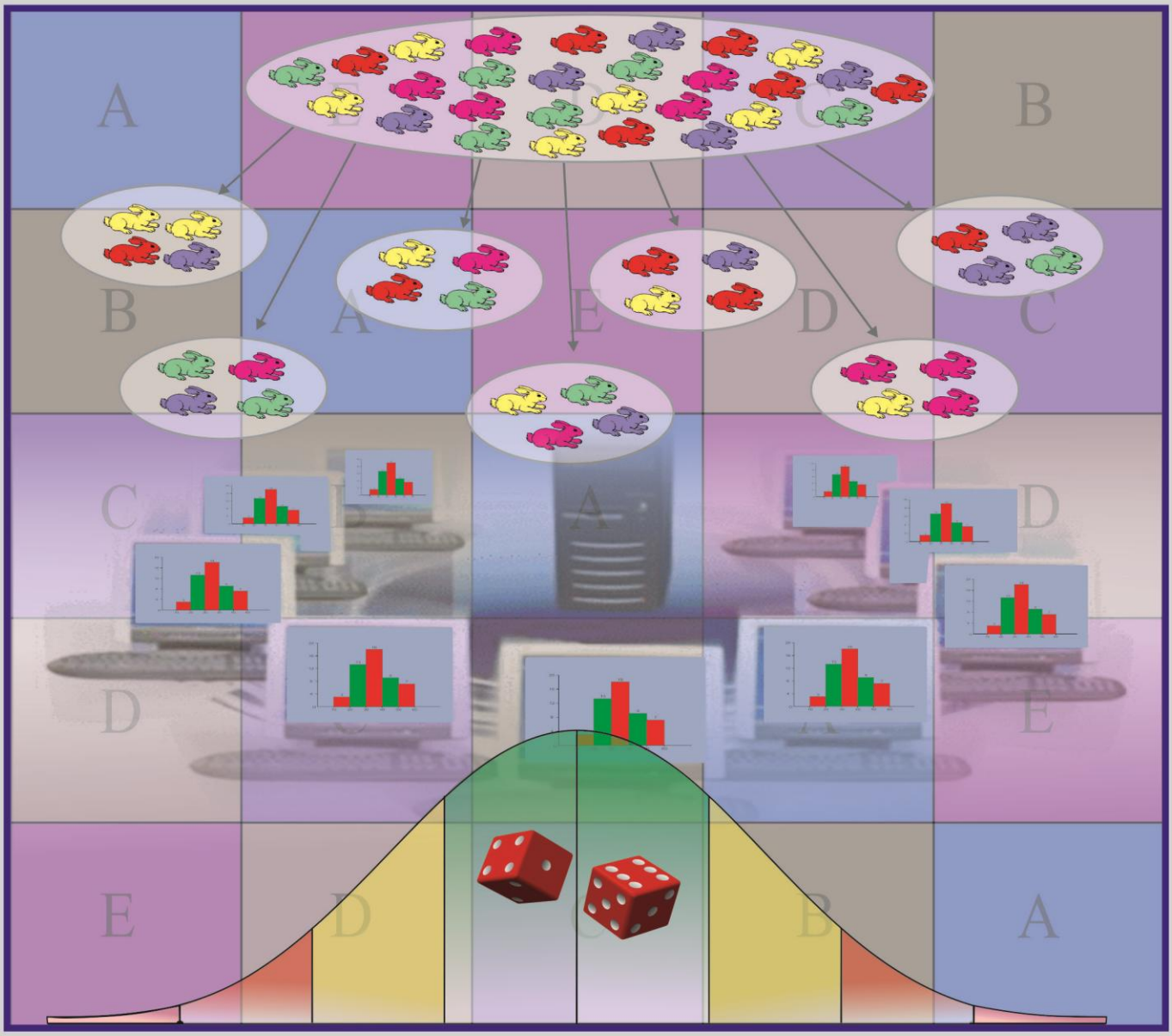
FORMULAE AND STATISTICAL TABLES BOOKLET

for

Post Graduate Diploma in Applied Statistics

(PGDAST)



School of Sciences

Indira Gandhi National Open University

New Delhi – 110068

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IMPORTANT

The **Formulae and Statistical Tables Booklet** contains the main formulae of the courses of the PGDAST programme and Statistical Tables.

The **Formulae and Statistical Tables Booklet** will be available at the examination centres in the Term-End Examinations for the courses of the PGDAST programme.

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CONTENTS

FORMULAE

1. Distributive Laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

De-Morgan's Laws

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

2. Important Relations Between Different Sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$$
$$n(A - B) = n(A) - n(A \cap B)$$
$$n(B - A) = n(B) - n(A \cap B)$$
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Laws of Logarithm

$$\log_a mn = \log_a m + \log_a n$$
$$\log_a \frac{m}{n} = \log_a m - \log_a n$$
$$\log_a m^n = n \log_a m$$
$$\log_a b = \frac{\log_n b}{\log_n a}$$
$$a^{\log_a m} = m$$
$$\log_a a = 1$$
$$\log_a b = \frac{1}{\log_b a}$$

3. Arithmetic Progression (A.P.)

For an A.P. having first term **a** and common difference **d**

n^{th} term $= a_n = T_n = a + (n-1)d$

Sum of first n terms $= S_n = \frac{n}{2}[2a + (n-1)d]$

or $S_n = \frac{n}{2}(a + l)$, l = last term

Geometric Progression (G.P.)

For a G.P. having first term **a** and common ratio **r**

n^{th} term $= a_n = T_n = ar^{n-1}$

Sum of first n terms $= S_n = \frac{a(1-r^n)}{1-r}$, $r \neq 1$

Sum of infinite G.P. $= S = \frac{a}{1-r}$, $|r| < 1$

Sum of Special Sequences

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2};$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

4. Permutation of things not all distinct

$\frac{\lfloor n}{\lfloor p_1 \times \lfloor p_2 \times \lfloor p_3 \times \ldots \times \lfloor p_k}$, where $p_1 + p_2 + \ldots + p_k = n$ and $p_i \geq 1$, $1 \leq k \leq n$

where p_i are the i^{th} kind of thing out of n things.

Permutation when repetition is allowed

Total number of permutations of n things taken r at a time anything can repeat any number of times $= n^r$.

Circular permutation

- Number of permutations of n distinct things $= (n-1)!$ (when anti-clock wise and clock wise order of arrangements makes different permutations)
- Number of permutations of n distinct things $= \frac{(n-1)!}{2}$ (when anti-clock wise and clock wise order of arrangements does not give distinct permutations)

Combination

The total number of combinations of n things taken r ($1 \leq r \leq n$) at a time $= {}^nC_r = \frac{n!}{(n-r)! \times r!}$

Result I: Total number of permutations of n distinct things taken r at a time such that

i) s ($0 < s < r$) particular things are always included $= {}^{n-s}P_{r-s} \times {}^rP_s$

ii) s ($0 < s < r$) particular things are always excluded $= {}^{n-s}P_r$

Result II: Total number of combination of n distinct things taken r at a time such that

i) s ($0 < s < r$) particular things are always included $= {}^{n-s}C_{r-s}$

ii) s ($0 < s < r$) particular things are always excluded $= {}^{n-s}C_r$

5. Binomial theorem for positive integral index

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + {}^nC_3\, a^{n-3}b^3 + \ldots + {}^nC_{n-1}\, ab^{n-1} + {}^nC_n\, b^n$$

Binomial theorem for any index

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-(r-1))}{r!}x^r + \ldots$$

6 Some Standard Results on Limit

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=\lim_{\theta\to 0}\frac{\theta}{\sin\theta}=1$$

$$\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=\lim_{\theta\to 0}\frac{\theta}{\tan\theta}=1$$

$$\lim_{\theta\to 0}\cos\theta=1$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\log_e|a|,$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

$$\lim_{x\to 0}(1+x)^{1/x}=e$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=\log_e e=1$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x}=1$$

$$\lim_{x\to\infty}\frac{1}{x^n}=0,\quad \text{where } n>0$$

7 Derivatives of Some Functions

$$\frac{d}{dx}(k)=0,\ k \text{ is constant}$$

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

$$\frac{d}{dx}(ax+b)^n=na(ax+b)^{n-1}$$

$$\frac{d}{dx}(cu)=c\frac{d}{dx}(u)$$

where c is constant and u is a function of x

Exponential function

$$\frac{d}{dx}(a^{bx})=ba^{bx}\log a$$

$$\frac{d}{dx}(e^{bx})=be^{bx}$$

Logarithmic function

$$\frac{d}{dx}(\log_a x)=\frac{1}{x}\log_a e$$

$$\frac{d}{dx}(\log_e x)=\frac{1}{x}$$

$$\frac{d}{dx}(u\pm v)=\frac{du}{dx}\pm\frac{dv}{dx}$$

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}\ \text{(Product Rule)}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}\ \text{(Quotient Rule)}$$

where u, v are functions of x

If y = f(u), u = g(w), w = h(x) then

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dw}\frac{dw}{dx}\ \text{(Chain Rule)}$$

Parametric functions

x = f(t), y = g(t)

$$\frac{dy}{dx}=\frac{dy}{dt}\Big/\frac{dx}{dt}$$

8 Integration of Some Functions

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+c,\ n\neq -1;$$

$$\int\frac{1}{x}dx=\log|x|+c;$$

$$\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+c,\ n\neq -1$$

$$\int\frac{1}{ax+b}dx=\frac{1}{a}\log|ax+b|+c;$$

$$\int a^{mx}\,dx=\frac{a^{mx}}{m\log_e|a|}+c;$$

$$\int a^{mx+n}\,dx=\frac{a^{mx+n}}{m\log|a|}+c;$$

$$\int e^{ax}\,dx=\frac{e^{ax}}{a}+c;$$

$$\int e^{ax+b}\,dx=\frac{e^{ax+b}}{a}+c$$

9 Integration by Parts

$$\int u(x)\,v(x)\,dx=u(x)\int v(x)\,dx-\int\left[\frac{d}{dx}(u(x))\int v(x)\,dx\right]dx$$

10 Elementary Properties of Definite Integral

$$\int_a^b f(x)dx=-\int_b^a f(x)dx$$

$$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$$

$$\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$\int_a^b f(x)dx=\int_a^{c_1} f(x)dx+\int_{c_1}^{c_2} f(x)dx+\ldots+\int_{c_{n-1}}^{c_n} f(x)dx+\int_{c_n}^{b} f(x)dx,\ \text{where } a<c_1<c_2<\ldots.<c_n<b$$

$$\int_{-a}^{a} f(x)dx=\begin{cases}2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function}\\ 0, & \text{if } f(x) \text{ is an odd function}\end{cases}$$

$$\int_0^{2a} f(x)dx=\begin{cases}2\int_0^a f(x)dx, & \text{if } f(2a-x)=f(x)\\ 0, & \text{if } f(2a-x)=-f(x)\end{cases}$$

11 Matrices and Determinants

$A^0 = I$, where I is the identity matrix of the same order as A

$(A')' = A$

$(kA)' = kA'$,

$(A+B)' = A' + B'$

tr (A + B) = tr (A) + tr (B)

tr (kA) = k tr (A)

	$(A+B)^2 = A^2 + AB + BA + B^2$ $(A+B)^2 = A^2 + 2AB + B^2$ if and only if AB = BA	$(A-B)' = A' - B'$ $(AB)' = B'A'$	tr (AB) = tr (BA) tr (AB) $\neq$ tr (A) tr (B)
	$\text{AdjA} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & \dots & A_{1n} \\ A_{21} & A_{22} & A_{23} & \dots & A_{2n} \\ A_{31} & A_{32} & A_{33} & \dots & A_{3n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ A_{n1} & A_{n2} & A_{n3} & \dots & A_{nn} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} & A_{31} & \dots & A_{n1} \\ A_{12} & A_{22} & A_{32} & \dots & A_{n2} \\ A_{13} & A_{23} & A_{33} & \dots & A_{n3} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ A_{1n} & A_{2n} & A_{3n} & \dots & A_{nn} \end{bmatrix}$ where A_{ij} represents cofactor of $(i, j)^{th}$ element of the matrix A $\lvert A' \rvert = \lvert A \rvert$, $(A^{-1})' = (A')^{-1}$; $(AB)^{-1} = B^{-1}A^{-1}$ If $\lvert A \rvert \neq 0$ then $A^{-1} = \frac{1}{\lvert A \rvert}(\text{adj}A)$		
12	**Graphical Presentation**		
	Histogram **Number of classes** K = 1 + 3.322 Log N, where K = the approximate number of classes, N = total number of observations Log = common logarithm (logarithm to the base 10) **Frequencies of the unequal class-intervals** $\frac{\text{Given frequency}}{\text{Width of its class-interval}} \times (\text{The least width})$ **Stem-and-leaf display** j m^{ths} quantile $Q_{j/m} = x_i$ where x_i is that value of the variable below which j m^{ths} observation lie and $i = \left(\frac{j \times n}{m}\right) + \frac{1}{2}$.	**Box Plot** Lower hing = first quartile, Upper hing = third quartile H-spread = Upper hing – Lower hing, Step = (H-Spread) $\times$ 1.5 Lower inner fence = Lower hing – One step Upper inner fence = Upper hing + One step Lower outer fence = Lower hing – Two steps Upper outer fence = Upper hing + Two steps Lower adjacent = Smallest observation Upper adjacent = Largest observation	

S. No. 1 — FORMULAE

Measures of Central Tendency

Mean for Ungrouped Data

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{X} = A + \frac{\sum_{i=1}^{n} d_i}{n}$$

Mean for Grouped Data

$$\bar{X} = \frac{\sum_{i=1}^{k} f_i x_i}{\sum_{i=1}^{k} f_i}$$

$$\bar{X} = A + \frac{\sum_{i=1}^{k} f_i d_i}{\sum_{i=1}^{k} f_i},$$

Weighted Mean

$$\bar{X}_W = \frac{\sum_{i=1}^{k} x_i w_i}{\sum_{i=1}^{k} w_i}$$

Combined Mean

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

In General

$$\bar{X} = \frac{\sum_{i=1}^{k} n_i \bar{X}_i}{\sum_{i=1}^{k} n_i}$$

Median for Ungrouped/Discrete Data

When n is odd

$$\text{Median}\,(M_d) = \left(\frac{n+1}{2}\right)^{th} \text{observation}$$

When n is even

$$M_d = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2};$$

Median for Continuous Data

$$\text{Median} = L + \frac{\left(\frac{N}{2} - C\right)}{f} \times h$$

Mode for Ungrouped/Discrete Data

Mode = value of the variable corresponding to the maximum frequency

Median for Continuous Data

$$M_0 = L + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \times h$$

Empirical Formula

Mode = 3 Median – 2 Mean

Geometric Mean for Ungrouped Data

$$GM = \sqrt[n]{x_1 x_2 \ldots x_n} = (x_1 x_2 \ldots x_n)^{\frac{1}{n}}$$

Geometric Mean for Grouped Data

$$GM = \left(x_1^{f_1} x_2^{f_2} \ldots x_k^{f_k}\right)^{\frac{1}{\sum f_i}}$$

Harmonic Mean for Ungrouped Data

$$HM = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Harmonic Mean for Grouped Data

$$HM = \frac{N}{\sum_{i=1}^{k} \frac{f_i}{x_i}}$$

Quartiles

$$Q_i = L + \frac{\left(\frac{iN}{4} - C\right)}{f} \times h$$

for $i = 1, 2, 3$

Deciles

$$D_i = L + \frac{\left(\frac{iN}{10} - C\right)}{f} \times h$$

for $i = 1, 2, \ldots, 9$

Percentiles

$$P_i = L + \frac{\left(\frac{iN}{100} - C\right)}{f} \times h$$

for $i = 1, 2, \ldots, 99$

Measures of Dispersion

Range

$$R = X_{Max} - X_{Min}$$

$$\text{Cofficient of range} = \frac{X_{Max} - X_{Min}}{X_{Max} + X_{Min}}$$

Quartile Deviation

$$QD = \frac{Q_3 - Q_1}{2}$$

$$\text{Cofficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Mean Deviation About any Arbitrary Point A

$$MD = \frac{1}{n}\sum_{i=1}^{n} |x_i - A|$$

			$MD = \dfrac{\sum_{i=1}^{k} f_i \lvert x_i - A \rvert}{\sum_{i=1}^{k} f_i}$
	Variance	**Combined Variance for Several Groups**	
	$\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ $\sigma^2 = \dfrac{1}{N}\sum_{i=1}^{k} f_i (x_i - \bar{x})^2$ **Standard deviation** $SD = \sqrt{\text{Variance}}$	$\sigma^2 = \dfrac{1}{(n_1 + n_2 + ... + n_k)}\left[n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + ... + n_k(\sigma_k^2 + d_k^2) \right]$ where $d_i = \bar{x}_i - \bar{x}$ and $\bar{x} = \dfrac{\sum_{i=1}^{k} n_i \bar{x}_i}{\sum_{i=1}^{k} n_i}$	
	Root mean square deviation (RMSD) for an arbitrary point A $RMSD = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - A)^2}{n}}$ and $RMSD = \sqrt{\dfrac{\sum_{i=1}^{k} f_i (x_i - A)^2}{\sum_{i=1}^{k} f_i}}$		**Coefficient of variation** $CV = \dfrac{\sigma}{\bar{x}} \times 100$
3	**Raw Moments (Moments about Arbitrary Point A)**	**Central Moments (Moments about Mean)**	
	$\mu'_r = \dfrac{\sum_{i=1}^{n}(x_i - A)^r}{n}$; for r = 1, 2, … $\mu'_r = \dfrac{\sum_{i=1}^{k} f_i (x_i - A)^r}{N}$; r = 1, 2, …	$\mu_r = \dfrac{\sum_{i=1}^{k}(x_i - \bar{x})^r}{N}$; for r = 0, 1, 2, … $\mu_r = \dfrac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^r}{N}$; for r = 0, 1, 2 …	
	Recurrence relation $\mu_r = \mu'_r - {}^rC_1 \mu'_{r-1}\mu'_1 + {}^rC_2 \mu'_{r-2}(\mu'_1)^2 - {}^rC_3 \mu'_{r-3}(\mu'_1)^3 + ... + (-1)^r (\mu'_1)^r$ $\mu_2 = \mu'_2 - 2(\mu'_1)^2$ $\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3$ $\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^4$		
4	**Coefficients of Skewness**	**Coefficients of Kurtosis**	
	$\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$; $\gamma_1 = \pm\sqrt{\beta_1}$ $S_k = \dfrac{(Q_3 - 2Q_2 + Q_1)}{(Q_3 - Q_1)}$; $S_k = \dfrac{(P_{90} - 2P_{50} + P_{10})}{(P_{90} - P_{10})}$ $S_k = \dfrac{(D_9 - 2D_5 + D_1)}{D_9 - D_1}$	$\beta_2 = \dfrac{\mu_4}{\mu_2^2}$ $\gamma_2 = \beta_2 - 3$	
5	**Curve Fitting**		
	Normal least square equations for straight line Y = a + bX : $\sum y = na + b\sum x$ $\sum yx = a\sum x + b\sum x^2$	**Normal least square equations for exponential curve $Y = ab^X$:** $\sum u = nA + B\sum x$ $\sum ux = A\sum x + B\sum x^2$	

Normal least square equations for second degree parabola $Y = a + bX + cX^2$:

$$\sum y = na + b\sum x + c\sum x^2$$
$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$
$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Normal least square equations for power curve $Y = aX^b$:

$$\sum u = nA + b\sum v$$
$$\sum uv = A\sum v + b\sum v^2$$

where u = log y, v = log x and A = log a

where log y = u, log a = A and log b = B

Normal least square equations for exponential curve $Y = ae^{bX}$:

$$\sum u = nA + B\sum x$$
$$\sum ux = A\sum x + B\sum x^2$$

where log y = u, log a = A and blog e = B
log - logarithm to the base 10.

6 Simple, Multiple and Partial Correlation Analysis

Correlation coefficient between X and Y

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left(\sum_{i=1}^{n}(x_i - \bar{x})^2\right)\left(\sum_{i=1}^{n}(y_i - \bar{y})^2\right)}}$$

Correlation coefficient between X and Y by short cut method

$$r = \frac{n\sum d_x d_y - \sum d_x \sum d_y}{\sqrt{\left\{n\sum d_x^2 - \left(\sum d_x\right)^2\right\}\left\{n\sum d_y^2 - \left(\sum d_y\right)^2\right\}}}$$

Correlation Coefficient between X and Y for bivariate data

$$r = \frac{\sum f_{xy} d_x d_y - \frac{\sum f_x d_x \sum f_y d_y}{N}}{\sqrt{\left\{\sum f_x d_x^2 - \frac{\left(\sum f_x d_x\right)^2}{N}\right\}\left\{\sum f_y d_y^2 - \frac{\left(\sum f_y d_y\right)^2}{N}\right\}}}$$

Rank correlation coefficient $r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$

Rank correlation coefficient for tied ranks

$$r_s = 1 - \frac{6\left\{\sum d^2 + \frac{m(m^2-1)}{12} + \ldots\right\}}{n(n^2-1)}$$

$$(r_s)_c = \frac{\frac{(n^3-n)}{6} - \left\{\sum d_{ci}^2 + T_x + T_y\right\}}{\sqrt{\left(\frac{n^3-n}{6} - 2T_x\right)\left(\frac{n^3-n}{6} - 2T_y\right)}}$$

$$T_x = \sum_{i=1}^{p}\frac{(m_{xi}^3 - m_{xi})}{12} \text{ and } T_y = \sum_{i=1}^{q}\frac{(m_{yi}^3 - m_{yi})}{12}$$

Coefficient of concurrent deviation

$$r_c = \pm\sqrt{\pm\frac{(2c-k)}{k}}$$

Correlation ratio for discrete data

Correlation ratio for continuous data

$$\eta_{yx}^2 = \frac{\sum_{i=1}^{m} f_i(\bar{y}_i - \bar{y})^2}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}(y_i - \bar{y})^2} \text{ or } \eta_{yx}^2 = \left[\sum_{i=1}^{m}\left(\frac{T_i^2}{n_i}\right) - \frac{T^2}{N}\right] \Big/ N\sigma_y^2$$

where $T_i = \sum_{j=1}^{n} f_{ij} y_{ij}$ and $T = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij} y_{ij}$

Intra-class correlation coefficient

$$r_{ic} = \frac{\sum_{i=1}^{n} k_i^2(\bar{x}_i - \bar{x})^2 - \sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij} - \bar{x})^2}{\sum_{i=1}^{n}\sum_{j=1}^{k_i}(k_i - 1)(x_{ij} - \bar{x})^2}$$

$$r_{ic} = \frac{1}{(k-1)}\left\{\frac{k\sigma_m^2}{\sigma_x^2} - 1\right\}$$

Multiple correlation coefficient

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}}$$

$$R_{2.13} = \sqrt{\frac{r_{12}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{21}^2}}$$

$$R_{3.12} = \sqrt{\frac{r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{12}^2}}$$

Multiple correlation coefficient in terms of partial correlation coefficient

$$R_{1.23} = \sqrt{1 - (1 - r_{12}^2)(1 - r_{13.2}^2)}$$

$$R_{2.13} = \sqrt{1 - (1 - r_{12}^2)(1 - r_{23.1}^2)}$$

$$R_{3.12} = \sqrt{1 - (1 - r_{13}^2)(1 - r_{32.1}^2)}$$

Partial correlation coefficient

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

	$\eta^2_{yx} = \dfrac{\sum_{i=1}^{m} n_i(\bar{y}_i - \bar{y})^2}{\sum_{i=1}^{m}\sum_{j=1}^{n}(y_{ij} - \bar{y})^2}$	$r_{13.2} = \dfrac{r_{13} - r_{12}r_{32}}{\sqrt{(1-r_{12}^2)(1-r_{32}^2)}}$ $r_{23.1} = \dfrac{r_{23} - r_{21}r_{31}}{\sqrt{(1-r_{21}^2)(1-r_{31}^2)}}$
7	**Regression Analysis**	
	Regression line of Y on X $(y-\bar{y}) = b_{yx}(x-\bar{x})$ where $b_{yx} = \dfrac{r\sigma_y}{\sigma_x}$ **Regression line of X on Y** $(x-\bar{x}) = b_{xy}(y-\bar{y})$ where $b_{xy} = \dfrac{r\sigma_x}{\sigma_y}$ **Angle between two lines of regression** $\tan\theta = \left\{\dfrac{(1-r^2)}{r}\left(\dfrac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)\right\}$	**Partial Regression Coefficients** $b_{12.3} = \dfrac{\sigma_1(r_{12} - r_{13}r_{23})}{\sigma_2(1-r_{23}^2)}$ $b_{13.2} = \dfrac{\sigma_1(r_{13} - r_{12}r_{23})}{\sigma_3(1-r_{23}^2)}$ $b_{23.1} = \dfrac{\sigma_2(r_{23} - r_{21}r_{31})}{\sigma_3(1-r_{31}^2)}$ **Variance of residual** $\sigma^2_{1.23} = \dfrac{\sigma_1^2}{1-r_{23}^2}(1 - r_{23}^2 - r_{12}^2 - r_{13}^2 + 2r_{12}r_{23}r_{13})$
8	**Association of Attributes**	
	Yule's coefficient of association $Q = \dfrac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$ and $Q = \dfrac{2\gamma}{1+\gamma^2}$ **Coefficient of colligation** $\gamma = \dfrac{1 - \sqrt{\dfrac{(A\beta)(\alpha B)}{(AB)(\alpha\beta)}}}{1 + \sqrt{\dfrac{(A\beta)(\alpha B)}{(AB)(\alpha\beta)}}}$	**Chi-square statistic** $\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\dfrac{(O_{ij} - E_{ij})^2}{E_{ij}}$ **Coefficient of contingency** $C = \sqrt{\dfrac{\chi^2}{\chi^2 + N}}$

S. No. 1 — Probability

Laws of probability

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

If A, B, C are mutually exclusive events, then

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Conditional probability

$P(A \cap B) = P(A)\, P(B|A), \quad P(A) > 0$

$= P(B)\, P(A|B), \quad P(B) > 0$

Happening of at least one of the events $A_1, A_2, A_3, ..., A_n$ is

$P(A_1 \cup A_2 \cup ... \cup A_n) = 1 - \left[P(\bar{A}_1) P(\bar{A}_2) \ldots P(\bar{A}_n) \right]$

Law of total probability

$P(A) = P(E_1)\, P(A|E_1) + P(E_2)\, P(A|E_2) + \ldots + P(E_n)\, P(A|E_n)$

$= \sum_{i=1}^{n} P(E_i) P(A \mid E_i)$

Bayes' theorem

$P(E_i \mid A) = \frac{P(E_i) P(A \mid E_i)}{P(A)}, \quad i = 1, 2, ..., n$

where $P(A) = \sum_{i=1}^{n} P(E_i) P(A \mid E_i)$

2 — Univariate random variable X

Distribution function or cumulative distribution function

$F(x) = P[X \le x]$

$$F(x) = P[X \le x] = \begin{cases} \sum_{k=0}^{x} P[X = k], & \text{in the case of discrete random variable} \\ \int_{-\infty}^{x} f(x)dx, & \text{in the case of continuous random variable} \end{cases}$$

3 — Bivariate Random Variable (X, Y)

For Discrete Random Variable (X, Y)

Joint probability mass function

$p(x_i, y_j) = P\left[X = x_i, Y = y_j \right]$ or

$p(x_i, y_j) = P\left[X = x_i \cap Y = y_j \right]$

where $p(x_i, y_j) \ge 0$ and $\sum_i \sum_j p(x_i, y_j) = 1$

Marginal probability mass function of X

$p(x_i) = \sum_j p(x_i, y_j)$

Marginal probability mass function of Y

$p(y_j) = \sum_i p(x_i, y_j)$

Conditional probability mass function of X given Y = y

$p(x \mid y) = P[X = x \mid Y = y] = \frac{P[X = x \cap Y = y]}{P[Y = y]}$

provided $P[Y = y] \ne 0$

Conditional probability mass function of Y given X = x

$p(y \mid x) = P[Y = y \mid X = x] = \frac{P[Y = y \cap X = x]}{P[X = x]}$

provided $P[X = x] \ne 0$

For Continuous Random Variable (X, Y)

Joint probability density function

$F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(x, y) dy dx$

where $f(x, y) \ge 0$ and $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dy dx = 1$

Marginal distribution function of X

$F(x) = P[X \le x] = \int_{-\infty}^{x} \left[\int_{-\infty}^{\infty} f(x, y)\, dy \right] dx$

Marginal distribution function of Y

$F(y) = P[Y \le y] = \int_{-\infty}^{y} \left[\int_{-\infty}^{\infty} f(x, y)\, dx \right] dy$

Marginal probability density function of X

$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy \quad \text{or} \quad f(x) = \frac{d}{dx}\left(F(x)\right)$

Marginal probability density function of Y

$f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx \quad \text{or} \quad f(y) = \frac{d}{dy}\left(F(y)\right)$

Conditional probability density function of X given Y = y

$f(x \mid y) = \frac{f(x, y)}{f(y)}$, where $f(y) > 0$ is the marginal density of Y

Marginal distribution function of X

$$F(x) = P[X \le x] = \sum_j P[X \le x, Y = y_j]$$

Marginal distribution function of Y

$$F(y) = P[Y \le y] = \sum_i P[X = x_i, Y \le y]$$

Condition for independence

$$P[X = x_i \cap Y = y_j] = P[X = x_i]P[Y = y_j]$$

Conditional probability density function of Y given X = x

$f(y \mid x) = \dfrac{f(x,y)}{f(x)}$, where f(x) > 0 is the marginal density of X

Conditional distribution function of X given Y = y

$$F(x \mid y) = P[X \le x \mid Y = y] = \int_{-\infty}^{x} f(x \mid y)\,dx, \text{ for all y such that } f(y) > 0$$

Conditional distribution function of Y given X = x

$$F(y \mid x) = P[Y \le y \mid X = x] = \int_{-\infty}^{y} f(y \mid x)\,dy,$$

for all x such that $f(x) > 0$

Condition for independence

$f(y \mid x) = f(y)$ and $f(x \mid y) = f(x)$

or f(x, y) = f(x) f(y)

4 Expectation

Properties of mathematical expectation

E(k) = k

E(kX) = kE(X)

E(aX + b) = aE(X) + b

where k, a and b are constant

r^{th} order moment about any point 'A'

$$\mu_r' = E(X - A)^r$$

$$\mu_r' = \begin{cases} \sum_i p_i (x_i - A)^r, & \text{if X is a discrete r.v.} \\ \int_{-\infty}^{\infty} (x - A)^r f(x)dx, & \text{if X is a continous r.v} \end{cases}$$

r^{th} order moment about mean (μ)

$$\mu_r = E(X - \mu)^r = E[X - E(X)]^r$$

$$\mu_r = \begin{cases} \sum_i p_i (x_i - \mu)^r, & \text{if X is a discrete r.v.} \\ \int_{-\infty}^{\infty} (x - \mu)^r f(x)dx, & \text{if X is a continous r.v.} \end{cases}$$

Addition theorem of expectation

$$E(X + Y) = E(X) + E(Y)$$

Multiplication theorem of expectation

E(XY)= E(X) E(Y)

where X and Y are independent random variable (r.v.)

