# UNIT 7 SINGLE SAMPLING PLANS

## Structure



## 7.1 INTRODUCTION

In Units 5 and 6, you have learnt the various features of a sampling inspection plan such as AQL, LTPD, producer's risk, consumer's risk, OC curve, ASN, ATI, etc. The same features characterise different types of sampling plans. In Unit 5, you have also learnt that the main types of acceptance sampling plans for attributes are:

i) Single sampling plan,

ii) Double sampling plan,

iii) Multiple sampling plan, and

iv) Sequential sampling plan

In this unit, we focus on the **single sampling plans for attributes**. In Sec. 7.2, we explain the single sampling plan and its implementation. We describe various features of the single sampling plan such as the operating characteristic (OC) curve, producer's risk, consumer's risk, average sample number (ASN) and average total inspection (ATI) in Secs. 7.3 to 7.6. Finally, we describe the design of the single sampling plans in Sec. 7.7. In the next unit, we shall discuss the double sampling plans for attributes.

### Objectives

After studying this unit, you should be able to:

- describe a single sampling plan;
- compute the probability of accepting or rejecting a lot of given incoming quality in a single sampling plan;
- construct the operating characteristics (OC) curve of a single sampling plan;
- compute the consumer's risk and producer's risk in a single sampling plan;

- compute the average sample number (ASN) and the average total inspection (ATI) for a single sampling plan; and
- design single sampling plans.

## 7.2 SINGLE SAMPLING PLAN

A sampling plan in which a decision about the acceptance or rejection of a lot is based on a single sample that has been inspected is known as a **single sampling plan**. For example, suppose a buyer purchases cricket balls in lots of 500 from a company manufacturing cricket balls. To check the quality of the lots, the buyer draws a random sample of size 20 from each lot and takes a decision about accepting or rejecting of the lot on the basis of the information provided by this sample. Since the buyer takes the decision about the lot on the basis of a single sample, this sampling plan is a single sampling plan.

A single sampling plan requires the specification of two quantities which are known as **parameters** of the single sampling plan. These parameters are

n – size of the sample, and

c – acceptance number for the sample.

Let us suppose that the lots are of the same size (N) and are submitted for inspection one at a time. The procedure for implementing the single sampling plan to arrive at a decision about the lot is described in the following steps:

**Step 1:** We draw a random sample of size n from the lot received from the supplier or the final assembly.

**Step 2:** We inspect each and every unit of the sample and classify it as defective or non-defective. At the end of the inspection, we count the number of defective units found in the sample. Suppose the number of defective units found in the sample is d.

**Step 3:** We compare the number of defective units (d) found in the sample with the stated acceptance number (c).

**Step 4:** We take the decision of acceptance or rejection of the lot on the basis of the sample as follows:

**Under acceptance sampling plan**

If the number of defective units (d) in the sample is less than or equal to the stated acceptance number (c), i.e., if $d \le c$, we accept the lot and if $d > c$, we reject the lot.

**Under rectifying sampling plan**

If $d \le c$, we accept the lot and replace all defective units found in the sample by non-defective units and if $d > c$, we accept the lot after inspecting the entire lot and replacing all defective units in the lot by non-defective units.

The steps described above are shown in Fig. 7.1.

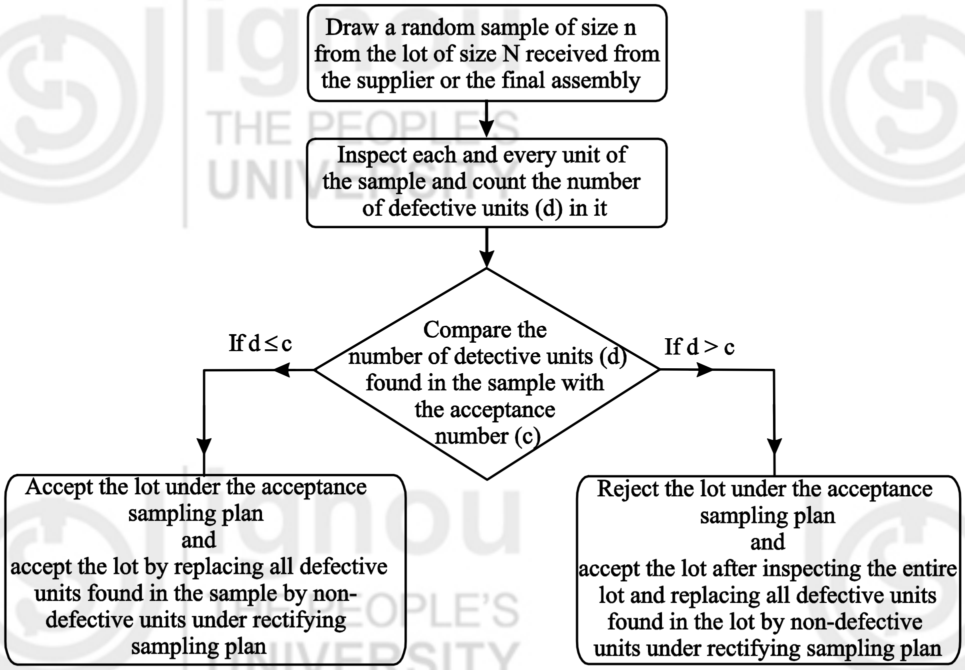


**Fig. 7.1: Procedure for implementing a single sampling plan.**

Let us explain these steps further with the help of an example.

**Example 1:** Suppose a mobile phone company produces mobiles phones in lots of 100 phones. To check the quality of the lots, the quality inspector of the company uses a single sampling plan with n = 15 and c = 1. Explain the procedure for implementing it.

**Solution:** For implementing the single sampling plan, the quality inspector of the company randomly draws a sample of 15 mobile phones from each lot and classifies each mobile of the sample as defective or non-defective. At the end of the inspection, he/she counts the number of defective mobiles (d) found in the sample and compares it with the acceptance number (c). If $d \le c$ (= 1), he/she accepts the lot and if $d > c$ (= 1), he/she rejects the lot under the acceptance sampling plan. Under rectifying sampling plan, if $d \le c$ (= 1), he/she accepts the lot by replacing all defective mobiles found in the sample by non-defective mobiles and if $d > c$, he/she accepts the lot by inspecting the entire lot and replacing all defective mobiles in the lot by non-defective mobiles.

You may like to explain the procedure for implementing a single sampling plan yourself. Try the following exercise.

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**E1)** A manufacturer of silicon chip produces lots of 1000 chips for shipment. A buyer uses a single sampling plan with n = 50 and c = 2 to test for bad outgoing lots. Explain the procedure for implementing it under acceptance sampling plan.

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So far you have learnt about the single sampling plan and how it is implemented. We describe various features of the single sampling plan in Secs.7.3 to 7.6.

## 7.3 OPERATING CHARACTERISTIC (OC) CURVE

You have studied in Unit 6 that the operating characteristics (OC) curve is an important aspect of an acceptance sampling plan. This curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected. In this section, we discuss how to construct the OC curve for a single sampling plan.

You have learnt that for constructing an OC curve, we require the probabilities of accepting a lot corresponding to different quality levels. Therefore, we first compute the probability of accepting a lot of incoming quality p for a single sampling plan.

You have studied in Sec. 7.2 that in a single sample plan, we accept the lot if the number of defective units (d) in the sample is less than or equal to the acceptance number (c). It means that if X represents the number of defective units in the sample, we accept the lot if $X \le c$, i.e., X = 0 or 1 or 2, …, or c. Therefore, the probability of accepting the lot of incoming quality p is given by

You have studied in Unit 3 of MST-003 that if A and B are mutually exclusive events then $P[A \text{ or } B] = P[A] + P[B]$

$$P_a(p) = P[X \le c] = P[X = 0 \text{ or } 1, ..., \text{ or } c]$$

$$= P[X = 0] + P[X = 1] + ... + P[X = c] \quad \left(\because X = 0, 1, 2, ..., c \text{ are mutually exclusive}\right)$$

$$= \sum_{x=0}^{c} P[X = x] \qquad \text{… (1)}$$

We can calculate this probability if we know the distribution of X. Generally, in quality control, a random sample is drawn from a lot of finite size without replacement. So in such situations, the number of defective units (X) in the sample follows a hypergeometric distribution. In a lot of size N and incoming quality p, the number of defective units is Np and the number of non-defective units is N – Np. Therefore, the probability of getting exactly x defective units in a sample of size n from this lot is given by

The notation ${}^{Np}C_x$ can also be repersened as $\binom{Np}{x}$.

$$P[X = x] = \frac{{}^{Np}C_x \; {}^{N-Np}C_{n-x}}{{}^{N}C_x}; \; x = 0, 1, ..., \min(Np, n) \qquad \text{… (2)}$$

Thus, we can obtain the probability of accepting a lot of quality p in a single sampling plan by putting the value of P[X = x] in equation (1) as follows:

$$P_a(p) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n-x}}{{}^{N}C_x} \qquad \text{… (3)}$$

We know from industrial experience that n is usually small for any economically worthwhile production process. Therefore, when sample size n is small compared to lot size (N), i.e., when $N \ge 10n$, we know that the hypergeometric distribution is approximated by the binomial distribution with parameters n and p where p is the lot quality. It is far easier to calculate the probabilities with the help of the binomial distribution in comparison with the hypergeometric distribution.

