UNIT 6 RECTIFYING SAMPLING PLANS

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6.1 INTRODUCTION

In Unit 5, you have learnt about the acceptance sampling plans and how these are implemented in industry. You have also learnt about different terms related to it such as acceptance quality level (AQL), lot tolerance percent defective (LTPD), producer's risk and consumer's risk. In an acceptance sampling plan, if the consumer rejects the lot on the basis of the information provided by the inspected sample and the producer is not able to sell the rejected lot, he/she suffers loss. If this continues, the producer might face a huge loss and not be able to continue with production. He/she may even have to shut it down. To avoid such situations, the concept of the rectifying sampling inspection plan was introduced.

In this unit, you will learn about the rectifying sampling plans for attributes. In Sec. 6.2, we discuss what is the rectifying sampling plan and how it implement in industry. In Secs. 6.3 to 6.6, we introduce some more parameters such as average outgoing quality (AOQ), operating characteristic (OC) curve, average sample number (ASN) and average total inspection (ATI). In the next unit, you will study single sampling plans.

Objectives

After studying this unit, you should be able to:

- explain why rectifying sampling plans are needed;
- describe a rectifying sampling plan;
- define average outgoing quality (AOQ) and the average outgoing quality limit (AOQL);
- describe the operating characteristic (OC) curve; and
- define average sample number (ASN) and average total inspection (ATI).

6.2 RECTIFYING SAMPLING PLAN

In an acceptance sampling plan, the consumer accepts the lot if the acceptance criteria are satisfied. Otherwise, he/she rejects the lot. This increases the producer's risk. Rectifying sampling plans are used to reduce the producer's risk. In such a plan, the entire lot is not rejected. Instead, each and every unit/item of the lot is inspected. It means that 100% inspection of the rejected lot is carried out and the defective units found in the lot are replaced by non-defective units. This procedure is known as **rectifying** or **screening** or **detailing** the rejected lots.

The sampling plan in which 100% inspection is carried out for rejected lots is called the **rectifying sampling inspection plan**. It is also called the **rectifying sampling plan** or **rectifying inspection plan**. Let us explain the procedure for implementing this plan.

6.2.1 Implementation of Rectifying Sampling Plan for Attributes

Suppose that lots of the same size (N) are received from the supplier or the final assembly line and submitted for inspection one at a time. The procedure for implementing the rectifying sampling plan to arrive at a decision about the lot is described in the following steps:

Step 1: We draw a random sample of size n from the lot received from the supplier or the final assembly.

Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective on the basis of certain criteria. At the end of the inspection, we count the number of defective units found in the sample.

Step 3: We compare the number of defective units found in the sample with the acceptance criteria.

Step 4: If acceptance criteria are satisfied, we accept the entire lot by replacing all defective units **in the sample** by non-defective units. If the criteria are not satisfied, we accept the lot by inspecting **the entire lot** and replacing all defective units in it by non-defective units.

The steps described above are shown in Fig. 6.1.

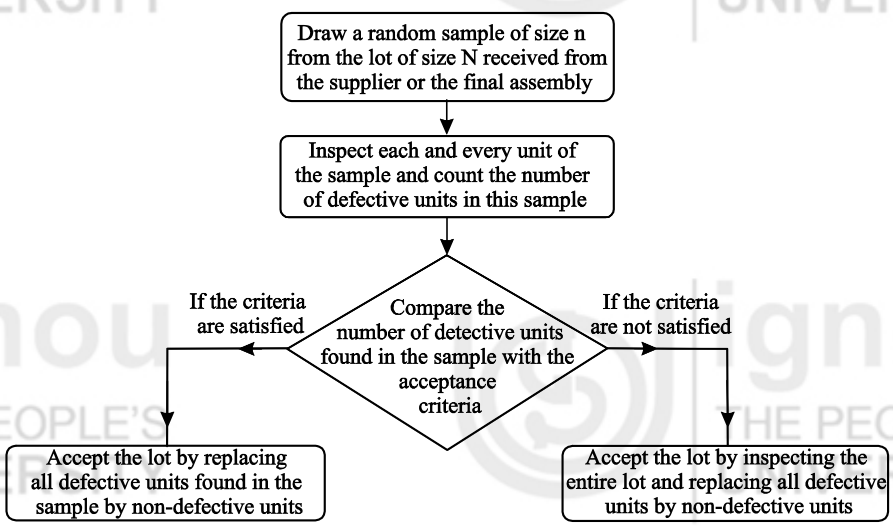

Fig. 6.1: Procedure for implementation of the rectifying sampling plan.

Let us explain these steps further with the help of an example.

Example 1: Suppose a cricket ball manufacturing company supplies lots of 200 balls. To check the quality of the lots, the buyer and the manufacturing company decide that a sample of size 20 will be drawn from each lot. The lot would be accepted if the inspected sample contains at most one defective ball. Otherwise, the lot would be rejected. If they use the rectifying sampling plan, explain the procedure for implementing it.

