UNIT 9 INTRODUCTION TO DECISION THEORY

Structure

9.1 Introduction

Objectives

9.2 Decision Making: Is it a Science or an Art?

9.3 Types of Decisions

9.4 Components of Decision Making

Courses of Action (Alternatives)

States of Nature (Events)

Payoff Values and Payoff Matrix

Opportunity Loss Table or Regret Table

9.5 Types of Environment

Decision Making under Certainty

Decision Making under Uncertainty

9.6 Summary

9.7 Solutions/Answers

9.1 INTRODUCTION

Decision making is an integral part of our lives. Some of these decisions are related to routine matters. For example, a vegetable seller has to decide which vegetables to purchase so that he can make maximum profit, a student has to decide which trouser and shirt to wear while going to a farewell party, a home maker has to decide whether to serve tea, coffee or cold drinks to her guests, and so on! Similarly, a manager of an organisation also has to take decisions to fulfil a given set of objectives such as increase in profit and turnover, reduction in cost, advertisement, etc. Moreover, decisions may affect an individual or a group of people or an organisation or the entire society. For example, decisions taken by business managers affect their consumers, shareholders and the employees of the organisation. In brief, we can say that all of us are involved in decision making either in our capacity as decision makers or suffer the consequences of decisions made by other people!

Some decisions are very important and require a great deal of consideration and thought. For instance, we would like to decide whether to invest in a particular share or mutual funds only after giving it careful thought. In the same way, a company requires extensive planning while taking a decision about whether to launch a new product in a market or not. In such cases, decisions have to be based on some criteria and standard tools and we can use techniques which increase the chances that our decisions are correct. Therefore, in this unit we present an introduction to decision theory.

Now-a-days, business related decisions are increasingly being taken on the basis of analysis and interpretation of data using statistical concepts and techniques. Especially, business managers and professionals need suitable data to help them in making decisions in different situations. You will understand this better when you go through the situations discussed in examples and exercises in this unit as well as in the next unit. The use of statistical concepts and techniques in decision making enables us to:

- solve problems in a systematic manner;
- add substance to decision making; and
- reduce the guess factor and obtain reliable results.

In effect, the decision maker needs to know and use decision support systems based on statistical models. Units 9 and 10 are devoted to the discussion of some methods which will benefit the people who have to make decisions in different situations. However, before we discuss these methods, we would like to answer the question "Is decision making a science or an art?" This is what we do in Sec. 9.2. In Sec. 9.3, we describe various types of decisions. We introduce the components of decision making in Sec. 9.4. A knowledge of the decision making environment helps the decision maker in arriving at good decisions. In Sec. 9.5 we discuss two types of environments in which the decision maker may have to take decisions, namely, decision making under certainty and decision making under uncertainty. We also discuss certain criteria that may be used for decision making under an environment of uncertainty which provide solutions of the problems in certain situations.

In the next unit, we shall discuss the environment of decision making under risk and decision tree analysis.

Objectives

After studying this unit, you should be able to:

- answer the question: "Is decision making a science or an art?";
- explain the concepts of structured and unstructured decisions;
- define the components of decision making;
- explain different types of environment in which a decision maker may have to make decisions;
- state the criteria which are applicable in the environment of "Decision Making under Certainty"; and
- solve numerical problems using different criteria in the environment of "Decision Making under Uncertainty".

9.2 DECISION MAKING: IS IT A SCIENCE OR AN ART?

The first and foremost question we would like to answer is:

Is decision making a science or an art?

This question arises because in decision making we make inferences about the **unknown**, and unknown phenomena may be predicted using two approaches: one based on objective application of appropriate scientific knowledge and the other using a subjective approach. Many a times, it is felt that the choice of a particular decision is related to the subjectivity of the decision maker. It may be argued that experienced people make right decisions without the aid of scientific tools and techniques and that decision making is largely a matter of intuition based on experience. For example, experienced fishermen can forecast the weather conditions by simply looking at the sky without applying any tools or techniques. Their decisions are completely based on their experience. Decision making in situations like this may be termed as an **art** because decisions are taken on the basis of the experience of an individual, rather than applying scientific tools or techniques.

On the other hand, there are many aspects of the decision making process, especially in industries and business, which lend themselves to scientific analysis and treatment. In such situations, a statistician breaks a decision making problem into small components and makes an effort to detect the pattern, which is relevant to a particular decision making process. He/she tries to analyse whether the pattern conforms to any set of rules or can be evaluated against a set of standards. These standards are set on the basis of knowledge about past events and experiences. There are no concrete rules and policies, which would enable a statistician to foresee events before they happen. The task of a decision maker in an industry or business becomes easier when he/she uses standards and tools. Statisticians try to develop standards and tools of measurement to solve a particular problem. Decision making that involves the application of scientific tools and techniques to the situation at hand rather than being entirely based on the intuition or experiences of individuals is known as a **science**.

From the above discussion, we may infer that decision making in the field of industry or business need not be exclusively subjective in character. There are many areas where it is possible to apply statistical tools and techniques and thereby make decision making far easier. In general, we can say that, as such, there are no specific criteria in decision making suitable for all situations. Hence, decision making may be treated as a **science** or an **art** depending upon the situations and objectives of the problem to be solved.

9.3 TYPES OF DECISIONS

Since we are taking decisions all the time as individuals, workers, managers or owners of companies, there exists a huge variety of decisions all around us. As you may know, for any scientific study, we first try to classify the subject of investigation. So we first need to classify decisions and select the types of decisions that we shall study in decision theory.

Classification of decisions is not unique. Decisions are sometimes categorised as:

- Personal Decisions, and
- Organisational Decisions,

and also as:

- Routine Decisions, and
- Basic Decisions.

In **decision theory,** decisions are generally classified as follows:

- Structured Decisions, and
- Unstructured Decisions.

Decisions related to routine situations or problems or preplanned decisions belong to the category of **structured decisions.** On the other hand, decisions related to problems arising accidentally or unplanned decisions belong to the category of **unstructured decisions**.

Let us further explain both these types of decisions.

Structured Decisions

Structured decisions are decisions which can be programmed (planned) in advance and are made under established situations. Structured or programmed decisions are the ones which an organisation or its employees take repeatedly on the basis of past data and advance planning.

For instance, the decisions of an organisation for reducing the cost of a product fall under the category of structured decisions because these can be planned in advance. Structured decisions are repetitive in nature and involve definite procedures and processes for handling problems. Managers make structured decisions for routine tasks all the time. For example, they routinely give additional facilities/benefits on the purchase of products to promote sales. A manager of a fast food outlet may incentivise sales by offering a free drink to his/her customers with their orders. Such promotional activities can always be preplanned or programmed in advance to fulfil the needs of the customers and maximise profit.

Structured decisions are deterministic in nature and the outcome of the decision can be determined with certainty if a specified sequence of activities is performed. Therefore, it is possible to make use of decision procedures or decision rules or algorithm to make structured decisons.

Unstructured Decisions

Sometimes, organisations face problems that have not arisen before and hence, there are no fixed processes for solving them. Decisions taken to solve such unforeseen problems are called **unstructured decisions**. These types of decisions are taken for the problems which arise occasionally and are unique in nature. Since such problems appear occasionally, there is no pre-defined method or predetermined procedure for solving them and a great deal of creativity may be required for decision making. Such decisions are quite complex the and risk factor involved in making unstructured decisions is quite high. Unstructured decisions are non-repetitive and vital. In fact, higher level managers face such situations quite often. For instance, they may have to handle the grievances of employees related to wages in an organisation or face a sudden labour strike in a factory. There are no predetermined procedures to resolve such problems and unstructured decisions need to be taken. We can say that unstructured decisions are taken in situations that are uncertain, unclear and not preplanned. An important feature of unstructured decisions is that they cannot be delegated.

So far you have learnt about the types of decisions we shall be dealing with in decision theory. We now discuss the components of decision making.

9.4 COMPONENTS OF DECISION MAKING

Decision theory teaches ways of assessing and managing a real life situation generally faced by decision makers working in a small or big organisation at the executive level or any other level. To apply techniques of decision theory to any real-life situation, we first need to define the following components:

- Courses of Action (Alternatives),
- States of Nature (Events), and
- Payoff Values and Payoff Matrix.

