UNIT 8 INVENTORY MODELS

Structure

8.1 INTRODUCTION

In Unit 7, you have studied the sequencing problem, which involves determining the optimum order or sequence of jobs for a process to optimise the total time. We have discussed two types of sequencing problems: the ones with n number of jobs to be completed through 2 machines and those with 2 jobs to be completed through m number of machines, in some pre-assigned order.

In this unit, we discuss various inventory models. Inventory refers to a stock of goods, materials, human resources or financial resources or any other idle resource having some economic value, which is stocked in order to meet the demand expected in future. Almost every business must maintain an inventory for running its operations efficiently and smoothly.

Although inventories are essential for business, maintenance of inventories also costs money by way of expenses on stores, equipment, personnel, insurance, etc. Thus, excess inventories are undesirable. This calls for controlling the inventories in the most profitable way. In the present unit, we discuss the models for inventory control known as **economic order quantity** models. These models help in deciding as to how much quantity should be kept in stock in order to balance the costs of holding too much stock vis-à-vis the costs of ordering in small quantities.

In this unit, we discuss inventory control and various factors involved in inventory analysis in Sec. 8.2. In Secs. 8.3 to 8.7, we describe five models for determining the economic order quantity: i) when demand is uniform; ii) when rates of demand are different in different cycles; iii) when shortages are allowed; iv) when replenishment is uniform; and v) when price (or quantity) discounts are given.

Objectives

After studying this unit, you should be able to:

- explain the concept of inventory control;
- determine the economic order quantity when demand is uniform;
- determine the economic order quantity when rates of demand are different in different cycles;

- determine the economic order quantity when shortages are allowed;
- determine the economic order quantity when replenishment is uniform; and
- determine the economic order quantity when there are price discounts.

8.2 INVENTORY CONTROL

An inventory means a physical stock of idle resources of any kind having some economic value kept for the purpose of meeting future demand. It indicates the raw material required before production, the finished goods after production ready for delivery to consumers, human resources, financial resources, etc., which are stocked in order to meet an expected demand in the future. Almost every business must maintain an inventory for running its operations efficiently and smoothly. If an enterprise does not maintain an inventory, it may suddenly find at some point in its operations that it has no materials or goods to supply to its customers. Then on receiving a sales order, it will first have to place order for purchase of raw materials, wait for their receipt and then start production. The customer will, thus, have to wait for a long time for the delivery of the goods and may turn to other suppliers, resulting in loss of business/goodwill for the enterprise.

Maintaining an inventory is necessary because of the following reasons:

i) It helps in smooth and efficient running of an enterprise.

ii) It provides service to the customer at short notice. Timely delivery can fetch more goodwill and orders.

iii) In the absence of the inventory, an enterprise may have to pay high prices because of piecemeal purchasing. Maintaining an inventory may earn price discounts because of bulk purchasing. Such purchases entail less orders and, therefore, less clerical costs.

iv) It also takes advantage of favourable market.

v) It acts as a buffer stock when raw materials are received late and shop rejections are too many.

vi) Process and movement inventories (also called pipeline stocks) are quite necessary in big enterprises wherein a significant amount of time is required to ship items from one location to another.

Though inventories are essential, their maintenance also costs money by way of expenses on stores, equipment, personnel, insurance, etc. Thus, excess inventories are undesirable. So, only that quantity should be kept in stock, which balances the costs of holding too much stock vis-à-vis the costs of ordering in small quantities. This calls for controlling the inventories in the most profitable way and that is why we need inventory analysis. We now discuss various factors involved in inventory analysis.

1. **Inventory related costs**

   Various costs associated with inventory control are often classified as follows:

   i) **Set-up cost**: This is the cost associated with the setting up of machinery before starting production. The set-up cost is generally assumed to be independent of the quantity ordered for.

   ii) **Ordering cost**: This is the cost incurred each time an order is placed. This cost includes the administrative costs (paper work, telephone calls, postage), transportation, receiving and inspection of goods, etc.

iii) **Purchase (or production) cost**: It is the actual price at which an item is purchased (or produced). It may be constant or variable. It becomes variable when quantity discounts are allowed for purchases above a certain quantity.

iv) **Carrying (or holding) cost**: The cost includes the following costs for maintaining the inventory: i) Rent for the space; ii) cost of equipment or any other special arrangement for storage; iii) interest of the money blocked; iv) the expenses on stationery; v) wages of the staff required for the purpose; vi) insurance and depreciation; and vii) deterioration and obsolescence, etc.

v) **Shortage (or Stock-out) cost**: This is the penalty cost for running out of stock, i.e., when an item cannot be supplied on the customer's demand. These costs include the loss of potential profit through sales of items demanded and loss of goodwill in terms of permanent loss of the customer.

2. **Demand**

Demand is the number of units required per period and may either be known exactly or known in terms of probabilities. Problems in which demand is known and fixed are called **deterministic problems** whereas problems in which demand is known in terms of probabilities are called **probabilistic problems**.

3. **Selling Price**

The amount which one gets on selling an item is called its selling price. The unit selling price may be constant or variable, depending upon whether quantity discount is allowed or not.

4. **Order Cycle**

The period between placement of two successive orders is referred to as an order cycle. The order may be placed on the basis of either of the following two types of inventory review systems:

a) The record of the inventory level is checked continuously until a specified point is reached where a new order is placed. This is called continuous review.

b) The inventory levels are reviewed at equal intervals of time and orders are placed accordingly at such levels. This is called periodic review.