Solution: For implementing the rectifying sampling plan, the buyer draws 20 balls from each lot, inspects each and every ball of the sample and classifies it as defective or non-defective on the basis of certain criteria. At the end of the inspection, he/she counts the number of defective balls found in the sample and compares the number of defective balls with the acceptance criteria. If the number of defective balls in the sample is less than or equal to the acceptance number, he/she accepts the lot by replacing all defective balls in the sample by non-defective balls. If the number of defective balls is greater than the acceptance number, he/she carries out 100% inspection for this lot instead of rejecting the lot. The defective balls found in the lot are replaced by non-defective balls and the lot is accepted.

You may like to explain the procedure for implementing a rectifying sampling plan yourself. Try the following exercise.

E1) A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with n = 25 and c = 2 is being used for accepting the lots. Explain the procedure for implementing the rectifying sampling plan in this case.

So far you have learnt about the rectifying sampling plan. In Secs. 6.3 to 6.6, we explain various terms associated with the rectifying sampling plans.

6.3 AVERAGE OUTGOING QUALITY (AOQ)

In a sampling inspection plan, the items or units produced/manufactured by the producer are formed in lots. The average quality level of the lots is set by the producer and the consumer through negotiation and the producer sends the lots to the consumer for inspection. The quality of the lots before the inspection is known as **incoming quality** and the quality of the lots which have been accepted after the inspection is known as **outgoing quality**. In an acceptance sampling plan, the lots are either accepted or rejected. So the outgoing quality is the same as the incoming quality. However, in a rectifying sampling inspection plan, the rejected lots are rectified or screened. So the outgoing quality will differ from the incoming quality. Therefore, the concept of **average outgoing quality** (AOQ) is particularly useful for the evaluation of a rectifying sampling plan.

You have learnt in Sec.6.2 that the consumer takes a sample from each lot to check its quality. He/she inspects each item or unit of the sample for defects. If the number of defective units in the sample is less than the acceptance number, the lot is accepted by replacing all defective units found in the sample by non-defective units. If the number of defective units is greater than the acceptance number, each and every unit of the lot is inspected. It means that 100% inspection is carried out for each rejected lot and all defective units found in the lot are replaced by non-defective units. Therefore, these lots are accepted after 100% inspection with zero percent defective. As a result, the accepted stores will consist of lots of varying quality level, ranging from quality levels lower than acceptance quality level to lots with zero defective. Therefore, we need to define the concept **average outgoing quality**.

When all lots are considered together, their average quality level may be considerably different from the incoming quality. The average outgoing quality (AQL) is defined as follows:

The expected quality of the lots after the application of sampling inspection is called the **average outgoing quality**. It is calculated as follows:

$$\text{AOQ} = \frac{\text{Number of defective units in the lot after inspection}}{\text{Total number of units in the lot}} \qquad \ldots(1)$$

But we do not know the number of defective units in the entire lot after the inspection of samples. So we have an alternative way of calculating AOQ. If N is the size of each lot, n is the sample size inspected in each lot, p is the incoming quality of the lots and P_a is the probability of accepting the lot of incoming quality p, AOQ for single sampling plan is given by

$$\text{AOQ} = \frac{p(N-n)P_a}{N} \qquad \ldots(2)$$

We shall derive this formula in Unit 7. The concept of AOQ is shown in Fig. 6.2.

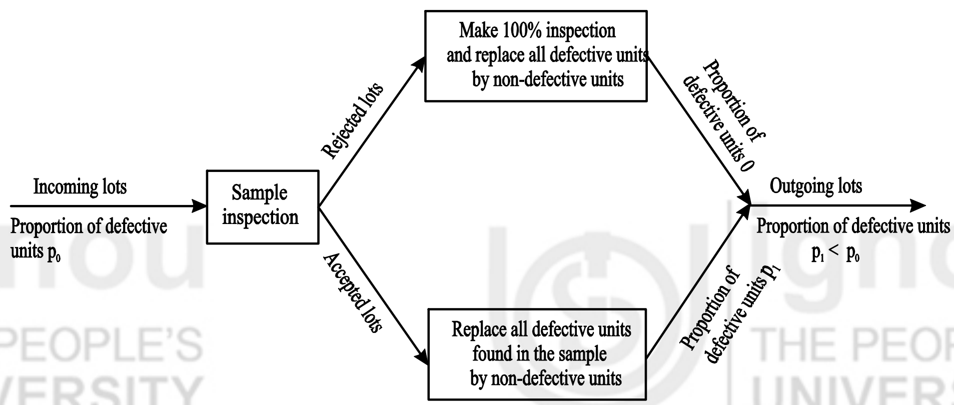

Fig. 6.2: Concept of average outgoing quality.

For the acceptance sampling plan in which rectification is not done, the AOQ is the same as the incoming quality.

If we draw a graph of AOQ versus the incoming quality, the curve so obtained is known as the **AOQ curve**. A typical AOQ curve is shown in Fig. 6.3.