In order to apply some of the decision criteria, we also need to define the **opportunity loss matrix.**

We now explain each one of these components.

9.4.1 Courses of Action (Alternatives)

The problem of decision making arises in a situation when we have more than one option. If there is only one course of action with no alternative then we have no choices and decision making is simple: We just follow that course of action! For example, if a student has a single pair of shoes, he/she wears just that pair and does not have to decide which pair of shoes to wear. So the problem of decision making arises in situations where only one course of action has to be taken out of two or more available courses of action or alternatives. For example, if a person is given the option of drinking tea or coffee, he will have to decide on only one of them! A manager in a fast food outlet may like to decide on whether to give the offer of 'buy one, get one free' or offer a free drink with an order or give a coupon for future redemption to increase sales. So, all **possible options** available in a given situation of decision making, which are **under the control** of the decision maker are known as **courses of action** or **alternatives**.

If A_1 and A_2 denote the courses of action in a given situation with two options, the **action space** is denoted by A and given by

$$A = \{A_1, A_2\}$$

In general, if n options are available to the decision maker in a given situation, which are under his/her control, the action space is given by

$A = \{A_1, A_2, ..., A_n\}$ where $A_i, (i = 1, 2, ..., n)$ are n courses of action.

A situation with three courses of action available to the decision maker is shown in Fig. 9.1.

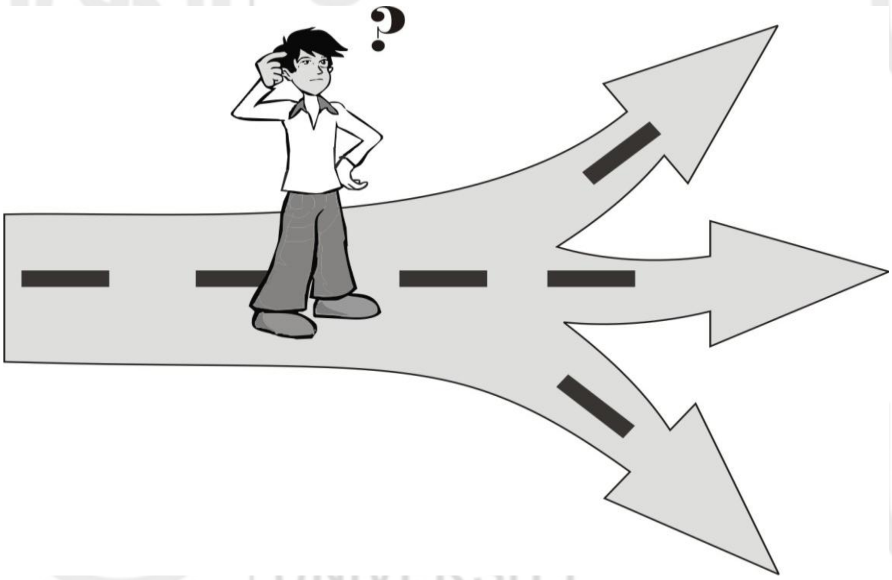

Fig. 9.1: A typical situation for three courses of action.

Let us explain the concept of courses of action (alternatives) in a given situation with the help of an example.

Example 1: Suppose a fruit seller buys apples at the rate of Rs 50 per kg and sells them at the rate of Rs 60 per kg. Assume that an apple not sold during the day is treated as rotten and thrown away. The daily sales of apples in the past has never been less than 48 kg. It has also not been more than 50 kg. State the courses of action in this situation and determine the action space.

Solution: On the basis of the given information, it is clear that the fruit seller should not buy less than 48 kg of apples. He should also not buy more than 50 kg of apples. Thus, he/she has three options or courses of action (alternatives): Buying 48 kg, 49 kg or 50 kg of apples and these options are under his/her control. If we denote these three courses of action by A_1, A_2, A_3, respectively, we have

$A_1 = 48,\ A_2 = 49,\ A_3 = 50$ and the action space is given by $A = \{48, 49, 50\}$.

9.4.2 States of Nature (Events)

You have learnt that courses of action are alternatives presently available to the decision maker, which are under his/her control. But a situation may have an outcome, which may not be under the control of the decision maker. **States of nature or events** are related to future outcomes, conditions or situations, which are **not under the control** of the decision maker. For example, consider Example 1. In this example, the sale of apples is the future outcome, which is not under control of the fruit seller. Hence, the sales of apples (in kg), correspond to the states of nature for Example 1.

In Example 1, the sales of apples may be 48 kg, 49 kg or 50 kg. If we denote these possible states of nature or events by N_1, N_2, N_3, respectively, we have

$N_1 = 48,\ N_2 = 49,\ N_3 = 50$

Note that the fruit seller cannot predict which of the three events N_1, N_2 or N_3 would take place, that is, these behave like random events.

A decision maker can, at most, associate probabilities with the outcomes of the events based on the experience of past data. But he/she may face two situations:

1) Situations where **data are not available to associate probabilities,** and
2) Situations where **data are available to associate probabilities**.

Decision making in the fitst situation is known as **decision making under uncertainty**. In the second situation, it is known as **decision making under risk.**

Let us further explain both these categories of decision making.

Decision making under uncertainty: This refers to decision making in situations where the probabilities associated with the occurrence of different states of nature are **not known** to the decision maker. Thus, decision making in situations where the decision maker does not have any prior data that could provide the probabilities of occurrence of different states of nature is known as **decision making under uncertainty**. For example, decision making for launching a new product lies under this category because the probabilities of success or failure of the product are not known in advance. Since the product is being launched for the first time, no past data about its performance in the market are available and the probabilities of its success or failure are unknown.

Decision making under risk: Decision making in situations where the probabilities associated with the occurrence of different states of nature are **known** from the past experience or past records falls under the category of **decision making under risk**. If the probabilities of the occurrence of the states of nature are given, the best decision to be selected is the one that gives the highest expected output among the available alternatives.

9.4.3 Payoff Values and Payoff Matrix

The possible outcomes from the combination of each course of action and each state of nature represented in numerical form (or in rupees) are termed as **payoff values**. In other words, a value of an output when represented in terms of gain or loss is called a **payoff value**. Payoff values may be presented in the form of a table or a matrix as shown in Tables 9.1 and 9.2. The resulting table or matrix is called the **payoff table** or **payoff matrix**. Payoff values are also known as **conditional profits** if they represent profits and **conditional losses** if they represent losses. The word conditional is added because of the states of nature, which are not under the control of the decision maker.

Let us consider Example 1 again to explain these concepts. The courses of action (apples bought) are $A_1 = 48,\ A_2 = 49, A_3 = 50,$ and the states of nature (future sales of apples) are $N_1 = 48, N_2 = 49, N_3 = 50$. Also, the cost price is Rs 50 per kg and the sale price is Rs 60 per kg.

Therefore, the profit on sold apples = Rs $(60-50)$ = Rs 10 per kg.

Loss on unsold apples = Rs 50 per kg.

Therefore, the payoff values for this example are given by:

$$\begin{aligned}\text{Payoff value} &= (\text{Profit per kg}) \times (\text{Number of kg of apples sold}) \\ &\quad - (\text{Loss per kg}) \times (\text{Number of kg of apples unsold}) \\ &= 10 \times (\text{Number of kg of apples sold}) \\ &\quad - 50 \times (\text{Number of kg of apples unsold})\end{aligned}$$

