

# Introduction to Intelligent Systems

University of Groningen (The Netherlands)

## Morphological Image Processing



Enrique Alegre

Artificial Vision & Pattern Recognition Group  
Dpto. Electrical, Automatic and Systems  
Engineering

University of Leon (Spain)

## Credits

Most of the slides of this presentation have been adapted from the An-Najah National University, presentation “Chapter 9. Morphological Image Processing” in <http://elearning.najah.edu/>



# Contents

---

## 1. Mathematic Morphology

Usefulness,  $Z^2$  and  $Z^3$ , Basic Set Theory,  
Logic Operations, Structuring Element (SE)

## 2. Basic Morphological operations

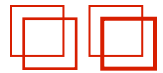
Erosion, Dilation, Opening, Closing

## 3. Applications

Filtering, Hit-or-Miss Tranf., Boundary  
Extraction, Region Filling, Extraction of  
connected components, Convex hull,  
Thinning, Thickening, Skeletons, Pruning



# 1. Mathematic Morphology



## 1.1 Usefulness

- used to extract image components that are useful in the **representation and description of region shape**, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning



# ... 1. Mathematic Morphology

---

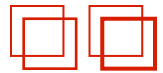
## ...1.1 Usefulness

mathematical framework used for:

- pre-processing
  - noise filtering, shape simplification, ...
- enhancing object structure
  - skeletonization, convex hull...
- Segmentation
  - watershed,...
- quantitative description
  - area, perimeter, ...



# ... 1. Mathematic Morphology



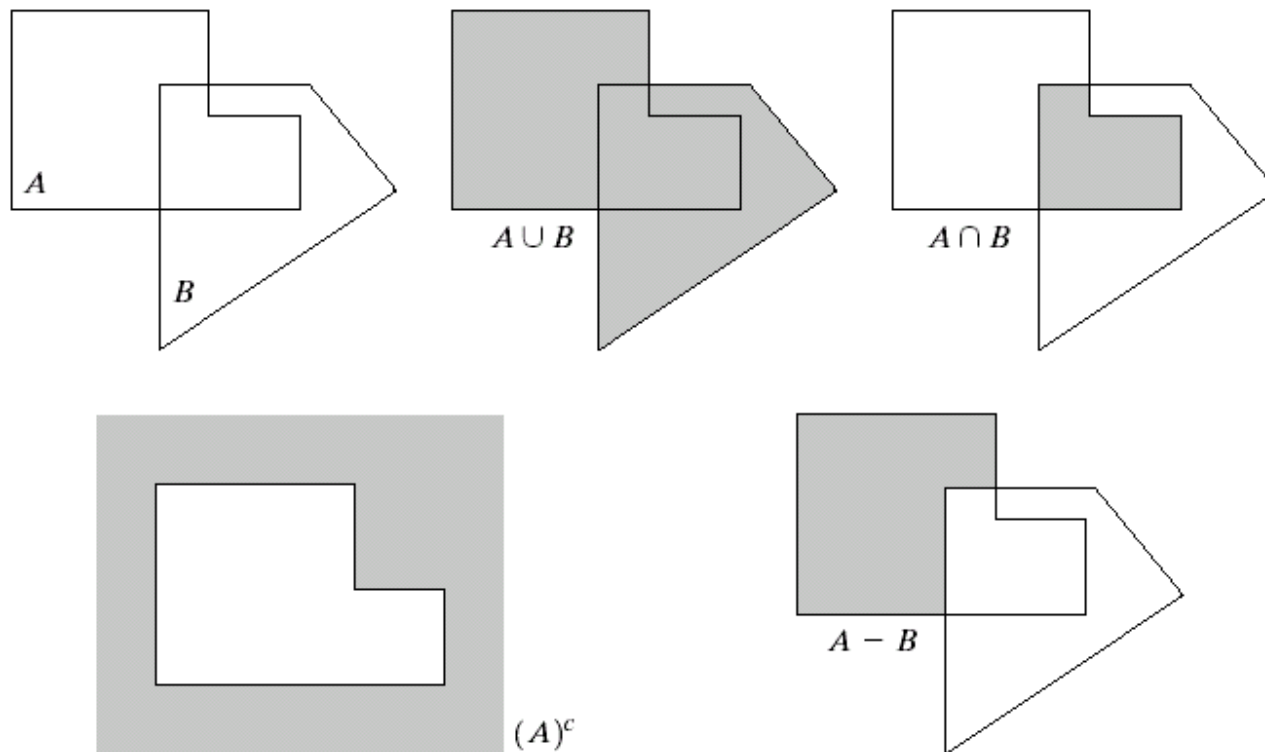
## 1.2 $Z^2$ and $Z^3$

- Set in mathematic morphology represent objects in an image
  - binary image (0 = black, 1 = white) : the element of the set is the coordinates (x,y) of pixel belong to the object  $\Rightarrow Z^2$
  - Gray-scaled image : the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels  $\Rightarrow Z^3$

# ... 1. Mathematic Morphology

## 1.3 Basic Set Theory

Union, Intersection, Complement, Difference



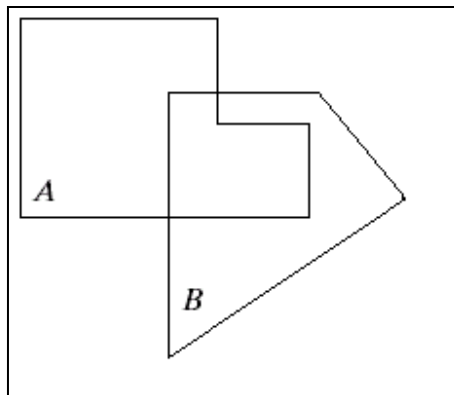
a	b	c
d	e	

**FIGURE 9.1**  
(a) Two sets  $A$  and  $B$ . (b) The union of  $A$  and  $B$ . (c) The intersection of  $A$  and  $B$ . (d) The complement of  $A$ . (e) The difference between  $A$  and  $B$ .

# ... 1. Mathematic Morphology

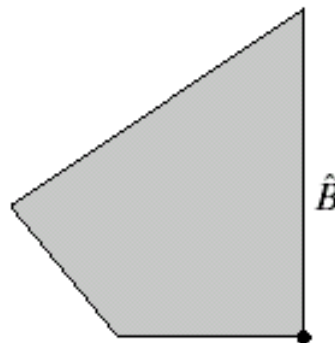
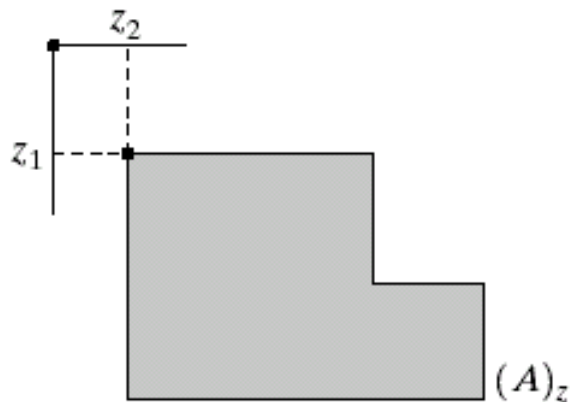
## ...1.3 Basic Set Theory

### Translation and Reflection



$$(A)_z = \{c \mid c \in a + z, \text{ for } a \in A\}$$

$$\hat{B} = \{w \mid w \in -b, \text{ for } b \in B\}$$



a b

**FIGURE 9.2**

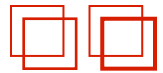
(a) Translation of  $A$  by  $z$ .

(b) Reflection of  $B$ . The sets  $A$  and  $B$  are from Fig. 9.1.





