UNIT 14 RELIABILITY EVALUATION OF SIMPLE SYSTEMS

Structure

14.1 INTRODUCTION

In Unit 13, we have defined reliability and basic reliability functions, namely, reliability function, cumulative failure distribution function, failure density function and hazard rate function. We have also discussed how to estimate these functions for a population of identical components through test-generated data.

One of the main objectives of getting estimates of these functions is to get an estimate of the reliability of the component, of which a sample is tested.

We now consider how to estimate the reliability of a system. By a system, we mean assemblages of, say, n identifiable components that perform some function. The main objective of this unit is to discuss some techniques of predicting/estimating the reliability of systems, from the reliabilities of the components of the system. We restrict our discussion to simple systems, which are defined in Sec. 14.2. In Sec. 14.3, we explain what is meant by the reliability evaluation of a system.

Techniques of reliability evaluation of a system depend upon the configuration of its components. In practice, several types of configurations are used. But we shall discuss the reliability evaluation techniques only for the following configurations of components of a system:

- Series configuration
- Parallel configuration
- Mixed configuration
- k-out-of-n configuration
- Standby configuration
- Complex configuration

Reliability evaluation of series and parallel systems is discussed in Secs. 14.4 and 14.5, respectively. In Sec. 14.6, we discuss the reliability of systems with mixed configuration of components. In Unit 15, we discuss reliability evaluation of k-out-of-n and standby configurations. Reliability evaluation of complex configurations is discussed in Unit 16.

Objectives

After studying this unit, you should be able to:

- define a system, a simple system and reliability of the system; and
- evaluate the reliability of a system when its components are in series, in parallel and in mixed configurations.

14.2 DEFINITION OF A SIMPLE SYSTEM

The definition of a simple system involves the concept of series and parallel configurations. So, to understand it better, you should first learn the definitions of series and parallel configurations. These are given below.

Series Configuration

Suppose a system has n components. From a reliability point of view, the components of a system are said to be in **series configuration** if they are connected in such a way that

(i) for the successful operation of the system all n components must perform their intended function, and

(ii) only one needs to fail for system failure.

The string of lights (bulbs or LEDs) used in festivals and weddings is a good example of a series system. The system performs successfully only when all bulbs/LEDs are working and only one of them needs to fail for the system to fail.

The reliability block diagram for logical connectivity of a series configuration generally looks as shown in Fig. 14.1, where each of the n components is illustrated by a block.

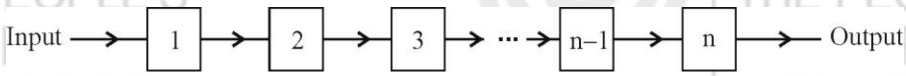

Fig. 14.1: Series configuration for n components.

In reliability literature, logical connectivity of n components, namely, 1, 2, …, n, in series configuration is also illustrated by a reliability graph as shown in Fig. 14.2.

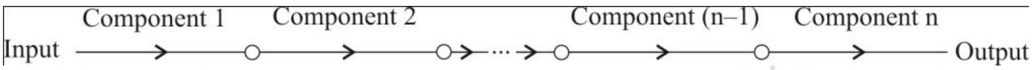

Fig. 14.2: Series configuration for n components.

Parallel Configuration

Let a system have n components. From a reliability point of view, the components of a system are said to be in parallel configuration if they are connected in such a way that

(i) only one component needs to work for the successful operation of the system, and

(ii) all components must fail for system failure.

The phone line is a good example of a parallel system.

The reliability block diagram for logical connectivity of the parallel configuration generally looks as shown in Fig. 14.3.

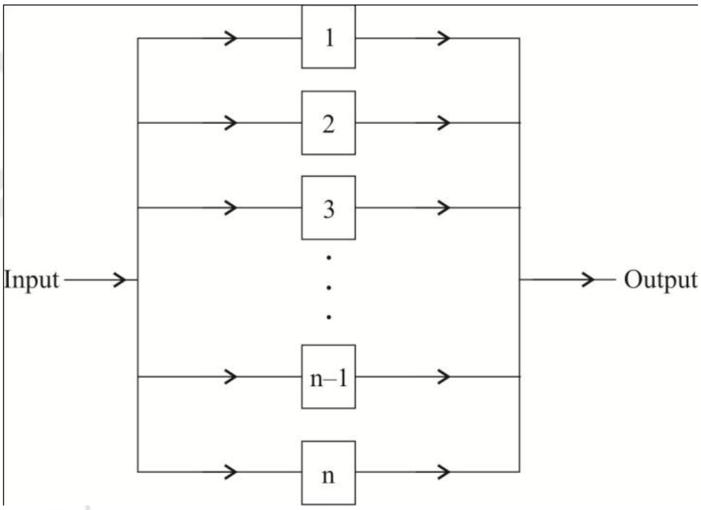

Fig. 14.3: Parallel configuration for n components.

In reliability literature, logical connectivity of n components, namely, 1, 2, …, n in parallel configuration is also illustrated by a reliability graph as shown in Fig. 14.4.

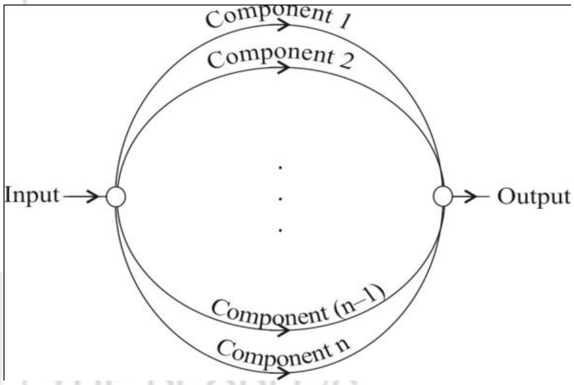

Fig. 14.4: Parallel configuration for n components.

In this unit, we shall use reliability block diagrams for series and parallel configurations.

Let us now define a simple system.

Simple System

A system is said to be simple if either its components are connected in parallel, in series or in a combination of both. In other words, a system is said to be simple if its reliability block diagram can be reduced into subsystems having independent components connected either in parallel or in series.

14.3 RELIABILITY EVALUATION OF A SYSTEM

In reliability theory,

A system is an assemblage of, say, n identifiable components that perform some function.