Now, if x_{ij}, $(i, j = 1, 2, 3)$ denotes the payoff value corresponding to the state of nature N_i and course of action A_j, then we have:

x_{11} = Payoff value corresponding to state of nature N_1 and course of action A_1

$$= \text{Rs } (10 \times 48 - 50 \times 0) = \text{Rs } 480 \quad \left[\begin{array}{l}\because N_1 = 48 \text{ means that demand is of} \\ 48 \text{ kg and } A_1 = 48 \text{ means he/she has} \\ \text{already bought 48 kg. So there is no} \\ \text{loss of unsold apples.}\end{array}\right]$$

x_{12} = Payoff value corresponding to state of nature N_1 and course of action A_2

$$= \text{Rs } (10 \times 48 - 50 \times 1) = \text{Rs } 430 \quad \left[\begin{array}{l}\because N_1 = 48 \text{ means that demand is of} \\ 48 \text{ kg and } A_2 = 49 \text{ means he/she has} \\ \text{already bought 49 kg. So he/she is} \\ \text{facing a loss for 1 kg.}\end{array}\right]$$

x_{13} = Payoff value corresponding to state of nature N_1 and course of action A_3

$$= \text{Rs } (10 \times 48 - 50 \times 2) = \text{Rs } 380 \quad \left[\begin{array}{l}\because N_1 = 48 \text{ means that demand is of} \\ 48 \text{ kg and } A_3 = 50 \text{ means that he/she} \\ \text{has already bought 50 kg. So he/she} \\ \text{is facing a loss for 2 kg.}\end{array}\right]$$

x_{21} = Payoff value corresponding to state of nature N_2 and course of action A_1

$$= \text{Rs } (10 \times 48 - 50 \times 0) = \text{Rs } 480 \quad \left[\begin{array}{l}\because N_2 = 49 \text{ means that demand is of} \\ 49 \text{ kg and } A_1 = 48 \text{ means that he/she} \\ \text{has already bought 48 kg. So there is} \\ \text{no loss of unsold apples.}\end{array}\right]$$

Similarly, we can calculate the payoff values $x_{22}, x_{23}, x_{31}, x_{32}$ and x_{33} for the different states of nature and different courses of action and present them in a tabular form as shown in Table 9.1.

Table 9.1: Payoff Table for the Fruit Seller

States of Nature (Sale)	Courses of Action (Purchase of Apples)		
	A_1 (=48)	A_2 (=49)	A_3 (=50)
N_1 (=48)	$x_{11} = 480$	$x_{12} = 430$	$x_{13} = 380$
N_2 (=49)	$x_{21} = 480$	$x_{22} = 10 \times 49 - 50 \times 0 = 490$	$x_{23} = 10 \times 49 - 50 \times 1 = 440$
N_3 (=50)	$x_{31} = 10 \times 48 - 50 \times 0 = 480$	$x_{32} = 10 \times 49 - 50 \times 0 = 490$	$x_{33} = 10 \times 50 - 50 \times 0 = 500$

In general, if in a given situation there are m states of nature $N_1, N_2, ..., N_m$ and n courses of action $A_1, A_2, ..., A_n$, the format of **a payoff table** (also called the **payoff matrix**) is as shown in Table 9.2. Here x_{ij}, $(i = 1, 2, ..., m;\ j = 1, 2, ..., n)$ denotes the value of payoff corresponding to the i^{th} state of nature N_i and j^{th} course of action A_j.

Table 9.2: General Format of a Payoff Table

States of Nature (Events)	Courses of Action (Alternatives)						
	A_1	A_2	A_3	…	A_j	…	A_n
N_1	x_{11}	x_{12}	x_{13}	…	x_{1j}	…	x_{1n}
N_2	x_{21}	x_{22}	x_{23}	…	x_{2j}	…	x_{2n}
N_3	x_{31}	x_{32}	x_{33}	…	x_{3j}	…	x_{3n}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_i	x_{i1}	x_{i2}	x_{i3}	…	x_{ij}	…	x_{in}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_m	x_{m1}	x_{m2}	x_{m3}	…	x_{mj}	…	x_{mn}

Let us take another example to help you understand how to calculate payoff values.

Example 2: A shopkeeper gets the following offer from a company:

- The company will provide 4 TV sets for Rs 40000 to the shopkeeper, where the retail price of each TV is Rs 15000.
- The shopkeeper cannot return the unsold TV set, if any, to the company.
- The shopkeeper has only two options: he can either accept the offer or reject it.

Assume that the unsold TV is a direct loss to the shopkeeper and prepare the payoff table for the shopkeeper.

Solution: First of all, we have to define the courses of action and the states of nature. Here the shopkeeper has two courses of action: he can either accept the offer or reject it. These are under his/her control. Let A_1 and A_2 denote these courses of action, respectively. Now, as far as the states of nature are concerned, there are five possibilities: he is able to sell none, one, two, three or all four TV sets. These are future outcomes and therefore, not under the control of the shopkeeper. So these form the states of nature. Let N_0, N_1, N_2, N_3, N_4 denote these five states of nature, respectively. If he accepts the offer and cannot sell any TV (corresponding to the state of nature N_0), he faces a loss of Rs 40000. We can represent it as a gain of INR –40000. Similarly, if he accepts the offer and only one TV is sold (corresponding to the state of nature N_1), he faces a loss of INR 25000 (= 40000 – 15000) or a gain of

–25000 and so on. The complete payoff table where each output is gain is shown in Table 9.3.

Table 9.3: Payoff Table for the Shopkeeper

States of Nature	**Courses of Action**	
	Accept the Offer (A_1)	**Reject the Offer (A_2)**
$\mathbf{N_0}$	–40000	0
$\mathbf{N_1}$	–25000	0
$\mathbf{N_2}$	–10000	0
$\mathbf{N_3}$	5000	0
$\mathbf{N_4}$	20000	0

From Table 9.3, note that to earn the maximum profit, the shopkeeper should accept the offer and should be able to sell all four TV sets. In this way, he will earn the maximum profit of Rs 20000.

So far, we have defined three components of decision making: courses of action, states of nature and payoff values. We have also explained how to construct a payoff table. We have to define these components in all decision making criteria to solve decision making problems. But some criteria of solving decision making problems such as the Salvage Criterion (discussed in this unit) and Expected Opportunity Loss (EOL) criterion (discussed in the next unit) use one more component of decision making known as **opportunity loss values** or **regret values.** We now discuss how to calculate opportunity loss values or regret values and construct the opportunity loss table or regret table.

9.4.4 Opportunity Loss Table or Regret Table

To obtain the opportunity loss table or regret table, we first need to form the payoff table. Then we have to examine whether the payoff values represent profits or losses. If the payoff values represent profits, the opportunity loss table is obtained as follows:

Step 1: We select the maximum payoff values for each state of nature.

Step 2: We subtract the payoff values of different courses of action for each state of nature from the maximum payoff value selected for that state of nature.

The opportunity loss table corresponding to the payoff Table 9.2 assuming that the payoff values represent profits is shown in Table 9.4, where

M_i = Maximum payoff value corresponding to the i^{th} state of nature, i = 1, 2, …, m.

Table 9.4: Opportunity Loss or Regret Table when Payoff Values x_{ij} Represent Profits or Gains

States of Nature	**Courses of Action (Alternatives)**						
	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	…	$\mathbf{A_j}$	…	$\mathbf{A_n}$
$\mathbf{N_1}$	$M_1 - x_{11}$	$M_1 - x_{12}$	$M_1 - x_{13}$	…	$M_1 - x_{1j}$	…	$M_1 - x_{1n}$
$\mathbf{N_2}$	$M_2 - x_{21}$	$M_2 - x_{22}$	$M_2 - x_{23}$	…	$M_2 - x_{2j}$	…	$M_2 - x_{2n}$
$\mathbf{N_3}$	$M_3 - x_{31}$	$M_3 - x_{32}$	$M_3 - m_{33}$	…	$M_3 - x_{3j}$	…	$M_3 - x_{3n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$\mathbf{N_i}$	$M_i - x_{i1}$	$M_i - x_{i2}$	$M_i - x_{i3}$	…	$M_i - x_{ij}$	…	$M_i - x_{in}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$\mathbf{N_m}$	$M_m - x_{m1}$	$M_m - x_{m2}$	$M_m - x_{m3}$	…	$M_m - x_{mj}$	…	$M_m - x_{mn}$

But if payoff values of the payoff table represent losses, then the opportunity loss table is obtained as follows:

Step 1: Select minimum payoff value for each state of nature.

Step 2: For each state of nature, subtract the minimum payoff value from all payoff values corresponding to different courses of action.

The opportunity loss table corresponding to payoff Table 9.2 assuming that the payoff values represent losses is shown in Table 9.5, where

$m_i =$ min payoff value corresponding to the i^{th} state of nature, i =1, 2, 3, …, m.

