
UNIT 13 INTRODUCTION TO RELIABILITY

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13.1 INTRODUCTION

Due to the rapid advancement of science and technology and to meet the ever-increasing needs of the society, industries are increasingly introducing automation for producing goods ranging from the simplest to highly complex systems. However, all processes and products are prone to failure. For example, a television remote control may stop functioning or a malfunction may occur in any other household appliance; an automobile engine starter may fail, an aeroplane may crash due to the failure of some of its component(s), credit card transactions may fail, and so on. You can think of many more similar examples from your daily life. So we would like to understand failures, present them in a different manner and see how we can try to prevent the interruption due to failure! Reliability is nothing but a science of better presentation of failure. For example, the statement that 20% students of the class failed the test can be better presented as 80% students of the class passed the test!

The mathematical foundations of reliability were laid down when queueing theory and renewal theory were developed in the early 1930s and 1940s. You will study some basic concepts of queueing theory in Unit 6 of MSTE-002. However, reliability as a subject gained importance only after World War II because a high failure rate of electronic equipment was observed during the war. The first journal on the subject, Institute of Electrical and Electronics Engineers (IEEE) – Transactions on Reliability, came out in 1963. A number of textbooks on the subject were published in the 1960s.

We give a formal definition of reliability in Sec. 13.2. Four basic functions in reliability, namely, the reliability function, cumulative failure distribution function, failure density function and the hazard rate and their relations are described in Sec. 13.3. We also explain the terms mean time to failure and median of the random variable, time to failure in Sec. 13.3. When a population of identical components is tested, the curve of the relative failure rate of the entire population of the components without replacement is generally of the shape of a bathtub. This curve, called the bathtub curve, is discussed in

It is obvious that without the possibility of one type of failure or another insurance industry would not exist. The science which deals with the concepts used in the insurance industry is known as actuarial science. That is why many concepts of reliability and actuarial science are the same except the terminology used there.

Sec. 13.4. We explain how the basic reliability functions can be estimated from failure data in Sec. 13.5.

In the next unit, we shall discuss reliability evaluation of simple systems.

Objectives

After studying this unit, you should be able to:

- define reliability;
- explain the basic functions in reliability, namely, reliability function, cumulative failure distribution function, failure density function and hazard rate;
- explain three phases of the bathtub curve; and
- estimate basic reliability functions from complete failure data.

13.2 DEFINITION OF RELIABILITY

In articles on reliability, many authors suggest that the word ‘reliability’ was first coined by the poet Samuel Taylor Coleridge when he wrote a piece in praise of his friend, the poet Robert Southey in 1816. However, reliability as a subject of study was taken up only after World War II when high failure rate of electronic equipment used in the war was observed. Today it has taken a new shape by blending itself in all phases of the product life cycle from proposal and design to manufacturing. So in the literature on reliability we find that the definitions of reliability vary from one practitioner or researcher to another. Still a comprehensive and widely accepted definition of reliability may be stated as follows:

Definition of Reliability: Reliability of a component/device/equipment/unit/system is the **probability** that it performs its **intended function adequately** for a specified period of **time** under the **given operating conditions**.

To understand what this definition actually means, we have to clearly understand the meaning of the above-mentioned bold words, which are the key words in this definition. Let us explain what we mean by these key words:

Probability: When a number of identical components operate under similar conditions, the failure times of components generally vary from component to component and we cannot predict their failure times exactly in advance. But we can describe the phenomenon of failure in probabilistic terms. For example, suppose we put N_0 identical components in operation at time $t = 0$ under identical conditions. Suppose after t units of time, we have

$N_s(t)$ = number of components that are operating at time t

$N_f(t)$ = number of components that have failed at time t

Then reliability $R(t)$ at time t is defined as

$$R(t) = \frac{N_s(t)}{N_0} \quad \dots (1)$$

Thus, reliability of a component/system at time t is the probability that it performs its function without failure. Being a probability, its value lies between 0 and 1. From equation (1), we can see that

if $N_s(t) = 0$, $R(t) = 0$ and

if $N_s(t) = N_0, R(t) = 1$.

By definition $N_s(t)$ satisfies the following relation:

$$0 \leq N_s(t) \leq N_0 \quad \text{for any time interval } [0, t]$$

$$\Rightarrow \frac{0}{N_0} \leq \frac{N_s(t)}{N_0} \leq \frac{N_0}{N_0} \quad [\text{on dividing by } N_0]$$

$$\Rightarrow 0 \leq R(t) \leq 1$$

What does this mean? Suppose that the reliability of the refrigerator of a particular company at 10 years is $R(t) = 0.9452$. Then this can be interpreted as follows: Out of 10000 refrigerators of this particular company, approximately 9452 refrigerators worked without failure for 10 years.

Intended Function: The intended function of a product must be described in unambiguous terms early on in the design process so that the expected requirements of the customers can be ensured/addressed. Thus, **intended function** of a product is the function/work/job which it is expected to perform when we put it in operation under stated conditions. For example, if a pump is designed to deliver at least 300 gallons of water per minute, the intended function of the pump is to deliver 300 gallons or more water per minute.

Adequately: A product is said to perform **adequately** if it is performing its intended function as expected. For example, if the intended function of a water pump is to deliver 300 gallons or more water and it is delivering 300 gallons or more water per minute, it means that it is performing adequately. But if the pump delivers less than 300 gallons of water per minute we say that it has failed or is not performing adequately.

Time: You should always keep in mind that reliability can be meaningful only if it is related to a time interval or period of time. For example, we can say that the reliability of a component is 0.98 for a mission time of 100 hours. But the statement 'reliability of a component is 0.95' is meaningless since the time interval is unknown. From equation (1), reliability at time 't' can be defined as follows:

$$R(t) = \frac{\text{Number of components performing intended function at time 't'}}{\text{Number of component at start (i.e., when 't' = 0)}} \quad \dots (2)$$

In the above definition, time 't' means the interval $[0, t]$.

In equation (2), $R(0) = 1$ since no component fails at $t = 0$, and $R(t \rightarrow \infty) = 0$ since no component can survive forever. This point will become clearer to you in the next section when we discuss the wear out period of a component.

Given Operating Condition: By given operating condition, we mean that the performance of the product should be observed under normal stated conditions in which it is expected to perform. The environmental conditions (such as temperature, humidity, shock, vibration, altitude, etc.), design loads (such as voltages, pressure, etc.) and operating conditions (such as maintenance, storage, etc.) affect the reliability of a product. So a product will perform better in the field if its design takes into account and is representative of how it is actually used by the customers. For example, a car which is designed for smooth roads will not perform well if a customer uses it on rough roads.

You may like to try the following exercises to check your understanding of the definition of reliability.

[illegible]

B) Intended function

D) Certainty

E2) The maximum possible value of reliability for a given period of time is

A) 1 B) 0 C) 0.1 D) 100

D) 100

13.3 BASIC FUNCTIONS IN RELIABILITY AND THEIR RELATIONSHIP

The goal of this section is to define the following basic functions and explain the relationship between them:

- Reliability function,
- Cumulative failure distribution function,
- Failure density function, and
- Hazard rate function.

In this section, we also obtain expressions for the mean and median of the random variable called the **time to failure** or failure time.

Let us explain these functions one at a time.

13.3.1 Reliability Function

In Sec. 13.2, you have learnt that reliability is a probability. If reliability is a probability, then a random variable has to be associated with it because probability is always associated with a random variable. You are familiar with the concept of a random variable as it has been discussed in detail in the course MST-003. In MST-003, you have also studied some discrete as well as continuous probability distributions. Now, can you guess what the random variable in the case of reliability is? Think for a while.

We hope that you have given this a thought and arrived at the following idea: Generally, the random variable in the case of reliability is generally time – the time in which a component/system fails. Let this random variable, called **time to failure** or **failure time** of the component/system, be denoted by T. Then, by definition, reliability [R(t)] at time t, of the component/system is given by

$R(t)$ = Probability that the time to failure of the component/system is $> t$

$$= P[T > t] \geq 0 \quad \left[\begin{array}{l} \because \text{probability of any event A} \\ \text{always satisfies } 0 \leq P(A) \leq 1 \end{array} \right] \dots (3)$$

The reliability of a component/system when expressed in terms of time t is known as **reliability function** and is generally denoted by $R(t)$.