Therefore, we can take

$$P[X = x] = {}^{n}C_x \, p^x (1-p)^{n-x}$$

Thus, the probability of accepting a lot of quality p using binomial approximation is given by

$$P_a(p) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} {}^nC_x\, p^x (1-p)^{n-x} \qquad \ldots (4)$$

However, for rapid calculation, we can use Table I entitled **Cumulative Binomial Probability Distribution** which is given at the end of this block. We can also approximate the binomial distribution to another distribution. When p is small and n is large such that np is finite, we know that the binomial distribution approaches the Poisson distribution with parameter $\lambda$ = np. Therefore, the probability of accepting a lot of quality p using the Poisson approximation is given by

$$P_a(p) = \sum_{x=0}^{c} \frac{e^{-\lambda}\lambda^x}{\lfloor x} \qquad \ldots (5)$$

For Poisson distribution

$$P[X = x] = \frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

We can use Table II entitled **Cumulative Poisson Probability Distribution** given at the end of this block for calculating this probability.

**Note:** Table I and Table II do not list a tabulated value for each value of p and $\lambda$, respectively. In such cases, we interpolate it as we have discussed in Unit 4 of MST-004 or use a scientific calculator to calculate the probability of accepting a lot.

We now illustrate how to compute the probability of accepting a lot in different situations with the help of an example.

**Example 2:** A manufacturer of silicon chip produces chips in lots of 1000. A single sampling plan is used to test for bad outgoing lots. If the quality of incoming lot is 0.01, calculate the probability of accepting the lot in the following cases:

i) n = 12 and c = 1, and

ii) n = 60 and c = 2.

**Solution:**

i) It is given that

$$N = 1000, n = 12, c = 1 \text{ and } p = 0.01$$

If X represents the number of defective chips in the sample, the lot is accepted if X ≤ c, i.e., X ≤ 1. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le c] = P[X \le 1] = P[X = 0] + P[X = 1]$$

$$= \sum_{x=0}^{1} P[X = x] = \sum_{x=0}^{1} \frac{{}^{Np}C_x\; {}^{N-Np}C_{n-x}}{{}^NC_x}$$

Since $N \ge 10n$, we can use the binomial distribution (with parameters n and p where p is the lot quality) as the approximation of the hypergeometric distribution. Therefore, the probability of accepting the lot of quality p is given by

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^nC_x\, p^x (1-p)^{n-x}$$

For rapid calculation, we can use Table I for obtaining this probability.

From Table I, for n = 12, x = c = 1 and p = 0.01, we have

$$\sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.9938$$

Therefore, the probability of accepting the lot of quality p = 0.01 is given by

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.9938$$

ii) It is given that

$$N = 1000, n = 60, c = 2 \text{ and } p = 0.01$$

Since p is small, n is large and np = 0.65 is finite, we can use the Poisson distribution (with parameters $\lambda$ = np) as the approximation of the hypergeometric distribution. Therefore, the probability of accepting the lot of quality p is given by

$$P_a(p) = P[X \le 2] = \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

where $\lambda = np = 60 \times 0.01 = 0.60.$

For rapid calculation, we can use Table II for obtaining this probability.

From Table II, for $\lambda$ = 0. 60 and x = c = 2, we have

$$\sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{\lfloor x} = 0.9769$$

Hence, the probability of accepting a lot of quality p = 0.01 is given by

$$P_a(p) = P[X \le 2] = \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{\lfloor x} = 0.9769$$

You can calculate the probability of acceptance of a lot for different lot qualities in the same way.

We now take up the construction of the OC curve for a single sampling plan.

As you know, the OC curve is constructed by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis. So for construction of the OC curve, we first consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding probability of acceptaning a lot as discussed in Example 2.

Let us consider an example to demonstrate the construction of the OC curve.

**Example 3:** Suppose a consumer receives lots of 500 candles from a new supplier. To check the quality of the lot, the consumer draws one sample of size 20 and accepts the lot if the inspected sample contains at most one defective cnadle. Otherwise, he/she rejects the lot. Construct the OC curve for this plan.

**Solution**: It is given that

$$N = 500, n = 20, c = 1$$

For constructing the OC curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If X represents the number of defective candles in the sample, the consumer accepts the lot if $X \le c$, i.e., $X \le 1$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le c] = P[X \le 1] = \sum_{x=0}^{1} P[X = x]$$

Since $N \ge 10n$, we can use binomial distribution. Therefore,

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^{n}C_x \, p^x (1-p)^{n-x}$$

We use Table I for calculating the probabilities of accepting the lot corresponding to different quality levels such as $p = 0.01, 0.02, 0.03\ldots$ as we have discussed in Example 2. These probabilities are shown in the table given below:

| **Incoming Lot Quality** | **Probability of Accepting the Lot** |
|---|---|
| 0 | 1 |
| 0.01 | 0.9831 |
| 0.02 | 0.9401 |
| 0.04 | 0.8103 |
| 0.06 | 0.6605 |
| 0.08 | 0.5169 |
| 0.10 | 0.3917 |
| 0.12 | 0.2891 |
| 0.14 | 0.2084 |
| 0.16 | 0.1471 |
| 0.18 | 0.1018 |
| 0.20 | 0.0692 |

We now construct the OC curve by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis as shown in Fig. 7.2.

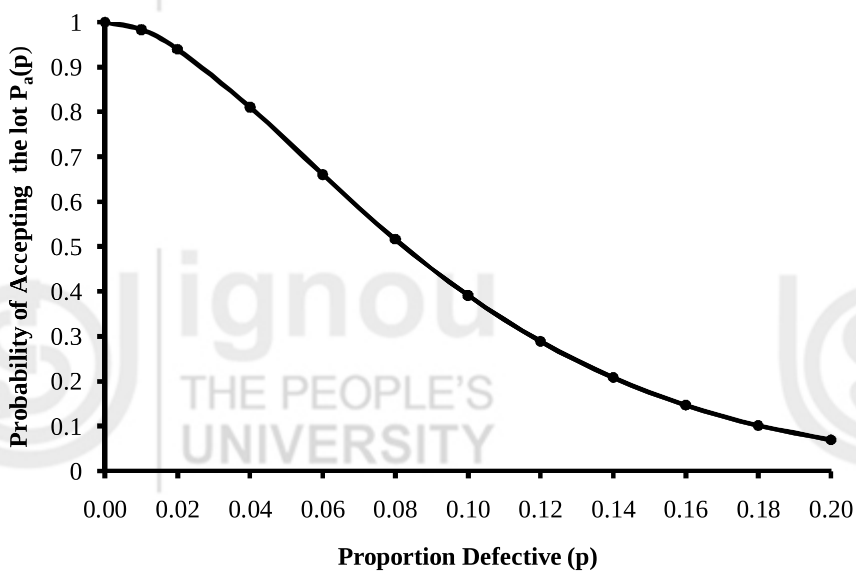


**Fig. 7.2: The OC curve for the single sampling plan.**

You may now like to construct the OC curve for the following exercise.

**E2)** A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with n = 25 and c = 2 is being used for inspection by the quality inspector of the hospital. Construct the OC curve for this plan.

You have learnt in Unit 5 that the acceptance or rejection of the entire lot depends on the conclusions drawn from the sample. Thus, there is always a chance of making a wrong decision. It means that a lot of good quality may be rejected and a lot of poor quality may be accepted. This leads to two kinds of risks:

1. Producer's risk, and
2. Consumer's risk.

We now discuss these risks for the single sampling plan.

## 7.4 PRODUCER'S RISK AND CONSUMER'S RISK

In Unit 5, we have defined the producer's risk as follows:

The probability of rejecting a lot of acceptance quality level (AQL) $p_1$ is known as the producer's risk.

Therefore, the producer's risk for a single sampling plan is given by

$$P_p = P[\text{rejecting a lot of acceptance quality level } p_1]$$

$$= 1 - P[\text{accepting a lot of acceptance quality level } p_1]$$

$$= 1 - P_a(p_1) \qquad \ldots (6)$$

We can compute $P_a(p_1)$ from equation (3) by replacing the quality level p with $p_1$ as follows:

$$P_a(p_1) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n-x}}{{}^{N}C_x}$$

Therefore, from equation (6), the producer's risk is given by

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n-x}}{{}^{N}C_x} \qquad \ldots (7)$$

For rapid calculation of the producer's risk for a single sampling plan, we can also use approximations as we have discussed in Sec. 7.4. Therefore, if we use the approximation of the hypergeometric distribution to the binomial distribution with parameters n and $p_1$, the producer's risk is given by

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} {}^{n}C_x \, p_1^x \, (1 - p_1)^{n-x} \qquad \ldots (8)$$

We now explain the **Consumer's Risk** for a single sampling plan:

By definition, the probability of accepting a lot of unsatisfactory quality (LTPD) $p_2$ is known as the consumer's risk.

Therefore, the consumer's risk for a single sampling plan is given by

$$P_c = P[\text{accepting a lot of quality } p_2]$$

We can compute the consumer's risk for the single sampling plan from equation (4) by replacing the quality level p with $p_2$ as follows:

$$P_c = P_a(p_2) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{{}^{Np_2}C_x \, {}^{N-Np_2}C_{n-x}}{{}^{N}C_x} \qquad \ldots (9)$$

If we approximate the hypergeometric distribution to the binomial distribution with parameters n and $p_2$, the consumer's risk is given by

$$P_c = P_a(p_2) = \sum_{x=0}^{c} {}^{n}C_x \, p_2^x (1-p_2)^{n-x} \qquad \ldots (10)$$

Let us take up an example from real life to explain these concepts.