Table 9.5: Opportunity Loss or Regret Table when Payoff Values x_{ij} Represent Losses or Costs

States of Nature (Events)	Courses of Action (Alternatives)						
	A_1	A_2	A_3	…	A_j	…	A_n
N_1	$x_{11}-m_1$	$x_{12}-m_1$	$x_{13}-m_1$	…	$x_{1j}-m_1$	…	$x_{1n}-m_1$
N_2	$x_{21}-m_2$	$x_{22}-m_2$	$x_{23}-m_2$	…	$x_{2j}-m_2$	…	$x_{2n}-m_2$
N_3	$x_{31}-m_3$	$x_{32}-m_3$	$x_{33}-m_3$	…	$x_{3j}-m_3$	…	$x_{3n}-m_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_i	$x_{i1}-m_i$	$x_{i2}-m_i$	$x_{i3}-m_i$	…	$x_{ij}-m_i$	..	$x_{in}-m_i$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_m	$x_{m1}-m_m$	$x_{m2}-m_m$	$x_{m3}-m_m$	…	$x_{mj}-m_m$	…	$x_{mn}-m_m$

Let us consider an example to explain the calculations involved in obtaining an opportunity loss table.

Example 3: Obtain the opportunity loss table for the data given in Example 1.

Solution: We have already obtained the payoff table for the data of Example 1 in Sec. 9.4.3 and the same is shown in Table 9.1.

Since payoff values in the payoff table shown in Table 9.1 represent profit of the fruit seller, the opportunity loss table is obtained by following the two steps given below.

Step 1: We select the maximum payoff value (Max PV) for each state of nature given in Table 9.1:

$$\therefore \text{Max PV for the state of nature } N_1 = \max(N_1)$$
$$= \max\{480, 430, 380\}$$
$$= 480$$

Similarly,

$$\max(N_2) = \max\{480, 490, 440\} = 490$$
$$\max(N_3) = \max\{480, 490, 500\} = 500$$

Step 2: We subtract all payoff values corresponding to different courses of action for each state of nature from the maximum payoff value for that state of nature. That is, for the state of nature N_1, we subtract all payoff values from 480. Similarly, we subtract all payoff values from 490 and 500, respectively, for the states of nature N_2 and N_3 as shown in Table 9.6:

Table 9.6: Opportunity Loss Table for Example 3

States of Nature	Courses of Action		
	$A_1(48)$	$A_2(49)$	$A_3(50)$
$N_1(48)$	480 – 480 = 0	480 – 430 = 50	480 – 380 = 100
$N_2(49)$	490 – 480 = 10	490 – 490 = 0	490 – 440 = 50
$N_3(50)$	500 – 480 = 20	500 – 490 = 10	500 – 500 = 0

You may now like to try the following exercises based on the components of decision making.

E1) A bangle seller gets the following offer:

- The cost of a pack of 10 dozen bangles is Rs 150.
- The bangle seller can either accept the offer or reject it.
- Each pack of 10 dozen bangles may have 40%, 30%, 20% or 10% defective bangles. Defective bangles cannot be returned by the bangle seller. Thus, defective bangles in a pack mean a direct loss to the bangle seller.

The bangle seller sells them at the rate of Rs 20 per dozen. Assume that if he/she accept the offer, all the bangles are sold out. On the basis of this information,

(i) Identify the courses of action,

(ii) Identify states of nature, and

(iii) Obtain the payoff table.

E2) Obtain the opportunity loss table for the data of E1.

The challenge for the decision maker is to select one of the possible courses of action in a given situation. The ultimate goal of this unit and the next unit is to enable you to face this challenge. An important aspect of decision making is to know the environment in which a decision is to be made. This is what we discuss in Sec. 9.5.

9.5 TYPES OF ENVIRONMENT

For a decision maker to arrive at a good decision in a given situation, he/she has to consider all the available data, courses of action or alternatives and states of nature. He/she should have a knowledge of the decision making environment, and also know how to use the relevant quantitative approach for decision making. In this section, we discuss different types of environment under which a decision maker may have to make a decision. This discussion is important because the selection of decision criteria depends on the environment under which a decision is to be taken. The following types of environment are categorised on the basis of the degree of certainty:

- Decision Making under Certainty,
- Decision Making under Uncertainty, and
- Decision Making under Risk.

We discuss the first two types of environment in this section and the third one in detail in Unit 10. We shall discuss one more type of environment, namely, "Decision Making under Conflict" in Units 11 and 12 on Game Theory since this type of environment is present in games between two or more players.

9.5.1 Decision Making under Certainty

In certain problems, the information available to the decision maker is almost complete so that he/she knows all the facts about the states of nature and also knows which state of nature would occur. He/she also knows the consequences. In such a situation, decision making is easy. The decision maker has to choose that alternative, which will give him/her the maximum payoff in terms of utility or output under the state of nature. In the payoff table, there will be only one column indicating the state of nature and he/she will pick up the alternative which optimises his/her payoff. For instance, the decision of purchasing Kisan Vikas Patra (KVP) or National Saving Certificate (NSC) ensures the future benefit the stakeholder will get at the time of maturity. In such investments, a stakeholder knows the amount of maturity in advance. Decision making under certainty is simple and easy for decision makers.

At first sight, it appears to be a simple problem, to select an alternative which optimises the payoff when there is a single state of nature. However, when the courses of action are large, the number of rows would be very large. For example, if there are 25 jobs to be assigned to 25 machines with different outputs and different costs of production, the total number of alternatives/rows would be in millions. In such complex situations, the decision making method becomes less efficient. Then the tools of operations research, such as linear/non-linear programming, etc. need to be used to arrive at decisions. You will study linear programming problems in detail in Block 1 of the course MSTE-002.

9.5.2 Decision Making under Uncertainty

We have already defined decision making under uncertainty in Sec. 9.4.2. Recall that under this scenario, the decision maker has no prior information about the probabilities of different states of nature. Undoubtedly, he/she can collect information, if possible, about the probabilities of the states of nature (e.g., by conducting market surveys).

In business situations, there are many problems of this nature and the choice of a course of action depends on the personality or judgment of the decision maker. The responses of decision makers may be termed as optimistic, pessimistic or those of least regret, etc. Consider the example of a half-filled glass of water. An optimist would say that the glass is half-filled while a pessimist would say that the glass is half-empty. And the Manager would say that the glass is twice as large as it needs to be!

Some criteria used for decision making under such situations are as follows:

- Optimistic (Maximax/Minimin) Criterion
- Pessimistic (Maximin) Criterion
- Hurwicz (Coefficient of optimism or realism) Criterion
- Regret (Salvage) Criterion
- Laplace (Equal Probability or Rationality) Criterion

We now discuss each one of these criteria with examples.

i) **Optimistic (Maximax/Minimin) Criterion**

Under this criterion, the decision maker first selects the maximum payoff value in case of profit or gain and the minimum payoff value in case of loss or cost from among the payoff values for each course of action. Then he/she selects the maximum in case of gains (or minimum in case of losses) from among the selected payoff values. Since the approach of the decision maker is completely positive, it is called the optimistic criterion. Thus, if payoff values represent profits or gains, the decision maker selects the maximum from among maximum payoff values for each course of action. Hence, it is known as the **maximax criterion**. Similarly, if payoff values represent losses or costs, the decision maker selects the minimum from among the minimum payoff values for each course of action. Hence, it is known as a **minimin criterion**. For solving the problems under optimistic criterion, we follow the steps given below:

Step 1: We select the maximum payoff value in case of profit or gain or the minimum payoff value in case of loss or cost for each course of action.

Step 2: We select the maximum from among the payoff values selected in Step 1 in case of profit or gain or the minimum from among the payoff values selected in Step 1 in case of loss or cost.

Let us explain the procedure with the help of an example.

Example 4: The profit of a milk booth of a particular brand under small, medium and large orders subject to low, moderate and high demands are shown in Table 9.7.

Table 9.7: Profits of the Owner of the Milk Booth

Order for Milk	Demand of Milk at the Booth		
	Low	**Moderate**	**High**
Small	1200	1200	1200
Medium	1000	1500	1500
Large	800	1300	1800

Using the optimistic criterion, take a decision on which order is more beneficial for the owner.

Recall from Unit 13 of MST-001 that **stubs** are row headings and explain what information the rows present. Stubs are placed at the extreme left of the table.

