UNIT 1 INTRODUCTION TO OPERATIONS RESEARCH

Structure

1.1 INTRODUCTION

Optimisation is the act of obtaining the best result under given circumstances. We come across many practical situations in which optimisation is observed. For example, suppose we are dealing with problems involving costs and want to minimise a cost. Then the least value of the cost is nothing but the optimised value. Similarly, maximisation of profit is the optimisation of profit. Further, suppose that different jobs are to be assigned to different persons and only one job is to be given to one person. We know that every person will complete each job in different times. Then the optimal solution is to assign each job to a person in such a way that the total time taken in completing all the jobs is the least.

We observe many other situations of optimisation around us. In design, construction and maintenance of any system, decision makers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is to either minimise the effort required or to maximise the desired profit/revenue. Optimisation can be defined as the process of finding the conditions that give the maximum or minimum value of a function. **Operations Research** is concerned with the application of scientific methods and techniques to decision making problems and with establishing the optimal solutions.

In this unit, we discuss the scope, applications, uses and limitations of Operations Research (O.R.). You will learn about various models in O.R. We also discuss the concept of convex set and basic feasible solutions. In the next unit, we deal with linear programming problems.

Objectives

After studying this unit, you should be able to:

- define Operations Research;
- describe the scope, uses and limitations of Operations Research;
- explain the models in Operations Research;
- define convex set; and
- determine basic feasible solutions of Linear Programming Problems.

1.2 ORIGIN AND DEFINITION OF OPERATIONS RESEARCH

You will agree that human beings have the tendency of getting maximum gains with minimum effort. The complexities in every aspect of life are increasing by the day. As a result, we would like to create/use scientific methods and techniques that help us attain optimum solutions in decision making problems. Operation Research (O.R.) is concerned with the applications of such scientific methods and techniques.

'Operation' implies some action applied in any area of interest. 'Research' means some organised process of getting and analysing information about the problem environment. The term Operations Research came into existence and gained prominence during the Second World War when military planners were faced with logistical tasks requiring prompt and effective solutions. Hence, a group of scientists with diverse educational backgrounds including mathematics, statistics and physics became involved in applying a scientific approach to deal with strategic and tactical problems of various military operations. This initial research on military operations soon found applications in other decision making problems in business and industry. Hence, the word 'military' was dropped and it was named as 'Operations Research'. In India, Operations Research came into existence in 1949 with the opening of an Operations Research unit at the Regional Research Laboratory in Hyderabad and also in the Defence Science Laboratory.

Operations Research gained momentum in the market place in the nineteen fifties and sixties. Solutions to problems involving manufacturing processes, personnel and material planning, inventory, scheduling and others were sought through O.R. Though Operations Research had its roots in statistics, it has now acquired its own identity. Operations Research is used to solve the real world problems that involve a lot of mathematics, statistics, computer science, accounting, etc.

Operations Research has been defined in different ways by different scientists from time to time, but none of those definitions has been accepted by all. The more general and comprehensive definition of Operations Research is as follows:

"Operations Research is a branch of science which is concerned with the application of scientific methods and techniques to decision making problems and with establishing the optimal solutions."

1.3 SCOPE OF OPERATIONS RESEARCH

O.R. has a wide scope in everyday life as it provides better solutions to various decision-making problems with great speed and competence. It finds applications in a wide range of areas including defence operations, planning, agriculture, industry (finance, marketing, personal management, production management), research and development. We now describe the applications briefly.

In Defence Operations

Since the Second World War, Operations Research techniques have been used for defence operations with the objective of getting maximum gains with minimum effort. It has been used for coordinating various activities of

Air Force, Army and Navy. Decisions regarding formulation and selection of strategies of the various available courses of action are taken by a team of scientists.

In Planning for Economic Development

Careful planning is necessary for economic development of any country. Operations Research is used to frame future economic and social policies.

In Agriculture

Agricultural output needs to be increased due to increasing needs for adequate quantity and quality of food for our increasing population. But there are a number of restrictions under which agricultural production is studied. Problems of agricultural production under various restrictions such as optimum allocation of land to various crops in accordance with the climatic conditions, optimum distribution of water from various resources for irrigation purposes can easily be solved by application of Operations Research techniques.

