
UNIT 2 MEASURES OF DISPERSION

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2.1 INTRODUCTION

Different measures of central tendency, discussed in Unit 1 of this block, give a value around which the data is concentrated. But it gives no idea about the nature of scatter or spread. For example, the observations 10, 30 and 50 have mean 30 while the observations 28, 30, 32 also have mean 30. Both the distributions are spread around 30. But it is observed that the variability among units is more in the first than in the second. In other words, there is greater variability or dispersion in the first set of observations in comparison to other. Measure of dispersion is calculated to get an idea about the variability in the data.

In Section 2.2, the concepts of measures of dispersion are described. Significance and properties of a good measure of dispersion are also explored in Sub-sections 2.2.1 and 2.2.2. The basic idea about the range is explained in Section 2.3 whereas the measures of quartile deviation are introduced in Section 2.4. Mean deviation is described in Section 2.5. Methods of calculation of variance and standard deviation for grouped and ungrouped data are explained in Section 2.6 whereas in Section 2.7 the basis idea about root mean square deviation is provided. Coefficient of variation, which is a relative measure of variation is described in Section 2.8.

Objectives

After reading this unit, you would be able to

- conceptualize dispersion;
- explain the utility of a measure of dispersion;
- explain the properties of a good measure of dispersion;
- explain methods of calculation of different types of measures of dispersion along with their merits and demerits; and
- solve the numerical problems related to the measures of dispersion.

2.2 MEASURES OF DISPERSION

According to Spiegel, the degree to which numerical data tend to spread about an average value is called the variation or dispersion of data. Actually, there are two basic kinds of a measure of dispersion (i) Absolute measures and (ii) Relative measures. The absolute measures of dispersion are used to measure the variability of a given data expressed in the same unit, while the relative measures are used to compare the variability of two or more sets of observations. Following are the different measures of dispersion:

1. Range
2. Quartile Deviation
3. Mean Deviation
4. Standard Deviation and Variance

2.2.1 Significance of Measures of Dispersion

Measures of dispersion are needed for the following four basic purposes:

1. Measures of dispersion determine the reliability of an average value. In other words, measures of variation are pointed out as to how far an average is representative of the entire data. When variation is less, the average closely represents the individual values of the data and when variation is large; the average may not closely represent all the units and be quite unreliable.
2. Another purpose of measuring variation is to determine the nature and causes of variations in order to control the variation itself. For example the variation in the quality of product in the process form of production can be checked by quality control department by identifying the reason for the variations in the quality of product. Thus, measurements of dispersion are helpful to control the causes of variation.
3. The measures of dispersion enable us to compare two or more series with regard to their variability. The relative measures of dispersion may also determine the uniformity or consistency. Smaller value of relative measure of dispersion implies greater uniformity or consistency in the data.
4. Measures of dispersion facilitate the use of other statistical methods. In other words, many powerful statistical tools in statistics such as correlation analysis, the testing of hypothesis, the analysis of variance, techniques of quality control, etc. are based on different measures of dispersion.

2.2.2 Properties of Good Measure of Dispersion

The properties of a good measure of dispersion are similar to the properties of a good measure of average. So, a good measure of dispersion should possess the following properties:

1. It should be simple to understand;
2. It should be easy to compute;
3. It should be rigidly defined;
4. It should be based on each and every observations of data;
5. It should be amenable to further algebraic treatment;

6. It should have sampling stability; and
7. It should not be unduly affected by extreme observations.

The detailed description of the properties has been discussed in Unit 1 of this block. Different measures of dispersions are discussed in following sections.

2.3 RANGE

Range is the simplest measure of dispersion. It is defined as the difference between the maximum value of the variable and the minimum value of the variable in the distribution. Its merit lies in its simplicity. The demerit is that it is a crude measure because it is using only the maximum and the minimum observations of variable. However, it still finds applications in Order Statistics and Statistical Quality Control. It can be defined as

$$R = X_{\text{Max}} - X_{\text{Min}}$$

where, X_{Max} : Maximum value of variable and

X_{Min} : Minimum value of variable.

Example 1: Find the range of the distribution 6, 8, 2, 10, 15, 5, 1, 13.