5. **Time Horizon**

The period over which the time cost will be minimised and inventory level will be controlled is termed as time horizon. This can be finite or infinite depending on the nature of demand.

6. **Stock Replenishment**

The rate at which items are added to the inventory is called the rate of replenishment. The actual replenishment of items may occur at a uniform rate or be instantaneous over time. Usually uniform replacement occurs in cases when the item is manufactured within the factory while instantaneous replacement occurs in cases when the items are purchased from outside sources.

7. **Lead Time**

The time gap between placing an order for an item and actually receiving the item into the inventory is referred to as lead time.

8. **Reorder Level**

   The lower limit for the stock is fixed at which the purchasing activities must be started for replenishment. With this replenishment, the stock reached at a level is known as maximum stock. The level between maximum and minimum stock is known as the reorder level.

9. **Economic Order Quantity (EOQ)**

   The order in quantity that balances the costs of holding too much stock vis-à-vis the costs of ordering in small quantities too frequently is called Economic Order Quantity (or Economic lot size).

10. **Reorder Quantity**

    The quantity ordered at the level of minimum stock is known as the reorder quantity. In certain cases it is the 'Economic Order Quantity'.

In Secs. 8.3 to 8.7, we shall discuss the following inventory models for obtaining economic order quantity:

i) EOQ Model with Uniform Demand

ii) EOQ Model with Different Rates of Demand in Different Cycles

iii) EOQ Model when Shortages are Allowed

iv) EOQ Model with Uniform Replenishment

v) EOQ Model with Price (or Quantity) Discounts

However, before discussing these models, we give the notations that we shall use in the development of the models.

The notation used in the Models

Q = Number of units ordered (supplied) per order

D = Demand in units of inventory per year

N = Number of orders placed per year

TC = Total Inventory cost

C_O = Ordering cost per order

C = Purchase or manufacturing price per unit inventory

C_h = Carrying or holding cost per unit per period of time the inventory is kept

C_s = Shortage cost per unit of inventory

t = The elapsed time between placement of two successive orders

r_p = Replenishment rate at which lot size Q is added to inventory.

8.3 ECONOMIC ORDER QUANTITY (EOQ) MODEL WITH UNIFORM DEMAND

The objective of the **EOQ model with uniform demand** is to determine an optimum economic order quantity such that the total inventory cost is minimised. We make the following assumptions for this model:

1. Demand rate (D) is constant and known;
2. Replenishment rate (r_p) is instantaneous;
3. Lead time is constant and zero;

4. Purchase price is constant, i.e., discounts are not allowed;
5. Carrying cost and ordering cost are known and constant; and
6. Shortages are not allowed.

The situation can be graphically represented as shown in Fig. 8.1.

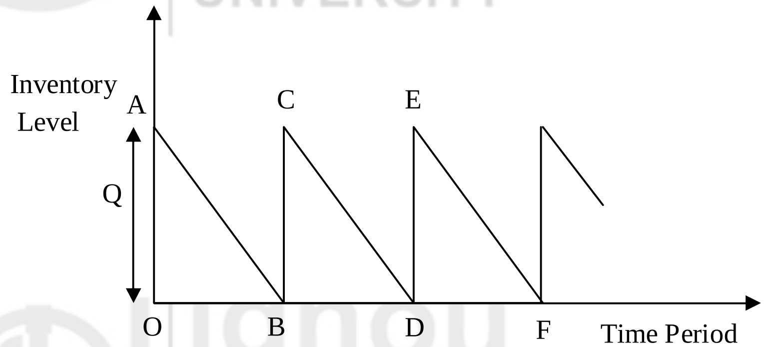

Fig. 8.1

The graph in Fig. 8.1 shows that initially there were Q units in the stock. The number of units goes on decreasing with respect to time to meet the demand and this is represented by the line AB in the graph. When the stock vanishes, i.e., the point B is reached, the stock level rises to Q instantaneously as the replenishment is instantaneous. Since the demand is uniform, the rate of decrease of the quantity remains the same as earlier. Therefore, this is represented by the line CD in the graph, which is parallel to AB. Similarly, EF is parallel to AB and CD due to uniform demand, and so on.

Since the demand is uniform, the average inventory is simply the arithmetic mean of the maximum and the minimum levels of inventory. Let Q be the quantity ordered (or replenished) when the minimum level, i.e., zero is reached.

Therefore, the average inventory = $\frac{Q+0}{2} = \frac{Q}{2}$

The ordering cost = Number of orders per year $\times$ ordering cost per order

$$= N \times C_O = \frac{D}{Q} \times C_O$$

The carrying cost = Average units in inventory $\times$ carrying cost per unit

$$= \frac{Q}{2} \times C_h$$

The total inventory cost is the ordering cost plus the carrying cost. Therefore,

$$TC = \frac{D}{Q} C_O + \frac{Q}{2} C_h$$

The total inventory cost (TC) is minimum at that value of Q where the derivative of TC with respect to Q is zero. Differentiating TC with respect to Q and then equating it to zero, we get

$$-\frac{D}{Q^2} C_O + \frac{1}{2} C_h = 0$$

$$\Rightarrow \frac{D}{Q} C_O = \frac{Q}{2} C_h \qquad \text{(i.e., ordering cost = carrying cost)}$$

$$\Rightarrow Q^2 = \frac{2D\,C_O}{C_h} \quad \Rightarrow Q = \sqrt{\frac{2DC_O}{C_h}}$$

This value of Q minimises the total inventory cost (TC) and hence it is the economic order quantity. Let us denote it by Q*. Thus, the EOQ for this model is

$$Q^* = \sqrt{\frac{2DC_O}{C_h}}$$

Hence, the optimum number of orders placed per year $(N^*) = \dfrac{D}{Q^*}$

The minimum total yearly inventory cost

$$(TC^*) = \frac{D}{Q^*} C_O + \frac{Q^*}{2} C_h = \frac{2DC_O + (Q^*)^2 C_h}{2Q^*}$$

$$= \sqrt{2DC_OC_h} \quad \text{(on simplification)}$$

and the total minimum cost = $(TC^*) +$ cost of material.