In Industry

Now-a-days, due to complexities of operations and huge sizes of industries, important decisions regarding various sections of the organisation, e.g., planning, procurement, marketing, finance, etc. have to be taken division wise. For example, the production department needs to minimise the cost of production, but maximise output; the finance department needs to optimise capital investment; the personnel department needs to appoint competent work force at minimum cost. Each department has to plan its own objectives which may be in conflict with the objectives of other departments and may not conform to the overall objectives of the organisation. For example, the sales department of an organisation may want to keep sufficient stocks in the inventory, whereas the finance department may want to have minimum investment. In that case, both departments would be in conflict with each other. The applications of O.R. techniques to such situations help in overcoming this difficulty by evolving an optimal strategy and serving efficiently the interest of the organisation as a whole.

Some of the problems faced by various divisions of an industry, which can be solved by the application of Operations Research techniques are as follows:

Finance department of an organisation needs to optimise capital investment, determine optimal replacement strategies, apply cash flow analysis for long range capital investments, formulate credit policies, credit risks, breakeven analysis. All these can be attained by applying Operations Research techniques.

The marketing department of any organisation has to face various problems like product selection, formulation of competitive strategies, sales forecasting, distribution strategies, selection of advertising media with respect to cost and time, finding the optimal number of salesmen, finding optimum time to launch a product. All such problems can be overcome using Operations Research methods.

Personnel Management of an organisation needs to find the best combination of workers in different categories with respect to costs, skills, age and nature of jobs. It also needs to forecast the work force requirement, frame recruitment policies, assign jobs to machines or workers, negotiate in a bargaining situation, etc. This can be achieved very easily by the application of Operations Research techniques.

Problems related to **production management** of an organisation, i.e., determination of the optimal product mix, selection of the location and design of the sites for production plant, scheduling and sequencing the production run by proper allocation of machines, location and size of warehouse/new plant, etc. can very easily be solved by applying Operations Research techniques.

In Research and Development

Operations Research helps in planning and control of new research and development projects. It also helps in planning the launch of new products. Operations Research helps in solving many other problems faced by public as well as private sectors such as the ones in economic and social planning, management of natural resources, energy, housing, pollution control, waiting lines and administrative problems, insurance policies, and many more.

1.4 MODELS IN OPERATIONS RESEARCH

A model in Operations Research is a mathematical or theoretical description of the various variables of a system representing the basic aspects or the most important features of a typical problem under investigation. The objective of the model is to identify the significant factors and interrelationships. It helps in deciding how the changes in one or more variables of a model may affect other variables or the system as a whole. Operations Research models are broadly classified as follows:

i) Mathematical and descriptive models, and

ii) Static and dynamic models

In **mathematical models,** various variables/parameters explaining different operations of a system are expressed in mathematical terms and the relations are explained by means of mathematical equations or inequalities. The variables can be exact (deterministic) or probabilistic. In deterministic models, the variables and their relationships are stated exactly. The probabilistic models are developed for problems involving risk and uncertainty. Therefore, in these models, decision variables take the form of probability distributions. For example, the variables in linear programming, transportation and assignment problems, which you will study in Units 3 and 4 of this Block and Unit 5 of Block 2 are deterministic. The models discussed in Unit 6 of Block 2 of this course are probabilistic models. The models in which various operations are explained in non-mathematical language are called **descriptive models** and serve as preliminary models for the development of mathematical models.

A probabilistic mathematical model is called **static** or **dynamic** according as the distribution of parameters remains unchanged or changes with time. An inventory model is a static model wherein the re-ordering of goods is determined using average demand and average time and not on the basis of changes that take place in any particular period. However, an inventory model in which the re-order point is determined by a certain stock level at any point of time is **dynamic**. You will learn about inventory models in Unit 8 of Block 2 of this course.