Let us consider two particular cases of the reliability function:
when $t = 0$ and when $t \rightarrow \infty$.

$$R(0) = P[T > 0] = 1 \quad \left[\because \text{we assume that at } t = 0, \text{ the component / system is performing its intended function.} \right]$$

$$R(t \rightarrow \infty) = P[T > t(t \rightarrow \infty)] = 0 \left[\begin{array}{l} \because \text{no component / system can perform} \\ \text{its intended function forever.} \end{array} \right]$$

The probability of survival of a component decreases as the life time of the component increases. Ultimately this probability will approach zero, since no

component can perform its intended function forever. A typical shape of the reliability function is shown in Fig. 13.1.

From Fig. 13.1, you can see that the reliability function is a non-increasing function in time (t).

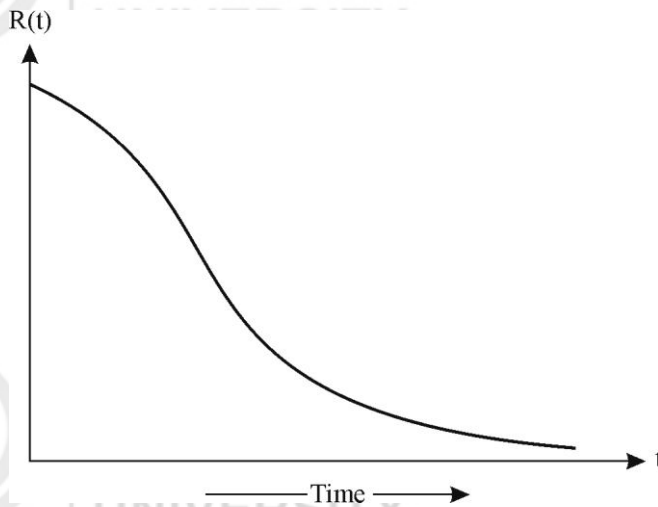


Fig. 13.1: Typical shape of reliability function $R(t)$.

13.3.2 Cumulative Failure Distribution Function

Let us define the function $F(t)$ by

$$F(t) = 1 - R(t) \quad \dots (4)$$

$$\therefore F(0) = 1 - R(0) = 1 - 1 = 0$$

$$\text{and } F(t)_{t \rightarrow \infty} = 1 - R(t)_{t \rightarrow \infty} = 1 - 0 = 1$$

Using equation (3), we can write equation (4) as

$$\begin{aligned} F(t) &= 1 - R(t) = 1 - P[X > t] \\ &= P[X \leq t] \end{aligned} \quad \left[\begin{array}{l} \because \text{from Unit 5 of MST-003, you know that} \\ \text{events } P[X > t] \text{ and } P[X \leq t] \text{ are complementary} \\ \text{to each other. Hence, } P[X > t] + P[X \leq t] = 1. \end{array} \right]$$

$$\text{or } F(t) = P[T \leq t] \quad \dots (5)$$

$$\text{Also, as shown above } F(0) = 0, F(\infty) = 1 \quad \dots (6)$$

Study equations (5) and (6). Can you identify a probability distribution function given in Unit 5 of MST-003, which has similar characteristics? Recall that the cumulative distribution function has these characteristics. But in reliability terminology, this distribution function is known as **cumulative failure distribution function** and gives **unreliability** of the component up to time t . So if we want to calculate the probability of failure of a component at time t (known as **unreliability** of the component), then we have to simply obtain the value of the function $F(t)$. Therefore, if we denote reliability and unreliability of a component by R and Q , respectively, then they satisfy the relation:

$$R + Q = 1 \quad \dots (7)$$

Note that the probability of failure of a component increases after its useful life period is over. Ultimately this probability will approach 1. A typical shape of cumulative failure distribution is shown in Fig. 13.2.

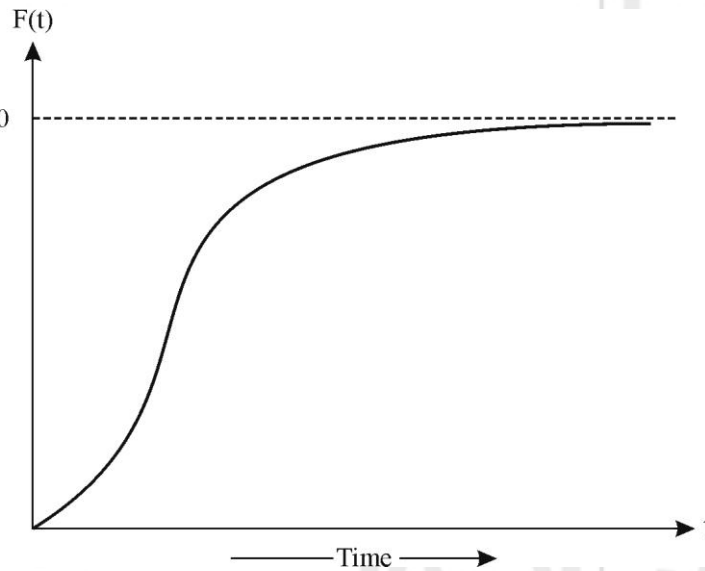


Fig. 13.2: Typical shape of cumulative failure distribution.

13.3.3 Failure Density Function

You have learnt in Unit 5 of MST-003 that the derivative of cumulative distribution function of a random variable is known as the probability density function (pdf) of the random variable. But in the terminology of reliability, the probability density function is known as **failure density function** (fdf). As explained in Sec. 13.3.2, the cumulative distribution function in reliability terminology is the cumulative failure distribution function. So if $f(t)$ denotes failure density function (life time density function), then

$$f(t) = \frac{d}{dt}(F(t)) \quad \dots (8a)$$

From Unit 5 of MST-003, recall that the following relationship exists between $f(t)$ and $F(t)$:

$$F(t) = \int_{-\infty}^t f(t)dt \quad \dots (8b)$$

Since t represents time and so $t \geq 0$, we can write equation (8b) as

$$F(t) = \int_0^t f(t)dt \quad \dots (9)$$

From equation (4), we have

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t)dt \quad \dots (10a)$$

Since $f(t)$ is the pdf of the random variable T , $\int_0^{\infty} f(t)dt = 1$, i.e., the sum of all probabilities of a random variable is always equal to 1. So we can replace 1 in equation (10a) by the integral $\int_0^{\infty} f(t)dt$ and write

$$R(t) = \int_0^{\infty} f(t)dt - \int_0^t f(t)dt \quad \dots (10b)$$

$$R(t) = \left[\int_0^t f(t)dt + \int_t^\infty f(t)dt \right] - \int_0^t f(t)dt \left[\because \text{from Unit 8 of MST-001, we know that} \right. \\ \left. \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, a < c < b \right]$$

or $R(t) = \int_t^\infty f(t)dt$... (10c)

Thus, we have

$$F(t) = \int_0^t f(t)dt \quad \dots (11a)$$

and $R(t) = \int_t^\infty f(t)dt \quad \dots (11b)$

If use of t within the integral sign as well as in the limits of the integral creates confusion for you, then the variable t inside the integral sign can be changed to any other dummy variable, say u . Then we can write $F(t)$ and $R(t)$ as:

$$F(t) = \int_0^t f(u)du \quad \dots (11c)$$

and $R(t) = \int_t^\infty f(u)du \quad \dots (11d)$

These functions may be interpreted geometrically as areas under the curve of $f(t)$ as shown in Fig 13.3.

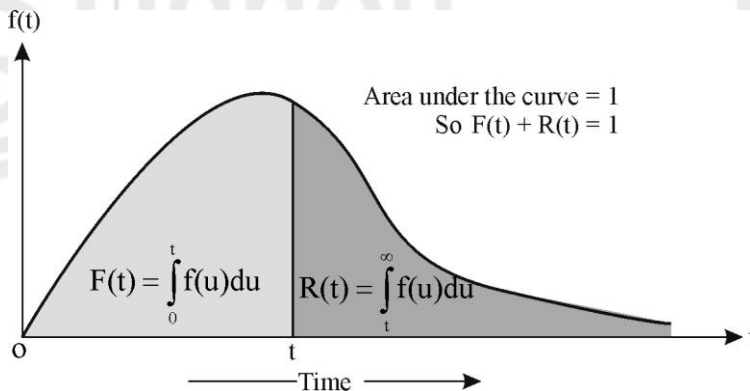


Fig. 13.3: Typical shape of failure density function and geometrical interpretation of $F(t)$ and $R(t)$.

