UNIT 7 SEQUENCING PROBLEMS

Structure

7.1 INTRODUCTION

In Unit 6, we have discussed the queueing system wherein customers are served after waiting in a queue for some time. In fact, waiting for service is an integral part of our daily life and that too at considerable cost most of the time. The purpose of queueing analysis is to provide information to determine an acceptable level of service. However, the appropriate order in which the waiting customers should be served also needs to be studied to minimise the total elapsed time. This aspect of the problem is not considered in queueing theory. It is covered in the study of sequencing problems, which are discussed in this unit.

Sequencing problems are quite common in real life. They arise whenever there is a choice of determining the order in which a number of jobs can be performed. These problems play a very important role in manufacturing set-ups for the optimal use of resources and/or the customer's satisfaction. Many-a-times, manufacturing firms produce goods made up of several components, which are sourced from other manufacturing firms. The final product undergoes many processes before it is taken to the market for sale. Since several machines are used in the production process, it is important to sequence the production process optimally so that the performance measures of the firm are satisfied.

In Sec.7.2, we discuss some procedures that help in determining the optimum order or sequence of jobs for a process. In Secs.7.3 and 7.4, we discuss how to solve the sequencing problems with n number of jobs to be completed through 2 machines and with 2 jobs to be completed through m machines, in some pre-assigned order, so as to optimise the total time involved.

In the next unit, we shall discuss inventory control and various factors, involved in inventory analysis. We shall also describe the models for determining the economic order quantity considering the situations: i) when demand is uniform; ii) when rates of demand are different in different cycles; iii) when shortages are allowed; iv) when replenishment is uniform, and v) when price (or quantity) discounts are given.

Objectives

After studying this unit, you should be able to:

- explain what a sequencing problem is;
- determine the appropriate order for the jobs to be done so as to minimise the elapsed time for 2-Machine, n-Job problems;

- determine the appropriate order for the jobs to be done so as to minimise the elapsed time for 2-Job, m-Machine problems; and
- obtain the minimum elapsed times for sequencing problems.

7.2 SEQUENCING PROBLEMS

In this section, we discuss the sequencing problems, which occur commonly in our real life whenever n number of tasks are to be performed in a predefined time schedule. The aim is to determine the best available possible sequences of jobs. A sequencing problem is the determination of the best sequence among all possible sequences. Here 'best' may be defined with respect to set objectives or performance measure for the concerned problem.

Let us consider a common experience that most grown-up men face everyday: whether to shave first and then take a bath, or to shave after taking bath. Clearly, the problem is to decide on the sequence (or order) of the two tasks: shaving and bathing. Now the choice lies between two alternative sequences, i.e., (1. Shaving, 2. Bathing) and (1. Bathing, 2. Shaving). The first option ensures more cleanliness, whereas the second one gives rise to ease in shaving. The solution to the problem depends on the objective or performance measure, i.e., cleanliness or ease in shaving, of the concerned person. Once the performance measure is specified, the solution is easy.

The selection of the appropriate order in which a number of jobs (customers) are to be performed (served) is called sequencing. In sequencing problems, there are two or more customers (jobs) to be served and one or more facilities (machines) available for this purpose. We wish to know when each job is to begin and what its due date or time is. We also wish to know which facilities are required to do each job, in which order these facilities are required and how long each operation is to take.

The following assumptions are usually made while dealing with sequencing problems:

i) Only one operation is carried out on a machine at a time.

ii) Processing times are known and do not change.

iii) Processing times are independent of the order of processing the jobs.

iv) The time involved in moving jobs from one machine to another is negligible.

v) Each operation, once started, must be completed.

vi) An operation must be completed before its succeeding operation can start.

vii) Only one machine of each type is available.

viii) A job is processed as soon as possible, but only in the order specified.

ix) No passing rule is followed strictly, i.e., the same order of jobs is maintained over each machine.