Properties of variance

$$V(aX + b) = a^2 V(X)$$

$$V(aX \pm bY) = a^2 V(X) + b^2 V(Y) \pm 2ab\,Cov(X, Y)$$

$$Cov(X, Y) = \begin{cases} \sum_i p_{ij}(x_i - \bar{x})(y_i - \bar{y}), & \text{if (X,Y) is discrete r.v.} \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - \bar{x})(y - \bar{y}) f(x, y)dydx, & \text{if (X,Y) is continuous r.v.} \end{cases}$$

$$= E(X - \mu_x)(Y - \mu_y)$$

M.D. = E|X – Mean|= E|X – E(X)|

$$= \begin{cases} \sum p_i \mid x - \text{Mean} \mid & \text{for discrete r.v.} \\ \int_{-\infty}^{\infty} \mid x - \text{Mean} \mid f(x)dx & \text{for continuous r.v.} \end{cases}$$

5 Discrete and Continuous Probability Distributions

Probability Distribution	Probability Function	Mean and Variance	Other Properties
Discrete Probability Distributions			
Bernoulli (with parameter p)	$P[X = x] = \begin{cases} p^x (1-p)^{1-x}; & x = 0,1 \\ 0; & \text{elsewhere} \end{cases}$	Mean = p and Variance = p(1 – p)	$\mu_3 = p(2p-1)(p-1)$ $\mu_4 = p[1 - 4p + 6p^2 - 3p^3]$
Binomial (with parameters n and p)	$P[X = x] = \begin{cases} {}^nC_x p^x q^{n-x}; & x = 0, 1, 2, \ldots, n \\ 0; & \text{elsewhere} \end{cases}$ **Recurrence relation** $p(x+1) = \dfrac{n-x}{x+1}.\dfrac{p}{q}.p(x)$, x = 0, 1, 2, …, n – 1 The **expected frequencies**	Mean = np and Variance = npq	$\mu_3 = npq(q-p)$ $\mu_4 = npq[1 + 3(n-2)pq]$ $\beta_1 = \dfrac{(q-p)^2}{npq}$ $\beta_2 = 3 + \dfrac{1-6pq}{npq}$

	$f(x) = N \times P[X = x]$ $= N.\,{}^{n}C_x p^x q^{n-x}\,; x = 0, 1, 2, ..., n$		$\gamma_1 = \frac{1-2p}{\sqrt{npq}}$, and $\gamma_2 = \frac{1-6pq}{npq}$
Poisson (with parameter $\lambda > 0$)	$P[X = x] = \begin{cases} \frac{e^{-\lambda}\lambda^x}{\lfloor x}; & x = 0, 1, 2, 3, ... \\ 0; & \text{elsewhere} \end{cases}$ **Recurrence relation** $p(x+1) = \frac{\lambda}{x+1}.p(x),\ x = 0, 1, 2, 3, ...$ **Expected frequencies** $f(x) = N.P[X = x] = N.\frac{e^{-\lambda}\lambda^x}{\lfloor x}; x = 0, 1, 2, ...$	Mean $= \lambda$ and Variance $= \lambda$	$\mu_3 = \lambda$, $\mu_4 = 3\lambda^2 + \lambda$ $\beta_1 = \frac{1}{\lambda}$, $\gamma_1 = \frac{1}{\sqrt{\lambda}}$, $\beta_2 = 3 + \frac{1}{\lambda}$, $\gamma_2 = \frac{1}{\lambda}$
Discrete Uniform (Rectangular) (with parameter n)	$P[X = x] = \begin{cases} \frac{1}{n} & \text{for } x = 1, 2, ..., n \\ 0, & \text{otherwise} \end{cases}$ **Expected frequency** $f(x) = N.P[X = x] = N.\frac{1}{n}; x = 1, 2, 3, ..., n.$	Mean $= \frac{n+1}{2}$ and Variance $= \frac{n^2-1}{12}$	
Hypergeometric (with parameters N, M and n)	$P[X = x] = \begin{cases} \frac{{}^{M}C_x . {}^{N-M}C_{n-x}}{{}^{N}C_n} & \text{for } x = 0, 1, 2, ..., \min\{n, M\} \\ 0, & \text{otherwise} \end{cases}$ where n, M, N are positive integers such that $n \le N$, $M \le N$	Mean $= \frac{nM}{N}$ and Variance $= \frac{NM(N-M)(N-n)}{N^2(N-1)}$	
Geometric (with parameter p)	$P[X = x] = \begin{cases} q^x p & \text{for } x = 0, 1, 2, ... \\ 0, & \text{otherwise} \end{cases}$	Mean $= \frac{q}{p}$ and Variance $= \frac{q}{p^2}$	
Negative Binomial (with parameters r and p)	$P[X = x] = \begin{cases} {}^{x+r-1}C_{r-1}\, p^r q^x & \text{for } x = 0, 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$ where r is a positive integer and $0 < p < 1$	Mean $= \frac{rq}{p}$ and Variance $= \frac{rq}{p^2}$	
Continuous Probability Distributions			
Normal (with parameters μ and σ)	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $-\infty < x < \infty$ where $-\infty < \mu < \infty$ and $\sigma^2 > 0$ • $P[\mu - \sigma < X < \mu + \sigma] = P[-1 < Z < 1] = 68.27\%$ • $P[\mu - 2\sigma < X < \mu + 2\sigma] = P[-2 < Z < 2] = 95.44\%$ • $P[\mu - 3\sigma < X < \mu + 3\sigma] = P[-3 < Z < 3] = 99.73\%$ Sum of independent normal variables is also a normal variable. **Standard normal distribution** $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$, $-\infty < z < \infty$	Mean $= \mu$ and Variance $= \sigma^2$	$\mu_1 = 0$, $\mu_2 = \sigma^2$ $\mu_3 = 0$, $\mu_4 = 3\sigma^4$ $\beta_1 = 0$, $\beta_2 = 3$, Q.D. $\approx \frac{2}{3}\sigma$, M.D. $\approx \frac{4}{5}\sigma$ $\gamma_1 = 0$, $\gamma_2 = 0$

	Continuous Uniform (Rectangular) (with parameters a and b)	$f(x)=\begin{cases}\frac{1}{b-a} & \text{for } a<x<b\\ 0, & \text{otherwise}\end{cases}$ **Cumulative distribution function** $F(x)=\begin{cases}0 & \text{for } x\le a\\ \frac{x-a}{b-a} & \text{for } a<x<b\\ 1 & \text{for } x\ge b\end{cases}$	Mean $=\frac{a+b}{2}$ and Variance $=\frac{(b-a)^2}{12}$	
	Exponential (with parameter λ**)**	$f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{for } x\ge 0\\ 0, & \text{elsewhere}\end{cases}$ where $\lambda > 0$ **Cumulative distribution function** $F(x)=\begin{cases}1-e^{-\lambda x} & \text{for } x\ge 0\\ 0, & \text{elsewhere}\end{cases}$	Mean $=\frac{1}{\lambda}$ and Variance $=\frac{1}{\lambda^2}$	**Memory less** (Lack of memory) Property $P[X \le x + a \mid X \ge a] = P[X \le x]$
	Gamma (with parameters r and λ)	**Gamma function with parameter n** $\Gamma n=\int_0^\infty x^{n-1}e^{-x}dx$ **Gamma probability distribution with parameters r > 0 and λ > 0** $f(x)=\begin{cases}\frac{\lambda^r e^{-\lambda x}x^{r-1}}{\Gamma r}, & x>0\\ 0, & \text{elsewhere}\end{cases}$ **Gamma distribution with single parameter r > 0** $f(x)=\frac{e^{-x}x^{r-1}}{\Gamma r}, x>0, r>0$	Mean $=\frac{r}{\lambda}$ and Variance $=\frac{r}{\lambda^2}$ In the case of single parameter r > 0 Mean $= r$ and Variance $= r$	• If $n > 1$, $\Gamma n = (n-1)\Gamma(n-1)$ • If n is a positive integer, then $\Gamma n = \underline{n-1}$ • $\Gamma\frac{1}{2}=\sqrt{\pi}$ **Additive property** of Gamma Distribution • If X_i are independent and $X_i \sim G(\lambda, r_i)$ then $\sum_{i=1}^n X_i \sim G(\lambda, \sum_{i=1}^n r_i)$ • If X_i are independent and $X_i \sim G(r_i)$ then $\sum_{i=1}^n X_i \sim G(\sum_{i=1}^n r_i)$
	Beta Distribution of first kind (with parameters m and n)	**Beta function** $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$ where $m > 0$, $n > 0$ **Beta distribution of first kind** $f(x)=\begin{cases}\frac{1}{\beta(m,n)}x^{m-1}(1-x)^{n-1}, & 0<x<1\\ 0, & \text{otherwise}\end{cases}$ where $m > 0$, $n > 0$	Mean $=\frac{m}{m+n}$ and Variance $=\frac{mn}{(m+n)^2(m+n+1)}$	• Beta function is **symmetric** function, i.e., $\beta(m, n) = \beta(n, m)$ • $\frac{\beta(p,q+1)}{q}=\frac{\beta(p+1,q)}{p}$ • $\beta(p,q)=\beta(p+q,q)\times\beta(p,q+1)$ • $\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ **Relationship between beta and gamma functions** If $m > 0$, $n > 0$, then $\beta(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma m+n}$

	Beta Distribution of second kind (with parameters m and n)	$f(x)=\begin{cases}\dfrac{x^{m-1}}{\beta(m,n)(1+x)^{m+n}}, & 0<x<\infty\\ 0, & \text{elsewhere}\end{cases}$ where m > 0, n > 0.	Mean $=\dfrac{m}{n-1}$, n > 1 and Variance = $\dfrac{m(m+n-1)}{(n-1)^2(n-2)}$, n > 2	

MST-004 STATISTICAL INFERENCE

S. No.	FORMULAE		
1	**Sampling Distribution**		
	Sample mean ($\bar{X}$) and sample variance (S^2) $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$		
	Sampling Distribution of Sample Mean	**Sampling Distribution of Sample Median**	**Sampling Distribution of Sample Proportion**
	$E(\bar{X})=\mu$ $Var(\bar{X})=\frac{\sigma^2}{n}$ $SE(\bar{X})=SD(\bar{X})=\frac{\sigma}{\sqrt{n}}$	$E(\tilde{X})=\mu$ $Var(\tilde{X})=\frac{\pi\sigma^2}{2n}$ $SE(\tilde{X})=\sqrt{\frac{\pi\sigma^2}{2n}}$	$E(p)=P$, $Var(p)=\frac{PQ}{n}$ and $SE(p)=\sqrt{\frac{PQ}{n}}$ **For finite population** $E(p)=P$, $Var(p)=\frac{N-n}{N-1}\frac{PQ}{n}$ and $SE(p)=\sqrt{\frac{N-n}{N-1}\frac{PQ}{n}}$
	Sampling Distribution of Difference of two Sample Means (Independent Populations)	**Sampling Distribution of Difference of two Sample Proportions**	**Central Limit Theorem and Law of Large Numbers**
	$E(\bar{X}-\bar{Y})=\mu_1-\mu_2$ $Var(\bar{X}-\bar{Y})=\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}$ $SE(\bar{X}-\bar{Y})=\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$	$E(p_1-p_2)=P_1-P_2$ $Var(p_1-p_2)=\frac{P_1Q_1}{n_1}+\frac{P_2Q_2}{n_2}$ $SE(p_1-p_2)=\sqrt{\frac{P_1Q_1}{n_1}+\frac{P_2Q_2}{n_2}}$	If $X_1, X_2, ..., X_n$ is a random sample of size n taken from a population with mean μ and variance σ^2 then $\bar{X}\sim N\left(\mu,\frac{\sigma^2}{n}\right)$ for n > 30. **Law of large numbers** $P\left[\lvert\bar{X}-\mu\rvert<\varepsilon\right]\geq 1-\eta$ $n\geq\frac{\sigma^2}{\varepsilon^2\eta}$ where $\varepsilon>0$ and $0<\eta<1$.
	Chi-Square Distribution	**t-Distribution**	**F-Distribution**

Chi-square statistic $= \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$ $f(\chi^2) = \dfrac{1}{2^{n/2}\left\lvert\dfrac{n}{2}\right.} e^{-\chi^2/2}(\chi^2)^{(n/2)-1}$ $;0 < \chi^2 < \infty$ $\mu'_r = \dfrac{1}{2^{n/2}\left\lvert\dfrac{n}{2}\right.} \dfrac{\left\lvert\dfrac{n}{2}+r\right.}{\left(\dfrac{1}{2}\right)^{\frac{n}{2}+r}}$	t-statistic $= \dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{(n-1)}$ t-distribution with n df $f(t) = \dfrac{1}{\sqrt{n}\,B\left(\dfrac{1}{2},\dfrac{n}{2}\right)\left(1+\dfrac{t^2}{n}\right)^{(n+1)/2}}$ $;-\infty < t < \infty$ $\mu'_{2r+1} = 0;$ for all $r = 0, 1, 2, 3...$ $\mu'_{2r} = n^r \dfrac{\left\lvert\dfrac{n}{2}-r\right. \left\lvert r+\dfrac{1}{2}\right.}{\left\lvert\dfrac{1}{2}\right. \left\lvert\dfrac{n}{2}\right.};$ $n > 2r$	F-statistic $= \dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{(n_1-1, n_2-1)}$ $F = \dfrac{\chi^2_{(n_1-1)}/(n_1-1)}{\chi^2_{(n_2-1)}/(n_2-1)} \sim F_{(n_1-1, n_2-1)}$ $f(F) = \dfrac{(\nu_1/\nu_2)^{\nu_1/2}}{B\left(\dfrac{\nu_1}{2},\dfrac{\nu_2}{2}\right)} \dfrac{F^{(\nu_1/2)-1}}{\left(1+\dfrac{\nu_1}{\nu_2}F\right)^{(\nu_1+\nu_2)/2}}$ $;0 < F < \infty$ $\mu'_r = \left(\dfrac{\nu_2}{\nu_1}\right)^r \dfrac{\left\lvert\dfrac{\nu_1}{2}+r\right. \left\lvert\dfrac{\nu_2}{2}-r\right.}{\left\lvert\dfrac{\nu_1}{2}\right. \left\lvert\dfrac{\nu_2}{2}\right.};$ for $\nu_2 > 2r$ where $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$. $F_{(\nu_1,\nu_2),(1-\alpha)} = \dfrac{1}{F_{(\nu_2,\nu_1),\alpha}}$
Relation between t, F and chi-square distribution $t^2 \sim F(1, \nu);$ $\chi^2 = \nu_1 F$ when $\nu_2 \to \infty$		

2 Point Estimation

$f(x_1, x_2, ..., x_n, \theta) = P[X_1 = x_1, X_2 = x_2, ..., X_n = x_n]$

Unbiasedness

$E(T) = \theta$ for all values of θ

Consistency

$T_n \xrightarrow{p} \theta$ as $n \to \infty$ for every $\theta \in \Theta$

$\lim_{n\to\infty} P[|T_n - \theta| < \varepsilon] = 1$ where $\varepsilon > 0$

$P[|T_n - \theta| < \varepsilon] > 1 - \eta$; $n \geq m$, $\varepsilon > 0$ and $\eta > 0$

Sufficient conditions for consistency

$E(T_n) \to \theta$ as $n \to \infty$ and $Var(T_n) \to 0$ as $n \to \infty$

Efficiency

T_1 is said to be more efficient than T_2 if $Var(T_1) < Var(T_2)$

Absolute efficiency of estimator T

$$e = \frac{Var(T^*)}{Var(T)}$$

Minimum variance unbiased estimator (T)

i) $E(T) = \theta$ for all values of θ

ii) $Var(T) \leq Var(T')$ for all values of θ

Sufficiency

$f(x_1, x_2, ..., x_n / T = t) = g(x_1, x_2, ..., x_n)$

Factorization theorem of sufficiency

$f(x_1, x_2, ..., x_n, \theta) = g[t(x), \theta].h(x_1, x_2, ..., x_n)$

Maximum likelihood estimation

$L(\theta) = f(x_1, x_1, ..., x_n, \theta)$

For discrete case

$L(\theta) = P[X_1 = x_1].P[X_2 = x_2]... P[X_n = x_n]$

For continuous case

$L(\theta) = f(x_1, \theta).f(x_2, \theta) ... f(x_n, \theta)$

For maximum likelihood estimation

$$\frac{\partial}{\partial\theta}(\log L) = 0 \quad \text{provided} \quad \frac{\partial^2}{\partial\theta^2}(\log L)\Big|_{\theta=\hat{\theta}} < 0$$

$$\frac{\partial}{\partial\theta_i}(\log L) = 0; \text{ for all } i = 1, 2, ..., k$$

provided, the matrix of derivatives

$$\frac{\partial^2}{\partial\theta_i \partial\theta_j}(\log L)\Big|_{\theta_i=\hat{\theta}_i \ \& \ \theta_j=\hat{\theta}_j} < 0; \text{ for all } i \neq j = 1, 2, ..., k$$

For moment estimation

$\mu_1' = M_1'$, $\mu_r = M_r$; $r = 2, 3, ..., k$

r^{th} sample moment about origin = $M_r' = \frac{1}{n}\sum_{i=1}^{n} X_i^r$

r^{th} sample moment about mean = $M_r = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^r$

r^{th} population moment about origin = $\mu_r' = E(X)^r$

r^{th} population moment about mean = $\mu_r = E(X - \mu)^r$

3 Confidence Interval

Length of confidence interval (L) = $T_2 - T_1$
where T_1 – lower confidence limit, and
T_2 – upper confidence limit.

Confidence interval for population mean

When population variance is known

$$\left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

When population variance is unknown

$$\left[\bar{X} - t_{(n-1),\,\alpha/2}\frac{S}{\sqrt{n}}, \quad \bar{X} + t_{(n-1),\,\alpha/2}\frac{S}{\sqrt{n}}\right]$$

$$\left[\bar{X} - z_{\alpha/2}\frac{S}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{S}{\sqrt{n}}\right] ; n > 30$$

For non-normal population

$$P\left[-z_{\alpha/2} \leq \frac{\bar{X} - E(\bar{X})}{\sqrt{Var(\bar{X})}} \leq z_{\alpha/2}\right] = 1 - \alpha$$

Confidence interval for population proportion

$$\left[p - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} \ , \ p + z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right]$$

Confidence interval for population variance

When population mean is known

$$\left[\frac{\sum_{i=1}^{n}(X_i - \mu)^2}{\chi^2_{(n),\,\alpha/2}}, \frac{\sum_{i=1}^{n}(X_i - \mu)^2}{\chi^2_{(n),\,(1-\alpha/2)}}\right]$$

When population mean is unknown

$$\left[\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\chi^2_{(n-1),\,\alpha/2}}, \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\chi^2_{(n-1),\,(1-\alpha/2)}}\right] \text{ or}$$

$$\left[\frac{(n-1)S^2}{\chi^2_{(n-1),\,\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{(n-1),\,(1-\alpha/2)}}\right]$$

Confidence interval for difference of two population means (independent populations)

$$\left[\left(\bar{X}-\bar{Y}\right)-z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\ ,\ \left(\bar{X}-\bar{Y}\right)+z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]$$

$$\left[\left(\bar{X}-\bar{Y}\right)-t_{(n_1+n_2-2),\,\alpha/2}\,S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}},\ \left(\bar{X}-\bar{Y}\right)+t_{(n_1+n_2-2),\,\alpha/2}\,S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right]$$

where $S_p^2=\frac{1}{n_1+n_2-2}\left[\sum_{i=1}^{n_1}\left(X_i-\bar{X}\right)^2+\sum_{i=1}^{n_2}\left(Y_i-\bar{Y}\right)^2\right]$ or

$$S_p^2=\frac{1}{n_1+n_2-2}\left[\left(n_1-1\right)S_1^2+\left(n_2-1\right)S_2^2\right]$$

and

$$S_1^2=\frac{1}{(n_1-1)}\sum_{i=1}^{n_1}\left(X_i-\bar{X}\right)^2\ ,\ S_2^2=\frac{1}{(n_2-1)}\sum_{i=1}^{n_2}\left(Y_i-\bar{Y}\right)^2$$

Short-cut approach

$$\bar{X}=a+\frac{1}{n_1}\sum d_1,\ \bar{Y}=b+\frac{1}{n_2}\sum d_2 \text{ and}$$

$$S_p^2=\frac{1}{n_1+n_2-2}\left[\left\{\sum d_1^2-\frac{\left(\sum d_1\right)^2}{n_1}\right\}+\left\{\sum d_2^2-\frac{\left(\sum d_2\right)^2}{n_2}\right\}\right]$$

where $d_1=(X-a)$ and $d_2=(Y-b)$, 'a' and 'b' are assumed arbitrary values.

When $n_1, n_2 > 30$

$$\left[\left(\bar{X}-\bar{Y}\right)-z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\ ,\ \left(\bar{X}-\bar{Y}\right)+z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right]$$

Confidence interval for difference of two population means (dependent populations)

$$\left[\bar{D}-t_{(n-1),\,\alpha/2}\frac{S_D}{\sqrt{n}}\ ,\ \bar{D}+t_{(n-1),\,\alpha/2}\frac{S_D}{\sqrt{n}}\right]$$

where $D_i=X_i-Y_i;\ \bar{D}=\frac{1}{n}\sum_{i=1}^{n}D_i$ and

$$S_D^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(D_i-\bar{D}\right)^2=\frac{1}{n-1}\left[\sum_{i=1}^{n}D_i^2-\frac{\left(\sum_{i=1}^{n}D_i\right)^2}{n}\right]$$

Confidence interval for difference of two population proportions (independent populations)

$$\left[\left(p_1-p_2\right)-z_{\alpha/2}\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}\ ,\left(p_1-p_2\right)+z_{\alpha/2}\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}\ \right]$$

Confidence interval for two population variances (independent populations)

$$\left[\frac{S_1^2/S_2^2}{F_{(n_1-1,n_2-1),\,\alpha/2}}\ ,\ F_{(n_2-1,n_1-1),\,\alpha/2}\cdot\frac{S_1^2}{S_2^2}\right]$$

4 Sample Size

$$n=\frac{z_{\alpha/2}^2\sigma^2}{E^2}$$

where E – sampling error or margin of error

$$n=\frac{z_{\alpha/2}^2P(1-P)}{E^2}$$

For finite population of size N

$$n=\frac{N\,z_{\alpha/2}^2\sigma^2}{E^2\left(N-1\right)+z_{\alpha/2}^2\sigma^2}$$

$$n=\frac{N\,z_{\alpha/2}^2P(1-P)}{E^2\left(N-1\right)+z_{\alpha/2}^2P(1-P)}$$

5 Parametric Tests

Type of error

$\alpha = P$ [reject H_0 when H_0 is true]

$\beta = P$[do not reject H_0 when H_0 is false]

Power of a test

$1-\beta = P$[Reject H_0 when H_1 is true]

p-values

p-value = P[Test Statistic (T) ≥ observed value of the test statistic] (for right-tailed test)

p-value = P[Test Statistic (T) ≤ observed value of the test statistic] (for left-tailed test)

p-value = $2P\left[T\geq\left|\text{observed value of the test statistic}\right|\right]$ (for two-tailed test)

Test statistic for population mean

$$Z=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}\ \sim\ N(0,1)\ ;$$

$$t=\frac{\bar{X}-\mu_0}{S/\sqrt{n}}\ \sim t_{(n-1)}\ ;\ Z=\frac{\bar{X}-\mu_0}{S/\sqrt{n}}\sim N(0,1);\ n>30$$

Test statistic for population proportion

$$Z=\frac{p-P_0}{\sqrt{\frac{P_0Q_0}{n}}}\ \sim\ N(0,1)$$

Test statistic for population variance

$$Z=\frac{S^2-\sigma_0^2}{\sigma_0^2\sqrt{\frac{2}{n}}}\sim N(0,1);\ n>30$$

$$\chi^2=\frac{\sum\left(X_i-\bar{X}\right)^2}{\sigma_0^2}=\frac{(n-1)S^2}{\sigma_0^2}\sim\chi^2_{(n-1)}$$

Test statistic for difference of two population means (independent populations)

$$Z = \frac{\bar{X} - \bar{Y}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

$$Z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$$

$$t = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}$$

where $S_p^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_{i=1}^{n_1}(X_i - \bar{X})^2 + \sum_{i=1}^{n_2}(Y_i - \bar{Y})^2\right]$ or

$$S_p^2 = \frac{1}{n_1 + n_2 - 2}\left[(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2\right]$$

$$Z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim N(0, 1)\ ;\ n_1, n_2 > 30$$

Test statistic for difference of two population means (dependent populations)

$$t = \frac{\bar{D}}{S_D / \sqrt{n}} \sim t_{(n-1)}$$

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i \text{ and } S_D^2 = \frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2$$

Test statistic for population correlation coefficient (ρ)

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{(n-2)}$$

Test statistic for difference of two population proportions (independent populations)

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad (\text{when P is known})$$

where Q = 1−P.

$$Z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad (\text{when P is unknown})$$

where

$$\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{X_1 + X_2}{n_1 + n_2} \text{ and } \hat{Q} = 1 - \hat{P}$$

Test statistic for two population variances (independent populations)

$$Z = \frac{S_1^2 - S_2^2}{\sigma^2\sqrt{2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1);\ n_1 \text{ and } n_2 > 30$$

$$Z = \frac{S_1^2 - S_2^2}{\sqrt{\left(\frac{2S_1^4}{n_1} + \frac{2S_2^4}{n_2}\right)}} \sim N(0,1);\ n_1 \text{ and } n_2 > 30$$

$$F = \frac{S_1^2}{S_2^2} \sim F_{(n_1-1, n_2-1)}$$

$$F_{(\nu_1,\nu_2),(1-\alpha)} = \frac{1}{F_{(\nu_2,\nu_1),\alpha}}$$

6 Non-parametric Tests

Sign test/Paired sign test

$$\text{Test statistic(S)} = \begin{cases} S^- & [\text{for right tail test}] \\ S^+ & [\text{for left tail test}] \\ \min\{S^-, S^+\} & [\text{for two tails test}] \end{cases}$$

where S^- – number of minus signs and

S^+ – number of plus signs.

p-value = $P[S \le \text{test statistic}]$

Test statistic (for n > 20) $Z = \frac{S - \frac{n}{2}}{\sqrt{\frac{n}{4}}} \sim N(0,1)$

Wilcoxon signed-rank test/Wilcoxon matched-pair signed-rank test

$$\text{Test statistic(T)} = \begin{cases} T^- & [\text{for right tail test}] \\ T^+ & [\text{for left tail test}] \\ \min\{T^-, T^+\} & [\text{for two tail test}] \end{cases}$$

where T^- – sum of ranks of minus signs and

T^+ – sum of ranks of plus signs.

Test statistic (for n > 25)

$$Z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0,1)$$

Run test

Test statistic = R = number of runs

Test statistic (for n > 40) $Z = \frac{R - \frac{2n_1n_2}{n} - 1}{\sqrt{\frac{2n_1n_2(2n_1n_2 - n)}{n^2(n-1)}}} \sim N(0,1)$

Kolmogorov-Smirnov goodness of fit test

$$\text{Test statistic}(D_n) = \sup_x |S(x) - F_0(x)|$$

$$S(x) = \frac{\text{Number of sample observations less than or equal to x}}{\text{Total number of observations}}$$

Mann-Whitney U test

$$\text{Test statistic}(U) = \begin{cases} S - \dfrac{n_1(n_1+1)}{2}; & \text{if } n_1 \text{ is small} \\ S - \dfrac{n_2(n_2+1)}{2}; & \text{if } n_2 \text{ is small} \end{cases}$$

where S – sum of the ranks assigned to the sample observations of smaller sized sample.

For large n_1 or $n_2 > 20$

$$\text{Test statistic } Z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} \sim N(0,1)$$

Kolmogorov-Smirnov two-sample test

$$\text{Test statistic}(D) = \sup_x |S_1(x) - S_2(x)|$$

$$S_1(x) = \frac{\text{Number of first sample observations less than or equal to x}}{n_1}$$

$$S_2(x) = \frac{\text{Number of second sample observations less than or equal to x}}{n_2}$$

Kruskal-Wallis test

$$\text{Test statistic}(H) = \frac{12}{n(n+1)}\left(\sum_{i=1}^{k}\frac{R_i^2}{n_i}\right) - 3(n+1)$$

where R_i – sum of ranks of i^{th} sample.

If tie occurs, the adjustment factor is

$$C = 1 - \frac{1}{(n^3 - n)}\sum_{i=1}^{r}(t_i^3 - t_i)$$

where r – number of groups of different tied ranks, and t_i – number of tied values within i^{th} group that are tied at a particular value.

$$\text{Test statistic}(H_C) = H / C$$

Friedman test

$$\text{Test statistic}(F) = \frac{12}{kn(n+1)}\sum_{j=1}^{n}R_j^2 - 3k(n+1)$$

where R_i – sum of all ranks for i^{th} treatment.

If tie occurs, the adjustment factor is

$$C = 1 - \frac{1}{k(n^2 - 1)}\sum_{i=1}^{k}(t_i^3 - t_i)$$

where t_i – number of tied observations in i^{th} block (sample).

$$\text{Test statistic}(F_C) = F / C$$

Chi-square test for goodness of fit

$$\text{Test statistic } (\chi^2) = \sum_{i=1}^{k}\frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{(k-1)}$$

$$E_i = Np_i$$

where O_i – observed frequency of i^{th} class, and

E_i – expected frequency of i^{th} class.