13.3.4 Hazard Rate

The instantaneous rate of failure at time t is known as the **hazard rate** and is generally denoted by $\lambda(t)$.

To define it mathematically, let T denote the random variable, time to failure of the component/system. Let us consider the time interval $[t, t + \Delta t]$. Then, we have

$$P[t \leq T \leq t + \Delta t] = F(t + \Delta t) - F(t) \quad \left[\because P[a \leq X \leq b] = F(b) - F(a) \text{ as you} \right. \\ \left. \text{have learnt in Unit 5 of MST-003} \right] \\ = [1 - R(t + \Delta t)] - [1 - R(t)] \quad [\because R(t) + F(t) = 1] \\ = R(t) - R(t + \Delta t) \quad \dots (12)$$

Conditional probability of failure in the time interval $[t, t + \Delta t]$ given that the component/system has performed its intended function till time t is given by

$$P[t \leq T \leq t + \Delta t | T > t] = \frac{P[t \leq T \leq t + \Delta t]}{P[T > t]}$$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$. If you want to refresh your memory, you may refer to Unit 3 of MST-003.

Using equation (12) and $R(t) = P[T > t]$, we get

$$P[t \leq T \leq t + \Delta t | T > t] = \frac{R(t) - R(t + \Delta t)}{R(t)}$$

Then the conditional probability of failure per unit time is given by

$$\frac{R(t) - R(t + \Delta t)}{R(t)\Delta t},$$

and instantaneous rate of failure at time t (i.e., hazard rate $\lambda(t)$) is given by

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{R(t) - R(t + \Delta t)}{R(t)\Delta t} = -\frac{1}{R(t)} \lim_{\Delta t \rightarrow 0} \frac{R(t + \Delta t) - R(t)}{\Delta t}$$

$$= -\frac{1}{R(t)} \frac{d}{dt}(R(t)) \quad \left[\text{on using the definition of derivative; refer to Unit 6 of MST-001.} \right]$$

$$\text{Since } f(t) = \frac{d}{dt}(F(t)) = \frac{d}{dt}(1 - R(t)) = -\frac{d}{dt}(R(t)),$$

$$\lambda(t) = \frac{-\frac{d}{dt}(R(t))}{R(t)} = \frac{f(t)}{R(t)} \quad \dots (13)$$

13.3.5 Relationship between the Functions $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$

So far we have defined the functions $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$. In this section, you will learn how these functions can be obtained from the failure data collected by testing a fixed number of identical components. And finally, we form a table showing the relationship between them, which will be very useful when we solve numerical problems.

Suppose that initially a fixed number (N_0) of identical components are put to test. Let

$N_s(t)$ = number of components that are performing the intended function adequately at time t

$N_f(t)$ = number of components that have failed at time t

$$\text{where } N_s(t) + N_f(t) = N_0 \quad \dots (14a)$$

Now, from equation (1), we know that reliability at time t is given by

$$R(t) = \frac{N_s(t)}{N_0} = \frac{N_0 - N_f(t)}{N_0} \quad [\because N_s(t) + N_f(t) = N_0]$$

$$\text{or } R(t) = 1 - \frac{N_f(t)}{N_0} \quad \dots (14b)$$

Differentiating $R(t)$ with respect to t , we get

$$\frac{d}{dt}(R(t)) = -\frac{1}{N_0} \frac{d}{dt}(N_f(t)) \quad \dots (15)$$

Also, from equations (8a) and (4), we know that

$$\begin{aligned} f(t) &= \frac{d}{dt}(F(t)) = \frac{d}{dt}(1 - R(t)) \\ &= -\frac{d}{dt}(R(t)) \quad \dots (16a) \end{aligned}$$

$$= -\left(-\frac{1}{N_0} \frac{d}{dt}(N_f(t))\right) \quad [\text{Using equation (15)}]$$

$$\text{or } f(t) = \frac{1}{N_0} \frac{d}{dt}(N_f(t)) \quad \dots (16b)$$

By definition, hazard rate is given by

$$\lambda(t) = \frac{\text{number of failures per unit time at time } t}{\text{number of components which are exposed to failure at time } t}$$

Since derivative $\frac{dy}{dx}$ = rate of change of y with respect to x , therefore, the numerator in terms of derivative can be written as follows :

$$\lambda(t) = \frac{\frac{d}{dt}(N_f(t))}{N_s(t)} \quad \dots (17)$$

$$\begin{aligned} \text{or } \lambda(t) &= \frac{N_0}{N_s(t)} \frac{1}{N_0} \frac{d}{dt}(N_f(t)) \quad [\text{on multiplying and dividing by } N_0] \\ &= \frac{1}{R(t)} f(t) \quad \left[\begin{array}{l} \because R(t) = \frac{N_s(t)}{N_0} \text{ from equation} \\ (1) \text{ and using equation (16b)} \end{array} \right] \end{aligned}$$

$$\text{or } \lambda(t) = \frac{f(t)}{R(t)} \quad \dots (18)$$

$$= -\frac{1}{R(t)} \frac{d}{dt}(R(t)) \quad [\text{using equation (16a)}]$$

$$\therefore \lambda(t) = -\frac{1}{R(t)} \frac{d}{dt}(R(t)) \quad \dots (19)$$

Integrating $\lambda(t)$ with respect to t on both sides within the limits $t = 0$ to $t = t$, we get

$$\begin{aligned} \int_0^t \lambda(t) dt &= -\int_0^t \frac{1}{R(t)} \frac{d}{dt}(R(t)) dt = -\int_0^t \frac{\frac{d}{dt}(R(t))}{R(t)} dt \\ &= -[\ln|R(t)|]_0^t \quad \left[\because \int_a^b \frac{\frac{d}{dx}(f(x))}{f(x)} dx = [\ln|f(x)|]_a^b \right] \\ &= -(\ln|R(t)| - \ln|R(0)|) \\ &= -\ln|R(t)| - \ln 1 \quad [\because R(0) = 1] \end{aligned}$$

$$\begin{aligned}
 &= -\ln(R(t)) - 0 \\
 &= -\ln R(t) \\
 &\Rightarrow -\int_0^t \lambda(t) dt = \ln R(t) \quad \dots (20a)
 \end{aligned}$$

$$\Rightarrow R(t) = e^{-\int_0^t \lambda(t) dt} \quad \left[\because \ln a = m \Rightarrow a = e^m \right]$$

$$\text{or } R(t) = \exp\left(-\int_0^t \lambda(t) dt\right) \quad \dots (20b)$$

Note that if $\lambda(t)$ is constant and $\lambda(t) = \lambda$, then equation (20b) gives

$$R(t) = e^{-\lambda t} \quad \dots (20c)$$

We now put together the relations between $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$ in Table 13.1 as follows:

Table 13.1: Relationship between the Functions $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$

| | $R(t)$ | $F(t)$ | $f(t)$ | $\lambda(t)$ |
|--------------|---------------------------|---------------------------------------|---------------------------------|---|
| $R(t)$ | — | $1 - F(t)$ | $\int_t^\infty f(t) dt$ | $\exp\left(-\int_0^t \lambda(t) dt\right)$ |
| $F(t)$ | $1 - R(t)$ | — | $\int_0^t f(t) dt$ | $1 - \exp\left(-\int_0^t \lambda(t) dt\right)$ |
| $f(t)$ | $-\frac{d}{dt}(R(t))$ | $\frac{d}{dt}(F(t))$ | — | $\lambda(t) \exp\left(-\int_0^t \lambda(t) dt\right)$ |
| $\lambda(t)$ | $-\frac{d}{dt}(\ln R(t))$ | $\frac{\frac{d}{dt}(F(t))}{1 - F(t)}$ | $\frac{f(t)}{\int_0^t f(t) dt}$ | — |

