UNIT 13 TREND COMPONENT ANALYSIS

Structure

13.1 INTRODUCTION

In the previous block, we have discussed simple and multiple linear regression analysis where we have dealt with bivariate as well as multivariate data. While studying that block, you have learnt how useful regression analysis is in decision making. If we carefully analyse most decisions and actions of the Government, an institution, an industrial organisation or an individual, we find that, to a large extent, these depend on the situations expected to arise in future. For example, suppose the Delhi Government wishes to frame a housing development policy for providing houses to all families of the central government employees in Delhi over the next five years. Then the Government would like to know: What would the number of families of government employees in Delhi be in the next five years? A similar assessment is required while formulating the employment policy, and so on.

Planning for future is an essential aspect of managing an organisation. This requires that we should be able to **forecast** the future requirements of that organisation. For example, suppose we are asked to provide quarterly forecasts of the sales volume for a particular product during the coming one year period. Our forecasts for sales would also affect production schedules, raw material requirements, inventory policies, and sales quota. A good forecast of the future requirements will result in good planning. A poor forecast results in poor planning and may result in increased cost. In order to provide such forecasts, we use historical data of the past few years to assess the average requirement, trend (if any) over the years and seasonal variations. Based on these features observed from the past data, we try to understand their role in causing variability and use them for forecasting requirements.

This exercise is done with the help of **time series analysis** which is a collection of observations made sequentially over a period of time. **The main objectives of time series analysis are description, explanation and forecasting.** It has applications in many fields including economics, engineering, meteorology, etc.

In this unit, we discuss the concept of time series and explain different types of time series in Sec. 13.2. In Secs. 13.3 and 13.4, we describe different components and basic models of time series. We explain the methods of smoothing and filtering the time series data along with the estimation of trend by the curve fitting and curvilinear methods in Secs. 13.5 and 13.6. Finally, in Sec. 13.7, we describe the methods of measurement of trend and cyclic effect in time series data.

In the next unit, we shall discuss some methods for estimating the seasonal component (S). We shall also discuss the method of estimating the trend component from deseasonalised time series data.

Objectives

After studying this unit, you should be able to:

- explain the concept of time series;
- describe the components of time series;
- explain the basic models of time series;
- decompose a time series into its different components for further analysis;
- describe the trend component of the time series;
- describe different types of trends;
- explain various methods for smoothing time series and estimation of trends; and
- describe the centred moving average method of measuring the trend effect.

13.2 INTRODUCTION TO TIME SERIES

Generally it is seen that forecasting involves studying the behaviour of a characteristic over time and examining data for any pattern. The forecasts are made by assuming that, in future, the characteristic will continue to behave according to the same pattern. The data gathered could be of sales per day, units of productions per week, the running cost of a machine per month, etc.

A time series (TS) is a collection of observations made sequentially over a period of time. In other words, the data on any characteristic collected with respect to time over a span of time is called a time series. Normally, we assume that observations are available at equal intervals of time, e.g., on an hourly, daily, monthly or yearly basis. Some time series cover a period of several years.

The methods of analysing time series constitute an important area of study in statistics. But before we discuss time series analysis, we would like to show you the plots of some time series from different fields. In the next three sub-sections, we look at the plots of **three time series**, namely, **time series with trend effect**, **time series with seasonal effect** and **time series with cyclic effect**. These plots are called **time plots**.

13.2.1 Time Series with Trend Effect

A **trend** is a long term smooth variation (increase or decrease) in the time series. When values in a time series are plotted in a graph and, on an average, these values show an increasing or decreasing trend over a long period of time, the time series is called the **time series with trend effect**. You should note that all time series do not show an increasing or decreasing trend. In some cases, the values of the time series fluctuate around a constant reading and do not show any trend with respect to time. You should also remember that an increase or decrease may not necessarily be *in the same direction* throughout the given period. Time series may show an upward trend, a downward trend or have no trend at all. Let us explain all three cases with the help of examples.

Time Series with Upward Trend

When values in a time series are plotted in a graph and these show an upward trend with respect to time, we call it a **time series with upward trend**. For example, the profit of a company plotted for the time period 1981 to 2012 in Fig. 13.1 shows an upward trend.

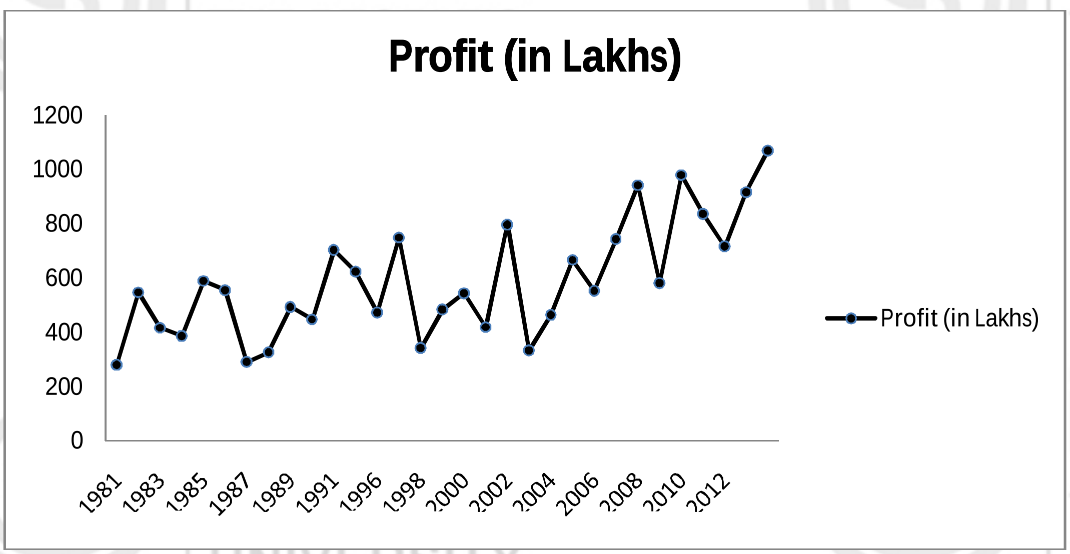


Fig. 13.1: Profit of a company from 1981 to 2012.

Time Series with Downward Trend

If data in a time series plotted in a graph show a downward trend with respect to time, it is called the **time series with downward trend**. For example, the values of mortality rate of a developing country from 1991 to 2009 show a downward trend in the time plot given in Fig. 13.2.

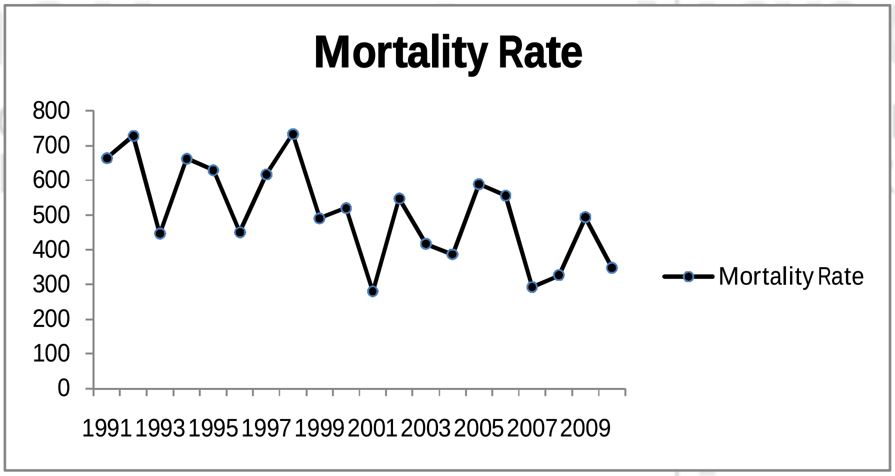


Fig. 13.2: Mortality rates of a developing country from 1991 to 2009.

Time Series with No Trend

If data of a time series is plotted on the graph paper and does not show any trend, that is, neither an upward nor a downward trend is reflected in the time plot, the time series is called the **time series with no trend**. For example, Fig. 13.3 shows the time plot of the production of a commodity in a factory from 1988 to 2012. Notice that the time series shows no trend.