Let us consider an example to obtain the total minimum cost in this model.

Example 1: An enterprise requires 1000 units per month. The ordering cost is estimated to be ` 50 per order. In addition to ` 1, the carrying costs are 5% per unit of average inventory per year. The purchase price is ` 20 per unit. Find the economic lot size to be ordered and the total minimum cost.

Solution: We are given that

$$D = \text{Monthly demand} \times 12$$

$$= 1000 \times 12 = 12000 \text{ units per year}$$

C_O = ` 50 per order; C = ` 20 per unit

$C_h = 1 + 5\%$ of ` 20

= 1 + 1 = ` 2 per unit of average inventory

The economic lot size, therefore, is given by

$$Q^* = \sqrt{\frac{2DC_O}{C_h}} = \sqrt{\frac{2 \times 12000 \times 50}{2}} = 775 \text{ units}$$

Total minimum cost = TC* + Cost of material

$$= \sqrt{2DC_OC_h} + (12000 \times 20)$$

$$= \sqrt{2 \times 12000 \times 50 \times 2} + 240000$$

$$= \sqrt{2400000} + 240000 = \sqrt{240} \times 100 + 240000$$

= 15.5 $\times$ 100 + 240000 = ` 241550

You may now like to solve the following problems to assess your understanding.

E1) The XYZ manufacturing company uses 12000 units of raw material annually, which costs `1.25 per unit. Placing each order costs `15 and the carrying costs are 15% per year per unit of the average inventory. Find the economic order quantity.

E2) The XYZ manufacturing company has determined from an analysis of its accounting and production data for a part, that its cost to purchase is `36 per order and `2 per part. Its inventory carrying charge is 18% of the average inventory. The demand of this part is 10000 units per annum.

a) What should the economic order quantity be?

b) What is the optimum number of days supply per optimum order?

8.4 EOQ MODEL WITH DIFFERENT RATES OF DEMANDS IN DIFFERENT CYCLES

In this EOQ model, the same quantity is ordered each time when the stock vanishes with the assumption that the replenishment is instantaneous. The demand is not uniform and the stock vanishes in different periods of time. The situation can be graphically represented as shown in Fig. 8.2.

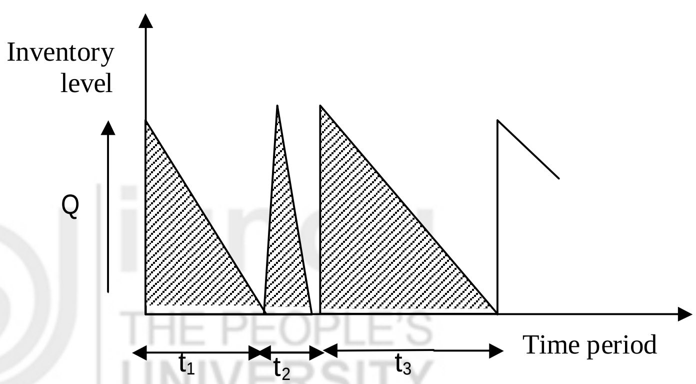

Fig. 8.2

Let the demand in different periods of time t_1, t_2, …, t_n be D_1, D_2, …, D_n, respectively, so that the total demand in time T is given by

$$D = D_1 + D_2 + \ldots + D_n \qquad \text{where } T = t_1 + t_2 + \ldots + t_n$$

The cost of ordering in time T

$$= \text{No. of orders per year} \times \text{ordering cost per order}$$

$$= N C_O$$

$$= \frac{\text{Total Demand in time T} \times \text{Ordering cost per order}}{\text{No. of units ordered per order}}$$

$$= \frac{D}{Q} \times C_O$$

The carrying cost for time T

$$= \text{Average unit in inventory} \times \text{carrying cost per unit}$$

$$= \frac{Q \times t_1}{2} C_h + \frac{Q \times t_2}{2} C_h + \ldots + \frac{Q \times t_n}{2} C_h$$

$$= \frac{Q}{2} C_h \left[t_1 + t_2 + \ldots + t_n \right] = \frac{QC_h T}{2}$$

$\therefore$ The total inventory cost (TC) = ordering cost + carrying cost

$$TC = \frac{DC_O}{Q} + \frac{QC_h T}{2}$$

This cost is minimum when its derivative with respect to Q is zero,
i.e., if

$$-\frac{D}{Q^2} C_O + \frac{1}{2} TC_h = 0$$

$$\Rightarrow \frac{D}{Q} C_O = \frac{Q}{2} TC_h \qquad \text{(i.e., ordering cost = carrying cost)}$$

$$\Rightarrow Q^2 = \frac{2D\ C_O}{C_h T} \quad \text{Þ} \quad Q^* = \sqrt{\frac{2D\ C_O}{C_h T}}$$

This result is similar to that of the model discussed in Sec. 8.3. The only difference is that the uniform demand (D) is replaced by the average demand (D/T).

$$\left(TC^*\right) = \frac{DC_O}{Q^*} + \frac{Q^* TC_h}{2} = \frac{2DC_O + (Q^*)^2 TC_h}{2Q^*}$$

$$= \sqrt{2DC_O C_h T} \quad \text{(on simplification);}$$

The total minimum cost = (TC*) + Cost of material

If T = 1 year, the result of this model is exactly the same as the EOQ model with uniform demand.