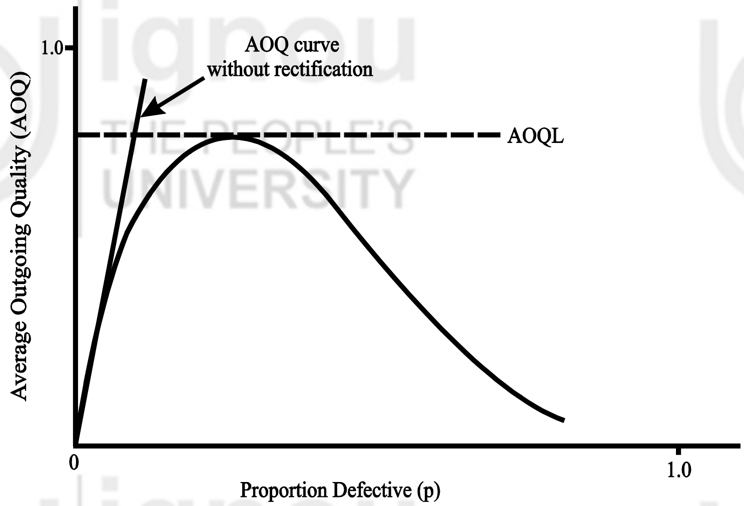

Fig. 6.3: The AOQ curve.

If we analyse the AOQ curve (shown in Fig. 6.3), we observe that for p = 0, the incoming lots have no defective units and hence there are no defective units in the outgoing lots. The consumer accepts all lots. So in this case, AOQ = 0. When the incoming quality is good, a large proportion of the lots will be accepted by the rectifying sampling plan and only a smaller fraction will be screened. Hence, the outgoing quality will be good. However, when the incoming quality is not good, a large proportion of the lots will be screened. In such cases, the outgoing quality will also be good because defective units or items will either be replaced or rectified. Between these extremes, the AOQ increases up to a maximum and then decreases. The maximum value of AOQ represents the worst possible average for the outgoing quality. It is known as the **average outgoing quality limit (AOQL)**. The AOQL tells us that no matter how poor the incoming quality is, on an average, the outgoing quality will never be poorer than AOQL.

Let us take up an example based on AOQ.

Example 2: Suppose a cricket ball manufacturing company formed lots of 500 balls. To check the quality of the lots, the buyer draws 20 balls from each lot and accepts the lot if the sample contains at most one defective ball. Otherwise, he/she rejects it. Suppose the quality of the submitted lots is 0.03. Calculate the AOQ for this sampling plan. If rejected lots are screened and all defective balls are replaced by non-defective ones, calculate the AOQ again.

Solution: It is given that

$$N = 500, n = 20, c = 1 \text{ and } p = 0.03$$

We know that in an acceptance sampling plan, the lots are either accepted or rejected and the average outgoing quality is the same as the incoming quality. Therefore, in the first case, when the lot is either accepted or rejected, the AOQ is the same as the quality of the submitted lot, i.e., AOQ = 0.03.

In the second case, when the rejected lots are screened and all defective balls are replaced by non-defective ones, we calculate the AOQ using equation (2).

For calculating the AOQ in this case, we have to calculate the probability of accepting a lot (P_a). Recall the method of fining it described in Sec. 5.5 of Unit 5.

If X represents the number of defective balls in the sample, the buyer accepts the lot if $X \le c$. Here c = 1. Therefore, the probability of accepting the lot is given by

$$P_a = P[X \le c = 1] = \sum_{x=0}^{1} P[X = x]$$

Since $N \ge 10n$, we can use the binomial distribution and Table I given at the end of this block to obtain P_a.

From Table I, for n = 20, x = c = 1 and p = 0.03, we have

$$P_a = P[X \le 1] = \sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.8802$$

On putting the values of N, n, p and P_a in equation (2), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.03 \times (500-20) \times 0.8802}{500} = 0.025$$

You may like to calculate the AOQ in the following exercises.

E2) Suppose, in Example 2, the quality of the submitted lots is 0.01, calculate AOQ for this sampling plan.

E3) Suppose, in E1, the quality of the submitted lot is 0.05, calculate AOQ for this sampling plan.

We now describe another feature of the sampling plan known as the operating characteristic curve.

6.4 OPERATING CHARACTERISTIC (OC) CURVE

You have learnt that in acceptance sampling, the lot is either rejected or accepted on the basis of conclusions drawn from the sample. Sometimes, a good lot may be rejected and a bad lot may be accepted because we infer the quality of all units in the lot on the basis of a sample (small part of the lot).

So we require an acceptance sampling plan which ensures that good lots are always accepted and the bad lots are always rejected. It means that the plan should perfectly discriminate between good and bad lots. Therefore, we introduce a tool called the **operating characteristic curve**, which measures the discriminating ability of a plan. The operating characteristic (OC) curve of an acceptance sampling plan reflects the ability of the plan to distinguish between good and bad lots.

The OC curve for a sampling plan is a graph of the probability of accepting the lot versus the proportion defective (proportion of defective units) in the lot. It is plotted by taking the proportion defective (p) along the X-axis and the probability of accepting the lot along the Y-axis.