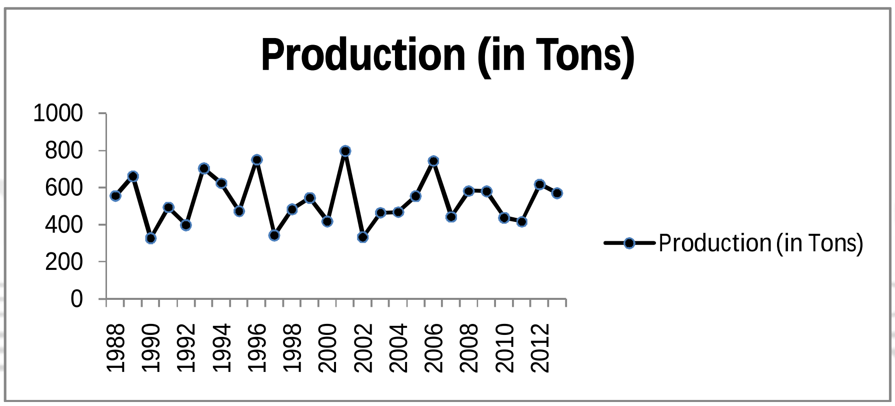
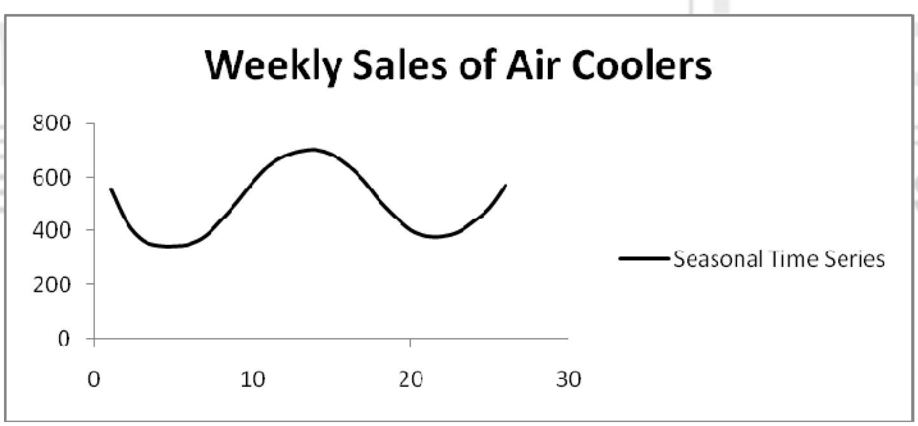


Fig. 13.3: Production of a commodity from 1988 to 2012.

13.2.2 Time Series with Seasonal Effect

If values in a time series reflect seasonal variation with respect to a given period of time such as a quarter, a month or a year, the time series is called a **time series with seasonal effect**. For example, the time plot of the data of weekly sales of air coolers shows a seasonal effect (Fig. 13.4).



13.2.3 Time Series with Cyclic Effect

If the time plot of data in a time series exhibits a cyclic trend, the time series is called a **time series with cyclic effect**. For example, time series data of the number of employees in software industry in different phases, i.e., phases of prosperity, recession, depression and recovery shows a cyclic pattern, that is, the pattern repeats itself over an almost fixed period of time (Fig. 13.5).

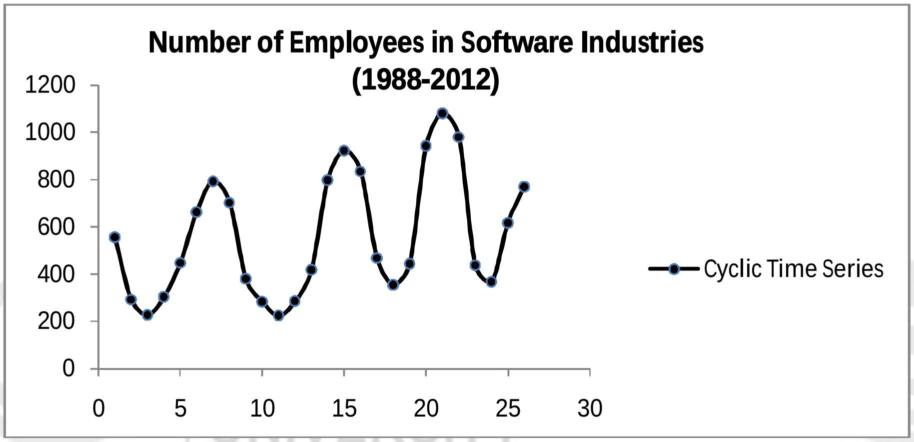


Fig. 13.5: Number of employees in software industries for the last 25 years.

So far, you have learnt about different types of time series plots which exhibit different trends in data. These trends arise due to the effect of various factors on the variations in data. The variations in the values or data are also described in terms of components of time series. Let us learn about them.

13.3 COMPONENTS OF TIME SERIES

In Sec. 13.2, you have learnt that the variations in the time series values are of different types and arise due to a variety of factors. These different types of variations in the values of the data in a time series are also called **components** of the time series.

In the past, time series analysis was mainly concerned with decomposing the variations in a time series into **components** representing **(i) Trend (ii) Seasonal (iii) Cyclic** and **(iv) Remaining variations** attributed to **Irregular fluctuations (sometimes referred to as the Random component)**. This approach is not necessarily the best one and we shall discuss the modern approach in later units of this block. Usually, some or all components may be present in a time series in varying amounts and can be classified in the above-mentioned four categories. For the sake of completeness, we discuss these components in some detail.

13.3.1 Trend Component (T)

Usually time series data show random variation, but over a long period of time, there may be a gradual shift in the mean level to a higher or a lower level. This gradual shift in the level of time series is known as the **trend**. In other words, **the general tendency of values of the data to increase or decrease during a long period of time is called the trend**. Some time series show an upward trend while some show a downward trend as you have learnt in the previous section. For example, upward trends are seen in the data of population growth, currency in circulation, etc., while data of that some time series may not show any trend at all. The shifting in level is usually the result of changes in the population, demographic characteristic of the population, technology, consumer preferences, purchasing power of the population, and so on.

You should clearly understand that **a trend is a general, smooth, long term and average tendency of a time series data**. The increase or decrease may not necessarily be in the same direction throughout the given period. A time series may show a linear or a nonlinear (curvilinear) trend. If the time series data are plotted on a graph and the points on the graph cluster more or less around a straight line, the tendency shown by the data is called **linear trend in time series**. But if the points plotted on the graph do not cluster more or less around a straight line, the tendency shown by the data is called **nonlinear or curvilinear trend**. Trend need not always be a straight line. It can be quadratic, exponential or may not be present at all.

Trend is also known as long term variation. However, do understand that long term or long period of time is a relative term which cannot be defined uniformly. In some situations, a period of one week may be fairly long while in others, a period of 2 years may not be long enough.

13.3.2 Seasonal Component (S)

In a time series, variations which occur due to rhythmic or natural forces/factors and operate in a regular and periodic manner over a span of **less than or equal to one year** are termed as **seasonal variations**. Although we generally think of seasonal movement in time series as occurring over one year, it can also represent any regularly repeating pattern that is **less than one year** in duration. For example, daily traffic volume data show seasonal behaviour within the same day, with peak level occurring during rush hours, moderate flow during the rest of the day, and light flow from midnight to early morning. Thus, in a time series, seasonal variations may exist if data are recorded on a quarterly, monthly, daily or hourly basis.

However, do remember this point: **Although the data may be recorded over a span of three months, one month, a week or a day, the total period should be one year to assess seasonal variation properly**. Note that the amplitudes of the seasonal variation are different for different spans of time over which data is recorded (quarterly, monthly, daily, etc.). Most of the time series data in the fields of economics or business show a seasonal pattern.

The seasonal pattern in a time series may be either due to natural forces or manmade conventions. Variations in time series that arise due to changes in seasons or weather conditions and climatic changes are known as **seasonal variations due to natural forces**. For example, the sales of umbrellas, rain coats and gumboots pick up very fast in the rainy season; the demand for air conditioners goes up in summers; the sale of woollens goes up in winter. All these arise due to natural factors. Variations in time series that arise due to changes in habits, fashions, customs and conventions of people in any society are termed as **seasonal variations from manmade conventions**. For example, in our country, the sale of gold and silver ornaments goes up in Diwali, Dussehra (Durga Puja) or the marriage season.

13.3.3 Cyclic Component (C)

Apart from seasonal effects, some time series exhibit variation in a fixed period of time due to other physical causes. For example, economic data are sometimes thought to be affected by business cycles with a period varying from three to ten years (see Fig. 13.5). The cycles could be caused by period of moderate inflation followed by a period of high inflation. However, the existence of such business cycles leads to some confusion about cyclic, trend and seasonal effects. To avoid this confusion, **we shall term a pattern in the time series as cyclic component only when its duration is longer than one year.**

The cyclic variations in a time series are usually called "business cycle" and comprise four phases of a business, namely, prosperity (boom), recession, depression and recovery. These are normally over a span of seven to eleven years. Thus, **the oscillatory variations with a period of oscillation of more than one year are called cyclic variations or the cyclic component in a time series**. **One oscillation period is called one cycle**.

13.3.4 Irregular Component (I)

The long term variations, i.e., the trend component and short term variations, i.e., the seasonal and cyclic component are known as **regular variations**. Apart from these regular variations, **random or irregular variations**, which are not accounted for by trend, seasonal or cyclic components, exist in almost all time series. In most cases, these irregular variations are random, irregular and unpredictable and are caused by short-term, unanticipated and non-recurring factors that affect time series in some cases. In Unit 16, we shall model the irregular component using probability models such as auto-regressive (AR) and moving average (MA) models.

So far, you have learnt about four components of time series. These components may be present individually or jointly in any time series.

We now take up an example of a time series, which has both trend and seasonal components. Consider the quarterly sales data of washing machines for the period 2001-2007 given in Table 1.