8.5 EOQ MODEL WHEN SHORTAGES ARE ALLOWED

The assumptions made for this model are the same as those for the EOQ model with uniform demand discussed in Sec. 8.3, except that shortages are allowed, which may occur frequently. The cost of shortages is assumed to be directly proportional to the number of units short. The assumptions for the model are:

i) Demand is constant and known;

ii) Replenishment is instantaneous;

iii) Lead time is zero;

iv) Purchasing cost is constant and known;

v) Carrying cost and ordering cost are constant and known; and

vi) Shortage is allowed and the shortage cost is directly proportional to the number of units short.

The situation can be represented graphically as shown in Fig. 8.3.

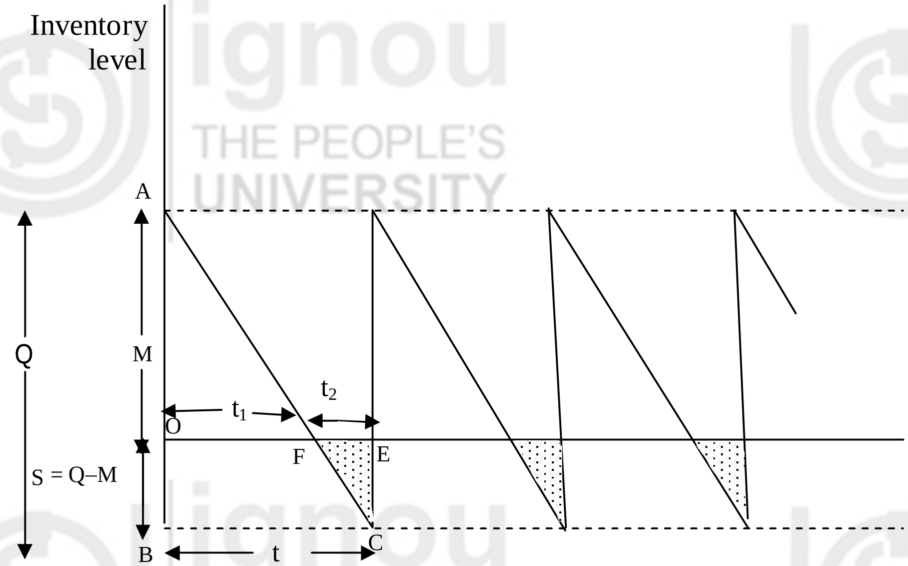

Fig. 8.3

Here,

Q = No. of units ordered per order

D = No. of units required per year

t_1 = Time when there are no shortages

t_2 = Time period during which there is shortage

t = Total cycle time, i.e., $t = t_1 + t_2$

S = Maximum shortage

The same quantity Q is ordered and received every time. The maximum shortage is equal to S. The remaining quantity M is placed in the inventory as surplus to satisfy the demand in the next cycle. Note that S units out of Q are always in the shortage list, i.e., these are never placed in the inventory. Thus, it results in a saving on the inventory carrying cost.

No. of orders placed per year $= N = \dfrac{D}{Q}$

Ordering cost = No. of orders placed per year × ordering cost per order

$$= NC_O = \frac{D}{Q}C_O$$

Average inventory $= \dfrac{M}{2}$

Carrying cost per cycle $= C_h\left(\dfrac{M}{2} \times t_1\right)$

Note from Fig. 8.3 that Δ ABC is similar to ΔAOF. Therefore,

$$\frac{t_1}{t} = \frac{M}{Q} \quad \Rightarrow t_1 = \frac{M}{Q}t$$

Therefore, carrying cost $= \left(\dfrac{M^2 \times t}{2Q}\right)C_h$

Carrying cost per unit time $= \dfrac{M^2}{2Q} C_h$

Shortage cost = Average no. of units short × time period of shortage × shortage cost per unit per time period

$$= \frac{S}{2} \times t_2 \times C_S = \left(\frac{S}{2} t_2\right) C_S$$

Now, ΔABC is similar to ΔCEF in Fig. 8.3. Therefore,

$$\frac{t_2}{t} = \frac{S}{Q} \quad \Rightarrow t_2 = \frac{S}{Q} t$$

Hence, shortage cost $= \left(\dfrac{S^2 \times t}{2Q}\right) C_S$

Shortage cost per unit time $= \dfrac{S^2}{2Q} C_S$

Therefore, the total inventory cost is given as:

TC = ordering cost + carrying cost + shortage cost

or $$TC = \frac{D}{Q} C_O + \frac{M^2}{2Q} C_h + \frac{S^2}{2Q} C_S$$

or $$TC = \frac{D}{Q} C_O + \frac{M^2 C_h}{2Q} + \frac{(Q-M)^2 C_S}{2Q} \qquad ...(1)$$