When the acceptance sampling plan is used, there is a conflict of interest between the producer and the consumer. The producer wants that all acceptable lots to be accepted and the consumer wants that all unsatisfactory lots to be rejected. A sampling plan which perfectly discriminates between good and bad lots is called an **ideal sampling plan**. The OC curve for an ideal sampling plan is called an **ideal OC curve** and is shown in Fig. 6.4.

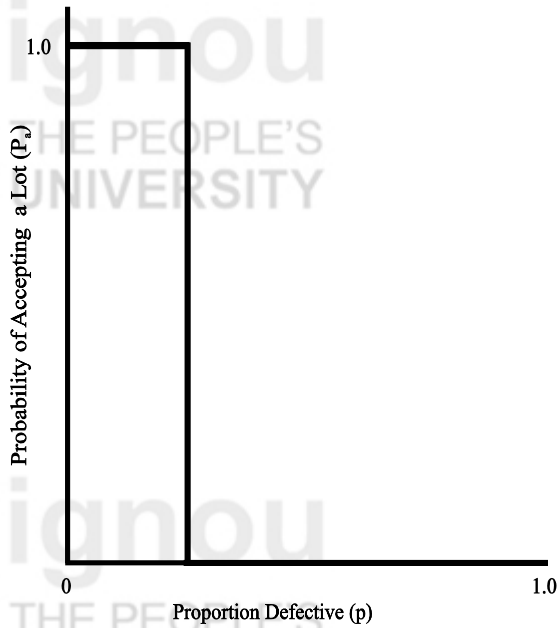

Fig. 6.4: Ideal OC curve.

The ideal OC curve runs horizontally at a probability of acceptance $P_a = 1.0$ until a level of lot quality that is considered **bad** is reached. At that point, the curve drops vertically to a probability of acceptance $P_a = 0$. Then the curve runs horizontally again for all lot proportion defectives greater than the undesirable level. If such a sampling plan could be employed, all lots of **bad** quality would be rejected and all lots of **good** quality would be accepted. But, the ideal OC curve can almost never be obtained in practice. In theory, it could be realised by 100% inspection, if the inspections were error free. But for 100% inspection, the cost is high and the time of inspection is much more. So we use sampling plans. The ideal OC curve shape can be approached by increasing sample size. A typical OC curve is shown in Fig. 6.5.

The OC curve shows the acceptance probability of a lot of certain quality. It represents the relationship between the probability of acceptance and the quality of the lot. If the lot has no defective units, it is certain to be accepted and if it has all defective units then it is equally certain to be rejected. Hence, the OC curve has to start with $P_a = 1$ when $p = 0$. As the proportion defective (p) of the lot increases, the probability P_a of lot acceptance decreases until it reaches 0. Thus, $P_a = 0$ when $p = 1$.

The OC curve for a particular combination of n and c shows how well the plan discriminates between good and bad lots. In Fig. 6.5, we have shown an OC curve for n = 100 and c = 2. In this plan, if 0, 1, or 2 defective units are found in the sample of size n = 100, the lot would be considered acceptable and if more than 2 defective units are found, it would be rejected. Suppose, the actual lot quality is 0.01, i.e., there are 1 percent defective units in the lot, the OC curve shows that the probability of accepting the lot is approximately 0.99. It means that this sampling plan accepts the lot about 99% of the time and rejects the lot about 1% of the time. However, if the actual quality of a lot is 0.05, i.e., there are 5 percent defective units, the OC curve shows that the probability of accepting the lot is approximately 0.19.

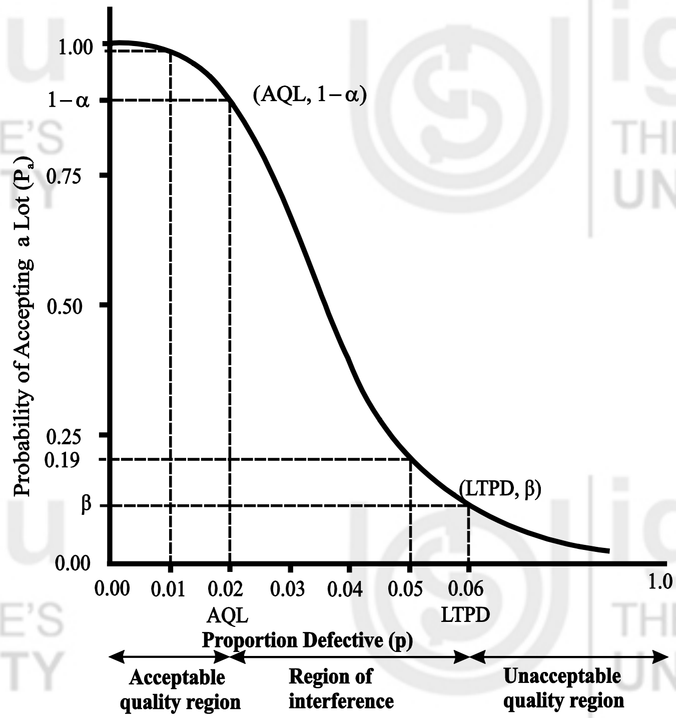

Fig. 6.5: the OC curve.