Sequencing problems have been most commonly encountered in production shops where different products are to be processed over various combinations of machines. A general sequencing problem may be defined as follows:

Let there be n jobs (1, 2, 3, . . . , n), each of which has to be processed, one at a time, on each of m machines (A, B, C, …). The time required for processing each job on each machine is also given. The order of processing each job through these machines may be any of the $(n!)^m$ possible sequences. But, we are interested in finding that technologically feasible sequence for processing the jobs which gives the minimum total elapsed time for all the jobs. For example, suppose we have a problem of 2 jobs (J_1, J_2) and 2 machines (A, B) with the following processing times (in hours):

Job	:	J_1	J_2
Machine A	:	2	4
Machine B	:	3	6

The possible sequences for processing of the jobs will be $(2!)^2 = 4$, which are:

i) First process J_1, J_2 in this order on machines A and B, respectively;

ii) First process J_1, J_2 in this order on machines B and A, respectively;

iii) First process J_2, J_1 in this order on machines A and B, respectively;

iv) First process J_2, J_1 in this order on machines B and A, respectively.

For the **first sequence**, the various times can be calculated as in the following table:

Jobs	**Machine A**		**Machine B**	
	Time in	Time out	Time in	Time out
J_1	0	2	2	2 + 3 = 5
J_2	2	2 + 4 = 6	6	6 + 6 = 12

Here, the total elapsed time is 12 hours.

The idle time for machine A

= Total elapsed time – Time when the last job is out of the machine A,
= 12 – 6 = 6 hrs

The idle time for machine B

= Time of starting the first job at B

+ (Time of starting the second job at B

– time of finishing the first job at B)

= 2 + (6 – 5) = 3 hrs

For the **second sequence**, the various times can be calculated as in the following table:

Jobs	**Machine B**		**Machine A**	
	Time in	Time out	Time in	Time out
J_1	0	3	3	3 + 2 = 5
J_2	3	3 + 6 = 9	9	9 + 4 = 13

Here, the total elapsed time is 13 hours.

The idle time for machine B

= Total elapsed time – Time when the last job is out of machine B

= 13 – 9 = 4 hrs

The idle time for machine A

= Time of starting the first job at A
+ (Time of starting the second job at A
– time of finishing the first job at A)
= 3 + (9 – 5) = 7 hrs

For **the third sequence**, the various times can be calculated as in the following table:

Jobs	**Machine A**		**Machine B**	
	Time in	Time out	Time in	Time out
J_2	0	4	4	4 + 6 = 10
J_1	4	4 + 2 = 6	10	10 + 3 = 13

Here, the total elapsed time is 13 hours.

The idle time for machine A

= Total elapsed time – Time when the last job is out of the machine A

= 13 – 6 = 7 hrs

The idle time for machine B

= Time of starting the first job at machine B
+ (Time of starting the second job at machine B
– time of finishing the first job at machine B)
= 4 + (10 –10) = 4 hrs

For **the fourth sequence**, the various times can be calculated as in the following table:

Jobs	**Machine B**		**Machine A**	
	Time in	Time out	Time in	Time out
J_2	0	6	6	6 + 4 = 10
J_1	6	6 + 3 = 9	10	10 + 2 = 12

Here, the total elapsed time is 12 hours.

The idle time for machine B

= Total elapsed time – Time when the last job is out of machine B

= 12 – 9 = 3 hrs

The idle time for machine A

= Time of starting the first job at machine A

+ (Time of starting the second job at machine A

– time of finishing the first job at machine A)

= 6 + (10 – 10) = 6 hrs

So, in the above example, the technologically feasible sequence for processing the jobs, which gives the minimum total elapsed time for all the jobs is either the first sequence or the fourth sequence. Idle times of the machines are also less for these sequences. The processing, therefore, should be done adopting either the first or the fourth sequence.

