Control Chart for Fraction Defective

SESSION 6

Structure

6.1 INTRODUCTION

In Lab Sessions 1 to 5, you have learnt how to construct the $\bar{X}$-chart, R-chart and S chart. $\bar{X}$-chart is used for controlling the mean of the quality characteristic whereas R-chart and S-chart are used for monitoring and controlling the process variability. We have already explained in Sec. 3.3 of Unit 3 of MSTE-001 (Industrial Statistics-I) that the p-chart is applied to control the **fraction or proportion defective** in the process.

In this lab session, you will learn how to prepare the control chart for fraction defective, i.e., the p-chart in MS Excel 2007 using a specific problem. In the next lab session, we shall explain how to construct the p-chart for **variable sample size**.

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Sessions 1 to 5 of MSTL-002 (Industrial Statistics Lab).
- Unit 3 of MSTE-001 (Industrial Statistics-I).

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for fraction defective;
- construct the control chart for fraction defective; and
- interpret the control chart.

6.2 PROBLEM DESCRIPTION

Let us consider a company which manufactures cream and is facing the problem of leakage in tubes of cream. Suppose these tubes are packed in boxes of 50 tubes each.

A quality control inspector in this company selected a sample of 30 boxes having 50 tubes each. The numbers of leaky tubes per box are recorded in Table 1.

Table 1: Number of leaky tubes

Sampled Box	Total number of Tubes in the Box	Number of Leaky Tubes	Sampled Box	Total number of Tubes in the Box	Number of Leaky Tubes
1	50	6	16	50	4
2	50	5	17	50	3
3	50	4	18	50	6
4	50	7	19	50	5
5	50	2	20	50	4
6	50	3	21	50	1
7	50	4	22	50	4
8	50	2	23	50	2
9	50	1	24	50	2
10	50	6	25	50	4
11	50	6	26	50	8
12	50	3	27	50	6
13	50	4	28	50	2
14	50	2	29	50	2
15	50	3	30	50	7

The quality control inspector needs to construct a control chart to check whether the process is under statistical control or not and also compute the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for fraction defective, i.e., p-chart for the data given in Table 1.

6.3 PROCEDURE FOR THE CONSTRUCTION OF p-CHART

You have already studied the detailed procedure for the construction of p-chart in Sec. 3.3 of Unit 3 of MSTE-001. It is similar to the control charts for variables discussed in the lab sessions of Part A. The main steps involved in the construction of the p-chart are as follows:

Step 1: We select k samples (subgroups) randomly from the process at different times and each selected item or unit is classified as defective or non-defective with respect to certain defects.

Step 2: After that, we count the number of defective items in each sample. Suppose there are k samples of the same size n and $d_1, d_2, \ldots, d_k$ are the numbers of defectives in 1^{st}, 2^{nd}, …, k^{th} sample of size n, respectively. Then for each sample, we calculate the fraction defective given by

$$\text{Fraction defective} = \frac{\text{Number of defectives}}{\text{Total number of items inspected}}$$

$$p_i = \frac{d_i}{n}; \quad (i = 1, 2, \ldots, k) \qquad \ldots(1)$$

Step 3: We find the average of all fraction defectives as follows:

$$\bar{p} = \frac{1}{k}(p_1 + p_2, \ldots, p_k) = \frac{1}{k}\sum_{i=1}^{k} p_i \qquad \ldots(2)$$

Step 4: We set up the control limits to check whether the process is under control or out-of-control. The control limits for the p-chart are:

✓ Centre line (CL) $= \bar{p}$ ...(3)

✓ Upper control limit (UCL) $= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ ...(4)

✓ Lower control limit (LCL) $= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ ...(5)

Note : If the value of LCL comes out to be negative, we should consider it as zero because negative fraction defective is not possible.

Step 5: Having set the centre line and control limits, we construct the p-chart by taking sample numbers on the X-axis and sample fraction defectives (p) on the Y-axis.

Step 6: Interpretation of the p-chart.