Chi-square test for independence of attributes

$$\text{Test statistic } (\chi^2) = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij} - E_{ij})^2}{E_{ij}} \sim \chi^2_{(r-1)(c-1)}$$

$$E_{ij} = \frac{R_i \times C_j}{N}$$

where R_i – sum of i^{th} row, and

C_i – sum of j^{th} column.

S. No. 1 — Simple Random Sampling

Sample		Population		
Mean	**Mean Square**	**Mean**	**Mean Square**	**Variance**
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$	$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$	$S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$	$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2$

Simple Random Sampling without Replacement (SRSWOR)	Simple Random Sampling with Replacement (SRSWR)
Number of possible samples = ${}^{N}C_{n}$	Number of possible samples = N^n
$E(\bar{x}) = \bar{X}$	$E(\bar{x}) = \bar{X}$
$Var(\bar{x}) = \left(\frac{1}{n} - \frac{1}{N}\right) S^2$	$Var(\bar{x}) = \frac{(N-1)}{nN} S^2$

Simple Random Sampling for Attributes

Sample			Population	
Proportion	**Mean Square**	**Variance**	**Proportion**	**Mean Square**
$p = \frac{a}{n}$	$s^2 = \frac{npq}{n-1}$	$Var(p) = \frac{N-n}{N-1}.\frac{\pi(1-\pi)}{n}$	$\pi = \frac{A}{N}$	$S^2 = \frac{N\pi(1-\pi)}{N-1}$

Sample Size in Simple Random Sampling

$n = \frac{NS^2 z_\alpha^2}{Nd^2 + S^2 z_\alpha^2}$ (for large population)	$n = \frac{NS^2 t_\alpha^2}{Nd^2 + S^2 t_\alpha^2}$ (for small population)

2 — Stratified Random Sampling

Sample			Population	
Mean	**Mean Square/Variance**	**Proportion**	**Mean**	**Mean Square**
$\bar{x}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}$ $\bar{x}_n = \frac{1}{n_i}\sum_{i=1}^{k} n_i \bar{x}_i$ $\bar{x}_{st} = \frac{1}{N}\sum_{i=1}^{k} N_i \bar{x}_i$ $= \sum_{i=1}^{k} W_i \bar{x}_i$	$s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2$ $Var(\bar{x}_{st})$ $= \frac{1}{N^2}\sum_{i=1}^{k} N_i(N_i - n_i)\frac{S_i}{n_i}$ $= \sum_{i=1}^{k}\left(\frac{1}{n_i} - \frac{1}{N_i}\right) W_i^2 S_i^2$	$p_{st} = \sum_{i=1}^{k} W_i p_i$ $E(p_{st}) = \pi$ $Var(p_{st})$ $= \frac{1}{N^2}\sum_{i=1}^{k}\frac{N_i^2(N_i - n_i)\pi_i(1-\pi_i)}{(N_i - 1)n_i}$ $= \frac{1}{N^2}\sum_{i=1}^{k} N_i(N_i - n_i)\frac{\pi_i(1-\pi_i)}{n_i}$	$\bar{X}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} X_{ij}$ $\bar{X} = \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i} X_{ij}$ $\bar{X} = \frac{1}{N}\sum_{i=1}^{k} N_i \bar{X}_i$	$S_i^2 = \frac{1}{N_i - 1}\sum_{j=1}^{N_i}(X_{ij} - \bar{X}_i)^2$

Allocation of Sample Size

Equal	Proportion	Neyman/Optimum
$n_i = \frac{n}{k}$	$n_i = \frac{n}{N} N_i$ $Var(\bar{x}_{st})_{PROP} = \left(1 - \frac{n}{N}\right)\sum_{i=1}^{k}\frac{W_i S_i^2}{n}$	$n_i = n\frac{N_i S_i}{\sum_{i=1}^{k} N_i S_i}$; $n_i = n\frac{(N_i S_i/\sqrt{c_i})}{\sum_{i=1}^{k}(N_i S_i/\sqrt{c_i})}$ $Var(\bar{x}_{st})_{NEY} = \frac{\left(\sum_{i=1}^{k} W_i S_i\right)^2}{n} - \frac{\sum_{i=1}^{k} W_i S_i^2}{N}$

		$=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{i=1}^{k} W_i S_i^2$	

3 Systematic Random Sampling

Sample Mean	Variance	Population Mean	Population Mean Square
$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$ $\bar{x}_{sys} = \frac{1}{k}\sum_{i=1}^{k} \bar{x}_i$	$Var(\bar{x}_{sys}) = \frac{N-1}{N}S^2 - \frac{(n-1)k}{N}S_{sys}^2$	$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}$ $\bar{X}_{..} = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n} X_{ij}$ $= \frac{1}{k}\sum_{i=1}^{k} \bar{X}_i$	$S^2 = \frac{1}{(nk-1)}\sum_{i=1}^{k}\sum_{j=1}^{n}(X_{ij}-\bar{X})^2$ $S_{sys}^2 = \frac{1}{k(n-1)}\sum_{i=1}^{k}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$

4 Cluster Sampling

Cluster Mean	Sample Mean	Population Mean	Population Mean Square
$\bar{x}_i = \frac{1}{M}\sum_{j=1}^{M} x_{ij}$	$\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} \bar{x}_i$	$\bar{X}_i = \frac{1}{M}\sum_{j=1}^{M} X_{ij}$ $\bar{X} = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M} X_{ij} = \frac{1}{N}\sum_{i=1}^{N} \bar{X}_i$	$S_i^2 = \frac{1}{(M-1)}\sum_{j=1}^{M}(X_{ij}-\bar{X}_i)^2$ $S_w^2 = \frac{1}{N}\sum_{i=1}^{N} S_i^2$ (within cluster) $S_b^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2$ (between clusters) $S^2 = \frac{1}{(NM-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}(X_{ij}-\bar{X})^2$

Variance

$$Var(\bar{x}_n) = \left(\frac{1-f}{n}\right)S_b^2 \left(\frac{1-f}{n}\right)S_b\left[1+(M-1)\rho\right] \quad \text{where } \rho = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k\neq j=1}^{M}(X_{ij}-\bar{X})(X_{ik}-\bar{X})}{(M-1)(NM-1)S^2}$$

5 Two Stage Sampling

Sample Mean	Population Mean	Population Mean Square
$\bar{x}_i = \frac{1}{m}\sum_{j=1}^{m} x_{ij}$ $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} \bar{x}_i$	$\bar{X}_i = \frac{1}{M}\sum_{j=1}^{M} X_{ij}$ $\bar{X} = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M} X_{ij}$	$S_b^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2$ (between samples) $S_w^2 = \frac{1}{N(M-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}(X_{ij}-\bar{X}_i)^2$ (within sample)

Variance

$$Var(\bar{x}) = \frac{(N-n)}{N}\frac{S_b^2}{n} + \frac{M-m}{M}\frac{S_w^2}{mn}$$

6 One Way ANOVA / Completely Randomised Design (CRD)

Sum of Squares (SS)	Mean Sum of Squares (MSS)	Variance Ratio
$SST = \sum_{i=1}^{k} T_{i.}^2 - CF$ $SSE = TSS - SST$ $TSS = RSS - CF$	$MSST = \frac{SST}{k-1}$ $MSSE = \frac{SSE}{N-k}$	$F = \frac{MSST}{MSSE}$

where $CF = \frac{G^2}{N}$, $RSS = \sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}^2$, G – grand total or sum of all observations, and

$T_{i.} = \sum_{j=1}^{n_i} T_{ij}$ – sum of the observations of i^{th} sample or i^{th} level.

Critical Difference in CRD/ One Way ANOVA

For Equal Size of Samples	For Unequal Sizes of Samples
$CD = \left\|t\right\|_{\alpha/2}$ (for error df) $\times \sqrt{\frac{2MSSE}{n}}$	$CD = t_{\alpha/2}$ (for error df) $\times \left[MSSE \left(\frac{1}{n_i} + \frac{1}{n_j} \right) \right]^{1/2}$

7 Two Way ANOVA / Randomised Block Design (RBD)

Sum of Squares	Mean Sum of Squares	Variance Ratio
$SSA = \frac{1}{q}\sum_{i=1}^{p} y_{i.}^2 - CF$ $SSB = \frac{1}{p}\sum_{j=1}^{q} y_{.j}^2 - CF$ $SSE = TSS - SSA - SSB$ $TSS = RSS - CF$	$MSSA = \frac{SSA}{p-1}$ $MSSB = \frac{SSB}{q-1}$ $MSSE = \frac{SSE}{(p-1)(q-1)}$	$F_A = \frac{MSSA}{MSSE}$ $F_B = \frac{MSSB}{MSSE}$

where $CF = \frac{G^2}{N}$, $RSS = \sum_{i=1}^{p}\sum_{j=1}^{q} y_{ij}^2$, $N = pq$

Critical Difference in RBD/ Two Way ANOVA

$CD = \left\|t\right\|_{\alpha/2}$ (for error df) $\times \sqrt{\frac{2MSSE}{q}}$

8 Two Way ANOVA with m Observations per Cell

Sum of Squares	Mean Sum of Squares	Variance Ratio
$SSA = \frac{1}{qm}\sum_{i=1}^{p} y_{i..}^2 - CF$ $SSB = \frac{1}{pm}\sum_{j=1}^{q} y_{.j.}^2 - CF$ $SSAB = \frac{1}{m}\sum_{i=1}^{p}\sum_{j=1}^{q} y_{ij.}^2 - CF - SSA - SSB$ $SSE = TSS - SSA - SSB - SSAB$ $TSS = RSS - CF$	$MSSA = \frac{SSA}{p-1}$ $MSSB = \frac{SSB}{q-1}$ $MSSAB = \frac{SSAB}{(p-1)(q-1)}$ $MSSE = \frac{SSE}{pq(m-1)}$	$F = \frac{MSSA}{MSSE}$ $F = \frac{MSSB}{MSSE}$ $F = \frac{MSSAB}{MSSE}$

where $CF = \frac{G^2}{N}$, $RSS = \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{m} y_{ijk}^2$, $N = pqm$

9 Latin Square Design (LSD)

Sum of Squares	Mean Sum of Squares	Variance Ratio
$SSR = \frac{1}{m}\sum_{i=1}^{m} y_{i..}^2 - CF$ $SSC = \frac{1}{m}\sum_{j=1}^{m} y_{.j.}^2 - CF$ $SST = \frac{1}{m}\sum_{k=1}^{m} y_{..k}^2 - CF$ $SSE = TSS - SSR - SSC - SST$ $TSS = RSS - CF$	$MSSR = \frac{SSR}{m-1}$ $MSSC = \frac{SSC}{m-1}$ $MSST = \frac{SST}{m-1}$ $MSSE = \frac{SSE}{(m-1)(m-2)}$	$F_R = \frac{MSSR}{MSSE}$ $F_C = \frac{MSSC}{MSSE}$ $F_T = \frac{MSST}{MSSE}$

where $CF = \frac{G^2}{N}$, $RSS = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} y_{ijk}^2$, $N = m^2$, $y_{i..} = \sum_{j=1}^{m}\sum_{k=1}^{m} y_{ijk}$ – sum of the observations of i^{th} row

$y_{.j.} = \sum_{i=1}^{m}\sum_{k=1}^{m} y_{ijk}$ – sum of the observations of j^{th} column, $y_{..k} = \sum_{i=1}^{m}\sum_{j=1}^{m} y_{ijk}$ – sum of the observations of k^{th} treatment

Relative Efficiency LSD over RBD	**Relative Efficiency LSD over CRD**
When rows are taken as blocks Relative efficiency = $\frac{MSSC + (m-1)MSSE}{m \times MSSE}$ When columns are taken as blocks Relative efficiency = $\frac{MSSR + (m-1)MSSE}{m \times MSSE}$	Relative efficiency = $\frac{MSSR + MSSC + (m-1)MSSE}{(m+1)MSSE}$
Missing Value in RBD	**Missing Value in LSD**
$\hat{Y} = \frac{pT_i' + qB_j' - G'}{(p-1)(q-1)}$	$\hat{Y} = \frac{m(R_i' + C_j' + T_k') - 2G}{(m-1)(m-2)}$

10 Factorial Experiments

Main and interaction effects for 2^2

[A] = [ab] + [a] – [b] – [1]

[B] = [ab] +[b] – [a] – [1]

[AB] = [ab] + [1] – [a] – [b]

Sum of squares due to

$A = [A]^2 / 4r$

$B = [B]^2 / 4r$

$AB = [AB]^2 / 4r$

Main and interaction effects for 2^3

$[A] = [abc] - [bc] + [ac] - [c] + [ab] - [b] + [a] - [1]$

$[B] = [abc] + [bc] - [ac] - [c] + [ab] + [b] - [a] - [1]$

$[AB] = [abc] - [bc] - [ac] + [c] + [ab] - [b] - [a] + [1]$

$[C] = [abc] + [bc] + [ac] + [c] - [ab] - [b] - [a] - [1]$

$[AC] = [abc] - [bc] + [ac] - [c] - [ab] + [b] - [a] + [1]$

$[BC] = [abc] + [bc] - [ac] - [c] - [ab] - [b] + [a] + [1]$

$[ABC] = [abc] - [bc] - [ac] + [c] - [ab] + [b] + [a] - [1]$

Sum of squares due to

$A = [A]^2 / 8r$; $B = [B]^2 / 8r$; $AB = [AB]^2 / 8r$; $C = [C]^2 / 8r$

$AC = [AC]^2 / 8r$; $BC = [BC]^2 / 8r$; $ABC = [ABC]^2 / 8r$

Standard errors for testing the difference between means at different levels

SE of difference between main effect means = $\sqrt{\frac{2MSSE}{r.2^{n-1}}}$

SE of difference between A means at same level of B

= SE of difference between B means at same level of A = $\sqrt{\frac{2MSSE}{r.2^{n-2}}}$

SE for testing the difference between means in case of r-factor interaction = $\sqrt{\frac{2MSSE}{r.2^{n-r}}}$

S. No.	FORMULAE			
1	**Process Control**			
	Control Chart	**Control Line (CL)**	**Lower Control Line (LCL)**	**Upper Control Line (UCL)**
	$\bar{X}$	μ	$\mu - A\sigma$	$\mu + A\sigma$
		$\bar{\bar{X}}$	$\bar{\bar{X}} - A_2\bar{R}$	$\bar{\bar{X}} + A_2\bar{R}$
		$\bar{\bar{X}}_{new}$	$\bar{\bar{X}}_{new} - A_2\bar{R}_{new}$	$\bar{\bar{X}}_{new} + A_2\bar{R}_{new}$
		$\bar{\bar{X}}$	$\bar{\bar{X}} - A_3\bar{S}$	$\bar{\bar{X}} + A_3\bar{S}$
		$\bar{\bar{X}}_{new}$	$\bar{\bar{X}}_{new} - A_3\bar{S}_{new}$	$\bar{\bar{X}}_{new} + A_3\bar{S}_{new}$
		where $\bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k}\bar{X}_i$; $\bar{\bar{X}}_{new} = \frac{\sum_{i=1}^{k}\bar{X}_i - \sum_{j=1}^{d}\bar{X}_j}{k-d}$; $\bar{R}_{new} = \frac{\sum_{i=1}^{k}R_i - \sum_{j=1}^{d}R_j}{k-d}$; $\bar{S}_{new} = \frac{\sum_{i=1}^{k}S_i - \sum_{j=1}^{d}S_j}{k-d}$		
	R	$d_2\sigma$	$D_1\sigma$	$D_2\sigma$
		$\bar{R}$	$D_3\bar{R}$	$D_4\bar{R}$
		$\bar{R}_{new}$	$D_3\bar{R}_{new}$	$D_4\bar{R}_{new}$
	S	$c_4\sigma$	$B_5\sigma$	$B_6\sigma$
		$\bar{S}$	$B_3\bar{S}$	$B_4\bar{S}$
		$\bar{S}_{new}$	$B_3\bar{S}_{new}$	$B_4\bar{S}_{new}$
	p	P	$P - 3\sqrt{\frac{P(1-P)}{n}}$	$P + 3\sqrt{\frac{P(1-P)}{n}}$
		$\bar{p}$	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
		$\bar{p}_{new}$	$\bar{p}_{new} - 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{n}}$	$\bar{p}_{new} + 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{n}}$
		where $p_i = \frac{d_i}{n}$; $\bar{p} = \frac{1}{k}\sum_{i=1}^{k}p_i$ or $\bar{p} = \frac{1}{nk}\sum_{i=1}^{k}d_i$; $\bar{p}_{new} = \frac{\sum_{i=1}^{k}p_i - \sum_{j=1}^{d}p_j}{k-d}$ or $\bar{p}_{new} = \frac{\sum_{i=1}^{k}d_i - \sum_{j=1}^{d}d_j}{n(k-d)}$		
	p-chart for variable sample size	P	$P - 3\sqrt{\frac{P(1-P)}{n_i}}$	$P + 3\sqrt{\frac{P(1-P)}{n_i}}$
		$\bar{p}$	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$
		$\bar{p}_{new}$	$\bar{p}_{new} - 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{n_i}}$	$\bar{p}_{new} + 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{n_i}}$
		P	$P - 3\sqrt{\frac{P(1-P)}{\bar{n}}}$	$P + 3\sqrt{\frac{P(1-P)}{\bar{n}}}$
		$\bar{p}$	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$
		$\bar{p}_{new}$	$\bar{p}_{new} - 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{\bar{n}}}$	$\bar{p}_{new} + 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{\bar{n}}}$

		where $p_i = \frac{d_i}{n_i}$; $\bar{p} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} = \frac{\sum_{i=1}^{k} d_i}{k\bar{n}}$; $\bar{n} = \frac{1}{k}\sum_{i=1}^{k} n_i$; $\bar{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{\sum_{i=1}^{k} n_i - \sum_{j=1}^{d} n_j}$		
	np	nP	$nP - 3\sqrt{nP(1-P)}$	$nP + 3\sqrt{nP(1-P)}$
		$n\bar{p}$	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
		$n\bar{p}_{new}$	$n\bar{p}_{new} - 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})}$	$n\bar{p}_{new} + 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})}$
	c	λ	$\lambda - 3\sqrt{\lambda}$	$\lambda + 3\sqrt{\lambda}$
		$\bar{c}$	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{c} + 3\sqrt{\bar{c}}$
		$\bar{c}_{new}$	$\bar{c}_{new} - 3\sqrt{\bar{c}_{new}}$	$\bar{c}_{new} + 3\sqrt{\bar{c}_{new}}$
		where $\bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i$; $\bar{c}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{k-d}$		
	u	$\bar{u}$	$\bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$	$\bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$
		$\bar{u}_{new}$	$\bar{u}_{new} - 3\sqrt{\frac{\bar{u}_{new}}{n_i}}$	$\bar{u}_{new} + 3\sqrt{\frac{\bar{u}_{new}}{n_i}}$
		$\bar{u}$	$\bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$	$\bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$
		$\bar{u}_{new}$	$\bar{u}_{new} - 3\sqrt{\frac{\bar{u}_{new}}{\bar{n}}}$	$\bar{u}_{new} + 3\sqrt{\frac{\bar{u}_{new}}{\bar{n}}}$
		where $u_i = \frac{c_i}{n_i}$; $\bar{u} = \frac{\sum_{i=1}^{k} c_i}{\sum_{i=1}^{k} n_i} = \frac{\sum_{i=1}^{k} d_i}{k\bar{n}}$; $\bar{u} = \frac{1}{k}\sum_{i=1}^{n} u_i$; $\bar{n} = \frac{1}{k}\sum_{i=1}^{k} n_i$; $\bar{u}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{\sum_{i=1}^{k} n_i - \sum_{j=1}^{d} n_j}$		
2	**Product Control**			
	Lot quality or **proportion defective** $p = \frac{\text{Number of defective units in a lot}}{\text{lot size}}$ **Probability of accepting a lot** $P_a(p) = P_a = P[X \le c] = P[$ X =1 or 2 or 3, ..., or c$]$		**Producer's risk** $P_p(p) = P_p = P[\text{rejecting a lot of acceptance quality level}] = \alpha$ **Consumer's risk** $P_c(p) = P[\text{accepting a lot of quality = LTPD}] = \beta$ **Acceptance outgoing quality (AOQ)** $AOQ = \frac{\text{Number of defective units in the lot after inspection}}{\text{Total number of units in the lot}}$	
3	**Single Sampling Plans**			
	Probability of accepting a lot $P_a(p) = P[X \le c] = \sum_{x=0}^{c} \frac{^{Np}C_x \; ^{N-Np}C_{n-x}}{^{N}C_x}$ Binomial approximation $P_a(p) = \sum_{x=0}^{c} {}^{n}C_x \, p^x (1-p)^{n-x}$ when $N \ge 10n$ Poisson approximation		**Consumer's risk** $P_c = P_a(p_2) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{^{Np_2}C_x \; ^{N-Np_2}C_{n-x}}{^{N}C_x}$ Binomial approximation $P_c = \sum_{x=0}^{c} {}^{n}C_x \, p_2^x (1-p_2)^{n-x}$ Acceptance outgoing quality (AOQ) $= \frac{p(N-n)P_a}{N}$;	

$$P_a(p) = \sum_{x=0}^{c} \frac{e^{-\lambda}\lambda^x}{\lfloor x} \text{ when np is finite}$$

Producer's risk

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n-x}}{{}^{N}C_x}$$

Binomial approximation

$$P_p = 1 - \sum_{x=0}^{c} {}^{n}C_x \; p_1^x (1-p_1)^{n-x}$$

$$AOQ = p\left(1 - \frac{n}{N}\right) P_a \cong pP_a \quad \text{when } \frac{n}{N} \leq 30$$

Average sample number (ASN) = n

Average total inspection (ATI) $= n + (1 - P_a)(N - n)$

Design of single sampling plans

$n = \frac{np_1}{p_1}$ (where p_1 = AQL); $n = \frac{np_2}{p_2}$ (where p_2 = LTPD)

$R = \frac{p_2}{p_1}$; $n = \frac{np_1}{p_1}$ and $p_2 = \frac{np_2}{n}$; $n = \frac{np_2}{p_2}$ and $p_1 = \frac{np_1}{n}$

4 Double Sampling Plans

Probability of accepting a lot

$$P_a(p) = P_{a1} + P_{a2} = \sum_{x=0}^{c_1} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n_1-x}}{{}^{N}C_{n_1}} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n_1-x}}{{}^{N}C_{n_1}} \cdot \frac{{}^{Np-x}C_y \; {}^{N-n_1-(Np-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}}$$

Binomial approximation

$$P_a(p) = \sum_{x=0}^{c_1} {}^{n_1}C_x \; p^x (1-p)^{n_1-x} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x \; p^x (1-p)^{n_1-x} \; {}^{n_2}C_y \; p^y (1-p)^{n_2-y}$$

Producer's risk

$$P_p = 1 - \sum_{x=0}^{c_1} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n_1-x}}{{}^{N}C_{n_1}} - \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n_1-x}}{{}^{N}C_{n_1}} \cdot \frac{{}^{Np_1-x}C_y \; {}^{N-n_1-(Np_1-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}}$$

Binomial approximation

$$P_p = 1 - \sum_{x=0}^{c_1} {}^{n_1}C_x p_1^x (1-p_1)^{n_1-x} - \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x p_1^x (1-p_1)^{n_1-x} \; {}^{n_2}C_y p_1^y (1-p_1)^{n_2-y}$$

Consumer's risk

$$P_c = \sum_{x=0}^{c_1} \frac{{}^{Np_2}C_x \; {}^{N-Np_2}C_{n_1-x}}{{}^{N}C_{n_1}} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np_2}C_x \; {}^{N-Np_2}C_{n_1-x}}{{}^{N}C_{n_1}} \cdot \frac{{}^{Np_2-x}C_y \; {}^{N-n_2-(Np_2-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}}$$

Binomial approximation

$$P_c = \sum_{x=0}^{c_1} {}^{n_1}C_x \; p_2^x (1-p_2)^{n_1-x} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x \; p_2^x (1-p_2)^{n_1-x} \; {}^{n_2}C_y \; p_2^y (1-p_2)^{n_2-y}$$

$$AOQ = p\left[\left(1 - \frac{n_1}{N}\right)(P_{a1} + P_{a2}) - \frac{n_2}{N} P_{a2}\right]$$

$AOQ = p(P_{a1} + P_{a2})$ when $\frac{n_1}{N} \leq 0$ and $\frac{n_2}{N} \leq 0$

$ASN = n_1 P_I + (n_1 + n_2)(1 - P_I)$

P_I = P[lot is accepted on the first sample]
+ P[lot is rejected on the first sample]

$ATI = n_1 P_{a1} + (n_1 + n_2) P_{a2} + N[1 - P_a]$

Design of double sampling plans

$n_1 = \frac{n_1 p}{p_1}$ (where p_1 = AQL, $n_1 = n_2$ or $n_2 = 2n_1$);

$n_1 = \frac{n_1 p}{p_2}$ (where p_2 = LTPD, $n_1 = n_2$ or $n_2 = 2n_1$)

$R = \frac{p_2}{p_1}$

5 Game Theory

Algebraic Method

Every 2×2 two-person zero-sum game without a saddle point:

Payoff matrix for player A

		Player B	
		B_1	B_2
Player A	A_1	a	b
	A_2	c	d

Mixed strategies (p_1, p_2) for player A

$$p_1 = \frac{d-c}{(a-b)+(d-c)}, \qquad p_2 = \frac{a-b}{(a-b)+(d-c)}$$

Mixed strategies (q_1, q_2) for player B

$$q_1 = \frac{d-b}{(a-b)+(d-c)}, \qquad q_2 = \frac{a-c}{(a-b)+(d-c)}$$

The value (v) of the game

		$v = \dfrac{ad - bc}{(a-b)+(d-c)}$
6	**Reliability Theory**	**Relations Between R(t), F(t), f(t) and $\lambda(t)$**
	$R(t) = \dfrac{N_s(t)}{N_0}$, $F(t) = \dfrac{N_f(t)}{N_0}$, $F(t) = 1 - R(t)$ where $N_s(t)$ – number of components that are operating at time t, and $N_f(t)$ – number of components that have failed at time t.	$R(t) = 1 - F(t) = \int_t^\infty f(t)\,dt = \exp\left(-\int_0^t \lambda(t)\,dt\right)$ $F(t) = 1 - R(t) = \int_0^t f(t)\,dt = 1 - \exp\left(-\int_0^t \lambda(t)\,dt\right)$ $f(t) = -\dfrac{d}{dt}(R(t)) = \dfrac{d}{dt}(F(t)) = \lambda(t) \times \exp\left(-\int_0^t \lambda(t)\,dt\right)$ $\lambda(t) = -\dfrac{d}{dt}(\ln R(t)) = \dfrac{\frac{d}{dt}(F(t))}{1 - F(t)} = \dfrac{f(t)}{\int_0^t f(t)dt}$
7	**Mean Time To Failure (MTTF)**	**Reliability of Series and Parallel Systems**
	$MTTF = E(T) = \int_0^\infty t\,f(t)\,dt$ $MTTF = \int_0^\infty R(t)dt$ If hazard rate follows exponential distribution then $MTTF = \dfrac{1}{\lambda}$ or $\lambda = \dfrac{1}{MTTF}$	Reliability (R) of a series system = $R = \prod_{i=1}^{n} R_i$ Reliability (R) of a parallel system $= R = 1 - (1-R_1)(1-R_2)(1-R_3)...(1-R_n) = 1 - \prod_{i=1}^{n}(1-R_i)$ where R_i – the reliability of the i^{th} component.
8	**Standby system with perfect switching** Let Q_i denote the unreliability of the component i, given that components $1 \text{ to } (i-1)$ have failed. Further, if R and Q denote the reliability and unreliability of the standby system, then $Q = Q_1Q_2Q_3...Q_n$ and $R = 1 - Q$ **Standby system with imperfect switching** $Q = P_s Q_A Q_B + \bar{P}_s Q_A$, where, P_s – probability of successful changeover, $\bar{P}_s = 1 - P_s$ – probability of unsuccessful changeover, Q_A – unreliability of component A, and Q_B – unreliability of component B given that component A has failed.	**Decomposition method** or **conditional probability approach** Reliability of the system (R_s) $= P(\text{system success} \mid \text{component K is good})\,P(\text{component K is good})$ $+ P(\text{system success} \mid \text{component K is bad})\,P(\text{component K is bad})$ **Cut set method** Unreliability of the system $(Q_s) = P(C_1 \cup C_2 \cup C_3 \cup ... \cup C_k)$ where $C_1, C_2, ..., C_k$ are minimal cut sets **Tie set method** Reliability of the system $(R_s) = P(T_1 \cup T_2 \cup T_3 \cup ... \cup T_k)$ where $T_1, T_2, ..., T_k$ are minimal tie sets

S. No.	FORMULAE

1 Operating Characteristics for M/M/1 Queueing Model

$P_n(t) = P[N(t) = n]$

$P_n = (1-\rho)\rho^n, \quad n \geq 0$

$P[n > k] = \rho^{k+1}$

$L_s = \sum_{n=1}^{\infty} nP_n$

$L_q = \sum_{n=c+1}^{\infty} (n-c)P_n$

$L_s = \lambda W_s \; ; \quad L_q = \lambda W_q$

$L_s = \frac{\lambda}{\mu - \lambda}$

$L_s = L_q + \frac{\lambda}{\mu}$

$W_s = W_q + \frac{1}{\mu}$

$W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}$

$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)}$

$W_q = W_s - \frac{1}{\mu}$

$\rho = \frac{\lambda}{\mu}$

2 EOQ Models for Inventory Control

Economic order quantity (EOQ) model with uniform demand

Ordering cost = $\frac{D}{Q} \times C_O$

Economic order quantity $Q^* = \sqrt{\frac{2DC_O}{C_h}}$

Minimum total yearly inventory cost $TC^* = \sqrt{2DC_OC_h}$

EOQ model with different rates of demand in different cycles

The carrying cost for time T $= \frac{QC_hT}{2}$

Economic order quantity $Q^* = \sqrt{\frac{2D\,C_O}{C_hT}}$

Minimum total yearly inventory cost $TC^* = \sqrt{2DC_OC_hT}$

3 EOQ Model when shortages are allowed

Carrying cost per cycle $= C_h\left(\frac{M}{2} \times t_1\right)$

Carrying cost $= \left(\frac{M^2 \times t}{2Q}\right)C_h$

Carrying cost per unit time $= \frac{M^2}{2Q}C_h$

Shortage cost $= \left(\frac{S^2 \times t}{2Q}\right)C_S$

Shortage cost per unit time $= \frac{S^2}{2Q}C_S$

Maximum inventory level

$M^* = \left(\frac{C_S}{C_h + C_S}\right)Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{C_S}{C_h + C_S}\right)}$