Table 1: **Quarterly sales of washing machines for the period 2001-2007**

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2001	556	662	327	494
2002	398	704	624	473
2003	750	343	484	545
2004	419	798	334	465
2005	468	554	744	443
2006	582	581	437	417
2007	618	571	517	754

Fig. 13.6 shows the plot of this quarterly data. Note that there are 28 quarters from the year 2001 to 2007 and so 28 values are plotted in Fig. 13.6. These have been numbered from 1 onwards on the horizontal axis. We have connected the data points by a dotted curve to obtain a time series plot of the data. From the plot, we note that the values exhibit an upward linear trend over the long term. We show this trend by a thin straight line in Fig. 13.6. So the thin straight line in Fig. 13.6 reflects the presence of a long-term linear trend. We also notice a seasonal variation in the data, which

we show by a smooth thick free-hand curve. This thick curve shows the approximate movement around the straight trend line.

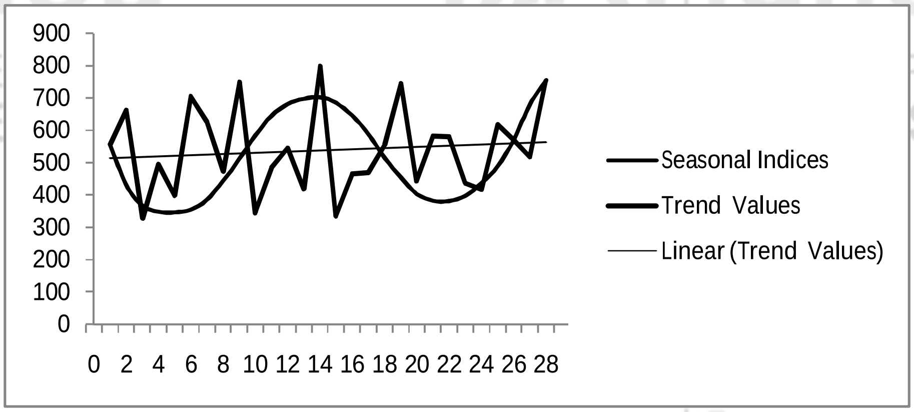


Fig. 13.6: Trend and seasonal variation in quarterly sales of washing machine.

In Fig. 13.6, in most years, the first quarter is a low point and then there is a rise in the second quarter, a decline in the third quarter and a rise in the fourth quarter. This could be due to changes in seasons and festival offers.

13.4 BASIC MODELS OF TIME SERIES

In Sec. 13.3, we have discussed different components of time series with examples. You have learnt of many factors (natural and manmade) that affect a time series. We would now like to describe the effect of these factors and the components of a time series mathematically. In this section, we discuss two commonly used mathematical models, which explain time series data reasonably well. While discussing these models, we shall use the notation y_t for the value of the time series at time t. We shall use serial numbering for time t since all time series are in chronological order. We now describe the two basic time series models, namely, the additive model and the multiplicative model.

13.4.1 Additive Model

The **additive model** is one of the most widely used models. It is based on the assumption that at any time t, the time series value Y_t is the sum of all the components. According to the additive model, a time series can be expressed as

$$Y_t = T_t + C_t + S_t + I_t$$

where T_t, C_t, S_t and I_t are the trend, cyclic, seasonal and irregular variations, respectively, at time t. In this model, we make the following assumptions:

- cyclic effects remain constant for all cycles;
- seasonal effects remain constant during any year or the corresponding period; and
- I_t is an i.i.d. normal variable with mean 0, i.e., the effect of irregular variation remains constant throughout. This implies that the term S_t does not appear in a time series of annual data.

The additive model implies that seasonal variations in different years, cyclic variations in different cycles and irregular variations in different trends show equal absolute effects irrespective of the trend value. In the previous sections, you have learnt that all four components need not necessarily be exhibited in every time series. For example, the time series of annual production data of a yield does not have seasonal variations. Similarly, a time series for the annual rainfall does not contain cyclic variations.

In additive model, we have assumed that the time series is the sum of the trend, cyclic, seasonal and irregular components. Generally, the additive model is appropriate when seasonal variations do not depend on the trend of the time series. However, there are a number of situations where the seasonal variations exhibit an increasing or decreasing trend over time. In such cases, we use the multiplicative model.

13.4.2 Multiplicative Model

When seasonal variations exhibit any change over time in terms of an increasing or decreasing trend, we can use the **multiplicative model** to describe the time series data. The multiplicative model is appropriate if various components in a time series operate proportionately to the general level of the series. The multiplicative model is based on the assumption that the time series value Y_t at time t is the product of the trend, cyclic, seasonal and irregular component of the series:

$$Y_t = T_t \times C_t \times S_t \times I_t$$

where T_t, C_t, S_t and I_t denote the trend, cyclic, seasonal and irregular variations, respectively. The multiplicative model is found to be appropriate for many business and economic data. Some examples are the time series for production of electricity, time series for number of passengers opting for air travel, time series for sales of soft drinks.

For estimation of any trend in a time series, smoothing (or filtering) the effect of irregular fluctuations present in it becomes important so that trend and seasonal effects may be easily estimated. We now discuss the smoothing or filtering of time series.

13.5 SMOOTHING OR FILTERING TIME SERIES

In this section, we discuss the methods of moving averages and exponential smoothing for smoothing or filtering the time series data. There are two methods of moving averages: the **equal weight** or **simple moving average method** and **the weighted (unequal) moving average method**. We discuss these two moving average methods in the next two sub-sections.

13.5.1 Equal Weight (Simple) Moving Average (MA) Method

In this method, we find the simple moving averages of time series data over m periods of time, called **m-period moving averages**. You can calculate them in the following way:

1. Calculate the average of the first m values of the time series.
2. Then discard the first value and take the average of the next m values again.
3. Repeat this process till all data are exhausted.

These steps yield a new time series of **m-period moving averages**.

Let us explain this method with the help of an example.

Example 1: Compute the 3-year simple moving averages for the time series of annual output of a factory for the period 2006-2011 given in Table 2.

Table 2: Annual output of a factory from 1976 to 1981

Year	1976	1977	1978	1979	1980	1981
Output (in thousands)	17	22	18	26	16	27

Solution: In this case, m = 3 years. The first value of the moving averages for m = 3 years is the average of 17, 22 and 18, which is 19. The second value of moving averages is obtained by discarding the first value, i.e., 17 and taking the average of the next 3 values in the time series, i.e., 22, 18 and 26. So we take the average of 22, 18 and 26, which is 22. Again, we discard the first value, i.e., 22 and take the average of the next 3 values in the time series, i.e., 18, 26 and 16. It is 20. We repeat this procedure for calculating the remaining 3-year moving averages. Table 3 gives the 3-year simple moving averages for the data given in Table 2. Note that each moving average is tabulated at the average (centre) of the time period for which it is computed. This method is, therefore, called the **centred moving averages**.

Table 3: Centred simple moving averages for time series data of Table 2

Year	1976	1977	1978	1979	1980	1981
Output (in thousands)	17	22	18	26	16	27
Moving Averages	-	19	22	20	23	-

Remember that moving averages vary less than the data values from which they are calculated as they smooth (or filter) out the effect of irregular component. This helps us in appreciating the effect of trend more clearly. For the given data the original time series varies between 16 and 27 whereas the moving averages vary between 19 and 23, which is much smoother than the original series. Fig.13.7 shows the output of the original time series and the 3-year moving averages series. In this figure, you can clearly see the smoothing property of moving averages.

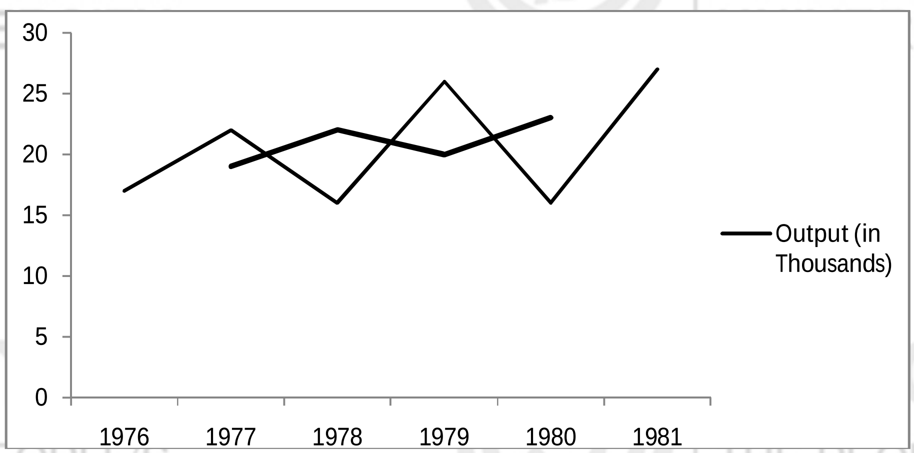


Fig. 13.7: Three-year centred moving average of the time series.

You may ask: What should the value of m be? If m is increased, the series becomes much smoother and it may also smooth out the effect of cyclical and seasonal components, which are our main interest of study. Sometimes 3-year, 5-year or 7-year moving averages are used to expose the combined trend and cyclical movement of time series. But as we shall see in Sec. 13.6, four quarter or 12-months moving averages are more useful for estimating

13.5.2 Weighted (Unequal) Moving Average Method

The simple moving average method described in Sec.13.5.1 is not generally recommended for measuring trend although it can be useful for removing seasonal variation. It may also not lie close to the latest values. Therefore, the weighted (unequal) moving averages method is used. In this method, instead of giving equal weights to all values, unequal weights are given in such a way that all the weights are positive and their sum is equal to 1. If w_i denotes the weight of the i^{th} observation, the weighted moving average value y_t is given by

$$y_t = \sum_{i=-q}^{q} w_i x_{t+i} \qquad w_i \geq 0, \ \sum_{i=-q}^{q} w_i = 1 \qquad \ldots (1)$$

where x_t is the original series.