**Example 4:** Suppose a tyre supplier ships tyres in lots of size 400 to the buyer. A single sampling plan with n = 15 and c = 0 is being used for the lot inspection. The supplier and the buyer's quality control inspector decide that AQL = 0.01 and LTPD = 0.10. Compute the producer's risk and consumer's risk for this single sampling plan.

**Solution:** It is given that

$$N = 400, n = 15, c = 0, AQL(p_1) = 0.01 \text{ and } LTPD(p_2) = 0.10$$

Since $N \geq 10n$, we can use the binomial distribution. Therefore, we can use equation (8) to calculate the producer's risk for the single sampling plan.

We first calculate the probability of accepting a lot of quality $p = p_1 = 0.01$ using Table I.

From Table I, for n = 15, x = c = 0 and $p = p_1 = 0.01$, we have

$$P_a(p_1) = P[X \leq 0] = \sum_{x=0}^{0} {}^{n}C_x \, p_1^{x} (1-p_1)^{n-x} = 0.8601$$

Therefore, from equation (8), we calculate the producer's risk as follows:

$$P_p = 1 - P_a(p_1) = 1 - 0.8601 = 0.1399$$

It means that if there are several lots of the same quality p = 0.01, about 13.99% of these will be rejected. This is obviously a risk for the supplier because it was agreed upon by both that lots of quality 0.01 will be accepted whereas the quality inspector is rejecting 13.99% of those.

Similarly, we can calculate the consumer's risk using equation (10).

We first calculate the probability of accepting a lot of quality $p = p_2 = 0.10$ using Table I.

From Table I, for n = 15, x = c = 0 and p = 0.10, we have

$$P_a(p_2) = P[X \leq 0] = \sum_{x=0}^{0} {}^{n}C_x \, p_2^x (1-p_2)^{n-x} = 0.2059$$

Therefore, from equation (10), the consumer's risk is given by

$$P_c = P_a(p_2) = 0.2059$$

It means that if there are several lots of the same quality p = 0.10, about 20.59% of these will be accepted by the quality inspector even though this quality is unsatisfactory. This is obviously the buyer's risk.

For practice you can also compute the producer's risk and consumer's risk in the following exercise.

**E3)** Suppose in E2, the acceptance quality level (AQL) and lot tolerance percent defective (LTPD) are 0.04 and 0.10, respectively. Calculate the producer's risk and consumer's risk for this plan.

## 7.5 AVERAGE OUTGOING QUALITY (AOQ)

You have studied in Unit 6 that the concept of average outgoing quality is particularly useful in the rectifying sampling plan where the rejected lots are inspected 100% and all defective units are replaced by non-defective units. The AOQ is defined as follows:

The expected quality of the lots after the applications of sampling inspection is called the **average outgoing quality (AOQ)**. It is calculated from the formula given below:

$$\text{AOQ} = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}} \quad \ldots (11)$$

So in a single sample plan, we can obtain the formula for average outgoing quality by considering the following situations:

i) If the lot of size N is accepted on the basis of a sample of size n, (N – n) units remain un-inspected. If the incoming quality of the lot is p, we expect that p(N – n) defective units are left in the lot after the inspection. However, the probability that the lot will be accepted by the sampling plan is $P_a$. Therefore, the expected number of defective units per lot in the outgoing stage is $p(N - n) P_a$.

ii) If the lot is rejected, all units of the lot go for 100% inspection and all defective units found in the lot are replaced by non-defective units. So there is no defective unit at the outgoing stage. The probability that the lot will be rejected is $(1 - P_a)$. Therefore, the expected number of defective units per lot at the outgoing stage is $0 \times (1 - P_a) = 0$.

Thus, the expected number of defective units per lot after sampling inspection is $p(N - n)P_a + 0 = p(N - n)P_a$.

Hence, the average proportion defective in the outgoing stage or average outgoing quality (AOQ) is given by

$$\text{AOQ} = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}}$$

or $$\text{AOQ} = \frac{p(N - n)P_a}{N} \quad \ldots (12)$$

If the sample size n is very small in proportion to the lot size N, i.e., $n/N \approx 0$, equation (12) for AOQ becomes

$$\text{AOQ} = p\left(1 - \frac{n}{N}\right)P_a \approx pP_a \quad \ldots (13)$$

Let us now discuss the construction of the AOQ curve for a single sampling plan.

As you know, the AOQ curve is constructed by taking the quality level (proportion defective) on the X-axis and the AOQ on the Y-axis. So for constructing the AOQ curve, we first consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding AOQ.

Let us consider some examples for calculating AOQ and constructing the AOQ curve.

**Example 5:** Suppose in Example 3 the submitted lot quality is p = 0.02. The rejected lots are screened and all defective candles are replaced by the non-defective candles. Calculate the average outgoing quality (AOQ) for this plan.

**Solution**: The submitted lot quality is p = 0.02 and we have to calculate the AOQ for this single sampling plan.

It is given that

$$N = 500, n = 20, c = 1 \text{ and } p = 0.02$$

From equation (12), the AOQ for the single sampling plan is

$$AOQ = \frac{p(N-n)P_a}{N}$$

where $P_a$ is the probability of accepting the lot of quality p. Therefore, for calculating AOQ, we have to calculate $P_a$. We have already calculated this probability in Example 3. So we directly use the result:

$$P_a = 0.9401$$

On putting the values of N = 500, n = 20, p = 0.02 and $P_a$ = 0.9401in equation (12), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.02\times(500-20)\times 0.9401}{500} = 0.018$$

In the same way, you can calculate AOQ for different submitted lot qualities.

**Example 6:** Suppose in Example 4, the rejected lots are screened and all defective tyres are replaced by non-defective tyres. Construct the AQO curve for this plan.

**Solution**: It is given that

$$N = 400, n = 15, c = 0$$

For constructing the AOQ curve, we first calculate the probabilities of accepting the lot corresponding to different quality levels using Table I. Then we calculate the AOQ for each quality level using equation (12).

The probabilities of accepting the lot and the AOQs corresponding to different quality levels are given in the following table:

| Incoming Lot Quality | Probability of Accepting the Lot | AOQ |
|---|---|---|
| 0 | 1 | 0 |
| 0.01 | 0.8601 | 0.0083 |
| 0.02 | 0.7386 | 0.0142 |

| | | |
|---|---|---|
| 0.04 | 0.5421 | 0.0209 |
| 0.06 | 0.3953 | 0.0228 |
| 0.08 | 0.2863 | 0.0220 |
| 0.10 | 0.2059 | 0.0198 |
| 0.12 | 0.1470 | 0.0170 |
| 0.14 | 0.1041 | 0.0099 |
| 0.16 | 0.0731 | 0.0079 |
| 0.18 | 0.0510 | 0.0061 |
| 0.20 | 0.0352 | 0.0000 |

We now construct the AOQ curve by taking the quality level (proportion defective) on the X-axis and the corresponding AOQ values on the Y-axis as shown in Fig.7.3.

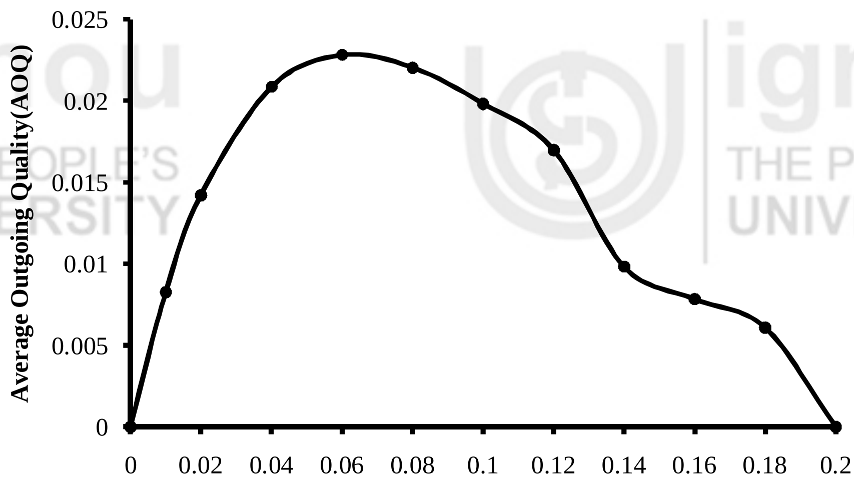


**Fig. 7.3: The AOQ curve for Example 5.**

You may now like to calculate the AOQ and construct the AOQ curve. Try the following exercises.

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**E4)** Assuming that the lot size is large relative to the sample size, calculate the approximate average outgoing quality (AOQ) for the single sampling plan with n = 10 and c = 0 containing 20% defective units.

**E5)** A computer manufacturer purchases computer chips from a company in lots of 200. Twelve computer chips are sampled at random and inspected for defects. The computer manufacturer accepts the lot if the inspected sample contains at most one defective chip. Otherwise, he/she rejects the lot. If the rejected lots are screened and all defective computer chips are replaced by non-defective chips. Construct the AOQ curve for this plan.

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You have learnt about the OC curve, producer's risk, consumer's risk and AOQ for a single sampling plan. We now discuss ASN and ATI of the plan.

## 7.6 AVERAGE SAMPLE NUMBER (ASN) AND AVERAGE TOTAL INSPECTION (ATI)

Two other features that are also useful for any sampling plan are the average sample number (ASN) and the average total inspection (ATI). We now discuss these for a single sampling plan in some detail.

