UNIT 5 ACCEPTANCE SAMPLING PLANS

Structure

5.1 INTRODUCTION

In many situations, the product is so complex that all components/parts of the product are not made by a manufacturer. In such cases, one or more components of the product are purchased from an outside agent or supplier and the manufacturer does not have direct control over the quality of such components. Since the final product is produced by the manufacturer, he/she faces problems such as:

- How to control the quality of components received from others?
- How to ensure that the lots produced do not contain an excessively large proportion of defective products?
- Do the products meet the desired specifications?

Such problems belong to the category of **product control**.

Product control refers to control the products in such a way that these are free from defects and conform to specifications.

You have learnt in Unit 1 that product control can be done by 100% inspection, i.e., each and every unit produced or received from the outside suppliers is inspected. This type of inspection has the advantage of ensuring that all defective units are eliminated. However, it is very time consuming and costly. Also, if the unit is destroyed under investigation, e.g., light bulb, cracker, ammunition, etc., 100% inspection is not practicable.

In 1920, Harold F. Dodge and Harry G. Roming developed statistical methods for product control known as **acceptance sampling** or **sampling inspection** as an alternative of 100% inspection. Now-a-days, product control is achieved through acceptance sampling.

In this unit, we introduce the acceptance sampling plans and the related terminology. In Sec. 5.2, we explain the meaning of the term **inspection** as used in industry. We discuss the concept of acceptance sampling plan, its advantages and limitations and the types of acceptance sampling plans in Sec. 5.3. In Sec 5.4, we discuss the implementation of the acceptance sampling plan. In Secs. 5.5 and 5.6, we explain the basic terminology related to acceptance sampling plan, such as lot, probability of accepting a lot, acceptance quality level (AQL), lot tolerance percent defective (LTPD), producer's risk and consumer's risk. In the next unit, we shall discuss the rectifying sampling plans.

Objectives

After studying this unit, you should be able to:

- distinguish between 100% inspection and sampling inspection;
- define acceptance sampling plan and discuss the procedure of its implementation;
- compute the probability of accepting or rejecting a lot;
- define acceptance quality level (AQL) and lot tolerance percent defective (LTPD) of the lot; and
- compute the producer's risk and consumer's risk for an acceptance sampling plan.

5.2 INSPECTION

You have learnt briefly about 100% inspection and sampling inspection in Sec. 1.3.1 of Unit 1. In this section, we discuss these concepts in some more detail.

When a manufacturer produces a product or buys some parts of the product from outside agents or suppliers, he/she would like to ensure that the final product is as per specifications. For this purpose, he/she inspects the lot at every strategic point. The method of checking, measuring or testing one or more quality characteristics of the product or the parts and determining whether it satisfies the required specifications or not, is called **inspection**.

Inspection is of two types:

i) Inspection of Variables

In this type of inspection, the quality characteristic (s) of an item/unit is (are) measured and compared with the required specifications. For example, the desired specification for the diameter of a ball bearing is 50 mm, length of the refill of a ball pen is 14 cm, weight of a cricket ball is 162 gram, and so on.

ii) Inspection of Attributes

In inspection of attributes, actual measurements are not taken. Instead, the item/units are categorised as defective or non-defective on the basis of Go-No-Go gauges. It means that if a unit fulfils all required quality characteristics, it is categorised as non-defective and if not, it is categorised as defective. In such type of inspection, the number of defective units (or defects) is counted.

A Go-No-Go gauge is an inspection tool which is used to check an item or a unit or a piece against its allowed tolerances. The name **Go-No-Go** derives from its use. It means that we check the item and if the item is acceptable (fulfil the specifications), we say **Go** and if unacceptable, we say **No-Go**.

Methods of Inspection

There are two methods of inspection:

i) 100% Inspection

In this method of inspection, each and every item/unit of any given lot is inspected. A decision regarding the quality of the entire lot is taken on the basis of **all** inspected units of the lot. This procedure needs huge expenditure of time, money, labour and resources. Also, if the product is such that it is completely destroyed under the process of inspection (e.g., a cracker, ammunition, etc.) then in such cases, 100% inspection is neither practicable nor economical.

As an alternative, we use sampling inspection.

ii) Sampling Inspection

In sampling inspection method, some items/units (called sample) are randomly selected from a lot in such a way that the selected sample is a true representative of the entire lot. Then each and every unit of the selected sample is inspected. A decision regarding the quality of the entire lot is taken on the basis of the information obtained from the sampled units.

The question now is: How do we take any decision about the quality of a lot on the basis of sampling inspection? For this, we need to learn about the **acceptance sampling plan**.

5.3 ACCEPTANCE SAMPLING PLAN

Let us first define acceptance sampling.

A sampling inspection in which a decision about acceptance or rejection of a lot is based on one or more samples that have been inspected is known as **acceptance sampling**.

In other words:

Acceptance sampling is a technique in which a small part or a fraction of the units/items is selected randomly from a lot and the selected units are inspected to decide whether the lot should be accepted or rejected on the basis of the information provided by the sample inspection.