In practice, fraction defective (P) of the process is not known. So it is necessary to estimate the fraction defective of the process. We estimate it using the sample fractions. The best estimator of process fraction defective is the average fraction defective of the samples.

6.4 STEPS INVOLVED IN THE CONSTRUCTION OF p-CHART IN EXCEL 2007

We now describe the procedure for construction of the p-chart with the help of the problem given in Sec. 6.2. For plotting a p-chart in Excel 2007, we follow the steps given below:

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 6.1.

	A	B	C
1			
2	Sampled Box	Tubes in Box	Leaky Tubes
3	1	50	6
4	2	50	5
5	3	50	4
6	4	50	7
7	5	50	2
8	6	50	3
9	7	50	4
10	8	50	2
11	9	50	1
12	10	50	6
13	11	50	6
14	12	50	3
15	13	50	4
16	14	50	2

p Chart

Fig. 6.1: Partial screenshot of the spreadsheet for the given data.

Fraction defective is always less than or equal to unity and expressed as a decimal fraction.

Step 2: To calculate the fraction defective for each sample, we use equation (1). For example, the 1st sample contains 50 tubes and each tube is inspected for leakage. After the inspection, 6 tubes are found defective (leaky). The fraction defective for the 1st sample is obtained as:

$$\text{Fraction defective (p)} = \frac{\text{Number of leaky tubes}}{\text{Total number of tubes inspected}}$$

$$= \frac{6}{50} = 0.12$$

To calculate the fraction defective of the 1st sample in Excel 2007, we type "=C3/B3" in Cell D3 as shown in Fig. 6.2. We drag it down up to the last sample, i.e., Cell D32 to find the fraction defective for the remaining samples (Fig. 6.2).

D3 | =C3/B3

	A	B	C	D
1				
2	Sampled Box	Tubes in Box	Leaky Tubes	Fraction Defective (p)
3	1	50	6	0.12
4	2	50	5	0.1
5	3	50	4	0.08
6	4	50	7	0.14
7	5	50	2	0.04
8	6	50	3	0.06
9	7	50	4	0.08
10	8	50	2	0.04
11	9	50	1	0.02
12	10	50	6	0.12
13	11	50	6	0.12
14	12	50	3	0.06
15	13	50	4	0.08
16	14	50	2	0.04
17	15	50	3	0.06
18	16	50	4	0.08
19	17	50	3	0.06
20	18	50	6	0.12

p Chart

Fig. 6.2

Step 3: We now use equation (2) to compute average fraction defective $\overline{p}$, i.e., the average of 30 sample fraction defectives (given in Cells D3 to D32) as shown in Fig. 6.3.

D33 | =AVERAGE(D3:D32)

	A	B	C	D
29	27	50	6	0.12
30	28	50	2	0.04
31	29	50	2	0.04
32	30	50	7	0.14
33			Average	0.0787
34				

Fig. 6.3

Step 4: Here we also use Columns E, F and G for putting the values of the centre line, upper and lower control limits, respectively, as discussed in Lab Sessions 1 to 5. We can calculate these limits from equations (3) to (5) as follows:

i) The centre line is $CL = \overline{p}$. Since the value of $\overline{p}$ is given in Cell D33 (see Fig. 6.3), we type "=$D$33" in Cell E3 to find the centre line as shown in Fig. 6.4a.

ii) The upper control limit is $UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$. We type "= $D$33+3*Sqrt(($D$33*(1-$D$33))/(B3))" in Cell F3 (Fig. 6.4b).

iii) Similarly, we calculate the lower control limit

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$ by typing

"=$D$33–3*Sqrt($D$33*(1-$D$33))/(B3))" in Cell G3 (Fig. 6.4c).

The formula with dollar sign ($) is used for an absolute reference.