Operations Research models are mainly of three kinds: **Iconic, Symbolic** and **Analogue.** We briefly explain them as under:

Iconic models are physical replicas of real life systems and are based on a smaller scale than the original. In many cases, they provide a pictorial presentation of various aspects of a system. Such models are designed for the purpose of understanding the behaviour of the operation of the system when conducting experiments on the real life systems is risky and/or costly affair. Flight simulators, missile firing simulators are examples of iconic models. Photographs, paintings, maps, drawings, clay, wooden or metallic models of systems are also iconic models. For example, a toy car can be considered as an iconic model of an automobile. Similarly, a blueprint representing the floor of a building is an iconic model, and the globe is an iconic model of earth.

Symbolic models reflect the structure of a system, denoting various components of a system and their interrelationships employing letters, numbers and various other types of mathematical symbols. For example, the buyers' behaviour at varying price level can be represented symbolically by the demand curve in Economics. A model for processing inventory cost in inventory problems is another example of symbolic model. Such models are very well suited for determination of various changes in the system.

Analogue models are helpful in representing all the significant properties of a system which are not represented by iconic or symbolic models. They are also physical, but not the exact replica of the system. They are used to explain the system by analogy. For example, the geological structure of earth cannot be represented by the globe, an iconic model of earth, if different colours are not used. If different colours are taken on the globe to represent the geological structure of the earth, it is known as Analogue model.

There is no unique method for solving all mathematical models. The nature of the method of solution depends on the type and complexity of the mathematical model. In Operations Research, the solutions are generally determined by algorithms which provide fixed computational rules. These rules are applied repetitively/iteratively to the problem, with each repetition/ iteration moving the solution closer to the optimum. The three main methods for the solution of a mathematical model are:

i) Analytic or deductive methods,

ii) Numerical or inductive methods, and

iii) Monte Carlo techniques or simulations.

Analytic methods involve graphs and elementary differential calculus. In **numerical methods**, numerical values are substituted for various variables involved in the model by trial and error. Then the set of values, which maximises the effectiveness of the system is taken as the required solution. **Monte Carlo techniques** are applied on systems, which are not represented adequately by theoretical models. In these techniques, the knowledge of the important characteristics and rules along with random sampling is used to determine the probability distributions of various components of the model.

Some mathematical models may be so complex that it is impossible to solve them by any of the available optimisation algorithms. In such cases, it may be necessary to abandon the search for the optimal solution and simply seek a better solution using heuristics. **Heuristic** models use intuitive rules or guidelines to find the solution to any problem. These models do not guarantee an optimum solution and give solutions depending on the assumptions based on past experience. These models, however, operate faster and are very useful for solving large size problems. However, they require a good amount of creativity and experience on the part of decision makers.

1.5 USES AND LIMITATIONS OF OPERATIONS RESEARCH

There are numerous advantages of Operations Research. Some of these are:

- Operations Research develops a model, which provides logical and systematic approach for understanding, solving and controlling a problem.
- It helps in optimum use of resources. For example, linear programming techniques in Operations Research suggest most effective methods and efficient ways of optimality. It also helps in finding the limitations and scope of an activity.
- Operations Research techniques can be used for improving the quality of decisions. A decision-maker can use a well-formulated mathematical model representing a real life situation to find the changes in the variables as per requirement. Such changes can be incorporated even without disturbing the system or problem under consideration.
- It helps, in preparing future managers. Operations Research methods constitute a means for improving the knowledge and skill of young managers.
- It modifies mathematical solutions before these are applied. Managers may accept or modify the mathematical solutions obtained using Operations Research techniques.
- It helps suggest alternative solutions for the same optimum profit if the management wants so.

To sum up, Operations Research is a very powerful method of getting the best out of limited resources. However, it does have some limitations, which emerge only due to the time and cost involved.

Some limitations of Operations Research are as follows:

- Formulation of mathematical models may take into account all possible factors for defining a real life problem and hence is difficult. As a result, the help of computers is required for the large number of cumbersome computations for such problems. This discourages small companies and other organisations from using O.R. techniques.
- Some problems may involve a large number of intangible factors such as human emotions, human relationship, etc. which cannot be quantified. Hence, the best solution cannot be determined for such problems because such factors have to be excluded.
- A specialist, who may be a mathematician or a statistician, is needed to understand the formulation of models, find solutions and recommend their implementation. Managers, who deal with such problems, may not have such specialisation and hence the results may not be optimal.
- Models are abstractions of real life situations and not the reality.