For example, suppose a manufacturer of cricket balls supplies them in lots of 500. A buyer wants to inspect 20 balls from each lot before accepting the lot and takes a decision about acceptance or rejection of the lot on the basis of the information provided by this sample. This is an example of acceptance sampling because the decision about the lot is taken on the basis of a sample.

Consider another example. Suppose a mobile phone company packs the mobile phones it produces in lots of 100 units. To check the quality of the lots, the quality inspector of the company draws a random sample of size 10 from each lot. He/she takes a decision about acceptance or rejection of the lot on the basis of the information provided by this sample.

When a lot is rejected on the basis of the acceptance sampling, it does not mean that all units of the lot are of no use or all units are defective. It means that all units of the lot have been carefully examined and the defective units have been either repaired or removed. Thus, the remaining units in the lot conform to a particular quality level. The rejected units may be sold at a lower price or the defects in the units may be removed so that it can be retained. For instance, the clearance sales of, e.g., ready-made clothes or shoes, etc. are carried out to sell those items which are not up to the specifications or have been rejected. This is a loss to the producer or the manufacturer, which can be prevented if he/she could maintain the desired level of quality. Therefore, manufacturers should be very careful about the quality of goods, which they supply to the customers.

We can now answer the question: **What is an acceptance sampling plan?**

An acceptance sampling plan is a specific plan that clearly states the rules for sampling and the associated criteria for acceptance or rejection of a lot.

Acceptance sampling plans can be used for the inspection of:

1. Manufactured units/items,
2. Components,
3. Raw materials,
4. Operations,
5. Materials in process,
6. Supplies in storage,
7. Maintenance operations,
8. Data or records, and
9. Administrative procedures.

Acceptance sampling has many advantages and some limitations, which we now describe.

5.3.1 Advantages and Limitations of Acceptance Sampling

The main **advantages of acceptance sampling** are as follows:

i) It is less expensive in terms of money, time and labour in comparison to 100% inspection.

ii) For items, which cannot be used after single inspection, such as crackers, bulbs, tube lights, food, etc., 100% inspection is not practicable. Sampling inspection is the only way for inspecting such items.

iii) In acceptance sampling, a sample of a small number of items or units is inspected and hence smaller inspection staff is required.

iv) In many cases, acceptance sampling provides better outgoing quality. In general, it is seen and agreed that **good 100% inspection** removes only **85% to 90%** of the defective items, whereas **very good 100% inspection** removes only **99%** of the defective items. However, due to human error, it usually does not reach the 100% mark. In other words, we can say that 100% inspection is not always reliable because it involves too much routine work for the persons inspecting each and every item. Due to this, defective items may also be labelled as satisfactory and may also be accepted at times when these persons are distracted. Hence, an appropriate sampling plan is preferable.

v) Due to quick inspection through the acceptance sampling, the scheduling and delivery times are saved.

Acceptance sampling has some **limitations** which are given below:

i) Since, in acceptance sampling, the entire lot is accepted or rejected on the basis of conclusions drawn from one or more samples, there is always some risk of making wrong inference about the quality of the lot. These risks are termed the producer's risk and consumer's risk. You will learn more about these in Sec. 5.6.

ii) The success of acceptance sampling depends on the randomness of the sample, quality characteristics to be tested, lot size, acceptance criteria, etc. Therefore, it is a specialised job requiring careful planning and execution and every one cannot undertake it.

So far you have learnt about the acceptance sampling plan and its advantages and limitations. It is also important for you to know about different types of acceptance sampling plans used in industries.

5.3.2 Types of Acceptance Sampling Plans

There are several types of acceptance sampling plans based on different approaches that can influence the decision about a lot. These are categorised as follows:

1. Acceptance sampling plans for attributes, and
2. Acceptance sampling plans for variables.

1. Acceptance Sampling Plans for Attributes

Acceptance sampling plans in which **actual measurements** of the quality characteristics are **not made**, but the units/items are categorised as **defective or non-defective** on the basis of Go-No-Go gauges are called **acceptance sampling plans for attributes**. These plans are of two types:

A. Lot-By-Lot Acceptance Sampling Plans for Attributes

Lot-by-lot acceptance sampling plans for attributes are the most commonly used sampling plans and, therefore, are simply called acceptance sampling plans for attributes. Such plans are used whenever the units to be inspected can be conveniently grouped into **batches** or **lots**.

In lot-by-lot acceptance sampling plan for attributes, generally, we use the following plans:

i) Single sampling plans

In the single sampling plan, the decision about the acceptance or rejection of a lot is based on a single sample that has been inspected. This is the simplest type of sampling plan.

ii) Double sampling plans

In the double sampling plan, the decision about the acceptance or rejection of a lot requires the evidence of two samples drawn from the lot. If the lot quality is good (or bad), the lot is accepted (or rejected) on the basis of the first sample. If the first sample shows an intermediate quality, the decision about the lot is taken on the evidence of the first and second sample combined. This is more complicated than the single sampling plan.

iii) Multiple sampling plans

A multiple sampling plan is an extension of the double sampling plan. This sampling plan may require more than two samples to reach a decision about the acceptance or rejection of a lot.

iv) Sequential sampling plans

In the sequential sampling plan, the sample size is not pre-fixed as in the case of single, double and multiple sampling plans. The units/items are drawn from the lot, one at a time and inspected. The decision about accepting or rejecting of the lot or continuing with the inspection by taking one more unit from the lot, is made on the basis of information available up to that stage.