Simple moving average becomes a particular case of weighted moving average for

$$m = 2q+1 \text{ and } w_i = \frac{1}{(2q+1)}$$

In Example 5, q =1, m = 2q+1 = 3 and w_i = 1/(2q+1) = 1/3.

Let us consider an example to illustrate this method.

Example 2: Compute the 3-years weighted moving averages for the time series given in Example 1.

Solution: Generally, the most recent observation receives the largest weight and the weights decrease for older data values. For the data given in Example 1, suppose we give 3 times more weight to the most recent observation than the first observation and 2 times more weight to the second observation. Then the weights for each year in every 3-year period are:

w_1=1/6, w_2=2/6, w_3=3/6

After assigning the weights as above, we get the smoothened trend values as

$$y_t = (x_{t-1} + 2x_t + 3x_{t+1}) / 6 \qquad \ldots (2)$$

The procedure for getting the smoothened trend values by weighted moving average method is as follows:

1. Calculate the weighted average of the first m values given in time series as explained above.
2. Now discard the first value and include the next one and take the average of the next m values by following the weight structure given in equation (2).
3. Repeat this process till all values are exhausted.

These steps yield a new time series of m-period weighted moving averages and the weighted moving average values are given in the following table:

Table 4: Weighted moving averages for time series data of Table 2

Year	1976	1977	1978	1979	1980	1981
Output (in thousands)	17	22	18	26	16	27
Weighted Moving Averages	-	19.16	22.66	19.66	23.16	-

The reason for giving larger weight to the latest observation rather than the earlier observations is that the latest observation is a better predictor of the trend than the earlier one. However, this may not be true for the cases in which the data contains a very large irregular component.

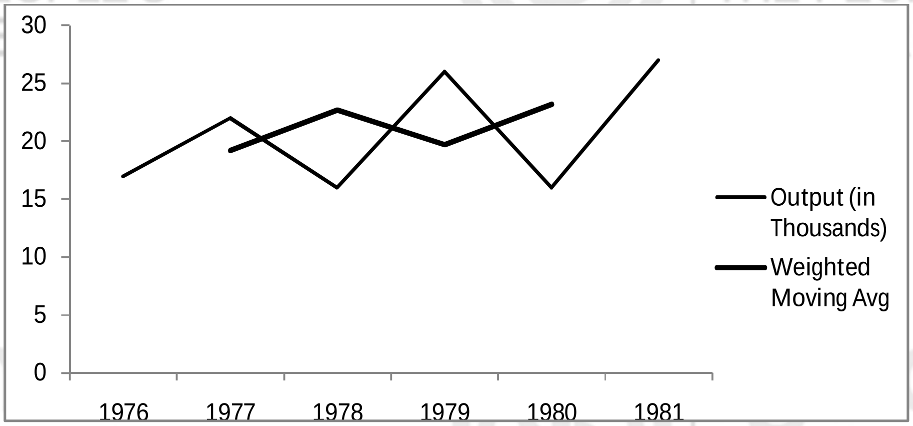


Fig. 13.8: Three-year weighted moving average of the time series.

13.5.3 Exponential Smoothing

We now discuss another technique of smoothing the time series, namely, the **exponential smoothing** technique, which is very popular. In Secs. 13.5.1 and 13.5.2, you have learnt that in the method of moving averages, weights (equal or unequal) are attached to each time series value that is considered. But in this method, the weights assigned to current as well as past time series values are different fixed positive numbers.

In this smoothing technique, weights decrease exponentially, except the last one, and the trend value is given by

$$y'_t = wy_t + (1-w)\ y'_{t-1} \qquad \text{for } t = 1, 2, \ldots \qquad \ldots(3)$$

where $0 < w < 1$. The value of 'w' is chosen as per the requirements. Equation (3) gives weighted average of $y_1, y_2, \ldots, y_t$ for calculating moving average series y'_t. You can see that the latest observation y'_{t-1} gets the maximum weight and then weights decrease exponentially. Note that the sum of weights is equal to one. This is a very popular technique for forecasting purposes. Let us take up an example to explain this method.

Example 3: Find the forecast for the time series given in Example 1.

Solution: On plotting the given data, it appears that there is no trend in the time series.

This is the same as saying that if $T = a + bt$ is the trend then $b = 0$. Therefore, if we fit a straight line to the given data with $b = 0$, the least squares estimate of a would be the simple average of the time series values, i.e.,

$$\hat{a} = \frac{\sum y_t}{6} = 20.66$$

In this way, the forecast for output for all t would be $\hat{a}$ as the model is

$$T = a + (\text{forecast error})$$

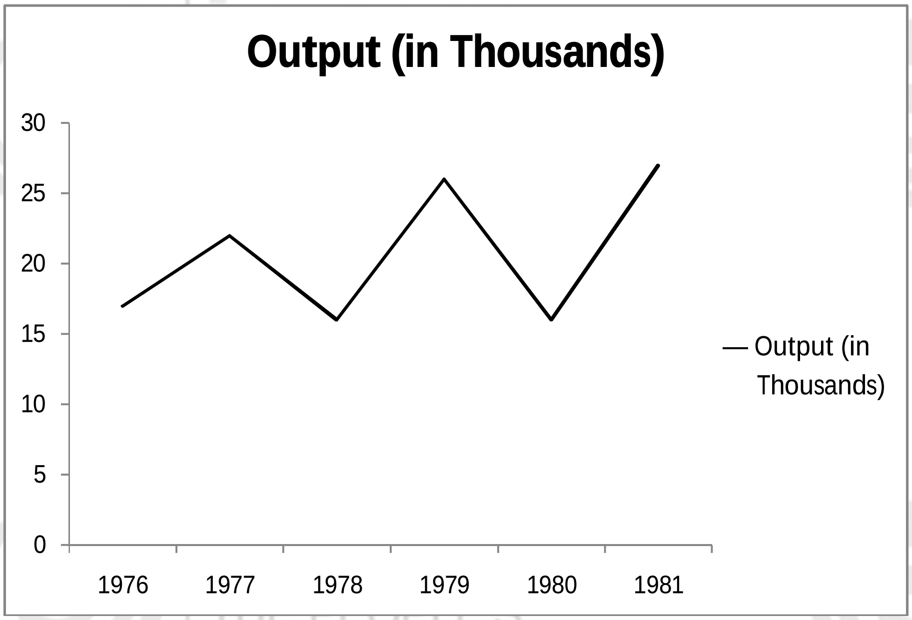


Fig. 13.9: Original time series values output (in thousands).

The result seems somewhat unreasonable because even though there is no trend, the time series values are not the same at all times t. The values of $\hat{a}$ seem to be higher in the year 2007, 09 and 2011 than the values in the year 2006, 08 and 2010. Therefore, it is logical to assume that the value of $\hat{a}$ is gradually changing over time. Therefore, the value of $\hat{a}$ should be denoted by $\hat{a}_t$ rather than '$\hat{a}$'.

According to this smoothing technique, we select a single weight w which is called exponential smoothing constant, where w lies between 0 and 1 and there is a method of choosing this constant. We can compute an exponential smoothing series y'_t as follows:

$$y'_1 = wy_1 + (1-w)\,y_1 = y_1$$

$$y'_2 = w\,y_2 + (1-w)\;y'_1$$

$$y'_3 = w\,y_3 + (1-w)\;y'_2$$

. .

. .

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$$y'_t = w\,y_t + (1-w)\;y'_{t-1}$$

We should know the value of y_1 for the computation of $y'_1, y'_2, y'_3 \ldots$

According to this technique, we also select a single weight w, which lies between 0 and 1. To start with, let us take w = 0.01. Then from equation (3), we get

$$y'_1 = 0.01\,y_1 + 0.99\,y_1 = 0.01(17) + (0.99)(17) = 17$$

Our forecast for y_1 at t = 0 is y'_1. Therefore, the forecast error is

$$e_1 = y_1 - y'_1 = 17 - 17 = 0.00$$

We get the second forecast value as follows:

$$y'_2 = 0.01\,y_2 + 0.99\,y'_1 = 0.01(22) + (0.99)(17) = 17.05$$

The forecast error at t = 2 is

$$e_2 = y_2 - y'_2 = 22 - 17.05 = 4.95$$

Proceeding in this way, we calculate the forecast value y'_t and forecast error e_t for all t. These are calculated and given in the following table:

Table 5: Forecast values and forecast errors for time series given in Table 2

Year	Output (in thousand) y_t	Forecasts y'_t	Forecast errors e_t
1976	17	17.00	+0.00
1977	22	17.05	+4.95
1978	18	17.06	+0.94
1979	26	17.15	+8.85
1980	16	17.14	−1.14
1981	27	17.24	+9.76

After calculating the forecasts y'_t and errors e_t for all t, we plot the forecast values with the original time series values (Fig. 13.10). The graph of time series shows that there are no peaks in the time series after smoothing.

