Rightarrow P[-2 < X < 2] = P[X < -2] + P[X > 2] \quad \begin{bmatrix} \text{By Addition law of} \\ \text{probability for mutually} \\ \text{exclusive events} \end{bmatrix}$$

$$\Rightarrow \int_{-2}^{2} \frac{1}{2a} dx = \int_{-a}^{-2} \frac{1}{2a} dx + \int_{2}^{a} \frac{1}{2a} dx$$

$$\Rightarrow \frac{1}{2a}[4] = \frac{1}{2a}[-2+a] + \frac{1}{2a}[a-2]$$

$$\Rightarrow 4 = (-2+a) + (a-2)$$

$$\Rightarrow 4 = -4 + 2a$$

$$\Rightarrow 2a = 8$$

$$\Rightarrow a = 4$$

E2) As $X \sim U[-2, 2]$,

$$\therefore f(x) = \frac{1}{4}, -2 < x < 2.$$

Now $P[X > k] = \frac{1}{2}$

$\Rightarrow \int_k^2 \frac{1}{4} dx = \frac{1}{2}$

$\Rightarrow \frac{2-k}{4} = \frac{1}{2}$

$\Rightarrow 2 - k = 2$

$\Rightarrow k = 0.$

E3) Comparing it with the exponential distribution given by

$f(x) = \lambda e^{-\lambda x}, x \geq 0$

We have $\lambda = 3$

$\therefore$ Mean $= \frac{1}{\lambda} = \frac{1}{3}$ and Variance $= \frac{1}{\lambda^2} = \frac{1}{9}$

E4) As the given function is exponential distribution i.e. a p.d.f.,

$\therefore \int_0^\infty f(x) dx = 1$

$\Rightarrow k = 2$ [On simplification]

Alternatively, you may compare the given function with exponential distribution

$f(x) = \lambda e^{-\lambda x},$

we have

$\lambda = 2$ and $\lambda = k$

$\therefore k = 2$

E5) Here $P[X \leq x] = F(x) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{20}x}$

i) P[First week is accident free] = P[Accident occurs after six days]

$= P[X > 6] = 1 - P[X \leq 5]$

$= 1 - \left[1 - e^{-5/20}\right] = e^{-\frac{1}{4}} = e^{-0.25} = 0.7788.$

ii) P[First accident occurs on second week from starting of working day on Tuesday till end of working day on Wednesday]

=P[First accident occurs after 7 working days and before the end of 9 working days]

$= P[7 < X \leq 9]$

$= P[X \leq 9] - P[X \leq 7]$

$$= \left(1 - e^{-\frac{9}{20}}\right) - \left(1 - e^{-\frac{7}{20}}\right)$$

$$= -e^{-\frac{9}{20}} + e^{-\frac{7}{20}}$$

$$= e^{-\frac{7}{20}} - e^{-\frac{9}{20}}$$

$$= e^{-0.35} - e^{-0.45}$$

= 0.7047 – 0.6376 [See the table give at the end of Unit 10]

= 0.0671.