The **multiple** and **sequential sampling plans** are beyond the scope of this course. However, in Units 7 and 8, we shall discuss the single sampling plans and double sampling plans in detail.

B. Continuous Production Acceptance Sampling Plans for Attributes

Continuous production acceptance sampling plans for attributes are used when the units/items to be inspected cannot be grouped into lots or batches. Many manufacturing operations do not create lots because in these operations, the units are produced in a continuous process on a conveyor belt or other straight-line systems.

Continuous production acceptance sampling plans for attributes are beyond the scope of this course.

2. Acceptance Sampling Plans for Variables

The acceptance sampling plans in which **actual measurements** of the quality characteristics are taken are called **acceptance sampling plans for variables**.

The acceptance sampling plans for variables are beyond the scope of this course. So we now focus on the acceptance sampling plans for attributes and describe the general procedure for implementing it.

5.4 IMPLEMENTATION OF ACCEPTANCE SAMPLING PLAN FOR ATTRIBUTES

Suppose that lots of the same size, say, N, are received from the supplier or the final assembly line and submitted for inspection one at a time. The procedure for implementing the acceptance sampling plan to arrive at a decision about the lot is described in the following steps:

Step 1: We draw a random sample of size n from the lot received from the supplier or the final assembly.

Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective on the basis of certain criteria. At the end of the inspection, we count the number of defective units found in the sample.

Step 3: We compare the number of defective units found in the sample with the acceptance criteria.

Step 4: If the acceptance criteria are satisfied, we accept the entire lot. Otherwise, we reject the entire lot.

The steps described above are shown in Fig. 5.1.

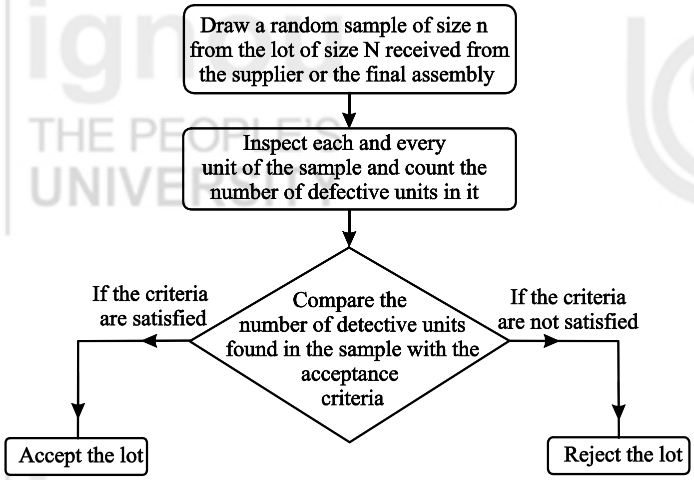

Fig. 5.1: Procedure for implementation of the acceptance sampling plan.

Let us explain these steps further with the help of an example.

Suppose a buyer of cricket balls wants to inspect a sample of 20 balls from each lot to check their quality. The acceptance criterion is that if the sample contains at most one defective ball, the lot would be accepted. Otherwise, it would be rejected. The buyer draws 20 balls from each lot and inspects each and every ball of the sample and classifies each ball as defective or non-defective on the basis of certain defects. At the end of the inspection, he/she counts the number of defective balls found in the sample and compares the number of defective balls with the acceptance criterion. If the number of defective balls in the sample is more than 1, he/she rejects the lot. If the number of defective balls is zero or 1, he/she accepts the lot.

We shall discuss different types of sampling inspection plans in detail in the remaining units. But, before studying these plans, you should be aware about the related terminology. This is what we discuss in Sec. 5.5.

5.5 TERMS USED IN ACCEPTANCE SAMPLING PLANS

In this section, we define some terms used in acceptance sampling plan.

1. Lot

A lot is the collection of units or items from which a sample is taken and inspected to determine its acceptability.

It is found that **lot formation** influences the effectiveness of the acceptance sampling plan. Therefore, formation of the lot is important for the success of an acceptance sampling plan. Some guidelines for the lot formation are as follows:

i) Units in the lot should be homogeneous. It means that the lot should consist of units produced by the same machine, same operators, using the same raw materials and approximately during the same time period.

ii) Units in the lots should be packed so that the shipping costs and handling risks are minimum. This also makes the selection of units in the sample easy.

The decision about the acceptance or rejection of all units of the lot is taken on the basis of the results of inspection of the sample. Therefore, the sample should be such that it is a true representative of the lot.

2. **Sentencing**

   The act of accepting or rejecting the entire lot is called **sentencing** the lot.

3. **Lot Size**

   The number of units in a lot is called **lot size**. It is denoted by N.