7.3 PROCESSING OF n JOBS THROUGH 2 MACHINES

The n-job, 2-machine sequencing problem is described as follows:

Let there be n jobs where each job is to be processed by two machines, say A and B. Each job is processed in a pre-specified order AB, i.e., each job goes to machine A first and then to machine B. In other words, passing is not allowed. The actual or expected processing times for the i^{th} job on machine A and machine B, respectively, are known where i= 1, 2, 3, …, n.

The problem is now to determine the sequence (order) of jobs which minimises the total elapsed time (T) for processing all the jobs from the start of the first job to the completion of the last job. The procedure suggested by S.M. Johnson and R. Bellman for finding such a sequence consists of the following steps:

Step 1: Select the minimum processing time among A_i and B_i (i = 1, 2, . . ., n) where A_i and B_i are the processing times of i^{th} job on machines A and B, respectively. If there are two or more minimum processing times, then select any one of them arbitrarily.

Step 2: a) If the minimum processing time is one of A_i, say A_r, then process the job r first and place it at the beginning of the sequence.

b) If the minimum processing time is one of B_i, say B_s, then process the job s last and place it at the end of the sequence.

c) If in step 2(a) and 2(b) above, $A_r = B_s$, then process job r first and job s last.

d) If there is a tie for minimum among A_i (i = 1, 2, …, n), then process any one of these jobs first.

e) If there is a tie for a minimum among B_i (i = 1, 2, …, n), then process any one of these jobs last.

Step 3: Eliminate the job which has already been assigned from further consideration and repeat Steps 1 and 2, placing the remaining jobs next to the first job or before the last job as the case may be.

Calculations of Various Times

1. The time at which the i^{th} job in a sequence finishes on machine A

   = Time at which the $(i – 1)^{th}$ job in a sequence finishes on machine A

   + Processing time for the i^{th} job on machine A for i = 1, 2, . . ., n

2. The time when the i^{th} job in a sequence starts on machine A

   = Time when the $(i – 1)^{th}$ job in a sequence finishes on machine A

3. The time the first job in a sequence starts on machine B

   = Time when the first job in a sequence finishes on machine A

4. The time the i^{th} job in a sequence finishes on machine B

   = Time when the i^{th} job in a sequence starts on machine B

   + Processing time of the i^{th} job on machine B for i = 1, 2, . . . , n

5. The time at which the $(i + 1)^{th}$ job in a sequence starts on machine B

   = Max{Time when the $(i+1)^{th}$ job in a sequence finishes at machine A; time when the i^{th} job in a sequence finishes at machine B}

6. Total elapsed time

   = Time when the n^{th} job in a sequence finishes on machine B

7. Idle time for Machine A

   = (Time when the n^{th} job in a sequence finishes on machine B)

   – (Time when the n^{th} job in a sequence finishes on machine A)

   OR

   = (Total elapsed time)

   – (Time when the last job in the sequence is out of machine A)

8. Idle time for Machine B

   =Time at which the first job in a sequence finishes on machine A

   $+ \sum_{i=2}^{n}$ [(Time when the i^{th} job in a sequence starts on machine B)

   – (Time when the $(i – 1)^{th}$ job in a sequence finishes on machine B)]

Let us consider an example to explain this sequencing process.

Example 1: There are 5 jobs, each of which has to go through the machines A and B in the order AB. The processing times (in hours) are given as

Job	:	J_1	J_2	J_3	J_4	J_5
Machine A	:	2	4	5	7	1
Machine B	:	3	6	1	4	8

Determine a sequence of these jobs that will minimise the total elapsed time T. Also obtain:

i) the minimum elapsed time; and

ii) the idle time for each of the machines.