Solution: For the given distribution, the maximum value of variable is 15 and the minimum value of variable is 1. Hence range = 15 - 1 = 14.

E1) Calculate the range for the following data:

60, 65, 70, 12, 52, 40, 48

E2) Calculate range for the following frequency distribution:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	2	6	12	7	3

2.3.1 Merits and Demerits of Range

Merits of Range

1. It is the simplest to understand;
2. It can be visually obtained since one can detect the largest and the smallest observations easily and can take the difference without involving much calculations; and
3. Though it is crude, it has useful applications in areas like order statistics and statistical quality control.

Demerits of Range

1. It utilizes only the maximum and the minimum values of variable in the series and gives no importance to other observations;
2. It is affected by fluctuations of sampling;

3. It is not very suitable for algebraic treatment;
4. If a single value lower than the minimum or higher than the maximum is added or if the maximum or minimum value is deleted range is seriously affected; and
5. Range is the measure having unit of the variable and is not a pure number. That's why sometimes coefficient of range is calculated by

$$\text{Coefficient of Range} = \frac{X_{\text{Max}} - X_{\text{Min}}}{X_{\text{Max}} + X_{\text{Min}}}$$

2.4 QUARTILE DEVIATION

As you have already studied in Unit 1 of this block that Q_1 and Q_3 are the first quartile and the third quartile respectively. $(Q_3 - Q_1)$ gives the inter quartile range. The semi inter quartile range which is also known as Quartile Deviation (QD) is given by

$$\text{Quartile Deviation (QD)} = (Q_3 - Q_1) / 2$$

Relative measure of Q.D. known as Coefficient of Q.D. and is defined as

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Example 2: For the following data, find the quartile deviation:

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	03	05	07	09	04

Solution: We have $N/4 = 28/4 = 7$ and 7th observation falls in the class 10-20. This is the first quartile class.

Similarly, $3N/4 = 21$ and 21st observation falls in the interval 30-40. This is the third quartile class.

Class Interval	Frequency	Cumulative Frequency
0-10	3	3
10-20	5	8
20-30	7	15
30-40	9	24
40-50	4	28

Using the formulae of first quartile and third quartile we found

$$Q_1 = 10 + \frac{(7-3)}{5} \times 10 = 18$$

$$Q_3 = 30 + \frac{(21-15)}{9} \times 10 = 36.67$$

Hence

$$\text{Quartile Deviation} = (36.67 - 18)/2 = 9.335$$

E3) Calculate the Quartile Deviation for the following data:

Class	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	12	24	36	32	24	8

2.5 MEAN DEVIATION

Mean deviation is defined as average of the sum of the absolute values of deviation from any arbitrary value viz. mean, median, mode, etc. It is often suggested to calculate it from the median because it gives least value when measured from the median.

The deviation of an observation x_i from the assumed mean A is defined as $(x_i - A)$.

Therefore, the mean deviation can be defined as

$$MD = \frac{1}{n} \sum_{i=1}^n |x_i - A|$$

The quantity $\sum |x_i - A|$ is minimum when A is median.

We accordingly define mean deviation from mean as

$$MD = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$$

and from the median as

$$MD = \frac{\sum_{i=1}^n |x_i - \text{median}|}{n}$$

For frequency distribution, the formula will be

$$MD = \frac{\sum_{i=1}^k f_i |x_i - \bar{x}|}{\sum_{i=1}^k f_i}$$

$$MD = \frac{\sum_{i=1}^k f_i |x_i - \text{median}|}{\sum_{i=1}^k f_i}$$

where, all symbols have usual meanings.

Example 3: Find mean deviation for the given data

1, 2, 3, 4, 5, 6, 7

Solution: First of all we find Mean

$$\bar{x} = \frac{1+2+3+4+5+6+7}{7} = \frac{28}{7} = 4$$

Then, we will find $|x_i - \bar{x}| : 3, 2, 1, 0, 1, 2, 3$

So, $\sum |x_i - \bar{x}| = 12$

Therefore,

$$MD = \frac{12}{7} = 1.71$$

Example 4: Find mean deviation from mean for the following data:

x	1	2	3	4	5	6	7
f	3	5	8	12	10	7	5

Solution: First of all we have to calculate the mean from the given data

x	f	fx	$ x - \bar{x} $	$f x - \bar{x} $
1	3	3	3.24	9.72
2	5	10	2.24	11.20
3	8	24	1.24	9.92
4	12	48	0.24	2.88
5	10	50	0.76	7.60
6	7	42	1.76	12.32
7	5	35	2.76	13.80
	50	212	12.24	67.44