4. **Sample Size**

   The number of units inspected to sentence a lot is called **sample size**. It is denoted by n.

5. **Lot Quality**

   The proportion of defective units in a lot is called **lot quality** or **proportion defective**. It is denoted by p and defined as

$$p = \frac{\text{Number of defective units in a lot}}{\text{lot size}} \quad \text{... (1)}$$

   From the definition of p given by equation (1), you should note that for the same lot size, as p increases, the lot quality decreases.

6. **Acceptance Number**

   When a deal is finalised between a seller and a buyer, they decide on the maximum number of allowable defective units in a sample. This number is called **acceptance number** and is denoted by c. The rule for acceptance or rejection of a lot is that if the number of defective units observed in the sample is less than or equal to the acceptance number c, the lot will be accepted. Otherwise, it will be rejected.

Let us explain the above terminology with the help of an example.

Example 1: Suppose a cricket ball manufacturing company supplies lots of 500 balls. To check the quality of the lots, a buyer draws a random sample of size 20 balls from each lot and accepts the lot if the inspected sample contains at most one defective ball. Otherwise, he/she rejects the lot. If the lot consists of 10 defective balls, find N, n, p and c.

Solution: Since the lot contains 500 balls, the **lot size** N = 500.

Since the buyer draws a sample of size 20 balls from each lot to take the decision about the lot, the **sample size** n = 20.

The buyer accepts the lot if the inspected sample contains at most one defective ball. So the **acceptance number** c = 1.

The lot consists of 10 defective balls. So from equation (1), the **lot quality** is given as

$$p = \frac{\text{Number of defective units in a lot}}{\text{lot size}} = \frac{10}{500} = 0.02$$

Now that you have learnt about terms such as lot size, lot quality, sample size, etc., you may like to know: **How can we be confident that a lot of good quality will be accepted and a lot of bad quality will be rejected?** We can answer such questions by calculating the probability of accepting a lot of specified quality. This probability will be 1 if all units in the lot are accepted and 0, if all units in the lot are rejected. The probability of accepting a lot is denoted by $P_a(p)$ or in short as P_a.

7. Probability of accepting a lot (P_a)

Suppose that the lots of the same size, say, N, are received from the supplier or the final assembly line and submitted for inspection one at a time. The lot is accepted if the number of defective units observed in the sample is less than acceptance number (c). Otherwise, it is rejected. Suppose a random sample of size n is drawn from each lot for inspection.

If X represents the number of defective units in the sample, the lot is accepted if $X \le c$. It means that we accept the lot if X = 0 or 1 or 2, ... or c. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P_a = P[X \le c] = P[\ X = 1 \text{ or } 2 \text{ or } 3, ..., \text{ or } c] \qquad ... (2)$$

You have studied in Unit 3 of MST-003 that if A and B are mutually exclusive events then $P[A \text{ or } B] = P[A] + P[B]$

Since X = 0, 1,..., c are mutually exclusive events, from the addition theorem of probability, we have

$$P_a(p) = P[X \le c] = P[X = 0] + P[X = 1] + ... + P[X = c]$$

or $$P_a(p) = \sum_{x=0}^{c} P[X = x] \qquad ... (3)$$

We can easily calculate this probability if we know the probability distribution of X. Generally, in quality control, a random sample is drawn without replacement. So the number of defective units (X) in the sample follows a hypergeometric distribution. However, we know that when the lot size (N) is large compared to the sample size, i.e, $N \ge 10n$, the hypergeometric distribution approximates a binomial distribution with parameters n and p where p is the lot quality. It is easier to calculate the probabilities with the help of the binomial distribution rather than with the hypergeometric distribution.

Therefore, the probability of getting exactly x defective units in a sample of size n using the binomial distribution is given by

The notation ${}^nC_x$ can also be repersened as $\binom{n}{x}$.

$$P[X = x] = {}^nC_x\, p^x (1-p)^{n-x} \qquad ... (4)$$

Hence, from equation (3), we have

$$P_a(p) = P[X \le c] = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} {}^nC_x\, p^x (1-p)^{n-x} \qquad ... (5)$$

However, for rapid calculation, we can use **Table I** entitled **Cumulative Binomial Probability Distribution** given at the end of this block.

Let us now compute $P_a(p)$ for Example 1.

In this example, we have

$$N = 500,\ n = 20,\ c = 1,\ p = 0.02$$

Suppose X represents the number of defective balls in the sample. The buyer accepts the lot if the number of defective balls (X) in the sample is at most one, i.e., c = 1. Therefore, the probability of accepting the lot of quality p = 0.02 is given by

$$P_a(p) = P[X \le c] = P[X \le 1] = P[X = 0] + P[X = 1]$$

$$= \sum_{x=0}^{1} P[X = x]$$

Since the lot size (N) is large compared to the sample size (n), i.e., $N \geq 10n$, X approximately follows the binomial distribution with parameters n and p where p is the lot quality. Therefore, the probability of accepting the lot of quality p is given by

$$P_a(p) = \sum_{x=0}^{1} P[X = x] = \sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x}$$

For rapid calculation, we can use Table I for obtaining this probability.