Solution: From the given information, we notice that the minimum processing time is 1 hour, which occurs for two jobs J_3 and J_5. For J_3, it is on machine B and for J_5, it is on machine A. Thus, job J_5 is placed first in the sequence and job J_3, the last. The partial allocation for the jobs will appear as follows:

J_5				J_3

The two jobs J_3 and J_5 are now deleted from further consideration. The reduced set of processing times will be as follows:

Job	:	J_1	J_2	J_4
Machine A	:	2	4	7
Machine B	:	3	6	4

In the above set, the minimum processing time is 2 hours for job J_1 on machine A. Hence, this job is placed next to the first job (i.e., next to J_5) in the sequence. The allocation of jobs till this stage would be as follows:

J_5	J_1			J_3

Proceeding as above, the optimal sequence will be as follows:

J_5	J_1	J_2	J_4	J_3

The various times can now be calculated as in the following table:

Jobs	**Machine A**		**Machine B**	
	Time in	Time out	Time in	Time out
J_5	0	1	1	9
J_1	1	3	9	12
J_2	3	7	12	18
J_4	7	14	18	22
J_3	14	19	22	23

a) Thus, the total elapsed time = 23 hours

b) The idle time for machine A
= (Total elapsed time) – (Time when the last job is out of machine A)
= 23 – 19 = 4 hours

c) The idle time for machine B
= Time of starting of first job at B
+ Total of time of starting of 2nd job onwards
– Total of time of completion of 1st job to $(n – 1)^{th}$ job
= 1 + (9 + 12 + 18 + 22) – (9 + 12 + 18 + 22) = 1 hour

You may now like to solve the following exercises to check your understanding.

E1) A ready-made garments manufacturer has to process five items through 2 stages of production, viz. cutting and sewing. The time taken for each of these items at the different stages is given below (in hours):

	Item :	1	2	3	4	5
Processing time (hours) {	**Cutting**:	5	7	3	4	6
	Sewing :	2	6	7	5	9

Find an order in which these items should be processed so as to minimise the total processing time. Also calculate the various idle times.

E2) There are 7 jobs, each of which has to go through the machines A and B in the order AB. Processing times (in hours) are given as follows:

Job :	1	2	3	4	5	6	7
Machine A :	3	12	15	6	10	11	9
Machine B :	8	10	10	6	12	1	3

Determine a sequence of these jobs that will minimise the total elapsed time. Also calculate the various idle times.

7.4 PROCESSING OF 2 JOBS THROUGH m MACHINES

For these types of problems, there are two jobs J_1, J_2 each of which has to be processed on m machines M_1, M_2, …, M_m with different sequences of machines for processing each of the jobs. Each machine can perform only one job at a time. The exact or expected processing times on all the given machines are known. The objective is to find a sequence of processing the jobs, which minimises the total elapsed time from the start of the first job to the completion of the last job.

A problem of this type can be solved with the help of graphical method. We have to use the following steps in this method:

Step 1: Represent the processing times of job 1 on different machines along the x-axis and processing times of job 2 on different machines along the y-axis.

Step 2: Draw the vertical/horizontal lines through processing times as given in Step 1 and shade the common area for processing of two jobs on the same machine at different times since each machine can perform only one job at a time.

Step 3: Starting from the origin, we move diagonally, horizontally or vertically through various stages of completion of the processing of both the jobs through different machines at the same time until all the processing times are finished. If both the jobs are being processed simultaneously at different machines at the same time, we move diagonally. If Job 2 is under process at any machine and Job 1 is idle, i.e., Job 1 is waiting to be processed at the same machine through which Job 2 is being processed, we move vertically; and we move horizontally when Job 1 is being processed and Job 2 is idle.

Step 4: An optimal sequence for the jobs is the shortest line consisting of combinations of horizontal, vertical and diagonal (45°) lines from the origin. Since both the jobs are being processed simultaneously on the diagonal line, effort should be made to select a path on which diagonal movement is maximum.

So, we first try to move diagonally but keep in mind that diagonal movement through the blocked out (shaded) areas is not allowed since both the jobs cannot be processed simultaneously on a machine. Then horizontal or vertical movement is chosen keeping in view that total elapsed time should be minimum. The elapsed time can be obtained by adding the idle time for either job to the processing time for that job.