Economic order quantity $Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{C_S + C_h}{C_s}\right)}$

Total cycle time $t^* = \frac{Q^*}{D} = \sqrt{\frac{2C_O}{DC_h}\left(\frac{C_S + C_h}{C_S}\right)}$

Minimum total yearly inventory cost

$TC^* = \sqrt{2DC_OC_h\left(\frac{C_S}{C_h + C_S}\right)}$

EOQ Model with uniform replenishment

Average inventory $= \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)$

Carrying cost = $\frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$

Length of each lot size production run $t_1 = \frac{Q^*}{r_p} = \sqrt{\frac{2DC_O}{C_hr_p(r_p - r_d)}}$

Optimum number of production runs per year

$N^* = \frac{D}{Q^*} = \sqrt{\frac{DC_h(r_p - r_d)}{2C_o\, r_p}}$

Economic order quantity $Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{r_p}{r_p - r_d}\right)}$

Minimum total yearly inventory cost

$TC^* = \sqrt{2DC_OC_h\left(1 - \frac{r_d}{r_p}\right)}$

EOQ model with price (or quantity) discounts

Economic order quantity $Q_n^* = \sqrt{\frac{2C_OD}{ip_n}}$

Economic order quantity $Q_{n-1}^* = \sqrt{\frac{2C_OD}{ip_{n-1}}}$

Minimum total yearly inventory cost

		$TC^*(Q_j) = Dp_j + \frac{C_O D}{Q_j} + \frac{1}{2}C_h Q_j$
4	**Simple Linear Regression Modelling**	
	Estimates of parameters $\hat{a} = \bar{Y} - \hat{b}\bar{X} = \frac{1}{n}\left[\sum Y - \hat{b}\sum X\right]$ $\hat{b} = \frac{SS_{XY}}{SS_X}$ **Sum of squares** $SS_X = \sum_{i=1}^{n} X_i^2 - \frac{\left(\sum X_i\right)^2}{n}$ $SS_{XY} = \sum Y_i X_i - \frac{\left(\sum Y_i\right)\left(\sum X_i\right)}{n}$ r^{th} residual = $r_i = Y_i - \hat{Y}_i$; $i = 1, 2, ..., n$ i^{th} standard residual = $d_i = \frac{r_i}{\sqrt{SS_{Res}}}$; $i = 1, 2, ..., n$ $\hat{\sigma}^2 = SS_{Res} = SS_Y - \frac{(SS_{XY})^2}{SS_X}$ or $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(r_i - \bar{r})^2}{n-k} = \frac{\sum_{i=1}^{n} r_i^2}{n-k}$ **Variance of estimates** $Var(\hat{a}) = \sigma^2\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)$ $Var(\hat{b}) = \frac{\sigma^2}{SS_x}$ $Var(\bar{Y}) = \frac{\sigma^2}{n}$	**Test statistic** $t = \frac{(\hat{a} - a_0)}{\sqrt{SS_{Res}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}} \sim t_{(n-2)}$ $t = \frac{(\hat{b} - b)}{\sqrt{\frac{SS_{Res}}{SS_X}}} \sim t_{(n-2)}$ **Lower and upper confidence limits for b** $b_L = \hat{b} - t_{\alpha/2}\sqrt{\frac{SS_{Res}}{SS_X}}$ $b_U = \hat{b} + t_{\alpha/2}\sqrt{\frac{SS_{Res}}{SS_X}}$ **Lower and upper limits** $\hat{Y}_L = \hat{Y} - t_{\alpha/2}\sqrt{V(\hat{Y})} = \hat{Y} - t_{\alpha/2}\sqrt{SS_{Res}\left(\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{SS_X}\right)}$ $\hat{Y}_U = \hat{Y} + t_{\alpha/2}\sqrt{V(\hat{Y})} = \hat{Y} + t_{\alpha/2}\sqrt{SS_{Res}\left(\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{SS_X}\right)}$ **Lower and upper limits for new value of $Y=Y_0$** $\hat{Y}_L = \hat{Y}_0 - t_{\alpha/2}\sqrt{V(\hat{Y}_0)} = \hat{Y}_0 - t_{\alpha/2}\sqrt{SS_{Res}\left(1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{SS_X}\right)}$ $\hat{Y}_U = \hat{Y}_0 + t_{\alpha/2}\sqrt{V(\hat{Y}_0)} = \hat{Y}_0 + t_{\alpha/2}\sqrt{SS_{Res}\left(1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{SS_X}\right)}$ **Coefficient of determination** $R^2 = \frac{\left(SS_{XY}\right)^2}{SS_X \times SS_Y}$ $\tilde{R}^2 = 1 - (1 - R^2)\left(\frac{n-1}{n-k-1}\right)$
5	**Multiple Linear Regression Modelling**	
	j^{th} normal equation $\sum Y_j X_{ij} = B_0 \sum X_{1j} X_{ij} + B_2 \sum X_{2j} X_{ij} + ... + B_p \sum X_{pj} X_{ij}$ **For matrix method** $\hat{B} = (X'X)^{-1} X'Y$ $\hat{Y} = X\hat{B} = X(X'X)^{-1} X'Y$ **Test statistics** $t_j = \frac{\hat{B}_j}{S.E.(\hat{B}_j)}$; $j = 0, 1, ...p$	**Sum of squares** $SS_{Res} = \sum(r_i - \bar{r})^2 = Y'Y - Y'X\hat{B}$ $SS_{Res} = \left(SS_Y - (SS_{XY})^2 / SS_X\right)$ **For matrix method** $SS_T = \sum Y_i^2 - n\bar{Y}^2$ $= Y'Y - \frac{\left(\sum Y_i\right)^2}{n}$ $SS_{Reg} = \sum \hat{B}_j Y' X_j - n\bar{Y}^2$ $SS_{Res} = SS_T - SS_{Reg}$	**Variance of estimates** $V(\hat{Y}) = \hat{\sigma}^2\left(\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{SS_X}\right)$ $V(\hat{Y}_0) = \sigma^2\left(1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{SS_X}\right)$ $\hat{\sigma}^2 = \frac{SS_{Res}}{(n-p-1)}$ $V(\hat{B}_j) = \sigma_{jj}$ $SE(\hat{B}_j) = \sqrt{\sigma_{jj}}$ $V(\hat{B}) = \hat{\sigma}^2 (X'X)^{-1} = (\sigma_{jk})$; $j, k = 0, 1, 2...p$ $Cov(\hat{B}_j, \hat{B}_k) = \sigma_{jk}$; $j \neq k$
	Probabilities for normal probability plot **Probability for i^{th} ordered residual**	**Adjusted R^2**

$$p_i = \frac{\left(i-\frac{1}{2}\right)}{n} \qquad i = 1, 2, ..., n$$

Percentage probability for i[th] ordered residual

$$P_i = \frac{\left(i-\frac{1}{2}\right)}{n} \times 100; \qquad i = 1, 2, ..., n$$

Coefficient of determination

$$R^2 = \frac{\sum_{j=0}^{p} \hat{B}_j Y'X_j - n\bar{Y}^2}{Y'Y - n\bar{Y}^2} = \frac{SS_{Reg}}{SS_T}$$

$$R^2_{Adj} = 1 - \frac{(n-1)(1-R^2)}{(n-p-1)}$$

Extra sum of squares

$$SS(B_1/B_0) = SS_{Res}(B_0) - SS_{Res}(B_0, B_1) = SS(B_0, B_1) - SS(B_0)$$

$$SS(B_2/B_0) = SS_{Res}(B_0) - SS_{Res}(B_0, B_2) = SS(B_0, B_2) - SS(B_0)$$

$$SS(B_1/B_0, B_2) = SS_{Res}(B_0, B_2) - SS_{Res}(B_0, B_1, B_2)$$
$$= SS(B_0, B_1, B_2) - SS(B_0, B_2)$$

$$SS(B_2/B_0, B_1) = SS_{Res}(B_0, B_1) - SS_{Res}(B_0, B_1, B_2)$$
$$= SS(B_0, B_1) - SS(B_0, B_1, B_2)$$

$$SS(B_1, B_2/B_0) = SS_{Res}(B_0) - SS_{Res}(B_0, B_1, B_2)$$
$$= SS(B_0, B_1, B_2) - SS(B_0)$$

6 Time Series Modelling

Time series values

Weighted moving average $y_t = \sum_{i=-q}^{q} w_i x_{t+i}$

Estimated time series values $y'_t = w\, y_t + (1-w)\, y'_{t-1}$

Estimated time series values in linear form

$$y_t = \sum_{j=0}^{p} b_j\, t^j$$

Estimated time series values in non linear form

$$y_t = \alpha_0 \alpha_1^t$$

Seasonal indices and seasonal relatives

$$\text{Seasonal index} = \frac{\bar{y}_i}{\bar{\bar{y}}} \times 100, \quad \text{for } i = 1, 2, ..., 12.$$

$$\text{Seasonal index} = \text{Median} \times \frac{k \times 100}{\text{Total of Indices}}$$
$$\text{for } i = 1, 2, 3, ..., k$$

Deseasonalised time series value $Z_t = \frac{y_t}{S_t} = T_t \times C_t \times I_t$

Seasonal relative = S × I × 100 = $(y_t / MA(2)) \times 100$

Auto-covariance and autocorrelation coefficient

Auto-covariance coefficient at lag k

$$c_k = \sum_{t=1}^{N-k} (y_t - \bar{y})(y_{t+k} - \bar{y}) / (N-k)$$

Auto-covariance coefficient at lag 0

$$c_0 = \sum_{t=1}^{N} (y_t - \bar{y})(y_t - \bar{y}) / N = \sigma_Y^2, \text{ for all k}$$

Auto-correlation coefficient $r_k = \frac{c_k}{c_0}$

Moving average processes

Variance of X_t in MA(q) : $V(X_t) = \sigma_a^2 \left(1 + \sum_{i=1}^{q} \beta_i^2\right)$

Autocorrelation function for MA(q)

$$\rho_k = \frac{\left(\beta_k + \beta_1 \beta_{k+1} + ... + \beta_{q-k} \beta_q\right)}{\left(1 + \sum_{i=1}^{q} \beta_i^2\right)}$$

Variance of X_t in MA(1) : $V(X_t) = \sigma_a^2 (1 + \beta_1^2)$

Variance of X_t in MA(2) : $V(X_t) = \sigma_a^2 (1 + \beta_1^2 + \beta_2^2)$

Autoregressive processes

Variance of X_t in AR(1)

$$\sigma_x^2 = \sigma_a^2 / (1 - \alpha_1^2)$$

Variance of X_t in AR(2)

$$\sigma_x^2 = \sigma_a^2 / (1 - \rho_1 \alpha_1 - \rho_2 \alpha_2)$$

Autocorrelation function of AR (2) process

$$\rho_k = \alpha_1 \rho_{k-1} + \alpha_2 \rho_{k-2}$$

Partial auto-covariance function of AR(1)

pacf (1) $= \rho_1 = \alpha_1$

Partial auto-covariance function of AR(1)

Autoregressive moving average processes

Auto-covariance function for ARMA at lag 0

$$\gamma_0 = \frac{\sigma_a^2 (1 + \beta^2 + 2\alpha\beta)}{(1 - \alpha^2)}$$

Auto-covariance function for ARMA at lag 1

$$\gamma_1 = \alpha\gamma_0 + \beta\sigma_a^2$$

Auto-covariance function for ARMA at lag k

$$\gamma_k = \alpha\gamma_{k-1}, \; k \geq 2$$

Auto-correlation function for ARMA at lag 0

$$\rho_1 = \frac{(1 + \alpha\beta)(\alpha + \beta)}{(1 + \beta^2 + 2\alpha\beta)}$$

Auto-correlation function for ARMA at lag k

$$\rho_k = \alpha\rho_{k-1}, \; k \geq 2$$

	pacf (2) $= \frac{\left(\rho_2 - \rho_1^2\right)}{\left(1 - \rho_1^2\right)} = \alpha_2$	Limits $= \pm \frac{2}{\sqrt{N}}$

Statistical Tables

TABLE 1 LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0	43	86	128	170	212	253	294	334	374	4	8	12	17	21	25	29	33	37
11	414	453	492	531	569	607	645	682	719	755	4	8	11	15	19	23	26	30	34
12	792	828	864	899	934	969	1004	1038	1072	1106	3	7	10	14	17	21	24	26	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2696	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3180	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5566	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6345	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7078	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7869	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8178	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8383	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8542	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	6937	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	3	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9674	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9766	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9681	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9843	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

TABLE 2 ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
0	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
1	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
2	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
3	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
4	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
5	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
6	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
7	1175	1178	1180	1183	1188	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
8	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
9	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1304	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
27	1862	1868	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
32	2089	2094	2090	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2480	2466	2472	2477	2483	2489	2495	2500	2508	1	1	2	2	3	3	4	5	5
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	3	3	4	4	5	5
42	2630	2836	2642	2849	2655	2661	2667	2673	2679	2685	1	1	2	3	3	4	4	5	6
43	2892	2898	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3096	3076	3083	1	1	2	3	4	4	5	6	6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3238	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4176	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4686	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5459	5470	5438	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5823	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6299	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6715	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8358	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

TABLE 3 VALUES OF $e^{-\lambda}$ (FOR COMPUTING POISSON PROBABILITIES) $(0 < \lambda < 1)$

λ	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
0.1	0.9048	0.8958	0.8860	0.8781	0.8694	0.8607	0.8521	0.8437	0.8353	0.8270
0.2	0.7187	0.8106	0.8025	0.7945	0.7866	0.7788	0.7711	0.7634	0.7558	0.7483
0.3	0.7408	0.7334	0.7261	0.7189	0.7118	0.7047	0.6970	0.6907	0.6839	0.6771
0.4	06703	0.6636	0.6570	0.6505	0.6440	0.6376	0.6313	0.6250	0.6188	0.6125
0.5	0.6065	0.6005	0.5945	0.5886	0.5827	0.5770	0.5712	0.5655	0.5599	0.5543
0.6	0.5448	0.5434	0.5379	0.5326	0.5278	0.5220	0.5160	0.5113	0.5066	0.5016
0.7	0.4966	0.4916	0.4868	0.4810	0.4771	0.4724	0.4670	0.4630	0.4584	0.4538
0.8	0.4493	0.4449	0.4404	0.4360	0.4317	0.4274	0.4232	0.4190	0.4148	0.4107
0.9	0.4066	0.4026	0.3985	0.3946	0.3906	0.3867	0.3829	0.3791	0.3753	0.3716
(λ=1, 2, 3, ...,10)										
λ	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$e^{-\lambda}$	0.3679	0.1353	0.0498	0.0183	0.0070	0.0028	0.0009	0.0004	0.0001	0.00004

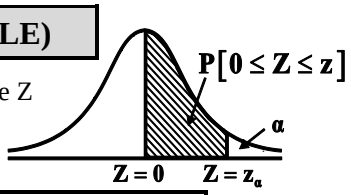
Note: To obtain values of $e^{-\lambda}$ for other values of λ, use the laws of exponents, i.e.,

$e^{-(a+b)} = e^{-a}.e^{-b}$ e.g. $e^{-2.25} = e^{-2}.e^{-0.25} = (0.1353)(0.7788) = 0.1054$

TABLE 4	COMMONLY USED VALUES OF STANDARD NORMAL VARIATE Z

Confidence Interval	99%	98%	95%	90%
Level of Significance (α)	**0.01 (1%)**	**0.02 (2%)**	**0.05 (5%)**	**0.10 (10%)**
Two-Tailed	$\pm z_{\alpha/2} = \pm 2.576$	$\pm z_{\alpha/2} = \pm 2.326$	$\pm z_{\alpha/2} = \pm 1.960$	$\pm z_{\alpha/2} = \pm 1.645$
One (right)-Tailed	$z_{\alpha} = 2.326$	$z_{\alpha} = 2.054$	$z_{\alpha} = 1.645$	$z_{\alpha} = 1.282$
One (left)-Tailed	$-z_{\alpha} = -2.326\ 2$	$-z_{\alpha} = -2.054$	$-z_{\alpha} = -1.645$	$-z_{\alpha} = -1.282$

TABLE 5 STANDARD NORMAL DISTRIBUTION (Z TABLE)

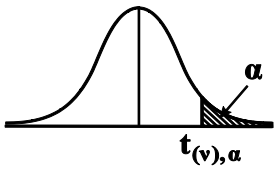


The first column and first row of the table indicate the values of standard normal variate Z at first and second place of decimal. The entry represents the upper tail area under the curve or probability, i.e., $P[0 \le Z \le z]$ for different values of Z.

↓ Z →	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3304	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Note: The area for negative values of Z is taken as the same as that for positive values, since the curve is symmetrical.

TABLE 6 STUDENT'S t DISTRIBUTION (t TABLE)

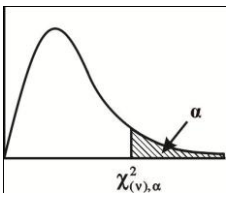


The first column of this table indicates the degrees of freedom and first row indicates a specified upper tail area (α). The entry represents the value of the t-statistic such that area under the curve of the t-distribution to its upper tail is equal to α.

One-Tailed Test					
α =	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
Two-Tailed Test					
α =	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**
ν =1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

Note: The area for negative values of Z is taken as the same as that for positive values, since the curve is symmetrical.

TABLE 7 CHI SQUARE DISTRIBUTION (χ^2 TABLE)

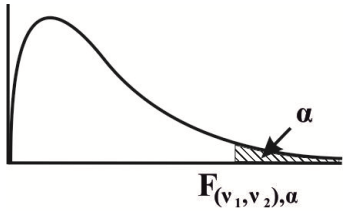


The first column of this table indicates the degrees of freedom and first indicates row a specified upper tail area (α). The entry represents the value of chi-square statistic such that the area under the curve of the chi square distribution to its upper tail is equal to α.

α =	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
ν = 1	---	---	---	---	0.02	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.05	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.56	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17
120	83.85	86.92	91.58	95.70	100.62	140.23	146.57	152.21	158.95	163.64

TABLE 8 F DISTRIBUTION (F TABLE)

F-table contains the values of the F-statistic for different set of degrees of freedom (ν_1, ν_2) of numerator and denominator such that the area under the curve of the F-distribution to its right (upper tail) is equal to α.



F values for α = 0.1

Degrees of Freedom for Denominator (ν_2)	Degrees of Freedom for Numerator(ν_1)																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	24	30	40	60	120	∞
1	39.86	49.50	53.60	55.83	57.23	58.21	58.91	59.44	59.86	60.20	60.47	60.70	60.91	61.07	61.22	61.35	61.47	61.57	61.66	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41	9.41	9.42	9.42	9.43	9.43	9.44	9.44	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22	5.21	5.20	5.20	5.20	5.19	5.19	5.19	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.96	3.94	3.92	3.91	3.90	3.89	3.88	3.87	3.86	3.86	3.85	3.85	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27	3.26	3.25	3.24	3.23	3.22	3.22	3.21	3.21	3.19	3.17	3.16	3.14	3.12	3.11
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90	2.89	2.88	2.87	2.86	2.86	2.85	2.84	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.79	2.75	2.72	2.70	2.68	2.67	2.65	2.64	2.63	2.62	2.61	2.61	2.60	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50	2.49	2.48	2.46	2.45	2.45	2.44	2.43	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38	2.36	2.35	2.34	2.33	2.32	2.31	2.31	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28	2.27	2.26	2.24	2.23	2.22	2.22	2.21	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21	2.19	2.18	2.17	2.16	2.15	2.14	2.13	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15	2.13	2.12	2.10	2.09	2.08	2.08	2.07	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10	2.08	2.07	2.05	2.04	2.03	2.02	2.01	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.07	2.05	2.04	2.02	2.01	2.00	1.99	1.98	1.97	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.95	1.94	1.93	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99	1.97	1.95	1.94	1.93	1.92	1.91	1.90	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96	1.94	1.93	1.91	1.90	1.89	1.88	1.87	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.95	1.93	1.92	1.90	1.89	1.87	1.86	1.85	1.85	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.93	1.91	1.89	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89	1.87	1.86	1.84	1.83	1.82	1.81	1.80	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.81	1.80	1.79	1.78	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86	1.84	1.83	1.81	1.80	1.79	1.78	1.77	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.87	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.84	1.82	1.80	1.79	1.77	1.76	1.75	1.74	1.73	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.83	1.81	1.79	1.77	1.76	1.75	1.73	1.72	1.71	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.82	1.80	1.78	1.76	1.75	1.74	1.72	1.71	1.70	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79	1.77	1.75	1.74	1.73	1.71	1.70	1.69	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.80	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.68	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77	1.75	1.74	1.72	1.71	1.70	1.69	1.68	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.71	1.70	1.68	1.66	1.65	1.64	1.62	1.61	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66	1.64	1.62	1.60	1.59	1.58	1.56	1.55	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60	1.58	1.56	1.55	1.53	1.52	1.50	1.49	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55	1.52	1.51	1.49	1.47	1.46	1.44	1.43	1.42	1.38	1.34	1.30	1.24	1.17	1.00

F values for $\alpha = 0.05$

Degrees of Freedom for Denominator (v_2)	Degrees of Freedom for Numerator(v_1)																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	24	30	40	60	120	∞
1	161	199	216	225	230	234	237	239	240	242	243	244	245	245	246	246	247	247	248	248	249	250	251	252	253	254
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40	19.40	19.41	19.42	19.43	19.43	19.43	19.44	19.44	19.44	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.72	8.70	8.69	8.68	8.67	8.67	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.84	5.83	5.82	5.81	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.60	4.59	4.58	4.57	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.99	2.97	2.96	2.95	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.79	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.51	2.50	2.48	2.47	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.44	2.43	2.41	2.40	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.29	2.27	2.26	2.24	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.20	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.18	2.17	2.15	2.14	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.11	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.13	2.11	2.10	2.08	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13	2.11	2.09	2.08	2.06	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.09	2.07	2.05	2.04	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09	2.07	2.05	2.04	2.02	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06	2.04	2.02	2.00	1.99	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03	2.01	1.99	1.97	1.96	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.76	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.80	1.78	1.75	1.73	1.71	1.69	1.67	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75	1.72	1.69	1.67	1.64	1.62	1.60	1.59	1.57	1.52	1.46	1.39	1.32	1.22	1.00

F values for $\alpha = 0.025$

Degrees of Freedom for Denominator (v_2)	Degrees of Freedom for Numerator(v_1)																									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	24	30	40	60	120	∞
1	648	800	864	900	922	937	948	957	963	969	973	977	980	983	985	987	989	990	992	993	997	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.41	39.42	39.43	39.43	39.44	39.44	39.44	39.45	39.45	39.46	39.47	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.37	14.34	14.30	14.28	14.25	14.23	14.21	14.20	14.18	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75	8.72	8.68	8.66	8.63	8.61	8.59	8.58	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52	6.49	6.46	6.43	6.40	6.38	6.36	6.34	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.41	5.37	5.33	5.30	5.27	5.24	5.22	5.20	5.18	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.71	4.67	4.63	4.60	4.57	4.54	4.52	4.50	4.48	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.24	4.20	4.16	4.13	4.10	4.08	4.05	4.03	4.02	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.91	3.87	3.83	3.80	3.77	3.74	3.72	3.70	3.68	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.66	3.62	3.58	3.55	3.52	3.50	3.47	3.45	3.44	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.47	3.43	3.39	3.36	3.33	3.30	3.28	3.26	3.24	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.32	3.28	3.24	3.21	3.18	3.15	3.13	3.11	3.09	3.07	3.02	2.96	2.91	2.85	2.79	2.73
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.09	3.05	3.01	2.98	2.95	2.92	2.90	2.88	2.86	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	3.01	2.96	2.92	2.89	2.86	2.84	2.81	2.79	2.77	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.93	2.89	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.87	2.82	2.79	2.75	2.72	2.70	2.67	2.65	2.63	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.81	2.77	2.73	2.70	2.67	2.64	2.62	2.60	2.58	2.56	2.50	2.45	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.76	2.72	2.68	2.65	2.62	2.59	2.57	2.55	2.53	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.72	2.68	2.64	2.60	2.57	2.55	2.52	2.50	2.48	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.68	2.64	2.60	2.56	2.53	2.51	2.48	2.46	2.44	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.65	2.60	2.56	2.53	2.50	2.47	2.45	2.43	2.41	2.39	2.33	2.27	2.21	2.15	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.62	2.57	2.53	2.50	2.47	2.44	2.42	2.39	2.37	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.59	2.54	2.50	2.47	2.44	2.41	2.39	2.36	2.35	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.56	2.51	2.48	2.44	2.41	2.38	2.36	2.34	2.32	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.54	2.49	2.45	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.51	2.47	2.43	2.39	2.36	2.34	2.31	2.29	2.27	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.49	2.45	2.41	2.37	2.34	2.32	2.29	2.27	2.25	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.46	2.48	2.43	2.39	2.36	2.30	2.27	2.25	2.23	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.33	2.41	2.37	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.26	2.29	2.25	2.21	2.18	2.15	2.13	2.11	2.09	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.22	2.17	2.13	2.09	2.06	2.03	2.01	1.98	1.96	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.10	2.05	2.01	1.98	1.94	1.92	1.89	1.87	1.84	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.99	1.94	1.90	1.87	1.83	1.80	1.78	1.75	1.73	1.71	1.64	1.57	1.48	1.39	1.27	1.00

F values for $\alpha = 0.01$

Degrees of Freedom for Denominator (v_2)	Degrees of Freedom for Numerator (v_1)																									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	24	30	40	60	120	∞
1	4063	4992	5404	5637	5760	5890	5890	6025	6025	6025	6025	6167	6167	6167	6167	6167	6167	6167	6167	6167	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.15	99.27	99.30	99.34	99.34	99.38	99.38	99.38	99.42	99.42	99.42	99.42	99.42	99.42	99.46	99.46	99.46	99.46	99.46	99.47	99.47	99.48	99.49	99.50
3	34.11	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	27.13	27.05	26.98	26.92	26.87	26.83	26.79	26.75	26.72	26.69	26.60	26.51	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.45	14.37	14.31	14.25	14.20	14.15	14.11	14.08	14.05	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89	9.83	9.77	9.72	9.68	9.64	9.61	9.58	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.66	7.60	7.56	7.52	7.48	7.45	7.42	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47	6.41	6.36	6.31	6.28	6.24	6.21	6.18	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67	5.61	5.56	5.52	5.48	5.44	5.41	5.38	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	5.05	5.01	4.96	4.92	4.89	4.86	4.83	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.65	4.60	4.56	4.52	4.49	4.46	4.43	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40	4.34	4.29	4.25	4.21	4.18	4.15	4.12	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16	4.10	4.05	4.01	3.97	3.94	3.91	3.88	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.91	3.86	3.82	3.78	3.75	3.72	3.69	3.66	3.59	3.51	3.43	3.34	3.26	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.75	3.70	3.66	3.62	3.59	3.56	3.53	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.61	3.56	3.52	3.49	3.45	3.42	3.40	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55	3.50	3.45	3.41	3.37	3.34	3.31	3.28	3.26	3.18	3.10	3.02	2.93	2.85	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46	3.40	3.35	3.31	3.27	3.24	3.21	3.19	3.16	3.08	3.00	2.92	2.84	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37	3.32	3.27	3.23	3.19	3.16	3.13	3.10	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.24	3.19	3.15	3.12	3.08	3.05	3.03	3.00	2.93	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.18	3.13	3.09	3.05	3.02	2.99	2.96	2.94	2.86	2.78	2.70	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.24	3.17	3.12	3.07	3.03	2.99	2.96	2.93	2.90	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.07	3.02	2.98	2.94	2.91	2.88	2.85	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	3.02	2.97	2.93	2.89	2.86	2.83	2.80	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.98	2.93	2.89	2.85	2.82	2.79	2.76	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	3.06	2.99	2.94	2.89	2.85	2.81	2.78	2.75	2.72	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	2.90	2.86	2.82	2.78	2.75	2.72	2.69	2.66	2.59	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.99	2.93	2.87	2.82	2.78	2.75	2.71	2.68	2.66	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	2.84	2.79	2.75	2.72	2.68	2.65	2.63	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93	2.87	2.81	2.77	2.73	2.69	2.66	2.63	2.60	2.57	2.50	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84	2.79	2.74	2.70	2.66	2.63	2.60	2.57	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	2.52	2.48	2.45	2.42	2.39	2.37	2.29	2.20	2.11	2.02	1.92	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.44	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34	2.28	2.23	2.19	2.15	2.12	2.09	2.06	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.19	2.13	2.08	2.04	2.00	1.97	1.94	1.91	1.88	1.79	1.70	1.59	1.47	1.33	1.00