The n individual components are known as **elementary components**. The reliability of a system depends on the reliability of elementary components. To evaluate the reliability of a system, we have to apply the rules of probability theory such as the addition rule, multiplication rule, conditional probability, independence of events or their combination. You are familiar with these rules as all these have been discussed in detail in Units 1 to 3 of MST-003. The choice of rules from among these for a given situation depends on the logical connectivity of the elementary components or subsystems made up of elementary components in the system. In general, we follow the steps given below to evaluate the reliability of a system:

Step 1: We identify the elementary components or subsystems, which constitute the given system such that either their individual reliabilities are given or can be estimated. Let us call them 'units' comprising the system.

Step 2: We evaluate the reliabilities of those units whose reliabilities are not directly given to us.

Step 3: We draw a reliability block diagram of the system to represent the logical connectivity of the units (i.e., elementary components and subsystems).

Step 4: We determine the constraints that should be fulfilled for the successful operation of the system.

For example, do we need successful operation of all units for the successful operation of the system? Is the successful operation of only one of them enough or some other combination of components needs to operate successfully?

Step 5: We apply the rules of probability theory such as addition rule, multiplication rule, conditional probability, independence of events or their combination to evaluate the reliability of the system.

Let us explain the 5 steps given above with the help of a reliability block diagram shown in Fig. 14.5. The number written inside each block indicates the numbering of the component and the number written at the top of each block represents reliability of the corresponding component.

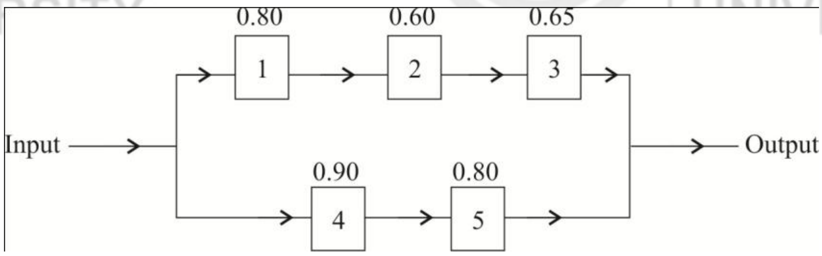

Fig. 14.5: A reliability block diagram.

Step 1: There are 5 elementary components in the system shown in Fig. 14.5, and these are numbered as 1, 2, 3, 4, 5.

Two sub-systems constitute the system, as explained below:

(i) Elementary components 1, 2 and 3 are in series configuration and these form a subsystem. Let us call it unit 1 and denote it by U_1.

(ii) Elementary components 4 and 5 are also in series configuration and form another subsystem. Let us call it unit 2 and denote it by U_2.

Step 2: To evaluate the reliability of the system, we first evaluate the reliabilities of the units U_1 and U_2. We discuss the methods of evaluating reliabilities in Secs. 14.4 to 14.6.

Step 3: The reliability block diagram of the system is shown in Fig. 14.5.

Step 4: The constraints that should be fulfilled for the successful operation of the system are listed below:

(i) For the successful operation of unit U_1, all components (1, 2 and 3) must work successfully as they are in series configuration.

(ii) For the successful operation of unit U_2, both components 4 and 5 must work successfully as they are also in series configuration.

(iii) For successful operation of the system, either unit U_1 or unit U_2 or both should work successfully as these units are arranged in parallel configuration.

Step 5: The reliability of the system is evaluated by calculating the probability of the event $\left(U_1 \cup U_2\right)$ using the addition law of probability theory. You have learnt these calculations in Unit 3 of MST-003.

In Secs. 14.4 to 14.6 and in Units 15 and 16, we discuss the reliability evaluation of systems with different configurations of elementary components. We begin by explaining reliability evaluation of a series system.

14.4 RELIABILITY OF A SERIES SYSTEM

In Sec. 14.2, we have defined series configuration of a number of components. A series system is nothing but a system, all of whose components are in series configuration. A series system made up of n components is represented as shown in Fig. 14.6 for the purpose of reliability evaluation.

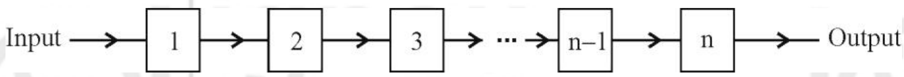

Fig. 14.6: Series system.

Note that Fig. 14.6 simply shows the logical connectivity of the system. Actual physical connectivity of the components in the system may vary. But always remember the following two characteristics of the series system (which have already been listed in Sec. 14.2):

(1) For the successful operation of the series system, it is necessary that all n components of the system perform their intended function successfully, and

(2) Only one component needs to fail for system failure.

Let us now obtain an expression for the reliability of the series system.

Let E_i be the event that component i performs its intended function successfully, where i = 1, 2, 3,…, n.

Let R_i denote the reliability of the component i, where i = 1, 2, 3, …, n for a mission of t units of time.

If R denotes the reliability of the series system, then, from the definitions of reliability and series system, we have

$$R = P\left(E_1 \cap E_2 \cap E_3 \cap ... \cap E_n\right) \qquad \text{… (1)}$$

In equation (1), we take the intersection of events because reliability is a probability and the definition of series system implies that for the successful operation of the system, all components must work.

If events $E_i, (i = 1, 2, ..., n)$ are not independent then we apply conditional probability and get

$$R = P(E_1)P(E_2|E_1)P(E_3|E_1E_2)...P(E_n|E_1E_2...E_{n-1}), \quad ... (2)$$

where $E_1E_2 = E_1 \cap E_2$, $E_1E_2E_3 = E_1 \cap E_2 \cap E_3$, etc.

But if events $E_i, (i = 1, 2, ..., n)$ are independent, then we have

$$R = P(E_1)P(E_2)P(E_3)...P(E_n) \quad \begin{bmatrix} \because P[E_i|E_j] = P[E_i] \text{ if events} \\ E_i \text{ and } E_j \text{ are independent, etc.} \end{bmatrix} \quad ... (3)$$

or $R = \prod_{i=1}^{n} R_i$, for a mission of t units of time, … (4a)

where $R_i = P(E_i)$ … (4b)

that is, the reliability of the series system is equal to the product of the individual reliabilities of the components of the system.

In particular, if a system has two components, then

$R = R_1R_2$ for a mission of t units of time … (5a)

and if the system has three components, then

$R = R_1R_2R_3$ for a mission of t units of time … (5b)

Let us consider some examples for evaluating the reliability of a series system.

Example 1: Evaluate the reliability of the series system having the reliability block diagram shown in Fig. 14.7 for a mission of 500 hours, where the given reliabilities of the components are for a mission of 500 hours. Assume that the failure of any component does not affect the functioning of the other components.