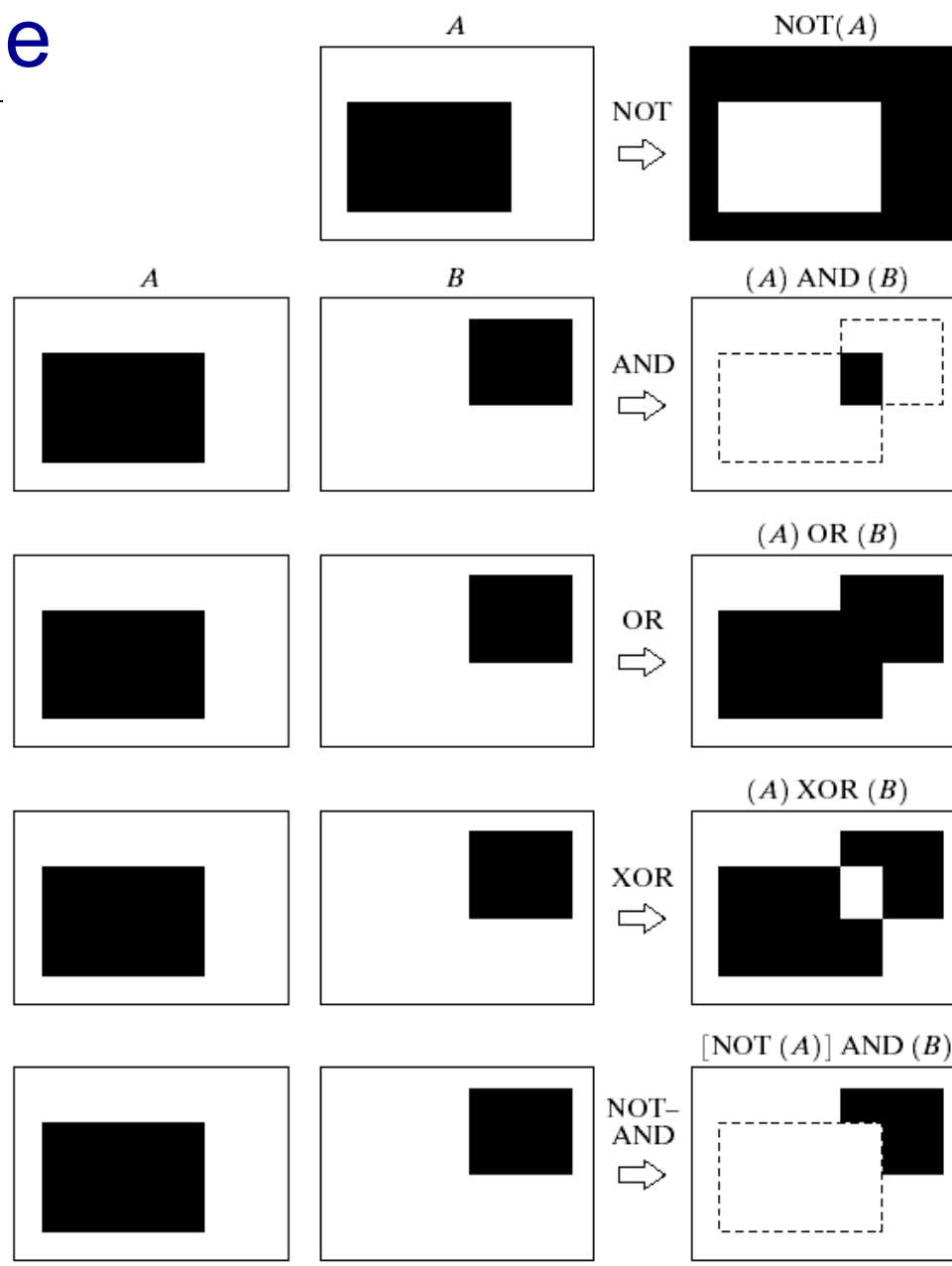
# ... 1. Mathematic Morphology



## 1.4 Logic Operations

$p$	$q$	$p \text{ AND } q$ (also $p \cdot q$ )	$p \text{ OR } q$ (also $p + q$ )	$\text{NOT } (p)$ (also $\bar{p}$ )
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

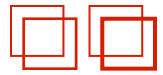
# Example



**FIGURE 9.3** Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

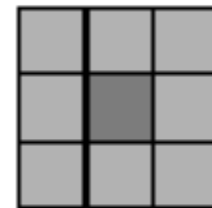
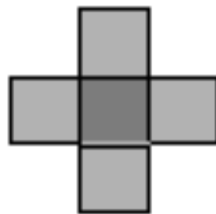


# ... 1. Mathematic Morphology



## 1.5 Structuring Element (SE)

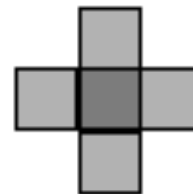
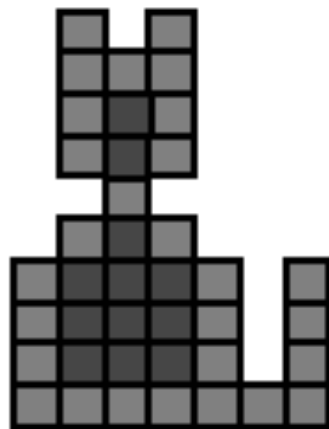
- small set to check with the image under study
- for each SE, define origo
- shape and size must be adapted to geometric properties for the objects



## ... 1. Mathematic Morphology ... 1.5 Structuring Element (SE)

### Basic idea

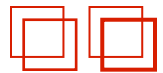
- in parallel for each pixel in binary image:
  - check if SE is "satisfied"
  - output pixel is set to 0 or 1 depending on used operation



pixels in output  
image if check is:  
*SE fits*



## ... 1. Mathematic Morphology ... 1.5 Structuring Element (SE)



# How to describe SE

- many different ways! -> see Matlab **strel**
- information needed:
  - position of origo for SE
  - positions of elements belonging to SE



line segment



line segment  
(origo is not in SE)



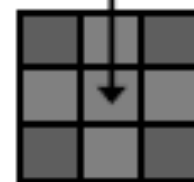
line segment  
(origo is not in SE)



pair of points  
(separated by one pixel)



origo





## 2. Basic Morphological operations

- Erosion

shrink

- Dilation

grow

- combine to

keep general shape but  
smooth with respect to

- Opening

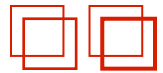
—————→ object

- Closing

—————→ background



## ... 2. Basic Morphological operations



### 2.1 Erosion

Does the structuring element **fit the set**?  
erosion of a set  $A$  by structuring element  $B$ :  
all  $z$  in  $A$  such that  $B$  is in  $A$  when origin  
of  $B=z$

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

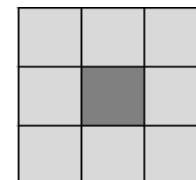
**shrink the object**



## ... 2. Basic Morphological operations ...2.1 Erosion

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0				0	0	0	0	0	0		0	0
4		0	0		<b>1</b>		0	0	0	0	0	0		0	0
5		0	0		<b>1</b>				0	0	0				0
6		0	0		<b>1</b>		0								0
7		0	0		<b>1</b>									0	0
8		0	0		<b>1</b>			0	0	0	0	0		0	0
9		0	0				0		0	0	0	0			0
10		0	0	0	0	0	0		0	0	0	0		0	0
11		0	0	0	0	0	0	0	0	0	0	0		0	0
12		0	0	0	0	0	0	0	0	0	0	0		0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0	0	1	0
4		0	0	1	1	1	0	0	0	0	0	0	0	1	0
5		0	0	1	1	1	1	1	0	0	0	0	1	1	0
6		0	0	1	1	1	0	1	1	1	1	1	1	1	0
7		0	0	1	1	1	1	1	1	1	1	1	1	1	0
8		0	0	1	1	1	1	0	0	0	0	0	0	1	0
9		0	0	1	1	1	0	1	0	0	0	0	0	1	0
10		0	0	0	0	0	0	1	0	0	0	0	0	1	0
11		0	0	0	0	0	0	0	0	0	0	0	0	1	0
12		0	0	0	0	0	0	0	0	0	0	0	0	1	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0