F values for $\alpha = 0.005$

Degrees of Freedom for Denominator (ν_2)	Degrees of Freedom for Numerator (ν_1)																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	24	30	40	60	120	∞
1	16211	19999	21615	22500	23056	23437	23715	23925	24091	24224	24334	24426	24505	24572	24630	24681	24727	24767	24803	24836	24940	25044	25148	25253	25359	25464
2	198.50	199.00	199.17	199.25	199.30	199.33	199.36	199.37	199.39	199.40	199.41	199.42	199.42	199.43	199.43	199.44	199.44	199.44	199.45	199.45	199.46	199.47	199.47	199.48	199.49	199.50
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.52	43.39	43.27	43.17	43.08	43.01	42.94	42.88	42.83	42.78	42.62	42.47	42.31	42.15	41.99	41.83
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.82	20.70	20.60	20.51	20.44	20.37	20.31	20.26	20.21	20.17	20.03	19.89	19.75	19.61	19.47	19.32
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.49	13.38	13.29	13.21	13.15	13.09	13.03	12.98	12.94	12.90	12.78	12.66	12.53	12.40	12.27	12.14
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25	10.13	10.03	9.95	9.88	9.81	9.76	9.71	9.66	9.62	9.59	9.47	9.36	9.24	9.12	9.00	8.88
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18	8.10	8.03	7.97	7.91	7.87	7.83	7.79	7.75	7.64	7.53	7.42	7.31	7.19	7.08
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01	6.94	6.87	6.81	6.76	6.72	6.68	6.64	6.61	6.50	6.40	6.29	6.18	6.06	5.95
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23	6.15	6.09	6.03	5.98	5.94	5.90	5.86	5.83	5.73	5.62	5.52	5.41	5.30	5.19
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66	5.59	5.53	5.47	5.42	5.38	5.34	5.31	5.27	5.17	5.07	4.97	4.86	4.75	4.64
11	12.23	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24	5.16	5.10	5.05	5.00	4.96	4.92	4.89	4.86	4.76	4.65	4.55	4.45	4.34	4.23
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91	4.84	4.77	4.72	4.67	4.63	4.59	4.56	4.53	4.43	4.33	4.23	4.12	4.01	3.90
13	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82	4.72	4.64	4.57	4.51	4.46	4.41	4.37	4.33	4.30	4.27	4.17	4.07	3.97	3.87	3.76	3.65
14	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60	4.51	4.43	4.36	4.30	4.25	4.20	4.16	4.12	4.09	4.06	3.96	3.86	3.76	3.66	3.55	3.44
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25	4.18	4.12	4.07	4.02	3.98	3.95	3.91	3.88	3.79	3.69	3.58	3.48	3.37	3.26
16	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27	4.18	4.10	4.03	3.97	3.92	3.87	3.83	3.80	3.76	3.73	3.64	3.54	3.44	3.33	3.22	3.11
17	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14	4.05	3.97	3.90	3.84	3.79	3.75	3.71	3.67	3.64	3.61	3.51	3.41	3.31	3.21	3.10	2.98
18	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03	3.94	3.86	3.79	3.73	3.68	3.64	3.60	3.56	3.53	3.50	3.40	3.30	3.20	3.10	2.99	2.87
19	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93	3.84	3.76	3.70	3.64	3.59	3.54	3.50	3.46	3.43	3.40	3.31	3.21	3.11	3.00	2.89	2.78
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68	3.61	3.55	3.50	3.46	3.42	3.38	3.35	3.32	3.22	3.12	3.02	2.92	2.81	2.69
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88	3.77	3.68	3.60	3.54	3.48	3.43	3.38	3.34	3.31	3.27	3.24	3.15	3.05	2.95	2.84	2.73	2.61
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70	3.61	3.54	3.47	3.41	3.36	3.31	3.27	3.24	3.21	3.18	3.08	2.98	2.88	2.77	2.66	2.55
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64	3.55	3.47	3.41	3.35	3.30	3.25	3.21	3.18	3.15	3.12	3.02	2.92	2.82	2.71	2.60	2.48
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42	3.35	3.30	3.25	3.20	3.16	3.12	3.09	3.06	2.97	2.87	2.77	2.66	2.55	2.43
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54	3.45	3.37	3.30	3.25	3.20	3.15	3.11	3.08	3.04	3.01	2.92	2.82	2.72	2.61	2.50	2.38
26	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.40	3.33	3.26	3.20	3.15	3.11	3.07	3.03	3.00	2.97	2.87	2.77	2.67	2.56	2.45	2.33
27	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.36	3.28	3.22	3.16	3.11	3.07	3.03	2.99	2.96	2.93	2.83	2.73	2.63	2.52	2.41	2.29
28	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.32	3.25	3.18	3.12	3.07	3.03	2.99	2.95	2.92	2.89	2.79	2.69	2.59	2.48	2.37	2.25
29	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.29	3.21	3.15	3.09	3.04	2.99	2.95	2.92	2.88	2.86	2.76	2.66	2.56	2.45	2.33	2.21
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18	3.11	3.06	3.01	2.96	2.92	2.89	2.85	2.82	2.73	2.63	2.52	2.42	2.30	2.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95	2.89	2.83	2.78	2.74	2.70	2.66	2.63	2.60	2.50	2.40	2.30	2.18	2.06	1.93
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74	2.68	2.62	2.57	2.53	2.49	2.45	2.42	2.39	2.29	2.19	2.08	1.96	1.83	1.69
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54	2.48	2.42	2.37	2.33	2.29	2.25	2.22	2.19	2.09	1.98	1.87	1.75	1.61	1.43
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36	2.29	2.24	2.19	2.14	2.10	2.06	2.03	2.00	1.90	1.79	1.67	1.53	1.36	1.36

TABLE 9 CRITICAL VALUES OF WILCOXON TEST

One-Tailed Test:					
α =	**0.005**	**0.01**	**0.025**	**0.05**	**0.10**
Two-Tailed Test:					
α =	**0.01**	**0.02**	**0.05**	**0.10**	**0.20**
n = 3	0	0	0	0	0
4	0	0	0	0	1
5	0	0	0	1	3
6	0	0	1	3	4
7	0	1	3	4	6
8	1	2	4	6	9
9	2	4	6	9	11
10	4	6	9	11	15
11	6	8	11	14	18
12	8	10	14	18	22
13	10	13	18	22	27
14	13	16	22	26	32
15	16	20	26	31	37
16	20	24	30	36	43
17	24	28	35	42	49
18	28	33	41	48	56
19	33	38	47	54	63
20	38	44	53	61	70
21	44	50	59	68	78
22	49	56	67	76	87
23	55	63	74	84	95
24	62	70	82	92	105
25	69	77	90	101	114

TABLE 10	CRITICAL VALUES OF RUNS TEST

Part-I (Lower Critical Values)
($\alpha = 0.025$)

n_1 \ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2	2	2	2	2	2	2	2	2
3					2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
4				2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
5			2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
6		2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
7		2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
8		2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
9		2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
10		2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
11		2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	1	11
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
16	2	3	4	4	5	6	6	7	8	8	9	9	12	10	11	1	11	12	12
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	1	12	12	13
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14

Part-II (Upper Critical Values)
($\alpha = 0.025$)

n_1 \ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2																			
3																			
4				9	9														
5			9	10	10	11	11												
6			9	10	11	12	12	13	13	13	13								
7				11	12	13	13	14	14	14	14	15	15	15					
8				11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9					13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10					13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11					13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12					13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13						15	16	17	18	19	19	20	20	21	21	22	22	23	23
14						15	16	17	18	19	20	20	21	22	22	23	23	23	24
15						15	16	18	18	19	20	21	22	22	23	23	24	24	25
16							17	18	19	20	21	21	22	23	23	24	25	25	25
17							17	18	19	20	21	22	23	23	24	25	25	26	26
18							17	18	19	20	21	22	23	24	25	25	26	26	27
19							17	18	20	21	22	23	23	24	25	26	26	27	27
20							17	18	20	21	22	23	24	25	25	26	27	27	28

TABLE 11	CRITICAL VALUES OF KOLMOGOROV-SMIRNOV TEST FOR ONE SAMPLE

One-Tailed Test:											
α =	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**α =**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
Two-Tailed Test:											
α =	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**α =**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**
n = 1	0.900	0.950	0.975	0.990	0.995	**n = 21**	0.226	0.259	0.287	0.321	0.344
2	0.684	0.776	0.842	0.900	0.929	**22**	0.221	0.253	0.281	0.314	0.337
3	0.565	0.636	0.708	0.785	0.829	**23**	0.216	0.247	0.275	0.307	0.330
4	0.493	0.565	0.624	0.689	0.734	**24**	0.212	0.242	0.269	0.301	0.323
5	0.447	0.509	0.563	0.627	0.669	**25**	0.208	0.238	0.264	0.295	0.317
6	0.410	0.468	0.519	0.577	0.617	**26**	0.204	0.233	0.259	0.290	0.311
7	0.381	0.436	0.483	0.538	0.576	**27**	0.200	0.229	0.254	0.284	0.305
8	0.358	0.410	0.454	0.507	0.542	**28**	0.197	0.225	0.250	0.279	0.300
9	0.339	0.387	0.430	0.480	0.513	**29**	1.193	0.221	0.246	0.275	0.295
10	0.323	0.369	0.409	0.457	0.489	**30**	0.190	0.218	0.242	0.270	0.290
11	0.308	0.352	0.391	0.437	0.468	**31**	0.187	0.214	0.238	0.266	0.285
12	0.296	0.338	0.375	0.419	0.449	**32**	0.184	0.211	0.234	0.262	0.281
13	0.285	0.325	0.361	0.404	0.432	**33**	0.182	0.208	0.231	0.258	0.277
14	0.275	0.314	0.349	0.390	0.418	**34**	0.179	0.205	0.227	0.254	0.273
15	0.266	0.304	0.338	0.377	0.404	**35**	0.177	0.202	0.224	0.251	0.269
16	0.258	0.295	0.327	0.366	0.392	**36**	0.174	0.199	0.221	0.247	0.265
17	0.250	0.286	0.318	0.355	0.381	**37**	0.172	0.196	0.218	0.244	0.262
18	0.244	0.279	0.309	0.346	0.371	**38**	0.170	0.194	0.215	0.241	0.258
19	0.237	0.271	0.301	0.337	0.361	**39**	0.168	0.191	0.213	0.238	0.255
20	0.232	0.265	0.294	0.329	0.352	**40**	0.165	0.189	0.210	0.235	0.252
						Approximation for n > 40	$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

TABLE 12 — CRITICAL VALUES OF KOLMOGOROV-SMIRNOV TEST FOR TWO SAMPLES OF EQUAL SIZE

One-Tailed Test:											
α =	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	α =	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
Two-Tailed Test:											
α =	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**α =**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**
n = 3	2/3	2/3				**n = 20**	6/2	7/2	8/20	9/20	10/20
4	3/4	3/4	3/4			**21**	6/21	7/21	8/21	9/21	10/21
5	3/5	3/5	4/5	4/5	4/5	**22**	7/22	8/22	8/22	10/22	10/22
6	3/6	4/6	4/6	5/6	5/6	**23**	7/23	8/23	9/23	10/23	10/23
7	4/7	4/7	5/7	5/7	5/7	**24**	7/24	8/24	9/24	10/24	11/24
8	4/8	4/8	5/8	5/8	6/8	**25**	7/25	8/25	9/25	10/25	11/25
9	4/9	5/9	5/9	6/9	6/9	**26**	7/26	8/26	9/26	10/26	11/26
10	4/10	5/10	6/10	6/10	7/10	**27**	7/27	8/27	9/27	11/27	11/27
11	5/11	5/11	6/11	7/11	7/11	**28**	8/28	9/28	10/28	11/28	12/28
12	5/12	5/12	6/12	7/12	7/12	**29**	8/29	9/29	10/29	11/29	12/29
13	5/13	6/13	6/13	7/13	8/13	**30**	8/30	9/30	10/30	11/30	12/30
14	5/14	6/14	7/14	7/14	8/14	**31**	8/31	9/31	10/31	11/31	12/31
15	5/15	6/15	7/15	8/15	8/15	**32**	8/32	9/32	10/32	12/32	12/32
16	6/16	6/16	7/16	8/16	9/16	**34**	8/34	10/34	11/34	12/34	13/34
17	6/17	7/17	7/17	8/17	9/17	**36**	9/36	10/36	11/36	12/36	13/36
18	6/18	7/18	8/18	9/18	9/18	**38**	9/38	10/38	11/38	13/38	14/38
19	6/19	7/19	8/19	9/19	9/19	**40**	9/40	10/40	12/40	13/40	14/40
						Approximation for n > 40					
							$\frac{1.52}{\sqrt{n}}$	$\frac{1.73}{\sqrt{n}}$	$\frac{1.92}{\sqrt{n}}$	$\frac{2.15}{\sqrt{n}}$	$\frac{2.303}{\sqrt{n}}$

TABLE 13 CRITICAL VALUES OF KOLMOGOROV-SMIRNOV TEST FOR TWO SAMPLES OF UNEQUAL SIZES

One -Tailed Test:		$\alpha = 0.10$	0.05	0.25	0.01	0.005
Two Ttailed Test:		$\alpha = 0.20$	0.1	0.05	0.25	0.01
$n_1 = 1$	$n_2 = 9$	17/18				
	10	9/10				
$n_1 = 2$	$n_2 = 3$	5/6				
	4	3/4				
	5	4/5	4/5			
	6	5/6	5/6			
	7	5/7	6/7			
	8	3/4	7/8	7/8		
	9	7/9	8/9	8/9		
	10	7/10	4/5	9/10		
$n_1 = 3$	$n_2 = 4$	3/4	3/4			
	5	2/3	4/5	4/5		
	6	2/3	2/3	5/6		
	7	2/3	5/7	6/7	6/7	
	8	5/8	3/4	3/4	7/8	
	9	2/3	2/3	7/9	8/9	8/9
	10	3/5	7/10	4/5	9/10	9/10
	12	7/12	2/3	3/4	5/6	11/12
$n_1 = 4$	$n_2 = 5$	3/5	3/4	4/5	4/5	
	6	7/12	2/3	3/4	5/6	5/6
	7	17/28	5/7	3/4	6/7	6/7
	8	5/8	5/8	3/4	7/8	7/8
	9	5/9	2/3	3/4	7/9	8/9
	10	11/20	13/20	7/10	4/5	4/5
	12	7/12	2/3	2/3	3/4	5/6
	16	9/16	5/8	11/16	3/4	13/16
$n_1 = 5$	$n_2 = 6$	3/5	2/3	2/3	5/6	5/6
	7	4/7	23/35	5/7	29/35	6/7
	8	11/20	5/8	47/40	4/5	4/5
	9	5/9	3/5	31/45	7/9	4/5
	10	1/2	3/5	7/10	7/10	4/5
	15	8/15	3/5	2/3	11/15	11/15
	20	1/2	11/20	3/5	7/10	3/4
$n_1 = 6$	$n_2 = 7$	23/42	4/7	29/42	5/7	5/6
	8	1/2	7/12	2/3	3/4	3/4
	9	1/2	5/9	2/3	13/18	7/9
	10	1/2	17/30	19/30	7/10	11/15
	12	1/2	7/12	7/12	2/3	3/4
	18	4/9	5/9	11/18	2/3	13/18
	24	11/24	1/2	7/12	5/8	2/3
$n_1 = 7$	$n_2 = 8$	27/56	33/56	5/8	41/56	3/4
	9	31/63	5/9	40/63	5/7	47/63
	10	33/70	39/70	43/70	7/10	5/7
	14	3/7	1/2	4/7	9/14	5/7
	28	3/7	13/28	15/28	17/28	9/14
$n_1 = 8$	$n_2 = 9$	4/9	13/24	5/8	2/3	3/4
	10	19/40	21/40	23/40	27/40	7/10
	12	11/24	1/2	7/12	5/8	2/3
	16	7/16	1/2	9/16	5/8	5/8
	32	13/32	7/16	1/2	9/16	19/32
$n_1 = 9$	$n_2 = 10$	7/15	1/2	26/45	2/3	31/45
	12	4/9	1/2	5/9	11/18	2/3
	15	19/45	22/45	8/15	3/5	29/45
	18	7/18	4/9	1/2	5/9	11/18
	36	13/36	5/12	17/36	19/36	5/9
$n_1 = 10$	$n_2 = 15$	2/5	7/15	1/2	17/30	19/30
	20	2/5	9/20	1/2	11/20	3/5
	40	7/20	2/5	9/20	1/2	
$n_1 = 12$	$n_2 = 15$	23/60	9/20	1/2	11/20	7/12
	16	3/8	7/16	23/48	13/24	7/12
	18	13/36	5/12	17/36	19/36	5/9
	20	11/30	5/12	7/15	31/60	17/30
$n_1 = 14$	$n_2 = 20$	7/20	2/5	13/30	29/60	31/60
$n_1 = 16$	$n_2 = 20$	27/80	33/80	17/40	19/40	41/80
Large-sample approximation		$1.07\sqrt{\frac{n_1+n_2}{n_1n_2}}$	$1.07\sqrt{\frac{n_1+n_2}{n_1n_2}}$	$1.36\sqrt{\frac{n_1+n_2}{n_1n_2}}$	$1.52\sqrt{\frac{n_1+n_2}{n_1n_2}}$	$1.63\sqrt{\frac{n_1+n_2}{n_1n_2}}$

TABLE 14 CRITICAL VALUES OF MANN WHITNEY U TEST

(Lower Critical Values)

n_1	α	$n_2 = 2$	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	0.001	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	0.005	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4
	0.01	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5
	0.025	3	3	3	3	3	3	4	4	4	5	5	5	5	5	5	6	6	6	6
	0.05	3	3	4	4	4	4	5	5	5	5	6	6	7	7	7	7	8	8	8
	0.10	3	4	6	5	5	5	6	6	7	7	8	8	8	9	9	10	10	11	11
3	0.001	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7
	0.005	6	6	6	6	6	6	6	7	7	7	8	8	8	9	9	9	9	10	10
	0.01	6	6	6	6	6	7	7	8	8	8	9	9	9	10	10	11	11	11	12
	0.025	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15
	0.05	6	7	7	8	9	9	10	11	11	12	12	13	14	14	15	16	16	17	18
	0.10	7	8	8	9	10	11	12	12	13	14	15	16	17	17	18	19	20	21	22
4	0.001	10	10	10	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	14
	0.005	10	10	10	10	11	11	12	12	13	13	14	14	15	16	16	17	17	18	19
	0.01	10	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	20	20	21
	0.025	10	10	11	12	13	14	15	15	16	17	18	19	20	21	22	22	23	24	25
	0.05	10	11	12	13	14	15	16	17	18	19	20	21	22	23	25	26	27	28	29
	0.10	11	12	14	15	16	17	18	20	21	22	23	24	26	27	28	29	31	32	33
5	0.001	15	15	15	15	15	15	16	17	17	18	18	19	19	20	21	21	22	23	23
	0.005	15	15	15	16	17	17	18	19	20	21	22	23	23	24	25	26	27	28	29
	0.01	15	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
	0.025	15	16	17	18	19	21	22	23	24	25	27	28	29	30	31	33	34	35	36
	0.05	16	17	18	20	21	22	24	25	27	28	29	31	32	34	35	36	38	39	41
	0.10	17	18	20	21	23	24	26	28	29	31	33	34	36	38	39	41	43	44	46
6	0.001	21	21	21	21	21	21	23	24	25	26	26	27	28	29	30	31	32	33	34
	0.005	21	21	22	23	24	25	26	27	28	29	31	32	33	34	35	37	38	39	40
	0.01	21	21	23	24	25	26	28	29	30	31	33	34	35	37	38	40	41	42	44
	0.025	21	23	24	25	27	28	30	32	33	35	36	38	39	41	43	44	46	47	49
	0.05	22	24	25	27	29	30	32	34	36	38	39	41	43	45	47	48	50	52	54
	0.10	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	56	58	60
7	0.001	28	28	28	28	29	30	31	32	34	35	36	37	38	39	40	42	43	44	45
	0.005	28	28	29	30	32	33	35	36	38	39	41	42	44	45	47	48	50	51	53
	0.01	28	29	30	32	33	35	36	38	40	41	43	45	46	48	50	52	53	55	57
	0.025	28	30	32	34	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63
	0.05	29	31	33	35	37	40	42	44	46	48	50	53	55	57	59	62	64	66	68
	0.10	30	33	35	37	40	42	45	47	50	52	55	57	60	62	65	67	70	72	75
8	0.001	36	36	36	37	38	39	41	42	43	45	46	48	49	51	52	54	55	57	58
	0.005	36	36	38	39	41	43	44	46	48	50	52	54	55	57	59	61	63	65	67
	0.01	36	37	39	41	43	44	46	48	50	52	54	56	59	61	63	65	67	69	71
	0.025	37	39	41	43	45	47	50	52	54	56	59	61	63	66	68	71	73	75	78
	0.05	38	40	42	45	47	50	52	55	57	60	63	65	68	70	73	76	78	81	84
	0.1	39	42	44	47	50	53	53	59	61	64	67	70	73	76	79	82	85	88	91
9	0.001	45	45	45	47	48	49	51	53	54	56	58	60	61	63	65	67	69	71	72
	0.005	45	46	47	49	51	53	55	57	59	62	64	66	68	70	73	75	77	79	82
	0.01	45	47	49	51	53	55	57	60	62	64	67	69	72	74	77	79	82	84	86
	0.025	46	48	50	53	56	58	61	63	66	69	72	74	77	80	83	85	88	91	94
	0.05	47	50	52	55	58	61	64	67	70	73	76	79	82	85	88	91	94	97	100
	0.10	48	51	55	58	61	64	68	71	74	77	81	84	87	91	94	98	101	104	108
10	0.001	55	55	56	57	59	61	62	64	66	68	70	73	75	77	79	81	83	85	88
	0.005	55	56	58	60	62	65	67	69	72	74	77	80	82	85	87	90	93	95	98
	0.01	55	57	59	62	64	67	69	72	75	78	80	83	86	89	92	94	97	100	103
	0.025	56	59	61	64	67	70	73	76	79	82	85	89	92	95	98	101	104	108	111
	0.05	57	60	63	67	70	73	76	80	83	87	90	93	97	100	104	107	111	114	118
	0.10	59	62	66	69	73	77	80	84	88	92	95	99	103	107	110	114	118	122	126

n_1	α	n_2 = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11	**0.001**	66	66	67	69	71	73	75	77	79	82	84	87	89	91	94	96	99	101	104
	0.005	66	67	69	72	74	77	80	83	85	88	91	94	97	100	103	106	109	112	115
	0.01	66	68	71	74	76	79	82	85	89	92	95	98	101	104	108	111	114	117	120
	0.025	67	70	73	76	80	83	86	90	93	97	100	104	107	111	114	118	122	125	129
	0.05	68	72	75	79	83	86	90	94	98	101	105	109	113	117	121	124	128	132	136
	0.10	70	74	78	82	86	90	94	98	103	107	111	115	119	124	128	132	136	140	145
12	**0.001**	78	78	79	81	83	86	88	91	93	96	98	102	104	106	110	113	116	118	121
	0.005	78	80	82	85	88	91	94	97	100	103	106	110	113	116	120	123	126	130	133
	0.01	78	81	84	87	90	93	96	100	103	107	110	114	117	121	125	128	132	135	139
	0.025	80	83	86	90	93	97	101	105	108	112	116	120	124	128	132	136	140	144	148
	0.05	81	84	88	92	96	100	105	109	111	117	121	126	130	134	139	143	147	151	156
	0.10	83	87	91	96	100	105	109	114	118	123	128	132	137	142	146	151	156	160	165
13	**0.001**	91	91	93	95	97	100	103	106	109	112	115	118	121	124	127	130	134	137	140
	0.005	91	93	95	99	102	105	109	112	116	119	123	126	130	134	137	141	145	149	152
	0.01	92	94	97	101	104	108	112	115	119	123	127	131	135	139	143	147	151	155	159
	0.025	93	96	100	104	108	112	116	120	125	129	133	137	142	146	151	155	159	164	168
	0.05	94	98	102	107	111	116	120	125	129	134	139	143	148	153	157	162	167	172	176
	0.10	96	101	105	110	115	120	125	130	135	140	145	150	155	160	166	171	176	181	186
14	**0.001**	105	105	107	109	112	115	118	121	125	128	131	135	138	142	145	149	152	156	160
	0.005	105	107	110	113	117	121	124	128	132	136	140	144	148	152	156	160	164	169	173
	0.01	106	108	112	116	119	123	128	132	136	140	144	149	153	157	162	166	171	175	179
	0.025	107	111	115	119	123	128	132	137	142	146	151	156	161	165	170	175	180	184	189
	0.05	109	113	117	122	127	132	137	142	147	152	157	162	167	172	177	183	188	193	198
	0.10	110	116	121	126	131	137	142	147	153	158	164	169	175	180	186	191	197	203	208
15	**0.001**	120	120	122	125	128	133	135	138	142	145	149	153	157	161	164	168	172	176	180
	0.005	120	123	126	129	133	137	141	145	150	154	158	163	167	172	176	181	185	190	194
	0.01	121	124	128	132	136	140	145	149	154	158	163	168	172	177	182	187	191	196	201
	0.025	122	126	131	135	140	145	150	155	160	165	170	175	180	185	191	196	201	206	211
	0.05	124	128	133	139	144	149	154	160	165	171	176	182	187	193	198	204	209	215	221
	0.10	126	131	137	143	148	154	160	166	172	178	184	189	195	201	207	213	219	225	231
16	**0.001**	136	136	139	142	145	148	152	156	160	164	168	172	176	180	185	189	193	197	202
	0.005	136	139	142	146	150	155	159	164	168	173	178	182	187	192	197	202	207	211	216
	0.01	137	140	144	149	153	158	163	168	173	178	183	188	193	198	203	208	213	219	224
	0.025	138	143	148	152	158	163	168	174	179	184	190	196	201	207	212	218	223	229	235
	0.05	140	145	151	156	162	167	173	179	185	191	197	202	208	214	220	226	232	238	244
	0.10	142	148	154	160	166	173	179	185	191	198	204	211	217	223	230	236	243	249	256
17	**0.001**	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220	224
	0.005	153	156	158	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235	240
	0.01	154	158	160	167	172	177	182	187	192	198	203	209	214	220	225	231	236	242	247
	0.025	156	160	163	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253	259
	0.05	157	163	166	174	180	187	193	199	205	211	218	224	231	237	243	250	256	263	269
	0.10	160	166	172	179	185	192	199	206	212	219	226	233	239	246	253	260	267	274	281
18	**0.001**	171	172	174	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243	248
	0.005	171	174	176	183	188	193	198	203	209	214	219	225	230	236	242	247	253	259	264
	0.01	172	176	179	186	191	196	202	208	213	219	225	231	237	242	248	254	260	266	272
	0.025	174	179	181	190	196	202	208	214	220	227	233	239	246	252	258	265	271	278	284
	0.05	176	181	185	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288	295
	0.10	178	185	191	199	206	213	220	227	234	241	249	256	263	270	278	285	292	300	307
19	**0.001**	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268	243
	0.005	191	194	195	203	208	213	219	224	230	236	242	248	254	260	265	272	278	284	290
	0.01	192	195	198	206	211	217	223	229	235	241	247	254	260	266	273	279	285	292	298
	0.025	193	198	201	210	216	223	229	236	243	249	256	263	269	276	283	290	297	304	310
	0.05	195	201	205	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314	321
	0.10	198	205	211	219	227	234	242	249	257	264	272	280	288	295	303	311	319	326	334
20	**0.001**	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293	299
	0.005	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310	316
	0.01	212	216	221	227	233	239	245	251	258	264	271	278	284	291	298	304	311	318	325
	0.025	213	219	225	231	238	245	2512	259	266	273	280	287	294	301	309	316	323	330	338
	0.05	215	222	229	236	243	250	258	265	273	280	288	295	303	311	318	326	334	341	349

	0.10	218	226	233	241	249	257	265	273	281	289	297	305	313	321	330	338	346	354	362

(Upper Critical Values)

n_1	α	n_2= 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	0.001	10	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
	0.005	10	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	40	42
	0.01	10	9	11	13	15	17	19	21	23	25	27	28	30	32	34	36	38	39	41
	0.025	10	9	11	13	15	17	18	20	22	23	25	27	29	31	33	34	36	38	40
	0.05	10	9	10	12	14	16	17	19	21	23	24	26	27	29	31	33	34	36	38
	0.10	10	8	8	11	13	15	16	18	19	21	22	24	26	27	29	30	32	33	35
3	0.001	18	15	18	21	24	27	30	33	36	39	42	45	48	51	54	56	59	62	65
	0.005	18	15	18	21	24	27	30	32	35	38	40	43	46	48	51	54	57	59	62
	0.01	18	15	18	21	24	26	29	31	34	37	39	42	45	47	50	52	55	58	60
	0.025	18	15	17	20	22	25	27	30	32	35	37	40	42	45	47	50	52	55	57
	0.05	18	14	17	19	21	24	26	28	31	33	36	38	40	43	45	47	50	52	54
	0.10	18	13	16	18	20	22	24	27	29	31	33	35	37	40	42	44	46	48	50
4	0.001	28	22	26	30	34	38	42	46	49	53	57	60	64	68	71	75	78	82	86
	0.005	28	22	26	30	33	37	40	44	47	51	54	58	61	64	68	71	75	78	81
	0.01	28	22	26	29	32	36	39	42	46	49	52	56	59	62	66	69	72	76	79
	0.025	28	22	25	28	31	34	37	41	44	47	50	53	56	59	62	66	69	72	75
	0.05	28	21	24	27	30	33	36	39	42	45	48	51	54	57	59	62	65	68	71
	0.10	28	20	22	25	28	31	34	36	39	42	45	48	50	53	56	59	61	64	67
5	0.001	40	30	35	40	45	50	54	58	63	67	72	76	81	85	89	94	98	102	107
	0.005	40	30	35	39	43	48	52	56	60	64	68	72	77	81	85	89	93	97	101
	0.01	40	30	34	38	42	46	50	54	58	62	66	70	74	78	82	86	90	94	98
	0.025	40	29	33	37	41	44	48	52	56	60	63	67	71	75	79	82	86	90	94
	0.05	40	28	32	35	39	43	46	50	53	57	61	64	68	71	75	79	82	86	89
	0.10	40	27	30	34	37	41	44	47	51	54	57	61	64	67	71	74	77	81	84
6	0.001	54	39	45	51	57	63	67	72	77	82	88	93	98	103	108	113	118	123	128
	0.005	54	39	44	49	54	59	64	69	74	79	83	88	93	98	103	107	112	117	122
	0.01	54	39	43	48	53	58	62	67	72	77	81	86	91	95	100	104	109	114	118
	0.025	54	37	42	47	51	56	60	64	69	73	78	82	87	91	95	100	104	109	113
	0.05	54	36	41	45	49	54	58	62	66	70	75	79	83	87	91	96	100	104	108
	0.10	54	35	39	43	47	51	55	59	63	67	71	75	79	83	87	91	94	98	102
7	0.001	70	49	56	63	69	75	81	87	92	98	104	110	116	122	128	133	139	145	151
	0.005	70	49	55	61	66	72	77	83	88	94	99	105	110	116	121	127	132	138	143
	0.01	70	48	54	59	65	70	76	81	86	92	97	102	108	113	118	123	129	134	139
	0.025	70	47	52	57	63	68	73	78	83	88	93	98	103	108	113	118	123	128	133
	0.05	70	46	51	56	61	65	70	75	80	85	90	94	99	104	109	113	118	123	128
	0.10	70	44	49	54	58	63	67	72	76	81	85	90	94	99	103	108	112	117	121
8	0.001	88	60	68	75	82	89	95	102	109	115	122	128	135	141	148	154	161	167	174
	0.005	88	60	66	73	79	85	92	98	104	110	116	122	129	135	141	147	153	159	165
	0.01	88	59	65	71	77	84	90	96	102	108	114	120	125	131	137	143	149	155	161
	0.025	88	57	63	69	75	81	86	92	98	104	109	115	121	126	132	137	143	149	154
	0.05	88	56	62	67	73	78	84	89	95	100	105	111	116	122	127	132	138	143	148
	0.10	88	54	60	65	70	75	83	85	91	96	101	106	111	116	121	126	131	136	141
9	0.001	108	72	81	88	96	104	111	118	126	133	140	147	155	162	169	176	183	190	198
	0.005	108	71	79	86	93	100	107	114	121	127	134	141	148	155	161	168	175	182	188
	0.01	108	70	77	84	91	98	105	111	118	125	131	138	144	151	157	164	170	177	184
	0.025	108	69	76	82	88	95	101	108	114	120	126	133	139	145	151	158	164	170	176
	0.05	108	67	74	80	86	92	98	104	110	116	122	128	134	140	146	152	158	164	170
	0.10	108	66	71	77	83	89	94	100	106	112	117	123	129	134	140	145	151	157	162
10	0.001	130	85	94	103	111	119	128	136	144	152	160	167	175	183	191	199	207	215	222
	0.005	130	84	92	100	108	115	123	131	138	146	153	160	168	175	183	190	197	205	212
	0.01	130	83	91	98	106	113	121	128	135	142	150	157	164	171	178	186	193	200	207
	0.025	130	81	89	96	103	110	117	124	131	138	145	151	158	165	172	179	186	192	199
	0.05	130	80	87	93	100	107	114	120	127	133	140	147	153	160	166	173	179	186	192
	0.10	130	78	84	91	97	103	110	116	122	128	135	141	147	153	160	166	172	178	184

n_1	α	n_2= 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11	0.001	154	99	109	118	127	136	145	154	163	171	180	188	197	206	214	223	231	240	248