Solution: The quantity of the milk ordered is under the control of the owner of the booth. So the courses of action are small (A_1), medium (A_2) and large (A_3). But the demand for any day is not under the control of the owner of the booth. So the states of nature are low (N_1), moderate (N_2) and high (N_3). This information can be expressed in the payoff table (Table 9.8) as follows:

Table 9.8: Payoff Table for the Owner of the Booth

States of Nature	Courses of Action		
	Small (A_1)	**Medium (A_2)**	**Large (A_3)**
Low (N_1)	1200	1000	800
Moderate (N_2)	1200	1500	1300
High (N_3)	1200	1500	1800

We now follow the two steps under the optimistic criterion:

Step 1: We look at the payoff values and assess whether these values represent profits or losses. In this case, the values represent profits. So we select the maximum payoff value for each course of action. These are shown in Table 9.9 against the stub 'Step 1'.

Step 2: We select the maximum among the payoff values selected in Step 1. Thus, $\max\{1200, 1500, 1800\} = 1800,$ and it corresponds to the course of action A_3 as shown in the Table 9.9 against the stub 'Step 2'. Hence, the decision maker should suggest to the owner that he/she should go with the large order.

Table 9.9: Payoff Table for the Owner of the Booth and Calculations of Step 1 and Step 2

States of Nature	Courses of Action		
	Small (A_1)	Medium (A_2)	Large (A_3)
Low (N_1)	1200	1000	800
Moderate (N_2)	1200	1500	1300
High (N_3)	1200	1500	1800
Step 1	1200	1500	1800
Step 2	$\max\{1200, 1500, 1800\} = 1800$		

ii) Pessimistic (Maximin) Criterion

Under this criterion, the decision maker first selects the minimum payoff value for each course of action. Then he/she selects the maximum among these selected payoff values. Thus, the approach of the decision maker is pessimistic and so this criterion is known as pessimistic (maximin, that is, maximum among minima) criterion. It is also known as Wald's criterion. For solving problems, we adopt the following two steps:

Step 1: We select the minimum payoff value for each course of action.

Step 2: We select the maximum from among the minimum payoff values selected in Step 1. The course of action corresponding to this maximum payoff value is selected under this criterion.

Let us apply this criterion to the data of Example 4 given in Table 9.10.

Step 1: We first select the minimum payoff value for each course of action as shown against the stub 'Step 1' in Table 9.10.

Step 2: We select the maximum from among the minimum payoff values selected in Step 1. Thus, we select $\max\{1200, 1000, 800\} = 1200$, which corresponds to the course of action A_1. It is also shown in Table 9.10 against the stub 'Step 2'. Hence, the owner of the booth should go with the small order under this criterion.

Table 9.10: Payoff Table for the Owner of the Booth and Calculation of Step 1 and Step 2

States of nature	Courses of action		
	Small (A_1)	Medium (A_2)	Large (A_3)
Low (N_1)	1200	1000	800
Moderate (N_2)	1200	1500	1300
High (N_3)	1200	1500	1800
Step 1	1200	1000	800
Step 2	$\max\{1200, 1000, 800\} = 1200$		

iii) Hurwicz Criterion

In the optimistic criterion, the approach of the decision maker was completely positive while in the pessimistic criterion, it was pessimistic. But Hurwicz criterion suggests that a decision maker should neither be too optimistic nor too pessimistic. For this purpose, Hurwicz introduced the idea of the **coefficient of optimism**. It is generally denoted by α. The value of α varies from 0 to 1 (i.e., $0 \le \alpha \le 1$), and measures the degree of optimism of the decision maker. The two extreme values $\alpha = 1$ and $\alpha = 0$ indicate completely optimistic and pessimistic approaches, respectively. Thus, the optimistic criterion and pessimistic criterion are particular cases of Hurwicz criterion. Also, $(1-\alpha)$ is known as the **coefficient of pessimism**.

The steps used to solve problems using the Hurwicz criterion are given below:

Step 1: We select the value of α as per the degree of optimism of the decision maker and consequently $1-\alpha$, the coefficient of pessimism is known.

Step 2: For each course of action, we identify the maximum as well as minimum payoff values and obtain the value of H, where

$$H = \alpha \times (\text{Max PV corresponding to a course of action})$$
$$+ (1-\alpha) \times (\text{Min PV corresponding to the same course of action})$$

Step 3: We select the course of action which corresponds to the maximum value of H obtained in Step 2. The course of action thus obtained will be the optimum course of action for the decision maker under Hurwicz criterion for the selected value of α.

Let us consider the data of Example 4 again to explain the calculation involved in this criterion for, say, $\alpha = 0.6$:

We have already defined the courses of action and states of nature for Example 4. So, we go directly to Step 1.

Step 1: We have selected $\alpha = 0.6$ and, therefore, $1-\alpha = 0.4$.

Step 2: Calculations of this step are shown in Table 9.11 against the stub 'Step 2'.

Table 9.11: Payoff Table for the Owner of the Booth and Calculation of Step 1and Step 2

States of Nature		Courses of Action		
		Small (A_1)	**Medium (A_2)**	**Large (A_3)**
Low (N_1)		1200	1000	800
Moderate (N_2)		1200	1500	1300
High (N_3)		1200	1500	1800
Step 2	**Max PV for each Course of action**	1200	1500	1800
	Min PV for each Course of action	1200	1000	800
	Value of H for each Course of action	0.6×1200 $+0.4\times1200$ $=1200$	0.6×1500 $+0.4\times1000$ $=1300$	0.6×1800 $+0.4\times800$ $=1400$
Step 3		$\max\{1200, 1300, 1400\} = 1400$		

Step 3: In this step we select the maximum among the values of H obtained in Step 2.

$\therefore \max\{1200, 1300, 1400\} = 1400,$ which corresponds to the course of action A_3. Hence, the large order (A_3) is the optimum course of action under this criterion.

iv) Regret (Salvage) Criterion

In this criterion, first of all we convert the payoff table into an **opportunity loss table** about which you have learnt in Sec. 9.4.4. Under this criterion, the goal of the decision maker is to minimise the opportunity loss which he/she may face by not selecting the best opportunity or by adopting a wrong course of action. For example, suppose two years ago you had surplus money and were planning to buy a flat from the investment point of view. Suppose at that time, flats were available within your budget at three locations, say X, Y and Z at the same price. You had to choose only one location and you chose location X. Now, after two years, the rates of the flats at the three locations X, Y, Z have increased by 30%, 20% and 42%, respectively. From the point of view of making a profit, you would regret your decision, because had you selected location Z, you would have earned (42 – 30)% = 12% more compared to what you earned for location X. This is an example of opportunity loss. You had an opportunity two years ago, which you missed because of making a wrong selection. We would like to minimise the opportunity loss due to the possibility of wrong decision. The regret criterion suggests **the course of action, which minimises our opportunity loss**. The following three steps are involved in using the regret criterion for solving problems:

Step 1: We form the opportunity loss matrix by calculating the amount of regret corresponding to each course of action and each state of nature. You have learnt these calculations in Sec. 9.4.4.

Step 2: We identify the maximum regret amount for each course of action from the opportunity loss table obtained in Step 1.

Step 3: We select the minimum opportunity loss value from among the regret amounts selected in Step 2. The optimum course of action corresponds to the minimum amount selected from among the regret amounts obtained in Step 2.

Let us consider the data of Example 4 to explain the calculations involved in this criterion.

We have already defined the courses of action and states of nature for Example 4. We have also obtained the payoff table for the data of Example 4. So, we go directly to Step 1.

Step 1: Since the payoff values represent profits, to obtain the opportunity loss table, we first calculate the maximum payoff value (Max PV) for each state of nature as shown below:

$$\text{Max PV for state of nature } N_1 = \max\{1200, 1000, 800\} = 1200$$

$$\text{Max PV for state of nature } N_2 = \max\{1200, 1500, 1300\} = 1500$$

$$\text{Max PV for state of nature } N_3 = \max\{1200, 1500, 1800\} = 1800$$

We then subtract all payoff values corresponding to different courses of action for the state of nature N_1 from 1200. We repeat the calculation for the states of nature N_2 and N_3 as shown in Table 9.12.