Now, TC is a function of two variables Q and M. Therefore, to find the minimum inventory cost, we differentiate TC with respect to Q and M separately and equate them to zero. Thus, we get

$$0 = \frac{d}{dQ}(TC) = -\frac{DC_O}{Q^2} - \frac{M^2 C_h}{2Q^2} + \frac{1}{2}\left[\frac{2Q(Q-M)^2 - (Q-M)^2}{Q^2}\right] C_S$$

$$0 = -\frac{DC_O}{Q^2} - \frac{M^2 C_h}{2Q^2} + \left(\frac{Q^2 - M^2}{2Q^2}\right) C_S$$

$$\Rightarrow -\frac{DC_O}{Q^2} - \frac{M^2}{2Q^2}(C_h + C_S) + \frac{C_S}{2} = 0$$

$$\Rightarrow \frac{C_S Q^2}{2} = \frac{M^2}{2}(C_h + C_S) + DC_O$$

$$\Rightarrow Q^2 = M^2 \left(\frac{C_h + C_S}{C_S}\right) + \frac{2DC_O}{C_S} \qquad \ldots (2)$$

and

$$\frac{d}{dM}(TC) = \frac{MC_h}{Q} - \frac{(Q - M)}{Q} C_S = 0$$

$$\Rightarrow \frac{M}{Q}(C_h + C_S) - C_S = 0$$

$$\Rightarrow M = \left(\frac{C_S}{C_h + C_S}\right)Q \qquad \text{….(3)}$$

Substituting the value of M in equation (2), we get

$$Q^2 = \left(\frac{C_S}{C_h + C_S}\right)^2 \times \left(\frac{C_h + C_S}{C_S}\right)Q^2 + \frac{2DC_O}{C_S}$$

$$Q^2\left[1 - \frac{C_S}{C_h + C_S}\right] = \frac{2DC_O}{C_S}$$

$$\Rightarrow Q^2\left[1 - \frac{C_S}{C_h + C_S}\right] = \frac{2DC_O}{C_S}$$

$$\Rightarrow Q^2\left(\frac{C_h}{C_h + C_S}\right) = \frac{2DC_O}{C_S}$$

$$\Rightarrow Q^2 = \frac{2DC_O}{C_h}\left(\frac{C_S + C_h}{C_S}\right)$$

Thus, the optimum value of Q is given by

$$Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{C_S + C_h}{C_S}\right)}$$

The optimum value of M is given by

$$M^* = \left(\frac{C_S}{C_h + C_S}\right)Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{C_S}{C_h + C_S}\right)}$$

Therefore, the minimum inventory cost

$$TC^* = \sqrt{2DC_OC_h\left(\frac{C_S}{C_h + C_S}\right)}$$

$$\text{Total Cycle time } = t^* = \frac{Q^*}{D} = \sqrt{\frac{2C_O}{DC_h}\left(\frac{C_S + C_h}{C_S}\right)}$$

Let us explain this model with the help of an example.

Example 2: A contractor has to supply engines to a truck manufacturer at the rate of 20 per day. The penalty in the contract is `100 per engine per day for missing the scheduled delivery date. The cost of holding an engine in stock for one month is `150. His production process is such that each month (30 days) he starts producing a batch of engines through the agencies and all are available for supply after the end of the month. Determine the maximum inventory level at the beginning of each month.

Solution: Here we have

$C_h =$ ` $\frac{150}{30}$ per engine per day = `5 per engine per day;

$C_s =$ ` 100 per day per engine; D = 20 engines per day and t* = 30 days.

The maximum inventory level M* at the beginning of each month will be

$$M^* = \frac{C_s}{C_h + C_s} Q^* = \frac{C_s}{C_h + C_s}\left(D \times t^*\right)$$

$$= \frac{100}{5+100} \times 20 \times 30 = 571 \text{ engines}$$

You may now like to solve the following problem to assess your understanding.

E3) A manufacturer has to supply his customer with 24000 units of his product every year. The demand is fixed and known. Since the unit is used by the customer in an assembly operation and the customer has no storage space for units, if the manufacturer fails to supply the required units, the shortage cost is `2 per unit per month. The inventory cost is `1 per unit per month and the set-up cost per run is ` 3500. Determine

i) the optimal run size (Q);

ii) the optimal level of inventory (M) at the beginning of any period;

iii) the optimal scheduling period; and

iv) the minimum inventory cost.

8.6 EOQ MODEL WITH UNIFORM REPLENISHMENT

For this model, it is assumed that the production run may take significant time to complete, that is, the replenishment is not instantaneous. The stock is added at a constant rate, i.e., the replenishment is uniform. Other assumptions are the same as the ones made for the EOQ model with uniform demand as explained in Sec. 8.3. The order size here is taken as the production size.