- Assumptions need to be made about the nature and importance of some factors in order to construct an Operations Research model.
- A reasonably good solution without the use of Operations Research may be preferred by the management as compared to a slightly better solution provided by using Operations Research since it is very expensive in terms of time and money.

So far we have introduced you to O.R. and its scope, uses, and limitations. From the next unit onwards, we shall introduce various O.R. techniques for obtaining optimal and feasible solutions. However, before you can learn these techniques, you must study certain related basic concepts such as the concepts of convex sets and basic feasible solutions. We discuss these concepts in the next two sections.

1.6 CONVEX SET

A region or a set R is convex if and only if for any two points on the set R, the line segment connecting these points lies entirely in R. Mathematically, set R is convex if and only if

$(x_1, y_1) \in R$ and $(x_2, y_2) \in R$

$$\Rightarrow \quad (\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2) \in R, \quad 0 \le \lambda \le 1$$

where $(\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2)$ gives all the points which lie on the line segment joining (x_1, y_1) and (x_2, y_2).

For example, let us consider Fig. 1.1.

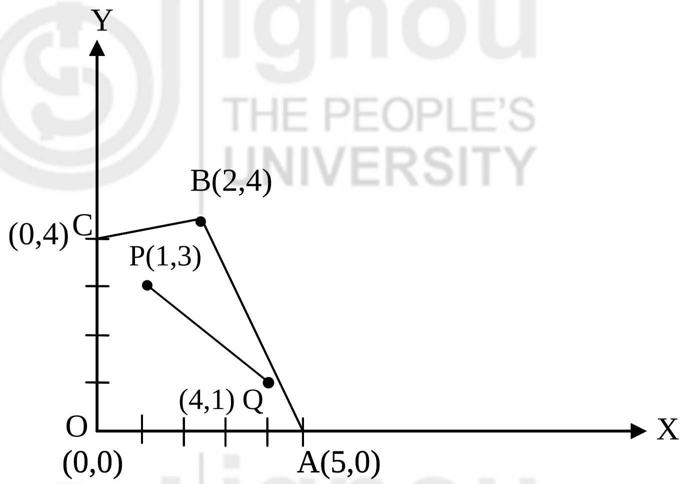


Fig. 1.1

Consider the region enclosed by OABC. Let us denote it by R. It is convex as the line segment joining any two points in this region lies wholly within it. As an example, let us take two points P (1, 3) and Q (4, 1). Then all points on the line segment joining P and Q are given by

$$(\lambda(1) + (1-\lambda)4, \lambda(3) + (1-\lambda)1)$$
$$= (\lambda + 4 - 4\lambda, 3\lambda + 1 - \lambda)$$
$$= (4 - 3\lambda, 1 + 2\lambda), \quad 0 \le \lambda \le 1.$$

Here $\lambda = 0$ gives the point Q (4, 1) and $\lambda = 1$ gives the point P (1, 3). Other points on the line segment PQ are given by

$$(4 - 3\lambda, 1 + 2\lambda), \text{ where } 0 < \lambda < 1$$

For example, let us take λ = 0.1, 0.3, 0.5, 0.7, 0.9; then the corresponding points (after substituting the values of λ = 0.1, 0.3, 0.5, 0.7, 0.9) are:

$(4 - 3(0.1), 1 + 2(0.1))$, $(4 - 3 \times 0.3, 1 + 2 \times 0.3)$, $(4 - 3 \times 0.5, 1 + 2 \times 0.5)$,

$(4 - 3 \times 0.7, 1 + 2 \times 0.7)$, $(4 - 3 \times 0.9, 1 + 2 \times 0.9)$

i.e., (3.7, 1.2), (3.1, 1.6), (2.5, 2), (1.9, 2.4), (1.3, 2.8)

All these points clearly lie on the line and also in the region R.

Similarly, all other points on the line segment PQ also lie inside the region R. Hence, the line segment PQ lies in R.

Therefore, R is convex in this example. Other examples of convex sets are shown in Fig. 1.2.