**Average Sample Number (ASN)**

You have studied in Unit 6 that the average sample number is the expected number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the acceptance sampling plan.

In acceptance single sampling plan, the decision about the acceptance or rejection of a lot is taken on the basis of a single sample that has been inspected. Therefore, the ASN in a single sampling plan is simply the sample size n. It means that ASN is constant in a single sampling plan.

Therefore,

$$\text{ASN} = n \qquad \ldots (14)$$

The curve drawn between the ASN and the lot quality (p) is known as the **ASN curve**.

The ASN curve for a single sampling plan is a straight line as shown in Fig. 7.4.

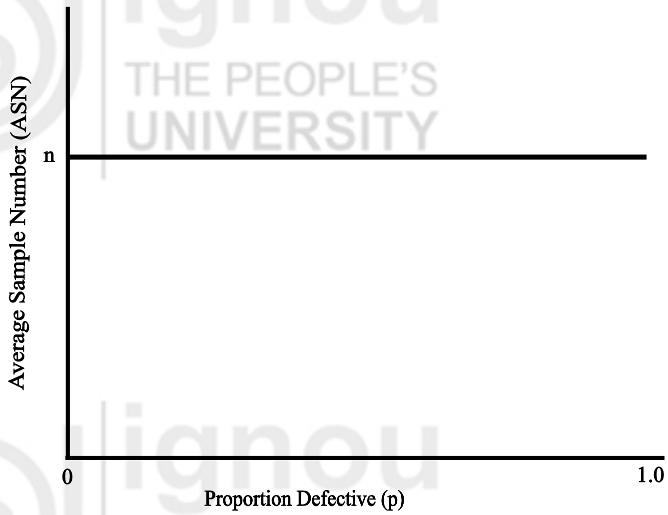


**Fig. 7.4: The ASN curve for a single sampling plan.**

Let us consider Example 4. In this example, the quality control inspector takes the decision of acceptance or rejection of the lot on the basis of the single sample of size n = 15. So the ASN for this single sampling plan is 15.

### Average Total Inspection (ATI)

You know that the concept of average total inspection (ATI) is considered under rectifying sampling plan in which rejected lots under go 100% inspection. It is defined as follows:

The average number of units inspected per lot under the rectifying sampling plan is called the **average total inspection** (ATI).

So in a rectifying single sampling plan, the number of units to be inspected will depend on two situations given below:

i) If the lot of size N is accepted on the basis of a sample of size n, the number of units inspected is n and the probability of the accepting the lot is $P_a$.

ii) If the lot is rejected on the basis of a sample, we inspect the entire lot of size N and the probability of rejecting the lot is $(1 – P_a)$.

Therefore, we can compute the ATI for a single sampling plan as follows:

ATI = Average number of units inspected per lot

$$= \sum (\text{inspected number of units} \times \text{probability of taking decision})$$

$$ATI = n \times P_a + N \times (1 - P_a)$$

This can also be written as

$$ATI = n + (1 - P_a)(N - n) \qquad \ldots (15)$$

The curve drawn between ATI and lot quality (p) is known as the **ATI curve**.

Let us take up an example to illustrate this concept.

**Example 7:** Calculate the ASN for the plan given in Example 4. If the rejected lots are screened and all defective tyres are replaced by non-defective tyres, construct the ATI curve for this plan.

**Solution:** It is given that

$$N = 400, n = 15, c = 0$$

In the first case, the decision of acceptance or rejection of the lot is taken only on a single sample of size n = 15. Therefore, the ASN for this plan is simply the sample size, i.e., ASN = n = 15.

For construction of the ATI curve, we first calculate the probabilities of accepting the lot corresponding to different quality levels using Table I. Then we calculate the ATI for each quality level using equation (15).

We have already calculated these probabilities in Example 6 and we use those results. Substituting the values of N, n, p and $P_a$ in equation (15) we can calculate ATI.

The probabilities of accepting the lot and the ATIs corresponding to different quality levels are given in the following table:

| Incoming Lot Quality | Probability of Accepting the Lot | ATI |
|---|---|---|
| 0 | 1 | 15.00 |
| 0.01 | 0.8601 | 68.86 |
| 0.02 | 0.7386 | 115.64 |
| 0.04 | 0.5421 | 191.29 |
| 0.06 | 0.3953 | 247.81 |
| 0.08 | 0.2863 | 289.77 |
| 0.10 | 0.2059 | 320.73 |
| 0.12 | 0.1470 | 343.41 |
| 0.14 | 0.1041 | 359.92 |
| 0.16 | 0.0731 | 371.86 |
| 0.18 | 0.0510 | 380.37 |
| 0.20 | 0.0352 | 386.45 |

We now construct the ATI curve by taking the quality level (proportion defective) on the X-axis and the corresponding ATI values on the Y-axis as shown in Fig.7.5.

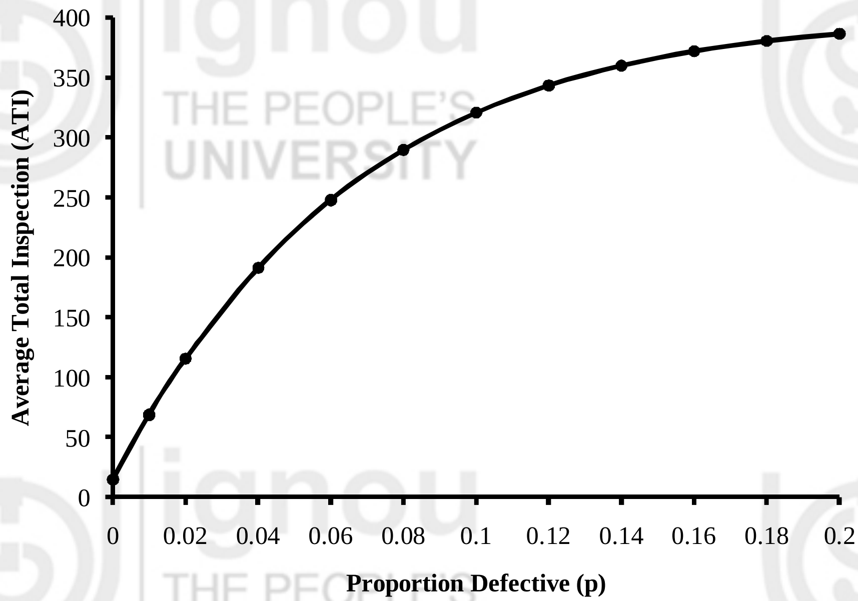


**Fig. 7.5: The ATI curve for Example 6.**

You can try the following exercise based on ASN and ATI for practice.

**E6)** Calculate the ASN and construct the ATI curve for the plan given in E5.

So far you have learnt the various features of the single sampling plan. We now discuss how to design a single sampling plan.

## 7.7 DESIGN OF SINGLE SAMPLING PLANS

The design of a single sampling plan implies the determination of the parameters of the plan, i.e., the sample size **n** and the acceptance number **c**. These numbers have to be decided in advance before applying the single sampling plan technique. There are several approaches for determining the parameters n and c for the single sampling. Sometimes, more than one plan will satisfy the criteria, but the best plan is the one with the largest sample size. It provides adequate protection to both producer and consumer. We now discuss some of the approaches.

### 7.7.1 Stipulated Producer's Risk

In this approach, the producer's risk α and its corresponding acceptance quality level (AQL) $p_1$ are specified. We would like to design a single sampling plan in such a way that the lots of quality level $p_1$ are accepted $100(1 - \alpha)\%$ of the time.

According to this approach, we first select an acceptance number (c) and then find the value of $np_1$ corresponding to c and α with the help of Table III given at the end of this block. Then the value of n is obtained by dividing $np_1$ by $p_1$ (= AQL) as follows:

$$n = \frac{np_1}{p_1} \qquad \ldots (16)$$

If the computed value of n is a fraction, it is rounded off to the next integer.

For different values of the acceptance number (c), we may get different values of sample size (n) (as you will see in Example 8). This means that for the same producer's risk and acceptance quality level, we have different single sampling plans. If we draw the OC curve for each sampling plan, we will see that each plan has a different consumer's risk. So out of these, we choose the sampling plan which gives the best protection to the consumer against acceptance of poor quality lots.

We now describe this procedure with the help of an example.

**Example 8:** Suppose a tyre supplier ships tyres in lots of size 400 to the buyer. The supplier and quality control inspector of the buyer decide the acceptance quality level (AQL) to be 2%. Design a sampling plan which ensures that the lots of quality 2% will be rejected 5% of the time.

**Solution:** The supplier and the quality control inspector desire the sampling plan for which AQL ($p_1$) = 2% and the producer's risk ($\alpha$) = 5%.

To find the desired sampling plan, first of all, we choose the acceptance number c as c = 1. Then we look up the value of $np_1$ corresponding to c = 1 and $\alpha = 0.05$ from Table III. We have

$$np_1 = 0.355$$

Therefore, we can obtain the sample size as follows:

$$n = \frac{np_1}{p_1} = \frac{0.355}{0.02} = 17.75 \simeq 18$$

Hence, the required single sampling plan is

$$n = 18 \text{ and } c = 1$$

Similarly, for c = 2 and $\alpha = 0.05$, the value of $np_1$ is 0.818.