Let us explain this method with the help of an example.

Example 2: A machine shop has five machines A, B, C, D and E. Two jobs must be processed through each of these machines. The time (in hours) taken on each of these machines and the necessary sequence of jobs through the shops are given below:

Job 1	**Sequence**	:	A	B	C	D	E
	Time	:	2	4	3	6	6
Job 2	**Sequence**	:	C	A	D	E	B
	Time	:	4	6	3	3	6

Use the graphical method to obtain the total minimum elapsed time.

Solution: We are given the job sequences and processing time of 2 jobs at 5 machines. We follow the graphical method to find the minimum total elapsed time from starting the first job at the first machine to completion of the second job at the last machine.

Refer to Fig. 7.1. We first represent the processing time of job 1 on different machines, i.e, 2, 4, 3, 6, 6 along the x-axis and the processing time of job 2, i.e., 4, 6, 3, 3, 6 along the y-axis.

Then we draw the vertical lines through processing times of Job 1. We draw the first vertical line at 2 hrs, the second at 2 + 4 = 6 hrs, the third at 6 + 3 = 9 hrs, and so on. Similarly, we draw the horizontal lines through processing times of Job 2. We draw the horizontal lines at 4, 10, 13, 16 and 22 hrs in this case. We now shade the common area for processing of jobs 1 and 2 on one machine at the same time as each machine can perform only one job at a time. In this case we find the common areas to be:

i) between 0 to 2 hours on the x-axis and 4 to 10 hours on the y-axis for machine A,

ii) between 2 to 6 hours on the x-axis and 16 to 22 hours on the y-axis for machine B,

iii) between 6 to 9 hours on the x-axis and 0 to 4 hours on the y-axis for machine C,

iv) between 9 to 15 hours on the x-axis and 10 to 13 hours on the y-axis for machine D,

v) between 15 to 21 hours in the x-axis and 13 to 16 hours on the y-axis for machine E.

All these areas are shaded in Fig. 7.1.

We now have to find an optimal sequence for processing of the jobs 1 and 2, i.e., the shortest line consisting of combinations of horizontal, vertical and diagonal (45°) lines from the origin. For this, we start from the origin and move diagonally first up to the point (9, 9) since both the jobs 1 and 2 are being processed simultaneously.

After being processed through machines A, B and C, job 1 becomes idle since job 2 is being processed in machine A and it is then being processed in machine D. Therefore, we move vertically which means Job 2 is under process and Job 1 is idle. The idle time for job 1 is 4 hours in this case.

Again we move diagonally up to the point (18, 22) since both the jobs 1 and 2 are being processed simultaneously. At this stage, job 2 is completed but job 1 is being processed through machine E. Therefore, we move horizontally, which means Job 1 is being processed and Job 2 is idle. The idle time for job 2 is 3 hours since job 1 is processed through machine E between 15 to 21 hours.

In this case, the idle time for job 1 is found to be 4 hours. Therefore, the total elapsed time for Job 1 is 21 + 4 = 25 hours.

Similarly, the idle time for job 2 is seen to be 3 hours when job 1 was under processing between 18 to 21 hrs and the processing for job 2 had been completed. Therefore, the total elapsed time for job 2 is 22 + 3 = 25 hours, which is shown in Fig. 7.1.