13.3.6 Mean Time to Failure and Median of the Random Variable T

In reliability terminology, the mean of the random variable T in the absence of repair and replacement is known as the **mean time to failure** (MTTF). Usually the users of a product/component are interested in knowing the average life of the product/component rather than the complete failure details. MTTF is a measure which simply gives a number (in units of time in which life of the product/component is measured) that tells us on an average how long the product/component performs its intended function successfully. It is calculated by taking the mean of the lifetimes obtained on the basis of results of a sample of such identical products/components tested under stated conditions. For example, suppose we put 10 identical components to test under the stated conditions. Let t_i , ($i = 1, 2, \dots, 10$) denote the time for which i^{th} component performs its intended function successfully. The results of the sample are shown in Fig. 13.4. Thus, mean time to failure (MTTF) of such components on the basis of the results of this sample is given by

$$\text{MTTF} = \frac{1}{10} \sum_{i=1}^{10} t_i = \frac{1}{10} (5 + 2 + 11 + 6 + 9 + 1 + 5 + 4 + 6 + 1) = \frac{50}{10} = 5 \text{ hr}$$

In general, if n components are put to test and $t_i, (i = 1, 2, 3, \dots, n)$ denotes the time for which the i^{th} component performs its intended function, then mean time to failure (MTTF) of such components is given by

$$\text{MTTF} = \frac{1}{n} \sum_{i=1}^n t_i$$

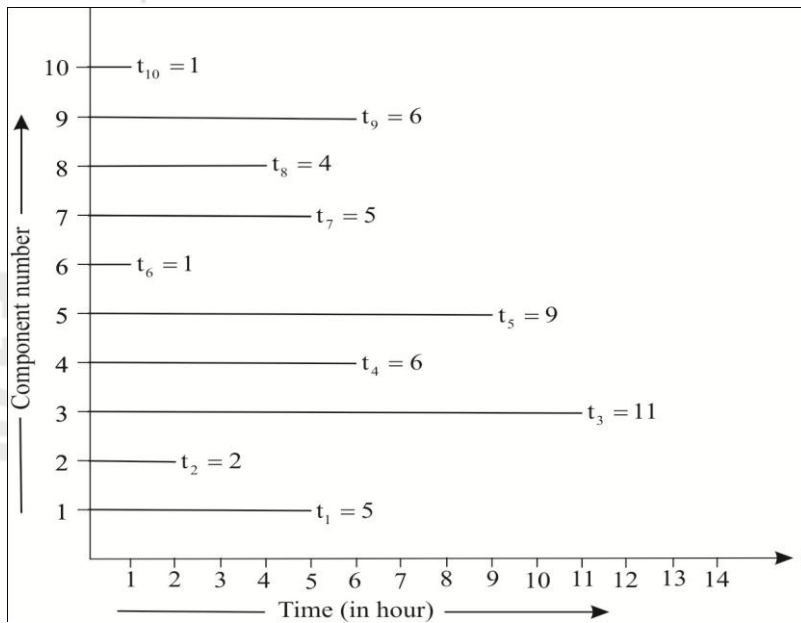


Fig. 13.4: Times for which each of the 10 components operates successfully.

However, if we are given the basic functions of a component then by definition, MTTF is given by

$$\begin{aligned} \text{MTTF} = E(T) &= \int_0^{\infty} t f(t) dt \quad [\text{refer to Unit 8 of MST-003}] \quad \dots (21) \\ &= \int_0^{\infty} t \left(-\frac{d}{dt} R(t) \right) dt \quad \left[\because f(t) = -\frac{d}{dt} R(t) \text{ from equation (16a)} \right] \end{aligned}$$

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Integrating by parts, we get

$$\text{MTTF} = t(-R(t)) \Big|_0^{\infty} - \int_0^{\infty} (1)(-R(t)) dt = tR(t) \Big|_{t \rightarrow \infty} - 0 + \int_0^{\infty} R(t) dt$$

Since $R(t) = e^{-\int_0^t \lambda(t) dt}$ and $\lim_{x \rightarrow \infty} x^n e^{-x} = 0$ for $n > -1$,

$$\text{MTTF} = \int_0^{\infty} R(t) dt \quad \dots (22)$$

To calculate MTTF, you should use equation (22) instead of equation (21), because, in general, dealing with equation (22) is easier.

Note 1: If failure rate is constant, i.e., $\lambda(t) = \lambda$, we have

$$\begin{aligned} R(t) &= e^{-\int_0^t \lambda dt} = e^{-\lambda t} \\ \therefore \text{MTTF} &= \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^{\infty} = -\frac{1}{\lambda} (0 - 1) = \frac{1}{\lambda} \end{aligned}$$

$$\text{i.e. } \boxed{\text{MTTF} = \frac{1}{\text{constant failure rate}}} \text{ or } \boxed{\text{Constant failure rate} = \frac{1}{\text{MTTF}}} \dots (23)$$

Now, to obtain an expression for median of the random variable T , we have to recall the definition of median from MST-002 or MST-003. Median is that value of the variate which divides the distribution of the variate into two equal parts. Therefore, if t_{md} denotes the median of the random variable T , then

$$R(t_{\text{md}}) = P[T > t_{\text{md}}] = 0.5 \quad \dots (24)$$

Let us now consider a few examples to apply the concepts discussed in this section to numerical problems.

Example 1: The failure density function of the random variable T is given by

$$f(t) = \begin{cases} 0.012 e^{-0.012t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Calculate

- Reliability of the component.
- Reliability of the component for a 100 hour mission time.
- Mean time to failure (MTTF).
- Median of the random variable T .
- What is the life of the component if a reliability of 0.96 is desired?

Solution: Failure density function of the random variable T is given as

$$f(t) = \begin{cases} 0.012 e^{-0.012t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- From equation (11b), reliability is given by

$$\begin{aligned} R(t) &= \int_t^{\infty} f(t) dt = \int_t^{\infty} 0.012 e^{-0.012t} dt \\ &= \left[\frac{0.012 e^{-0.012t}}{-0.012} \right]_t^{\infty} \quad \left[\because \int_a^b e^{kt} dt = \left[\frac{e^{kt}}{k} \right]_a^b \right] \\ &= - \left[0 - e^{-0.012t} \right] \quad \left[\because e^{-0.012t} \rightarrow 0 \text{ as } t \rightarrow \infty \right] \\ &= e^{-0.012t} \quad \dots (i) \end{aligned}$$

- The reliability of the component for a 100 hour mission time is obtained by replacing t by 100 in (i):

$$R(100) = e^{-0.012 \times 100} = e^{-1.2} \approx 0.3012$$

- We know from equation (22) that

$$\begin{aligned} \text{MTTF} &= \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-0.012t} dt = \left[\frac{e^{-0.012t}}{-0.012} \right]_0^{\infty} \\ &= - \frac{1}{0.012} [0 - 1] = \frac{1}{0.012} \approx 83.3333 \end{aligned}$$

- iv) Let t_{md} denote the median time to failure. Then from equation (24), we have

$$R(t_{md}) = P[T > t_{md}] = 0.5$$

$$\Rightarrow \int_{t_{md}}^{\infty} f(t) dt = 0.5$$

$$\Rightarrow e^{-0.012t_{md}} = 0.5 \quad [\text{replacing } t \text{ by } t_{md} \text{ in (i)}]$$

Taking natural logarithm on both sides

$$-0.012t_{md} = \ln(0.5) \Rightarrow -0.012t_{md} \approx -0.6931$$

$$\Rightarrow t_{md} \approx \frac{0.6931}{0.012} \approx 57.7583 \text{ hr}$$

- v) The life of the component for a reliability of 0.96 can be obtained by solving (i) for t and replacing $R(t)$ by 0.96 as follows:

$$0.96 = e^{-0.012t}$$

Taking natural logarithm on both sides, we get

$$\ln(0.96) = -0.012t \Rightarrow -0.0408 \approx -0.012t \Rightarrow t \approx \frac{0.0408}{0.012} = 3.4 \text{ hr}$$

Example 2: The hazard rate of a component is given by $\lambda(t) = 0.6 \times 10^{-6} t$, where t is in years.

Calculate the reliability of the component for the first 2 years.

Solution: It is given that the hazard rate of a component is $\lambda(t) = 0.6 \times 10^{-6} t$, where t is in years.