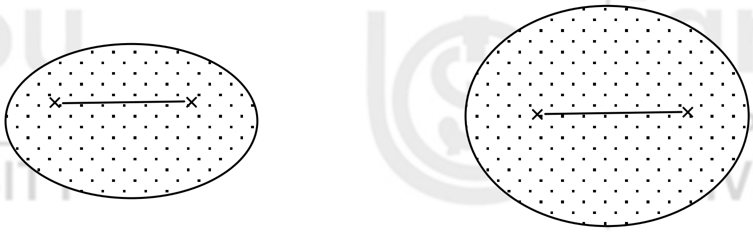
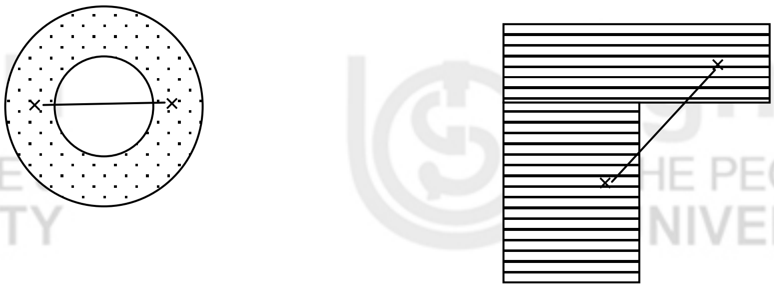


Fig. 1.2

Some examples of non-convex sets are the shaded regions in Figs. 1.3a and b.



(a) **(b)**

Fig. 1.3

In each of the regions, in Figs. 1.3a and b, the line segment joining two points has been shown. Note that these do not lie wholly in the region, whereas the two points lie in the regions.

Example 1: Show that the set $\$ = \{(x, y) : x^2 + y^2 \leq 1\}$ is a convex set.

Solution: Let us take any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in Fig. 1.4 such that:

$$x_1^2 + y_1^2 \leq 1, \text{ and } x_2^2 + y_2^2 \leq 1.$$

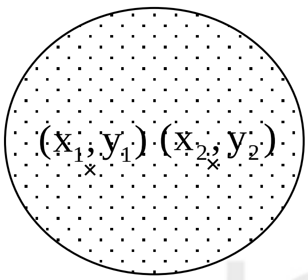


Fig. 1.4

Now, the line segment joining P and Q is the set

$$\{\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2); \quad 0 \le \lambda \le 1\}.$$

Let

$$u_1 = \lambda x_1 + (1-\lambda)x_2$$
$$u_2 = \lambda y_1 + (1-\lambda)y_2$$

Therefore, all points on the line segment PQ are given by (u_1, u_2).

Now, the line segment PQ lies wholly in \$ if

$$u_1^2 + u_2^2 \le 1$$

Since $u_1^2 + u_2^2 = [\lambda x_1 + (1-\lambda)x_2]^2 + [\lambda y_1 + (1-\lambda)y_2]^2$

$$= \lambda^2 x_1^2 + (1-\lambda)^2 x_2^2 + 2\lambda(1-\lambda)x_1x_2 + \lambda^2 y_1^2 + (1-\lambda)^2 y_2^2 + 2\lambda(1-\lambda)y_1y_2$$
$$= \lambda^2[x_1^2 + y_1^2] + (1-\lambda)^2[x_2^2 + y_2^2] + 2\lambda(1-\lambda)[x_1x_2 + y_1y_2]$$

We have

$$u_1^2 + u_2^2 \le \lambda^2 + (1-\lambda)^2 + 2\lambda(1-\lambda)[x_1x_2 + y_1y_2] \qquad \ldots (1)$$

Now, consider

$$(x_1x_2 + y_1y_2)^2 = x_1^2x_2^2 + y_1^2y_2^2 + 2x_1x_2y_1y_2$$
$$= x_1^2x_2^2 + y_1^2y_2^2 + x_1^2y_2^2 + x_2^2y_1^2 - x_1^2y_2^2 - x_2^2y_1^2 + 2x_1x_2y_1y_2$$
$$= (x_1^2 + y_1^2)(x_2^2 + y_2^2) - x_1y_2(x_1y_2 - x_2y_1) - x_2y_1(x_2y_1 - x_1y_2)$$
$$= (x_1^2 + y_1^2)(x_2^2 + y_2^2) - (x_1y_2 - x_2y_1)^2$$
$$\le (x_1^2 + y_1^2)(x_2^2 + y_2^2) \le 1$$
$$\Rightarrow (x_1x_2 + y_1y_2) \le 1 \qquad \ldots (2)$$