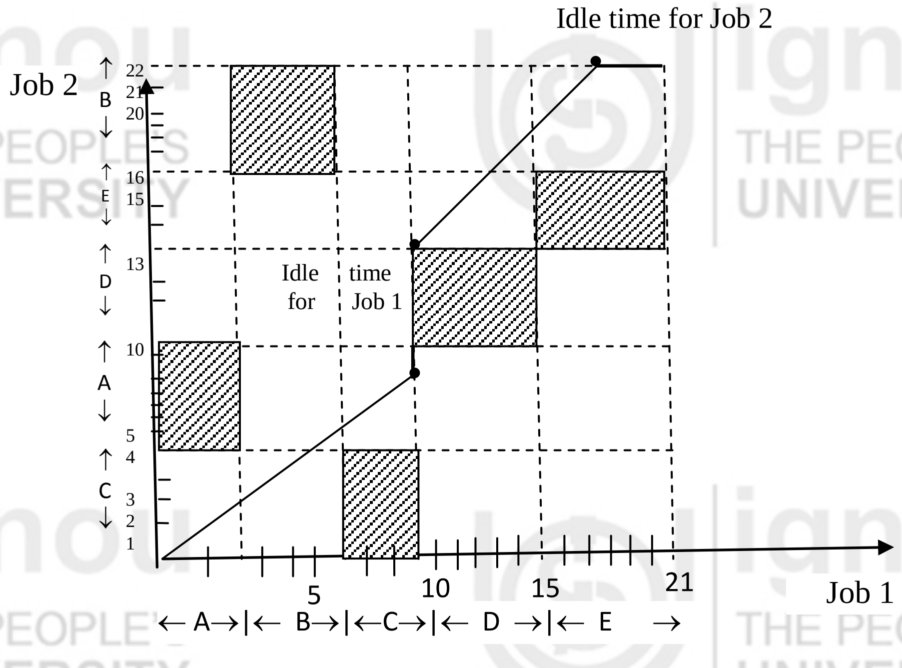


Fig. 7.1

You may now like to solve the following problems to check your understanding.

E3) Using the graphical method, calculate the minimum time needed to process job 1 and job 2 on five machines A, B, C, D and E, that is, for each machine find the job which should be done first. Also calculate the total time needed to complete both jobs.

Job 1	**Sequence**	:	A	B	C	D	E
	Time (in hrs)	:	1	2	3	5	1
Job 2	**Sequence**	:	C	A	D	E	B
	Time (in hrs)	:	3	4	2	1	5

E4) Use the graphical method to minimise the time required to process the following jobs on the machines, that is, for each machine specify the job that should be done first. Also calculate the total elapsed time for completing both jobs.

Job 1	**Sequence**	:	A	B	C	D	E
	Time (in hrs)	:	6	8	4	12	3
Job 2	**Sequence**	:	B	C	A	D	E
	Time (in hrs)	:	10	8	6	4	12

Let us now summarise the main points which have been covered in this unit.

7.5 SUMMARY

1. The selection of the appropriate order in which waiting customers are to be served is called **sequencing**. In sequencing problems, there are two or more jobs to be done on one or more machines available for this purpose. We are interested in finding when each job is to begin and what its due date or time is. We are also interested in determining the sequence (order) of jobs which minimises the total elapsed time (T) for processing all the jobs from the start of the first job to the completion of the last job.

2. In case of **processing of n-jobs through two machines**, the procedure suggested by S.M Johnson and R. Bellman for finding the sequence of jobs which minimises the total elapsed time is used. **Total elapsed time** in this case is the time when the n^{th} job in a sequence finishes on second machine. **Idle time for the first machine** is the time when the last job in a sequence finishes on the first machine subtracted from the time when the last job in a sequence finishes on the second machine. **Idle time for second machine** is

   = the time at which the first job in a sequence finishes on the first machine

   $+ \sum$[(Time when the ith job in a sequence starts on the second machine)

   – (Time when the $(i-1)^{th}$ job in a sequence finishes on the second machine)]

3. In case of **processing of 2 jobs through m machines**, each of the two jobs is to be processed on m machines $M_1, M_2, \ldots, M_m$ in different sequence of machines. The exact/ expected processing times on all the given machines are known. The objective is to find a sequence of processing the jobs, which minimises the total elapsed time from the start of the first job to the completion of the last job. A problem of this type can be solved with the help of the graphical method.