Therefore, the sample size is

$$n = \frac{np_1}{p_1} = \frac{0.818}{0.02} = 40.9 \simeq 41$$

Hence, the required single sampling plan is

$$n = 41 \text{ and } c = 2$$

Similarly, for c = 5 and $\alpha = 0.05$, the value of $np_1$ is 2.613.

Therefore,

$$n = \frac{np_1}{p_1} = \frac{2.613}{0.02} = 130.65 \simeq 131$$

Hence, the required single sampling plan is

$$n = 131 \text{ and } c = 5$$

The OC curves for the three sampling plans (as discussed in Sec. 7.4) are shown in Fig. 7.6.

| **Incoming Lot Quality** | **Probability of Accepting the Lot** | | |
|---|---|---|---|
| | **n = 18 and c = 1** | **n = 41 and c = 2** | **n = 131 and c = 5** |
| 0 | 1.0000 | 1.0000 | 1.0000 |
| 0.01 | 0.9862 | 0.9920 | 0.9978 |
| 0.02 | 0.9505 | 0.9514 | 0.9513 |
| 0.04 | 0.8393 | 0.7750 | 0.5731 |
| 0.06 | 0.7055 | 0.5505 | 0.1959 |

| | | | |
|---|---|---|---|
| 0.08 | 0.5719 | 0.3526 | 0.0445 |
| 0.10 | 0.4503 | 0.2086 | 0.0075 |
| 0.12 | 0.3460 | 0.1156 | 0.0010 |
| 0.14 | 0.2602 | 0.0607 | 0.0001 |
| 0.16 | 0.1920 | 0.0303 | 0.0000 |
| 0.18 | 0.1391 | 0.0145 | 0.0000 |
| 0.20 | 0.0991 | 0.0066 | 0.0000 |

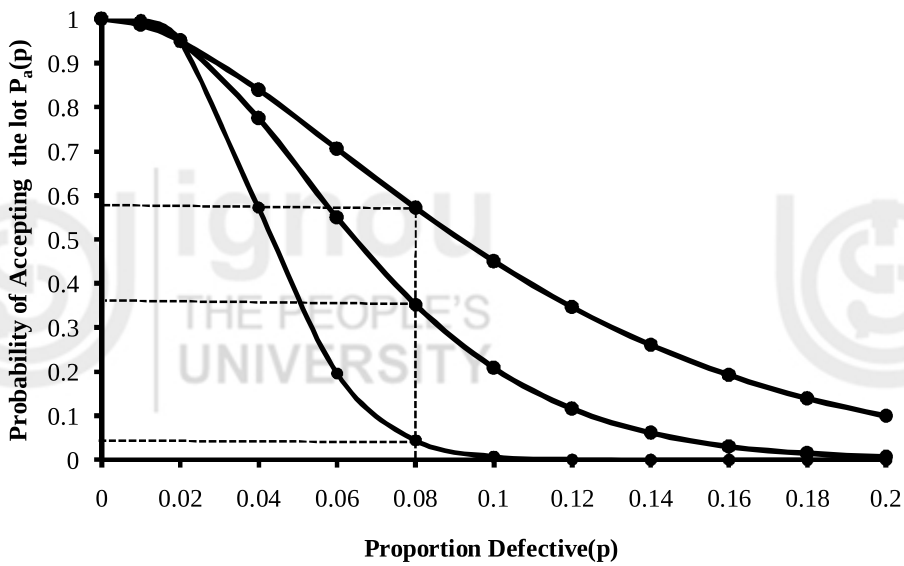


**Fig. 7.6: The OC curves for Example 8.**

From Fig. 7.6, we conclude that out of the three sampling plans, the sampling plan with n = 131, c = 5 provides the best protection to the consumer because it has the lowest probability of accepting poor quality lots. However, this sampling plan has the largest sample size (n = 131), which increases the inspection cost.

## 7.7.2 Stipulated Consumer's Risk

According to this approach, the consumer risk β and its corresponding lot tolerance percent defective (LTPD) $p_2$ are specified. We desire to determine a single sampling plan in such a way that we will be accepted lots of quality level $p_2$ 100β% of the time.

To design the plan using this approach, we first select an acceptance number (c) and then find the value of $np_2$ corresponding to c and β with the help of Table III. Then the value of n can be obtained by dividing $np_2$ by $p_2$ = LTPD as follows:

$$n = \frac{np_2}{p_2} \qquad \ldots (17)$$

For different values of c, a number of sampling plans may satisfy this criterion. If we draw the OC curve for each sampling plan, we will see that each plan has a different producer's risk. So out of these, we choose the sampling plan which gives the best protection to the producer against the rejection of good quality lots.

We now describe this procedure with the help of an example.

**Example 9:** Suppose, in Example 8, the supplier and the quality control inspector decide the lot tolerance percent defective (LTPD) to be 5%. Determine the single sampling plans using c = 1, 2 and 8 which ensure that the lots of quality 5% will be accepted 10% of the time.

**Solution:** It is given that

LTPD ($p_2$) = 5% and the consumer'risk ($\beta$) = 10%

To find the desired sampling plan, we first look up the value of $np_2$ corresponding to c = 1 and $\beta$ = 0.10 from Table III. We have

$$np_2 = 3.890$$

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{3.890}{0.10} = 38.90 \approx 39$$

Hence, the required single sampling plan is

$$n = 39, c = 1$$

Similarly, for c = 2 and $\beta$ = 0.10, the value of $np_2$ is 5.322.

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{5.322}{0.10} = 53.22 \approx 54$$

Hence, the required single sampling plan is

$$n = 54, c = 2$$

Similarly, for c = 8 and $\beta$ = 0.10 the value of $np_2$ is 12.995.

Therefore,

$$n = \frac{np_2}{p_2} = \frac{12.955}{0.10} = 129.55 \approx 130$$

Hence, the required single sampling plan is

$$n = 130, c = 8$$

### 7.7.3 Stipulated Producer's Risk and Consumer's Risk

According to this approach, the producer's risk with its corresponding acceptance quality level (AQL) and consumer's risk with its corresponding lot tolerance percent defective (LTPD) are specified. We have to design single sampling plans which satisfy both producer's and consumer's risks, such that the lots of AQL are to be rejected no more than 100$\alpha$% of the time and lots of LTPD are to be accepted no more than 100$\beta$% of the time.

Here the criteria are more stringent than the previous approaches and we may not have much flexibility in choosing the acceptance number and the associated sampling plan. So it may be difficult to design a sampling plan that will satisfy both producer's and consumer's risks.

In this approach, we first find the operating ratio R as follows:

$$R = \frac{p_2}{p_1} \qquad \dots (18)$$

The values of R corresponding to various acceptance number (c) and α and β are also listed in Table III. We choose a value of R from Table III which is exactly equal to the desired value of R corresponding to the desired α and β. Generally, the tabulated value of R is not equal to the desired value of R. So in such situations, we take the tabulated values of R between which the desired value of R lies. Then we look up the corresponding values of acceptance number (c) in Table III and find the value of sample size (n). The following two approaches are used to find n:

**1. Satisfy Producer's Risk Stipulation Exactly and come Close to Consumer's Risk**

According to this approach, we find the values of n as we have discussed in Sec. 7.7.1. It means that we first find the value of $np_1$ corresponding to c and α with the help of Table III. Then we find n from equation (16). Thus,

$$n = \frac{np_1}{p_1}$$

The values of n are obtained for both values of c. In this way, we get two sampling plans which satisfy producer's risk exactly.

Out of these, we choose the sampling plan which is close to satisfying the consumer's risk. For that we find the value of $p_2$ for each plan. We find the values of $np_2$ corresponding to the desired β and each c from Table III. Then we find the value of $p_2$ for each plan as follows:

$$p_2 = \frac{np_2}{n} \qquad \ldots (19)$$

We choose the sampling plan for which the calculated $p_2$ is closer to the desired $p_2$.

Another criterion for choosing the sampling plan is that we select the sampling plan which has the smallest sample size in order to minimize inspection costs. Alternatively, we can choose the sampling plan which has the largest sample size in order to get maximum information.

**2. Satisfy Consumer's Risk Stipulation Exactly and come Close to Producer's Risk**

According to this approach, we find the values of n as we have discussed in Sec. 7.7.2. It means that we first find the value of $np_2$ corresponding to c and β with the help of Table III. Then we find n from equation (17). Thus,

$$n = \frac{np_2}{p_2}$$

The values of n are obtained for both values of c. In this way, we get two sampling plans which satisfy consumer's risk exactly.

Out of these, we choose the sampling plan which is close to satisfying the producer's risk. For that we find the value of $p_1$ for each plan. We find the values of $np_1$ corresponding to the desired α and each c from Table III. Then we find the value of $p_1$ for each plan as follows:

$$p_1 = \frac{np_1}{n} \qquad \ldots (20)$$

We choose the sampling plan for which the calculated $p_1$ is closer to the desired $p_1$.

Another criterion for choosing the sampling plan is that we select the sampling plan which has the smallest sample size in order to minimize inspection costs. Alternatively, we can choose the sampling plan which has the largest sample size in order to get maximum information.

We now describe this procedure with the help of an example.

**Example 10:** Suppose, in Example 8, the supplier and the quality control inspector decide the acceptance quality level (AQL) to be 2% and the lot tolerance percent defective (LTPD) to be 8%. Design a sampling plan which ensures that lots of quality 2% will be rejected 5% of the time and lots of quality 8% will be accepted 5% of the time.