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{212}{50} = 4.24$$

$$MD = \frac{\sum_{i=1}^k f_i |x_i - \bar{x}|}{\sum_{i=1}^k f_i} = \frac{67.44}{50} = 1.348$$

Example 5: Consider the data given in Example 2 and find the Mean deviation from median.

Solution:

Class	Mid Value (x)	Frequency (f)	C.F.	$f_i x_i - \text{median} $
0-10	5	3	3	70.71
10-20	15	5	8	67.85
20-30	25	7	15	24.99
30-40	35	9	24	57.87
40-50	45	4	28	65.72
		$\sum f = 28$		$\sum f_i x_i - \text{median} = 287.14$

We have $N/2 = 28/2 = 14$

The 14th observation falls in the class 20-30. This is therefore the median class.

Using the formula of median, $\text{Median} = 20 + \frac{14-8}{7} \times 10 = 28.57$

$$\text{Mean Deviation from Median} = \frac{\sum_{i=1}^k f_i |x_i - \text{median}|}{\sum_{i=1}^k f_i} = 10.255$$

E4) Following are the marks of 7 students in Statistics:

16, 24, 13, 18, 15, 10, 23

Find the mean deviation from mean.

E5) Find mean deviation for the following distribution:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

2.5.1 Merits and Demerits of Mean Deviation

Merits of Mean Deviation

1. It utilizes all the observations;
2. It is easy to understand and calculate; and
3. It is not much affected by extreme values.

Demerits of Mean Deviation

1. Negative deviations are straightaway made positive;
2. It is not amenable to algebraic treatment; and
3. It can not be calculated for open end classes.

2.6 VARIANCE AND STANDARD DEVIATION

In the previous section, we have seen that while calculating the mean deviation, negative deviations are straightaway made positive. To overcome this drawback we move towards the next measure of dispersion called variance. Variance is the average of the square of deviations of the values taken from mean. Taking a square of the deviation is a better technique to get rid of negative deviations.

Variance is defined as

$$\text{Var}(x) = \sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

and for a frequency distribution, the formula is

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^k f_i (x_i - \bar{x})^2$$

where, all symbols have their usual meanings.

It should be noted that sum of squares of deviations is least when deviations are measured from the mean. This means $\sum (x_i - A)^2$ is least when $A = \text{Mean}$.

Example 6: Calculate the variance for the data given in Example 3.

Solution:

x	$(x - \bar{x})$	$(x - \bar{x})^2$
1	-3	9
2	-2	4
3	-1	1
4	0	0
5	1	1
6	2	4
7	3	9

We have $\sum_{i=1}^n (x_i - \bar{x})^2 = 28$

Therefore, $\text{Var}(x) = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{7} \times 28 = 4$

Example 7: For the data given in Example 4 of Unit 1, compute variance.

Solution: We have the following data:

Class	Mid Value	Frequency (f)	$f(x - \bar{x})^2$
0-10	5	3	1470.924
10-20	15	5	737.250
20-30	25	7	32.1441
30-40	35	9	555.606
40-50	45	4	1275.504
		$\sum f = 28$	$\sum f(x_i - \bar{x})^2$ =4071.428

$$\begin{aligned}\text{Variance} &= \frac{\sum_{i=1}^k f_i (x_i - \bar{x})^2}{\sum_{i=1}^k f_i} \\ &= 4071.429 / 28 = 145.408\end{aligned}$$

2.6.1 Merits and Demerits of Variance

Merits of Variance

1. It is rigidly defined;
2. It utilizes all the observations;
3. Amenable to algebraic treatment;
4. Squaring is a better technique to get rid of negative deviations; and
5. It is the most popular measure of dispersion.