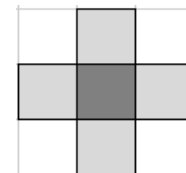




## ... 2. Basic Morphological operations ...2.1 Erosion

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

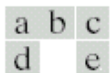
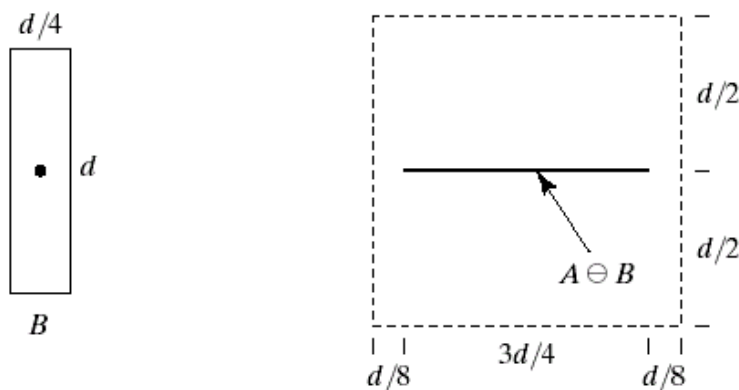
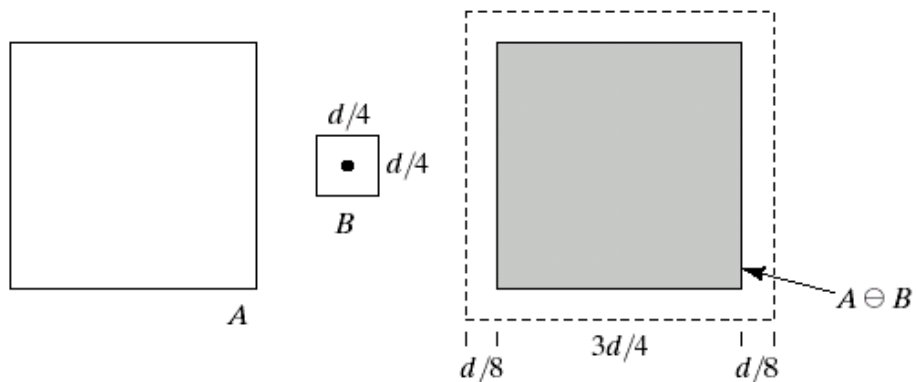
<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0				0	0	0	0	0	0		0	0
4		0	0		1		0	0	0	0	0	0		0	0
5		0	0		1	1			0	0	0		1		0
6		0	0		1		0					1	1		0
7		0	0		1	1								0	0
8		0	0		1	1		0	0	0	0	0		0	0
9		0	0				0		0	0	0	0			0
10		0	0	0	0	0	0		0	0	0	0		0	0
11		0	0	0	0	0	0	0	0	0	0	0		0	0
12		0	0	0	0	0	0	0	0	0	0	0		0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0	0	1	0
4		0	0	1	1	1	0	0	0	0	0	0	0	1	0
5		0	0	1	1	1	1	1	0	0	0	0	1	1	0
6		0	0	1	1	1	0	1	1	1	1	1	1	1	0
7		0	0	1	1	1	1	1	1	1	1	1	1	1	0
8		0	0	1	1	1	1	0	0	0	0	0	0	1	0
9		0	0	1	1	1	0	1	0	0	0	0	0	1	0
10		0	0	0	0	0	0	1	0	0	0	0	0	1	0
11		0	0	0	0	0	0	0	0	0	0	0	0	1	0
12		0	0	0	0	0	0	0	0	0	0	0	0	1	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0

## ... 2. Basic Morphological operations ...2.1 Erosion

$$A \ominus B = \{z | (B)_z \subseteq A\}$$



**FIGURE 9.6** (a) Set  $A$ . (b) Square structuring element. (c) Erosion of  $A$  by  $B$ , shown shaded. (d) Elongated structuring element. (e) Erosion of  $A$  using this element.

## ... 2. Basic Morphological operations

### 2.2 Dilation

- Does the structuring element **hit the set?**
- dilation of a set  $A$  by structuring element  $B$ : all  $z$  in  $A$  such that  $B$  hits  $A$  when origin of  $B=z$

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$$

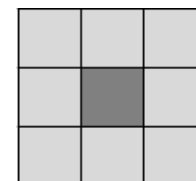
- **grow the object**



## ... 2. Basic Morphological operations ...2.2 Dilation

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$$

<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	1	1	1	1	1	0	0	0	0	1	1	1	0
3		0	1	1	1	1	1	0	0	0	0	1	1	1	0
4		0	1	1	1	1	1	1	1	0	1	1	1	1	1
5		0	1	1	1	1	1	1	1	1	1	1	1	1	1
6		0	1	1	1	1	1	1	1	1	1	1	1	1	1
7		0	1	1	1	1	1	1	1	1	1	1	1	1	1
8		0	1	1	1	1	1	1	1	1	1	1	1	1	1
9		0	1	1	1	1	1	1	1	0	0	1	1	1	1
10		0	1	1	1	1	1	1	1	0	0	1	1	1	1
11		0	0	0	0	0	1	1	1	0	0	1	1	1	0
12		0	0	0	0	0	0	0	0	0	0	1	1	1	0
13		0	0	0	0	0	0	0	0	0	0	1	1	1	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



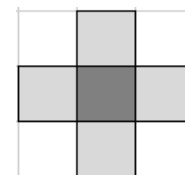
<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0	1	0	0
4		0	0	1	1	1	0	0	0	0	0	0	1	0	0
5		0	0	1	1	1	1	1	0	0	0	0	1	1	0
6		0	0	1	1	1	0	1	1	1	1	1	1	1	0
7		0	0	1	1	1	1	1	1	1	1	1	1	0	0
8		0	0	1	1	1	1	0	0	0	0	0	1	0	0
9		0	0	1	1	1	0	1	0	0	0	0	1	1	0
10		0	0	0	0	0	0	0	1	0	0	0	0	1	0
11		0	0	0	0	0	0	0	0	0	0	0	0	1	0
12		0	0	0	0	0	0	0	0	0	0	0	0	1	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



## ... 2. Basic Morphological operations ...2.2 Dilation

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$$

<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	1	1	1	0	0	0	0	0	0	1	0	0
3		0	1	1	1	1	1	0	0	0	0	1	1	1	0
4		0	1	1	1	1	1	1	0	0	0	1	1	1	0
5		0	1	1	1	1	1	1	1	1	1	1	1	1	1
6		0	1	1	1	1	1	1	1	1	1	1	1	1	1
7		0	1	1	1	1	1	1	1	1	1	1	1	1	0
8		0	1	1	1	1	1	1	1	1	1	1	1	1	0
9		0	1	1	1	1	1	1	1	0	0	1	1	1	1
10		0	0	1	1	1	1	1	1	0	0	1	1	1	0
11		0	0	0	0	0	0	1	0	0	0	1	1	1	0
12		0	0	0	0	0	0	0	0	0	0	1	1	1	0
13		0	0	0	0	0	0	0	0	0	0	0	1	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	1	1	0	0	0	0	0	0	1	0	0
4	0	0	1	1	1	0	0	0	0	0	0	1	0	0
5	0	0	1	1	1	1	1	0	0	0	0	1	1	0
6	0	0	1	1	1	0	1	1	1	1	1	1	1	0
7	0	0	1	1	1	1	1	1	1	1	1	1	0	0
8	0	0	1	1	1	1	0	0	0	0	0	1	0	0
9	0	0	1	1	1	0	1	0	0	0	0	0	1	0
10	0	0	0	0	0	0	1	0	0	0	0	1	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

## ... 2. Basic Morphological operations ...2.2 Dilation

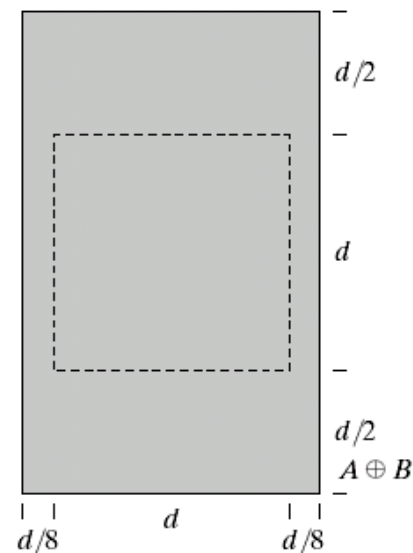
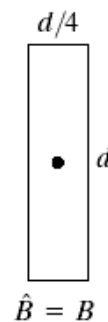
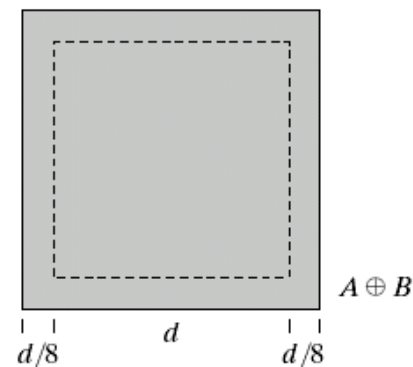
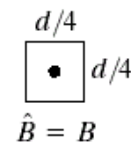
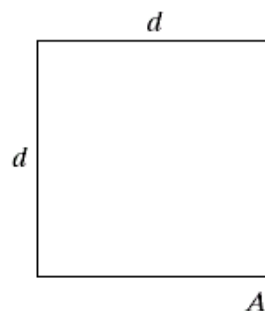
$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$$

$B$  = structuring element

a	b	c
d		e

**FIGURE 9.4**

- (a) Set  $A$ .  
 (b) Square structuring element (dot is the center).  
 (c) Dilation of  $A$  by  $B$ , shown shaded.  
 (d) Elongated structuring element.  
 (e) Dilation of  $A$  using this element.