**Solution:** It is given that

$$\text{AQL} = p_1 = 2\% = 0.02 \text{ and } \alpha = 5\% = 0.05$$

$$\text{LTPD} = p_2 = 8\% = 0.08 \text{ and } \beta = 5\% = 0.05$$

To design the desired sampling plan, we first calculate the operating ratio (R) from equation (18) as follows:

$$R = \frac{p_2}{p_1} = \frac{0.08}{0.02} = 4.0$$

From Table III, we see that the desired value of R = 4.0 lies between 4.023 and 3.604 for $\alpha = 0.05$ and $\beta = 0.05$. From Table III, the corresponding acceptance numbers (c) are 5 and 6. We have to find the value of sample size (n).

We first find the plans which satisfy the desired producer's risk exactly.

For this, we find the value of $np_1$ corresponding to c = 5 and $\alpha = 0.05$ with the help of Table III.

From Table III, for c = 5 and $\alpha = 0.05$, we have $np_1 = 2.613$. Therefore, from equation (16), we have

$$n = \frac{np_1}{p_1} = \frac{2.613}{0.02} = 130.65 \approx 131$$

Similarly, for c = 6 and $\alpha = 0.05$, the value of $np_1$ is 3.286. Therefore,

$$n = \frac{np_1}{p_1} = \frac{3.286}{0.02} = 164.3 \approx 165$$

So both the plans with n = 131, c = 5 and n = 165, c = 6 satisfy the producer's risk exactly. Out of these plans, we have to find the plan which is closer to satisfying the desired consumer's risk. For this we find $np_2$ corresponding to c and $\beta$.

From Table III, for c = 5, $\beta = 0.05$, we have $np_2 = 10.531$.

Therefore, from equation (19), we have

$$p_2 = \frac{np_2}{n} = \frac{10.513}{131} = 0.08$$

Similarly, for c = 6, $\beta = 0.05$, we have $np_2 = 11.842$.

Therefore,

$$p_2 = \frac{np_2}{n} = \frac{11.842}{165} = 0.07$$

Since the value of $p_2 = 0.08$ corresponding to the plan n = 131, c = 5 is equal to the desired value 0.08, the plan with n = 131, c = 5 is the best single sampling plan.

We now find the plans which satisfy the desired consumer's risk exactly.

From Table III, for c = 5 and $\beta = 0.05$, we have $np_2 = 10.513$. Therefore, from equation (17), we have

$$n = \frac{np_2}{p_2} = \frac{10.513}{0.08} = 131.4 \simeq 132$$

Similarly, for c = 6 and $\beta = 0.05$, we have $np_2 = 11.842$. Therefore,

$$n = \frac{np_2}{p_2} = \frac{11.842}{0.08} = 148.02 \simeq 149$$

So both the plans with n = 132, c = 5 and n = 149, c = 6 are satisfied the consumer's risk exactly. Out of these plans, we have to find the plan which is closer to the desired producer's risk. For this we find $np_1$ corresponding to c and $\alpha$.

From Table III, for c = 5, $\alpha = 0.05$, we have $np_1 = 2.613$

Therefore,

$$p_1 = \frac{np_1}{n} = \frac{2.613}{132} = 0.0198$$

Similarly, for c = 6, $\alpha = 0.05$, we have $np_1 = 3.286$.

Therefore,

$$p_1 = \frac{np_1}{n} = \frac{3.286}{149} = 0.022$$

Since the value of $p_1 = 0.0198$ corresponding to the plan n = 132, c = 5 is approximate equal to the desired value 0.02, the plan n = 132, c = 5 is the best single sampling plan.

### 7.7.4 Larson Binomial Nomograph

There is also a graphical method for designing the single sampling plans when the producer's risk with corresponding acceptance quality level (AQL) and consumer's risk with corresponding lot tolerance percent defective (LTPD) are specified.

This method is based on Larson binomial nomograph and is used when we take the binomial approximate to the hypergeometric distribution, i.e., when $N \geq 10n$. The Larson binomial nomograph (shown in Fig. 7.7) is a graph of the cumulative binomial distribution. It has two scales. On the left scale, **proportion defective (p)** is shown as **probability of occurences on a single trial (p)**. This scale is known as **p-scale**. On the right scale, the **probability of acceptance ($P_a$)** is shown as **probability of less than or equal to c occurences in n trials (p)**. This scale is known as **$P_a$-scale**.

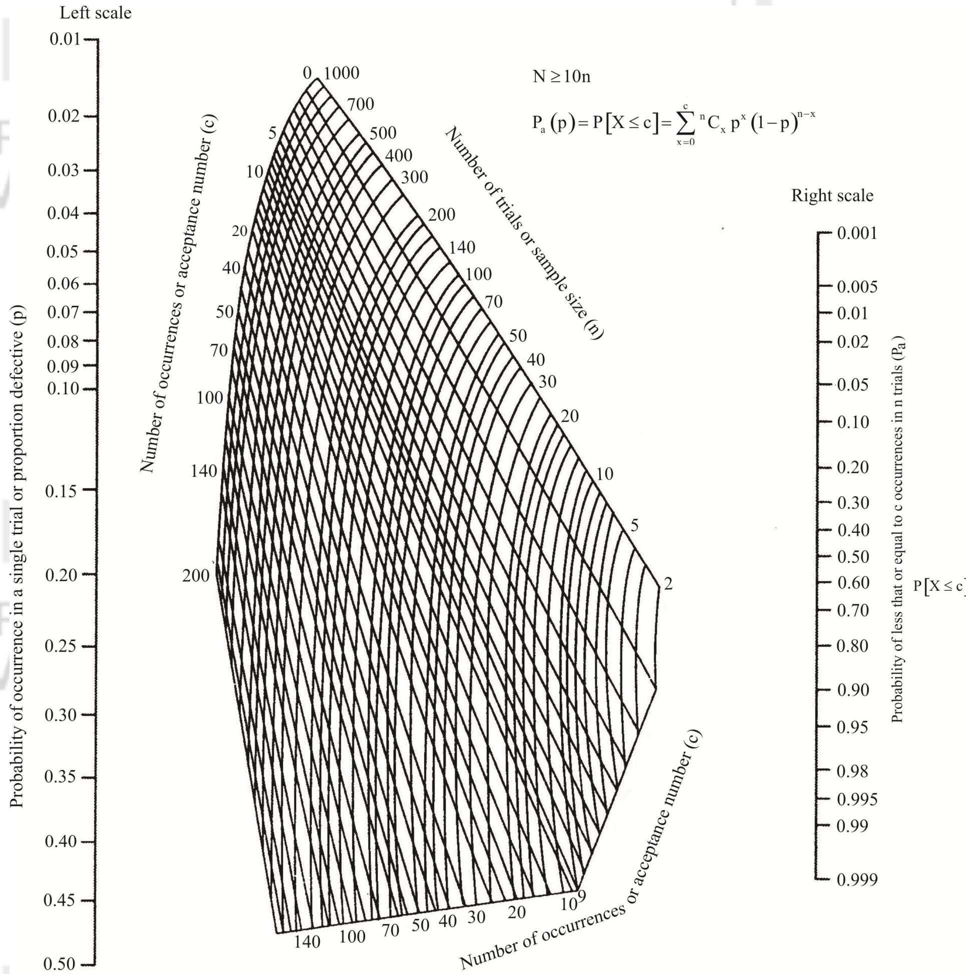


**Fig. 7.7: Larson binomial nomograph.**

The procedure of designing a single sampling plan by using the nomograph is quite simple. We plot the values of AQL ($p_1$) and LTPD ($p_2$) on the left scale and the corresponding values of $(1 - \alpha)$ and $\beta$ on the right scale. Then we join $p_1$ with $(1 - \alpha)$ and $p_2$ with $\beta$ by straight lines. We read the values of sample size (n) and acceptance number (c) at the intersection of the lines on the grid.

To demonstrate this method, we consider Example 10.

It is given that

$$\text{AQL} = p_1 = 2\% = 0.02 \text{ and } \alpha = 5\% = 0.05$$

$$\text{LTPD} = p_2 = 8\% = 0.08 \text{ and } \beta = 5\% = 0.05$$

For designing a single sampling plan by using Larson nomograph, we first plot $p_1 = 0.02$ and $p_2 = 0.08$ on the p-scale on the nomograph. Then we plot $1 - \alpha$ (= 0.95) and $\beta = 0.05$ on the $P_a$-scale on the nomograph. After plotting these points, we draw a straight line joining $p_1$ (= 0.02) and $1 - \alpha$ (= 0.95) and another straight line joining $p_2$ ( = 0.08) and $\beta$ ( = 0.05) as shown in Fig. 7.8. At the intersection of the two lines, we read the values of n and c.