$\therefore$ From (1) and (2), we have

$$u_1^2 + u_2^2 \le \lambda^2 + (1-\lambda)^2 + 2\lambda(1-\lambda)$$

or $\quad u_1^2 + u_2^2 \le [\lambda + (1-\lambda)]^2$

$$\Rightarrow u_1^2 + u_2^2 \le 1$$

$\therefore$ \$ is convex set.

Example 2: Show that the set \$ = {(x, y) : 0 ≤ y ≤ 5 when 0 ≤ x ≤ 2 and 3 ≤ y ≤ 5 when 2 ≤ x ≤7} is not a convex set.

Solution: Let us take two points P(1, 1) and Q(5, 4) in the given region \$ (see Fig. 1.5).

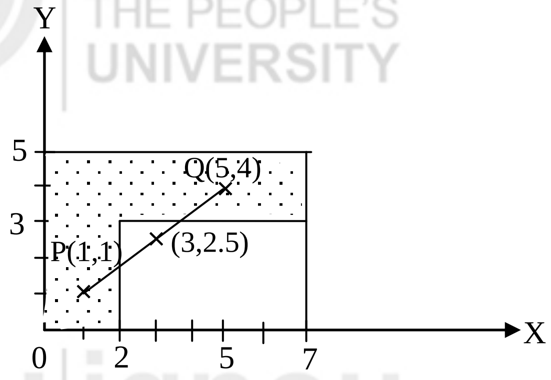


Fig. 1.5

Now, all the points on the line segment PQ are given as

$$\{l\,x_1 + (1-l)x_2\ ,\ \ l\,y_1 + (1-l)y_2\ ,\ \ 0 \pounds l \pounds 1\}$$

$$= \{l\,(1) + (1-l)5\ ,\ \ l\,(1) + (1-l)4\ ,\ \ 0 \pounds l \pounds 1\}$$

$$= \{5 - 4l\ \ ,\ \ 4 - 3l\ \ ,\ \ 0 \pounds l \pounds 1\}$$

Let us take one of these points and put $\lambda = \frac{1}{2}$. So the point is

$$\left(5 - 4 \times \frac{1}{2}, 4 - 3 \times \frac{1}{2}\right) = \left(5 - 2, 4 - \frac{3}{2}\right) = (3, 2.5)$$

This point is on the line, but does not belong to the given set \$ since for y = 2.5, x should lie between 0 and 2 but here it is 3.

Therefore, the given set is not convex.

Extreme Points of a Convex Set

A point (x, y) in a convex set \$ is called an extreme point if it is not possible to locate two distinct points in or on R so that the line joining them will include (x, y).

Mathematically, a point (x, y) is an extreme point of a convex set if it cannot be expressed as a convex combination of any two points (x_1, y_1) and (x_2, y_2) in \$, i.e., there do not exist other points (x_1, y_1), (x_2, y_2) [for $(x_1, y_1) \neq (x_2, y_2)$] in the set such that

$$x = \lambda x_1 + (1-\lambda)x_2$$

and $$y = \lambda y_1 + (1-\lambda)y_2 \qquad 0 < \lambda < 1.$$

For example,

i) The vertices of the polygons, which are convex sets, are the extreme points.

ii) Every point on the circumference of the region containing the portion in and on the circle is an extreme point.

Having explained briefly the concept of convex sets, we now discuss basic feasible solutions of linear programming problems. These will be used in Units 3 to 8.

1.7 BASIC FEASIBLE SOLUTIONS OF LINEAR PROGRAMMING PROBLEMS

We study feasible solution, basic solution, basic feasible solution of a system of equations less in number than the number of decision variables. Such solutions are required to be obtained for allocation problems which we discuss in Units 3 to 8.

Feasible Solution

Any non-negative value of the point corresponding to decision variables is a feasible solution for the system of equations representing the limited available resources for allocation problems.