From Table I, for n = 20, x = c = 1 and p = 0.02, we have

$$\sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.9401$$

Therefore, the probability of accepting the lot of quality p = 0.02 is given by

$$P_a(p) = \sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.9401 \qquad \text{.. (6)}$$

It means that if there are several lots of the same quality p = 0.02, about 94.01% of these will be accepted and about 5.99% will be rejected.

We can also calculate this probability manually using the scientific calculator for n = 20, p = 0.02 as follows:

$P[X = x] = {}^{n}C_x\, p^x (1-p)^{n-x}$

$$\begin{aligned} P_a(p) &= P[X \leq 1] = P[X = 0] + P[X = 1] \\ &= {}^{20}C_0 (0.02)^0 (1-0.02)^{20-0} + {}^{20}C_1 (0.02)^1 (1-0.02)^{20-1} \\ &= (0.98)^{20} + 20 \times 0.02 \times (0.98)^{19} \\ &= 0.6676 + 0.2725 = 0.9401 \end{aligned}$$

Now, we can see the effect of the lot quality on the probability of accepting the lot.

Suppose the lot in Example 1 contains 20 defective balls instead of 10. Then the lot quality is:

$$p = \frac{\text{Number of defective units in the lot}}{\text{lot size}} = \frac{20}{500} = 0.04$$

We can calculate the probability of accepting the lot of quality p = 0.04 with the help of Table I as discussed above.

From Table I, for n = 20, x = c = 1 and p = 0.04, we have

$$P_a(p) = P[X \leq 1] = 0.8103$$

In this case, about 81.03% lots will be accepted and about 18.97% lots will be rejected.

If we compare this probability with the probability obtained for p = 0.02, we observe that as the lot quality decreases (from p = 0.02 to p = 0.04), the probability of accepting the lot also decreases (from $P_a = 0.9401$ to $P_a = 0.8103$).

We can also see the effect of the acceptance number on the probability of accepting the lot.

Suppose the buyer accepts the lot if the inspected sample contains at most two defective balls, that is, c = 2. Then we can obtain the probability of accepting the lot using Table I.

From Table I, for n = 20, x = c = 2 and p = 0.02, we have

$$P_a(p) = P[X \le c] = P[X \le 2] = 0.9929$$

It means that if there are several lots of the same quality p = 0.02, then for the acceptance number c = 2, about 99.29% lots will be accepted and about 0.71% lots will be rejected.

If we compare this probability with the probability obtained for p = 0.02 and c = 1, we observe that when the acceptance number increases (from c = 1 to c = 2), the probability of accepting the lot also increases (form $P_a = 0.9401$ to $P_a = 0.9929$).

It is time for you to pause and solve some exercises for practice.

E1) A shopkeeper purchases pens from a pen company in cartons (lots) that usually contain one thousand pens. To check the quality of the pens, the shopkeeper selects 25 pens at random from each carton and visually inspects each selected pen for certain defects. The shopkeeper accepts the lot if the inspected sample contains at most two defective pens. Otherwise, he/she rejects the lot. If there are 40 defective pens in each carton, find N, n, p, c and P_a.

E2) For a sampling plan with n = 5 and c = 0, find the probability of accepting a lot that has 2% defective units by assuming that the number of defective units in a sample follows a binomial distribution.

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8. Acceptance Quality Level (AQL)

The seller and the buyer generally know that the supply of completely defect-free lots is not possible. So they usually negotiate and arrive at an agreement that the buyer will accept all lots which have at most a definite quality level or definite percentage of defective units. This definite quality level is known as **acceptance quality level (AQL)**. It is denoted by p_1. Hence, AQL can be defined as follows:

Acceptance quality level is the quality level decided in a negotiation of the seller and the buyer: If the proportion of defective units in a lot is less than or equal to AQL, the buyer will have to definitely accept the lot. Otherwise, the buyer may accept or reject the lot.

Suppose 100% inspection of all lots is carried out and the lot quality (p) for each lot is computed. Then all lots with lot quality p ≤ AQL will be accepted and all lots with p > AQL may either be accepted or rejected.

In Example 1, if the manufacturing company and the buyer decide that the buyer will accept all lots which have at most 2% defective units, then acceptance quality level (AQL) of this deal is 2%.

Even though the seller and the buyer decide that the buyer would accept all lots of AQL quality, this may not happen in practical situations. In Example 1, the probability of accepting the lot of the lot quality p = AQL = 0.02 was

$$P_a(p) = P[X \le 1] = 0.9401$$

It means that if there are several lots of the same quality p = 0.02 = AQL, about 94.01% of these would be accepted and about 5.99% would be rejected. This is obviously a risk of the manufacturer because it was agreed upon by both that all lots of quality 0.02 will be accepted whereas the buyer is rejecting 5.99% of them. This risk is known as producer's risk which is explained in detail in Sec.5.6.