E3 | =$D$33

	E	F	G	H	I
1		Control Limits			
2	Centre Line	UCL	LCL		
3	0.0787				
4					

(a)

F3 | =$D$33+3*SQRT(($D$33*(1-$D$33))/(B3))

	F	G	H	I	J
1	ontrol Limits				
2	UCL	LCL			
3	0.1929				
4					

(b)

G3 | =$D$33-3*SQRT(($D$33*(1-$D$33))/(B3))

	G	H	I	J	K
1	s				
2	LCL				
3	-0.0356				
4					

(c)

Fig. 6.4

Step 5: To plot these limits on the chart, we first select Cells E3:G3 and drag them down up to Row 32. After dragging, we get the result shown in Fig. 6.5.

	E	F	G
1		Control Limits	
2	Centre Line	UCL	LCL
3	0.0787	0.1929	-0.0356
4	0.0787	0.1929	-0.0356
5	0.0787	0.1929	-0.0356
6	0.0787	0.1929	-0.0356
7	0.0787	0.1929	-0.0356
8	0.0787	0.1929	-0.0356
9	0.0787	0.1929	-0.0356
10	0.0787	0.1929	-0.0356
11	0.0787	0.1929	-0.0356
12	0.0787	0.1929	-0.0356
13	0.0787	0.1929	-0.0356
14	0.0787	0.1929	-0.0356
15	0.0787	0.1929	-0.0356
16	0.0787	0.1929	-0.0356
17	0.0787	0.1929	-0.0356
18	0.0787	0.1929	-0.0356
19	0.0787	0.1929	-0.0356
20	0.0787	0.1929	-0.0356

p Chart

Fig. 6.5

Step 6: We notice from Fig. 6.5 that the value of LCL is negative, which is not possible. We set it to the value 0 (zero) and denote the limit by LCL* as shown in Fig. 6.6 to plot the control chart.

If the value of lower control limit is negative, we set the lower control limit as zero because a negative fraction defective is not possible.

	E	F	G	H
1	Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.0787	0.1929	-0.0356	0
4	0.0787	0.1929	-0.0356	0
5	0.0787	0.1929	-0.0356	0
6	0.0787	0.1929	-0.0356	0
7	0.0787	0.1929	-0.0356	0
8	0.0787	0.1929	-0.0356	0
9	0.0787	0.1929	-0.0356	0
10	0.0787	0.1929	-0.0356	0
11	0.0787	0.1929	-0.0356	0
12	0.0787	0.1929	-0.0356	0
13	0.0787	0.1929	-0.0356	0
14	0.0787	0.1929	-0.0356	0
15	0.0787	0.1929	-0.0356	0
16	0.0787	0.1929	-0.0356	0
17	0.0787	0.1929	-0.0356	0
18	0.0787	0.1929	-0.0356	0
19	0.0787	0.1929	-0.0356	0
20	0.0787	0.1929	-0.0356	0

p Chart

Fig. 6.6

Step 7: To obtain the p-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells D2:F32 and H2:H32 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. Thus, we obtain the p-chart shown in Fig. 6.7.

If the p-chart continually indicates an increase in the value of the average proportion of non-conforming items, the management should investigate the reasons behind this increase rather than constantly revising the centre line, upper and lower control limits. The possible reasons behind the increase in the non-conforming items may be poor incoming quality from vendors, tightening of specification limits, etc.