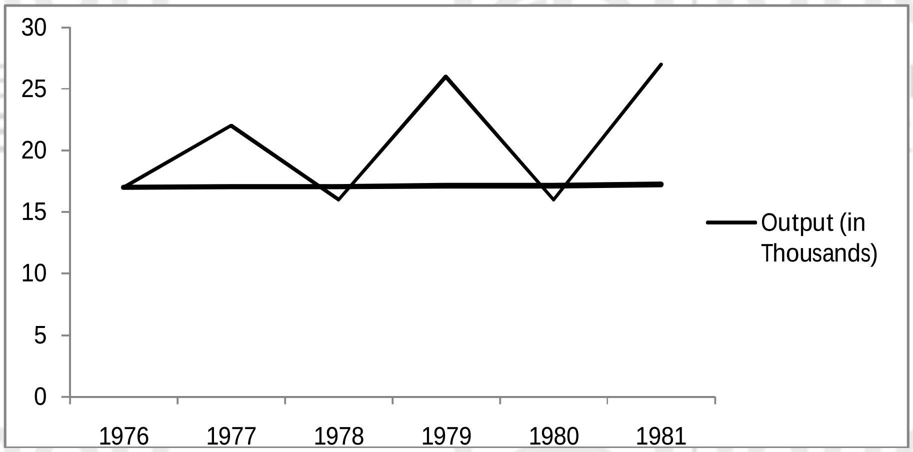

Fig. 13.10: Original and smoothed time series values of output (in thousands).

You should try to solve a few problems to check your understanding of the concepts discussed so far.

E1) Calculate the last two values of 3-year moving average for the data given in Example 1.

E2) Use exponential smoothing to obtain filtered values for the data given in Example 1 taking w = 0.5 and compare them with simple moving average values obtained in the same example.

E3) Obtain filtered values for the following data using exponential smoothing:

Year	Rainfall (in cm)	Year	Rainfall (in cm)	Year	Rainfall (in cm)	Year	Rainfall (in cm)
1970	664	1981	548	1991	624	2001	468
1971	728	1982	417	1992	473	2002	554
1972	447	1983	387	1993	750	2003	744
1973	663	1984	590	1994	343	2004	943
1974	630	1985	556	1995	484	2005	582
1975	451	1986	292	1996	545	2006	581
1976	617	1987	327	1997	419	2007	437
1977	734	1988	494	1998	798	2008	417
1978	491	1989	448	1999	334	2009	617
1979	520	1990	704	2000	465	2010	571
1980	280						

13.6 ESTIMATION OF TRENDS BY CURVE FITTING

An alternative approach to smoothing is to fit a polynomial to the data. This treats smoothing as a regression problem in which y_t is the trend value and integral powers of time t are the explanatory variables. The resulting smooth function is a polynomial

$$y_t = \sum_{j=0}^{p} b_j t^j \qquad \ldots(4)$$

where b_j is the j^{th} coefficient of the polynomial of degree p. The coefficients of the polynomial are estimated by the method of least squares by minimising the quantity

$$E = \sum_{i=1}^{n}\left(y_i - \sum_{j=0}^{p} b_j t_i^j\right)^2 \qquad \ldots(5)$$

We shall not discuss it here in detail as you have already studied the method of least squares and curve fitting in Unit 5 of MST-002. In the next section, we shall discuss the case when p = 1, i.e., the case of a linear trend equation.

13.6.1 Fitting a Linear Trend Equation

The trend equation given in equation (4) would be the special case of linear equation for p =1 and we get the equation of the straight line as

$$y_t = b_0 + b_1 t \qquad \ldots(6)$$

We can write it as

$$y_t = b_0 + b_1\bar{t} + b_1\left(t - \bar{t}\right) \qquad \ldots(7)$$

To simplify the calculations, we take $x_t = t - \bar{t}$, where $\bar{t}$ is the mean of all times t.

Equation (6) then becomes

$$y_t = a_1 + b_1 x_t \quad \text{where } a_1 = b_0 + b_1\bar{t} \qquad \ldots(8)$$

Using the method of least squares for minimising the error term, we obtain the following normal equations:

$$\sum_{t=1}^{n} y_t = na_1 + b_1 \sum_{t=1}^{n} x_t$$

$$\sum_{t=1}^{n} y_t x_t = a_1 \sum_{t=1}^{n} x_t + b_1 \sum_{t=1}^{n} x_t^2 \qquad \ldots(9a)$$

On solving the above equations, the least squares estimates of a_1 and b_1 are given by $\hat{a}_1$ and $\hat{b}_1$ as:

$$\hat{a}_1 = \bar{y} \quad \text{and} \quad \hat{b}_1 = \frac{\sum_{t=1}^{n} y_t x_t}{\sum_{t=1}^{n} x_t^2} \qquad \ldots(9b)$$

The estimate of b_0 is given by $\hat{b}_0$:

$$\hat{b}_0 = \bar{y} - \hat{b}_1\bar{t} \qquad \ldots(10)$$

The fitted trend line is given by

$$\hat{y}_t = \hat{b}_0 + \hat{b}_1\bar{t} + \hat{b}_1(t - \bar{t}) = \bar{y} + \hat{b}_1(t - \bar{t}) \qquad \ldots(11)$$

Let us explain this concept further with the help of an example.

Example 6: Fit a straight line trend for the data of annual profit of a company given below:

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Profit (in crores)	93	102.8	126.7	103.5	105.7	133.2	156.7	175.7	161.6

Solution: From the given data, we obtain linear trend values for the annual profit as follows:

Table 6: Trend values for the given time series

Year	**Profit (in crores)**	$x_t = t - \bar{t}$	$x_t y_t$	x_t^2	**Trend Values**
2003	93.0	–4	–372.0	16	89.915
2004	102.8	–3	–308.4	9	99.828
2005	126.7	–2	–253.4	4	109.351
2006	103.5	–1	–103.5	1	119.054
2007	105.7	0	0	0	128.767
2008	133.2	1	133.2	1	138.480
2009	156.7	2	313.4	4	148.193
2010	175.7	3	527.1	9	157.906
2011	161.6	4	646.4	16	167.619
	$\Sigma y_t = 1158.9$		$\Sigma x_t y_t = 582.8$	$\Sigma x_t^2 = 60$	

From the above table, we calculate the least squares estimates as follows:

$$\hat{a}_1 = \bar{y} = \frac{\sum y}{9} = 128.76 \qquad \hat{b}_1 = \frac{\sum_{t=1}^{n} y_t x_t}{\sum_{t=1}^{n} x_t^2} = \frac{582.8}{60} = 9.71$$

The equation of the fitted trend line is given by equation (11) as

$$\hat{y}_t = \hat{b}_0 + \hat{b}_1\bar{t} = \bar{y} + \hat{b}_1(t - \bar{t})$$

$$= 128.76 + 9.71\ (t - 2007)$$

The trend projection for 2013 is

$$Y_{2013} = 128.76 + 9.71(2013 - 1977) = 187.02$$

These points are used to plot the trend line along with the data and projected value for 2013 (Fig. 13.11).

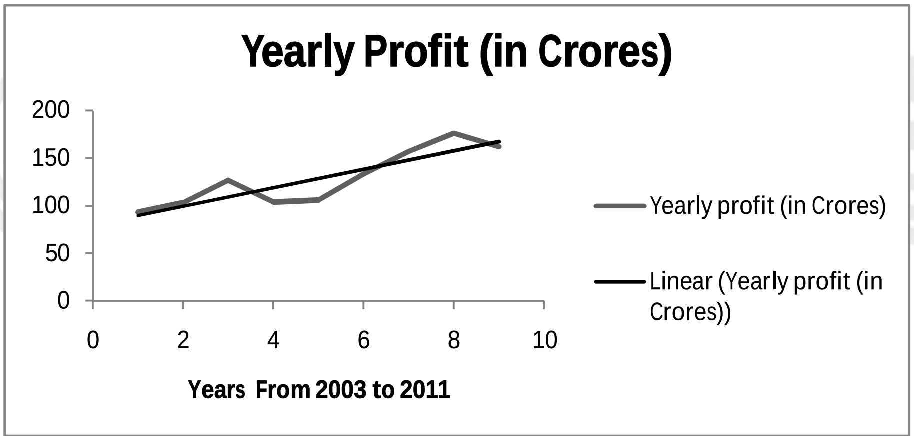

Fig. 13.11: Computed trend line for the data of profit of a company.

The projections are forecasts of future trend values but they do not take into account the cyclical effect. Sometimes cyclical effects are confused with trend curves which are of higher degree of polynomials.

13.6.2 Fitting a Quadratic Trend Equation

Sometimes trend is not linear and shows some curvature. The simplest curvilinear form is a second degree polynomial which can be obtained by taking p = 2 in equation (4).

$$y_t = b_0 + b_1 t + b_2 t^2 \qquad \ldots(12)$$

Proceeding in the same way as in Sec. 13.6.1 for linear trend equation, the equations for estimating b_0, b_1 and b_2 are given by

$$\begin{aligned}\sum y_t &= nb_0 + b_1\sum x_t + b_2\sum x_t^2\\ \sum x_t y_t &= b_0\sum x_t + b_1\sum x_t^2 + b_2\sum x_t^3\\ \sum x_t^2 y_t &= b_0\sum x_t^2 + b_1\sum x_t^3 + b_2\sum x_t^4\end{aligned} \qquad \ldots(13)$$

The values of $\sum y_t$, $\sum x_t$ $\sum x_t y_t$, $\sum x_t^2 y_t$, $\sum x_t^2$, $\sum x_t^3$ and $\sum x_t^4$ are obtained from the given data and the normal equations given in equation (13) can now be solved for the optimum values $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$. With these values, equation (12) gives the desired quadratic trend.