The graph representing this situation is shown in Fig. 8.4

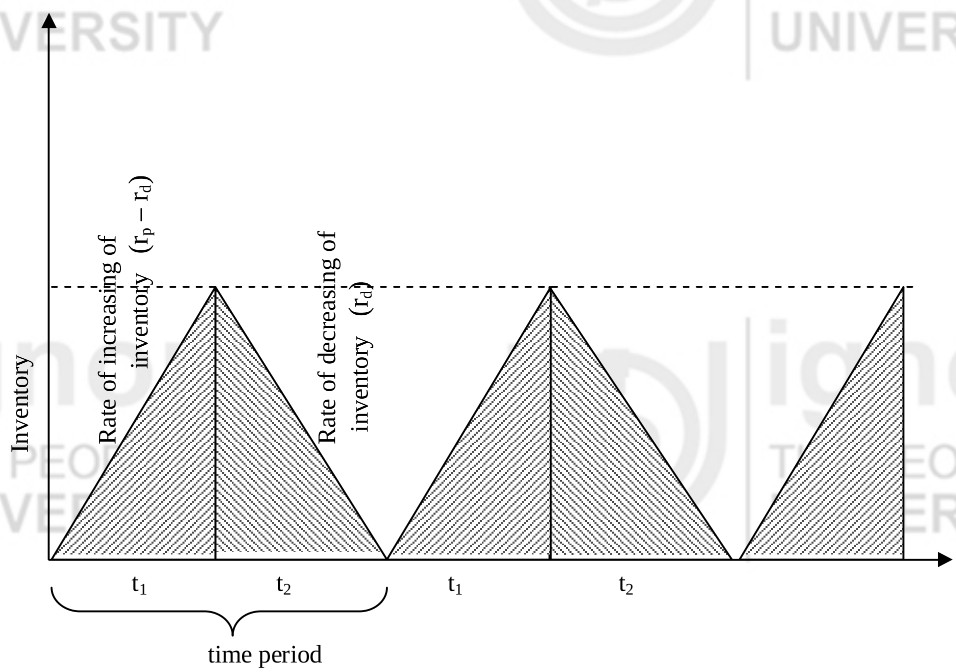

Fig. 8.4

Let r_d be the demand rate in units per time period, r_p, the replenishment rate per unit of time, t_1, the time of production run or time of replenishment, and t_2, the time required for the inventory to be exhausted. Then $t_1 + t_2 = t$ (say) is the time for each order cycle.

Let Q be the number of units produced per order cycle. Then

$$t_1 = \frac{Q}{r_p} \quad \left(\because \text{Time for producing 1 unit} = 1/\text{Rate of replenishment} = 1/r_p\right)$$

Inventory is building up at the rate of $r_p - r_d$.

$\therefore$ Maximum inventory level = $\left(r_p - r_d\right) t_1$

$$\text{Average inventory} = \frac{\left(r_p - r_d\right)t_1}{2} = \frac{\left(r_p - r_d\right)}{2}\frac{Q}{r_p} = \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)$$

Now, ordering (or set-up) cost = $\frac{D}{C}C_O$ and carrying cost = $\frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$

$\therefore$ the total annual inventory cost is given by

$$TC = \frac{D}{Q}C_O + \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$$

The cost will be minimum if $\frac{D}{Q}C_O = \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$

$$\Rightarrow Q^2 = \frac{2DC_O}{C_h}\left(\frac{r_p}{r_p - r_d}\right) \Rightarrow Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{r_p}{r_p - r_d}\right)}$$

Thus, the characteristics of the model are as follows:

1. Optimum number of production runs per year:

$$N^* = \frac{D}{Q^*} = \sqrt{\frac{DC_h\left(r_p - r_d\right)}{2C_o\, r_p}}$$

2. Length of each lot size production run:

$$t_1 = \frac{Q^*}{r_p} = \sqrt{\frac{2DC_O}{C_h r_p\left(r_p - r_d\right)}}$$

3. The total minimum inventory cost:

$$TC^* = \frac{D}{Q^*}C_O + \frac{Q^*}{2}\left(1 - \frac{r_d}{r_p}\right)C_h = \sqrt{2DC_OC_h\left(1 - \frac{r_d}{r_p}\right)}$$

We take up an example to apply this model.

Example 3: A tyre producer makes 1600 tyres per day and sells them at approximately half that rate. Accounting figures show that the production set-up cost is `1000 and carrying cost per unit is `5. If the annual demand is 160000 tyres, what is the optimal lot size and how many production runs should be scheduled per year?

Solution: We are given that the annual demand (D) = 160000 tyres, C_h = ` 5, $C_O$ = `1000, Production rate (r_p) = 1600 tyres per day and Demand rate (r_d) = 800 tyres per day.

$$\therefore \text{The optimal lot size } Q^* = \sqrt{\frac{2DC_O}{C_h}\frac{r_p}{r_p - r_d}}$$

$$Q^* = \sqrt{\frac{2\times160000\times1000}{5}\times\frac{1600}{1600-800}} = 11314 \text{ tyres}$$

Optimal production runs per year $N^* = \frac{D}{Q^*} = \frac{160000}{11314} = 14$ runs/year approx.

You may now like to solve the following problems to assess your understanding.

E4) A product is manufactured at the rate of 50 items per day. The demand occurs at the rate of 30 items per day. It is given that the set-up cost per order is `1000 and holding cost per unit time is `0.05. Find the economic lot size and the associated total cost per cycle assuming that no shortage is allowed.

E5) A company uses 100000 units of a particular item per year. Each item costs `2. The production engineering department estimates the holding cost as 12.5% of the value of the inventory per day. The replenishment rate is uniform at 500 units per day. Assuming 250 working days (for replenishment purpose), calculate the

a) optimal set-up quantity;

b) total cost on the basis of optimal policy; and

c) optimal number of set-ups.

8.7 EOQ MODEL WITH PRICE (OR QUANTITY) DISCOUNTS

If the items are purchased in bulk, some discount in price is usually offered by the supplier. The discount is called **all units discount** if it is applicable for all the units purchased. It is said to be **incremental discount** if discounts are offered only for the items which are in excess of the specified amount. In the incremental discount, the prices offered in different slabs are applicable in finding the total cost while in ‘all units discount’, only one price at any one slab is applicable for finding the total cost. Buying in large quantities may result in the following advantages:

Less unit price, less ordering costs, less transportation cost, fewer stock-outs and hence less chances of loss of goodwill, mass display by retailers and preferential treatment by the sellers.

At the same time, large quantity buying may involve the following disadvantages:

Higher carrying costs, older stocks, lower stock turnover, more capital required, blocked money, less flexibility, heavier deterioration and depreciation.

