
Algorithm 1: Standard preconditioned conjugate gradients (PCG).

Input : $\text{mvm_A}()$ – function for matrix-vector multiplication (MVM) with matrix A
 \mathbf{b} – vector to solve against
 $P^{-1}()$ – function for preconditioner

Output : $A^{-1}\mathbf{b}$.

$\mathbf{u}_0 \leftarrow \mathbf{0}$ // Current solution

$\mathbf{r}_0 \leftarrow \text{mvm_A}(\mathbf{u}_0) - \mathbf{b}$ // Current error $\mathbf{r}_0 \leftarrow \mathbf{b} - \text{mvm_A}(\mathbf{u}_0)$

$\mathbf{z}_0 \leftarrow P^{-1}(\mathbf{r}_0)$ // Preconditioned error

$\mathbf{d}_0 \leftarrow \mathbf{z}_0$ // "Search" direction for next solution

for $j \leftarrow 0$ **to** T **do**

$\mathbf{v}_j \leftarrow \text{mvm_A}(\mathbf{d}_{j-1})$

$\alpha_j \leftarrow (\mathbf{r}_{j-1}^\top \mathbf{z}_{j-1}) / (\mathbf{d}_{j-1}^\top \mathbf{v}_j)$

$\mathbf{u}_j \leftarrow \mathbf{u}_{j-1} + \alpha_j \mathbf{d}_{j-1}$

$\mathbf{r}_j \leftarrow \mathbf{r}_{j-1} - \alpha_j \mathbf{v}_j$

if $\|\mathbf{r}_j\|_2 < \text{tolerance}$ **then return** \mathbf{u}_j ;

$\mathbf{z}_j \leftarrow P^{-1}(\mathbf{r}_j)$

$\beta_j \leftarrow (\mathbf{z}_j^\top \mathbf{z}_j) / (\mathbf{z}_{j-1}^\top \mathbf{z}_{j-1})$ $\beta_j \leftarrow (\mathbf{z}_j^\top \mathbf{r}_j) / (\mathbf{z}_{j-1}^\top \mathbf{r}_{j-1})$

$\mathbf{d}_j \leftarrow \mathbf{z}_j - \beta_j \mathbf{d}_{j-1}$ $\mathbf{d}_j \leftarrow (\mathbf{z}_j + \beta_j \mathbf{d}_{j-1})$

end

return \mathbf{u}_{j+1}