We know from equation (20b) that

$$\begin{aligned} R(t) &= \exp\left(-\int_0^t \lambda(t) dt\right) = \exp\left(-\int_0^t 0.6 \times 10^{-6} t dt\right) = \exp\left(\left[\frac{-0.6 \times 10^{-6} t^2}{2}\right]_0^t\right) \\ &= \exp(-0.3 \times 10^{-6} t^2 - 0) = \exp(-0.3 \times 10^{-6} t^2) \end{aligned}$$

Now reliability of the component for the first two years is given by just replacing t by 2. Therefore,

$$R(2) = e^{-0.3 \times 10^{-6} \times 2^2} = e^{-1.2 \times 10^{-6}}$$

Example 3: Show that if the hazard rate of a component is constant, say λ , then the failure distribution of the component follows exponential distribution. Also show that $MTTF = 1/\lambda$.

Solution: We are given that the hazard rate of a component = constant = λ . From equation (20b), we have

$$\begin{aligned} R(t) &= e^{-\int_0^t \lambda(t) dt} = e^{-\int_0^t \lambda dt} \quad [\because \lambda(t) = \lambda = \text{constant}] \\ &= e^{-[\lambda t]_0^t} = e^{-\lambda(t-0)} = e^{-\lambda t} \end{aligned}$$

Using equation (4), we have

$$F(t) = 1 - R(t) = 1 - e^{-\lambda t}$$

which is nothing but the cumulative distribution function of an exponential distribution. Hence, failure distribution of the component follows exponential distribution.

Now we calculate mean time to failure (MTTF) of the component. From equation (22),

$$\begin{aligned} \text{MTTF} &= \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^{\infty} \left[\because \int_a^b e^{kt} dt = \left[\frac{e^{kt}}{k} \right]_a^b \right] \\ &= -\frac{1}{\lambda} [0 - 1] \quad \left[\because \text{as } t \rightarrow \infty, e^{-\lambda t} \rightarrow 0 \right] \\ &= \frac{1}{\lambda} \end{aligned}$$

$$\text{Hence MTTF} = \frac{1}{\lambda} = \frac{1}{\text{constant hazard rate}}$$

Note 2: Thus we see that if hazard rate = constant = λ = then

$$\text{MTTF} = \frac{1}{\lambda} \text{ or } \lambda = \frac{1}{\text{MTTF}}.$$

Now, you may like to solve the following numerical problems to obtain MTTF.

E3) Calculate MTTF using the information given in Example 2.

E4) If hazard rate of a component is 0.2×10^{-4} , then obtain MTTF.

13.4 BATHTUB CURVE

In Sec. 13.3.4, we have discussed the hazard rate of a component. Consider a population of new identical components and suppose that all of them enter the field at some point in time, say, $t = 0$. In general, the curve of relative failure rate of the entire population of the components without replacement has the typical shape of a bathtub. Therefore, it is known as the **bathtub curve** as shown in Fig. 13.5. The bathtub curve is nothing but a graphical representation of the failure rate of a population of identical components versus time.

The shape of this curve has three different phases, which are explained as follows:

Early Failure or Infant Mortality

When a population of new identical components starts to perform its intended function, a high failure rate is observed in initial stages due to many factors such as manufacturing errors, poor quality or substandard items, incorrect adjustment or positioning, bad assembly, human factors, improper design, etc. This phase of the bathtub curve is known as the period of **early failure** or **infant mortality**. It is also known as the burn-in or decreasing failure rate, or debugging period. Most failures in this period are due to manufacturing errors, design problems or poor quality for which manufacturer may be responsible. So this period is generally covered by a warranty period by the manufacturer. The duration of this period varies from component to component. However, it is typically for the period of time till failure rate decreases.

The hazard rate in this phase can be reduced by increasing quality control at the production level (you have studied about quality control in detail in the first eight units of this course). However, even increased quality control at the production stage cannot completely eliminate infant mortality. Therefore, components should be tested at the factory before delivering them to the customers. That is why, good companies generally test the components before supplying them to users.

Normal Life, Useful Life or Chance Failure Period

Most components having manufacturing error, improper design, bad assembly, etc. have been failed in the early failure period. So for a long period of time (length of the period may vary from component to component), fewer failures are reported and failure rate remains almost at a constant level. This phase of the life of the component is known as **normal life, useful life or chance failure period** as shown in Fig. 13.5.

Usually failures in this period are due to stress-related causes, random fluctuations, etc. But these failures cannot be predicted exactly and happen randomly. That is why this phase is known as chance failure period. The appropriate distribution used in this phase is the exponential distribution as shown in Example 3. We know that in the case of constant failure rate, the failure distribution of the component follows an exponential distribution.

Wear out Failures

Since no component is perfect, it cannot last forever. So after the span of useful life, the failure rate of components starts increasing due to ageing of the components. This phase of the bathtub curve when the components begin to deteriorate is known as the period of **wear out failures** as shown in Fig. 13.5. In the wear out phase, failure rate increases with time. So if we wish to minimise this interruption in the smooth working of a real life system, one of the strategies which can be helpful is replacement of the component when it reaches mean time to failure of its life. The probability distributions generally used to study the failure characteristics in this phase are normal (due the shape of the failure curve in this phase shown in Fig. 13.8), Weibull and Gamma (due to the presence of shape parameter in these distributions).

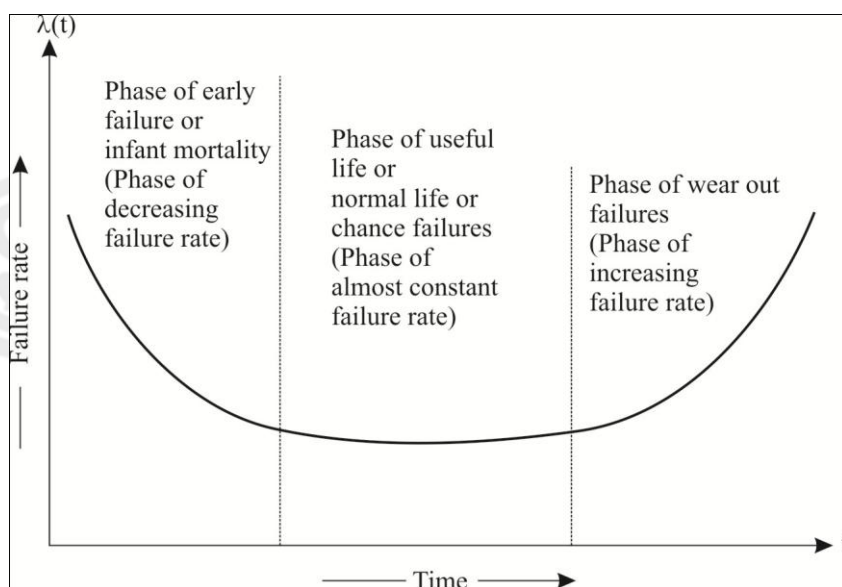


Fig. 13.5: Bathtub curve.

Now, you can try the following exercises based on the concept of the bathtub curve.

- E5)** In different phases of the bathtub curve, which one is the phase of increasing failure rate? Choose the most appropriate alternative.
 A) Infant mortality B) Useful life C) Wear out D) Early failures
- E6)** In the phase of useful life of a component, which distribution is the best suited to study the failure phenomenon? Choose the most appropriate alternative.
 A) Exponential B) Normal C) Weibull D) Gamma

13.5 ESTIMATION OF RELIABILITY FUNCTIONS FROM FAILURE DATA

In Sec. 13.3, we have discussed the basic reliability functions, namely, reliability, cumulative failure distribution, failure density and hazard rate function. It is a real challenge for the people working in the field of reliability to estimate the basic reliability functions from the failure data of a sample of identical components obtained either from test-generated failures or collected from actual field of operation of these components.

The following two approaches are generally used for this purpose:

- (i) Fitting a theoretical distribution, which is appropriate to the failure data.
- (ii) Deriving a function for hazard rate directly from the failure data and consequently other reliability functions using the relations mentioned in Table 13.1.

Here we follow the second approach as the first approach is beyond the scope of the course. Further, the choice of the method in the second approach also varies depending upon whether we have complete failure data or censored (incomplete) failure data. Let us first explain what we mean by complete and censored failure data.

Complete and Censored Failure Data: Suppose a sample of n identical components numbered from 1 to n is put to test. We record the lifetime of the components that fail. If the test is allowed to run until the failure of all n components, we shall have a record of the lifetimes of all n components. The data thus obtained is known as **complete failure data**. But if the test is not allowed to run until the failure of all n components due to some reason, we shall not have a record of the lifetimes of all n components. The data thus obtained is known as **censored failure data**.