9. **Lot Tolerance Percent Defective (LTPD)**

In order to reduce the producer's risk, the buyer agrees to tolerate a lot quality worse than acceptance quality level (AQL) up to a certain limit but not beyond it. This limiting value is known as the **lot tolerance percent or proportion defective (LTPD)**. It is also known as **rejectable quality level (RQL), unsatisfactory quality level** or **limiting quality level (LQL)**.

LTPD is also decided at the time when the acceptance quality level is decided in the negotiation between the producer and the buyer. They make an agreement that the buyer will definitely reject the lot of quality equal to or greater than LTPD. Therefore, LTPD can be defined as follows:

Lot tolerance percent or proportion defective is the quality level decided in negotiation between the producer and the buyer: If the proportion of defective units in a lot is equal to or greater than this level, the buyer will definitely reject the lot. Otherwise, the buyer may accept or reject the lot.

The lot tolerance percent defective (LTPD) of a sampling plan is the level of quality at which the lot is routinely rejected by the sampling plan. LTPD is greater than AQL.

In Example 1, if the manufacturing company and the buyer decide that the buyer will accept all lots which have at most 2% defective units and reject all lots which have 5% or more defective units, then for this plan the AQL is 2% and the LTPD is 5%.

This level represents the dividing line between good and bad lots. Therefore, lot tolerance percent defective is the quality level of a lot that the buyer considers bad and he/she would like to reject all lots that have this level of quality. LTPD is denoted by p_2.

In acceptance sampling, the acceptance or rejection of the entire lot depends on the conclusions drawn from the sample. Thus, there is always a chance of making a wrong decision. It means that a lot of good quality may be rejected and a lot of poor quality may be accepted. This leads to two kinds of risks:

i) Producer's risk, and

ii) Consumer's risk.

We now discuss these risks in some details.

5.6 PRODUCER'S RISK AND CONSUMER'S RISK

In quality control, a producer may be defined as follows:

A **person** or a **firm** or an **organisation** that **produces/manufactures** goods or provides services for use or consumption of another person or firm or organisation is known as a **producer**.

In Example 1, the manufacturing company that produces the cricket balls, is the producer. Let us explain what is meant by producer's risk.

Producer's Risk

It may happen in practice that a sampling inspection plan leads to the rejection of a lot of satisfactory or good quality. This means that there is a possibility of rejecting a lot having a quality level less than or equal to the acceptance quality level (AQL) due to sampling inspection. If a lot of good quality is rejected, the producer suffers loss. Therefore, the producer always faces the risk of a good lot being rejected. Such a risk is known as **producer's risk** and is defined as follows:

The probability of rejecting a lot of acceptance quality level (AQL) is known as the producer's risk. It is denoted by $P_p(p)$ or in short as P_p and given by

$$P_p(p) = P_p = P[\text{rejecting a lot of acceptance quality level}] = \alpha$$

This can be written as

$$P_p(p) = 1 - P[\text{accepting a lot of acceptance quality level}]$$

$$= 1 - P_a(p - p_1) \qquad \ldots (7)$$

The producer's risk is equivalent to the type I error in hypothesis testing discussed in Unit 8 of MST-004 entitled Statistical Inference. Therefore, it is also denoted by α.

For computing the producer's risk, we have to compute the probability of accepting a lot of quality p = AQL. Then we use equation (7) to compute the producer's risk.

Let us explain how to compute the producer's risk with the help of an example.

Example 2: Suppose in Example 1, the manufacturing company and the buyer agree that AQL = 0.02. Find the producer's risk for this plan.

Solution: We know that the producer's risk is

$$P_p(p) = 1 - P[\text{accepting a lot of acceptance quality level}] = 1 - P_a(p)$$

For computing the producer's risk, we first compute the probability of accepting a lot of quality (p) = AQL = 0.02. We have already calculated this probability in Example 1. So here we use the result:

$$P_a(p) = 0.9401$$

Therefore, the producer's risk for this plan is

$$P_p(p) = 1 - P_a(p) = 1 - 0.9401 = 0.0599$$

It means that if there are several lots of the same quality p = 0.02 as AQL, out of these lots, about 5.99% will be rejected. This is obviously a risk of the manufacturer (producer) because it was agreed upon by both that all lots of quality p =AQL = 0.02 will be accepted whereas the buyer is rejecting 5.99% of the lots.

Let us now explain the consumer's risk.

Consumer's Risk

A **person** or a **firm** or an **organisation** that **purchases** goods for its own need or consumption or for use in the production of other goods (not for resale to another person or firm or organization directly or indirectly) is known as a **consumer**.

In Example 1, since the buyer purchases the cricket balls from the manufacturer, he/she is the consumer.

Just as the producer has a risk of a lot of good quality being rejected, a consumer also has a risk of buying a lot of unsatisfactory quality. Such a risk is known as the **consumer's risk**. If p_2 is the maximum proportion of defective (LTPD) in the lot, which the consumer is ready to tolerate, the consumer's risk may be defined as follows:

The probability of accepting a lot of unsatisfactory quality, i.e., LTPD is known as consumer's risk. It is denoted by $P_c(p)$ or in short as P_c.