Basic Solution

Suppose there are m equations representing constraints (limited available resources) containing m + n variables in an allocation problem. The solution obtained by setting any n variables equal to zero and solving for the remaining m variables is called the basic solution. These m variables are the basic variables and the remaining n variables are non-basic variables.

The maximum number of possible basic solutions is given by the formula

$$^{m+n}C_n$$

For example, if there are 2 equations in 3 variables, then the maximum number of possible basic solutions is

$$^{3}C_2 = \frac{3!}{2!(3-2)!} = 3.$$

Basic Feasible Solution

A basic solution for which all the basic variables are non-negative is called the basic feasible solution.

Example 3: Determine all basic solutions and basic feasible solutions of the equations:

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

Solution: Here the maximum number of possible basic solutions is

$$^{4}C_2 = \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = \frac{4 \times 3 \times 2}{2 \times 2} = 6.$$

We obtain these as follows:

Setting $x_1 = 0, x_2 = 0$, we have $2x_3 + x_4 = 3$ and $4x_3 + 6x_4 = 2$

$$\Rightarrow \; x_3 = 2, \; x_4 = -1$$

Setting $x_1 = 0, x_3 = 0$, we have $6x_2 + x_4 = 3$ and $4x_2 + 6x_4 = 2$

$$\Rightarrow \; x_2 = \frac{1}{2}, \; x_4 = 0$$

Setting $x_1 = 0, x_4 = 0$, we have $6x_2 + 2x_3 = 3$ and $4x_2 + 4x_3 = 2$

$$\Rightarrow \; x_2 = \frac{1}{2}, \; x_3 = 0$$

Setting $x_2 = 0, x_3 = 0$, we have $2x_1 + x_4 = 3$ and $6x_1 + 6x_4 = 2$

$$\Rightarrow \; x_1 = \frac{8}{3} \text{ and } x_4 = -\frac{7}{3}$$

Setting $x_2 = 0, x_4 = 0$, we have $2x_1 + 2x_3 = 3$ and $6x_1 + 4x_3 = 2$

$$\Rightarrow \; x_1 = -2, x_3 = \frac{7}{2}$$

Setting $x_3 = 0, x_4 = 0$, we have $2x_1 + 6x_2 = 3$ and $6x_1 + 4x_2 = 2$

$$\Rightarrow \; x_1 = 0, x_2 = \frac{1}{2}$$

Thus, all basic solutions of the given system of equations are:

$$(0,0,2,-1),\ (0,\frac{1}{2},0,0),\ (0,\frac{1}{2},0,0),\ (\frac{8}{3},0,0,-\frac{7}{3}),\ (-2,0,\frac{7}{2},0),\ (0,\frac{1}{2},0,0).$$

Here $(0,\frac{1}{2},0,0)$ is repeated thrice and hence the basic solutions are

$$(0,\ 0,\ 2,\ -1),\ (0,\frac{1}{2},0,0),\ (\frac{8}{3},0,0,-\frac{7}{3}),\ (-2,0,\frac{7}{2},0).$$

Of these solutions, the basic feasible solution is $(0,\frac{1}{2},0,0)$ as in the other solutions, all decision variables are not positive.

Example 4: Find all basic solutions for the system of simultaneous equations:

$$2x_1 + 3x_2 + 4x_3 = 5 \qquad \text{and} \qquad 3x_1 + 4x_2 + 5x_3 = 6$$

Solution: The maximum number of possible basic solutions is

$$^3C_2 = \frac{3!}{2!(3-2)!} = 3.$$

Now, putting $x_1 = 0$ and solving for x_2 and x_3, we get

$$x_2 = -1, x_3 = 2$$

Again, putting $x_2 = 0$ and solving for x_1 and x_3, we get

$$x_1 = -\frac{1}{2},\ x_3 = \frac{3}{2}$$

Finally, putting $x_3 = 0$ and solve for x_1 and x_2, we get

$$x_1 = -2,\ x_2 = 3.$$

This verifies that there are only three basic solutions but none of them are feasible solutions because in all solutions (–2, 0, 3/2), (–1,0, 3/2) and (–2, 3, 0), the values of the decision variables are not positive.

