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**ANALYSIS OF VARIANCE**

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## ANALYSIS OF VARIANCE

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The impact of the ever increasing number of applications of analysis of variance in the field of Physical Sciences, Life Sciences, Horticulture, Agriculture, Engineering, Management, Medical Sciences, Pharmaceutical and Social Science makes it indispensable that uses of these disciplines are offered a basic training and knowledge of Statistics specially analysis of variance. This block on analysis of variance is designed in such a way to serve as a basic course on text, meeting the sufficient requirements of a learner or researcher of the above discipline.

It is a fact that before understanding the concept of analysis of variance, one has to have some idea or basic knowledge of Statistics and procedure of estimation and testing of hypothesis. This course is basically designed for applied purpose. So, each concept is introduced in an intuitive way. The relevance of all the procedures and techniques is derived by considering different types of examples. The aim of this course is to enable you to successfully handle and solve any statistical problem to analysis of variance in your field or discipline. We have included many practical exercises and examples, solved and unsolved, on various places in the course.

This block is divided into four units. In Unit 5, we introduced the concept of analysis of variances, its definition, terminology used, assumptions of analysis of variance and its uses. The concept of linear models in analysis of variance technique is also explained in this unit.

In Unit 6, one-way classified data with assumptions and its analysis is explained. The basic assumptions are explored and the expectation of various sums of squares is also derived in this unit.

The procedure for analysis of two-way classified data is described in Unit 7. We shall be dealing with analysis of variance technique in two-way classified data with  $m$ -observations per cell in Unit 8.

It is advised that the learners should do a lot of practice for commanding on the techniques and the exercises, given in the block, by using calculator or computer. If you are interested to learn more, you may look up or consider or consult some more books on the subject analysis of variance or you can see on the internet.

### **Suggested Readings:**

- Goon, A. M., Gupta, M. K. and Das Gupta, B.; Fundamentals of Statistics, Vol II, World Press, Calcutta.
- Gupta, S. C. and Kapoor, V. K.; Fundamentals of Applied Statistics, Sultan Chand & Sons.
- Goulden, C. H.; Methods of Statistical Analysis (Ch. 5), Asia Publishing House, 1959.
- Guenther, W. C.; The Analysis of Variance, Prentice-Hall, 1964.
- Scheffe, H.; The Analysis of Variance (Chs. 3, 4, 7, 8), John Wiley, 1961.

## Notations and Symbols

$Y$	: Response variable/ Dependent variable
$X$	: Explanatory variable/Independent variable/Predictor variable/Treatment/Factor/Effect
$\bar{y}$	: Mean of response variable
$\bar{x}$	: Mean of explanatory variable
$y_{ij}$	: $j^{\text{th}}$ observation in the $i^{\text{th}}$ level of a factor A
$y_{ijk}$	: $k^{\text{th}}$ observation under $i^{\text{th}}$ level of factor A and $j^{\text{th}}$ level of factor B
$\mu$	: An over all mean or Grand mean
$\mu_i$	: Mean of $i^{\text{th}}$ level of a factor A
$\alpha_i$	: Effect of $i^{\text{th}}$ level of a factor A
$e_{ij}$	: Error term
$n_i$	: Number of observations in $i^{\text{th}}$ level of a factor
$E$	: Residual sum of squares
$H_0$	: Null hypothesis
$H_1$	: Alternative hypothesis
$df/DF$	: Degrees of freedom
$V(e_{ij})$	: Variance of error $e_{ij}$
$F$	: F- statistic / Variation ratio
$\alpha$	: Level of significance
$CD$	: Critical difference
$\beta_j$	: Effect due to $j^{\text{th}}$ level of factor B
$(\alpha\beta)_{ij}$	: Interaction effect between $i^{\text{th}}$ level of factor A and $j^{\text{th}}$ level of factor B
$p$	: Number of levels of factor A
$q$	: Number of levels of factor B
$SSA$	: Sum of Squares due to factor A
$SSB$	: Sum of Squares due to factor B
$TSS$	: Total Sum of Squares
$SST$	: Sum of Squares due to Treatments or different levels of a factor
$SSE$	: Sum of Squares due to Error or Residual
$N(\mu, \sigma^2)$	: Normally distributed with mean ( $\mu$ ) and variance ( $\sigma^2$ )
i.i.d.	: Independently and identically distributed.
$RSS$	: Raw Sum of Squares
$CF$	: Correction Factor
$SS$	: Sum of Squares
$MSS$	: Mean Sum of Squares
$SV$	: Source of Variation