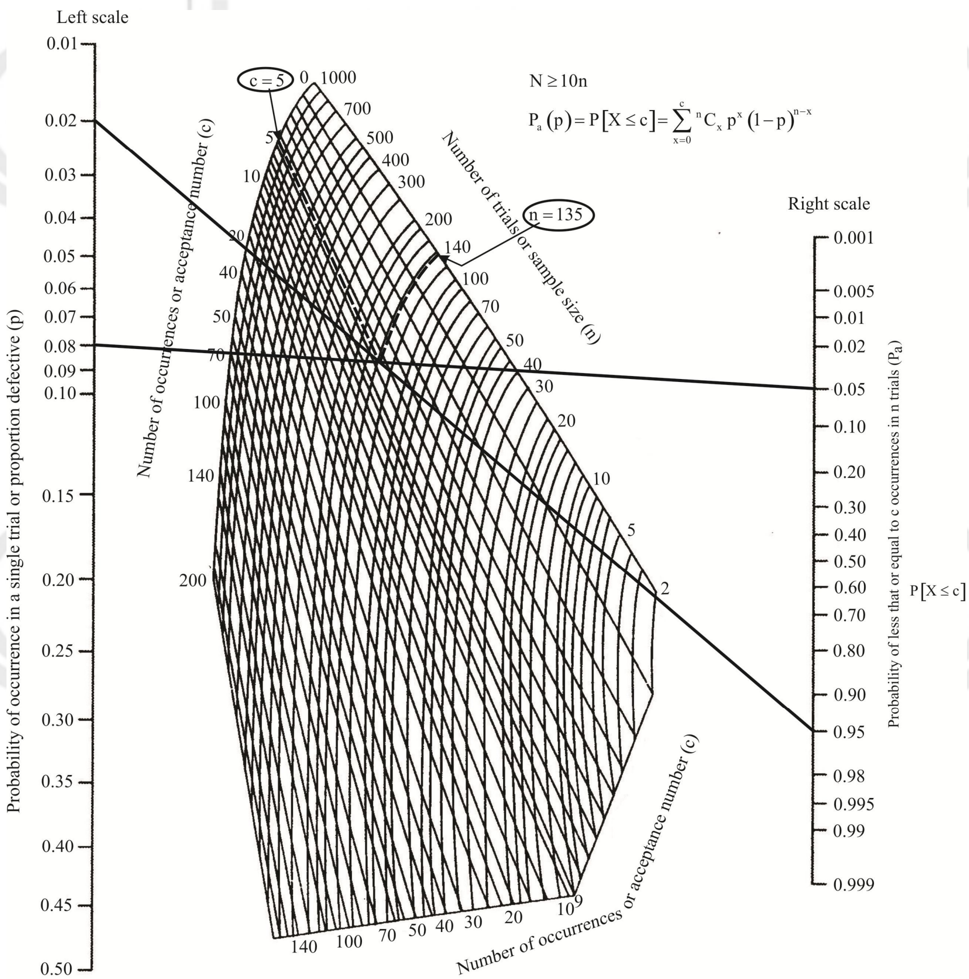


**Fig. 7.8**

From Fig. 7.8 we have, n = 135 and c = 5.

You can now check your understanding of how to design a single sampling plan by answering the following exercises.

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**E7)** A ball bearing supplier and an automobile company have decided to check the quality of ball bearings in lots of size 1000 with acceptance quality level (AQL) as 1%. Design the single sampling plans using c = 2, 4 and 6 which ensure that lots of quality 1% will be rejected 1% of the time.

**E8)** A consumer receives lots of 5000 candles from a new supplier. To check the quality of lots, the consumer and supplier want to use the single sampling plan which satisfies a consumer's risk of 5% for lots of quality 5% . Determine sampling plans for the specified consumer's risk and LTPD for acceptance number c = 3 and 6.

---

We end this unit by giving a summary of what we have covered in it.

## 7.8 SUMMARY

1. The main acceptance sampling plans for attributes are:

   i) Single sampling plan,

   ii) Double sampling plan,

iii) Multiple sampling plan, and

iv) Sequential Sampling Plan.

2. A sampling plan in which a decision about the acceptance or rejection of a lot is based on a single sample that has been inspected is known as a **single sampling plan**. There are two parameters of a single sampling plan:

   n – size of the sample, and

   c – acceptance number for the sample.

3. In a single sampling plan, if number of defective units (d) in the sample is less than or equal to the stated acceptance number (c), i.e., if $d \le c$, we accept the lot and if $d > c$, we reject the lot under acceptance sampling plan.

4. In a single sampling plan, if $d \le c$, we accept the lot and replace all defective units found in the sample by non-defective units and if $d > c$, we accept the lot by inspecting the entire lot and replacing all defective units in the lot by non-defective units under rectifying sampling plan.

5. The probability of accepting a lot of quality p for a single sampling plan is given by

$$P_a(p) = P[X \le c] = \sum_{x=0}^{c} {}^nC_x\, p^x (1-p)^{n-x}$$

6. The produce's risk and consumer's risk for a single sampling plan are given by

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} {}^nC_x\, p_1^x (1-p_x)^{n-x} \text{ and}$$

$$P_c = P_a(p_2) = \sum_{x=0}^{c} {}^nC_x\, p_2^x (1-p_2)^{n-x}$$

7. The AOQ for a single sampling plan is

$$AOQ = \frac{p(N-n)P_a}{N}$$

8. The ASN and ATI for a single sampling plan are

$$ASN = n \text{ and } ATI = n + (1-P_a)(N-n)$$

9. Designing a single sampling plan implies determining the sample size (n) and acceptance number (c).

## 7.9 SOLUTIONS/ANSWERS

**E1)** To check the quality of the lots, the buyer randomly draws 50 silicon chips from each lot. After that he/she inspects each and every chip drawn from the lot for certain defects and classifies each chip of the sample as defective or non-defective. At the end of the inspection, he/she counts the number of defective chips (d) found in the sample and then compares the number of defective chips (d) with the acceptance number (c). If $d \le c = 2$, he/she accepts the lot and if $d > c = 2$, he/she rejects the lot on the basis of the inspected sample. It means that if the buyer finds 0 or 1 or 2 defective chips in the sample, he/she accepts the lot. Otherwise, he/she rejects the lot.

**E2)** It is given that

$$N = 2000, n = 25, c = 2$$

For constructing the OC curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If X represents the number of defective syringes in the sample, the quality inspector accepts the lot if $X \le c = 2$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le c] = P[X \le 2] = \sum_{x=0}^{2} P[X = x]$$

Since $N \ge 10n$, we use the binomial distribution. We can use Table I for calculating the probabilities of accepting the lot corresponding to different quality levels such as $p = 0.01, 0.02, 0.03\ldots$ .These probabilities are shown in the table given below:

| **Incoming Lot Quality** | **Probability of Accepting the Lot** |
|---|---|
| 0 | 1 |
| 0.01 | 0.9980 |
| 0.02 | 0.9868 |
| 0.04 | 0.9235 |
| 0.06 | 0.8129 |
| 0.08 | 0.6768 |
| 0.10 | 0.5371 |
| 0.12 | 0.4088 |
| 0.14 | 0.3000 |
| 0.16 | 0.2130 |
| 0.18 | 0.1467 |
| 0.20 | 0.0982 |

We construct the OC curve by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis as shown in Fig. 7.9.

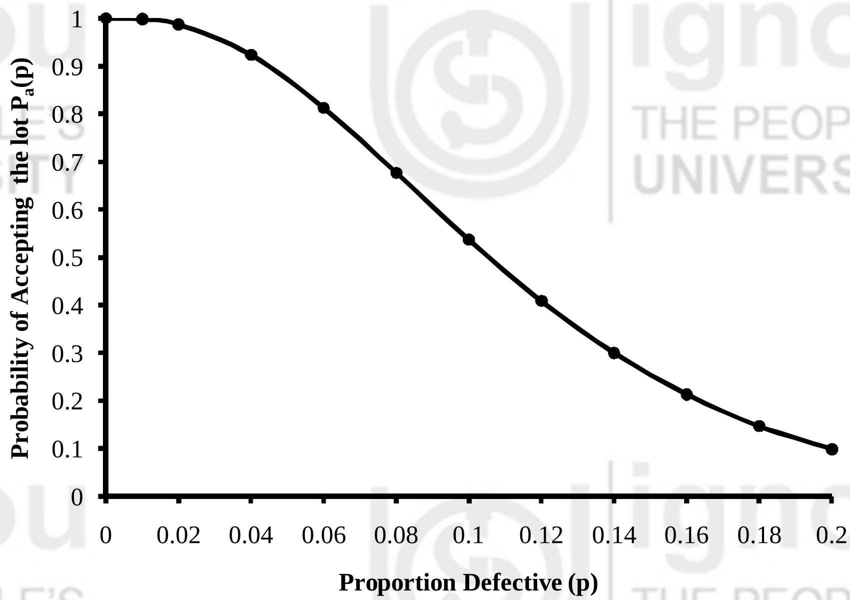


**Fig. 7.9: The OC curve for E2.**

**E3)** It is given that

$$N = 2000, n = 25, c = 2, AOL(p_1) = 0.04 \text{ and } LTPD(p_2) = 0.10$$

Since $N \geq 10n$, we use the binomial distribution. Therefore, we use equation (8) to calculate the producer's risk for the single sampling plan.

We first calculate the probability of accepting the lot of quality $p = p_1 = AQL = 0.04$ using Table I.

From Table I, for n = 25, x = c = 2 and $p = p_1 = 0.04$, we have

$$P_a(p_1) = P[X \leq 2] = \sum_{x=0}^{2} {}^nC_x \, p_1^x (1-p_1)^{n-x} = 0.9235$$

Therefore, from equation (8), we have

$$P_p = 1 - P_a(p_1) = 1 - 0.9235 = 0.0765$$

It means that if there are several lots of the same quality p = 0.04, about 7.65% of these will be rejected. This is obviously a risk for the supplier because it was agreed upon by both that lots of quality 0.04 will be accepted whereas the quality inspector is rejecting 7.65% of them.