Table 9.12: Opportunity Loss Table

Status of Nature	Courses of Action		
	Small (A_1)	**Medium (A_2)**	**Large (A_3)**
Low (N_1)	1200 – 1200 = 0	1200 – 1000 = 200	1200 – 800 = 400
Moderate (N_2)	1500 – 1200 = 300	1500 – 1500 = 0	1500 – 1300 = 200
High (N_3)	1800 – 1200 = 600	1800 – 1500 = 300	1800 – 1800 = 0
Step 2	600	300	400
Step 3	$\min\{600, 300, 400\} = 300$		

Step 2: Maximum amount of regret for the course of action $A_1 = \max\{0, 300, 600\} = 600$. Similarly, the maximum amount of regret for the courses of action A_2 and A_3 are 300 and 400, respectively, as shown in Table 9.12 against the stub 'Step 2'.

Step 3: We select the minimum regret amount from among the amounts obtained in Step 2.

$$\therefore \min\{600, 300, 400\} = 300.$$

This corresponds to the course of action A_2. Hence, A_2 is the optimum course of action under this criterion. It is also shown in Table 9.12 against the stub 'Step 3'.

v) **Laplace Criterion**

This criterion is a method to solve the problems that involve "decision making under uncertainty". You have learnt in Sec. 9.4.2 that in the situations of decision making under uncertainty, we do not have any information about the probabilities of occurrence of different states of nature. But this criterion works by giving equal probabilities to each state of nature. The point to be noted here is that we apply this criterion in a given situation only if we do not have sufficient information about the inequality of the probabilities of different states of nature. This is because we are assuming equal probabilities for different states of nature for each course of action.

The following steps are involved in the calculations for this criterion:

Step 1: We assign an equal probability to each state of nature. That is, if there are m states of nature, then we assign the probability 1/m to each state of nature.

Step 2: We calculate the expected payoff value for each course of action. For example, if $x_{1j}, x_{2j}, ..., x_{mj}$ denote the payoff values for the j^{th} course of action corresponding to m states of nature, then the **expected payoff value** (EPV) of the j^{th} course of action is given by:

$$\text{EPV for } j^{th} \text{ course of action } = \frac{1}{m}\left(x_{1j} + x_{2j} + ... + x_{mj}\right)$$

Step 3: We select the maximum expected payoff value from among the values calculated in Step 2. The optimum course of action corresponds to this maximum expected payoff value.

If, in Example 4, we do not have information that the three states of demands (i.e., low, moderate and high) have different probabilities, we can apply this criterion. The calculations involved in this criterion are explained below:

Step 1: Here we have three states of nature and we know that under this criterion each state of nature is assigned equal probability. So each state of nature will be assigned the probability 1/3.

Step 2: The expected payoff value (EPV) for each course of action is given by:

$$\text{EPV for course of action } A_1 = \frac{1}{3}(1200 + 1200 + 1200) = \frac{3600}{3} = 1200$$

$$\text{EPV for course of action } A_2 = \frac{1}{3}(1000 + 1500 + 1500) = \frac{4000}{3} \simeq 1333.33$$

$$\text{EPV for course of action } A_3 = \frac{1}{3}(800 + 1300 + 1800) = \frac{3900}{3} = 1300$$

Step 3: We select the maximum EPV from among the values obtained in Step 2. $\max\{1200, 1333.33, 1300\} = 1333.33$, which corresponds to the course of action A_2. Hence, A_2 is the optimum course of action under this criterion.

Let us consider another example.

Example 5: Consider the following payoff table

States of Nature	**Courses of Action**			
	A_1	**A_2**	**A_3**	**A_4**
N_1	400	900	900	1000
N_2	200	400	700	–300
N_3	600	200	500	700

Identify the optimum course of action under:

(i) Optimistic criterion (assume that payoff values represent profits),

(ii) Pessimistic criterion,

(iii) Hurwicz criterion $\left(\text{for}\,\alpha = 0.8\right)$,

(iv) Regret criterion (assume that payoff values represent losses), and

(v) Laplace criterion (assume that we do not have sufficient information that the states of nature N_1, N_2, N_3 have different probabilities).

Solution:

(i) The payoff values represent profits. So under the optimistic criterion we follow the two steps given below:

Step 1: We select a maximum payoff value (Max PV) for each course of action:

$$\text{Max PV for course of action } A_1 = \max\{400, 200, 600\} = 600$$

The maximum payoff values for the courses of action A_2, A_3, A_4 are 900, 900, 1000, respectively.

Step 2: We select the maximum from among the payoff values selected in Step 1. Thus, $\max\{600, 900, 900, 1000\} = 1000$, which corresponds to the course of action A_4. It is shown in Table 9.13 against the stub 'Step 2'. Hence A_4 is the optimum course of action.

Table 9.13: Calculation for Optimistic Criterion

States of Nature	Courses of Action			
	A_1	A_2	A_3	A_4
N_1	400	900	900	1000
N_2	200	400	700	–300
N_3	600	200	500	700
Step 1	600	900	900	1000
Step 2	$\max\{600, 900, 900, 1000\} = 1000$			

(ii) To identify an optimum course of action under the pessimistic criterion, we follow the two steps given below:

Step 1: We select the minimum payoff value for each course of action:

$$\text{Min PV for the course of action } A_1 = \min\{400, 200, 600\} = 200.$$

Similarly, the minimum payoff values for the courses of action A_2, A_3, A_4 are 200, 500, –300, respectively. These values are shown in Table 9.14 against the stub 'Step 1'.

Step 2: We select the maximum from among the payoff values selected in Step 1. Thus, $\max\{200, 200, 500, -300\} = 500$, which corresponds to the course of action A_3. It is shown in Table 9.14 against the stub 'Step 2'. Hence, A_3 is the optimum course of action under the pessimistic criterion.

Table 9.14: Calculation for the Pessimistic Criterion

States of Nature	Courses of Action			
	A_1	A_2	A_3	A_4
N_1	400	900	900	1000
N_2	200	400	700	–300
N_3	600	200	500	700
Step 1	200	200	500	–300
Step 2	$\max\{200, 200, 500, -300\} = 500$			

(iii) To identify an optimum course of action under the Hurwicz criterion for $\alpha = 0.8$, we follow the three steps given below:

Step 1: Here we are given that $\alpha = 0.8$ and so $1 - \alpha = 0.2$.

Step 2: We identify the maximum as well as the minimum payoff values for each course of action and obtain H, where

$$H = \alpha \times (\text{Max PV corresponding to a course of action})$$
$$+(1-\alpha) \times (\text{Min PV corresponding to the same course of action})$$
$$= 0.8 \times (\text{Max PV corresponding to a course of action})$$
$$+0.2 \times (\text{Min PV corresponding to the same course of action})$$

Calculations of this step for each course of action are shown in Table 9.15 against 'Step 2'.

Table 9.15: Calculation for Hurwicz Criterion

States of Nature		Courses of Action			
		A_1	A_2	A_3	A_4
N_1		400	900	900	1000
N_2		200	400	700	–300
N_3		600	200	500	700
Step 2	**Max PV**	600	900	900	1000
	Min PV	200	200	500	– 300
	Value of H	$0.8 \times 600 + 0.2 \times 200 = 520$	$0.8 \times 900 + 0.2 \times 200 = 760$	$0.8 \times 900 + 0.2 \times 500 = 820$	$0.8 \times 1000 + 0.2 \times (-300) = 740$
Step 3		$\max\{520, 760, 820, 740\} = 820$			

Step 3: We select the maximum value of H from among the values obtained in Step 2.

$\therefore$ Max $\{520, 760, 820, 740\} = 820$, which corresponds to the course of action A_3. It is also shown against the stub 'Step 3' in Table 9.15. Hence, A_3 is the optimum course of action under the Hurwicz criterion.

(iv) It is given that payoff values represent losses. So, under the regret criterion, we follow the three steps given below:

Step 1: We first obtain the opportunity loss matrix. Since it is given that the payoff values represent losses, to obtain opportunity loss table, we calculate the minimum payoff value (Min PV) for each state of nature as shown below:

For the state of nature N_1, it is $\min\{400, 900, 900, 1000\} = 400$, For the state of nature N_2, it is $\min\{200, 400, 700, -300\} = -300$, and

For the state of nature N_3, it is $\min\{600, 200, 500, 700\} = 200$.