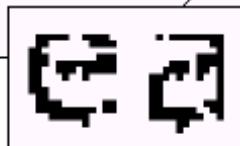


## ... 2. Basic Morphological operations ...2.2 Dilation

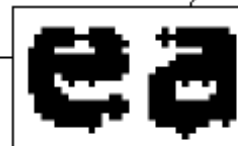


# Dilation: Bridging gaps

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



**Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.**



a b c

**FIGURE 9.5**  
(a) Sample text of poor resolution with broken characters (magnified view).  
(b) Structuring element.  
(c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0



## ... 2. Basic Morphological operations ...2.2 Dilation



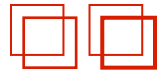
### Usefulness

- Erosion  
removal of structures of certain shape and size, given by SE
- Dilation  
filling of holes of certain shape and size, given by SE





## ... 2. Basic Morphological operations



# Combining erosion and dilation

### WANTED:

- remove structures / fill holes
- without affecting remaining parts

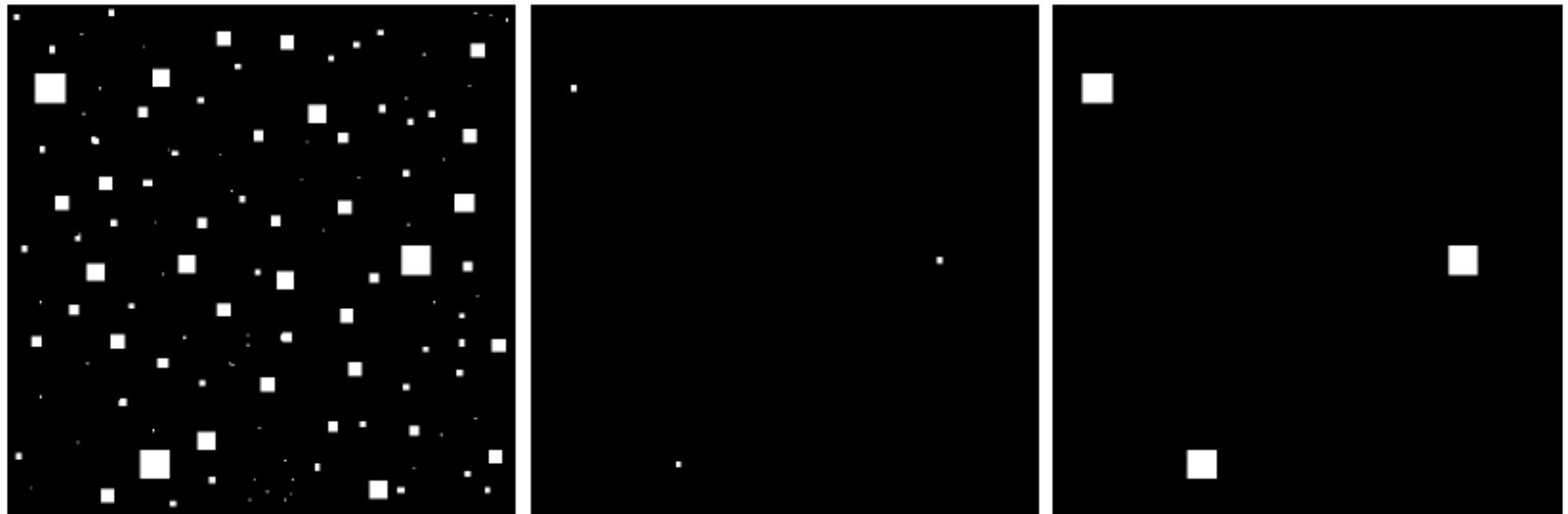
### SOLUTION:

- combine erosion and dilation
- (using same SE)



## ... 2. Basic Morphological operations

# Erosion : eliminating irrelevant detail



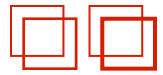
a b c

**FIGURE 9.7** (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element  $B = 13 \times 13$  pixels of gray level 1



## ... 2. Basic Morphological operations



### 2.3 Opening

erosion followed by dilation, denoted  $\circ$

$$A \circ B = (A \ominus B) \oplus B$$

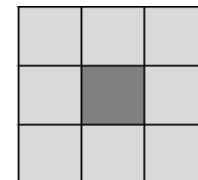
- eliminates protrusions
- breaks necks
- smoothes contour



## ... 2. Basic Morphological operations ...2.3 Opening

$$A \circ B = (A \ominus B) \oplus B$$

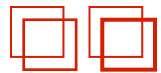
<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0		0	0
4		0	0	1	1	1	0	0	0	0	0	0		0	0
5		0	0	1	1	1			0	0	0				0
6		0	0	1	1	1	0								0
7		0	0	1	1	1								0	0
8		0	0	1	1	1		0	0	0	0	0		0	0
9		0	0	1	1	1	0		0	0	0	0			0
10		0	0	0	0	0	0		0	0	0	0		0	0
11		0	0	0	0	0	0	0	0	0	0	0		0	0
12		0	0	0	0	0	0	0	0	0	0	0		0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0	1	0	0
4		0	0	1	1	1	0	0	0	0	0	0	1	0	0
5		0	0	1	1	1	1	1	0	0	0	0	1	1	0
6		0	0	1	1	1	0	1	1	1	1	1	1	1	0
7		0	0	1	1	1	1	1	1	1	1	1	1	0	0
8		0	0	1	1	1	1	0	0	0	0	0	1	0	0
9		0	0	1	1	1	0	1	0	0	0	0	1	1	0
10		0	0	0	0	0	0	1	0	0	0	0	1	0	0
11		0	0	0	0	0	0	0	0	0	0	0	1	0	0
12		0	0	0	0	0	0	0	0	0	0	0	1	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0

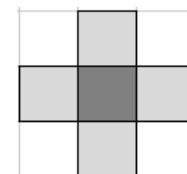


## ... 2. Basic Morphological operations ...2.3 Opening



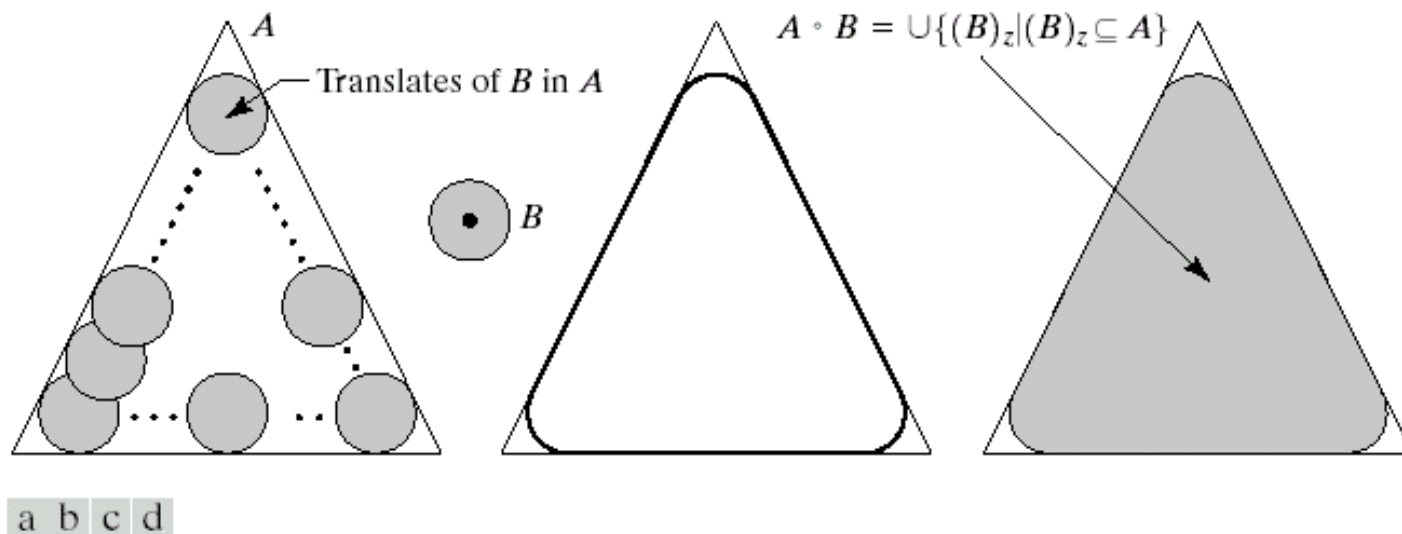
$$A \circ B = (A \ominus B) \oplus B$$

<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0		1		0	0	0	0	0	0		0	0
4		0	0	1	1	1	0	0	0	0	0	0	1	0	0
5		0	0	1	1	1	1		0	0	0	1	1	1	0
6		0	0	1	1	1	0				1	1	1	1	0
7		0	0	1	1	1	1					1	1	0	0
8		0	0	1	1	1	1	0	0	0	0	0		0	0
9		0	0		1	1	0		0	0	0	0			0
10		0	0	0	0	0	0		0	0	0	0		0	0
11		0	0	0	0	0	0	0	0	0	0	0		0	0
12		0	0	0	0	0	0	0	0	0	0	0		0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0	1	0	0
4		0	0	1	1	1	0	0	0	0	0	0	1	0	0
5		0	0	1	1	1	1	1	0	0	0	0	1	1	0
6		0	0	1	1	1	0	1	1	1	1	1	1	1	0
7		0	0	1	1	1	1	1	1	1	1	1	1	0	0
8		0	0	1	1	1	1	0	0	0	0	0	1	0	0
9		0	0	1	1	1	0	1	0	0	0	0	1	1	0
10		0	0	0	0	0	0	1	0	0	0	0	1	0	0
11		0	0	0	0	0	0	0	0	0	0	0	1	0	0
12		0	0	0	0	0	0	0	0	0	0	0	1	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0