Here we discuss only the case of complete failure data as censored failure data is beyond the scope of the course. Let us explain the procedure of estimating the basic reliability functions with the help of the following example. The data considered in this example is purely hypothetical.

Example 4: The failure data for 1000 electronic components is shown in Table 13.2.

Table 13.2: Failure Data of 1000 Electronic Components

| Operating Time (in hours) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---------------------------|-------|--------|---------|---------|---------|---------|---------|-------|
| Number of Failures | 167 | 116 | 98 | 76 | 65 | 56 | 44 | 36 |
| Operating Time (in hours) | 80-90 | 90-100 | 100-110 | 110-120 | 120-130 | 130-140 | 140-150 | |
| Number of Failures | 48 | 64 | 77 | 61 | 46 | 27 | 19 | |

Estimate and plot the reliability, cumulative failure distribution, failure density and failure rate functions.

Solution: In this example, the number of components in the sample is large. So instead of recording the time to failure of each component individually, it is a common practice to record the number of failures in some suitable intervals of time. In this example, the interval of time is taken as 10 hours. The estimate of reliability, cumulative failure distribution, failure density and failure/hazard rate functions are given in columns 5, 6, 7 and 8 of Table 13.3. The calculations for different columns are explained below.

Calculations of Columns 1 and 2

In column 1, we simply enter time in hours starting from the lower bound of first class interval up to the upper bound of last class interval for the data given in Table 13.2. Entries in the second column represent number of failures observed during test in each interval. The number of failures (frequencies) listed against each class interval of time in Table 13.2 is the sum of the numbers of all components that fail during that time interval. In the second column of Table 13.3, this frequency is entered against the upper bound of the class interval during which it is observed.

Calculation of Column 3

Entries in this column simply represent the cumulative frequencies of the frequencies written in column 2. You are familiar with the concept of cumulative frequencies as it has been explained and used by you a number of times in other courses of this programme. If $N_f(t)$ denotes the number of components that have failed by time t , then the first three entries of the column are calculated as follows:

First entry $N_f(0) = 0$ = same as the first entry of column 2

Second entry $N_f(10) = 0 + 167 = 167$ = sum of first two entries of column 2

Third entry $N_f(20) = 167 + 116 = 283$ = sum of first three entries of column 2

You can calculate the remaining cumulative frequencies as explained above.

Calculation of Column 4

Entries of column 4 represent the number of components that are performing their intended function adequately at time t . Therefore if $N_s(t)$ denotes the number of components that are performing their intended function at time t , then

$$N_s(0) = 1000 \quad \left[\begin{array}{l} \because \text{initially we started with 1000} \\ \text{components and it is assumed that} \\ \text{no component fails at time } t = 0 \end{array} \right]$$

$$N_s(10) = 1000 - 167 = 833 \quad \left[\begin{array}{l} \because 167 \text{ components have failed within the} \\ \text{first 10 hours of their operational life.} \end{array} \right]$$

$$N_s(20) = 833 - 116 = 717 \quad \left[\begin{array}{l} \because \text{out of the remaining 833 components,} \\ 116 \text{ have failed within the next} \\ 10 \text{ hours of their operational life} \end{array} \right]$$

The other entries of this column are calculated in the same way.

Calculations of Column 5

Entries in column 5 represent values of reliabilities at different points in time.

If $R(t)$ denotes the reliability at time t , then by definition [equation (1)]

$$R(t) = \frac{N_s(t)}{N_0} \quad \left[\begin{array}{l} \text{where } N_0 = \text{the size of the sample that} \\ \text{we start with and } N_s(t) \text{ is the same as} \\ \text{explained in calculations of column 4} \end{array} \right]$$

The first three entries of this column are calculated as follows:

$$R(0) = \frac{N_s(0)}{N_0} = \frac{1000}{1000} = 1, \quad R(10) = \frac{N_s(10)}{N_0} = \frac{833}{1000} = 0.833$$

$$R(20) = \frac{N_s(20)}{N_0} = \frac{717}{1000} = 0.717$$

In fact, entries of this column can simply be calculated by dividing the corresponding entries of column 4 by 1000 (the total number of components in the beginning).

Calculations of Column 6

Entries in column 6 represent values of cumulative failure distribution at different points in time. Therefore, if $F(t)$ denotes cumulative failure distribution at time t , then

$$F(t) = \frac{N_f(t)}{N_0}, \quad \left[\begin{array}{l} \text{where } N_f(t) \text{ denotes the number of} \\ \text{components that have failed by time } t \end{array} \right]$$

The first three entries of this column are calculated as follows:

$$F(0) = \frac{N_f(0)}{N_0} = \frac{0}{1000} = 0, \quad F(10) = \frac{N_f(10)}{N_0} = \frac{167}{1000} = 0.167$$

$$F(20) = \frac{N_f(20)}{N_0} = \frac{283}{1000} = 0.283$$

In fact, entries of this column can simply be calculated by dividing the corresponding entries of column 3 by 1000 (the total number of components in the beginning).

Calculations of Column 7

Entries of this column represent values of failure density function at different points of time. If $f(t)$ denotes the value of failure density function at time t , then from equation (16b), we have

$$\begin{aligned} f(t) &= \frac{\frac{d}{dt}(N_f(t))}{N_0} \\ &= \lim_{\Delta t \rightarrow 0} \frac{N_f(t + \Delta t) - N_f(t)}{N_0 \Delta t} \quad [\text{by definition of derivative}] \end{aligned}$$

Thus, approximate values of $f(t)$ can be obtained from the expression

$$f(t) = \frac{N_f(t + \Delta t) - N_f(t)}{N_0 \Delta t}, \quad \text{where } \Delta t = 10 \text{ in this example}$$

$$= \frac{N_f(t + 10) - N_f(t)}{10 \times 1000}, \quad \text{where } t = 0, 10, 20, 30, \dots, \text{ and } N_0 = 1000$$

The first three entries of this column are calculated as follows:

$$f(0) = \frac{N_f(10) - N_f(0)}{10000} = \frac{167 - 0}{10000} = 0.0167$$

$$f(10) = \frac{N_f(20) - N_f(10)}{10000} = \frac{283 - 167}{10000} = \frac{116}{10000} = 0.0116$$

$$f(20) = \frac{N_f(30) - N_f(20)}{10000} = \frac{98}{10000} = 0.0098$$

In fact, entries of this column can simply be calculated by dividing the entries of column 2 by 10000.

Calculations of Column 8

Entries of this column represent values of failure rate or hazard rate at different points in time. If $\lambda(t)$ denotes the value of failure rate at time t , then from equation (17), we have

$$\lambda(t) = \frac{\frac{d}{dt}(N_f(t))}{N_s(t)}$$

$$= \lim_{\Delta t \rightarrow 0} \frac{N_f(t + \Delta t) - N_f(t)}{N_s(t) \Delta t} \quad [\text{by definition of derivative}]$$

Thus, approximate values of $\lambda(t)$ can be obtained from the expression

$$\lambda(t) = \frac{N_f(t + \Delta t) - N_f(t)}{N_s(t) \Delta t}, \quad \text{where } \Delta t = 10 \text{ in this example}$$

$$= \frac{N_f(t + 10) - N_f(t)}{10 N_s(t)}, \quad \text{where } t = 0, 10, 20, 30, \dots,$$

The first three entries of this column are calculated as follows:

$$\lambda(0) = \frac{N_f(10) - N_f(0)}{10 N_s(0)} = \frac{167 - 0}{10 \times 1000} = \frac{167}{10000} = 0.0167$$

$$\lambda(10) = \frac{N_f(20) - N_f(10)}{10 N_s(10)} = \frac{283 - 167}{10 \times 833} = \frac{116}{8330} \approx 0.0139$$

$$\lambda(20) = \frac{N_f(30) - N_f(20)}{10 N_s(20)} = \frac{98}{10 \times 717} = \frac{98}{7170} \approx 0.0137$$

In fact, entries of this column can be calculated by dividing the entries of column 2 by the preceding entries of column 4 and further dividing the resulting value by 10.