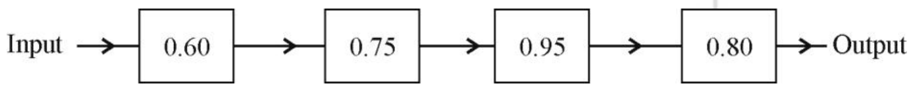

Fig. 14.7: Reliability block diagram for Example 1.

Solution: We know that the reliability of a series system is equal to the product of the individual reliabilities of the components. Therefore, the required reliability of the series system is given from equation (4a) as

$R = 0.60 \times 0.75 \times 0.95 \times 0.80$ for a mission of 500 hours

$= 0.342$ for a mission of 500 hours

Note 1: Note the answer of this example. Does it make you wonder why the reliability of the system is less than that of the worst component? Do not worry. The answer is correct. This problem arises due to limitations of the series system. One of the main limitations of the series system is that reliability of the series system can never be better than the reliability of the worst component in the system. The second major limitation of the series system is that reliability of the system decreases as the number of components in the system increases. These two limitations are illustrated in Examples 2 and 3.

Example 2: A system has three components connected in series having reliabilities 0.40, 0.70, 0.80, respectively, for a mission of 400 hours. What is the percentage increase in the reliability of the system in each of the following cases?

(i) Reliability of the first component is increased by 0.1 and that of the second and third components remains the same.

(ii) Reliability of the second component is increased by 0.1 and that of the first and third components remains the same.

(iii) Reliability of the third component is increased by 0.1 and that of the first and second components remains the same.

Solution: First of all, we calculate the reliability of the system as per the given reliabilities of the components. From equation (4a or 5b),

$$R = 0.4 \times 0.7 \times 0.8 \qquad \text{for a mission of 400 hours}$$
$$= 0.224 \qquad \text{for a mission of 400 hours}$$

Now, reliabilities of the system and percentage increase in the reliability of the system in the cases (i), (ii) and (iii) are shown in columns 5 and 6, respectively, of Table 14.1.

Table 14.1: Computation of System Reliability and Percentage Increase in the Reliability of the System for cases (i), (ii) and (iii) of Example 2

	Reliability of the First Component	Reliability of the Second Component	Reliability of the Third Component	Reliability of the System	Percentage Increase in the Reliability of the System
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
	0.4	0.7	0.8	0.224	–
i)	0.5	0.7	0.8	0.280	$\frac{0.280-0.224}{0.224}\times 100 = 25\%$
ii)	0.4	0.8	0.8	0.256	$\frac{0.256-0.224}{0.224}\times 100 \simeq 14.29\%$
iii)	0.4	0.7	0.9	0.252	$\frac{0.252-0.224}{0.224}\times 100 = 12.5\%$

Note 2: From column 6 of Table 14.1, note that the improvement in the reliability of the system (in percentage) is higher when reliability of the weakest component is increased by 0.1 in comparison with the cases when reliabilities of the other two components are increased one at a time by the same amount (0.1). This suggests that if a system engineer wants to improve the reliability of the series system, he/she should **concentrate** on the **improvement** of the reliability of the **poorest component**. You may have heard a saying that a chain is only as strong as its weakest link. This saying best fits the series system. So we can say that the unit (component/subsystem), which is the poorest in the system from a reliability point of view, dictates the reliability of a series system.

Example 3: Consider a component having reliability 0.95 for a mission of 100 hours. Evaluate the reliability of a series system in the cases having 2, 3, 4,…,10 identical components having reliability 0.95 for the same mission of 100 hours.

Solution: If the single component of a system has reliability 0.95 for a mission of 100 hours, then the system will also have the same reliability for a mission of 100 hours. But the reliability of the series system for a mission of 100 hours will vary if it has 2, 3, 4, …,10 such identical components connected in series from a reliability point of view. In fact, the reliability of the system decreases with an increase in the number of its components. The calculations are shown in Table 14.2.

Table 14.2: Calculation of Reliability as the Number of Components varies from 1 to 10

Number of Components	Reliability of Series System	% Decrease in the Reliability of the System Compared to the System having Single Component
1	0.95	–
2	$(0.95)^2 = 0.9025$	$\frac{0.95-0.9025}{0.95}\times 100 = 5\%$
3	$(0.95)^3 = 0.857375$	$\frac{0.95-0.857375}{0.95}\times 100 = 9.75\%$
4	$(0.95)^4 \approx 0.814506$	$\frac{0.95-0.814506}{0.95}\times 100 \approx 14.26\%$
5	$(0.95)^5 \approx 0.773781$	$\frac{0.95-0.773781}{0.95}\times 100 \approx 18.55\%$
6	$(0.95)^6 \approx 0.735092$	$\frac{0.95-0.735092}{0.95}\times 100 \approx 22.62\%$
7	$(0.95)^7 \approx 0.698337$	$\frac{0.95-0.698337}{0.95}\times 100 \approx 26.49\%$
8	$(0.95)^8 \approx 0.663420$	$\frac{0.95-0.663420}{0.95}\times 100 \approx 30.17\%$
9	$(0.95)^9 \approx 0.630249$	$\frac{0.95-0.630249}{0.95}\times 100 \approx 33.66\%$
10	$(0.95)^{10} \approx 0.598737$	$\frac{0.95-0.598737}{0.95}\times 100 \approx 36.98\%$

Note 3: From the third column of Table 14.2, note that the reliability of a series system decreases as we increase the number of components in it.

Before we end this section, we also define the unreliability (Q) of a system. It is simply given by

$$Q = 1 - R \qquad \dots (6)$$

You may like to solve the following exercises to evaluate the reliability of a series system.

E1) Three components of a system are connected in series configuration from a reliability point of view. The reliabilities of these components for a mission of 200 days are 0.80, 0.95 and 0.96, respectively. Evaluate the reliability of the system for a mission of 200 days. Assume that the components are independent.

E2) Evaluate the unreliability of the system given in E1.

E3) The design of a system is such that it requires 100 identical components in series. Further it is desired that the reliability of the system must be 0.95 for a mission of 800 hours. Determine the minimum reliability of each component.

14.5 RELIABILITY OF A PARALLEL SYSTEM

In Sec. 14.4, you have learnt that the reliability of the series system is worse than the reliability of its poorest component. It means that we cannot improve the reliability of a series system with less reliable components. But this can be done in the case of parallel system, which is discussed in this section.

We have defined parallel configuration in Sec. 14.2. A parallel system is nothing but a system all of whose components are in parallel configuration. A parallel system made up of n components is represented as shown in Fig. 14.8 for the purpose of reliability evaluation.

