

**Block****4****RANDOM NUMBER GENERATION AND  
SIMULATION TECHNIQUES**

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## **RANDOM NUMBER GENERATION AND SIMULATION TECHNIQUES**

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Simulation is becoming a very important tool for analysis of a wide variety of problems whose exact solution is very difficult, expensive or time taking. In the last few years it has been used for making optimum decisions in major manufacturing process design, air traffic control, bank teller scheduling, location of fire stations, computer networking, etc. Recently it has been used in the financial risk analysis and is of great help in obtaining optimum decisions of inventory problems.

In this block we have described different methods of generation of random numbers, which are required for generating data from some logical models with random components, for a system under study. The random number generation methods of generating data for discrete variables have been discussed in Unit 13 and the random number generation methods of generating data for continuous variables have been discussed in Unit 14. We also described how to conduct numerical experiments to obtain approximate solutions of some important problems.

Through simulation technique the operation of the model can be studied and from it, properties concerning the behavior of the actual system can be inferred like forest management, epidemics, traffic congestion, etc. In Unit 15, the different steps in setting up simulation are described. The Monte-Carlo simulation technique is also discussed for solving some deterministic and stochastic problem. Many exercises and examples have been given to make you familiar with simulation techniques for solving different types of problems. In Unit 16 we have described various applications of simulations with examples. Different methods of testing the randomness of the generated sequence are also described.

### **Suggested Readings:**

1. Fisher, R. A. and Yates, F. (1963); Statistical Tables (sixth edition), Longman, England.
2. Rand Corporation (1955); A million random digits with 100,000 Normal Deviates, The Free press, Glencoe, III.

### **Some Further Readings in Simulation:**

1. Fishman, G. S. (1995); Monte-Carlo Concepts, Algorithms and Applications, Springer-Verlag, New York.
2. Hoover S. V. and Perry, R. F. (1989); Simulation: A Problem Solving Approach, Addison-Wesley, Reading.
3. Law, A. M. and Kelton, W. D. (1991); Simulation Modeling and Analysis (2<sup>nd</sup> ed.), McGraw Hill, New York.
4. Morgan, B. J. T. (1984); Elements of Simulation, Chapman and Hall, London.
5. Ross, S. M. (2002); Simulation (3rd ed.), Academic Press, London.
6. Rubinstein, R. Y. and Melamed, B. (1997); Modern Simulation and Modeling, Wiley, New York.

## Notations and Symbols

$IPT$	: Inverse Probability Transformation
$PRN$	: Pseudo Random Numbers
$U(0,1)$	: Uniform random variable
$LCG$	: Linear Congruential Generator
$(n) \bmod m$	: Reminder part of $n/m$
$X$	: Random variable
$F(x) = P(X \leq x)$	: Cumulative distribution function of $X$
$f(x)$	: Probability density function of $X$
$P(X=x)$	: Probability mass function of $X$
$\mu_x$	: Mean of Normal distribution
$\sigma_x$	: Standard deviation of Normal distribution
$Z$	: Standard Normal variate
$Be(\alpha, \beta)$	: Beta distribution
$\chi_m^2$	: Chi-square variate
$g(x)$	: Known function of $x$
$\hat{\theta}$	: Estimate of $\theta$
$\theta$	: Deterministic integral function
$S_i$	: Service time for $i^{th}$ customer
$I_i$	: Inter arrival time between arrival of $i^{th}$ and $(i-1)^{th}$
$\mu$	: Mean service rate
$\lambda$	: Mean arrival rate
$\xi$	: Traffic intensity
$W_n$	: System of $n^{th}$ customer
$P_{j0}$	: Probability of an observations lying in $j^{th}$ class
$H_0$	: Null hypothesis
$H_1$	: Alternative hypothesis
$n_j$	: Observed numbers of $j^{th}$ category
$np_{j0}$	: Expected numbers of $j^{th}$ category
$D_n$	: Kolmogorov-Smirnov test statistic