## ... 2. Basic Morphological operations ...2.3 Opening



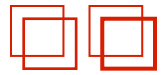
**FIGURE 9.8** (a) Structuring element  $B$  “rolling” along the inner boundary of  $A$  (the dot indicates the origin of  $B$ ). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$



## ... 2. Basic Morphological operations



### 2.4 Closing

dilation followed by erosion, denoted •

$$A \bullet B = (A \oplus B) \ominus B$$

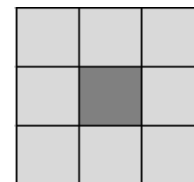
- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour



## ... 2. Basic Morphological operations ...2.4 Closing

$$A \bullet B = (A \oplus B) \ominus B$$

<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0						0	0	0	0				0
3		0		1	1	1		0	0	0	0		1		0
4		0		1	1	1				0			1		
5		0		1	1	1	1	1				1	1	1	
6		0		1	1	1	1	1	1	1	1	1	1	1	
7		0		1	1	1	1	1	1	1	1	1	1	1	
8		0		1	1	1	1	1					1	1	
9		0		1	1	1	1	1		0	0		1	1	
10		0						1		0	0		1		
11		0	0	0	0	0				0	0		1		0
12		0	0	0	0	0	0	0	0	0	0		1		0
13		0	0	0	0	0	0	0	0	0	0				0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	1	1	0	0	0	0	0	0	1	0	0
4	0	0	1	1	1	0	0	0	0	0	0	1	0	0
5	0	0	1	1	1	1	1	0	0	0	0	1	1	0
6	0	0	1	1	1	0	1	1	1	1	1	1	1	0
7	0	0	1	1	1	1	1	1	1	1	1	1	1	0
8	0	0	1	1	1	1	0	0	0	0	0	1	0	0
9	0	0	1	1	1	0	1	0	0	0	0	1	1	0
10	0	0	0	0	0	0	1	0	0	0	0	1	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

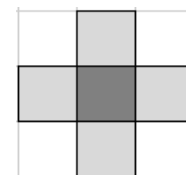




## ... 2. Basic Morphological operations ...2.4 Closing

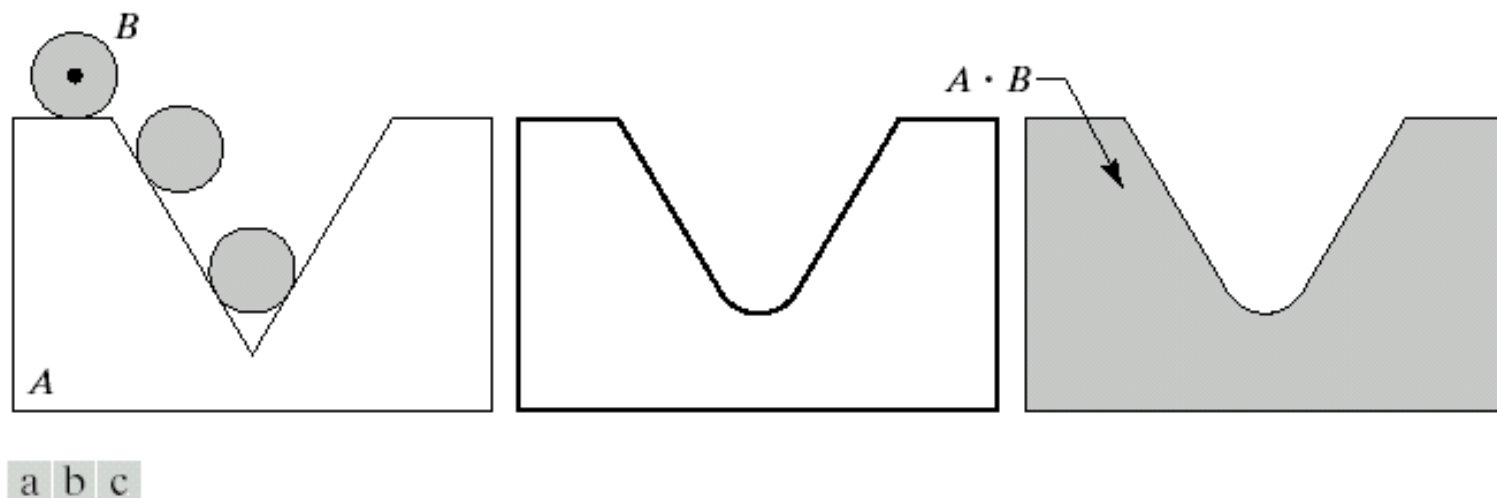
$$A \bullet B = (A \oplus B) \ominus B$$

<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0				0	0	0	0	0	0		0	0
3		0		1	1	1		0	0	0	0		1		0
4		0		1	1	1	1		0	0	0		1		0
5		0		1	1	1	1	1				1	1	1	
6		0		1	1	1	1	1	1	1	1	1	1	1	
7		0		1	1	1	1	1	1	1	1	1	1		0
8		0		1	1	1	1	1	1			1	1		0
9		0		1	1	1	1	1		0	0		1	1	
10		0	0					1		0	0		1		0
11		0	0	0	0	0	0		0	0	0		1		0
12		0	0	0	0	0	0	0	0	0	0		1		0
13		0	0	0	0	0	0	0	0	0	0	0		0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0



<b>A</b>		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	1	1	1	0	0	0	0	0	0	1	0	0
4		0	0	1	1	1	0	0	0	0	0	0	1	0	0
5		0	0	1	1	1	1	1	0	0	0	0	1	1	0
6		0	0	1	1	1	0	1	1	1	1	1	1	1	0
7		0	0	1	1	1	1	1	1	1	1	1	1	0	0
8		0	0	1	1	1	1	0	0	0	0	0	1	0	0
9		0	0	1	1	1	0	1	0	0	0	0	1	1	0
10		0	0	0	0	0	0	1	0	0	0	0	1	0	0
11		0	0	0	0	0	0	0	0	0	0	0	1	0	0
12		0	0	0	0	0	0	0	0	0	0	0	1	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0

## ... 2. Basic Morphological operations ...2.4 Closing



**FIGURE 9.9** (a) Structuring element  $B$  “rolling” on the outer boundary of set  $A$ . (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$$A \bullet B = (A \oplus B) \ominus B$$



## ... 2. Basic Morphological operations



# Properties

### Opening

- (i)  $A \circ B$  is a subset (subimage) of  $A$
- (ii) If  $C$  is a subset of  $D$ , then  $C \circ B$  is a subset of  $D \circ B$
- (iii)  $(A \circ B) \circ B = A \circ B$

### Closing

- (i)  $A$  is a subset (subimage) of  $A \bullet B$
- (ii) If  $C$  is a subset of  $D$ , then  $C \bullet B$  is a subset of  $D \bullet B$
- (iii)  $(A \bullet B) \bullet B = A \bullet B$