To illustrate this, let us take an example of fitting a quadratic trend for the data of gross revenue of a company.

Example 7: Fit a quadratic trend equation for the data given below:

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999
Gross Revenue (in Lakhs)	240	167	140	120	124	128	142	176	207
Year	2000	2001	2002	2003	2004	2005	2006	2007	
Gross Revenue (in Lakhs)	304	338	397	439	481	577	711	778	

Solution: Let us take the simplest curvilinear form of a second degree polynomial given as

$$y_t = b_0 + b_1 t + b_2 t^2 \qquad \ldots \text{(i)}$$

Proceeding in the same way as in Sec. 13.6.1, the normal equations for estimating b_0, b_1 and b_2 are given by

$$\sum y_t = nb_0 + b_1 \sum x_t + b_2 \sum x_t^2$$

$$\sum x_t y_t = b_0 \sum x_t + b_1 \sum x_t^2 + b_2 \sum x_t^3 \qquad \text{... (ii)}$$

$$\sum x_t^2 y_t = b_0 \sum x_t^2 + b_1 \sum x_t^3 + b_2 \sum x_t^4$$

The values of $\sum y_t$, $\sum x_t$, $\sum x_t y_t$, $\sum x_t^2 y_t$, $\sum x_t^2$, $\sum x_t^3$ and $\sum x_t^4$ are obtained from the given data as follows:

Table 7: Calculations for fitting of quadratic trend

Year	y_t	$x_t = t-1999$	$x_t y_t$	$x_t^2 y_t$	x_t^2	x_t^3	x_t^4
1991	240	–8	–1920	15360	64	–512	4096
1992	167	–7	–1169	8183	49	–343	2401
1993	140	–6	–840	5040	36	–216	1296
1994	120	–5	–600	3000	25	–125	625
1995	124	–4	–496	1984	16	–64	256
1996	128	–3	–384	1152	9	–27	81
1997	142	–2	–284	568	4	–8	16
1998	176	–1	–176	176	1	–1	1
1999	207	0	0	0	0	0	0
2000	304	1	304	304	1	1	1
2001	338	2	676	1352	4	8	16
2002	397	3	1191	3573	9	27	81
2003	439	4	1756	7024	16	64	256
2004	481	5	2405	12025	25	125	625
2005	577	6	3462	20772	36	216	1296
2006	711	7	4977	34839	49	343	2401
2007	778	8	6224	49792	64	512	4096
Total	5469	0	15126	165144	408	0	17544

Putting the values from the above table in the normal equations given in equation (ii), we get

$$17\, b_0 + 0\, b_1 + 408\, b_2 = 5469$$

$$0\, b_0 + 408\, b_1 + 0\, b_2 = 15126$$

$$408\, b_0 + 0\, b_1 + 17544\, b_2 = 165144$$

On solving the normal equations, we get the optimum values: $\hat{b}_0 = 373.22$, $\hat{b}_1 = -41.614$ and $\hat{b}_2 = 4.3715$.

With these values, we get the desired quadratic trend:

$$y_t = 373.22 - 41.614\left(t-\bar{t}\right) + 4.3715\left(t-\bar{t}\right)^2$$

Since we have used $x_t = \left(t-\bar{t}\right)$ in equestion (1)

Fig. 13.12 gives a plot of the above quadratic equation that fits the data (shown by continuous line) along with the actual values of the gross revenue of a company shown by dots. The values of time t are coded year values (t = 1, 2, ..., 17). The fitted model with coded year values is shown in Fig. 13.12.

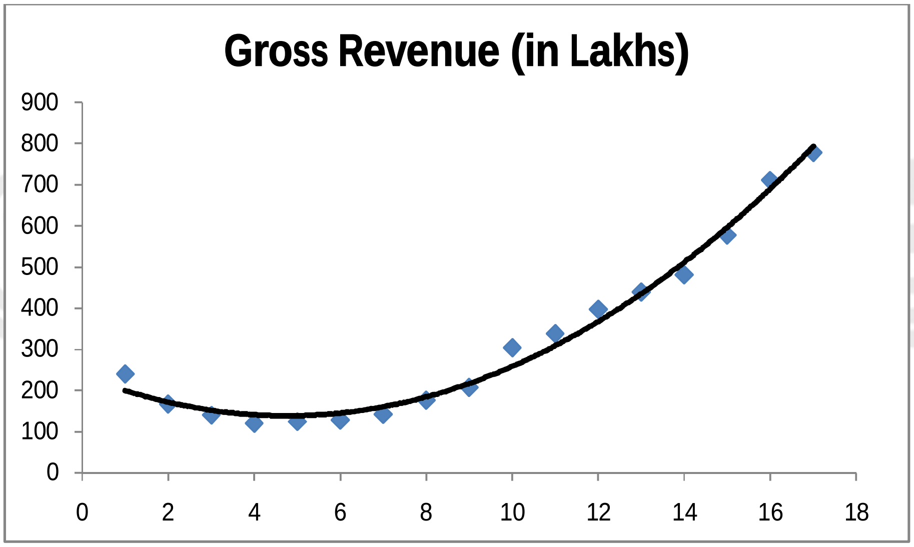

Fig. 13.12: Plot the quadratic trend model to the data of Example 7.

13.6.3 Fitting the Exponential Trend Equation

Sometimes data show that compound annual growth rate is constant over time rather than increasing annually as in the case of linear model. This can be represented by an exponential model

$$y_t = a_0 a_1^t$$

You can see that $(\alpha_1 - 1) \times 100\%$ is the annual compound growth rate (in %), which remains constant. As far as fitting is concerned, we transform this model to a linear trend model by taking natural logarithm of y_t:

$$\log_e y_t = \log_e \alpha_0 + t \log_e \alpha_1$$

$$Y_t = \beta_0 + \beta_1 t$$

where $Y_t = \log_e y_t$, $\beta_0 = \log_e \alpha_0$, $\beta_1 = \log_e \alpha_1$,

Now, we can fit the model with Y_t and t as described in Sec. 13.6.1. Once we know the estimates of β_0 and β_1, i.e., $\hat{\beta}_0$ and $\hat{\beta}_1$, we can obtain the estimates of α_0 and α_1, i.e., the values of $\hat{\alpha}_0$ and $\hat{\alpha}_1$ by taking anti-logarithm of $\hat{\beta}_0$ and $\hat{\beta}_1$, respectively. When we fit the exponential model to the data given in Example 7, we get the values $\hat{\beta}_0 = 2.57$ and $\hat{\beta}_1 = 0.0494$, respectively, and the estimated exponential trend equation is obtained as:

$$y_t = 371.535\,(1.1205)^t$$

The fitted and raw data are plotted in Fig. 13.13.

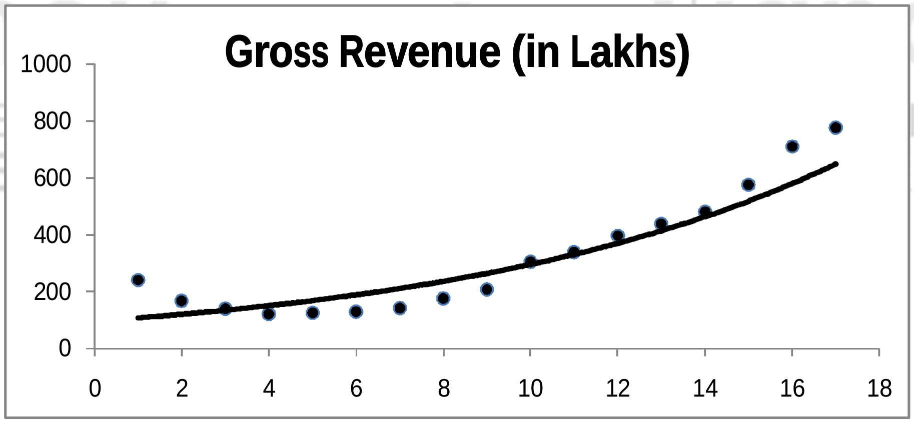

Fig. 13.13: Plot of fitted exponential trend model to the data of Example 7.

An exponential model can also be represented by

$$y_t = \alpha_0 \exp^{\beta_1 t}$$

As far as fitting is concerned, we transform this model to a linear trend model by taking natural logarithm of y_t:

$$\log_e y_t = \log_e \alpha_0 + \beta_1 t$$

$$\text{or } Y_t = \beta_0 + \beta_1 t$$

where $Y_t = \log_e y_t$, $\beta_0 = \log_e \alpha_0$.

On comparing Fig. 13.12 with Fig. 13.13, we find that the quadratic model is a better choice.

You may now like to solve the following exercise.

E4) Use the data given in Example 1 to fit a linear trend line and obtain the projection for the year 2012.