Thus,

$$P_c(p) = P[\text{accepting a lot of quality} = \text{LTPD}] = \beta \qquad \text{... (8)}$$

Note that $P_c(p)$ is the same as $P_a(p)$ for p = LTPD.

The consumer's risk is denoted by β because it is equivalent to the type II error described in Unit 8 of MST-004.

Let us explain how to compute the consumer's risk with the help of an example.

Example 3: Suppose in Example 1, the manufacturing company and the buyer agree that LTPD = 0.05. Find the consumer's risk for this plan.

Solution: We know that the consumer's risk is given by

$$P_c(p) = P[\text{accepting a lot of quality} = \text{LTPD}] = P_a(p = 0.05)$$

It is given that

$$N = 500, n = 20, c = 1 \text{ and } \text{LTPD} = 0.05$$

We first calculate the probability of accepting the lot of quality p = 0.05 as follows:

$$P_a(p) = P[X \le c] = P[X \le 1]$$

We can calculate this probability as we have discussed in Example 1.

From Table I, for n = 20, x = c = 1 and p = LTPD = 0.05, we have

$$P_a(p) = P[X \le 1] = 0.7358$$

Therefore the consumer's risk is given by

$$P_c(p) = P_a(p = 0.05) = 0.7358$$

It means that if there are several lots of the same quality p = 0.05, out of these about 73.58% of them will be accepted by the buyer even though this quality is unsatisfactory. This is obviously the buyer's risk.

We take up an example to further explain the terms AQL, LTPD, producer's risk and consumer's risk.

Example 4: A mobile manufacturing company has decided to purchase the mobile batteries from a battery manufacturing company. Both manufacturing companies have decided that the batteries are to be supplied in lots of 1000 batteries each. The lot will be accepted up to quality level p = 0.05 and rejected at more than quality level p = 0.20. Acceptance sampling plan is based on a sample of size 25 drawn from each lot and the lot is accepted if inspected sample contains at most one defective battery. Otherwise, the lot is rejected. Identify which company is the producer and which one is the consumer in the plan. Calculate the producer's risk and the consumer's risk.

Solution: Here the mobile manufacturing company purchases the mobile batteries from a battery manufacturing company. So it is a consumer. The battery manufacturing company supplies batteries to the mobile manufacturing company. So it is a producer.

It is given that

$$N = 1000, AQL = 0.05, LTPD = 0.20, n = 25 \text{ and } c = 1$$

We know that the producer's risk is defined as

$$P_p = P[\text{rejecting a lot of acceptance quality level}]$$

$$= 1 - P[\text{accepting a lot of quality} = AQL = 0.05] = 1 - P_a(p = 0.05) \ldots (i)$$

To calculate the producer's risk, we have to calculate the probability of accepting a lot of quality p = 0.05.

Suppose X denotes the number of defective mobiles in the sample. The mobile company accepts the lot if the number of defective mobiles (X) in the sample is at most one. Therefore, the probability of accepting the lot of quality p is given by

$$P_a(p) = P[X \le c] = P[X \le 1]$$

Since the lot size (N) is large compared to the sample size (n), i.e., $N \ge 10n$, we can use the binomial distribution and can use Table I for obtaining the probability $P[X \le 1]$.

From Table I, for n = 25, x = c = 1 and p = 0.05, we have

$$P[X \le 1] = 0.6424$$

Therefore, the probability of accepting the lot of quality p = 0.05 is given by

$$P_a(p = 0.05) = 0.6424$$

From (i), we calculate the producer's risk as follows:

$$P_p = 1 - P_a(p = 0.05) = 1 - 0.6424 = 0.3576$$

Similarly, we know that the consumer's risk is defined as

$$P_c = P[\text{accepting a lot of quality } p = LTPD = 0.20] = P_a(p = 0.20)$$

From Table I, for n = 25, x = c = 1and p = 0.2, we have

$$P_c = P_a(p = 0.20) = 0.0274$$

You may like to solve some exercises for practice.

E3) Identify the consumer and producer in the exercise E1. If the shopkeeper and the pen company have decided that AQL = 0.03 and LTPD = 0.10, calculate the producer's risk and the consumer's risk.

E4) If in the exercise E2, the acceptance quality level (AQL) and the Lot tolerance percent defective (LTPD) are 1% and 5%, respectively, calculate the producer's risk and the consumer's risk for this plan.

We end this unit by giving a summary of what we have covered in it.