Table 13.3: Estimation of $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$

| Time | Number of Failures | Cumulative Failures | Number of Survivors | Reliability | Cumulative Failure Distribution | Failure Density Function | Hazard Rate |
|------|--------------------|---------------------|---------------------|-------------|---------------------------------|--------------------------|--------------|
| t | f | $N_f(t)$ | $N_s(t)$ | $R(t)$ | $F(t)$ | $f(t)$ | $\lambda(t)$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 0 | 0 | 0 | 1000 | 1 | 0 | 0.0167 | 0.0167 |
| 10 | 167 | 167 | 833 | 0.833 | 0.167 | 0.0116 | 0.0139 |
| 20 | 116 | 283 | 717 | 0.717 | 0.283 | 0.0098 | 0.0137 |
| 30 | 98 | 381 | 619 | 0.619 | 0.381 | 0.0076 | 0.0123 |
| 40 | 76 | 457 | 543 | 0.543 | 0.457 | 0.0065 | 0.0120 |
| 50 | 65 | 522 | 478 | 0.478 | 0.522 | 0.0056 | 0.0117 |
| 60 | 56 | 578 | 422 | 0.422 | 0.578 | 0.0044 | 0.0104 |
| 70 | 44 | 622 | 378 | 0.378 | 0.622 | 0.0036 | 0.0095 |
| 80 | 36 | 658 | 342 | 0.342 | 0.658 | 0.0048 | 0.0140 |
| 90 | 48 | 706 | 294 | 0.294 | 0.706 | 0.0064 | 0.0218 |
| 100 | 64 | 770 | 230 | 0.230 | 0.770 | 0.0077 | 0.0335 |
| 110 | 77 | 847 | 153 | 0.153 | 0.847 | 0.0061 | 0.0399 |
| 120 | 61 | 908 | 92 | 0.092 | 0.908 | 0.0046 | 0.0500 |
| 130 | 46 | 954 | 46 | 0.046 | 0.954 | 0.0027 | 0.0587 |
| 140 | 27 | 981 | 19 | 0.019 | 0.981 | 0.0019 | 0.1000 |
| 150 | 19 | 1000 | 0 | 0 | 1 | — | — |

The plot of the reliability, cumulative failure distribution, failure density and hazard rate functions are shown in Figs. 13.6 to 13.9, respectively.

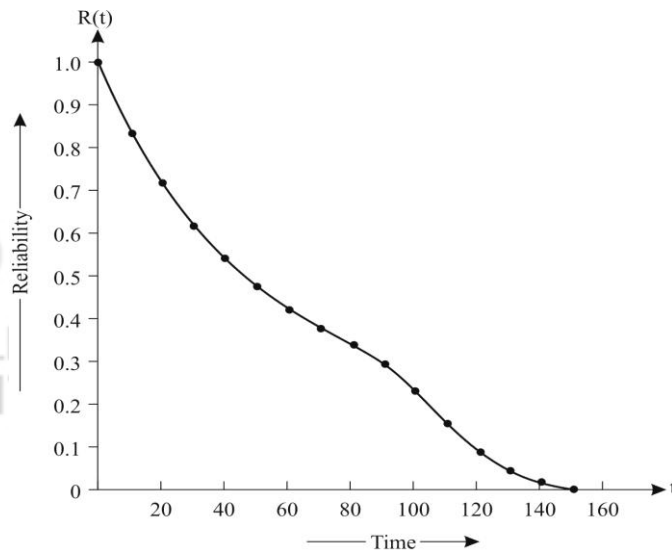


Fig. 13.6: Reliability function.

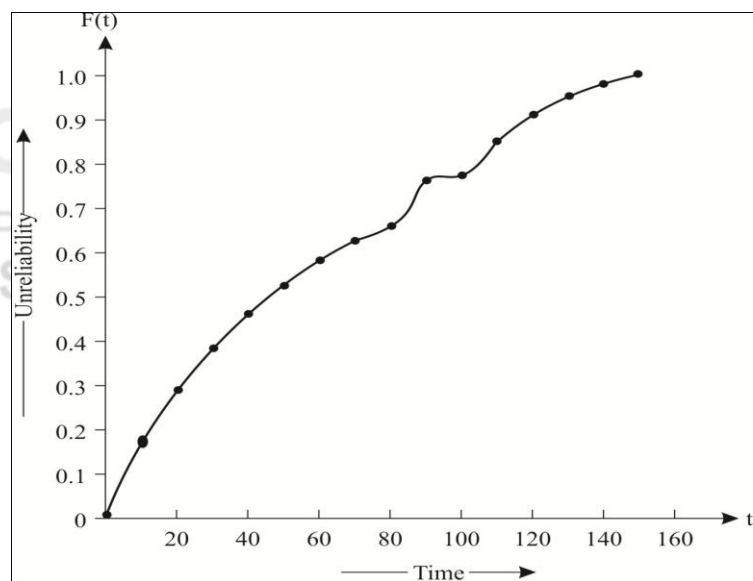


Fig. 13.7: Cumulative failure distribution function.

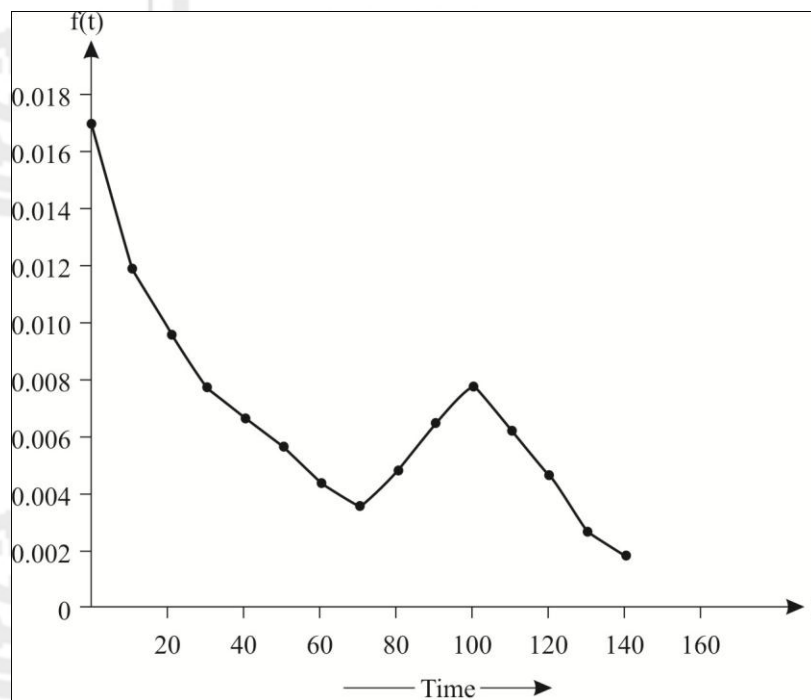


Fig.13.8: Failure density function

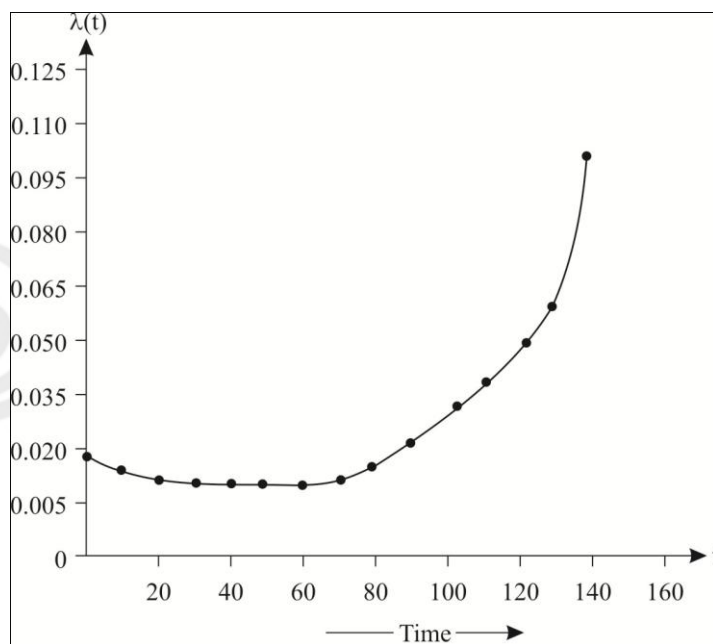


Fig.13.9: Failure rate.

Now, you can try the following exercises to obtain all these functions.