We now subtract the minimum payoff value (400) from all payoff values corresponding to different courses of action for the state of nature N_1. We do the same calculations for the states of nature N_2 and N_3 as shown in Table 9.16.

Step 2: We identify the maximum regret amount for each course of action from the opportunity loss table obtained in Step 1. This is shown in Table 9.16 against the stub 'Step 2'.

Step 3: We select the minimum regret amount from among the regret amounts obtained in Step 2. Thus, $\min\{500, 700, 1000, 600\} = 500$, which corresponds to the course of action A_1. It is also shown against the stub 'Step 3' in Table 9.16. Hence, A_1 is the optimum course of action under the regret criterion.

Table 9.16: Calculation for the Regret Criterion

States of Nature	**Courses of Action**			
	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$
$\mathbf{N_1}$	400 – 400 = 0	900 – 40 = 500	900 – 40 = 500	1000 – 40 = 600
$\mathbf{N_2}$	200 –(–300) = 500	400 – (–300) = 700	700 – (–300) = 1000	–300 – (– 300 = 0
$\mathbf{N_3}$	600 – 20 = 400	200 – 200 = 0	500 – 20 = 300	700 – 200 = 500
Step 2	500	700	1000	600
Step 3	$\min\{500, 700, 1000, 600\} = 500$			

(v) It is given that there is no sufficient information that the states of nature N_1, N_2 and N_3 have different probabilities. So, we can opt for the Laplace criterion. The calculations involved in this criterion are explained in the following three steps:

Step 1: We assign equal probability to each state of nature. Here, we have three states of nature. So each state of nature will be assigned the probability 1/3.

Step 2: We calculate the expected payoff value (EPV) for each course of action as follows:

$$\text{EPV for the course of action } A_1 = \frac{400 + 200 + 600}{3} = \frac{1200}{3} = 400$$

$$\text{EPV for the course of action } A_2 = \frac{900 + 400 + 200}{3} = \frac{1500}{3} = 500$$

$$\text{EPV for the course of action } A_3 = \frac{900 + 700 + 500}{3} = \frac{2100}{3} = 700$$

$$\text{EPV for the course of action } A_4 = \frac{1000 + (-300) + 700}{3} = \frac{1400}{3}$$

□ 466.67

Step 3: We select the maximum EPV from among the values obtained in Step 2. Thus, Max $\{400, 500, 700, 466.67\} = 700$, which corresponds to the course of action A_3. Hence, A_3 is the optimum course of action under the Laplace criterion.

You may now like to identify the optimum course of action for the following exercise.

E3) In the following payoff table, the payoff values represent losses:

Table 9.17: Payoff Table for E3 where Payoff Values Represent Losses

States of Nature	Courses of Action		
	A_1	**A_2**	**A_3**
N_1	1500	1800	2500
N_2	2400	1600	2000
N_3	1200	2000	3000
N_4	800	1000	400

On the basis of this information, identify the optimum course of action under each of the following criteria:

(i) Optimistic Criterion,

(ii) Pessimistic Criterion,

(iii) Hurwicz Criterion (for $\alpha = 0.7$),

(iv) Regret Criterion, and

(v) Laplace criterion (assuming that we have no information that the states of nature N_1, N_2, N_3, N_4 have different probabilities)

Let us summarise the main points that we have discussed in this unit:

9.6 SUMMARY

1) Decision making in situations where decisions are taken on the basis of the experience of an individual, rather than by applying scientific tools or techniques is known as an **art**.

2) Decision making in situations where decisions are taken by applying scientific tools or techniques rather than on the basis of the experiences of an individual is known as a **science**.

3) Structured decisions are made under established situations. **Structured** or **programmed decisions** are the ones where the organisation has already faced such decisions in the past and the employees are used to solving such problems.

4) Decisions, that are taken to solve problems which have not been faced before in the organisation and cannot be solved using any known procedures are called **unstructured decisions**.

5) To apply techniques of decision theory in any real life situation, we first need to break down the given situation into the following components:

   - **Courses of Action (Alternatives):** Possible options available in a given situation of decision making which are **under the control** of the decision maker are known as **courses of action** or **alternatives**.

   - **States of Nature (Events):** States of nature or events are related to **future outcomes**, conditions or situations which are **not under the control** of the decision maker.

   - **Payoff Values and Payoff Matrix:** Total possible outcomes which result from combination of each course of action and each state of nature in terms of money or some other value of interest are known as **payoff values**. We can represent payoff values in the form of a **payoff**

table or **payoff matrix**. Payoff values are also known as **conditional profits** if they represent profits and **conditional losses** if they represent losses (because of the states of nature which are not under the control of the decision maker).

6) The selection of a decision criterion depends on the environment in which decision is to be taken. The following types of environment are categorised on the basis of the degree of certainty:

- Decision Making under Certainty,
- Decision Making under Uncertainty, and
- Decision Making under Risk.

7) Several criteria are used for decision making under uncertainty. Some of these are:

   i) Optimistic (Maximax/Minimin) Criterion
   ii) Pessimistic (Maximin) Criterion
   iii) Regret (Salvage) Criterion
   iv) Hurwicz (Coefficient of optimism or realism) Criterion
   v) Laplace (Equal Probability or Rationality) Criterion

9.7 SOLUTIONS/ANSWERS

E1) (i) The bangle seller has two options: he/she can either accept the offer or reject it. Also accepting or rejecting the offer is under his/her control, and so the courses of action, A_1 and A_2, are

A_1 : accept the offer.

A_2 : reject the offer.

(ii) It is assumed that if he/she accepts the offer, there is no issue of demand as all the bangles are sold out. So, demand does not form states of nature. But the number of defective bangles is a future event and may be 40%, 30%, 20% or 10%. Since the number of defective bangles are not under the control of the bangle seller, these form the states of nature. If we denote these states of nature by N_1, N_2, N_3, N_4, respectively, then

N_1 : represents the state of 40% defective bangles

N_2 : represents the state of 30% defective bangles

N_3 : represents the state of 20% defective bangles

N_4 : represents the state of 10% defective bangles

iii) In the first two parts we have identified the courses of action and the states of nature. The payoff values are nothing but the outcomes in terms of money of all possible combinations of courses of action and states of nature. If he/she rejects the offer (i.e., he/she adopts the course of action A_2), then there will be no profit and no loss and so payoff values corresponding to all states of nature will be zero. But if he/she accepts the offer, then his/her profit will vary according to the states of nature as explained below:

Case I: For the state of nature N_1, The total number of bangles in one packing $= 10$ dozen

Number of defective bangles $= 40\%$ of 10 dozen $= 4$ dozen

Bangles which can be sold out = 10 – 4 = 6 dozen

Cost price of bangles $= \text{Rs}\,\frac{150}{10} = \text{Rs}\,15$ per dozen

Selling price of bangles = Rs 20 per dozen

Profit per dozen on good bangles = Rs (20 – 15) = Rs 5

$$\therefore \text{ Payoff value } = 5 \times (\text{Number of good bangles in dozen})$$

$$-15 \times \begin{pmatrix}\text{Number of defective} \\ \text{bangles in dozen}\end{pmatrix}$$

$$= 5\times 6 - 15\times 4 = 30 - 60 = -\text{Rs}\,30$$

Case II: For the state of nature N_2,

The number of defective bangles $= 30\%$ of 10 dozen

$= 3$ dozen

$$\therefore \text{Payoff value } = 5\times(10-3) - 15\times 3 = 35 - 45 = -\text{Rs}\,10$$

Case III: For the state of nature N_3

The number of defective bangles $= 20\%$ of 10 dozen

$= 2$ dozen

$$\therefore \text{ Payoff value } = 5\times(10-2) - 15\times 2 = 40 - 30 = \text{Rs } 10$$

Case IV: For the state of nature N_4,

The number of defective bangles $= 10\%$ of 10 dozen

$= 1$ dozen

$$\therefore \text{Payoff value } = 5\times(10-1) - 15\times 1 = 45 - 15 = \text{Rs } 30$$

Thus, the payoff table for the bangle seller is given as follows:

Table 9.18: Payoff Table for Bangle Seller

States of Nature	**Courses of Action**	
	A_1 (Accept the Offer)	**A_2 (Reject the Offer)**
N_1 (40%)	–30	0
N_2 (30%)	–10	0
N_3 (20%)	10	0
N_4 (10%)	30	0

E 2) We have already obtained the payoff table for the data of E1. Also, the payoff values in the payoff table represent profits of the bangle seller. So, the opportunity loss table is obtained by using the same two steps as explained in Example 3 and is given below.