Hence, if the actual quality of a lot is good, the plan provides high probability of acceptance and if its quality is poor, the probability of acceptance of the lot is low. Thus, the OC curve shows how well a given plan discriminates between good and bad quality.

The OC curve of a sampling plan passes through two points agreed upon by the producer and the consumer. These points are (AQL, 1– α) and (LTPD, β) where the average quality level (AQL) and the lot tolerance percent defective (LTPD) are decided by the producer and the consumer after negotiation with each other. You know from Unit 5 that α and β are the producer's risk and the consumer's risk, respectively. Since the OC curve represents the probability of accepting a lot, instead of taking α (probability of rejecting a lot of quality AQL) we take 1 – α (probability of accepting a lot of quality AQL).

In practice, the performance of acceptance sampling plan for discriminating good and bad lots depends mainly on the sample size (n) taken from the lot and the acceptance number (c) that can be permitted in the sample. Since n and c are different in different sampling plans, the OC curve will be different for different sampling plans. Let us see how the sample size (n) and the acceptance sampling number (c) affect the OC curve.

Effect of increasing the sample size n on OC curve

Study Fig. 6.6. It shows the effect of increasing sample size on OC curve. Here we have three OC curves for three sampling plans with n = 100, c = 1; n = 200, c = 2 and n = 300, c = 3. From Fig. 6.6, we note that even though each plan uses the same percent defective which can be allowed for an acceptance lot (1/100, 2/200, 3/300 = 0.01, i.e.,1 percent), the OC curve becomes steeper and lies closer to the origin as the sample size increases.

The greater the slope of the OC curve, greater the discriminatory power.

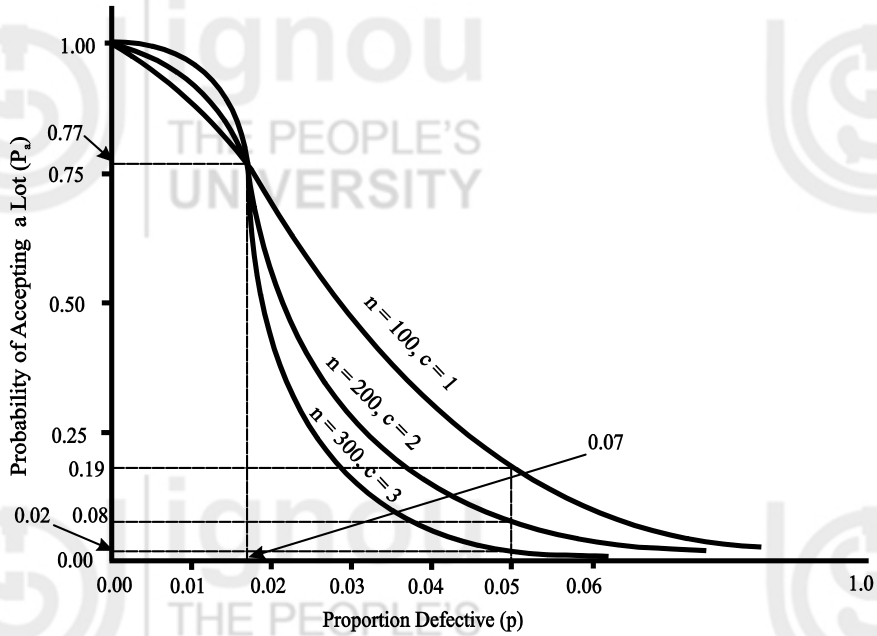

Fig. 6.6: The OC curves for n = 100, c = 1; n = 200, c = 2 and n = 300, c = 3.

If we compare the discrimination power of the three plans, we see that all three plans would accept lots with about 0.017, i.e., 1.7 percent defective units, about 77 percent of the time. However, if the actual quality falls to 5 percent defective units, the plan n = 100, c = 1 accepts the lot about 19 percent of the time, n = 200, c = 2 about 8 percent of the time and n = 300, c = 3 about 2 percent of the times. Therefore, the plans with larger sample sizes have definitely more discrimination power. It means that the plans with larger sample sizes are more effective in protecting the consumer against the acceptance of relatively bad lots and also give protection to the producer against rejection of relatively good lots.

Effect of decreasing the acceptance number (c) on OC curve

In Fig. 6.6, you have seen that the OC curve changes as the sample size changes. Generally, changing the acceptance number does not dramatically change the slope of the OC curve. As the acceptance number is decreased, the OC curve shifts to the left. For the same n, plans with smaller values of c provide discrimination at lower levels of the lot proportion defective than the plans with larger values of c. You can see this in Fig. 6.7, which shows the effect of acceptance number on the OC curve. It has three OC curves for n = 200 and c = 1, 2 and 3.