Demerits of Variance

1. In cases where mean is not a suitable average, standard deviation may not be the coveted measure of dispersion like when open end classes are present. In such cases quartile deviation may be used;
2. It is not unit free;
3. Although easy to understand, calculation may require a calculator or a computer; and
4. Its unit is square of the unit of the variable due to which it is difficult to judge the magnitude of dispersion compared to standard deviation.

2.6.2 Variance of Combined Series

In Section 2.6, we have discussed how to calculate the variance for a single variable. If there are two or more populations and the information about the means and variances of those populations are available then we can obtain the combined variance of several populations. If n_1, n_2, \dots, n_k are the sizes, $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k$ are the means and $\sigma_1^2, \sigma_2^2, \dots, \sigma_k^2$ are the variances of k populations, then the combined variance is given by

$$\sigma^2 = \frac{1}{(n_1 + n_2 + \dots + n_k)} \left[n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2) + \dots + n_k (\sigma_k^2 + d_k^2) \right]$$

where, $d_i = \bar{x}_i - \bar{x}$ and

$$\bar{x} = \frac{\sum_{i=1}^k n_i \bar{x}_i}{\sum_{i=1}^k n_i}$$

is the mean of the combined series.

Example 8: Suppose a series of 100 observations has mean 50 and variance 20. Another series of 200 observations has mean 80 and variance 40. What is the combined variance of the given series?

Solution: First we find the mean of the combined series

$$\begin{aligned}\bar{x} &= \frac{(100 \times 50 + 200 \times 80)}{(100 + 200)} \\ &= \frac{21000}{300} = 70\end{aligned}$$

Therefore, $d_1 = 50 - 70 = -20$ and $d_2 = 80 - 70 = 10$

Variance of the combined series

$$\begin{aligned}&= \frac{(100 \times (20 + 400) + 200 \times (40 + 100))}{(100 + 200)} \\ &\Rightarrow \frac{(42000 + 28000)}{(300)} = \frac{70000}{300} = 233.33\end{aligned}$$

E6) For a group containing 100 observations the arithmetic mean and standard deviation are 16 and $\sqrt{21}$ respectively. For 50 observations, the mean and standard deviation are 20 and 2 respectively. Calculate mean and standard deviation of other half.

2.6.3 Standard Deviation

Standard deviation (SD) is defined as the positive square root of variance. The formula is

$$SD = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

and for a frequency distribution the formula is

$$SD = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \bar{x})^2}{\sum_{i=1}^k f_i}}$$

where, all symbols have usual meanings. SD, MD and variance cannot be negative.

Example 9: Find the SD for the data in Example 4 of Unit 1.

Solution: In the Example 7, we have already found the

$$\text{Variance} = 145.408$$

So

$$SD = +\sqrt{(145.408)} = 12.04$$

2.6.4 Merits and Demerits of Standard Deviation

Merits of Standard Deviation

1. It is rigidly defined;
2. It utilizes all the observations;
3. It is amenable to algebraic treatment;

4. Squaring is a better technique to get rid of negative deviations; and
5. It is the most popular measure of dispersion.

Demerits of Standard Deviation

1. In cases where mean is not a suitable average, standard deviation may not be the appropriate measure of dispersion like when open end classes are present. In such cases quartile deviation may be used;
2. It is not unit free; and
3. Although it is easy to understand but calculation may require a calculator or a computer.

E7) Find the standard deviation for the following numbers:

10 27 40 60 33 30 10

E8) Calculate standard deviation for the following data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Remark:

SD is algebraically more amenable than MD. MD straightaway neglects the sign of observations which is also a useful information. In SD, the signs are considered but they get automatically eliminated due to squaring. However MD will be useful in situations where Median is more suitable than mean as an average.

2.7 ROOT MEAN SQUARE DEVIATION

As we have discussed in Section 2.6 that standard deviation is the positive square root of the average of the squares of deviations taken from mean. If we take the deviations from assumed mean then it is called Root Mean Square Deviation and it is defined as

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^n (x_i - A)^2}{n}}$$

where, A is the assumed mean.

For a frequency distribution, the formula is

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - A)^2}{\sum_{i=1}^k f_i}}$$

When assumed mean is equal to the actual mean \bar{x} i.e. $A = \bar{x}$ root mean square deviation will be equal to the standard deviation.