5.7 SUMMARY

1. The technique of controlling the quality of products in such a way that these are free from defects and conform to their specifications, is called **product control**.
2. The method of checking, measuring, testing one or more quality characteristics of a product (unit) to determine whether it satisfies the required specifications or not is called **inspection**.
3. If quality characteristic of a unit is measured on a continuous scale, the inspection is called **inspection by variables**.
4. If the actual measurements of the quality characteristic of a unit are not taken, but the unit is categorised as defective or non-defective on the basis of Go-No-Go gauges, the inspection is called **inspection by attributes**.
5. A **lot** is the collection of units or items from which a sample is taken and inspected to determine its acceptability.
6. If each and every unit of a lot is inspected, the inspection is known as **100% inspection**.
7. If some items or units are randomly selected from a lot in such a way that a selected sample is a true representative of the entire lot and each and every unit of the selected sample is inspected, the inspection is known as **sample inspection**.
8. **Acceptance sampling** is a technique in which a small part or a fraction of units is selected randomly from a lot and the selected units are inspected to decide whether the lot should be accepted or rejected on the basis of the information supplied by the sample inspection.
9. The proportion of defective units in a lot is called **lot quality** or **proportion defective**, which is denoted by p and defined as follows:

$$p = \frac{\text{Number of defective units in a lot}}{\text{lot size}}$$

10. The **acceptance quality level (AQL)** is the quality level decided mutually by the manufacturer and the buyer. If the proportion of defective units in a lot is less than or equal to AQL, the buyer has to accept the lot. Otherwise, the buyer may accept or reject the lot.
11. The **lot tolerance percent or proportion defective (LTPD)** is the quality level decided mutually by the manufacturer and the buyer. If the proportion of defects in a lot is greater than or equal to this level, the buyer will definitely reject the lot. Otherwise, the buyer may accept or reject the lot.

12. The probability of rejecting a lot of acceptance quality level (AQL) is known as producer's risk $P_p(p)$.
13. The probability of accepting a lot of unsatisfactory quality (LTPD) is known as consumer's risk $P_c(p)$.

5.8 SOLUTIONS/ANSWERS

E1) The carton contains 1000 pens and so the lot size N = 1000.

Since the shopkeeper draws a sample of size 25 pens, the sample size n = 25.

The shopkeeper accepts the lot if the inspected sample contains at most two defective pens. Otherwise, he/she rejects the lot.

$\therefore$ The acceptance number c = 2

The lot contains 40 defective pens

$$\therefore \quad p = \frac{\text{Number of defective pens in the lot}}{\text{lot size}} = \frac{40}{1000} = 0.04$$

If X represents the number of defective pens in the sample, then the shopkeeper accepts the lot if $X \le c = 2$. Therefore, the probability of accepting the lot is given by

$$P_a = P[X \le 2] = P[X = 0] + P[X = 1] + P[X = 2]$$

Since the lot size is large as compared to the sample size ($N \ge 10n$), X approximately follows the binomial distribution with parameters n and p. Therefore, we can obtain this probability by using Table I.

From Table I, for n = 25, x = c = 2, and p = 0.04, we have

$$P_a = P[X \le c = 2] = 0.9235$$

E2) It is given that

n = 5, c = 0 and p = 2% = 0.02

Suppose X represents the number of defective units in the sample, the probability of accepting the lot is given by

$$P_a = P[X \le c] = P[X \le 0]$$

Since X follows the binomial distribution, we can use Table I to obtain this probability.

From Table I, for n = 5, x = c = 0, and p = 0.02, we have

$$P[X \le 0] = 0.9039$$

Hence, the probability of accepting the lot is given by

$$P_a = P[X \le 0] = 0.9039$$

E3) The shopkeeper purchases pens from a pen company and, therefore, he/she is a consumer. The pen company is a producer since it sells pens.

We have

n = 25, c = 2, AQL = 0.03 and LTPD = 0.10

The producer's risk is given by

$$P_p = \alpha = P[\text{rejecting a lot of acceptance quality level}]$$
$$= 1 - P[\text{accepting a lot of AQL} = 0.03]$$
$$= 1 - P_a(p = 0.03)$$

` From Table I, for n = 25, x = c = 2 and p = AQL = 0.03, we have

$$P_a(p = 0.03) = 0.9620$$

$$\therefore \quad P_p = 1 - P_a(p = 0.03) = 1 - 0.9620 = 0.0380$$

The consumer's risk is given by

$$P_c = P[\text{accepting a lot of LTPD quality}]$$
$$= P_a(p = 0.10)$$

From Table I, for n = 25, x = c = 2 and p = LTPD = 0.10, we have

$$P_a(p = 0.10) = 0.5371$$

$$\therefore \quad P_c = P_a(p = 0.10) = 0.5371$$

E4) It is given that

n = 5, c = 0, AQL = 1% = 0.01 and LTPD = 5% = 0.05

The producer's risk is

$$P_p = \alpha = P[\text{rejecting a lot of acceptance quality level}]$$
$$= 1 - P_a(p = 0.01)$$

`From Table I, for n = 5, x = c = 0 and p = AQL = 0.01, we have

$$P_a(p = 0.01) = 0.9510$$

$$\therefore \quad P_p = 1 - P_a(p = 0.01) = 1 - 0.9510 = 0.0490$$

The consumer's risk is

$$P_c = P[\text{accepting a lot of LTPD quality}]$$
$$= P_a(p = 0.05)$$

From Table I, for n = 5, x = c = 0 and p = LTPD = 0.05, we have

$$P_a(p = 0.05) = 0.7738$$

$$\therefore \quad P_c = P_a(p = 0.05) = 0.7738$$