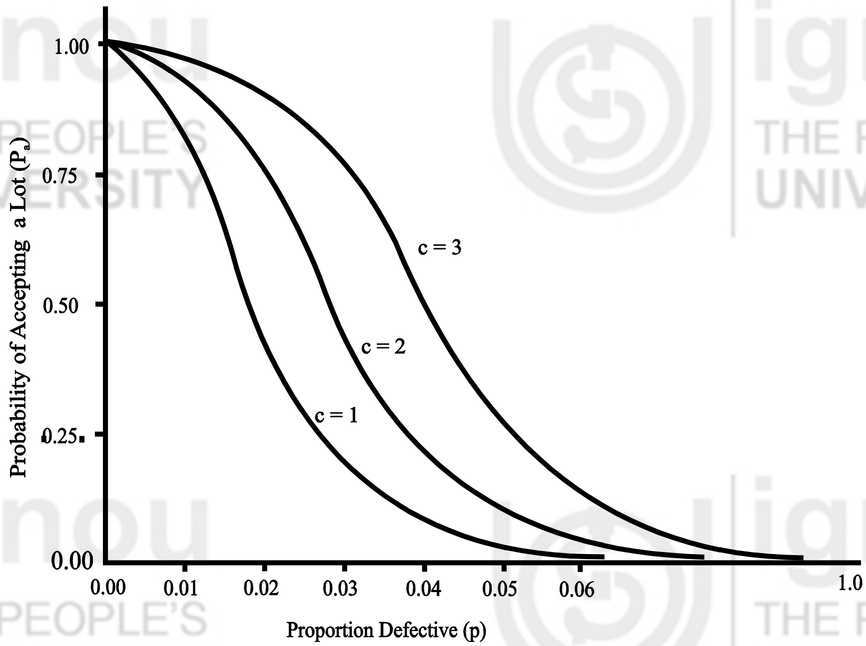

Fig. 6.7

Note that in Fig. 6.7, the effect of decreasing c is that the OC curve becomes steeper. Thus, when we decrease the acceptance number (c), the sampling plan becomes **tighter or stricter**. It means that the plan with smaller acceptance number has more discrimination power.

Now that you know what the OC curve of a sampling plan is and why it is used, let us summarize its properties.

Properties of the Operating Characteristic Curve

The OC curve possesses the following properties:

i) The OC curve of an acceptance sampling plan shows the ability of the plan to distinguish between good and bad lots.

ii) The OC curve with larger sample size is steeper. It means that the plan with larger sample size has more discrimination power. The plan with larger sample size protects the consumer against the acceptance of relatively bad lots and also gives protection to the producer against rejection of relatively good lots.

iii) The OC curve for a fixed sample size and smaller acceptance sample number is steeper. It means that the plan with a smaller acceptance number has more discrimination power.

iv) The OC curves with fixed sample sizes give constant quality protection, whereas the OC curves with relative sample sizes give different quality protections.

We now discuss another useful concept of sampling plans called the **average sample number (ASN)**.

6.5 AVERAGE SAMPLE NUMBER (ASN)

In acceptance sampling plan, the decision of acceptance or rejection of a lot is based on the information provided by the sample (s) drawn from the lot. So the average sample number can be defined as follows:

The **average sample number (ASN)** is defined as the average (expected) number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the **acceptance sampling plan**.

The curve drawn between the ASN and the lot quality (p) is known as the **ASN curve**.

In a single sampling plan, we take the decision of acceptance or rejection of the lot on the basis of only a single sample of size n. Hence, the ASN in a single sampling plan is simply the sample size n, which means that ASN is constant. Therefore, the ASN curve for a single sampling plan is a straight line as shown in Fig. 6.8.

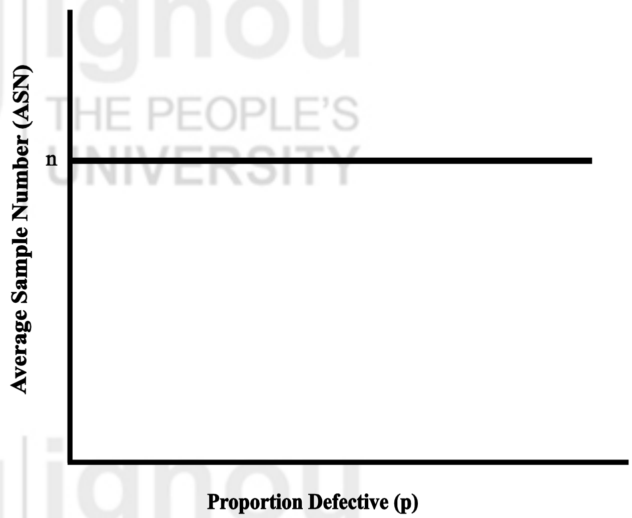

Fig. 6.8: The ASN curve for a single sampling plan.

ASN for a double sampling plan varies with the quality of the lot and is a function of the actual proportion defective (p) in the lot. We shall discuss the method of finding ASN and ASN curve for a double sampling plan in Unit 8.

The concept of average sample number is considered for the acceptance sampling plan. The average total inspection (ATI) plays the same role in a rectifying sampling plan.

6.6 AVERAGE TOTAL INSPECTION (ATI)

The average total inspection (ATI) for a rectifying sampling plan is defined as follows:

The average number of units inspected per lot under the rectifying sampling plan is called the average total inspection (ATI).