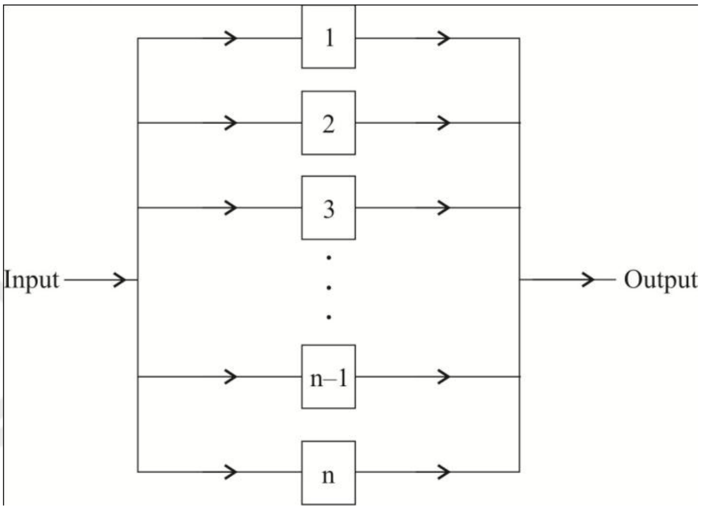

Fig. 14.8: Parallel system.

Note that Fig. 14.8 simply shows logical connectivity of the system. The actual physical connectivity of the components in the system may vary. But always remember the following two characteristics of a parallel system:

(1) Only one component needs to work successfully for the successful operation of the system, and

(2) All the components of the system must fail for system failure.

Let us now obtain an expression for the reliability evaluation of the parallel system.

Let E_i be the event that component i performs its intended function successfully, where i = 1, 2, 3, … , n. Let $\overline{E}_i$ (i = 1, 2, 3, …, n) denote the event complementary to the event E_i.

Let R_i denote the reliability of the component i, where i = 1, 2, 3, …, n for a mission of t units of time.

If R denotes the reliability of the parallel system for the mission of t units of time, then by definition of reliability and parallel system, we have

$$R = P\left(E_1 \cup E_2 \cup E_3 \cup ... \cup E_n\right) \qquad \text{… (7a)}$$

We take the union of events in equation (7a) because the definition of reliability implies that reliability is a probability and the definition of parallel system implies that for the successful operation of the parallel system only one component needs to work.

Further, from Unit 1 of MST-003 we know that for any event, say, A: $P(A) + P\left(\overline{A}\right) = 1$. Here $A = E_1 \cup E_2 \cup E_3 \cup ... \cup E_n$. Thus (7a) may be written as:

$$R = 1 - P\left(\overline{E_1 \cup E_2 \cup E_3 \cup ... \cup E_n}\right) \qquad \text{... (7b)}$$

Using De Morgan's law (which you have studied in Unit 1 of MST-001), we have

$$R = 1 - P\left(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap ... \cap \bar{E}_n\right) \qquad ... (7c)$$

If events $E_i, (i = 1, 2, ..., n)$ are not independent, then on applying conditional probability, we get

$$R = 1 - P\left(\bar{E}_1\right) P\left(\bar{E}_2 \middle| \bar{E}_1 \bar{E}_2\right) ... P\left(\bar{E}_n \middle| \bar{E}_1 \bar{E}_2 ... \bar{E}_{n-1}\right) \qquad ... (8a)$$

But if $E_i, (i = 1, 2, ..., n)$ are independent then $\bar{E}_i, (i = 1, 2, ..., n)$ are independent [this result has been proved in Unit 3 of MST-003]. Then, we get

$$R = 1 - P\left(\bar{E}_1\right) P\left(\bar{E}_2\right) P\left(\bar{E}_3\right) ... P\left(\bar{E}_n\right)$$

$$= 1 - \left(1 - P\left(E_1\right)\right)\left(1 - P\left(E_2\right)\right)\left(1 - P\left(E_3\right)\right) ... \left(1 - P\left(E_n\right)\right)$$

Since $R_i = P(E_i)$, therefore

$$R = 1 - \left(1 - R_1\right)\left(1 - R_2\right)\left(1 - R_3\right) ... \left(1 - R_n\right) \qquad ...(8b)$$

$$\text{or } R = 1 - \prod_{i=1}^{n} \left(1 - R_i\right) \qquad \text{for a mission of t units of time} \qquad ... (8c)$$

Thus, the reliability of the parallel system is equal to one minus the product of unreliabilities of the elementary components of the system.

In particular, if the parallel system has only two components, then

$$R = 1 - \left(1 - R_1\right)\left(1 - R_2\right) \qquad \text{for a mission of t units of time} \qquad ... (9)$$

and if the parallel system has three components, then

$$R = 1 - \left(1 - R_1\right)\left(1 - R_2\right)\left(1 - R_3\right) \qquad \text{for a mission of t units of time} \qquad ... (10)$$

Let us consider some examples for evaluating the reliability of parallel systems.

Example 4: Evaluate the reliability of the parallel system, which has the reliability block diagram shown in Fig. 14.9 for a mission of 90 days. The reliabilities of the components given in Fig. 14.9 are for a mission of 90 days. Assume that the failure of any component does not affect the functioning of the other components.

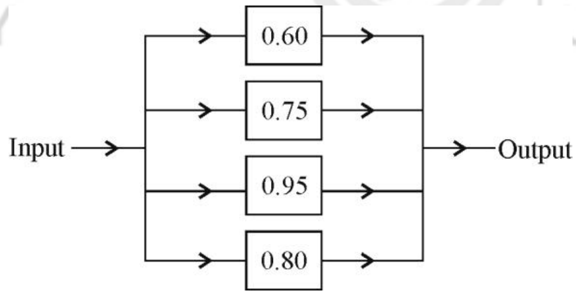

Fig. 14.9: Reliability block diagram for Example 4.

From equation (8b), we know that the reliability of a parallel system (R) having n components with reliabilities R_i, $i = 1, 2, 3, ..., n$, is given by

$$R = 1-(1-R_1)(1-R_2)(1-R_3)...(1-R_n)$$

In this case, $R_1 = 0.60, R_2 = 0.75, R_3 = 0.95$ and $R_4 = 0.80$.

$$\therefore R = 1-(1-0.60)(1-0.75)(1-0.95)(1-0.80)$$

$$= 1-0.40\times 0.25\times 0.05\times 0.20 = 1-0.001$$

$$= 0.999 \qquad \text{for a mission of 90 days}$$

Note 4: Note the answer of this example. It tells us that the reliability of the system is very good compared to the series system of Example 1 having components of the same reliability. Here the reliability of the system is even better than the reliability of the best component of the system. But you should keep in mind that for the successful operation of the parallel system only one component needs to work. So this improvement in the system reliability comes at the cost of an initial investment of three additional components. Thus, we have to balance the cost and reliability of a system. Whether to give importance to the cost or to reliability depends on the severity of consequence of system failure. For example, there should be at least one additional parallel engine in the aeroplane, whatever be the initial cost that the airline has to pay. So in this case reliability is more important than cost. But in the case of a component of, say TV, importance may be given to cost instead of reliability because failure of a TV has little consequence compared to the crash of an aeroplane, which may result in loss of lives of hundreds of people.