Table 9.19: Opportunity Loss Table

States of Nature	Courses of Action	
	A_1 (Accept the Offer)	A_1 (Reject the Offer)
N_1 (40%)	0 – (–30) = 30	0 – 0 = 0
N_2 (30%)	0 – (–10) = 10	0 – 0 = 0
N_3 (20%)	10 – 10 = 0	10 – 0 = 10
N_4 (10%)	30 – 30 = 0	30 – 0 = 30

E3) i) It is given that the payoff values represent losses. So under the optimistic criterion, we proceed as follows:

Step 1: We select a minimum payoff value for each course of action.

$\therefore$ For the course of action A_1, it is $\min\{1500, 2400, 1200, 800\}$ $= 800$

Similarly, the minimum payoff values for the courses of action A_2, A_3 are 1000, 400, respectively.

Step 2: We select the minimum from among the payoff values selected in Step 1. Thus, $\min\{800, 1000, 400\} = 400$, which corresponds to the course of action A_3. It is also shown in Table 9.20 against the stub 'Step 2'. Hence, A_3 is the optimum course of action.

Table 9.20: Calculation for Optimistic Criterion

States of Nature	Courses of Action		
	A_1	A_2	A_3
N_1	1500	1800	2500
N_2	2400	1600	2000
N_3	1200	2000	3000
N_4	800	1000	400
Step 1	800	1000	400
Step 2	$\min\{800, 1000, 400\} = 400$		

ii) To identify an optimum course of action under the pessimistic criterion, we follow the two steps given below:

Step 1: We select the maximum payoff value for each course of action.

$\therefore$ for the course of action A_1, it is $\max\{1500, 2400, 1200, 800\} = 2400$

Similarly, the maximum payoff values for the courses of action A_2, A_3 are 2000, 3000, respectively, and are shown in Table 9.21 against the stub 'Step 1'.

Step 2: We select the minimum from among the payoff values selected in Step 1. Min $\{2400, 2000, 3000\} = 2000$, which corresponds to the course of action A_2 and is shown in Table 9.21 against the stub 'Step 2'. Hence, A_2 is the optimum course of action under the pessimistic criterion.

Table 9.21: Calculation for Pessimistic Criterion

States of Nature	Courses of Action		
	A_1	A_2	A_3
N_1	1500	1800	2500

N_2	2400	1600	2000
N_3	1200	2000	3000
N_4	800	1000	400
Step 1	2400	2000	3000
Step 2	$\min\{2400, 2000, 3000\} = 2000$		

iii) To identify an optimum course of action under the Hurwicz criterion for $\alpha = 0.7$, we follow the three steps given below:

Step 1: We select the value of α as per the degree of optimism of the decision maker. It is given that $\alpha = 0.7$ and so the coefficient of pessimism is $1-\alpha = 0.3$.

Step 2: We identify the maximum as well as the minimum payoff values for each course of action and obtain H, where

$$H = \alpha \times (\text{Max PV corresponding to a course of action})$$
$$+ (1-\alpha) \times (\text{Min PV corresponding to the same course of action})$$
$$= 0.7 \times (\text{Max PV corresponding to a course of action})$$
$$+ 0.3 \times (\text{Min PV corresponding to the same course of action})$$

Calculations of H for each course of action are shown against the stub 'Step 2' in Table 9.22.

Table 9.22: Calculation for Hurwicz Criterion

<table>
<tr><th colspan="2">States of Nature</th><th colspan="3">Courses of Action</th></tr>
<tr><th colspan="2"></th><th>A_1</th><th>A_2</th><th>A_3</th></tr>
<tr><td colspan="2">N_1</td><td>1500</td><td>1800</td><td>2500</td></tr>
<tr><td colspan="2">N_2</td><td>2400</td><td>1600</td><td>2000</td></tr>
<tr><td colspan="2">N_3</td><td>1200</td><td>2000</td><td>3000</td></tr>
<tr><td colspan="2">N_4</td><td>800</td><td>1000</td><td>400</td></tr>
<tr><td rowspan="3">Step 2</td><td>Max PV</td><td>2400</td><td>2000</td><td>3000</td></tr>
<tr><td>Min PV</td><td>800</td><td>1000</td><td>400</td></tr>
<tr><td>Value of H</td><td>0.7×2400 $+ 0.3 \times 800$ $= 1920$</td><td>0.7×2000 $+ 0.3 \times 1000$ $= 1700$</td><td>0.7×3000 $+ 0.2 \times 400$ $= 2180$</td></tr>
<tr><td colspan="2">Step 3</td><td colspan="3">$\max\{1920, 1700, 2180\} = 2180$</td></tr>
</table>

Step 3: We select the maximum value of H from among the values obtained in Step 2.

Therefore, $\max\{1920, 1700, 2180\} = 2180$, which corresponds to the course of action A_3 and is also shown against the stub 'Step 3' in Table 9.22. Hence, A_3 is the optimum course of action under Hurwicz criterion.

iv) It is given that the payoff values represent losses. So, under the regret criterion we follow the three steps given below:

Step 1: In this step, we first obtain the opportunity loss matrix. Since it is given that payoff values represent losses, to obtain opportunity loss table, we calculate the minimum payoff value (Min PV) for each state of nature as follows:

$\text{Min PV for state of nature } N_1 = \{1500, 1800, 2500\} = 1500,$

$\text{Min PV for state of nature } N_2 = \{2400, 1600, 2000\} = 1600,$

$\text{Min PV for state of nature } N_3 = \{1200, 2000, 3000\} = 1200, \text{ and}$

$\text{Min PV for state of nature } N_4 = \{800, 1000, 400\} = 400.$

Now, we subtract the minimum payoff value (1500) from all payoff values corresponding to different courses of action for the state of nature N_1. Similarly, the values are calculated for the states of nature N_2, N_3 and N_4 and shown in Table 9.23.

Step 2: We identify the maximum regret amount for each course of action from the opportunity loss table obtained in Step 1. This is shown in Table 9.23 against the stub 'Step 2'.

Step 3: We select the minimum regret amount from among the regret amounts obtained in Step 2.

$\text{Min } \{800, 800, 1800\} = 800,$ which corresponds to the courses of action A_1 and A_2 and is also shown against the stub 'Step 3' in Table 9.23. Hence, both A_1 and A_2 are the optimum courses of action under the regret criterion.

Table 9.23: Calculation for Regret Criterion

States of Nature	**Courses of Action**		
	A_1	**A_2**	**A_3**
N_1	1500 – 1500 = 0	1800 – 1500 = 300	2500 – 1500 = 1000
N_2	2400 –1600 = 800	1600 – 1600 = 0	2000 – 1600 = 400
N_3	1200 – 1200 = 0	2000 – 1200 = 800	3000 – 1200 = 1800
N_4	800 – 400 = 400	1000 – 400 = 600	400 – 400 = 0
Step 2	800	800	1800
Step 3	$\min\{800, 800, 1800\} = 800$		

v) We do not have the infomation that the states of nature N_1, N_2, N_3 and N_4 have different probabilities. So we can use the Laplace criterion. The calculations are explained in the following three steps:

Step 1: We assign equal probability to each state of nature. Here we have four states of nature. So each state of nature is assigned the probability 1/4.

Step 2: We calculate the expected payoff value (EPV) for each course of action as follows:

$$\text{EPV for course of action } (A_1) = \frac{1500 + 2400 + 1200 + 800}{4} = 1475$$

$$\text{EPV for course of action } (A_2) = \frac{1800 + 1600 + 2000 + 1000}{4} = 1600$$

$$\text{EPV for course of action } (A_3) = \frac{2500 + 2000 + 3000 + 400}{4} = 1975$$

Step 3: We select the minimum EPV from among the values obtained in Step 2:

$\min\{1475, 1600, 1975\} = 1475,$ which corresponds to the course of action A_1. Hence, A_1 is the optimum course of action under the Laplace criterion.