**Note:** repeated openings/closings has no effect!



## ... 2. Basic Morphological operations



# Duality

- Opening and closing are dual with respect to complementation and reflection

$$(A \bullet B)^c = (A^c \circ \hat{B})$$



## ... 2. Basic Morphological operations



$A$

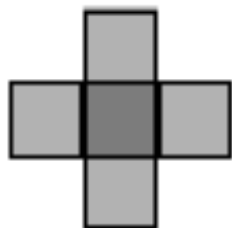


$A \ominus B$



$(A \ominus B)^C$

$$B = \hat{B}$$



$A^C$

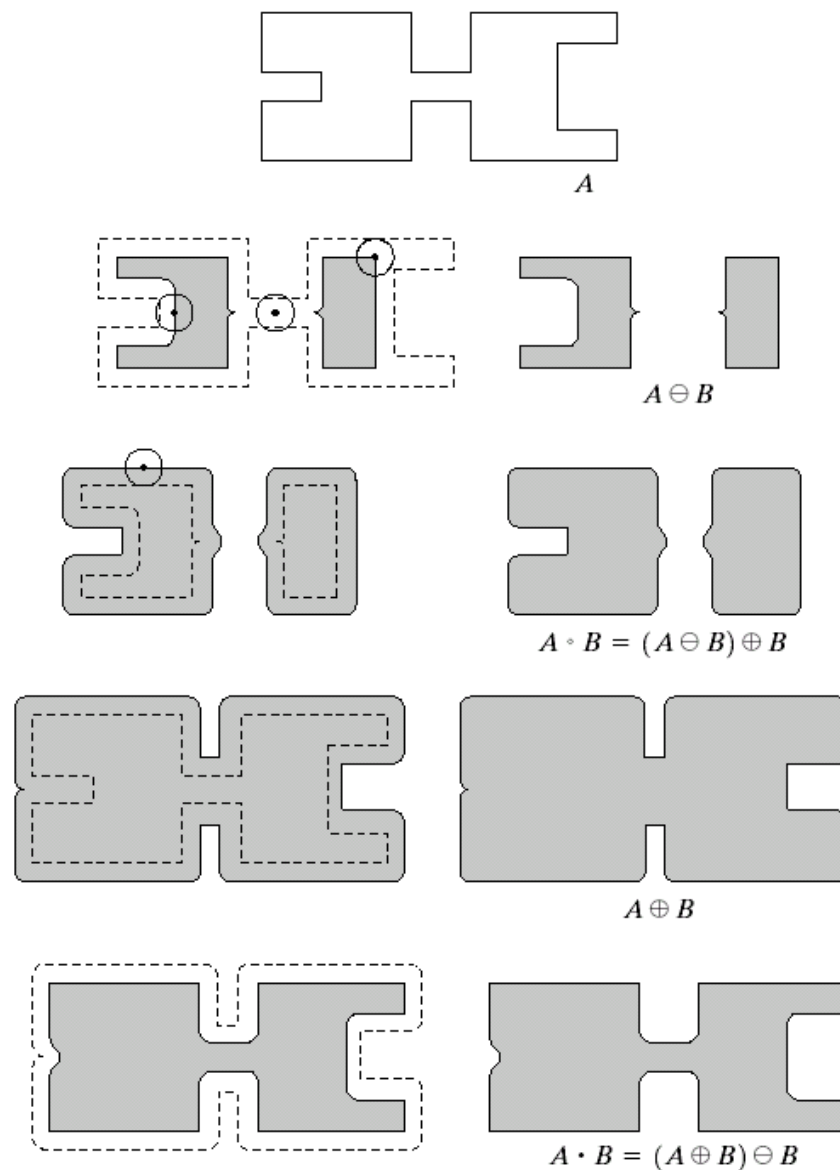


$A^C \oplus B$

## ... 2. Basic Morphological operations

a
b c
d e
f g
h i

**FIGURE 9.10**  
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.





## ... 2. Basic Morphological operations



# Usefulness: open & close



A



opening of A

→ removal of small protrusions, thin connections, ...



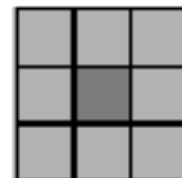
closing of A

→ removal of holes

# 3. Applications

## 3.1 Filtering

Application:  
filtering



1. erode  
 $A \ominus B$

3. dilate  
 $(A \circ B) \oplus B$



2. dilate  
 $(A \ominus B) \oplus B = A \circ B$

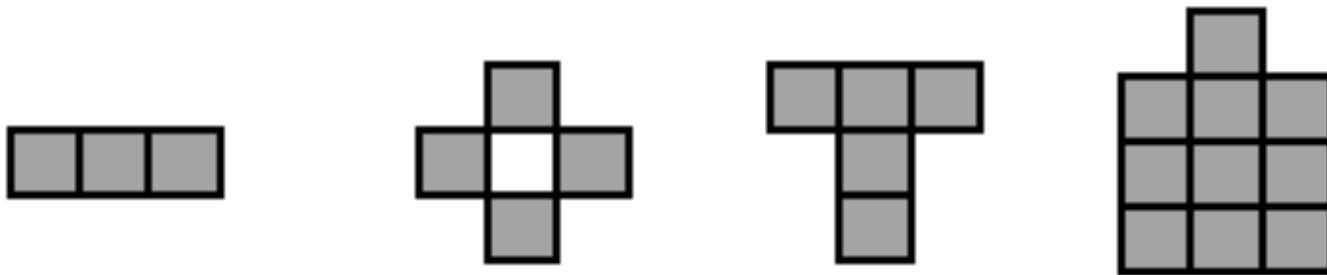
4. erode  
 $((A \circ B) \oplus B) \ominus B = (A \circ B) \bullet B$



## ... 3. Applications of Morphological Operations

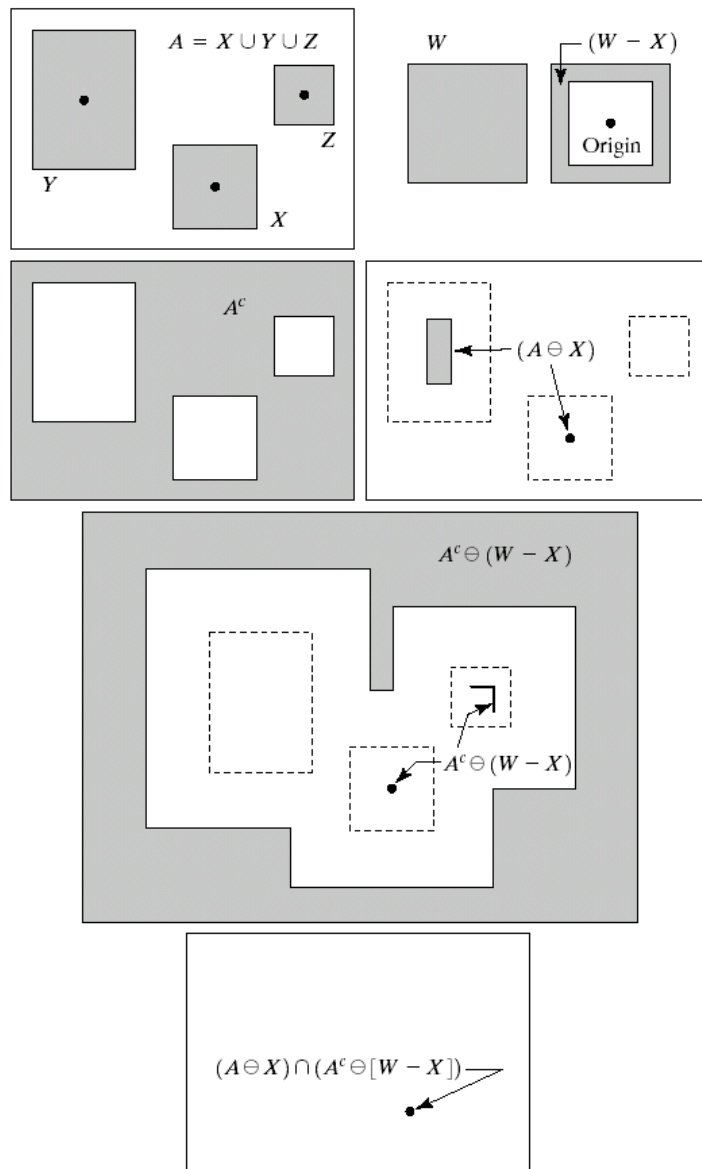
### 3.2. Hit-or-Miss Transformation $\circledast$ (HMT)

- find location of one shape among a set of shapes  
"template matching"

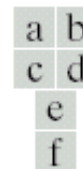


- composite SE: object part (B1) and background part (B2)
- does B1 *fits the object while, simultaneously,* B2 misses the object, i.e., *fits the background?*

# ...Hit-or-Miss Transformation



$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

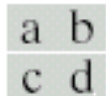


**FIGURE 9.12**

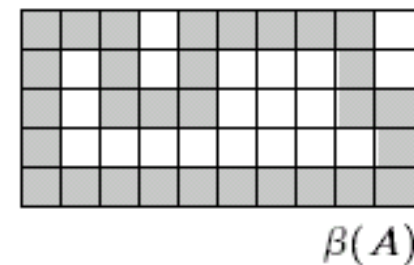
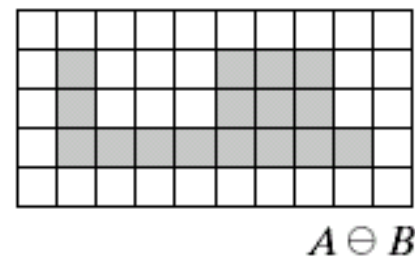
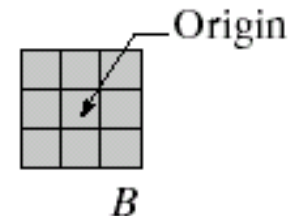
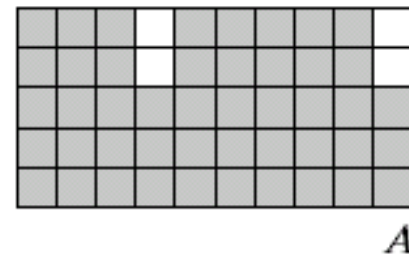
(a) Set  $A$ . (b) A window,  $W$ , and the local background of  $X$  with respect to  $W$ ,  $(W - X)$ . (c) Complement of  $A$ . (d) Erosion of  $A$  by  $X$ . (e) Erosion of  $A^c$  by  $(W - X)$ . (f) Intersection of (d) and (e), showing the location of the origin of  $X$ , as desired.