Let us see the effect of a component's reliability on the reliability of the parallel system with the help of the following example.

Example 5: A system has three components connected in parallel from a reliability point of view having reliabilities 0.20, 0.40, 0.50, respectively, for a mission of 400 hours. What is the percentage increase in the reliability of the system in each of the following cases?

(i) Reliability of the first component is increased by 0.1 and that of the second and third components remains the same.

(ii) Reliability of the second component is increased by 0.1 and that of the first and third components remains the same.

(iii) Reliability of the third component is increased by 0.1 and that of the first and second components remains the same.

Solution: We first calculate the reliability of the system as per the given reliabilities of the components using equation (8b):

$$R = 1-(1-R_1)(1-R_2)(1-R_3), \quad \text{where } R_1 = 0.2, R_2 = 0.4, R_3 = 0.5$$

$$= 1-(1-0.20)(1-0.4)(1-0.5) = 1-0.8\times 0.6\times 0.5 = 1-0.240$$

$$= 0.760 \qquad \text{for a mission of 400 hours}$$

Now, reliabilities of the system and percentage increase in the reliability of the system for the cases (i), (ii) and (iii) are shown in columns 5 and 6, respectively, in Table 14.3.

Table 14.3: Computation of System Reliability and Percentage Increase in the Reliability of the System for the Cases (i) , (ii) and (iii)

	Reliability of the I Component	Reliability of the II Component	Reliability of the III Component	Reliability of the System	Percentage Increase in the Reliability of the System
(1)	(2)	(3)	(4)	(5)	(6)
	0.2	0.4	0.5	0.760	–
i)	0.3	0.4	0.5	0.790	$\frac{0.790-0.760}{0.760}\times 100 \simeq 3.95\%$
ii)	0.2	0.5	0.5	0.800	$\frac{0.800-0.760}{0.760}\times 100 \simeq 5.26\%$
iii)	0.2	0.4	0.6	0.808	$\frac{0.808-0.760}{0.760}\times 100 \simeq 6.32\%$

Note 5: From column 6 of Table 14.3, you can see that the improvement in the reliability of the system (in percentage) is higher when reliability of the best component is increased by 0.1 as compared to the cases when reliabilities of the other two components are increased one at a time by the same amount (0.1). This suggests that if a system engineer wants to improve the reliability of a parallel system, he/she should **concentrate** on the **improvement** of the reliability of the **best component**. Thus, the best component dictates the reliability of the parallel system. Recall that the poorest component dictates the reliability of the series system.

Let us see the effect of the number of components on the reliability of a parallel system with the help of Example 6:

Example 6: Consider a component having reliability 0.75 for a mission of 100 hours. Evaluate the reliability of a parallel system when the system has 2, 3, 4, …, 10 identical components for the same mission of 100 hours.

Solution: If a system has a single component of reliability 0.75 for a mission of 100 hours, it will also have the same reliability for a mission of 100 hours. But the reliability of the parallel system for a mission of 100 hours will vary in the cases having 2, 3, 4 ,…,10 identical components connected in parallel from reliability point of view. In fact, it increases with the number of components as the calculations given in Table. 14.4 below show.

Table 14.4: Calculation of Reliability as Number of Components varies from 1 to 10

Number of Components	Reliability of Parallel System	% Increase in the Reliability of the System Compared to the System having Single Component
1	0.75	–
2	$1-(1-0.75)^2 = 0.9375$	$\frac{0.9375-0.75}{0.75}\times 100 = 25\%$
3	$1-(1-0.75)^3 = 0.984375$	$\frac{0.984375-0.75}{0.75}\times 100 = 31.25\%$
4	$1-(1-0.75)^4 \simeq 0.996094$	$\frac{0.996094-0.75}{0.75}\times 100 \simeq 32.81\%$
5	$1-(1-0.75)^5 \simeq 0.999023$	$\frac{0.999023-0.75}{0.75}\times 100 \simeq 33.20\%$
6	$1-(1-0.75)^6 \simeq 0.999756$	$\frac{0.999756-0.75}{0.75}\times 100 \simeq 33.30\%$
7	$1-(1-0.75)^7 \simeq 0.999939$	$\frac{0.999939-0.75}{0.75}\times 100 \simeq 33.33\%$
8	$1-(1-0.75)^8 \simeq 0.999985$	$\frac{0.999985-0.75}{0.75}\times 100 \simeq 33.33\%$
9	$1-(1-0.75)^9 \simeq 0.999996$	$\frac{0.999996-0.75}{0.75}\times 100 \simeq 33.33\%$
10	$1-(1-0.75)^{10} \simeq 0.999999$	$\frac{0.999999-0.75}{0.75}\times 100 \simeq 33.33\%$

Note 6: From the third column of Table 14.4, you can see that the reliability of a parallel system increases by a great extent on the addition of the first redundant component to the one-component system as compared to the addition of second redundant component, third redundant component, and so on. The same column further suggests that to provide more than two redundant components for a one-component system from a reliability point of view is not of much benefit. That is why an aeroplane which requires two engines for successful operation is generally provided two redundant engines to improve the reliability for successful operation.

Example 7: Suppose you are an industrial statistician in a company. Suppose a system engineer visits you and explains his/her problem to you as follows:

'I want to design a parallel system with an overall reliability of 0.98 by using identical components, each having individual reliability of 0.25. What is the minimum number of components that I should connect?'

What will your answer be?

Solution: Let R_s be the overall reliability of the system and R, the reliability of each component. If the required number of components is n, then from equation (8b), we have

$$R_s = 1-(1-R)^n$$

$$\Rightarrow 0.98 = 1-(1-0.25)^n \Rightarrow (1-0.25)^n = 1-0.98 \Rightarrow (0.75)^n = 0.02$$

Taking natural logarithm on both sides, we get

$n \ln(0.75) = \ln(0.02)$ $\qquad \left[\because \ln(m^n) = n(\ln m)\right]$

$\Rightarrow -0.2877n \approx -3.9120$ $\qquad$ [Using scientific calculator]

$$\Rightarrow n \approx \frac{3.9120}{0.2877} \approx 13.5975$$

Since the number of components cannot be in fraction, he/she should connect 14 components in parallel to achieve a reliability of 0.98.