	0.005	154	98	107	115	124	132	140	148	157	165	173	181	189	197	205	213	221	229	237
	0.01	154	97	105	113	122	130	138	146	153	161	169	177	185	193	200	208	216	224	232
	0.025	154	95	103	111	118	126	134	141	149	156	164	171	179	186	194	201	208	216	223
	0.05	154	93	101	108	115	123	130	137	144	152	159	166	173	180	187	195	202	209	216
	0.10	154	91	98	105	112	119	126	133	139	146	153	160	167	173	180	187	194	201	207
12	**0.001**	180	114	125	135	145	154	164	173	183	192	202	210	220	230	238	247	256	266	275
	0.005	180	112	122	131	140	149	158	167	176	185	194	202	211	220	228	237	246	254	263
	0.01	180	111	120	129	138	147	156	164	173	181	190	198	207	215	223	232	240	249	257
	0.025	180	109	118	126	135	143	151	159	168	176	184	192	200	208	216	224	232	240	248
	0.05	180	108	116	124	132	140	147	155	165	171	179	186	194	202	209	217	225	233	240
	0.10	180	105	113	120	128	135	143	150	158	165	172	180	187	194	202	209	216	224	231
13	**0.001**	208	130	141	152	163	173	183	193	203	213	223	233	243	253	263	273	282	292	302
	0.005	208	128	139	148	158	168	177	187	196	206	215	225	234	243	253	262	271	280	290
	0.01	208	127	137	146	156	165	174	184	193	202	211	220	229	238	247	256	265	274	283
	0.025	208	125	134	143	152	161	170	179	187	196	205	214	222	231	239	248	257	265	274
	0.05	208	123	132	140	149	157	166	174	183	191	199	208	216	224	233	241	249	257	266
	0.10	208	120	129	137	145	153	161	169	177	185	193	201	209	217	224	232	240	248	256
14	**0.001**	238	147	159	171	182	193	204	215	225	236	247	257	268	278	289	299	310	320	330
	0.005	238	145	156	167	177	187	198	208	218	228	238	248	258	268	278	288	298	307	317
	0.01	238	144	154	164	175	185	194	204	214	224	234	243	253	263	272	282	291	301	311
	0.025	238	141	151	161	171	180	190	199	208	218	227	236	245	255	264	273	282	292	301
	0.05	238	139	149	158	167	176	185	194	203	212	221	230	239	248	257	265	274	283	292
	0.10	238	136	145	154	163	171	180	189	197	206	214	223	231	240	248	257	265	273	282
15	**0.001**	270	165	178	190	202	212	225	237	248	260	271	282	293	304	316	327	338	349	360
	0.005	270	162	174	186	197	208	219	230	240	251	262	272	283	293	304	314	325	335	346
	0.01	270	161	172	183	194	205	215	226	236	247	257	267	278	288	298	308	319	329	339
	0.025	270	159	169	180	190	200	210	220	230	240	250	260	270	280	289	299	309	319	329
	0.05	270	157	167	176	186	196	206	215	225	234	244	253	263	272	282	291	301	310	319
	0.10	270	154	163	172	182	191	200	209	218	227	236	246	255	264	273	282	291	300	309
16	**0.001**	304	184	197	210	223	236	248	260	272	284	296	308	320	332	343	355	367	379	390
	0.005	304	181	194	206	218	229	241	252	264	275	286	298	309	320	331	342	353	365	376
	0.01	304	180	192	203	215	226	237	248	259	270	281	292	303	314	325	336	347	357	368
	0.025	304	177	188	200	210	221	232	242	253	264	274	284	295	305	316	326	337	347	357
	0.05	304	175	185	196	206	217	227	237	247	257	267	278	288	298	308	318	328	338	348
	0.10	304	172	182	192	202	211	221	231	241	250	260	269	279	289	298	308	317	327	336
17	**0.001**	340	203	218	232	245	258	271	284	297	310	322	335	347	360	372	384	397	409	422
	0.005	340	201	216	227	239	252	264	276	288	300	312	324	336	347	359	371	383	394	406
	0.01	340	199	214	224	236	248	260	272	284	295	307	318	330	341	353	364	376	387	399
	0.025	340	197	211	220	232	243	254	266	277	288	299	310	321	332	343	354	365	376	387
	0.05	340	194	208	217	228	238	249	260	271	282	292	303	313	324	335	345	356	366	377
	0.10	340	191	202	212	223	233	243	253	264	274	284	294	305	315	325	335	345	355	365
18	**0.001**	378	224	240	254	268	282	296	309	323	336	349	362	376	389	402	415	428	441	454
	0.005	378	222	238	249	262	275	288	301	313	326	339	351	364	376	388	401	413	425	438
	0.01	378	220	235	246	259	272	284	296	309	321	333	345	357	370	382	394	406	418	430
	0.025	378	217	233	242	254	266	278	290	302	313	325	337	348	360	372	383	395	406	418
	0.05	378	215	229	238	250	261	273	284	295	307	318	329	340	352	363	374	385	396	407
	0.10	378	211	223	233	244	255	266	277	288	299	309	320	331	342	352	363	374	384	395
19	**0.001**	418	246	262	277	292	307	321	335	350	364	377	391	405	419	433	446	460	473	517
	0.005	418	243	261	272	286	300	313	327	340	353	366	379	392	405	419	431	444	457	470
	0.01	418	242	258	269	283	296	309	322	335	348	361	373	386	399	411	424	437	449	462
	0.025	418	239	255	265	278	290	303	315	327	340	352	364	377	389	401	413	425	437	450
	0.05	418	236	251	261	273	285	297	309	321	333	345	356	368	380	392	403	415	427	439
	0.10	418	232	245	256	267	279	290	302	313	325	336	347	358	370	381	392	403	415	426
20	**0.001**	460	269	286	302	317	333	348	363	377	392	407	421	435	450	464	479	493	507	521
	0.005	460	266	281	296	311	325	339	353	367	381	395	409	422	436	450	463	477	490	504
	0.01	460	264	279	293	307	321	332	349	362	376	389	402	416	429	442	456	469	482	495
	0.025	460	261	275	289	302	315	330	341	354	367	380	393	406	419	431	444	457	470	482
	0.05	460	258	271	284	297	310	322	335	347	360	372	385	397	409	422	434	446	459	471
	0.10	460	254	267	279	291	303	315	327	339	351	363	375	387	399	410	422	434	446	458

TABLE 15 CRITICAL VALUES OF KRUSKAL-WALLIS TEST

n_1	n_2	n_3	$\alpha = 0.1$	0.05	0.025	0.01
2	**2**	**2**	4.571	–	–	–
3	**2**	**1**	4.286	–	–	–
3	**2**	**2**	4.500	4.714	–	–
3	**3**	**1**	4.571	5.143	–	–
3	**3**	**2**	4.556	5.361	5.556	–
3	**3**	**3**	4.622	5.600	5.956	7.200
4	**2**	**1**	4.500	–	–	–
4	**2**	**2**	4.458	5.333	5.500	–
4	**3**	**1**	4.056	5.208	5.833	–
4	**3**	**2**	4.511	5.444	6.000	6.444
4	**3**	**3**	4.709	5.791	6.155	6.745
4	**4**	**1**	4.167	4.967	6.167	6.667
4	**4**	**2**	4.555	5.455	6.327	7.036
4	**4**	**3**	4.545	5.598	6.394	7.144
4	**4**	**4**	4.654	5.692	6.615	7.654
5	**2**	**1**	4.200	5.000	–	–
5	**2**	**2**	4.373	5.160	6.000	6.533
5	**3**	**1**	4.018	4.960	6.044	–
5	**3**	**2**	4.651	5.251	6.004	6.909
5	**3**	**3**	4.533	5.648	6.315	7.079
5	**4**	**1**	3.987	4.985	5.858	6.955
5	**4**	**2**	4.541	5.273	6.068	7.205
5	**4**	**3**	4.549	5.656	6.410	7.445
5	**4**	**4**	4.668	5.657	6.673	7.760
5	**5**	**1**	4.109	5.127	6.000	7.309
5	**5**	**2**	4.623	5.338	6.346	7.338
5	**5**	**3**	4.545	5.705	6.549	7.578
5	**5**	**4**	4.523	5.666	6.760	7.823
5	**5**	**5**	4.560	5.780	6.740	8.000

Notes: 1. The critical values in the above table are approximated values.

2. When this table is not applicable, we use the Chi-square table for critical values of the Kruskal-Wallis test.

TABLE 16 CRITICAL VALUES OF FRIEDMAN TEST

n	k = 3		k = 4		k = 5		k = 6	
	α = 5%	α = 1%	α = 5%	α = 1%	α = 5%	α = 1%	α = 5%	α = 1%
2	–	–	–	6.00	–	–	9.14	9.71
3	–	6.00	7.40	9.00	7.60	8.00	9.86	11.76
4	6.50	8.00	7.80	9.60	8.53	10.13	10.29	12.71
5	6.40	8.40	7.80	9.96	8.80	11.20	10.49	13.23
6	7.00	9.00	7.60	10.20	8.96	11.68	10.57	13.62
7	7.14	8.86	7.80	10.54	9.07	11.87	10.67	13.86
8	6.25	9.00	7.65	10.50	9.14	12.11	10.71	14.00
9	6.22	9.56	7.67	10.73	9.20	13.20	10.78	14.14
10	6.20	9.60	7.68	10.68	9.24	12.44	10.80	14.23
11	6.55	9.46	7.69	10.75	9.28	12.48	10.84	14.32
12	6.50	9.50	7.70	10.80	9.31	12.58	10.86	14.38
13	6.62	9.39	7.80	10.85	9.33	12.60	10.89	14.45
14	6.14	9.14	7.71	10.89	9.35	12.68	10.90	14.49
15	6.40	8.93	7.72	10.92	9.37	12.74	10.92	14.54
16	6.50	9.38	7.80	10.95	9.39	12.80	10.96	14.57
17	6.12	9.29	7.80	10.05	9.40	12.85	10.95	14.61
18	6.33	9.00	7.73	10.93	9.42	12.89	10.95	14.63
19	6.42	9.58	7.86	11.02	9.43	12.88	11.00	14.67
20	6.30	9.30	7.80	11.80	9.40	12.92	11.00	14.66

Note: When this table is not applicable, we use the Chi-square table for critical values of the Freidman test.

TABLE 17 POISSON PROBABILITY

For a given value of λ entry indicates the probability of obtaining a specified value of X

X	λ = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647	0.1839
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613
4	0	0.0001	0.0003	0.0007	0.0016	0.003	0.005	0.0077	0.0111	0.0153
5	0	0	0	0.0001	0.0002	0.0004	0.0007	0.0012	0.002	0.0031
6	0	0	0	0	0	0	0.0001	0.0002	0.0003	0.0005
7	0	0	0	0	0	0	0	0	0	0.0001
X	λ = 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.3662	0.3614	0.3543	0.3452	0.3347	0.323	0.3106	0.2975	0.2842	0.2707
2	0.2014	0.2169	0.2303	0.2417	0.251	0.2584	0.264	0.2678	0.27	0.2707
3	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.171	0.1804
4	0.0203	0.026	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902
5	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.026	0.0309	0.0361
6	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.012
7	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.002	0.0027	0.0034
8	0	0	0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009
9	0	0	0	0	0	0	0.0001	0.0001	0.0001	0.0002
X	λ = 2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.055	0.0498
1	0.2572	0.2438	0.2306	0.2177	0.2052	0.1931	0.1815	0.1703	0.1596	0.1494
2	0.27	0.2681	0.2652	0.2613	0.2565	0.251	0.245	0.2384	0.2314	0.224
3	0.189	0.1966	0.2033	0.209	0.2138	0.2176	0.2205	0.2225	0.2237	0.224
4	0.0992	0.1082	0.1169	0.1254	0.1336	0.1414	0.1488	0.1557	0.1622	0.168
5	0.0417	0.0476	0.0538	0.0602	0.0668	0.0735	0.0804	0.0872	0.094	0.1008
6	0.0146	0.0174	0.0206	0.0241	0.0278	0.0319	0.0362	0.0407	0.0455	0.0504
7	0.0044	0.0055	0.0068	0.0083	0.0099	0.0118	0.0139	0.0163	0.0188	0.0216
8	0.0011	0.0015	0.0019	0.0025	0.0031	0.0038	0.0047	0.0057	0.0068	0.0081
9	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011	0.0014	0.0018	0.0022	0.0027
10	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008
11	0	0	0	0	0	0.0001	0.0001	0.0001	0.0002	0.0002
12	0	0	0	0	0	0	0	0	0	0.0001
X	λ = 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	0.045	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202	0.0183
1	0.1397	0.1304	0.1217	0.1135	0.1057	0.0984	0.0915	0.085	0.0789	0.0733
2	0.2165	0.2087	0.2008	0.1929	0.185	0.1771	0.1692	0.1615	0.1539	0.1465
3	0.2237	0.2226	0.2209	0.2186	0.2158	0.2125	0.2087	0.2046	0.2001	0.1954
4	0.1734	0.1781	0.1823	0.1858	0.1888	0.1912	0.1931	0.1944	0.1951	0.1954
5	0.1075	0.114	0.1203	0.1264	0.1322	0.1377	0.1429	0.1477	0.1522	0.1563
6	0.0555	0.0608	0.0662	0.0716	0.0771	0.0826	0.0881	0.0936	0.0989	0.1042
7	0.0246	0.0278	0.0312	0.0348	0.0385	0.0425	0.0466	0.0508	0.0551	0.0595
8	0.0095	0.0111	0.0129	0.0148	0.0169	0.0191	0.0215	0.0241	0.0269	0.0298
9	0.0033	0.004	0.0047	0.0056	0.0066	0.0076	0.0089	0.0102	0.0116	0.0132
10	0.001	0.0013	0.0016	0.0019	0.0023	0.0028	0.0033	0.0039	0.0045	0.0053
11	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0011	0.0013	0.0016	0.0019
12	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006
13	0	0	0	0	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
14	0	0	0	0	0	0	0	0	0	0.0001
X	λ = 4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	0.0166	0.015	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074	0.0067
1	0.0679	0.063	0.0583	0.054	0.05	0.0462	0.0427	0.0395	0.0365	0.0337
2	0.1393	0.1323	0.1254	0.1188	0.1125	0.1063	0.1005	0.0948	0.0894	0.0842
3	0.1904	0.1852	0.1798	0.1743	0.1687	0.1631	0.1574	0.1517	0.146	0.1404
4	0.1951	0.1944	0.1933	0.1917	0.1898	0.1875	0.1849	0.182	0.1789	0.1755
5	0.16	0.1633	0.1662	0.1687	0.1708	0.1725	0.1738	0.1747	0.1753	0.1755
6	0.1093	0.1143	0.1191	0.1237	0.1281	0.1323	0.1362	0.1398	0.1432	0.1462
7	0.064	0.0686	0.0732	0.0778	0.0824	0.0869	0.0914	0.0959	0.1002	0.1044
8	0.0328	0.036	0.0393	0.0428	0.0463	0.05	0.0537	0.0575	0.0614	0.0653
9	0.015	0.0168	0.0188	0.0209	0.0232	0.0255	0.028	0.0307	0.0334	0.0363
10	0.0061	0.0071	0.0081	0.0092	0.0104	0.0118	0.0132	0.0147	0.0164	0.0181
11	0.0023	0.0027	0.0032	0.0037	0.0043	0.0049	0.0056	0.0064	0.0073	0.0082
12	0.0008	0.0009	0.0011	0.0014	0.0016	0.0019	0.0022	0.0026	0.003	0.0034

X	λ = 4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
13	0.0002	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009	0.0011	0.0013
14	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005
15	0	0	0	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002

X	λ = 5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	0.0061	0.0055	0.005	0.0045	0.0041	0.0037	0.0033	0.003	0.0027	0.0025
1	0.0311	0.0287	0.0265	0.0244	0.0225	0.0207	0.0191	0.0176	0.0162	0.0149
2	0.0793	0.0746	0.0701	0.0659	0.0618	0.058	0.0544	0.0509	0.0477	0.0446
3	0.1348	0.1293	0.1239	0.1185	0.1133	0.1082	0.1033	0.0985	0.0938	0.0892
4	0.1719	0.1681	0.1641	0.16	0.1558	0.1515	0.1472	0.1428	0.1383	0.1339
5	0.1753	0.1748	0.174	0.1728	0.1714	0.1697	0.1678	0.1656	0.1632	0.1606
6	0.149	0.1515	0.1537	0.1555	0.1571	0.1584	0.1594	0.1601	0.1605	0.1606
7	0.1086	0.1125	0.1163	0.12	0.1234	0.1267	0.1298	0.1326	0.1353	0.1377
8	0.0692	0.0731	0.0771	0.081	0.0849	0.0887	0.0925	0.0962	0.0998	0.1033
9	0.0392	0.0423	0.0454	0.0486	0.0519	0.0552	0.0586	0.062	0.0654	0.0688
10	0.02	0.022	0.0241	0.0262	0.0285	0.0309	0.0334	0.0359	0.0386	0.0413
11	0.0093	0.0104	0.0116	0.0129	0.0143	0.0157	0.0173	0.019	0.0207	0.0225
12	0.0039	0.0045	0.0051	0.0058	0.0065	0.0073	0.0082	0.0092	0.0102	0.0113
13	0.0015	0.0018	0.0021	0.0024	0.0028	0.0032	0.0036	0.0041	0.0046	0.0052
14	0.0006	0.0007	0.0008	0.0009	0.0011	0.0013	0.0015	0.0017	0.0019	0.0022
15	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009
16	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003
17	0	0	0	0	0	0	0.0001	0.0001	0.0001	0.0001

X	λ = 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	0.0022	0.002	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.001	0.0009
1	0.0137	0.0126	0.0116	0.0106	0.0098	0.009	0.0082	0.0076	0.007	0.0064
2	0.0417	0.039	0.0364	0.034	0.0318	0.0296	0.0276	0.0258	0.024	0.0223
3	0.0848	0.0806	0.0765	0.0726	0.0688	0.0652	0.0167	0.0584	0.0552	0.0521
4	0.1294	0.1249	0.1205	0.1162	0.1118	0.1076	0.1034	0.0992	0.0952	0.0912
5	0.1579	0.1549	0.1519	0.1487	0.1454	0.142	0.1385	0.1349	0.1314	0.1277
6	0.1605	0.1601	0.1595	0.1586	0.1575	0.1562	0.1546	0.1529	0.1511	0.149
7	0.1399	0.1418	0.1435	0.145	0.1462	0.1472	0.148	0.1486	0.1489	0.149
8	0.1066	0.1099	0.113	0.116	0.1188	0.1215	0.124	0.1263	0.1284	0.1304
9	0.0723	0.0757	0.0791	0.0825	0.0858	0.0891	0.0923	0.0954	0.0985	0.1014
10	0.0441	0.0469	0.0498	0.0528	0.0558	0.0588	0.0618	0.0649	0.0679	0.071
11	0.0245	0.0265	0.0285	0.0307	0.033	0.0353	0.0377	0.0401	0.0426	0.0452
12	0.0124	0.0137	0.015	0.0164	0.0179	0.0194	0.021	0.0227	0.0245	0.0264
13	0.0058	0.0065	0.0073	0.0081	0.0089	0.0098	0.0108	0.0119	0.013	0.0142
14	0.0025	0.0029	0.0033	0.0037	0.0041	0.0046	0.0052	0.0058	0.0064	0.0071
15	0.001	0.0012	0.0014	0.0016	0.0018	0.002	0.0023	0.0026	0.0029	0.0033
16	0.0004	0.0005	0.0005	0.0006	0.0007	0.0008	0.001	0.0011	0.0013	0.0014
17	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006
18	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
19	0	0	0	0	0	0	0	0.0001	0.0001	0.0001

X	λ = 7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003
1	0.0059	0.0054	0.0049	0.0045	0.0041	0.0038	0.0035	0.0032	0.0029	0.0027
2	0.0208	0.0194	0.018	0.0167	0.0156	0.0145	0.0134	0.0125	0.0116	0.0107
3	0.0492	0.0464	0.0438	0.0413	0.0389	0.0366	0.0345	0.0324	0.0305	0.0286
4	0.0874	0.0836	0.0799	0.0764	0.0729	0.0696	0.0663	0.0632	0.0602	0.0573
5	0.1241	0.1204	0.1167	0.113	0.1094	0.1057	0.1021	0.0986	0.0951	0.0916
6	0.1468	0.1445	0.142	0.1394	0.1367	0.1339	0.1311	0.1282	0.1252	0.1221
7	0.1489	0.1486	0.1481	0.1474	0.1465	0.1454	0.1442	0.1428	0.1413	0.1396
8	0.1321	0.1337	0.1351	0.1363	0.1373	0.1382	0.1388	0.1392	0.1395	0.1396
9	0.1042	0.107	0.1096	0.1121	0.1144	0.1167	0.1187	0.1207	0.1224	0.1241
10	0.074	0.077	0.08	0.0829	0.0858	0.0887	0.0914	0.0941	0.0967	0.0993
11	0.0478	0.0504	0.0531	0.0558	0.0585	0.0613	0.064	0.0667	0.0695	0.0722
12	0.0283	0.0303	0.0323	0.0344	0.0366	0.0388	0.0411	0.0434	0.0457	0.0481
13	0.0154	0.0168	0.0181	0.0196	0.0211	0.0227	0.0243	0.026	0.0278	0.0296
14	0.0078	0.0086	0.0095	0.0104	0.0113	0.0123	0.0134	0.0145	0.0157	0.0169
15	0.0037	0.0041	0.0046	0.0051	0.0057	0.0062	0.0069	0.0075	0.0083	0.009
16	0.0016	0.0019	0.0021	0.0024	0.0026	0.003	0.0033	0.0037	0.0041	0.0045
17	0.0007	0.0008	0.0009	0.001	0.0012	0.0013	0.0015	0.0017	0.0019	0.0021
18	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009
19	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004
20	0	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
21	0	0	0	0	0	0	0	0	0	0.0001

X	λ = 8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
1	0.0025	0.0023	0.0021	0.0019	0.0017	0.0016	0.0014	0.0013	0.0012	0.0011
2	0.01	0.0092	0.0086	0.0079	0.0074	0.0068	0.0063	0.0058	0.0054	0.005
3	0.0269	0.0252	0.0237	0.0222	0.0208	0.0195	0.0183	0.0171	0.016	0.015
4	0.0544	0.0517	0.0491	0.0466	0.0443	0.042	0.0398	0.0377	0.0357	0.0337
5	0.0882	0.0849	0.0816	0.0784	0.0752	0.0722	0.0692	0.0663	0.0635	0.0607
6	0.1191	0.116	0.1128	0.1097	0.1066	0.1034	0.1003	0.0972	0.0941	0.0911
7	0.1378	0.1358	0.1338	0.1317	0.1294	0.1271	0.1247	0.1222	0.1197	0.1171
8	0.1395	0.1392	0.1388	0.1382	0.1375	0.1366	0.1356	0.1344	0.1332	0.1318
9	0.1256	0.1269	0.128	0.129	0.1299	0.1306	0.1311	0.1315	0.1317	0.1318
10	0.1017	0.104	0.1063	0.1084	0.1104	0.1123	0.114	0.1157	0.1172	0.1186
11	0.0749	0.0776	0.0802	0.0828	0.0853	0.0878	0.0902	0.0925	0.0948	0.097
12	0.0505	0.053	0.0555	0.0579	0.0604	0.0629	0.0654	0.0679	0.0703	0.0728
13	0.0315	0.0334	0.0354	0.0374	0.0395	0.0416	0.0438	0.0459	0.0481	0.0504
14	0.0182	0.0196	0.021	0.0225	0.024	0.0256	0.0272	0.0289	0.0306	0.0324
15	0.0098	0.0107	0.0116	0.0126	0.0136	0.0147	0.0158	0.0169	0.0182	0.0194
16	0.005	0.0055	0.006	0.0066	0.0072	0.0079	0.0086	0.0093	0.0101	0.0109
17	0.0024	0.0026	0.0029	0.0033	0.0036	0.004	0.0044	0.0048	0.0053	0.0058
18	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019	0.0021	0.0024	0.0026	0.0029
19	0.0005	0.0005	0.0006	0.0007	0.0008	0.0009	0.001	0.0011	0.0012	0.0014
20	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0005	0.0006
21	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003
22	0	0	0	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
X	**λ = 9.1**	**9.2**	**9.3**	**9.4**	**9.5**	**9.6**	**9.7**	**9.8**	**9.9**	**10.0**
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0
1	0.001	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005	0.0005	0.0005
2	0.0046	0.0043	0.004	0.0037	0.0034	0.0031	0.0029	0.0027	0.0025	0.0023
3	0.014	0.0131	0.0123	0.0115	0.0107	0.01	0.0093	0.0087	0.0081	0.0076
4	0.0319	0.0302	0.0285	0.0269	0.0254	0.024	0.0226	0.0213	0.0201	0.0189
5	0.0581	0.0555	0.053	0.0506	0.0483	0.046	0.0439	0.0418	0.0398	0.0378
6	0.0881	0.0851	0.0822	0.0793	0.0764	0.0736	0.0709	0.0682	0.0656	0.0631
7	0.1145	0.1118	0.1091	0.1064	0.1037	0.101	0.0982	0.0955	0.0928	0.0901
8	0.1302	0.1286	0.1269	0.1251	0.1232	0.1212	0.1191	0.117	0.1148	0.1126
9	0.1317	0.1315	0.1311	0.1306	0.13	0.1293	0.1284	0.1274	0.1263	0.1251
10	0.1198	0.121	0.1219	0.1228	0.1235	0.1241	0.1245	0.1249	0.125	0.1251
11	0.0991	0.1012	0.1031	0.1049	0.1067	0.1083	0.1098	0.1112	0.1125	0.1137
12	0.0752	0.0776	0.0799	0.0822	0.0844	0.0866	0.0888	0.0908	0.0928	0.0948
13	0.0526	0.0549	0.0572	0.0594	0.0617	0.064	0.0662	0.0685	0.0707	0.0729
14	0.0342	0.0361	0.038	0.0399	0.0419	0.0439	0.0459	0.0479	0.05	0.0521
15	0.0208	0.0221	0.0235	0.025	0.0265	0.0281	0.0297	0.0313	0.033	0.0347
16	0.0118	0.0127	0.0137	0.0147	0.0157	0.0168	0.018	0.0192	0.0204	0.0217
17	0.0063	0.0069	0.0075	0.0081	0.0088	0.0095	0.0103	0.0111	0.0119	0.0128
18	0.0032	0.0035	0.0039	0.0042	0.0046	0.0051	0.0055	0.006	0.0065	0.0071
19	0.0015	0.0017	0.0019	0.0021	0.0023	0.0026	0.0028	0.0031	0.0034	0.0037
20	0.0007	0.0008	0.0009	0.001	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019
21	0.0003	0.0003	0.0004	0.0004	0.0005	0.0006	0.0006	0.0007	0.0008	0.0009
22	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004
23	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
24	0	0	0	0	0	0	0	0.0001	0.0001	0.0001
X	**λ = 11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20.0**
0	0	0	0	0	0	0	0	0	0	0
1	0.0002	0.0001	0	0	0	0	0	0	0	0
2	0.001	0.0004	0.0002	0.0001	0	0	0	0	0	0
3	0.0037	0.0018	0.0008	0.0004	0.0002	0.0001	0	0	0	0
4	0.0102	0.0053	0.0027	0.0013	0.0006	0.0003	0.0001	0.0001	0	0
5	0.0224	0.0127	0.007	0.0037	0.0019	0.001	0.0005	0.0002	0.0001	0.0001
6	0.0411	0.0255	0.0152	0.0087	0.0048	0.0026	0.0014	0.0007	0.0004	0.0002
7	0.0646	0.0437	0.0281	0.0174	0.0104	0.006	0.0034	0.0018	0.001	0.0005
8	0.0888	0.0655	0.0457	0.0304	0.0194	0.012	0.0072	0.0042	0.0024	0.0013
9	0.1085	0.0874	0.0661	0.0473	0.0324	0.0213	0.0135	0.0083	0.005	0.0029
10	0.1194	0.1048	0.0859	0.0663	0.0486	0.0341	0.023	0.015	0.0095	0.0058
11	0.1194	0.1144	0.1015	0.0844	0.0663	0.0496	0.0355	0.0245	0.0164	0.0106
12	0.1094	0.1144	0.1099	0.0984	0.0829	0.0661	0.0504	0.0368	0.0259	0.0176
13	0.0926	0.1056	0.1099	0.106	0.0956	0.0814	0.0658	0.0509	0.0378	0.0271
14	0.0728	0.0905	0.1021	0.106	0.1024	0.093	0.08	0.0655	0.0514	0.0387