2.8 COEFFICIENT OF VARIATION

Coefficient of Variation (CV) is defined as

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

It is a relative measure of variability. If we are comparing the two data series, the data series having smaller CV will be more consistent. One should be careful in making interpretation with CV. For example, the series 10, 10, 10 has SD zero and hence CV is also zero. The series 50, 50 and 50 also has SD zero and hence CV is zero. But the second series has higher mean. So we shall regard the second series as more consistent than the first.

Example 10: Suppose batsman A has mean 50 with SD 10. Batsman B has mean 30 with SD 3. What do you infer about their performance?

Solution: A has higher mean than B. This means A is a better run maker.

However, B has lower CV ($3/30 = 0.1$) than A ($10/50 = 0.2$) and is consequently more consistent.

E9) If $n = 10$, $\bar{x} = 4$, $\sum x^2 = 200$, find the coefficient of variation.

E10) For a distribution, the coefficient of variation is 70.6% and the value of arithmetic mean is 16. Find the value of standard deviation.

2.9 SUMMARY

In this unit, we have discussed:

1. The measures of dispersion;
2. The utility of a measure of dispersion;
3. The properties of a good measure of dispersion;
4. The different types of measures of dispersion along with their merits and demerits; and
5. Coefficient of Variation.

While average gives the value around which a distribution is scattered, measure of dispersion tells how it is scattered. So if one suitable measure of average and one suitable measure of dispersion is calculated (say mean and SD), we get a good idea of the distribution even if the distribution is large.

2.10 SOLUTIONS /ANSWERS

E1) Maximum value (X_{Max}) = 70
Minimum value (X_{Min}) = 12

$$\begin{aligned} \text{Therefore, } R &= X_{\text{Max}} - X_{\text{Min}} \\ &= 70 - 12 = 58 \end{aligned}$$

E2) In the given frequency distribution, we have

Lower limit of first class = 0

Upper limit of last class = 50

Therefore,

$$\text{Range} = 50 - 0 = 50$$

E3) First we construct the following cumulative frequency distribution:

Class	f	Cumulative frequencies
0-5	6	6
5-10	8	14
10-15	12	26
15-20	24	50
20-25	36	86
25-30	32	118
30-35	24	142
35-40	8	150
	N = 150	

$$Q_1 = L + \frac{\left(\frac{N}{4} - C\right)}{f} \times h \quad \text{15-20 is the } Q_1 \text{ class}$$

$$\begin{aligned}
 &= 15 + \frac{\left(\frac{150}{4} - 26\right)}{24} \times 5 \\
 &= 15 + \frac{37.5 - 26}{24} \times 5 = 15 + \frac{57.5}{24} \\
 &= 17.40
 \end{aligned}$$

$$Q_3 = L + \frac{\left(\frac{3N}{4} - C\right)}{f} \times h \quad \text{25-30 is the } Q_3 \text{ class}$$

$$\begin{aligned}
 &= 25 + \frac{\left(\frac{3 \times 150}{4} - 86\right)}{32} \times 5 \\
 &= 25 + \frac{112.5 - 86}{24} \times 5 = 25 + \frac{132.5}{24} \\
 &= 30.52
 \end{aligned}$$

$$\text{Therefore, } QD = \frac{Q_3 - Q_1}{2} = \frac{30.52 - 17.40}{2} = \frac{13.12}{2} = 6.56$$

E4) We have the following distribution:

x	(x-17)	$x - \bar{x}$
16	-1	1
24	+7	7
13	-4	4
18	+1	1
15	-2	2
10	-7	7
23	+6	6
$\sum x = 119$		$\sum x - \bar{x} = 28$

Then
$$\bar{x} = \frac{\sum x}{n} = \frac{119}{7} = 17$$

Therefore,
$$MD = \frac{\sum |x - \bar{x}|}{n} = \frac{28}{7} = 4$$

E5) First we construct the following frequency distribution:

Class	x	f	xf	(x - \bar{x})	$x - \bar{x}$	f$x - \bar{x}$
0-10	5	5	25	-22	22	110
10-20	15	8	120	-12	12	96
20-30	25	15	375	-2	2	30
30-40	35	16	560	8	8	128
40-50	45	6	270	18	18	108
						$\sum f x - \bar{x} = 472$