You can now try the following exercises to evaluate the reliability of a parallel system.

E4) A system has three components, which are connected in parallel configuration from a reliability point of view. The reliabilities of these components for a mission of 300 days are 0.60, 0.55 and 0.70, respectively. Evaluate the reliability of the system for a mission of 300 days. Assume that the components are independent.

E5) Evaluate the unreliability of the system given in E4.

E6) Suppose you are an industrial statistician in a company. Suppose a system engineer visits you and explains his/her simple problem to you as follows:

'I want to design a parallel system with an overall reliability of the system as 0.95 for a mission of 900 hours. The system will have 10 identical components. How poor can a component in the system be'?

Assuming that the components are independent, what will your answer be?

In Secs. 14.4 and 14.5, you have learnt how to evaluate the reliability of series and parallel systems, respectively. But in practice, there are many real systems having components connected in series as well as in parallel. Such systems are known as mixed systems and we discuss them in the next section.

14.6 RELIABILITY OF A MIXED SYSTEM

A system is said to be a mixed system if the components of the system are connected both in series and in parallel configurations. The reliability block diagram shown in Fig. 14.5 corresponds to a mixed system where components 1, 2, 3 and 4, 5 are separately connected in series while the two subsystems – one consisting the components 1, 2, 3 and another consisting the components 4, 5 themselves – are in parallel.

To evaluate the reliability of a mixed system, we first break the reliability block diagram into series or parallel subsystems. Then we evaluate the reliability of each subsystem. Finally, we evaluate the reliability of the given mixed system by combining the reliabilities of the subsystems and applying appropriate probability law(s). The procedure is explained below with the help of an example.

Example 8: Evaluate the reliability of the system for which the reliability block diagram is shown in Fig. 14.10, for a mission of 100 hours. Assume that all components are independent and the reliability of each component is given for a mission of 100 hours as follows:

$R_1 = 0.80$, $R_2 = 0.75$, $R_3 = 0.50$, $R_4 = 0.65$, $R_5 = 0.76$, $R_6 = 0.60$, $R_7 = 0.95$, $R_8 = 0.90$,

where R_i denotes the reliability of the component i, (i = 1, 2, 3, …,8).

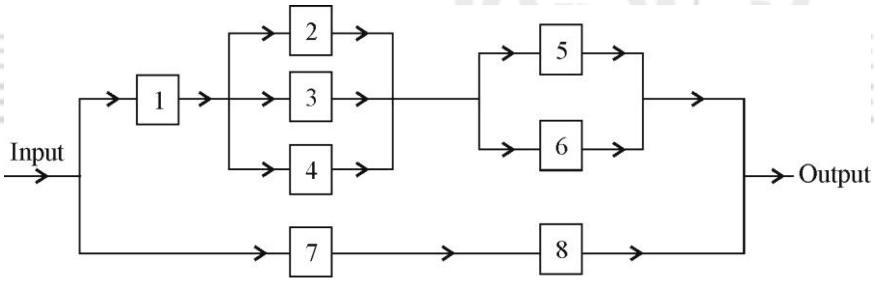

Fig. 14.10: Reliability block diagram for Example 8.

Solution: The components of the given system are connected in both series and parallel configuration. So it is a mixed system. To evaluate the reliability of this mixed system, we have to break the system into subsystems such that all components of a subsystem are either in series or in parallel. This can be done as follows:

- **Reduction I:** Combine the components 2, 3, 4 in parallel configuration to form an equivalent component 9 (see reduction I in Fig. 14.11).
- **Reduction II:** Combine the components 5, 6 in parallel configuration to form an equivalent component 10 (see reduction II in Fig. 14.11).
- **Reduction III:** Combine the components 1, 9, 10 in series configuration to form an equivalent component 11 (see reduction III in Fig. 14.11).

- **Reduction IV:** Combine the components 7, 8 in series configuration to form an equivalent component 12 (see reduction IV in Fig. 14.11).
- **Reduction V:** Finally, combine the components 11 and 12 in parallel configuration to form an equivalent component 13 (see reduction V in Fig. 14.11). The component 13 represents the complete system.

The step by step reduction process is shown in Fig. 14.11.

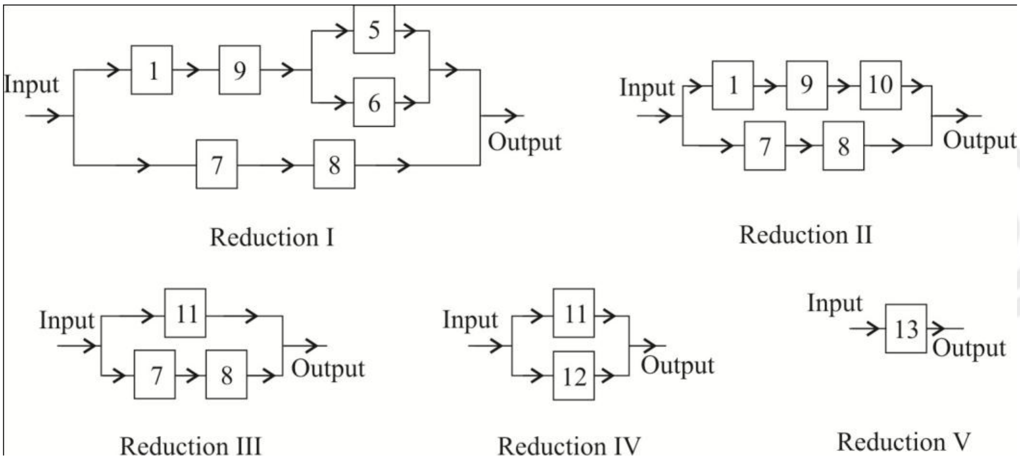

Fig. 14.11: Reductions I to V of Fig. 14.10.