- E7)** The failure data of 10 electronic components are shown in Table 13.4 given below. Estimate the reliability, cumulative failure distribution, failure density and failure rate functions.

Table 13.4: Failure Data of 10 Components

| Failure Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------------------------|---|---|----|----|----|-----|-----|-----|-----|-----|
| Operating Time (in hours) | 2 | 6 | 31 | 51 | 76 | 116 | 140 | 182 | 250 | 302 |

- E8)** Check whether the following statement is true or false. Give reason in support of your answer.

‘A sample of n identical components numbered from 1 to n is put to test. The test is allowed to run until all n components. The data of records of the lifetimes of all n components thus obtained is an example of censored failure data’.

13.6 SUMMARY

1. **Definition of Reliability:** Reliability of a component/device/equipment/unit/system is the **probability** that it performs its **intended function adequately** for a specified period of **time** under the given operating **conditions**.
2. Reliability of a component/system when expressed in terms of time t is known as **reliability function** and is generally denoted by $R(t)$.
3. In reliability terminology, distribution function of the random variable, time to failure (T), is known as **cumulative failure distribution function**.
4. In reliability terminology, probability density function of the random variable, time to failure (T), is known as **failure density function**.
5. Instantaneous rate of failure at time t is known as the **hazard rate function** at time t and is generally denoted by $\lambda(t)$.
6. Some commonly used relations between $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$ are given as follows:

$$R(t) = 1 - F(t) = \int_t^{\infty} f(t) dt = \exp\left(-\int_0^t \lambda(t) dt\right)$$

$$F(t) = 1 - R(t) = \int_0^t f(t) dt = 1 - \exp\left(-\int_0^t \lambda(t) dt\right)$$

$$f(t) = -\frac{d}{dt}(R(t)) = \frac{d}{dt}(F(t)) = \lambda(t) \times \exp\left(-\int_0^t \lambda(t) dt\right)$$

$$\lambda(t) = -\frac{d}{dt}(\ln R(t)) = \frac{\frac{d}{dt}(F(t))}{1 - F(t)} = \frac{f(t)}{\int_0^t f(t) dt}$$

7. The bathtub curve has three phases known as **Early Failure**, **Normal Life** and **Wear out Failure**.

13.7 SOLUTIONS/ANSWERS

- E1)** The definition of reliability is:

Reliability of a component/device/equipment/unit/system/ is the probability that it performs its intended function adequately for a specified period of time under the given operating conditions.

From the above definition, it is clear that the correct option is D.

- E2)** Reliability of a component/device/equipment/unit/system is the probability that it performs its intended function. Being a probability, its value will vary from 0 to 1. Hence, A is the correct option.

- E3)** In Example 2, we have calculated

$$R(t) = e^{-0.3 \times 10^{-6} t^2}$$

$$\therefore \text{MTTF} = \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-0.3 \times 10^{-6} t^2} dt$$

Putting $0.3 \times 10^{-6} t^2 = x$, we get

$$t = \frac{\sqrt{x}}{\sqrt{0.3 \times 10^{-3}}}$$

$$\text{or } dt = \frac{10^3}{\sqrt{0.3}} \frac{1}{2\sqrt{x}} dx$$

Also, when $t = 0, x = 0$

and when $t \rightarrow \infty, x \rightarrow \infty$

$$\therefore \text{MTTF} = \int_0^{\infty} e^{-x} \frac{10^3}{2\sqrt{0.3}} x^{-1/2} dx = \frac{10^3}{2\sqrt{0.3}} \int_0^{\infty} x^{1/2-1} e^{-x} dx$$

$$\begin{aligned} &= \frac{10^3}{1.0954} \left[\frac{1}{2} \right] \left[\because \int_0^{\infty} x^{n-1} e^{-x} dx, n > 0 \right] \\ &= \frac{10^7}{10954} \sqrt{\pi} \left[\because \left[\frac{1}{2} \right] = \sqrt{\pi} \right] \end{aligned}$$

E4) We know that if hazard rate of a component = constant = λ , then

$$\text{MTTF} = \frac{1}{\lambda}$$

Here $\lambda = 0.2 \times 10^{-4}$, which is a constant.

$$\therefore \text{MTTF} = \frac{1}{0.2 \times 10^{-4}} = \frac{10^4}{0.2} = \frac{10^5}{2} = 50000$$

E5) When a population of identical components is put to test, the failure rate of the components starts increasing due to ageing of the components. In the bathtub curve, this phase is known as wear out. Hence the correct option is (C).

E6) We know that in the phase of the useful life of a population of identical components, hazard rate = constant and constant hazard rate implies that the distribution of failures follows the exponential distribution. Hence, (A) is the correct alternative.

E7) Here the number of components under test is small in size. So we can record the failure time of each component easily, which is given in Table 13.5. Calculations of the entries of different columns have already been explained in Example 4 with necessary clarifications. You can follow similar steps and calculate the entries of different columns of Table 13.5. It is a small exercise given to you for practice.

Table 13.5: Estimation of $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$

| Time | Number of Failures | Cumulative Failures | Number of Survivors | Reliability | Cumulative Failure Distribution | Failure Density Function | Hazard Rate |
|------|--------------------|---------------------|---------------------|----------------------|---------------------------------|--|---|
| t | | $N_f(t)$ | $N_s(t)$ | $R(t)$ | $F(t)$ | $f(t)$ | $\lambda(t)$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 0 | 0 | 0 | 10 | $\frac{10}{10} = 1$ | 0 | $\frac{1}{10 \times (2 - 0)} = 0.05$ | $\frac{1}{10(2 - 0)} = 0.05$ |
| 2 | 1 | 1 | 9 | $\frac{9}{10} = 0.9$ | $\frac{1}{10} = 0.1$ | $\frac{1}{10(6 - 2)} = 0.025$ | $\frac{1}{9(6 - 2)} \square 0.0278$ |
| 6 | 1 | 2 | 8 | $\frac{8}{10} = 0.8$ | $\frac{2}{10} = 0.2$ | $\frac{1}{10(31 - 6)} = 0.004$ | $\frac{1}{8(31 - 6)} = 0.005$ |
| 31 | 1 | 3 | 7 | $\frac{7}{10} = 0.7$ | $\frac{3}{10} = 0.3$ | $\frac{1}{10(51 - 31)} = 0.005$ | $\frac{1}{7(51 - 31)} \square 0.007$ |
| 51 | 1 | 4 | 6 | $\frac{6}{10} = 0.6$ | $\frac{4}{10} = 0.4$ | $\frac{1}{10(76 - 51)} = 0.004$ | $\frac{1}{6(76 - 51)} \square 0.0067$ |
| 76 | 1 | 5 | 5 | $\frac{5}{10} = 0.5$ | $\frac{5}{10} = 0.5$ | $\frac{1}{10(116 - 76)} = 0.0025$ | $\frac{1}{5(116 - 76)} = 0.005$ |
| 116 | 1 | 6 | 4 | $\frac{4}{10} = 0.4$ | $\frac{6}{10} = 0.6$ | $\frac{1}{10(140 - 116)} \square 0.0042$ | $\frac{1}{4(140 - 116)} \square 0.0104$ |
| 140 | 1 | 7 | 3 | $\frac{3}{10} = 0.3$ | $\frac{7}{10} = 0.7$ | $\frac{1}{10(182 - 140)} \square 0.0024$ | $\frac{1}{3(182 - 140)} \square 0.0079$ |
| 182 | 1 | 8 | 2 | $\frac{2}{10} = 0.2$ | $\frac{8}{10} = 0.8$ | $\frac{1}{10(250 - 182)} \square 0.0015$ | $\frac{1}{2(250 - 182)} \square 0.0074$ |
| 250 | 1 | 9 | 1 | $\frac{1}{10} = 0.1$ | $\frac{9}{10} = 0.9$ | $\frac{1}{10(302 - 250)} \square 0.0019$ | $\frac{1}{1 \times (302 - 250)} \square 0.0192$ |
| 302 | 1 | 10 | | $\frac{0}{10} = 0$ | $\frac{10}{10} = 1$ | — | — |

E8) This statement is false because we know that if we have failure data of each component of the sample under test then data thus obtained are known as complete failure data, while failure data set is said to be censored if the test is not allowed to run until all n components fail due to one reason or another.