Then
$$\bar{x} = \frac{\sum xf}{\sum f} = \frac{1350}{50} = 27$$

and

$$MD = \frac{\sum f|x - \bar{x}|}{\sum f} = \frac{472}{50} = 9.44$$

E6) We have given

$$n = 100, \quad \bar{x} = 16, \quad \sigma = \sqrt{21}$$

$$n_1 = 50, \quad \bar{x}_1 = 20, \quad \sigma_1 = 2, \quad \text{and} \quad n_2 = 50$$

we have to find

$$\bar{x}_2 = ?, \quad \sigma_2 = ?$$

$$\text{Now, } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{50 \times 20 + 50 \bar{x}_2}{100}$$

$$16 = \frac{50 \times 20 + 50 \bar{x}_2}{100}$$

$$50 \bar{x}_2 = 16 \times 100 - 50 \times 20$$

$$50 \bar{x}_2 = 1600 - 1000 = 600$$

$$\Rightarrow \bar{x}_2 = 12$$

$$d_1 = \bar{x}_1 - \bar{x} = 20 - 16 = 4 \Rightarrow d_1^2 = 16$$

$$d_2 = \bar{x}_2 - \bar{x} = 12 - 16 = -4 \Rightarrow d_2^2 = 16$$

$$\sigma^2 = \frac{1}{n_1 + n_2} [n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2)]$$

$$(\sqrt{21})^2 = \frac{[50(4 + 16) + 50(\sigma_2^2 + 16)]}{100}$$

$$21 \times 100 - 20 \times 50 = 50\sigma_2^2 + 50 \times 16$$

$$50\sigma_2^2 = 2100 - 1000 - 800$$

$$50\sigma_2^2 = 2100 - 1000 - 800$$

$$50\sigma_2^2 = 300$$

$$\Rightarrow \sigma_2^2 = 6 \Rightarrow \sigma_2 = \sqrt{6}$$

E7) First we calculate the following distribution:

x	x ²
10	100
27	729
40	1600
60	3600
33	1089
30	900
10	100
$\sum x = 210$	$\sum x^2 = 8118$

Then

$$\bar{x} = \frac{\sum x}{n} = \frac{210}{7} = 30$$

and therefore,

$$\sigma = \sqrt{\frac{1}{n} \sum x^2 - (\bar{x})^2} = \sqrt{\frac{8118}{7} - 900}$$

$$= \sqrt{1159.7 - 900} = \sqrt{259.7} = 16.115$$

E8) Let us take $A = 25$ and calculate the following frequency distribution:

Class	x	f	d = x - A	f d	f d ²
0-10	5	5	-20	-100	2000
10-20	15	8	-10	-80	800
20-30	25	15	0	0	0
30-40	35	16	10	160	1600
40-50	45	6	20	120	2400
		N = 50		$\sum fd = 100$	$\sum fd^2 = 6800$

Therefore,

$$\begin{aligned}\sigma &= \sqrt{\frac{1}{N} \sum_{i=1}^k f_i d_i^2 - \left(\frac{\sum_{i=1}^k f_i d_i}{N} \right)^2} = \sqrt{\frac{1}{50} (6800) - \left(\frac{100}{50} \right)^2} \\ &= \sqrt{136 - 4} = \sqrt{132} = 11.49\end{aligned}$$

E9) We have

$$\begin{aligned}\sigma^2 &= \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2 \\ &= \frac{1}{n} \times 200 - (4)^2 = 20 - 16 = 4 \\ \Rightarrow \sigma^2 &= 4 \Rightarrow \sigma = 2\end{aligned}$$

Therefore, coefficient of variation

$$\begin{aligned}\text{C.V}(X) &= \frac{\sigma}{\bar{x}} \times 100 \\ &= \frac{2}{4} \times 100 = 50\%\end{aligned}$$

E10) We have the formulae for coefficient of variation

$$\text{CV} = \frac{\sigma}{\bar{x}} \times 100$$

By putting the given values, we have

$$\begin{aligned}\sigma &= \frac{\text{CV} \times \bar{x}}{100} = \frac{70.6 \times 16}{100} \\ &= 11.296\end{aligned}$$