7.6 SOLUTIONS/ANSWERS

E1) We first make an optimal sequence for processing on cutting and sewing machines. If minimum processing time is that of the cutting machine then we place it in the beginning of the sequence and if it is that of sewing machine, then it will be placed at the end of sequence. In this way, we obtain the following optimal sequence:

3	4	5	2	1

The various times can be calculated as follows:

Item	Cutting		Sewing	
	Time in	Time Out	Time in	Time Out
3	0	3	3	10
4	3	7	10	15
5	7	13	15	24
2	13	20	24	30
1	20	25	30	32

Total elapsed time = 32 hours.

Idle time for cutting machine = 32 – 25 = 7 hours.

Idle time for sewing machine = 3 hours.

E2) The optimal sequence is

1	4	5	3	2	7	6

OR

1	5	3	2	4	7	6

Here, either of the above 2 sequences can be taken as an optimal solution. We get the same answer using either of these sequences. Let us take the first sequence to obtain various elapsed times, which have been calculated in the following table :

Job	**Machine A**		**Machine B**	
	Time in	Time Out	Time in	Time Out
1	0	3	3	11
4	3	9	11	17
5	9	19	19	31
3	19	34	34	44
2	34	46	46	56
7	46	55	56	59
6	55	66	66	67

Total elapsed time = 67 hours

Idle time of Machine A = 67 – 66 = 1 hour

Idle time of Machine B = 3 + (11 + 19 + 34 + 46 + 56 + 66)
– (11 + 17 + 31 + 44 + 56 + 59)
= 3 + 232 – 218 = 17 hours

Let us take the second sequence to obtain various elapsed times, which have been calculated in the following table:

Job	**Machine A**		**Machine B**	
	Time in	Time Out	Time in	Time Out
1	0	3	3	11
4	3	13	11	23
5	13	28	28	38
3	28	40	40	50
2	40	46	50	56
7	46	55	56	59
6	55	66	66	67

Total elapsed time = 67 hours

Idle time of Machine A = 67 – 66 = 1 hour

Idle time of Machine B = 3 + (11 + 28 + 40 + 50 + 56 + 66)
– (11 + 23 + 38 + 50 + 56 + 59)
= 3 + 251 – 237 = 17 hours

E3) Let us draw a set of axes at right angle to each other where the x-axis represents the time of job 1 on different machines while job 2 remains idle and the y-axis represents the processing time of job 2 while job 1 remains idle. We draw vertical/horizontal lines through processing times as shown in the Fig. 7.2 and shade the common area for each machine.

Starting from the origin, we move diagonally, horizontally or vertically through various stages of completion until all the processing times are finished. Elapsed time obtained by adding the idle time for job 1 to its processing time is (12 + 3) = 15 hours.

Elapsed time obtained by adding the idle time for job 2 to its processing time is (15 + 0) = 15 hours.