If the items are purchased with quantity discounts, the price C may vary according to the following scheme:

$$C = p_o \text{ if purchased quantity } Q = Q_0 < b_1$$

$$= p_1 \text{ if purchased quantity } b_1 \leq Q_1 < b_2$$

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$$= p_n \text{ if purchased quantity } b_n < Q_n$$

where p_{j-1} is greater than p_j, for j = 1, 2, …, n. Here p_j is the price per unit for the j^{th} lot size. If the shortages are not allowed and supply is instantaneous, the total cost per year is given by the set of relations:

$$TC\left(Q_j\right) = Dp_j + \frac{C_O D}{Q_j} + \frac{1}{2} C_h Q_j, \text{ where } b_j \leq Q_j < b_{j+1}$$

and $C_h = ip_j$, where i is the percentage change for j = 0, 1, 2, …., n.

We follow the procedure given below to find the overall optimum lot size:

Step 1: We find EOQ for the lowest price, i.e., we calculate

$$Q_n^* = \sqrt{\frac{2C_O D}{ip_n}}$$

If $Q_n^{*3}\ b_n$, the optimum order quantity is Q_n^*. If $Q_n^* < b_n$, we go to Step 2.

Step 2: We compute $Q_{n-1}^* = \sqrt{\dfrac{2C_O D}{ip_{n-1}}}$ for the next lowest price.

If $Q_{n-1}^* > b_{n-1}$, then we compare the total cost TC_{n-1} for purchasing Q_{n-1}^* with the total cost TC_n for purchasing quantity b_n and select the one that gives the least cost.

If $Q_{n-1}^* < b_{n-1}$, we go to Step 3.

Step 3: We compute $Q_{n-2}^* = \sqrt{\dfrac{2C_O D}{ip_{n-2}}}$. If $Q_{n-2}^* \geq b_{n-2}$, we compare the total cost TC_{n-2}, TC_{n-1} and TC_n for purchase of quantities Q_{n-2}^*, b_{n-1} and b_n, respectively, and select the optimum purchase quantity.

If $Q_{n-2}^* < n_{n-2}$, then we go to Step 4.

Step 4: We continue in this fashion until $Q_{n-j}^* \geq b_{n-j}$. Then we compare total costs TC_{n-j} with $TC_{n-j+1}, \ldots, TC_{n-1}, TC_n$ for purchase quantities Q_{n-j}^*, $b_{n-j+1}, \ldots, b_{n-1}, b_n$, respectively, and select the optimum purchase quantity.

We now apply this method to an example.

Example 5: The annual demand for an item is 2400 units. The inventory carrying charge is 24% of the purchase price per year. Purchase prices are:

$p_1 =$ ` 10 for purchasing $Q_1 < 500$
$p_2 =$ ` 9.25 for purchasing $500 \le Q_2 < 750$
$p_3 =$ ` 8.75 for purchasing $750 \le Q_3$

Determine the optimum purchase quantity taking the ordering cost as

i) ` 437.5

ii) ` 87.5

Solution: To determine the optimum purchase quantity, we follow the procedure given below:

i) As per Step 1, $Q_3^* = \sqrt{\dfrac{2 \times 437.5 \times 2400}{(0.24)(8.75)}} = 1000$ units

Since 1000 is greater than 750, the optimum purchase quantity is 1000 units.

ii) Here $Q_3^* = \sqrt{\dfrac{2 \times 87.5 \times 2400}{(0.24)(8.75)}} = 447$ units

Since $447 < 750 = b_3$, we next compute

$$Q_2^* = \sqrt{\frac{2 \times 87.5 \times 2400}{(0.24)(9.25)}} = 435 \text{ units}$$

Since $435 < 500 = b_2$, we next compute

$$Q_1^* = \sqrt{\frac{2 \times 87.5 \times 2400}{(0.24)(10)}} = 418 \text{ units} > 0 \text{ and} < 500$$

We now compare the total cost for purchasing $Q_1^* = 418$, $b_2 = 500$ and $b_3 = 750$ units, respectively.

$$TC_1 (\text{for purchasing } 418) = 10 \times 2400 + \frac{87.5 \times 2400}{418} + \frac{1}{2}(0.24) \times (10) \times 418$$

$$= \text{`}\ 24504$$

$$TC_2 (\text{for } Q_2 = 500) = 9.25 \times 2400 + \frac{87.5 \times 2400}{500} + \frac{1}{2}(0.24) \times (9.25) \times (500)$$

$$= \text{`}23175$$

$$TC_3 (\text{for } Q_3 = 750) = 8.75 \times 2400 + \frac{87.5 \times 2400}{750} + \frac{1}{2}(0.24) \times (8.75) \times (750)$$

$$= \text{`}22067$$

As the total inventory cost is minimum for $Q_3 = 750$, therefore, the optimum purchase quantity is 750.

You may now like to solve the following problems to assess your understanding.

E6) Consider Example 5 and determine the optimum purchase quantity with the ordering cost as `175.

Let us now summarise the main points which have been covered in this unit.

8.8 SUMMARY

1. An inventory is a physical stock of idle resources of any kind having some economic value kept for the purpose of meeting future demand. It indicates the raw material required before production, the finished goods after production ready for delivery to consumers, human resources, financial resources, etc. stocked in order to meet an expected demand in the future.
2. Inventories are essential for almost all businesses for efficient and smooth operations. Hence, inventories need to be controlled in the most profitable way.
3. The quantity that is kept in stock in order to balance the costs of holding too much stock vis-à-vis the costs of ordering in small quantities is called the economic order quantity.
4. Five models have been discussed for obtaining the economic order quantity for the following situations:
   i) when demand is uniform,
   ii) when rates of demand are different in different cycles,
   iii) when shortages are allowed,
   iv) when replenishment is uniform, and
   v) when price discounts are given.