Now, you should try to solve the following exercise.

E1) Determine all the basic feasible solutions of the system of equations:

$$3x_1 + 5x_2 + x_3 = 15$$
$$5x_1 + 2x_2 + x_4 = 10$$

Match your answer to the above problem with the solution/answer given in Sec. 1.9. Let us now summarise the main points, which have been covered in this unit.

1.8 SUMMARY

1. Operations Research is a branch of science which is concerned with the application of scientific methods and techniques to decision making problems and establishing optimal solutions.
2. Operations Research has got a wide scope in everyday life as it provides better solutions to various decision-making problems with greater speed and competence. It finds applications in a wide range of areas including defence operations, planning, agriculture, industry (finance, marketing, personal management, production management), research and development.

3. A model in Operations Research is a mathematical or theoretical description of the various variables of a system representing the basic aspects or the most important features of a typical problem under investigation. The objective of the model is to identify the significant factors and their interrelationships. It helps in deciding how the changes in one or more variables of a model may affect other variables of the system as a whole. Operations Research models are broadly classified as mathematical and descriptive models, and also as static and dynamic models. The models are mainly of three kinds: Iconic, Symbolic and Analogue.

4. The three main methods for the solution of a mathematical model are: **Analytic** or **deductive method**, **numerical** or **inductive method** and **Monte Carlo techniques** or **simulation.**

5. Operations Research is a very powerful method of getting the best out of limited resources. However, it does have some limitations which emerge only due to time and cost involved.

6. A region or a set R is called **convex set** if and only if for any two points on the set R the line segment connecting these points lies entirely in R.

7. A point (x, y) in a convex set $ is called an **extreme point** if it is not possible to locate two distinct points in or on R so that the line joining them will include (x, y).

8. Any non-negative value of the point corresponding to decision variables is a **feasible solution** for the system of equations representing the limited available resources for allocation problems.

9. Suppose there are m equations representing constraints (limited available resources) containing m + n variables in an allocation problem. The solution obtained by setting any n variables equal to zero and solving for the remaining m variables is called the **basic solution**. These m variables are the basic variables and the remaining n variables are non-basic variables.

10. A basic solution for which all the basic variables are non-negative is called a **basic feasible solution.**

1.9 SOLUTIONS/ANSWERS

E1) Here, the maximum number of possible basic solutions will be

$$ {}^4C_2 = \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = \frac{4 \times 3 \times 2}{2 \times 2} = 6. $$

These are obtained as follows:

Putting $x_1 = 0, x_2 = 0,$ we have $x_3 = 15, x_4 = 10.$

Putting $x_1 = 0, x_3 = 0,$ we have $5x_2 = 15$ and $2x_2 + x_4 = 10$

Þ $x_2 = 3$ and $x_4 = 4.$

For $x_1 = 0, x_4 = 0, 5x_2 + x_3 = 15$ and $2x_2 = 10$

Þ $x_2 = 5$ and $x_3 = -10$

For $x_2 = 0, x_3 = 0, 3x_1 = 15$ and $5x_1 + x_4 = 10$

Þ $x_1 = 5$ and $x_4 = -15$

For $x_2 = 0, x_4 = 0, 3x_1 + x_3 = 15$ and $5x_1 = 10$

$$\Rightarrow \; x_1 = 2 \text{ and } x_3 = 9$$

For $x_3 = 0, x_4 = 0, 3x_1 + 5x_2 = 15$ and $5x_1 + 2x_2 = 10$

$$\Rightarrow \; x_1 = \frac{50}{19}, \; x_2 = \frac{45}{19} \text{ (solving the equation)}$$

Thus, the possible solutions are

(0, 0, 15, 10), (0, 3, 0, 4), (0, 5, −10, 0), (5, 0, 0, −15),

(2, 0, 9, 0) and $\left(\frac{50}{19}, \frac{45}{19}, 0, 0\right)$.

Of these, the basic feasible solutions are:

(0, 0, 15, 10), (0, 3, 0, 4), (2, 0, 9, 0) and $\left(\frac{50}{19}, \frac{45}{19}, 0, 0\right)$.