13.7 MEASUREMENT OF TREND EFFECT USING CENTRED MOVING AVERAGE METHOD

In Sec. 13.3, you have learnt that time series data usually have four components: Trend (T), Cyclical (C), Seasonal (S) and Irregular (I). Suppose a set of data are recorded on a monthly basis and there is a seasonal effect of one year. This means that after twelve months the data behaves in the same way as it did twelve months ago. This is called the **period** of seasonal effect. If we take a centred moving average with m = 12, then it will smooth out (eliminate) the effect of season. If data have been recorded on a quarterly basis and there is a seasonal effect with a period of twelve months, then a moving average with m = 4 will smooth out the seasonal effect. Not only this, it will also smooth out the effect of the irregular component I. Thus, the time series values of centred moving average will be nearly free from the effects of S and I. When the effects of S and I are removed, we are left with the effect of trend (T) and the cyclic effect (C).

In the next unit, we shall discus the method of estimating seasonal effect (S) by making use of the estimates of trend (T) and cycle (C).

Example 8: Compute the trend values for the given data for quarterly sales of washing machines by an appliance manufacturer for the period 2001-2009.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2001	935	1215	1045	1455
2002	990	1315	1350	1485
2003	1370	1815	1470	1680
2004	1160	1365	1205	1445
2005	1030	1475	1195	1585
2006	1185	1330	1500	2145
2007	1410	2120	1915	2390
2008	1875	2145	1965	2800
2009	1865	2115	1935	2165

Solution: We shall use the data of four years 2001-2004 from the given data to explain the calculations of moving averages, which are estimates of T and C. The given time series data is shown in Fig. 13.14.

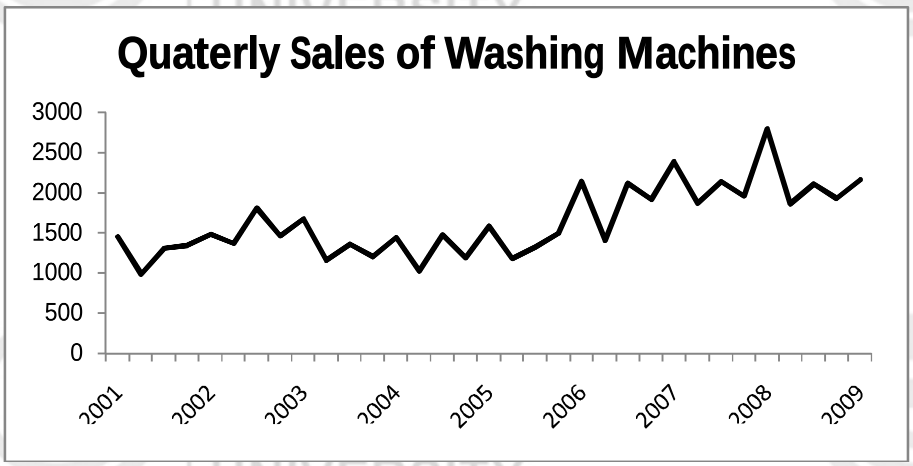

Fig. 13.14: Time series plot for quarterly sales of washing machines from 2001-2009.

We have seen from the plot (Fig. 13.14) that it has a very strong 12 monthly seasonal effect. Hence, to remove the seasonal effect, we have to take moving averages with m = 4.

Table 8: Calculation of centred moving averages

Year	Quarter	Sales (in Hundreds)	Centred MA(1)	Centred MA (2)
2001	1	935	-	
	2	1215		
			1162.5	
	3	1045		1169.375
			1176.25	
	4	1455		1188.75
			1201.25	
2002	1	990		1239.375
			1277.5	
	2	1315		1281.25
			1285.0	
	3	1350		1332.5
			1380.0	
	4	1485		1442.5

			1505.0	
2003	1	1370		1520
			1535.0	
	2	1815		1559.125
			1583.25	
	3	1470		1619.75
			1656.25	
	4	1680		1600
			1543.75	
2004	1	1660		1510.625
			1477.5	
	2	1365		1448.625
			1419.75	
	3	1205		-
			-	
	4	1445		-

The values in MA(1) are the moving averages for m = 4 but they do not correspond to any of the given four quarters as the average of 1, 2, 3 and 4 is 2.5. Hence, to make it correspond to a quarter, we usually calculate moving averages with m = 2 on MA(1) so that the centred values correspond to one of the four quarters. MA(2) gives the moving average of MA(1) series with m = 2 so that the values correspond to quarters 3, 4, 1, 2,.. etc.

You should try to solve the following exercises.

E5) Compute MA(1), the moving average values for m = 4 and MA(2), the moving average values for m = 2 of MA(1) for the remaining years of the period 2005-2009 for the data given in Example 8.

E6) Compute the moving average (MA) values for m = 3 for time series for the period 2001-2009 for the data given in Example 8.

Let us now summarise the concepts that we have discussed in this unit.

13.8 SUMMARY

1. A good forecast of the future requirements will result in good planning. A poor forecast results in poor planning and may lead to increased cost. In order to provide such forecasts, we use historical data of the past few years to assess the average requirement, trend (if any) over the years and seasonal variations. Based on these features observed from the past data, we try to understand their role in causing variability and use them for forecasting requirements.

2. **A time series** is a collection of observations made sequentially over a period of time. **The main objectives of time series analysis are description, explanation and forecasting.** It has applications in many fields including economics, engineering, meteorology, etc.

3. A **trend** is a long term smooth variation (increase or decrease) in the time series. When values in a time series are plotted in a graph and, on an average, these values show an increasing or decreasing trend over a long period of time, the time series is called the **time series with trend effect**.

4. If values in a time series reflect seasonal variation with respect to a given period of time such as a quarter, a month or a year, the time series is called a **time series with seasonal effect**. If the time plot of data in a time series exhibits a cyclic trend, it is called the **time series with cyclic effect**.

5. The long term variations, i.e., the trend component, and short term variations, i.e., the seasonal and cyclic component, are known as **regular variations**. Apart from these regular variations, **random or irregular variations**, which are not accounted for by trend, seasonal or cyclic variations, exist in almost all time series.

6. The **additive model** is one of the most widely used models. It is based on the assumption that at any time t, the time series value Y_t is the sum of all the components. According to the additive model, a time series can be expressed as

$$Y_t = T_t + C_t + S_t + I_t$$

where T_t, C_t, S_t and I_t are the trend, cyclic, seasonal and irregular variations, respectively, at time t.

9. The multiplicative model is based on the assumption that the time series value Y_t at time t is the product of the trend, cyclic, seasonal and irregular component of the series:

$$Y_t = T_t \times C_t \times S_t \times I_t$$

where T_t, C_t, S_t and I_t denote the trend, cyclic, seasonal and irregular variations, respectively. The multiplicative model is found to be appropriate for many business and economic data.

10. There are two methods of moving averages: the **equal weight** or **simple moving averages method** and **the weighted (unequal) moving average method**. The methods of moving averages and exponential smoothing are used for smoothing or filtering the time series data.

11. In exponential smoothing technique, where weights decrease exponentially, except the last one, the trend value is given by

$$y'_t = wy_t + (1 - w)\ y'_{t-1} \qquad \text{for } t = 1, 2, ...$$

where $0 < w < 1$. The value of 'w' is chosen as per the requirements.

12. An alternative approach to smoothing is to fit a polynomial to the data. This treats smoothing as a regression problem in which y_t is the trend value and integral powers of time t are the explanatory variables. The resulting smooth function is a polynomial

$$y_t = \sum_{j=0}^{p} b_j\, t^j$$

where b_j is the j^{th} coefficient of the polynomial of degree p. The coefficients of the polynomial are estimated by the method of least squares.

13.9 SOLUTIONS/ANSWERS

E1)

Year	MA
1979	(18 + 26 + 16)/3 = 20
1980	(26 + 16 + 27)/3 = 23

E2) Using equation (1), we obtain the exponential smoothing values as follows:

$y_2 = 0.5x_2 + 0.5x_1 = 0.5 \times 22 + 0.5 \times 17 = 19.50$

$y_3 = 0.5x_3 + 0.25x_2 + 0.25x_1$

$= 0.5 \times 18 + 0.25 \times 22 + 0.25 \times 17 = 18.75$

$y_4 = 0.5x_4 + 0.25x_3 + 0.125x_2 + 0.125x_1$

$= 0.5 \times 26 + 0.25 \times 18 + 0.125 \times 22 + 0.125 \times 17 = 22.375$

$y_5 = 0.5x_5 + 0.25x_4 + 0.125x_3 + 0.0625x_2 + 0.0625x_1$

$= 0.5 \times 16 + 0.25 \times 26 + 0.125 \times 18 + 0.0625 \times 22$

$+ 0.0625 \times 17 = 19.1875$

$y_6 = 0.5x_6 + 0.25x_5 + 0.125x_4 + 0.0625x_3 + 0.03125\, x_2 + 0.03125x_1$

$= 0.5 \times 27 + 0.25 \times 16 + 0.125 \times 26 + 0.0625 \times 18$

$+ 0.03125 \times 22 + 0.03125 \times 17 = 23.09375$

E3) If we plot the given data in a graph, there seems to be no trend in the time series.