In other words,

The **average total inspection (ATI)** is the number of units inspected per lot to take the decision for acceptance or rejection of the lot under rectifying sampling plan calling for 100% inspection of the rejected lots.

If the lot quality is p, the average total inspection (ATI) for a single sampling plan is given by

$$ATI = n + (1 - P_a)(N - n) \quad \ldots(3)$$

We shall derive this formula in Unit 7.

Therefore,

ATI = ASN + (average number of units inspected in the rejected lots)

Thus, if the lot is accepted on the basis of the rectifying sampling plan, then

ATI = ASN

Otherwise,

ATI > ASN

If a lot contains no defective unit, it will obviously be accepted by the sampling plan and only n items will be inspected. Therefore, in this case ATI will be equal to the sample size n. If all units of the lot are defective, the lot will be rejected and 100% inspection of the lot will be called. Therefore, in this case the ATI will be equal to the lot size N. If the lot quality lies between 0 and 1, i.e., $0 < p < 1$, the ATI will lie between the sample size n and the lot size N. This means that ATI is a function of the lot quality p.

The curve drawn between ATI and the lot quality (p) is known as **ATI curve**. A typical ATI curve for a single sample plan is shown in Fig. 6.9 for N = 2000, n = 30 and c = 2.

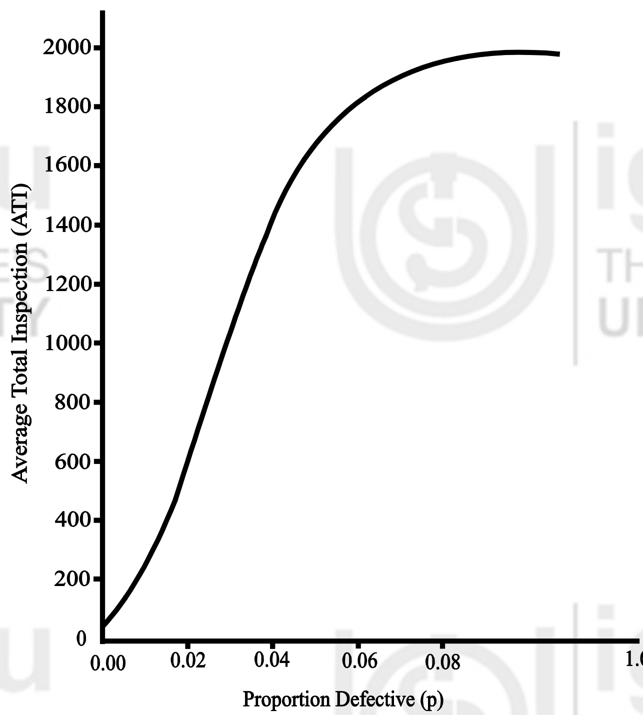

Fig. 6.9: The ATI curve.

We shall discuss the methods of obtaining ATI and ATI curves for single and double sampling plans in Units 7 and 8.

Let us now compute ASN and ATI for a real life situation.

Example 3: A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with n = 25 and c = 2 is being used for inspection. Calculate the average sample number (ASN). Suppose the incoming lots contain 5% defective units. Calculate the average total inspection (ATI) for this plan, if the rejected lots are 100% inspected and all defective syringes are replaced by non-defective syringes.

Solution: It is given that

$$N = 2000, n = 25, c = 2, p = 0.05$$

In the first case, the decision of acceptance or rejection of the lot is taken only on a single sample of size n = 25. Therefore, the ASN for this plan is simply the sample size, i.e., ASN = n = 25.

We calculate ATI using equation (3).

We first calculate the probability of accepting a lot of quality p = 0.05.

If X represents the number of defective injection syringes in the sample, the hospital accepts the lot if $X \le c = 2$. Therefore, the probability of accepting the lot is given by

$$P_a = P[X \le 2] = \sum_{x=0}^{2} P[X = x]$$

Since $N \ge 10n$, we can obtain this probability by using Table I.

From Table I, for n = 25, x = c = 2 and p = 0.05, we have

$$P_a = P[X \le 2] = \sum_{x=0}^{2} {}^{n}C_x \, p^x (1-p)^{n-x} = 0.8729$$

On putting the values of N, n and P_a in equation (3), we get

$$\begin{aligned} ATI &= n + (1 - P_a)(N - n) \\ &= 25 + (1 - 0.8729)(2000 - 25) \\ &= 25 + 251.02 = 276.02 \simeq 277 \end{aligned}$$

It is time for you to do the following exercises based on ASN and ATI.

E4) Explain the following terms:

i) Average sample number (ASN), and

ii) Average total inspection (ATI).

E5) A computer manufacturer purchases computer chips from a company in lots of 400. Twenty computer chips are sampled from each lot at random and inspected for defects. The computer manufacturer accepts the lot if inspected sample contains at most one defective chip. Otherwise, he rejects it. Suppose the incoming lots contain 3% defective chips. Calculate ASN for this plan. If rejected lots are screened and all defective computer chips are replaced by non-defective ones, calculate AOQ for this plan.