Similarly, we can calculate the consumer's risk using equation (10).

We first calculate the probability of accepting the lot of quality $p = p_2 = LTPD = 0.10$ using Table I.

From Table I, for n = 25, x = c = 2 and $p = p_2 = 0.10$, we have

$$P_a(p_2) = P[X \leq 2] = \sum_{x=0}^{2} {}^nC_x \, p_2^x (1-p_2)^{n-x} = 0.5371$$

Therefore, from equation (10), we have the consumer's risk

$$P_c = P_a(p_2) = 0.5371$$

It means that if there are several lots of the same quality p = 0.10, about 53.71% out of these will be accepted by the quality inspector even though this quality is unsatisfactory. This is obviously a risk for the quality inspector.

**E4)** It is given that

$$n = 10, c = 0, p = 20\% = 0.20$$

Since the lot size is large relative to the sample size, we can calculate the average outgoing quality (AOQ) for the single sampling plan using equation (13).

For calculating AOQ, we have to calculate the probability of accepting the lot corresponding to p = 0.20.

If X represents the number of defective units in the sample, the lot is accepted if $X \le c = 0$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P_a = P[X \le 0]$$

Since the lot size is large relative to the sample size, we can calculate this probability by using Table I.

From Table I, for n = 10, x = c = 0 and p = 0.20, we have

$$P_a = P[X \le 0] = \sum_{x=0}^{0} {}^nC_x\, p^x (1-p)^{n-x} = 0.1074$$

On putting the values of p and $P_a$ in equation (13), we get

$$AOQ = pP_a = 0.20 \times 0.1074 = 0.0215 = 2.15\%$$

**E5)** It is given that

$$N = 200, n = 12, c = 1$$

Since $N \ge 10n$, we can use the binomial distribution. Therefore, we use equation (12) to calculate the AOQ.

For constructing the AOQ curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If X represents the number of defective chips in the sample, the manufacturer accepts the lot if $X \le c = 1$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} P[X = x]$$

We can use Table I for calculating the probabilities of accepting the lot corresponding to different quality levels such as p = 0.01, 0.02, 0.03… Then we calculate AOQ for each quality level by using equation (12). The probabilities of accepting the lot and AOQs corresponding to different quality levels are given in the following table:

| Incoming Lot Quality | Probabilty of Accepting the Lot | AOQ |
|---|---|---|
| 0 | 1.0000 | 0 |
| 0.01 | 0.9938 | 0.0093 |
| 0.02 | 0.9769 | 0.0184 |

| | | |
|---|---|---|
| 0.04 | 0.9191 | 0.0346 |
| 0.06 | 0.8405 | 0.0474 |
| 0.08 | 0.7513 | 0.0565 |
| 0.10 | 0.6590 | 0.0619 |
| 0.12 | 0.5686 | 0.0641 |
| 0.14 | 0.4834 | 0.0636 |
| 0.16 | 0.4055 | 0.0610 |
| 0.18 | 0.3359 | 0.0568 |
| 0.20 | 0.2749 | 0.0517 |

We construct the AOQ curve by taking the quality level (proportion defective) on the X-axis and the corresponding AOQ values on the Y-axis as shown in Fig.7.10.

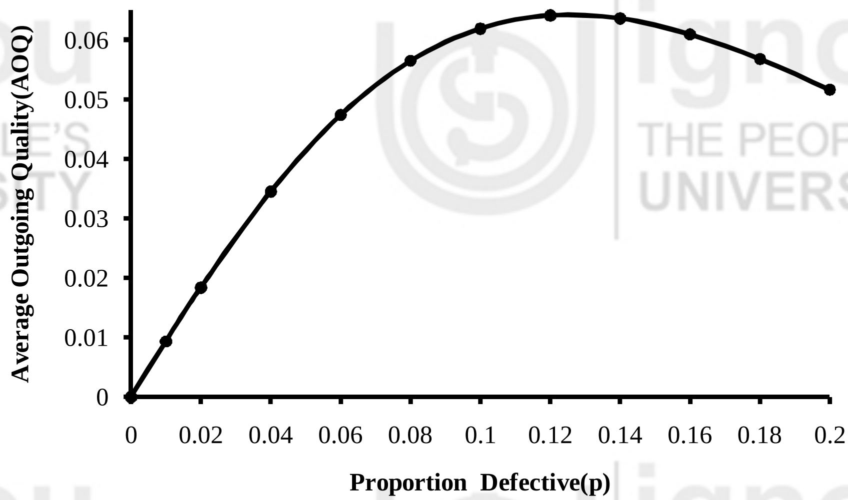


**Fig. 7.10: The AOQ curve for E5.**

**E6)** It is given that

$$N = 200, n = 12, c = 1$$

In the first case, the decision of acceptance or rejection of the lot is taken only on a single sample of size n = 12. Therefore, the ASN for this plan is simply the sample size, i.e., ASN = n = 12.

For construction of the ATI curve, we first calculate the probabilities of accepting the lot corresponding to different quality levels using Table I. Then we calculate the ATI for each quality level using equation (15).

We have already calculated these probabilities in E5 and we those results. Substituting the values of N, n, p and $P_a$ in equation (15) we can calculate ATI.

The probabilities of accepting the lot and the ATIs corresponding to different quality levels are given in the following table:

| Incoming Lot Quality | Probability of Accepting the Lot | ATI |
|---|---|---|
| 0 | 1.0000 | 12.00 |
| 0.01 | 0.9938 | 13.17 |
| 0.02 | 0.9769 | 16.34 |
| 0.04 | 0.9191 | 27.21 |
| 0.06 | 0.8405 | 41.99 |

| | | |
|---|---|---|
| 0.08 | 0.7513 | 58.76 |
| 0.10 | 0.6590 | 76.11 |
| 0.12 | 0.5686 | 93.10 |
| 0.14 | 0.4834 | 109.12 |
| 0.16 | 0.4055 | 123.77 |
| 0.18 | 0.3359 | 136.85 |
| 0.20 | 0.2749 | 148.32 |

We now construct the ATI curve by taking the quality level (proportion defective) on the X-axis and the corresponding ATI values on the Y-axis as shown in Fig.7.11.

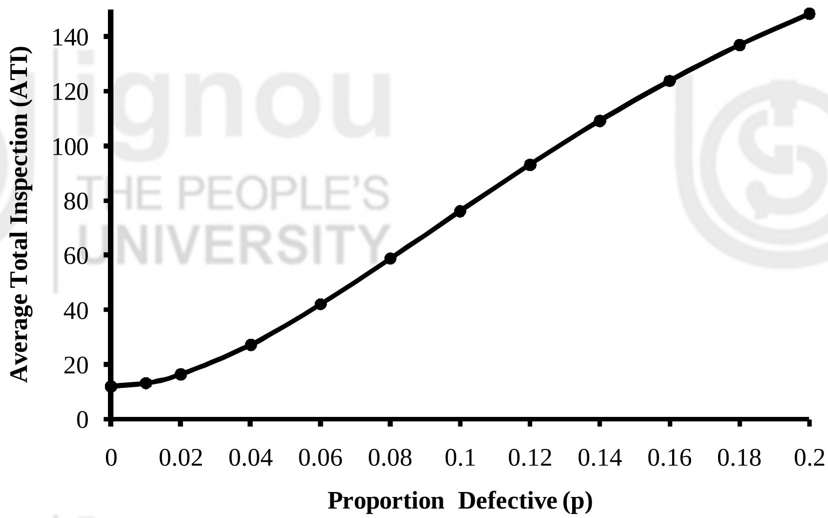


**Fig. 7.11: The ATI curve for E6.**

**E7)** We have

$$\text{AQL} = 1\% = 0.01 \text{ and } \alpha = 1\% = 0.01$$

To design the desired sampling plan, we first look up the value of $np_1$ corresponding to $c = 2$ and $\alpha = 0.01$ from Table III. We have

$$np_1 = 0.436$$

Therefore, the sample size is

$$n = \frac{np_1}{p_1} = \frac{0.436}{0.01} = 43.6 \approx 44$$

Hence, the required single sampling plan is

$$n = 44 \text{ and } c = 2$$

Similarly, for $c = 4$ and $\alpha = 0.01$, the value of $np_1$ is 1.279.

Therefore, the sample size is

$$n = \frac{np_1}{p_1} = \frac{1.279}{0.01} = 127.9 \approx 128$$

Hence, the required single sampling plan is

$$n = 128 \text{ and } c = 4$$

Similarly, for $c = 6$ and $\alpha = 0.01$, the value of $np_1$ is 2.330.

Therefore,

$$n = \frac{np_1}{p_1} = \frac{2.330}{0.01} = 233$$

Hence, the required single sampling plan is

$$n = 233 \text{ and } c = 6$$

**E8)** We have

$$\text{LTPD} = p_2 = 5\% = 0.05 \text{ and } \beta = 5\% = 0.05$$

To design the desired sampling plan, we look up the value of $np_2$ corresponding to $c = 3$ and $\beta = 0.05$ from Table III. We have

$$np_2 = 7.754$$

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{7.754}{0.05} = 155.08 \approx 156$$

Hence, the required single sampling plan is

$$n = 156 \text{ and } c = 3$$

Similarly, for $c = 6$ and $\beta = 0.05$, the value of $np_2$ is 11.842.

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{11.842}{0.05} = 236.84 \approx 237$$

Hence, the required single sampling plan is

$$n = 237 \text{ and } c = 6$$