We are given that $R_1 = 0.80$, $R_2 = 0.75$, $R_3 = 0.50$, $R_4 = 0.65$, $R_5 = 0.76$, $R_6 = 0.60$, $R_7 = 0.95$, $R_8 = 0.90$ where R_i denotes the reliability of the given component i, (i = 1, 2, 3,…, 8). Similarly, if R_9 to R_{13} denote the reliabilities of the equivalent components 9 to 13, then using equations (8b and 4a), we have

$$R_9 = 1-(1-R_2)(1-R_3)(1-R_4) \qquad \left[\begin{array}{l}\because \text{ components } 2,3,4 \text{ are} \\ \text{in parallel configuration}\end{array}\right]$$

$$= 1-(1-0.75)(1-0.50)(1-0.65)$$

$$= 1-0.25\times 0.50\times 0.35 \approx 1-0.0438 = 0.9562$$

$$R_{10} = 1-(1-R_5)(1-R_6) \qquad \left[\begin{array}{l}\because \text{ components } 5,6 \text{ are} \\ \text{in parallel configuration}\end{array}\right]$$

$$= 1-(1-0.76)(1-0.60)$$

$$= 1-0.24\times 0.40 = 1-0.096 = 0.904$$

$$R_{11} = R_1 R_9 R_{10} \qquad \left[\begin{array}{l}\because \text{ components } 1,9,10 \text{ are} \\ \text{in series configuration}\end{array}\right]$$

$$\approx 0.80\times 0.9562\times 0.904 \approx 0.6915$$

$$R_{12} = R_7 R_8 \qquad \left[\begin{array}{l}\because \text{ components } 7,8 \text{ are} \\ \text{in series configuration}\end{array}\right]$$

$$= 0.95\times 0.90 = 0.855$$

$$R_{13} = 1-(1-R_{11})(1-R_{12}) \qquad \left[\begin{array}{l}\because \text{ components } 11,12 \text{ are} \\ \text{in parallel configuration}\end{array}\right]$$

$$\approx 1-(1-0.6915)(1-0.855) = 1-0.3085\times 0.145 \approx 1-0.0447 = 0.9553$$

Hence, the reliability of the given mixed system is 0.9553.

You can now try the following exercise to evaluate the reliability of a mixed system.

E7) Evaluate the reliability of the system for which the reliability block diagram is shown in Fig. 14.12, for a mission of 500 hours. Assume that all components are independent. The reliability of each component is given below for a mission of 500 hours:

$R_1 = 0.40$, $R_2 = 0.30$, $R_3 = 0.60$, $R_4 = 0.80$, $R_5 = 0.85$, $R_6 = 0.60$,

$R_7 = 0.70$, $R_8 = 0.95$, $R_9 = 0.96$.

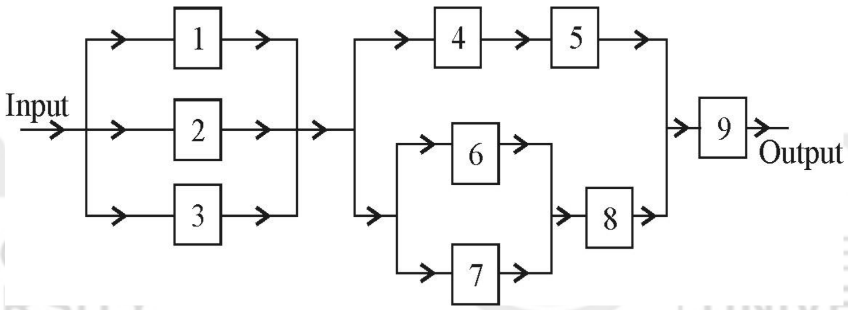

Fig. 14.12: Reliability block diagram for E7.

Let us summarise the main points that we have covered in this unit.

14.7 SUMMARY

1. An assemblage of, say n, identifiable components that perform some function is known as a **system** in reliability.

2. A system is said to have its components in **series configuration** from a reliability point of view if they are connected in such a way that:

   - for the successful operation of the system, it is necessary that all n components perform their intended function successfully, and

   - only one component needs to fail for the system failure.

3. A system is said to have its components in **parallel configuration** from a reliability point of view if they are connected in such a way that:

   - only one component needs to work for the successful operation of the system, and

   - all components must fail for system failure.

4. A system is said to be **simple** if either its components are connected in parallel, in series or in combinations of both. In other words, a system is said to be simple if its reliability block diagram can be reduced into subsystems having independent components either in parallel or in series.

5. In a **series system,** all components of the system are in series configuration.

6. In a **parallel system,** all components of the system are in parallel configuration.

7. In a **mixed system,** the components of the system are connected both in series and in parallel.

14.8 SOLUTIONS/ANSWERS

E1) The reliability block diagram of the given series system is shown in Fig. 14.13.

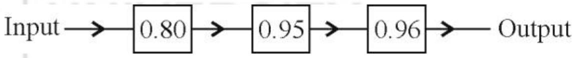

Fig.14.13: Reliability block diagram for E1.

We know that reliability of a series system is equal to the product of the individual reliabilities of the components [see equation (4a)].

Therefore, the required reliability of the series system is given by

$R = 0.80 \times 0.95 \times 0.96$ for a mission of 200 days

$= 0.7296$ for a mission of 200 days

E2) From equation (6), we know that unreliability (Q) of a system is given by.

$Q = 1 - R$

$\Rightarrow Q = 1 - 0.7296$ [using the result of E1]

$\Rightarrow Q = 0.2704$

Therefore, the probability that the system will fail before a mission of 200 days is 0.2704.

E3) Since all 100 components are identical, each component has the same reliability. Let it be denoted by R. If R_s denotes the reliability of the series system, then from equation (4a), we have

$R_s = R^{100}$

$\Rightarrow 0.95 = R^{100}$

$\Rightarrow R = (0.95)^{1/100}$

≈ 0.9995 [using scientific calculator]

E4) The reliability block diagram of the given parallel system is shown in Fig. 14.14.

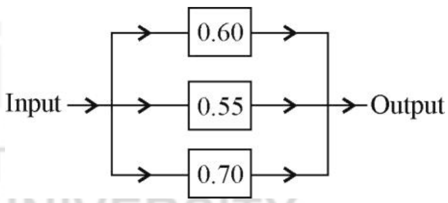

Fig. 14.14: Reliability block diagram for E4.

We know that the reliability (R) of a parallel system having n components with reliabilities R_i, i = 1, 2, 3, …, n is given by equation (8b) as

$$R = 1 - (1 - R_1)(1 - R_2)(1 - R_3)\ldots(1 - R_n)$$

In this case, $R_1 = 0.60$, $R_2 = 0.55$, $R_3 = 0.70$ and n = 3.

$\therefore R = (1 - 0.60)(1 - 0.55)(1 - 0.70) = 1 - 0.40 \times 0.45 \times 0.30 = 1 - 0.054$

$= 0.946$ for a mission of 300 days

E5) We know that the unreliability (Q) of a system is given by

$Q = 1 - R$

or $Q = 1 - 0.946$ [using result of E4]

$= 0.054$

Therefore, the probability that the system will fail before a mission of 300 days is 0.054.