X	λ = 11	12	13	14	15	16	17	18	19	20.0
15	0.0534	0.0724	0.0885	0.0989	0.1024	0.0992	0.0906	0.0786	0.065	0.0516
16	0.0367	0.0543	0.0719	0.0866	0.096	0.0992	0.0963	0.0884	0.0772	0.0646
17	0.0237	0.0383	0.055	0.0713	0.0847	0.0934	0.0963	0.0936	0.0863	0.076
18	0.0145	0.0256	0.0397	0.0554	0.0706	0.083	0.0909	0.0936	0.0911	0.0844
19	0.0084	0.0161	0.0272	0.0409	0.0557	0.0699	0.0814	0.0887	0.0911	0.0888
20	0.0046	0.0097	0.0177	0.0286	0.0418	0.0559	0.0692	0.0798	0.0866	0.0888
21	0.0024	0.0055	0.0109	0.0191	0.0299	0.0426	0.056	0.0684	0.0783	0.0846
22	0.0012	0.003	0.0065	0.0121	0.0204	0.031	0.0433	0.056	0.0676	0.0769
23	0.0006	0.0016	0.0037	0.0074	0.0133	0.0216	0.032	0.0438	0.0559	0.0669
24	0.0003	0.0008	0.002	0.0043	0.0083	0.0144	0.0226	0.0328	0.0442	0.0557
25	0.0001	0.0004	0.001	0.0024	0.005	0.0092	0.0154	0.0237	0.0336	0.0446
26	0	0.0002	0.0005	0.0013	0.0029	0.0057	0.0101	0.0164	0.0246	0.0343
27	0	0.0001	0.0002	0.0007	0.0016	0.0034	0.0063	0.0109	0.0173	0.0254
28	0	0	0.0001	0.0003	0.0009	0.0019	0.0038	0.007	0.0117	0.0181
29	0	0	0.0001	0.0002	0.0004	0.0011	0.0023	0.0044	0.0077	0.0125
30	0	0	0	0.0001	0.0002	0.0006	0.0013	0.0026	0.0049	0.0083
31	0	0	0	0	0.0001	0.0003	0.0007	0.0015	0.003	0.0054
32	0	0	0	0	0.0001	0.0001	0.0004	0.0009	0.0018	0.0034
33	0	0	0	0	0	0.0001	0.0002	0.0005	0.001	0.002
34	0	0	0	0	0	0	0.0001	0.0002	0.0006	0.0012
35	0	0	0	0	0	0	0	0.0001	0.0003	0.0007
36	0	0	0	0	0	0	0	0.0001	0.0002	0.0004
37	0	0	0	0	0	0	0	0	0.0001	0.0002
38	0	0	0	0	0	0	0	0	0	0.0001
39	0	0	0	0	0	0	0	0	0	0.0001

TABLE 18	CONSTANTS/FACTORS FOR VARIABLE CONTROL CHARTS

Sample Size (n)	c_4	d_2	d_3	A	A_2	A_3	B_3	B_4	D_1	D_2	D_3	D_4
2	0.798	1.128	0.853	2.121	1.880	2.659	0	3.267	0	3.686	0	3.267
3	0.886	1.693	0.888	1.732	1.023	1.954	0	2.568	0	4.358	0	2.574
4	0.921	2.059	0.880	1.500	0.729	1.628	0	2.266	0	4.698	0	2.282
5	0.940	2.326	0.864	1.342	0.577	1.427	0	2.089	0	4.918	0	2.114
6	0.952	2.534	0.848	1.225	0.483	1.287	0.030	1.970	0	5.078	0	2.004
7	0.959	2.704	0.833	1.134	0.419	1.182	0.118	1.882	0.204	5.204	0.076	1.924
8	0.965	2.847	0.820	1.061	0.373	1.099	0.185	1.815	0.388	5.306	0.136	1.864
9	0.969	2.970	0.808	1.000	0.337	1.032	0.239	1.761	0.547	5.393	0.184	1.816
10	0.973	3.078	0.797	0.949	0.308	0.975	0.284	1.716	0.687	5.469	0.223	1.777
11	0.975	3.173	0.787	0.905	0.285	0.927	0.321	1.679	0.811	5.535	0.256	1.744
12	0.978	3.258	0.778	0.866	0.266	0.886	0.354	1.646	0.922	5.594	0.283	1.717
13	0.979	3.336	0.770	0.832	0.249	0.850	0.382	1.618	1.025	5.647	0.307	1.693
14	0.981	3.407	0.763	0.802	0.235	0.817	0.406	1.594	1.118	5.696	0.328	1.672
15	0.982	3.472	0.756	0.775	0.223	0.789	0.428	1.572	1.203	5.741	0.347	1.653
16	0.984	3.532	0.750	0.750	0.212	0.763	0.448	1.552	1.282	5.782	0.363	1.637
17	0.985	3.588	0.744	0.728	0.203	0.739	0.466	1.534	1.356	5.820	0.378	1.622
18	0.985	3.640	0.739	0.707	0.194	0.718	0.482	1.518	1.424	5.856	0.391	1.608
19	0.986	3.689	0.734	0.688	0.187	0.698	0.497	1.503	1.487	5.891	0.403	1.597
20	0.987	3.735	0.729	0.671	0.180	0.680	0.510	1.490	1.549	5.921	0.415	1.585

TABLE 19 CUMULATIVE BINOMIAL PROBABILITY DISTRIBUTION $\left[P[X \le c] = \sum_{x=0}^{c} {}^{n}c_{x}\, p^{x} (1-p)^{n-x} \right]$

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0.50
1	0	0.9900	0.9800	0.9700	0.9600	0.9500	0.9400	0.9300	0.9200	0.9100	0.9000	0.8900	0.8800	0.8600	0.8400	0.8200	0.8000	0.5000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7921	0.7744	0.7396	0.7056	0.6724	0.6400	0.2500
	1	0.9999	0.9996	0.9991	0.9984	0.9975	0.9964	0.9951	0.9936	0.9919	0.9900	0.9879	0.9856	0.9804	0.9744	0.9676	0.9600	0.7500
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.7050	0.6815	0.6361	0.5927	0.5514	0.5120	0.1250
	1	0.9997	0.9988	0.9974	0.9953	0.9928	0.9896	0.9860	0.9818	0.9772	0.9720	0.9664	0.9603	0.9467	0.9314	0.9145	0.8960	0.5000
	2	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993	0.9990	0.9987	0.9983	0.9973	0.9959	0.9942	0.9920	0.8750
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.6274	0.5997	0.5470	0.4979	0.4521	0.4096	0.0625
	1	0.9994	0.9977	0.9948	0.9909	0.9860	0.9801	0.9733	0.9656	0.9570	0.9477	0.9376	0.9268	0.9032	0.8772	0.8491	0.8192	0.3125
	2	1.0000	1.0000	0.9999	0.9998	0.9995	0.9992	0.9987	0.9981	0.9973	0.9963	0.9951	0.9937	0.9902	0.9856	0.9798	0.9728	0.6875
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9996	0.9993	0.9990	0.9984	0.9375
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.5584	0.5277	0.4704	0.4182	0.3707	0.3277	0.0313
	1	0.9990	0.9962	0.9915	0.9852	0.9774	0.9681	0.9575	0.9456	0.9326	0.9185	0.9035	0.8875	0.8533	0.8165	0.7776	0.7373	0.1875
	2	1.0000	0.9999	0.9997	0.9994	0.9988	0.9980	0.9969	0.9955	0.9937	0.9914	0.9888	0.9857	0.9780	0.9682	0.9563	0.9421	0.5000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993	0.9991	0.9983	0.9971	0.9955	0.9933	0.8125
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9688
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.4970	0.4644	0.4046	0.3513	0.3040	0.2621	0.0156
	1	0.9985	0.9943	0.9875	0.9784	0.9672	0.9541	0.9392	0.9227	0.9048	0.8857	0.8655	0.8444	0.7997	0.7528	0.7044	0.6554	0.1094
	2	1.0000	0.9998	0.9995	0.9988	0.9978	0.9962	0.9942	0.9915	0.9882	0.9842	0.9794	0.9739	0.9605	0.9440	0.9241	0.9011	0.3438
	3	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9982	0.9975	0.9955	0.9925	0.9884	0.9830	0.6563
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9997	0.9995	0.9990	0.9984	0.8906
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9844
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4783	0.4423	0.4087	0.3479	0.2951	0.2493	0.2097	0.0078
	1	0.9980	0.9921	0.9829	0.9706	0.9556	0.9382	0.9187	0.8974	0.8745	0.8503	0.8250	0.7988	0.7444	0.6885	0.6323	0.5767	0.0625
	2	1.0000	0.9997	0.9991	0.9980	0.9962	0.9937	0.9903	0.9860	0.9807	0.9743	0.9669	0.9584	0.9380	0.9134	0.8846	0.8520	0.2266
	3	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9988	0.9982	0.9973	0.9961	0.9946	0.9906	0.9847	0.9769	0.9667	0.5000

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9991	0.9983	0.9971	0.9953	0.7734
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9375
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9922
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	**0**	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.3937	0.3596	0.2992	0.2479	0.2044	0.1678	0.0039
	1	0.9973	0.9897	0.9777	0.9619	0.9428	0.9208	0.8965	0.8702	0.8423	0.8131	0.7829	0.7520	0.6889	0.6256	0.5634	0.5033	0.0352
	2	0.9999	0.9996	0.9987	0.9969	0.9942	0.9904	0.9853	0.9789	0.9711	0.9619	0.9513	0.9392	0.9109	0.8774	0.8392	0.7969	0.1445
	3	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9987	0.9978	0.9966	0.9950	0.9929	0.9903	0.9832	0.9733	0.9603	0.9437	0.3633
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9996	0.9993	0.9990	0.9979	0.9962	0.9935	0.9896	0.6367
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9993	0.9988	0.8555
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9648
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9961
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	**0**	0.9135	0.8337	0.7602	0.6925	0.6302	0.5730	0.5204	0.4722	0.4279	0.3874	0.3504	0.3165	0.2573	0.2082	0.1676	0.1342	0.0020
	1	0.9966	0.9869	0.9718	0.9522	0.9288	0.9022	0.8729	0.8417	0.8088	0.7748	0.7401	0.7049	0.6343	0.5652	0.4988	0.4362	0.0195
	2	0.9999	0.9994	0.9980	0.9955	0.9916	0.9862	0.9791	0.9702	0.9595	0.9470	0.9328	0.9167	0.8798	0.8371	0.7895	0.7382	0.0898
	3	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9977	0.9963	0.9943	0.9917	0.9883	0.9842	0.9731	0.9580	0.9385	0.9144	0.2539
	4	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9991	0.9986	0.9979	0.9959	0.9925	0.9875	0.9804	0.5000
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9991	0.9983	0.9969	0.7461
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9102
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9980
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	**0**	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487	0.3118	0.2785	0.2213	0.1749	0.1374	0.1074	0.0010
	1	0.9957	0.9838	0.9655	0.9418	0.9139	0.8824	0.8483	0.8121	0.7746	0.7361	0.6972	0.6583	0.5816	0.5080	0.4392	0.3758	0.0107
	2	0.9999	0.9991	0.9972	0.9938	0.9885	0.9812	0.9717	0.9599	0.9460	0.9298	0.9116	0.8913	0.8455	0.7936	0.7372	0.6778	0.0547
	3	1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9964	0.9942	0.9912	0.9872	0.9822	0.9761	0.9600	0.9386	0.9117	0.8791	0.1719
	4	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9975	0.9963	0.9927	0.9870	0.9787	0.9672	0.3770
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9996	0.9990	0.9980	0.9963	0.9936	0.6230
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.8281
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9990
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
11	**0**	0.8953	0.8007	0.7153	0.6382	0.5688	0.5063	0.4501	0.3996	0.3544	0.3138	0.2775	0.2451	0.1903	0.1469	0.1127	0.0859	0.0005
	1	0.9948	0.9805	0.9587	0.9308	0.8981	0.8618	0.8228	0.7819	0.7399	0.6974	0.6548	0.6127	0.5311	0.4547	0.3849	0.3221	0.0059
	2	0.9998	0.9988	0.9963	0.9917	0.9848	0.9752	0.9630	0.9481	0.9305	0.9104	0.8880	0.8634	0.8085	0.7479	0.6836	0.6174	0.0327
	3	1.0000	1.0000	0.9998	0.9993	0.9984	0.9970	0.9947	0.9915	0.9871	0.9815	0.9744	0.9659	0.9440	0.9154	0.8803	0.8389	0.1133
	4	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9990	0.9983	0.9972	0.9958	0.9939	0.9881	0.9793	0.9666	0.9496	0.2744
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9982	0.9963	0.9932	0.9883	0.5000
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9980	0.7256
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.8867
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9673
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9941
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	**0**	0.8864	0.7847	0.6938	0.6127	0.5404	0.4759	0.4186	0.3677	0.3225	0.2824	0.2470	0.2157	0.1637	0.1234	0.0924	0.0687	0.0002
	1	0.9938	0.9769	0.9514	0.9191	0.8816	0.8405	0.7967	0.7513	0.7052	0.6590	0.6133	0.5686	0.4834	0.4055	0.3359	0.2749	0.0032
	2	0.9998	0.9985	0.9952	0.9893	0.9804	0.9684	0.9532	0.9348	0.9134	0.8891	0.8623	0.8333	0.7697	0.7010	0.6298	0.5583	0.0193
	3	1.0000	0.9999	0.9997	0.9990	0.9978	0.9957	0.9925	0.9880	0.9820	0.9744	0.9649	0.9536	0.9250	0.8886	0.8448	0.7946	0.0730
	4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9973	0.9957	0.9935	0.9905	0.9819	0.9690	0.9511	0.9274	0.1938
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9991	0.9986	0.9967	0.9935	0.9884	0.9806	0.3872
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9990	0.9979	0.9961	0.6128
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.8062
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9270
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9807
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9968
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	**0**	0.8775	0.7690	0.6730	0.5882	0.5133	0.4474	0.3893	0.3383	0.2935	0.2542	0.2198	0.1898	0.1408	0.1037	0.0758	0.0550	0.0001
	1	0.9928	0.9730	0.9436	0.9068	0.8646	0.8186	0.7702	0.7206	0.6707	0.6213	0.5730	0.5262	0.4386	0.3604	0.2920	0.2336	0.0017
	2	0.9997	0.9980	0.9938	0.9865	0.9755	0.9608	0.9422	0.9201	0.8946	0.8661	0.8349	0.8015	0.7296	0.6537	0.5769	0.5017	0.0112
	3	1.0000	0.9999	0.9995	0.9986	0.9969	0.9940	0.9897	0.9837	0.9758	0.9658	0.9536	0.9391	0.9033	0.8586	0.8061	0.7473	0.0461
	4	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9987	0.9976	0.9959	0.9935	0.9903	0.9861	0.9740	0.9562	0.9319	0.9009	0.1334
	5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9991	0.9985	0.9976	0.9947	0.9896	0.9817	0.9700	0.2905
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9992	0.9981	0.9962	0.9930	0.5000
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9988	0.7095
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.8666
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9539
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9888

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9983
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	**0**	0.8687	0.7536	0.6528	0.5647	0.4877	0.4205	0.3620	0.3112	0.2670	0.2288	0.1956	0.1670	0.1211	0.0871	0.0621	0.0440	0.0001
	1	0.9916	0.9690	0.9355	0.8941	0.8470	0.7963	0.7436	0.6900	0.6368	0.5846	0.5342	0.4859	0.3969	0.3193	0.2531	0.1979	0.0009
	2	0.9997	0.9975	0.9923	0.9833	0.9699	0.9522	0.9302	0.9042	0.8745	0.8416	0.8061	0.7685	0.6889	0.6068	0.5256	0.4481	0.0065
	3	1.0000	0.9999	0.9994	0.9981	0.9958	0.9920	0.9864	0.9786	0.9685	0.9559	0.9406	0.9226	0.8790	0.8258	0.7649	0.6982	0.0287
	4	1.0000	1.0000	1.0000	0.9998	0.9996	0.9990	0.9980	0.9965	0.9941	0.9908	0.9863	0.9804	0.9641	0.9406	0.9093	0.8702	0.0898
	5	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985	0.9976	0.9962	0.9918	0.9843	0.9727	0.9561	0.2120
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9985	0.9968	0.9936	0.9884	0.3953
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9976	0.6047
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.7880
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9102
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9713
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9935
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	**0**	0.8601	0.7386	0.6333	0.5421	0.4633	0.3953	0.3367	0.2863	0.2430	0.2059	0.1741	0.1470	0.1041	0.0731	0.0510	0.0352	0.0000
	1	0.9904	0.9647	0.9270	0.8809	0.8290	0.7738	0.7168	0.6597	0.6035	0.5490	0.4969	0.4476	0.3583	0.2821	0.2187	0.1671	0.0005
	2	0.9996	0.9970	0.9906	0.9797	0.9638	0.9429	0.9171	0.8870	0.8531	0.8159	0.7762	0.7346	0.6480	0.5608	0.4766	0.3980	0.0037
	3	1.0000	0.9998	0.9992	0.9976	0.9945	0.9896	0.9825	0.9727	0.9601	0.9444	0.9258	0.9041	0.8524	0.7908	0.7218	0.6482	0.0176
	4	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9972	0.9950	0.9918	0.9873	0.9813	0.9735	0.9522	0.9222	0.8833	0.8358	0.0592
	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9978	0.9963	0.9943	0.9879	0.9773	0.9613	0.9389	0.1509
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9976	0.9948	0.9898	0.9819	0.3036
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9996	0.9990	0.9979	0.9958	0.5000
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.6964
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.8491
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9408
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9824
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9963
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
16	**0**	0.8515	0.7238	0.6143	0.5204	0.4401	0.3716	0.3131	0.2634	0.2211	0.1853	0.1550	0.1293	0.0895	0.0614	0.0418	0.0281	0.0000
	1	0.9891	0.9601	0.9182	0.8673	0.8108	0.7511	0.6902	0.6299	0.5711	0.5147	0.4614	0.4115	0.3227	0.2487	0.1885	0.1407	0.0003
	2	0.9995	0.9963	0.9887	0.9758	0.9571	0.9327	0.9031	0.8689	0.8306	0.7892	0.7455	0.7001	0.6074	0.5162	0.4302	0.3518	0.0021
	3	1.0000	0.9998	0.9989	0.9968	0.9930	0.9868	0.9779	0.9658	0.9504	0.9316	0.9093	0.8838	0.8237	0.7540	0.6777	0.5981	0.0106
	4	1.0000	1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9889	0.9830	0.9752	0.9652	0.9382	0.9012	0.8542	0.7982	0.0384
	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9981	0.9967	0.9947	0.9918	0.9829	0.9685	0.9473	0.9183	0.1051
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9991	0.9985	0.9962	0.9920	0.9847	0.9733	0.2272
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9993	0.9984	0.9964	0.9930	0.4018
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9985	0.5982
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.7728
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8949
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9616
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9894
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9979
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	**0**	0.8429	0.7093	0.5958	0.4996	0.4181	0.3493	0.2912	0.2423	0.2012	0.1668	0.1379	0.1138	0.0770	0.0516	0.0343	0.0225	0.0000
	1	0.9877	0.9554	0.9091	0.8535	0.7922	0.7283	0.6638	0.6005	0.5396	0.4818	0.4277	0.3777	0.2901	0.2187	0.1621	0.1182	0.0001
	2	0.9994	0.9956	0.9866	0.9714	0.9497	0.9218	0.8882	0.8497	0.8073	0.7618	0.7142	0.6655	0.5676	0.4734	0.3867	0.3096	0.0012
	3	1.0000	0.9997	0.9986	0.9960	0.9912	0.9836	0.9727	0.9581	0.9397	0.9174	0.8913	0.8617	0.7935	0.7159	0.6331	0.5489	0.0064
	4	1.0000	1.0000	0.9999	0.9996	0.9988	0.9974	0.9949	0.9911	0.9855	0.9779	0.9679	0.9554	0.9222	0.8776	0.8225	0.7582	0.0245
	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9985	0.9973	0.9953	0.9925	0.9886	0.9766	0.9577	0.9305	0.8943	0.0717
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9944	0.9882	0.9780	0.9623	0.1662
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9989	0.9973	0.9943	0.9891	0.3145
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9974	0.5000
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.6855
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.8338
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9283
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9755
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9936
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9988
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
18	**0**	0.8345	0.6951	0.5780	0.4796	0.3972	0.3283	0.2708	0.2229	0.1831	0.1501	0.1227	0.1002	0.0662	0.0434	0.0281	0.0180	0.0000
	1	0.9862	0.9505	0.8997	0.8393	0.7735	0.7055	0.6378	0.5719	0.5091	0.4503	0.3958	0.3460	0.2602	0.1920	0.1391	0.0991	0.0001
	2	0.9993	0.9948	0.9843	0.9667	0.9419	0.9102	0.8725	0.8298	0.7832	0.7338	0.6827	0.6310	0.5287	0.4327	0.3462	0.2713	0.0007
	3	1.0000	0.9996	0.9982	0.9950	0.9891	0.9799	0.9667	0.9494	0.9277	0.9018	0.8718	0.8382	0.7618	0.6771	0.5888	0.5010	0.0038
	4	1.0000	1.0000	0.9998	0.9994	0.9985	0.9966	0.9933	0.9884	0.9814	0.9718	0.9595	0.9442	0.9041	0.8518	0.7884	0.7164	0.0154
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9979	0.9962	0.9936	0.9898	0.9846	0.9690	0.9449	0.9111	0.8671	0.0481
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9988	0.9979	0.9966	0.9919	0.9833	0.9694	0.9487	0.1189
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9983	0.9959	0.9914	0.9837	0.2403
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980	0.9957	0.4073
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9991	0.5927
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.7597
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8811
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9519
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9846
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9962
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	**0**	0.8262	0.6812	0.5606	0.4604	0.3774	0.3086	0.2519	0.2051	0.1666	0.1351	0.1092	0.0881	0.0569	0.0364	0.0230	0.0144	0.0000
	1	0.9847	0.9454	0.8900	0.8249	0.7547	0.6829	0.6121	0.5440	0.4798	0.4203	0.3658	0.3165	0.2331	0.1682	0.1191	0.0829	0.0000
	2	0.9991	0.9939	0.9817	0.9616	0.9335	0.8979	0.8561	0.8092	0.7585	0.7054	0.6512	0.5968	0.4911	0.3941	0.3090	0.2369	0.0004
	3	1.0000	0.9995	0.9978	0.9939	0.9868	0.9757	0.9602	0.9398	0.9147	0.8850	0.8510	0.8133	0.7292	0.6380	0.5451	0.4551	0.0022
	4	1.0000	1.0000	0.9998	0.9993	0.9980	0.9956	0.9915	0.9853	0.9765	0.9648	0.9498	0.9315	0.8842	0.8238	0.7524	0.6733	0.0096
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9971	0.9949	0.9914	0.9865	0.9798	0.9599	0.9300	0.8890	0.8369	0.0318
	6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9983	0.9970	0.9952	0.9887	0.9772	0.9589	0.9324	0.0835
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9991	0.9974	0.9939	0.9874	0.9767	0.1796
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9986	0.9968	0.9933	0.3238
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9984	0.5000
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.6762
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8204
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9165
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9682
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9904
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9978
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	**0**	0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516	0.1216	0.0972	0.0776	0.0490	0.0306	0.0189	0.0115	0.0000
	1	0.9831	0.9401	0.8802	0.8103	0.7358	0.6605	0.5869	0.5169	0.4516	0.3917	0.3376	0.2891	0.2084	0.1471	0.1018	0.0692	0.0000
	2	0.9990	0.9929	0.9790	0.9561	0.9245	0.8850	0.8390	0.7879	0.7334	0.6769	0.6198	0.5631	0.4550	0.3580	0.2748	0.2061	0.0002
	3	1.0000	0.9994	0.9973	0.9926	0.9841	0.9710	0.9529	0.9294	0.9007	0.8670	0.8290	0.7873	0.6959	0.5990	0.5026	0.4114	0.0013
	4	1.0000	1.0000	0.9997	0.9990	0.9974	0.9944	0.9893	0.9817	0.9710	0.9568	0.9390	0.9173	0.8625	0.7941	0.7151	0.6296	0.0059
	5	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9887	0.9825	0.9740	0.9493	0.9130	0.8644	0.8042	0.0207
	6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9976	0.9959	0.9933	0.9847	0.9696	0.9463	0.9133	0.0577
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9962	0.9912	0.9823	0.9679	0.1316
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9992	0.9979	0.9951	0.9900	0.2517
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9974	0.4119
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.5881
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.7483
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8684
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9423
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9793
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9941
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
21	**0**	0.8097	0.6543	0.5275	0.4243	0.3406	0.2727	0.2178	0.1736	0.1380	0.1094	0.0865	0.0683	0.0421	0.0257	0.0155	0.0092	0.0000
	1	0.9815	0.9347	0.8701	0.7956	0.7170	0.6382	0.5622	0.4906	0.4246	0.3647	0.3111	0.2637	0.1861	0.1285	0.0869	0.0576	0.0000
	2	0.9988	0.9919	0.9760	0.9503	0.9151	0.8716	0.8213	0.7663	0.7081	0.6484	0.5887	0.5302	0.4205	0.3243	0.2437	0.1787	0.0001
	3	0.9999	0.9993	0.9968	0.9911	0.9811	0.9659	0.9449	0.9181	0.8856	0.8480	0.8060	0.7604	0.6622	0.5604	0.4616	0.3704	0.0007
	4	1.0000	1.0000	0.9997	0.9988	0.9968	0.9930	0.9867	0.9775	0.9646	0.9478	0.9269	0.9017	0.8392	0.7629	0.6769	0.5860	0.0036
	5	1.0000	1.0000	1.0000	0.9999	0.9996	0.9988	0.9975	0.9950	0.9912	0.9856	0.9777	0.9672	0.9372	0.8940	0.8375	0.7693	0.0133
	6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9991	0.9982	0.9967	0.9944	0.9910	0.9797	0.9606	0.9316	0.8915	0.0392
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9994	0.9988	0.9980	0.9945	0.9877	0.9758	0.9569	0.0946
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9988	0.9968	0.9928	0.9856	0.1917
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.3318
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.5000
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.6682

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8083
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9054
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9608
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9867
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9964
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	**0**	0.8016	0.6412	0.5117	0.4073	0.3235	0.2563	0.2026	0.1597	0.1256	0.0985	0.0770	0.0601	0.0362	0.0216	0.0127	0.0074	0.0000
	1	0.9798	0.9290	0.8598	0.7808	0.6982	0.6163	0.5381	0.4652	0.3988	0.3392	0.2864	0.2403	0.1659	0.1120	0.0740	0.0480	0.0000
	2	0.9987	0.9907	0.9728	0.9441	0.9052	0.8576	0.8032	0.7442	0.6826	0.6200	0.5582	0.4983	0.3877	0.2929	0.2154	0.1545	0.0001
	3	0.9999	0.9991	0.9962	0.9895	0.9778	0.9602	0.9362	0.9059	0.8696	0.8281	0.7821	0.7328	0.6283	0.5226	0.4224	0.3320	0.0004
	4	1.0000	0.9999	0.9996	0.9985	0.9960	0.9913	0.9838	0.9727	0.9575	0.9379	0.9136	0.8847	0.8144	0.7305	0.6381	0.5429	0.0022
	5	1.0000	1.0000	1.0000	0.9998	0.9994	0.9985	0.9967	0.9936	0.9888	0.9818	0.9721	0.9593	0.9235	0.8730	0.8086	0.7326	0.0085
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9976	0.9956	0.9926	0.9881	0.9738	0.9499	0.9147	0.8670	0.0262
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9971	0.9925	0.9834	0.9679	0.9439	0.0669
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9994	0.9982	0.9954	0.9898	0.9799	0.1431
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9972	0.9939	0.2617
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.4159
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.5841
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.7383
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8569
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9331
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9738
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9915
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9978
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	**0**	0.7936	0.6283	0.4963	0.3911	0.3074	0.2410	0.1884	0.1469	0.1143	0.0886	0.0685	0.0529	0.0312	0.0181	0.0104	0.0059	0.0000
	1	0.9780	0.9233	0.8493	0.7658	0.6794	0.5947	0.5146	0.4408	0.3742	0.3151	0.2634	0.2186	0.1478	0.0976	0.0630	0.0398	0.0000
	2	0.9985	0.9895	0.9695	0.9376	0.8948	0.8431	0.7846	0.7219	0.6570	0.5920	0.5283	0.4673	0.3566	0.2640	0.1900	0.1332	0.0000

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	3	0.9999	0.9990	0.9955	0.9877	0.9742	0.9541	0.9269	0.8930	0.8528	0.8073	0.7575	0.7047	0.5946	0.4859	0.3851	0.2965	0.0002
	4	1.0000	0.9999	0.9995	0.9981	0.9951	0.9895	0.9805	0.9674	0.9496	0.9269	0.8991	0.8665	0.7884	0.6972	0.5993	0.5007	0.0013
	5	1.0000	1.0000	1.0000	0.9998	0.9992	0.9981	0.9958	0.9920	0.9860	0.9774	0.9656	0.9504	0.9082	0.8502	0.7779	0.6947	0.0053
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9984	0.9968	0.9942	0.9903	0.9847	0.9667	0.9376	0.8956	0.8402	0.0173
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9988	0.9977	0.9960	0.9899	0.9780	0.9583	0.9285	0.0466
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9991	0.9974	0.9934	0.9858	0.9727	0.1050
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9911	0.2024
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9975	0.3388
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.5000
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.6612
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7976
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8950
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9534
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9827
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9947
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	**0**	0.7857	0.6158	0.4814	0.3754	0.2920	0.2265	0.1752	0.1352	0.1040	0.0798	0.0610	0.0465	0.0268	0.0152	0.0085	0.0047	0.0000
	1	0.9761	0.9174	0.8388	0.7508	0.6608	0.5735	0.4918	0.4173	0.3508	0.2925	0.2420	0.1987	0.1315	0.0849	0.0535	0.0331	0.0000
	2	0.9983	0.9882	0.9659	0.9307	0.8841	0.8282	0.7657	0.6994	0.6316	0.5643	0.4992	0.4375	0.3274	0.2374	0.1671	0.1145	0.0000
	3	0.9999	0.9988	0.9947	0.9857	0.9702	0.9474	0.9170	0.8793	0.8352	0.7857	0.7323	0.6762	0.5613	0.4504	0.3500	0.2639	0.0001
	4	1.0000	0.9999	0.9994	0.9977	0.9940	0.9873	0.9767	0.9614	0.9409	0.9149	0.8835	0.8471	0.7612	0.6634	0.5607	0.4599	0.0008
	5	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9947	0.9900	0.9827	0.9723	0.9583	0.9403	0.8914	0.8257	0.7458	0.6559	0.0033
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9979	0.9958	0.9925	0.9876	0.9806	0.9585	0.9236	0.8744	0.8111	0.0113
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9983	0.9969	0.9947	0.9866	0.9716	0.9470	0.9108	0.0320
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9988	0.9963	0.9910	0.9809	0.9638	0.0758
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9991	0.9976	0.9941	0.9874	0.1537
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9994	0.9984	0.9962	0.2706
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.4194
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.5806
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7294
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8463