## ... 3. Applications of Morphological Operations

### 3.3. Boundary Extraction



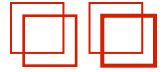
**FIGURE 9.13** (a) Set  $A$ . (b) Structuring element  $B$ . (c)  $A$  eroded by  $B$ . (d) Boundary, given by the set difference between  $A$  and its erosion.



$$\beta(A) = A - (A \ominus B)$$



# Example



a b

## FIGURE 9.14

(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

## ... 3. Applications of Morphological Operations

### 3.4. Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$

a	b	c
d	e	f
g	h	i

**FIGURE 9.15**

Region filling.

(a) Set  $A$ .

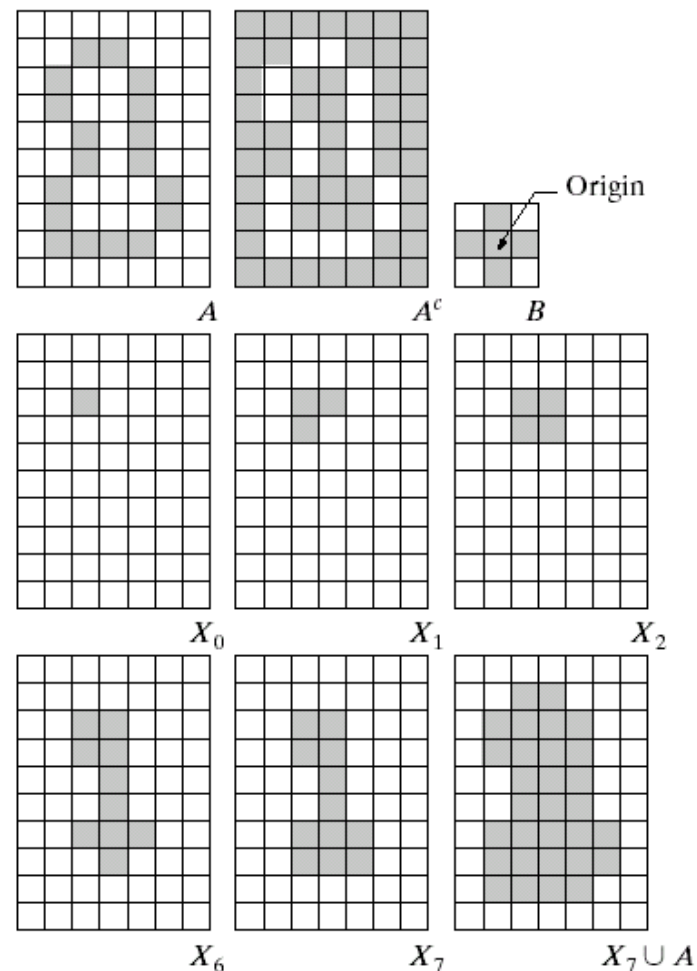
(b) Complement of  $A$ .

(c) Structuring element  $B$ .

(d) Initial point inside the boundary.

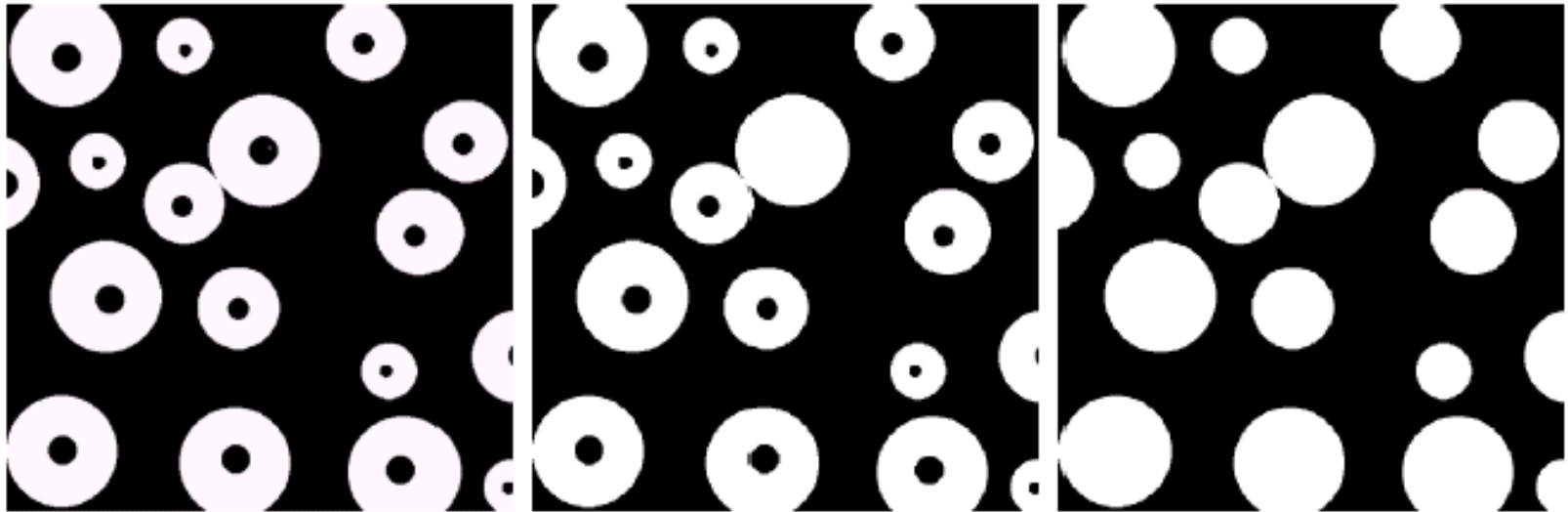
(e)–(h) Various steps of Eq. (9.5-2).

(i) Final result [union of (a) and (h)].