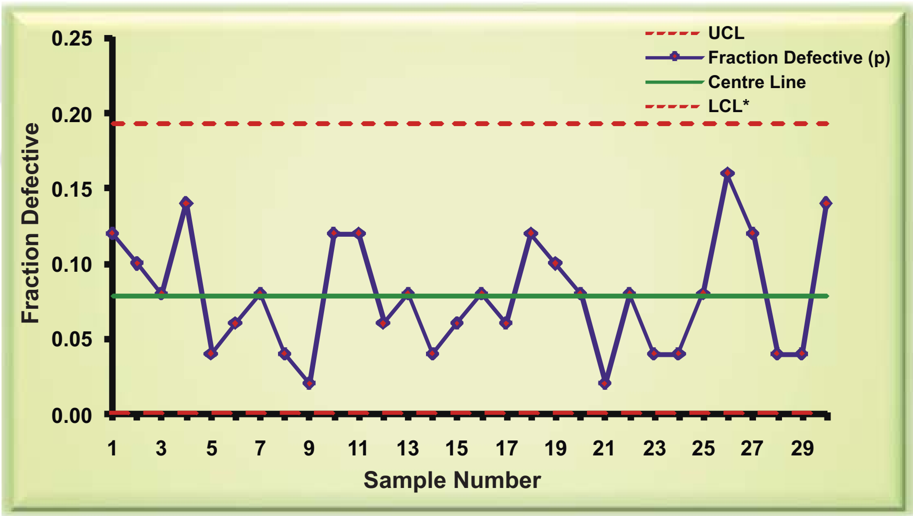

Fig. 6.7

Interpretation

Study Fig. 6.7. Since all points lie between the upper and lower control limits, we may conclude that the process of manufacturing cream tubes is under statistical control with respect to the fraction defective. So, there is no need to revise the control limits.

In the next section, we state the formulae and procedure of computing the revised control limits for the p-chart if you come across this situation.

6.5 REVISED p-CHART

If one or more points lie outside the upper or lower or both control limits, the control chart indicates that the process is not under statistical control. Some assignable causes may be present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once this is done, we remove out-of-control samples and calculate the revised centre line, upper and lower control limits for the p-chart using the remaining samples. These limits are known as revised control limits.

You have already studied various formulae and procedure for revising the p-chart in Sec. 3.3 of Unit 3 of MSTE-001. Here we are listing the steps to compute the revised p-chart:

Step 1: To calculate the revised control limits of p-chart, we first calculate the new $\overline{p}$ as follows:

$$\overline{p}_{new} = \frac{\sum_{i=1}^{k} p_i - \sum_{j=1}^{d} p_j}{k-d} \quad \text{or} \quad \overline{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{n(k-d)} \qquad \ldots (6)$$

where d – number of discarded samples,

$\sum_{j=1}^{d} p_j$ – sum of fraction defectives in the discarded samples, and

$\sum_{j=1}^{d} d_j$ – sum of number of defectives in the discarded samples.

Step 2: After finding $\overline{p}_{new}$, we reconstruct the centre line and control limits of the chart by replacing $\overline{p}$ by $\overline{p}_{new}$ as given below:

✓ Centre line $(CL) = \overline{p}_{new}$ ...(7)

✓ Upper control limit (UCL)

$$= \overline{p}_{new} + 3\sqrt{\frac{\overline{p}_{new}(1-\overline{p}_{new})}{n}} \qquad \ldots(8)$$

✓ Lower control limit (LCL)

$$= \overline{p}_{new} - 3\sqrt{\frac{\overline{p}_{new}(1-\overline{p}_{new})}{n}} \qquad \ldots (9)$$

Step 3: We plot the p-chart for the revised control limits until all points lie within UCL and LCL in the same way as discussed in the Lab Sessions 2 to 5.

Step 4: Interpretation of the p-chart.

You should now apply the methods discussed in Secs. 6.4 and 6.5 on other problems for practice.

Activity

Construct the control charts for fraction defective with the help of MS Excel 2007 and interpret the results for

A1) Examples 1, 2 and 3 given in Unit 3 of MSTE-001.

A2) Exercises E4 and E5 given in Unit 3 of MSTE-001.

Also match the results with the manual calculation done in Unit 3 of MSTE-001.

Continuous Assessment 6

To monitor the manufacturing of laptops, a quality controller randomly selected a fixed number of laptops from the production line, each day over a period of 25 days. The laptops were inspected for defectives and the number of defective laptops found each day was recorded. The data is given in Table 2.

Table 2: The number of defective laptops

Day	Number of Defective Laptops	Day	Number of Defective Laptops
1	3	14	5
2	4	15	2
3	4	16	4
4	11	17	3
5	4	18	6
6	2	19	2
7	4	20	4
8	4	21	3
9	5	22	1
10	4	23	5
11	6	24	3
12	6	25	4
13	6		

Construct the p-chart for fixed sample size and comment on the process. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 6.4 and 6.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 6" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.