E6) Let the reliability of each component be denoted by R and that of the system by R_s. We know that the reliability (R) of a parallel system having n components with reliabilities R_i, i = 1, 2, 3, …, n is given by equation (8b) as

$$R = 1 - (1 - R_1)(1 - R_2)(1 - R_3)\ldots(1 - R_n)$$

In this case $R = R_s$, $R_1 = R_2 = R_3 = \ldots = R_n = R$ and n = 10

$$\therefore R_s = 1 - (1 - R)^{10}$$

$$\Rightarrow 0.95 = 1 - (1 - R)^{10}$$

$$\Rightarrow (1 - R)^{10} = 1 - 0.95 \quad \text{as } R_s = 0.95(\text{given})$$

$$\Rightarrow (1 - R)^{10} = 0.05$$

$$\Rightarrow 1 - R = (0.05)^{1/10}$$

$\Rightarrow 1 - R \approx 0.7411$ [Using scientific calculator]

$\Rightarrow R \approx 1 - 0.7411 = 0.2589$

Hence, your answer to the system engineer should be that each component cannot be poorer than a component having reliability 0.2589.

E7) The components of the given system are connected both in series and in parallel. So it is a mixed system. To evaluate the reliability of this mixed system, we have to break the system into subsystems such that either all the components of a subsystem are in series or in parallel. This can be done as follows.

Reduction I: Combine the components 1, 2, 3 in parallel configuration to form an equivalent component 10 (see reduction I in Fig. 14.15).

Reduction II: Combine the components 4, 5 in series configuration to form an equivalent component 11 (see reduction II in Fig. 14.15).

Reduction III: Combine the components 6, 7 in parallel configuration to form an equivalent component 12 (see reduction III in Fig. 14.15).

Reduction IV: Combine the components 12, 8 in series configuration to form an equivalent component 13 (see reduction IV in Fig. 14.15).

Reduction V: Combine the components 11, 13 in parallel configuration to form an equivalent component 14 (see reduction V in Fig. 14.15).

Reduction VI: Combine the components 10, 14, 9 in series configuration to form an equivalent component 15 (see reduction VI in Fig. 14.15). The component 15 represents the complete system.

This step by step reduction process is shown in Fig. 14.15.

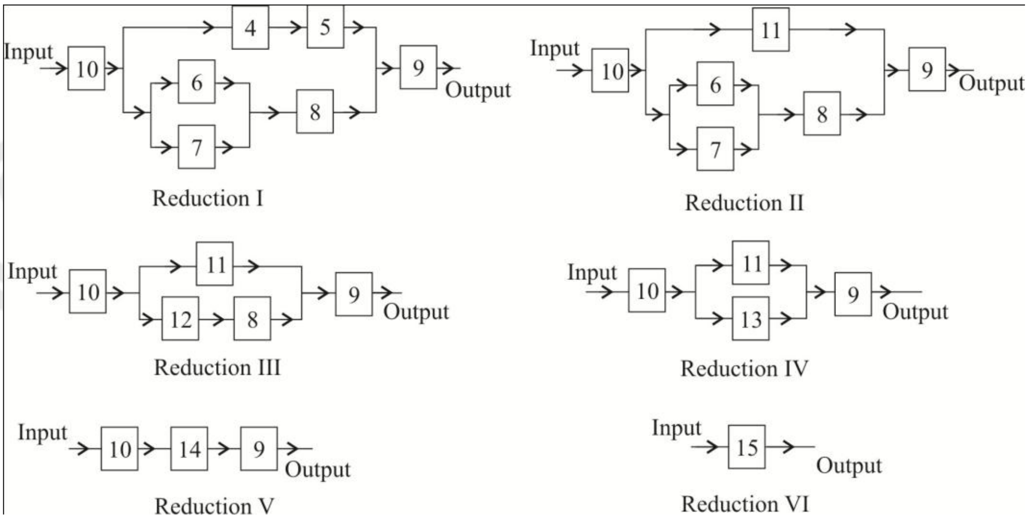

Fig. 14.15: Reliability block diagram for E7.

We are given that $R_1 = 0.40$, $R_2 = 0.30$, $R_3 = 0.60$, $R_4 = 0.80$, $R_5 = 0.85$, $R_6 = 0.60$, $R_7 = 0.70$, $R_8 = 0.95$, $R_9 = 0.96$.

Similarly, if R_{10} to R_{15} denote the reliabilities of the equivalent components 10 to 15, then from equations (4a) and (8b), we have

$$R_{10} = 1-(1-R_1)(1-R_2)(1-R_3) \qquad \begin{bmatrix} \because \text{ components } 1,2,3 \text{ are} \\ \text{in parallel configuration} \end{bmatrix}$$

$$= 1-(1-0.40)(1-0.30)(1-0.60)$$

$$= 1-0.60\times0.70\times0.40 = 1-0.168 = 0.832$$

$$R_{11} = R_4R_5 \qquad \begin{bmatrix} \because \text{ components } 4,5 \text{ are} \\ \text{in series configuration} \end{bmatrix}$$

$$= 0.80\times0.85 = 0.68$$

$$R_{12} = 1-(1-R_6)(1-R_7) \qquad \begin{bmatrix} \because \text{ components } 6,7 \text{ are} \\ \text{in parallel configuration} \end{bmatrix}$$

$$= 1-(1-0.60)(1-0.70)$$

$$= 1-0.40\times0.30 = 1-0.12 = 0.88$$

$$R_{13} = R_{12}R_8 \qquad \begin{bmatrix} \because \text{ components } 12,8 \text{ are} \\ \text{in series configuration} \end{bmatrix}$$

$$= 0.88\times0.95 = 0.836$$

$$R_{14} = 1-(1-R_{11})(1-R_{13}) \qquad \left[\because \begin{array}{l}\text{components 11,13 are} \\ \text{in parallel configuration}\end{array}\right]$$

$$= 1-(1-0.68)(1-0.836)$$

$$= 1-0.32\times 0.164 = 1-0.05248 = 0.94752$$

$$R_{15} = R_{10}R_{14}R_9 \qquad \left[\because \begin{array}{l}\text{components 10,14,9 are} \\ \text{in series configuration}\end{array}\right]$$

$$= 0.832\times 0.94752\times 0.96 = 0.7568032$$

Hence, reliability of the given system is 0.7568032 for a mission of 500 hours.