8.9 SOLUTIONS/ANSWERS

E1) We have $D = \frac{12000}{1.25} = \frac{12000}{125} \times 100 = 9600$; C = 1.25 per unit;

C_O = 15 and C_h = 15% of 1.25 $\Rightarrow \frac{15}{100} \times \frac{125}{100}$

$$Q^* = \sqrt{\frac{2DC_O}{C_h}} = \sqrt{\frac{2 \times 9600 \times 15 \times 100}{15}} = \sqrt{1920000} = 1385 \text{ units}$$

E2) Here C_O = 36, C = 2, D = 10000, and $C_h = 18\%$ of $2 = \frac{18 \times 2}{100} = \frac{9}{25}$

Therefore, a) $Q^* = \sqrt{\frac{2 \times 10000 \times 36 \times 25}{9}} = \sqrt{2000000} = 1414$ units

and b) $\frac{D}{Q^*} = \frac{10000}{1414} = 7.072$

Thus, the optimum no. of days = $\frac{365}{7} = 52$ days

E3) We have D = number of units supplied per year = 24000

Ordering cost per unit = ₹3500

Shortage cost = ₹2 per unit per month

= ₹24 per unit per year

Carrying cost = ₹1 per unit per month

= ₹12 per unit per year

i) The optimal value of Q is

$$Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{C_S + C_h}{C_S}\right)}$$

$$= \sqrt{\frac{2\times 24000\times 3500}{12}\left(\frac{24+12}{24}\right)}$$

$$= \sqrt{2\times 1000\times 3500\times 3} = \sqrt{210\times 100000}$$

$$= \sqrt{21}\times 1000 = 4.583\times 1000 = 4583 \text{ units}$$

ii) $$M^* = \left(\frac{C_S}{C_h + C_S}\right)Q^* = \left(\frac{24}{12+24}\right)\times 4583$$

$$= \frac{2}{3}\times 4583 = 3055 \text{ units}$$

iii) $$t^* = \frac{Q^*}{D} = \frac{4583}{24000} = \frac{4.583}{2} = 2.29 \text{ months}$$

iv) The minimum inventory cost = TC*

$$= \sqrt{2DC_OC_h\left(\frac{C_S}{C_h + C_S}\right)}$$

$$= \sqrt{2\times 24000\times 12\times\left(\frac{24}{12+24}\right)\times 3500}$$

$$= 576000 \times \left(\frac{24}{36}\right)\times 3500 = \text{`}\ 37200$$

E4) We have D = 30 items per day, r_p = 50 items per day,

C_O = `1000, $C_h$ = ` 0.05, r_d = 30 items per day

Therefore, $$Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{r_p}{r_p - r_d}\right)} = \sqrt{\frac{2\times 30\times 1000}{0.05}\left(\frac{50}{20}\right)}$$

$$= \sqrt{3000000} = 1732.05081 = 1732 \text{ units}$$

Total minimum inventory cost

$$TC^* = \sqrt{2DC_OC_h\left(1 - \frac{r_d}{r_p}\right)}$$

$$= \sqrt{2\times 30\times 1000\times 0.05\left(1 - \frac{30}{50}\right)}$$

$$= \sqrt{2\times 30\times 1000\times \frac{5}{100}\times\frac{20}{50}} = \sqrt{1200} = \text{`}\ 34.64$$

E5) We are given that

r_p = 500 × 250 units per year, r_d = 100000 units per year, C_O = 25

$$C_h = 12.5\% \text{ of `}2 = \frac{12.5 \times 2}{100} = \frac{1}{4} = 0.25$$

a) $$Q^* = \sqrt{\frac{2DC_O}{C_h}\left(\frac{r_p}{r_p - r_d}\right)}$$

$$= \sqrt{\frac{2 \times 100000 \times 25}{0.25}\left(\frac{125000}{25000}\right)}$$

$$= \sqrt{4000000 \times 25} = 10000$$

b) $$TC^* = \sqrt{2DC_OC_h\left(1 - \frac{r_d}{r_p}\right)}$$

$$= \sqrt{2 \times 100000 \times 0.25 \times 25\left(1 - \frac{100000}{125000}\right)}$$

$$= \sqrt{250000} = 500$$

Total cost on the basis of optimal policy = TC* + 2 × 100000
= 200500

c) Optimum number of set ups $= \dfrac{D}{Q^*} = \dfrac{100000}{10000} = 10$ set-ups

E6) Here $$Q_3^* = \sqrt{\frac{2 \times 175 \times 2400}{(.24)(8.75)}} = 632 \text{ units}$$

Since $632 < 750 = b_3$, we next compute

$$Q_2^* = \left(\frac{2 \times 175 \times 2400}{(.24)(9.25)}\right)^{1/2} = 615 \text{ units}$$

Since 615> 500 = b_2 we compare the total costs for purchasing $Q_2^* = 615$ and $b_3 = 750$ units respectively.

$$TC_2 (\text{for } 615 \text{units}) = 9.25 \times 2400 + \frac{100 \times 2400}{615} + \frac{1}{2} \times (0.24) \times (9.25) \times (615)$$
$$= \text{`}\ 23,273$$

$$TC_3 (\text{for } Q_3 = 750) = 8.75 \times 2400 + \frac{100 \times 2400}{750} + \frac{1}{2} \times (0.24) \times (8.75) \times (750)$$
$$= \text{`}\ 22,119.50$$

Therefore, the economic purchase quantity for this problem is

$$Q_3^* = 750 \text{ units}$$