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9242
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9680
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9887
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9967
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	**0**	0.7778	0.6035	0.4670	0.3604	0.2774	0.2129	0.1630	0.1244	0.0946	0.0718	0.0543	0.0409	0.0230	0.0128	0.0070	0.0038	0.0000
	1	0.9742	0.9114	0.8280	0.7358	0.6424	0.5527	0.4696	0.3947	0.3286	0.2712	0.2221	0.1805	0.1168	0.0737	0.0454	0.0274	0.0000
	2	0.9980	0.9868	0.9620	0.9235	0.8729	0.8129	0.7466	0.6768	0.6063	0.5371	0.4709	0.4088	0.3000	0.2130	0.1467	0.0982	0.0000
	3	0.9999	0.9986	0.9938	0.9835	0.9659	0.9402	0.9064	0.8649	0.8169	0.7636	0.7066	0.6475	0.5286	0.4163	0.3171	0.2340	0.0001
	4	1.0000	0.9999	0.9992	0.9972	0.9928	0.9850	0.9726	0.9549	0.9314	0.9020	0.8669	0.8266	0.7332	0.6293	0.5228	0.4207	0.0005
	5	1.0000	1.0000	0.9999	0.9996	0.9988	0.9969	0.9935	0.9877	0.9790	0.9666	0.9501	0.9291	0.8732	0.7998	0.7125	0.6167	0.0020
	6	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9987	0.9972	0.9946	0.9905	0.9844	0.9757	0.9491	0.9080	0.8512	0.7800	0.0073
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9989	0.9977	0.9959	0.9930	0.9827	0.9639	0.9339	0.8909	0.0216
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9991	0.9983	0.9950	0.9879	0.9748	0.9532	0.0539
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9987	0.9965	0.9917	0.9827	0.1148
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9976	0.9944	0.2122
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9994	0.9985	0.3450
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.5000
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.6550
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7878
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8852
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9461
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9784
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9927
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9980
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
26	**0**	0.7700	0.5914	0.4530	0.3460	0.2635	0.2001	0.1516	0.1144	0.0861	0.0646	0.0483	0.0360	0.0198	0.0107	0.0057	0.0030	0.0000
	1	0.9723	0.9052	0.8172	0.7208	0.6241	0.5323	0.4481	0.3731	0.3076	0.2513	0.2036	0.1637	0.1037	0.0640	0.0385	0.0227	0.0000
	2	0.9978	0.9852	0.9580	0.9160	0.8614	0.7973	0.7272	0.6543	0.5813	0.5105	0.4435	0.3814	0.2743	0.1907	0.1285	0.0841	0.0000
	3	0.9999	0.9983	0.9929	0.9811	0.9613	0.9326	0.8952	0.8499	0.7979	0.7409	0.6807	0.6189	0.4966	0.3838	0.2864	0.2068	0.0000
	4	1.0000	0.9999	0.9991	0.9967	0.9915	0.9823	0.9679	0.9477	0.9211	0.8882	0.8493	0.8051	0.7046	0.5952	0.4858	0.3833	0.0003
	5	1.0000	1.0000	0.9999	0.9995	0.9985	0.9962	0.9920	0.9851	0.9747	0.9601	0.9410	0.9168	0.8536	0.7725	0.6783	0.5775	0.0012
	6	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9965	0.9932	0.9881	0.9806	0.9701	0.9385	0.8906	0.8263	0.7474	0.0047
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9985	0.9970	0.9946	0.9909	0.9780	0.9550	0.9190	0.8687	0.0145
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9976	0.9932	0.9840	0.9674	0.9408	0.0378
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9982	0.9951	0.9886	0.9768	0.0843
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9966	0.9921	0.1635
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9977	0.2786
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.4225
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.5775
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7214
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8365
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9157
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9622
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9855
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9953
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9988
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	**0**	0.7623	0.5796	0.4394	0.3321	0.2503	0.1881	0.1409	0.1053	0.0784	0.0581	0.0430	0.0317	0.0170	0.0090	0.0047	0.0024	0.0000
	1	0.9703	0.8989	0.8063	0.7058	0.6061	0.5123	0.4274	0.3524	0.2876	0.2326	0.1865	0.1484	0.0919	0.0555	0.0326	0.0187	0.0000
	2	0.9976	0.9836	0.9538	0.9082	0.8495	0.7814	0.7076	0.6318	0.5567	0.4846	0.4171	0.3553	0.2504	0.1704	0.1123	0.0718	0.0000
	3	0.9999	0.9981	0.9918	0.9785	0.9563	0.9245	0.8834	0.8342	0.7784	0.7179	0.6546	0.5904	0.4654	0.3529	0.2580	0.1823	0.0000
	4	1.0000	0.9998	0.9989	0.9960	0.9900	0.9793	0.9628	0.9398	0.9100	0.8734	0.8307	0.7827	0.6755	0.5614	0.4499	0.3480	0.0002
	5	1.0000	1.0000	0.9999	0.9994	0.9981	0.9954	0.9903	0.9821	0.9699	0.9529	0.9309	0.9034	0.8327	0.7441	0.6437	0.5387	0.0008
	6	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9979	0.9956	0.9916	0.9853	0.9762	0.9637	0.9266	0.8717	0.7996	0.7134	0.0030
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9991	0.9980	0.9961	0.9931	0.9884	0.9725	0.9447	0.9023	0.8444	0.0096
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9983	0.9968	0.9911	0.9794	0.9587	0.9263	0.0261

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9975	0.9933	0.9848	0.9696	0.0610
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9981	0.9951	0.9890	0.1239
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9986	0.9965	0.2210
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.3506
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.5000
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6494
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7790
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8761
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9390
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9739
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9904
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9970
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
28	**0**	0.7547	0.5680	0.4262	0.3189	0.2378	0.1768	0.1311	0.0968	0.0713	0.0523	0.0383	0.0279	0.0147	0.0076	0.0039	0.0019	0.0000
	1	0.9682	0.8925	0.7953	0.6909	0.5883	0.4929	0.4073	0.3326	0.2688	0.2152	0.1707	0.1344	0.0814	0.0480	0.0276	0.0155	0.0000
	2	0.9973	0.9820	0.9494	0.9001	0.8373	0.7652	0.6880	0.6094	0.5325	0.4594	0.3917	0.3305	0.2282	0.1520	0.0979	0.0612	0.0000
	3	0.9998	0.9978	0.9907	0.9757	0.9509	0.9159	0.8711	0.8180	0.7585	0.6946	0.6285	0.5622	0.4353	0.3237	0.2318	0.1602	0.0000
	4	1.0000	0.9998	0.9987	0.9953	0.9883	0.9760	0.9573	0.9314	0.8982	0.8579	0.8114	0.7597	0.6461	0.5280	0.4153	0.3149	0.0001
	5	1.0000	1.0000	0.9998	0.9993	0.9977	0.9944	0.9884	0.9787	0.9645	0.9450	0.9199	0.8889	0.8107	0.7149	0.6088	0.5005	0.0005
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9974	0.9945	0.9896	0.9821	0.9713	0.9565	0.9135	0.8513	0.7716	0.6784	0.0019
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9988	0.9974	0.9950	0.9912	0.9855	0.9660	0.9330	0.8839	0.8182	0.0063
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9977	0.9958	0.9885	0.9738	0.9486	0.9100	0.0178
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9966	0.9911	0.9801	0.9609	0.0436
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9991	0.9974	0.9933	0.9851	0.0925
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.1725
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9985	0.2858
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.4253
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.5747
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7142
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8275

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9075
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9564
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9822
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9937
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9981
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
29	**0**	0.7472	0.5566	0.4134	0.3061	0.2259	0.1662	0.1219	0.0891	0.0649	0.0471	0.0341	0.0245	0.0126	0.0064	0.0032	0.0015	0.0000
	1	0.9660	0.8860	0.7842	0.6760	0.5708	0.4739	0.3880	0.3138	0.2510	0.1989	0.1562	0.1216	0.0721	0.0416	0.0233	0.0128	0.0000
	2	0.9970	0.9802	0.9447	0.8917	0.8249	0.7489	0.6684	0.5873	0.5087	0.4350	0.3674	0.3069	0.2077	0.1354	0.0853	0.0520	0.0000
	3	0.9998	0.9975	0.9894	0.9726	0.9452	0.9069	0.8583	0.8013	0.7381	0.6710	0.6024	0.5344	0.4064	0.2962	0.2077	0.1404	0.0000
	4	1.0000	0.9997	0.9984	0.9946	0.9864	0.9724	0.9512	0.9223	0.8856	0.8416	0.7912	0.7360	0.6166	0.4953	0.3823	0.2839	0.0001
	5	1.0000	1.0000	0.9998	0.9991	0.9973	0.9933	0.9862	0.9749	0.9585	0.9363	0.9079	0.8734	0.7877	0.6850	0.5740	0.4634	0.0003
	6	1.0000	1.0000	1.0000	0.9999	0.9995	0.9986	0.9967	0.9932	0.9873	0.9784	0.9656	0.9484	0.8991	0.8295	0.7423	0.6429	0.0012
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9984	0.9967	0.9938	0.9890	0.9820	0.9587	0.9199	0.8636	0.7903	0.0041
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9984	0.9970	0.9946	0.9854	0.9673	0.9369	0.8916	0.0121
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9986	0.9955	0.9883	0.9744	0.9507	0.0307
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9988	0.9964	0.9909	0.9803	0.0680
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9972	0.9931	0.1325
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.2291
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.3555
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.5000
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6445
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7709
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8675
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9320
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9693
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9879
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9959
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9988
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997

n	x	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20	0. 50
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	**0**	0.7397	0.5455	0.4010	0.2939	0.2146	0.1563	0.1134	0.0820	0.0591	0.0424	0.0303	0.0216	0.0108	0.0054	0.0026	0.0012	0.0000
	1	0.9639	0.8795	0.7731	0.6612	0.5535	0.4555	0.3694	0.2958	0.2343	0.1837	0.1427	0.1100	0.0638	0.0359	0.0197	0.0105	0.0000
	2	0.9967	0.9783	0.9399	0.8831	0.8122	0.7324	0.6487	0.5654	0.4855	0.4114	0.3442	0.2847	0.1887	0.1204	0.0741	0.0442	0.0000
	3	0.9998	0.9971	0.9881	0.9694	0.9392	0.8974	0.8450	0.7842	0.7175	0.6474	0.5766	0.5071	0.3785	0.2705	0.1856	0.1227	0.0000
	4	1.0000	0.9997	0.9982	0.9937	0.9844	0.9685	0.9447	0.9126	0.8723	0.8245	0.7705	0.7118	0.5871	0.4635	0.3509	0.2552	0.0000
	5	1.0000	1.0000	0.9998	0.9989	0.9967	0.9921	0.9838	0.9707	0.9519	0.9268	0.8951	0.8569	0.7637	0.6547	0.5395	0.4275	0.0002
	6	1.0000	1.0000	1.0000	0.9999	0.9994	0.9983	0.9960	0.9918	0.9848	0.9742	0.9593	0.9394	0.8835	0.8064	0.7120	0.6070	0.0007
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980	0.9959	0.9922	0.9865	0.9779	0.9503	0.9055	0.8418	0.7608	0.0026
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9961	0.9931	0.9816	0.9597	0.9237	0.8713	0.0081
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9981	0.9941	0.9850	0.9677	0.9389	0.0214
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9983	0.9951	0.9880	0.9744	0.0494
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9960	0.9905	0.1002
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9969	0.1808
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.2923
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.4278
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.5722
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7077
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8192
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8998
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9506
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9786
	21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9919
	22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9974
	23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993
	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 20 CUMULATIVE POISSON PROBABILITY DISTRIBUTION $\left[P[X \le c] = \sum_{x=0}^{c} \frac{e^{-\lambda}\lambda^{x}}{\lfloor x} \right]$

x	λ = 0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	1.0000	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2.0**
1	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
2	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
3	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
4	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
5	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
6	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
7	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
8	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 2.1**	**2.2**	**2.3**	**2.4**	**2.5**	**2.6**	**2.7**	**2.8**	**2.9**	**3.0**
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318	0.8153
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713	0.9665
7	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991	0.9989
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 3.1**	**3.2**	**3.3**	**3.4**	**3.5**	**3.6**	**3.7**	**3.8**	**3.9**	**4.0**
0	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202	0.0183
1	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992	0.0916
2	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531	0.2381
3	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532	0.4335
4	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484	0.6288
5	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006	0.7851
6	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995	0.8893
7	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546	0.9489
8	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815	0.9786

x	λ = 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
9	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931	0.9919
10	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977	0.9972
11	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991
12	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 4.1**	**4.2**	**4.3**	**4.4**	**4.5**	**4.6**	**4.7**	**4.8**	**4.9**	**5.0**
0	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074	0.0067
1	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439	0.0404
2	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333	0.1247
3	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793	0.2650
4	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582	0.4405
5	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335	0.6160
6	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767	0.7622
7	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769	0.8666
8	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382	0.9319
9	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717	0.9682
10	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880	0.9863
11	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953	0.9945
12	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983	0.9980
13	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993
14	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 5.1**	**5.2**	**5.3**	**5.4**	**5.5**	**5.6**	**5.7**	**5.8**	**5.9**	**6.0**
0	0.0061	0.0055	0.0050	0.0045	0.0041	0.0037	0.0033	0.0030	0.0027	0.0025
1	0.0372	0.0342	0.0314	0.0289	0.0266	0.0244	0.0224	0.0206	0.0189	0.0174
2	0.1165	0.1088	0.1016	0.0948	0.0884	0.0824	0.0768	0.0715	0.0666	0.0620
3	0.2513	0.2381	0.2254	0.2133	0.2017	0.1906	0.1800	0.1700	0.1604	0.1512
4	0.4231	0.4061	0.3895	0.3733	0.3575	0.3422	0.3272	0.3127	0.2987	0.2851
5	0.5984	0.5809	0.5635	0.5461	0.5289	0.5119	0.4950	0.4783	0.4619	0.4457
6	0.7474	0.7324	0.7171	0.7017	0.6860	0.6703	0.6544	0.6384	0.6224	0.6063
7	0.8560	0.8449	0.8335	0.8217	0.8095	0.7970	0.7841	0.7710	0.7576	0.7440
8	0.9252	0.9181	0.9106	0.9027	0.8944	0.8857	0.8766	0.8672	0.8574	0.8472
9	0.9644	0.9603	0.9559	0.9512	0.9462	0.9409	0.9352	0.9292	0.9228	0.9161
10	0.9844	0.9823	0.9800	0.9775	0.9747	0.9718	0.9686	0.9651	0.9614	0.9574
11	0.9937	0.9927	0.9916	0.9904	0.9890	0.9875	0.9859	0.9841	0.9821	0.9799
12	0.9976	0.9972	0.9967	0.9962	0.9955	0.9949	0.9941	0.9932	0.9922	0.9912
13	0.9992	0.9990	0.9988	0.9986	0.9983	0.9980	0.9977	0.9973	0.9969	0.9964
14	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991	0.9990	0.9988	0.9986
15	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9996	0.9996	0.9995
16	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 6.1**	**6.2**	**6.3**	**6.4**	**6.5**	**6.6**	**6.7**	**6.8**	**6.9**	**7.0**
0	0.0022	0.0020	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.0010	0.0009
1	0.0159	0.0146	0.0134	0.0123	0.0113	0.0103	0.0095	0.0087	0.0080	0.0073
2	0.0577	0.0536	0.0498	0.0463	0.0430	0.0400	0.0371	0.0344	0.0320	0.0296
3	0.1425	0.1342	0.1264	0.1189	0.1118	0.1052	0.0988	0.0928	0.0871	0.0818
4	0.2719	0.2592	0.2469	0.2351	0.2237	0.2127	0.2022	0.1920	0.1823	0.1730
5	0.4298	0.4141	0.3988	0.3837	0.3690	0.3547	0.3406	0.3270	0.3137	0.3007
6	0.5902	0.5742	0.5582	0.5423	0.5265	0.5108	0.4953	0.4799	0.4647	0.4497

X	λ = 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
7	0.7301	0.7160	0.7017	0.6873	0.6728	0.6581	0.6433	0.6285	0.6136	0.5987
8	0.8367	0.8259	0.8148	0.8033	0.7916	0.7796	0.7673	0.7548	0.7420	0.7291
9	0.9090	0.9016	0.8939	0.8858	0.8774	0.8686	0.8596	0.8502	0.8405	0.8305
10	0.9531	0.9486	0.9437	0.9386	0.9332	0.9274	0.9214	0.9151	0.9084	0.9015
11	0.9776	0.9750	0.9723	0.9693	0.9661	0.9627	0.9591	0.9552	0.9510	0.9467
12	0.9900	0.9887	0.9873	0.9857	0.9840	0.9821	0.9801	0.9779	0.9755	0.9730
13	0.9958	0.9952	0.9945	0.9937	0.9929	0.9920	0.9909	0.9898	0.9885	0.9872
14	0.9984	0.9981	0.9978	0.9974	0.9970	0.9966	0.9961	0.9956	0.9950	0.9943
15	0.9994	0.9993	0.9992	0.9990	0.9988	0.9986	0.9984	0.9982	0.9979	0.9976
16	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9993	0.9992	0.9990
17	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996
18	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 7.1**	**7.2**	**7.3**	**7.4**	**7.5**	**7.6**	**7.7**	**7.8**	**7.9**	**8.0**
0	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003
1	0.0067	0.0061	0.0056	0.0051	0.0047	0.0043	0.0039	0.0036	0.0033	0.0030
2	0.0275	0.0255	0.0236	0.0219	0.0203	0.0188	0.0174	0.0161	0.0149	0.0138
3	0.0767	0.0719	0.0674	0.0632	0.0591	0.0554	0.0518	0.0485	0.0453	0.0424
4	0.1641	0.1555	0.1473	0.1395	0.1321	0.1249	0.1181	0.1117	0.1055	0.0996
5	0.2881	0.2759	0.2640	0.2526	0.2414	0.2307	0.2203	0.2103	0.2006	0.1912
6	0.4349	0.4204	0.4060	0.3920	0.3782	0.3646	0.3514	0.3384	0.3257	0.3134
7	0.5838	0.5689	0.5541	0.5393	0.5246	0.5100	0.4956	0.4812	0.4670	0.4530
8	0.7160	0.7027	0.6892	0.6757	0.6620	0.6482	0.6343	0.6204	0.6065	0.5925
9	0.8202	0.8096	0.7988	0.7877	0.7764	0.7649	0.7531	0.7411	0.7290	0.7166
10	0.8942	0.8867	0.8788	0.8707	0.8622	0.8535	0.8445	0.8352	0.8257	0.8159
11	0.9420	0.9371	0.9319	0.9265	0.9208	0.9148	0.9085	0.9020	0.8952	0.8881
12	0.9703	0.9673	0.9642	0.9609	0.9573	0.9536	0.9496	0.9454	0.9409	0.9362
13	0.9857	0.9841	0.9824	0.9805	0.9784	0.9762	0.9739	0.9714	0.9687	0.9658
14	0.9935	0.9927	0.9918	0.9908	0.9897	0.9886	0.9873	0.9859	0.9844	0.9827
15	0.9972	0.9969	0.9964	0.9959	0.9954	0.9948	0.9941	0.9934	0.9926	0.9918
16	0.9989	0.9987	0.9985	0.9983	0.9980	0.9978	0.9974	0.9971	0.9967	0.9963
17	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9989	0.9988	0.9986	0.9984
18	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9993
19	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 8.1**	**8.2**	**8.3**	**8.4**	**8.5**	**8.6**	**8.7**	**8.8**	**8.9**	**9.0**
0	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
1	0.0028	0.0025	0.0023	0.0021	0.0019	0.0018	0.0016	0.0015	0.0014	0.0012
2	0.0127	0.0118	0.0109	0.0100	0.0093	0.0086	0.0079	0.0073	0.0068	0.0062
3	0.0396	0.0370	0.0346	0.0323	0.0301	0.0281	0.0262	0.0244	0.0228	0.0212
4	0.0940	0.0887	0.0837	0.0789	0.0744	0.0701	0.0660	0.0621	0.0584	0.0550
5	0.1822	0.1736	0.1653	0.1573	0.1496	0.1422	0.1352	0.1284	0.1219	0.1157
6	0.3013	0.2896	0.2781	0.2670	0.2562	0.2457	0.2355	0.2256	0.2160	0.2068
7	0.4391	0.4254	0.4119	0.3987	0.3856	0.3728	0.3602	0.3478	0.3357	0.3239
8	0.5786	0.5647	0.5507	0.5369	0.5231	0.5094	0.4958	0.4823	0.4689	0.4557
9	0.7041	0.6915	0.6788	0.6659	0.6530	0.6400	0.6269	0.6137	0.6006	0.5874
10	0.8058	0.7955	0.7850	0.7743	0.7634	0.7522	0.7409	0.7294	0.7178	0.7060
11	0.8807	0.8731	0.8652	0.8571	0.8487	0.8400	0.8311	0.8220	0.8126	0.8030
12	0.9313	0.9261	0.9207	0.9150	0.9091	0.9029	0.8965	0.8898	0.8829	0.8758
13	0.9628	0.9595	0.9561	0.9524	0.9486	0.9445	0.9403	0.9358	0.9311	0.9261
14	0.9810	0.9791	0.9771	0.9749	0.9726	0.9701	0.9675	0.9647	0.9617	0.9585
15	0.9908	0.9898	0.9887	0.9875	0.9862	0.9848	0.9832	0.9816	0.9798	0.9780

x	λ = 8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
16	0.9958	0.9953	0.9947	0.9941	0.9934	0.9926	0.9918	0.9909	0.9899	0.9889
17	0.9982	0.9979	0.9977	0.9973	0.9970	0.9966	0.9962	0.9957	0.9952	0.9947
18	0.9992	0.9991	0.9990	0.9989	0.9987	0.9985	0.9983	0.9981	0.9978	0.9976
19	0.9997	0.9997	0.9996	0.9995	0.9995	0.9994	0.9993	0.9992	0.9991	0.9989
20	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996
21	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 9.1**	**9.2**	**9.3**	**9.4**	**9.5**	**9.6**	**9.7**	**9.8**	**9.9**	**10.0**
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
1	0.0011	0.0010	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005	0.0005
2	0.0058	0.0053	0.0049	0.0045	0.0042	0.0038	0.0035	0.0033	0.0030	0.0028
3	0.0198	0.0184	0.0172	0.0160	0.0149	0.0138	0.0129	0.0120	0.0111	0.0103
4	0.0517	0.0486	0.0456	0.0429	0.0403	0.0378	0.0355	0.0333	0.0312	0.0293
5	0.1098	0.1041	0.0986	0.0935	0.0885	0.0838	0.0793	0.0750	0.0710	0.0671
6	0.1978	0.1892	0.1808	0.1727	0.1649	0.1574	0.1502	0.1433	0.1366	0.1301
7	0.3123	0.3010	0.2900	0.2792	0.2687	0.2584	0.2485	0.2388	0.2294	0.2202
8	0.4426	0.4296	0.4168	0.4042	0.3918	0.3796	0.3676	0.3558	0.3442	0.3328
9	0.5742	0.5611	0.5479	0.5349	0.5218	0.5089	0.4960	0.4832	0.4705	0.4579
10	0.6941	0.6820	0.6699	0.6576	0.6453	0.6329	0.6205	0.6080	0.5955	0.5830
11	0.7932	0.7832	0.7730	0.7626	0.7520	0.7412	0.7303	0.7193	0.7081	0.6968
12	0.8684	0.8607	0.8529	0.8448	0.8364	0.8279	0.8191	0.8101	0.8009	0.7916
13	0.9210	0.9156	0.9100	0.9042	0.8981	0.8919	0.8853	0.8786	0.8716	0.8645
14	0.9552	0.9517	0.9480	0.9441	0.9400	0.9357	0.9312	0.9265	0.9216	0.9165
15	0.9760	0.9738	0.9715	0.9691	0.9665	0.9638	0.9609	0.9579	0.9546	0.9513
16	0.9878	0.9865	0.9852	0.9838	0.9823	0.9806	0.9789	0.9770	0.9751	0.9730
17	0.9941	0.9934	0.9927	0.9919	0.9911	0.9902	0.9892	0.9881	0.9870	0.9857
18	0.9973	0.9969	0.9966	0.9962	0.9957	0.9952	0.9947	0.9941	0.9935	0.9928
19	0.9988	0.9986	0.9985	0.9983	0.9980	0.9978	0.9975	0.9972	0.9969	0.9965
20	0.9995	0.9994	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9986	0.9984
21	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0.9994	0.9993
22	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997
23	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**λ = 11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000	0.0000
5	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002	0.0001
6	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005	0.0003
7	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015	0.0008
8	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039	0.0021
9	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089	0.0050
10	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183	0.0108
11	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347	0.0214
12	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606	0.0390
13	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984	0.0661
14	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497	0.1049
15	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148	0.1565
16	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920	0.2211
17	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784	0.2970

x	λ = 11	12	13	14	15	16	17	18	19	20
18	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695	0.3814
19	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606	0.4703
20	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472	0.5591
21	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255	0.6437
22	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931	0.7206
23	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490	0.7875
24	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933	0.8432
25	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269	0.8878
26	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718	0.9514	0.9221
27	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827	0.9687	0.9475
28	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897	0.9805	0.9657
29	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9941	0.9882	0.9782
30	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9967	0.9930	0.9865
31	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9982	0.9960	0.9919
32	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978	0.9953
33	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9973
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9985
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 21	RANDOM NUMBER TABLE

03339	19233	50911	14209	39594	68368	97742	36252	27671	55091
97971	19968	31709	40197	16313	80020	01588	21654	50328	04577
16779	47712	33846	84716	49870	59670	46946	71716	50623	38681
12675	95993	08790	13241	71260	16558	83316	68482	10294	45137
55804	72742	16237	72550	10570	31470	92612	94917	48822	79794
16835	56263	53062	71543	67632	30337	28739	17582	40924	32434
84544	14327	07580	48813	30161	10746	96470	60680	63507	14435
63230	41243	90765	08867	08033	05038	10908	00633	21740	55450
33564	93563	10770	10595	71323	84243	09402	62877	49762	56151
57461	55618	40570	72906	30794	49144	65239	21788	38288	29180
91645	42451	83776	99246	45548	02457	74804	49536	89815	74285
78305	63797	26995	23146	56071	97081	22376	09819	56855	97424
97888	55122	65545	02904	40042	70653	24483	31258	96475	77668
67286	09001	09718	67231	54033	24185	52097	78713	95910	84400
53610	59459	89945	72102	66595	02198	26968	88467	46939	52318
52965	76189	68892	64541	02225	09603	59304	38179	75920	80486
25336	39735	25594	50557	96257	59700	27715	42432	27652	88151
73078	44371	77616	49296	55882	71507	30168	31876	28283	53424
31797	52244	38354	47800	48454	43304	14256	74281	82279	28882
47772	22798	36910	39986	34033	39868	24009	97123	59151	27583
54153	70832	37575	31898	39212	63993	05419	77565	73150	98537
93745	99871	37129	55032	94444	17884	27082	23502	06136	89476
81686	51330	58828	74199	87214	13727	80539	95037	73536	16862
79788	02193	33250	05865	53018	62394	56997	41534	01953	13763
92112	61235	68760	61201	02189	09424	24156	10368	26257	89107
87542	28171	45150	75523	66790	63963	13903	68498	02891	25219
37535	48342	48943	07719	20407	33748	93650	39356	01011	22099
95957	96668	69380	49091	90182	13205	71802	35482	27973	46814
34642	85350	53361	63940	79546	89956	96836	91313	80712	73572
50413	31008	09231	46516	61672	79954	01291	72278	55658	84893
53312	73768	59931	55182	43761	59424	79775	17772	41552	45236
16302	64092	76045	28958	21182	30050	96256	85737	86962	27067
96357	98654	01909	58799	87374	53184	87233	55275	59572	56476
38529	89095	89538	15600	33687	86353	61917	63876	52367	79032
45939	05014	06099	76041	57638	55342	41269	96173	94872	35605
02300	23739	68485	98567	77035	91533	62500	31548	09511	80252
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21285	53607	82657	22053	88931	84439	94747	77982	61932	21928
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13263	92240	89559	66545	06433	38634	36645	22350	81169	97417
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56348	05291	38713	82303	26293	61319	45285	72784	50043	44438

93108	77033	68325	10160	38667	62441	87023	94372	06164	30700
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21841	35545	11148	25255	50283	94037	57463	92925	12042	91414
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22604	56719	21784	68788	38358	59827	19270	99287	81193	43366
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63172	28010	29405	91554	75195	51183	65805	87525	35952	83204
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93185	69446	75253	51915	97839	75427	90685	60352	96288	34248
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55768	09210	52439	33355	57884	36791	00853	49969	74814	09270
38080	49460	48137	61589	42742	92035	21766	19435	92579	27683
22360	16332	05343	34613	24013	98831	17157	44089	07366	66196
40521	09057	00239	51284	71556	22605	41293	54854	39736	05113
19292	40078	06838	05509	68581	39400	85615	52314	83202	40313
64138	27983	84048	42635	58658	62243	82572	45211	37060	15017