If we fit a linear regression to this data and if T = a + bt is the equation, it seems to have b = 0. Therefore, the least squares estimates of all 41 time series values would be the same. Also our forecast for rainfall is a for all t because our model is

$$T = a + (\text{forecast error})$$

This result of forecast seems somewhat unreasonable because a particular place cannot have a constant amount of rainfall every year. From Fig. 13.15, you can see that the value of y_t seems to be higher during the period 1970 to 1980 compared to the value of a for the period 1980 to 1990. Therefore, it is logical to assume that the value of a is gradually changing over time and to denote it by y_t rather than a.

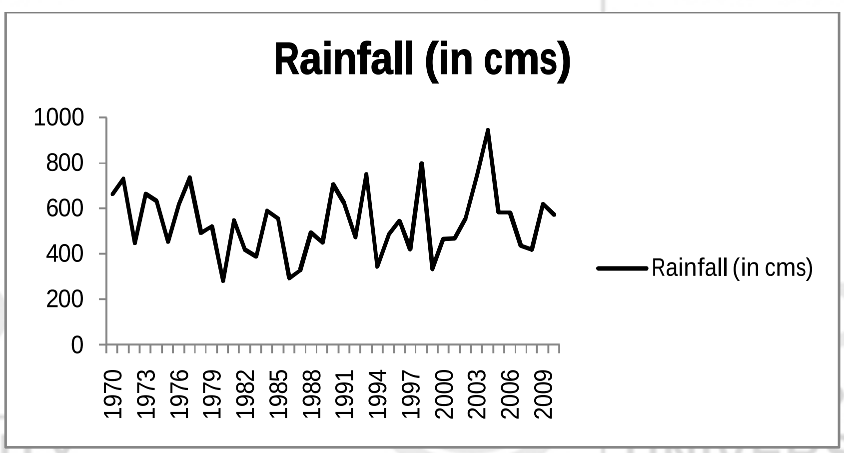

Fig. 13.15: Time series plot for rainfall (in cm) from 1970 to 2009.

According to this smoothing technique, we select a single weight w called the exponential smoothing constant. Here w lies between 0 and 1 and there is a method of choosing this constant. We can compute an exponential smoothing series y'_t as follows:

$$
\begin{aligned}
y'_1 &= wy_1 + (1-w)\,y_1 = y_1 \\
y'_2 &= w\,y_2 + (1-w)\;y'_1 \\
y'_3 &= w\,y_3 + (1-w)\;y'_2 \\
&\;\;\vdots \\
y'_t &= w\,y_t + (1-w)\;y'_{t-1}
\end{aligned}
\qquad \text{... (i)}
$$

We should know the value of y_t to initialise the computation of y_1, y_2, … For this example we take the initial value of y_t as y_1, the first and initial value of the given time series. Thus,

$$y'_1 = y_1$$

Now let us take w = 0.02 since w can take any value between 0 and 1. From experience it is suggested that the value of w be taken between 0.01 and 0.3. Then from the equation (i) given above, we get

$$y'_2 = 0.02\,y_2 + 0.98\,y'_1 = 0.02\ (728) + (0.98)\ (664) = 543.71$$

Our forecast for x_1 at time zero is y_1. Therefore, the forecast error is

$$e_1 = y_1 - y'_1 = 664 - 664 = 0.00$$

Similarly, the second forecast error is

$$e_2 = y_2 - y'_2 = 728 - 665.24 = 62.76$$

Following the same procedure, we get

$$y'_3 = (0.02)\,y_3 + (0.98)\,y'_2 = (0.02)\,447 + 0.98 \times 665.24 = 660.875$$

and the third forecast error is

$$e_3 = y_3 - y'_3 = 447 - 660.875 = -213.875$$

Proceeding in the same way, we calculate y'_t and the forecast errors e_t for all t. The calculated forecast values and errors are given in the following table:

Table 9: Forecast values and Forecast errors for given time series

Year	**Rainfall** y_t	**Forecast** y'_t	**Error** $e_t = y_t - y'_t$
1970	664	664.00	0.00
1971	728	665.24	+62.76
1972	447	660.875	−213.875
1973	663	660.92	+02.08
1974	630	660.30	−30.30
1975	451	656.11	−05.11
1976	617	655.33	−38.33
1977	734	656.91	+77.09

1978	491	653.58	−162.58
1979	520	650.92	−130.92
1980	280	643.50	−363.50
1981	548	641.59	−93.59
1982	417	637.1	−220.1
1983	387	632.1	−245.1
1984	590	631.25	−41.25
1985	556	629.75	−73.75
1986	292	623.0	−331.0
1987	327	617.07	−290.07
1988	494	614.61	−120.61
1989	448	611.28	−163.28
1990	704	613.13	+90.87
1991	624	613.35	+10.65
1992	473	610.54	−137.54
1993	750	613.33	+136.67
1994	343	607.93	−264.93
1995	484	605.45	−121.45
1996	545	604.24	−59.24
1997	419	600.53	−181.53
1998	798	604.48	+193.52
1999	334	599.07	−265.07
2000	465	596.40	−131.40
2001	468	593.82	−125.82
2002	554	593.03	−39.03
2003	744	596.05	+147.95
2004	943	602.99	+340.01
2005	582	602.56	−20.56
2006	581	602.13	−21.13
2007	437	598.83	−161.83
2008	417	595.20	−178.20
2009	618	595.65	+22.35
2010	571	595.16	−24.16

After calculating the forecast a_t and the error e_t for all t, we plot the time series values along with the forecasted values, which are the outcomes after smoothing the time series values and observe the change in the time series values before and after the smoothing (Fig. 13.16). We can also see that the peaks barely exist in the graph of the time series after smoothing.

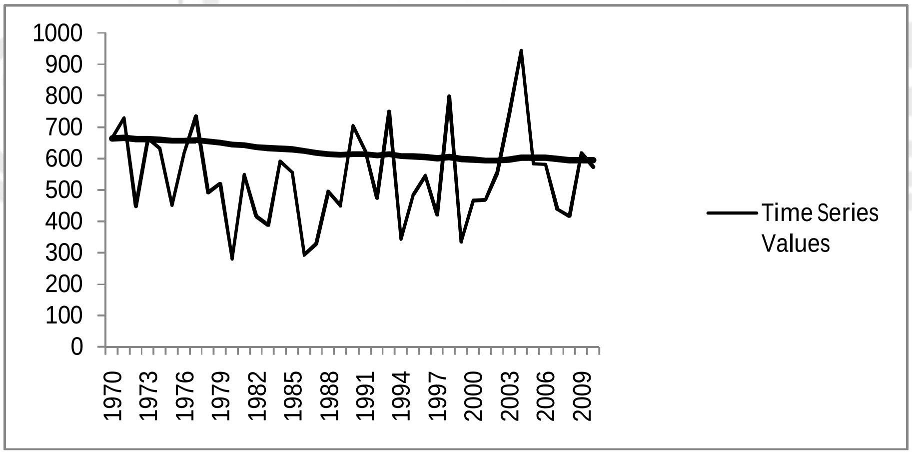

Fig. 13.16: Exponentially smoothed data for rainfall from 1971-2010.

E4) From the given data, we calculate the following values:

$$\bar{t} = 2008.5 \quad \sum x_t^2 = \sum \left(t - \bar{t}\right)^2 = 17.5,$$

$$\Sigma y_t x_t = 21.0 \text{ and } \bar{y} = 20.667$$

On solving the equations, we get

$$\hat{a}_1 = \bar{y} = 20.667$$

$$\hat{b}_1 = \sum y_t x_t \Big/ \sum x_t^2 = 1.20$$

The trend equation is

$$\hat{y}_t = \bar{y} + \hat{b}_1 (t - \bar{t}) = 20.667 + 1.20(t - 2008.5)$$

The projection for 2012 is:

$$\hat{Y}_{2012} = 20.667 + 1.20(2012 - 2008.5) = 20.667 + 1.20 \times 3.5$$

$$= 24.867$$

E5) The complete set of MA(2) values for m = 2 are as follows:

Table 10: Centred moving averages for given time series

Quarter Year	1	2	3	4
2001	-	-	1169.375	1188.75
2002	1239.375	1281.25	1332.5	1442.5
2003	1520.0	1559.125	1619.75	1600.0
2004	1510.625	1448.625	1340.5	1275.0
1977	1287.5	1303.75	1340.625	1341.875
2006	1361.875	1470	1568.125	1695.0
2007	1845.625	1928.125	2016.875	2078.125
2008	2087.5	2145.0	2195.0	2190.0
2009	2182.5	2099.375	-	-

E6) The complete set of MA(1) values for m = 3 are as follows:

Table 11: Centred moving averages for given time series

Quarter Year	1	2	3	4
2001	-	1065	1238.34	1163.34
2002	1253.34	1218.34	1383.34	1401.67
2003	1556.67	1551.67	1655.0	1436.67
2004	1401.67	1243.34	1338.34	1226.67
2005	1316.67	1233.34	1418.34	1321.67
2006	1366.67	1338.34	1658.34	1685.0
2007	1891.67	1815.0	2141.67	2060.0
2008	2136.67	1995.0	2303.34	2210.0
2009	2260.0	1971.67	2071.67	-