You have studied four curves, namely, AOQ, OC, ASN and ATI in Secs. 6.3 to 6.6. Their main purpose is to assess the efficiency and compare the costs of sampling plans. The AOQ and OC curves serve to assess protection of the consumer and the producer in terms of their risks given by the sampling plan and also give an idea of the efficiency of the sampling plan. The ASN and ATI curves give an indication of the cost of inspection. With the help of these curves, we arrive at a balance between the costs of inspection and the degree of protection. If inspection (sampling) costs are more than the costs of wrong decisions (without the aid of such inspection), then that particular inspection plan cannot be justified on economic considerations.

We now end this unit by giving a summary of what we have covered in it.

6.7 SUMMARY

1. An acceptance sampling plan in which rejected lots are 100% inspected is called a **rectifying sampling plan**.
2. The quality of the lots before inspection is known as **incoming quality** and the quality of the lots which have been accepted after the inspection is known as **outgoing quality**.
3. The expected quality of the lots after the application of sampling inspection is called **average outgoing quality (AOQ)** and is given by

$$AOQ = \frac{\text{Number of defective units in the lot after inspection}}{\text{Total number of units in the lot}}$$

4. The maximum value of AOQ represents the worst possible average for the outgoing quality and is known as the **average outgoing quality limit (AOQL)**.
5. The **OC curve** for a sampling plan is a graph of the probability of accepting the lot versus the proportion or fraction defective in the lot. It shows the ability of the plan to distinguish between good and bad lots.
6. A sampling plan which perfectly discriminates between good and bad lots is called an **ideal sampling plan** and the OC curve for this sampling plan is called an **ideal OC curve**.
7. The **precision** with which a sampling plan differentiates between good and bad lots increases with the **size of the sample**: The **greater the slope** of the OC curve, the **greater is the discrimination power** of the plan. The plan with a **smaller acceptance number** has **more discrimination power**.
8. The **average sample number (ASN)** is defined as the average (expected) number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under acceptance sampling plan. The curve drawn between ASN and lot quality (p) is known as the **ASN curve**.
9. The average number of units inspected per lot under the rectifying sampling plan is called the **average total inspection**. The curve drawn between ATI and lot quality (p) is known as the **ATI curve**.

6.8 SOLUTIONS/ANSWERS

E1) To check the quality of the lot, the quality inspector of the hospital draws 25 syringes randomly from each lot of disposable injection syringes and inspects each and every syringe of the sample. He/she classifies each syringe as defective or non-defective on the basis of certain defects. At the end of the inspection, he/she counts the number of defective syringes found in the sample and compares the number of defective syringes found in the sample with the acceptance number. If the number of defective syringes in the sample is greater than $c = 2$, say, 3, instead of rejecting the lot, he/she calls for 100% inspection of this lot and replaces all defective syringes found in the lot by non-defective syringes. Then he/she accepts the lot. If the number of defective syringes is less than or equal to c = 2, say, 1, he/she accepts the lot by replacing all defective syringes in the sample by non-defective syringes.

E2) It is given that

$$N = 500, n = 20, c = 1 \text{ and } p = 0.01$$

We calculate AOQ for the single sampling plan using equation (2). We first calculate P_a. Since $N \geq 10n$, we can obtain this probability by using Table I.

From Table I, for n = 20, x = c = 1 and p = 0.01, we have

$$P_a = P[X \leq 1] = \sum_{x=0}^{1} {}^nC_x\, p^x (1-p)^{n-x} = 0.9831$$

On putting the values of N, n, p and P_a in equation (2), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.01 \times (500-20) \times 0.9831}{500}$$

$$= \frac{4.7189}{500} = 0.009$$

E3) It is given that

$$N = 2000, n = 25, c = 2 \text{ and } p = 0.05$$

We calculate AOQ for the single sampling plan using equation (2).

Since here $N \geq 10n$, the probability of accepting the lot can be obtained as in E2.

$$\therefore\ P_a = P[X \leq 2] = \sum_{x=0}^{2} {}^nC_x\, p^x (1-p)^{n-x} = 0.8729$$

On putting the values of N, n, p and P_a in equation (2), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.05 \times (2000-25) \times 0.8729}{2000}$$

$$= \frac{86.1989}{2000} = 0.043$$

E4) Refer to Sec. 6.5 for (i) and Sec. 6.6 for (ii).

E5) In this sampling plan, the decision of acceptance or rejection of the lot is taken only using a single sample of size n = 20. Therefore, the ASN for this plan is simply the sample size n = 20.

It is given that

$$N = 400, n = 20, c = 1, p = 3\% = 0.03$$

We calculate ATI for the single sampling plan using equation (3).

Since $N \geq 10n$, we can obtain the probability of accepting a lot of quality p = 3% = 0.03 using Table I.

From Table I, for n = 20, x = c = 1 and p = 0.03, we have

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.8802$$

On putting the values of N, n and P_a in equation (3) we get

$$\begin{aligned} ATI &= n + (1 - P_a)(N - n) \\ &= 20 + (1 - 0.8802)(400 - 20) \\ &= 20 + 45.524 = 65.524 \approx 66 \end{aligned}$$