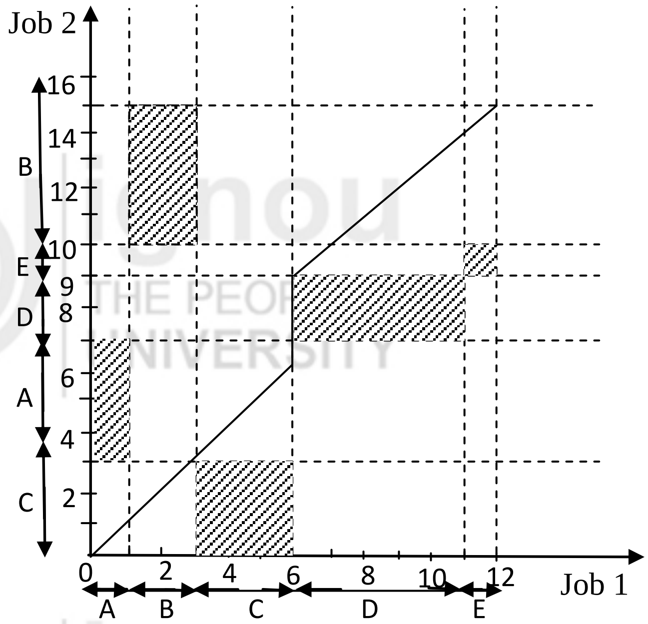


Fig. 7.2

E4) Let us draw a set of axes at right angle to each other where the x-axis represents the processing time of job 1 on different machines while job 2 remains idle and the y-axis represents the processing time of job 2 while job 1 remains idle. We draw vertical/horizontal lines through processing times as shown in Fig. 7.3 and shade the common area for each machine.

Starting from the origin, we move diagonally, horizontally or vertically through various stages of completion until all the processing times are finished.

Idle time of job 1 in hours = 4 + 6 = 10 hours

Idle time of job 2 in hours = 3 hours

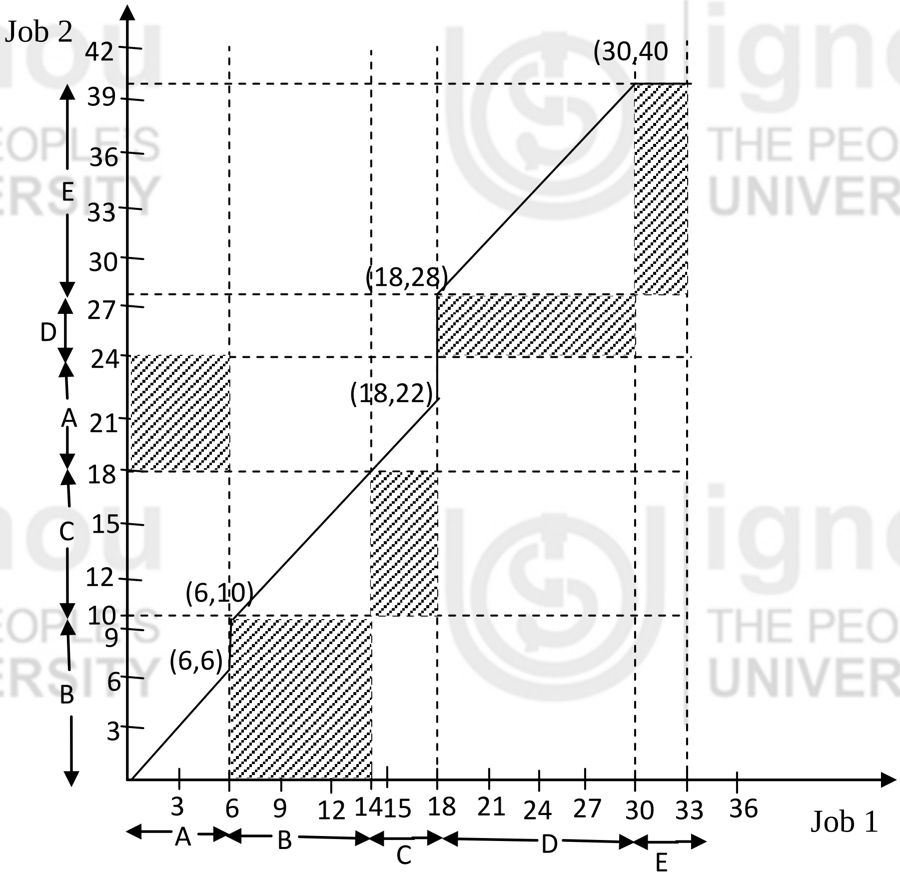
Elapsed time = Processing time of Job 1+ its idle time

= 33 + 10 = 43 hours

OR

= Processing time of job 2 + its idle time

= 40 + 3 = 43 hours


Job 2
42
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Job 1
A
B
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Fig. 7.3