# Example



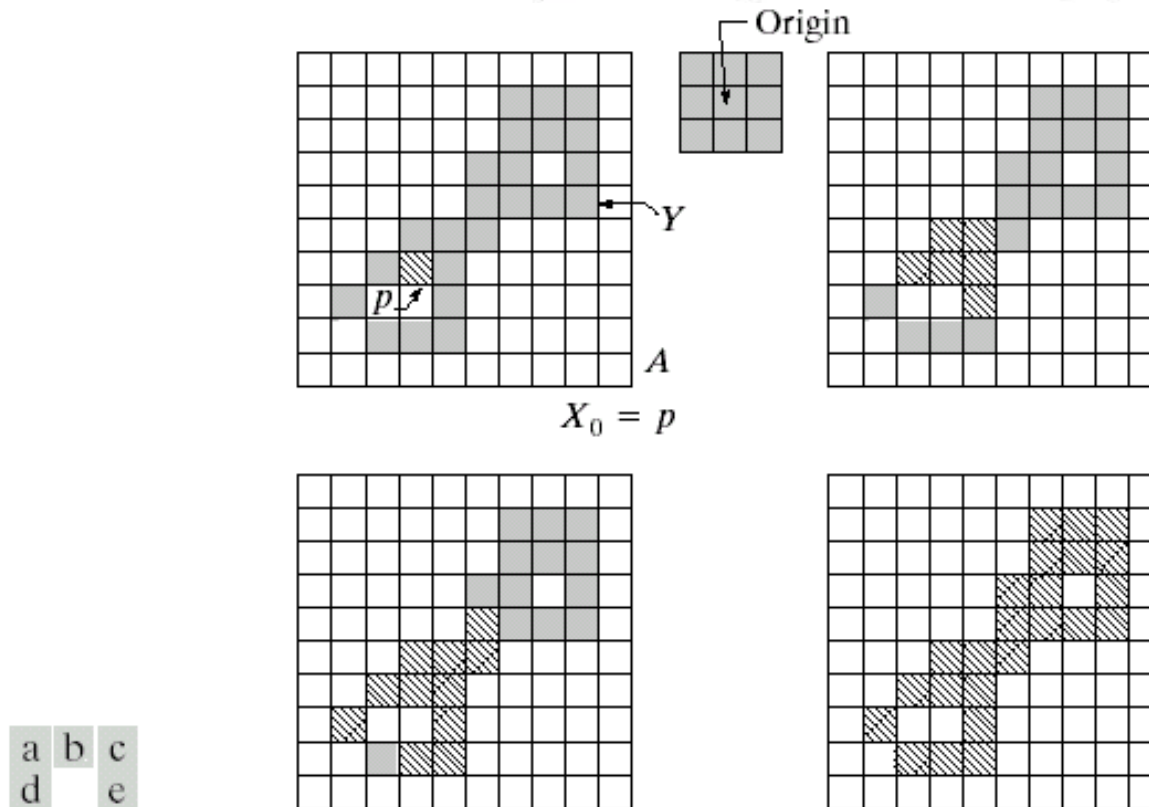
a b c

**FIGURE 9.16** (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

## ... 3. Applications of Morphological Operations

### 3.5. Extraction of connected components

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \dots$$



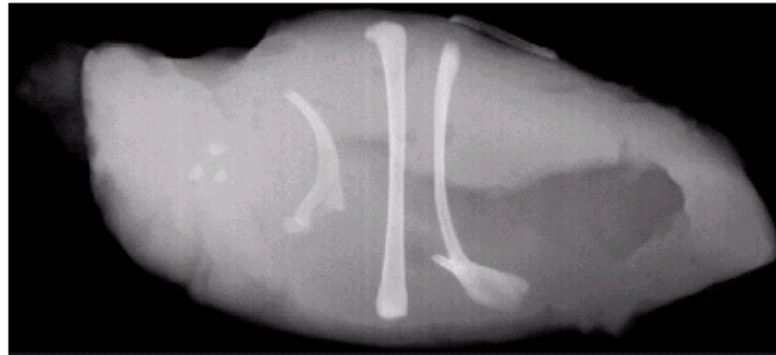
**FIGURE 9.17** (a) Set  $A$  showing initial point  $p$  (all shaded points are valued 1, but are shown different from  $p$  to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

# Example

a  
b  
c d

**FIGURE 9.18**

(a) X-ray image of chicken filet with bone fragments.  
(b) Thresholded image. (c) Image eroded with a  $5 \times 5$  structuring element of 1's.  
(d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, [www.ntbxray.com](http://www.ntbxray.com).)



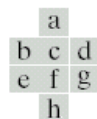
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85



## ... 3. Applications of Morphological Operations

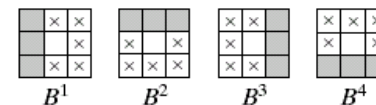
### 3.6. Convex Hull

A set  $A$  is said to be convex if the straight line segment joining any two points in  $A$  lies entirely within  $A$ .



**FIGURE 9.19**  
(a) Structuring elements. (b) Set  $A$ . (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

$$X_k^i = (X_k^{i-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \dots$$



$B^1$



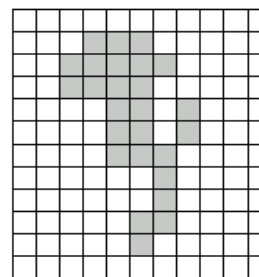
$B^2$



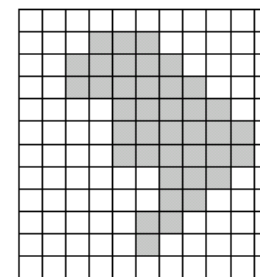
$B^3$



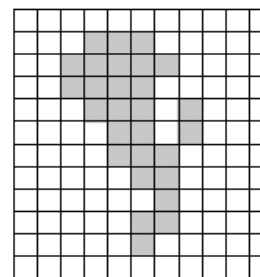
$B^4$



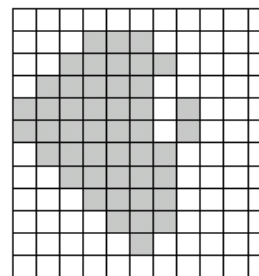
$X_0^1 = A$



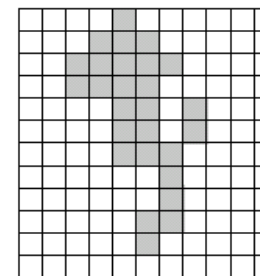
$X_4^1$



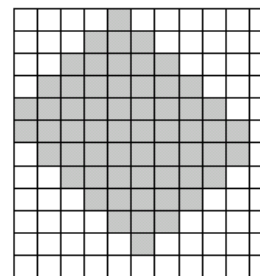
$X_2^2$



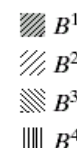
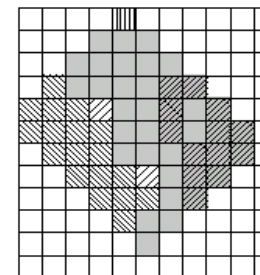
$X_8^3$



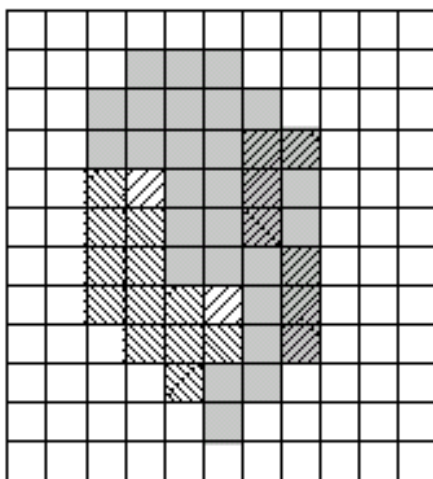
$X_2^4$



$C(A)$



$$C(A) = \bigcup_{i=1}^4 D^i$$



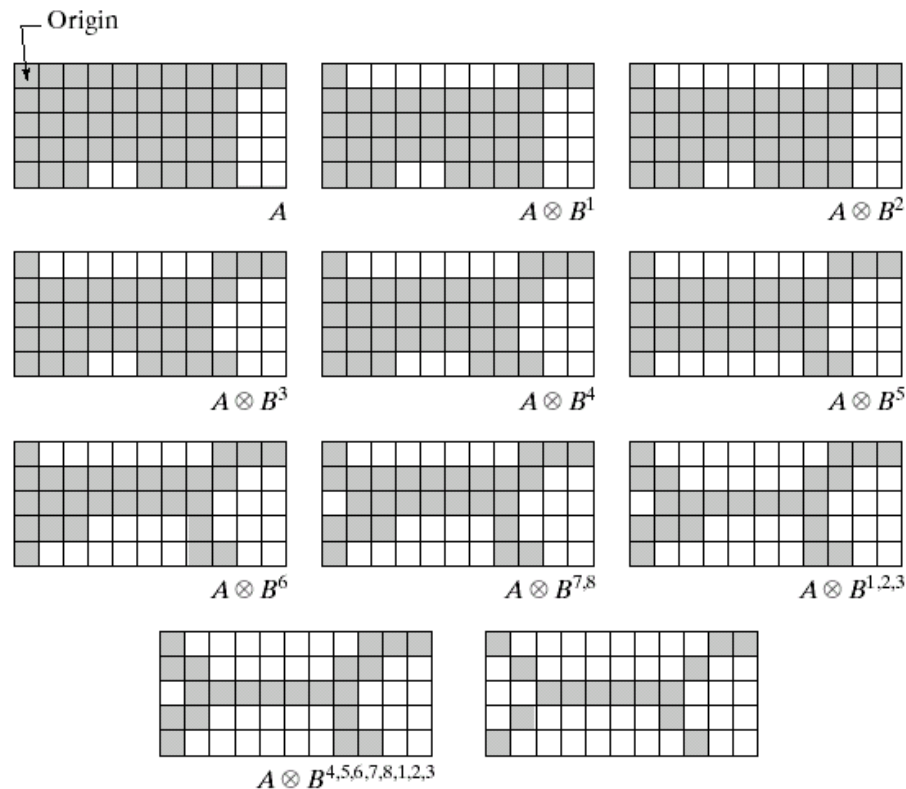
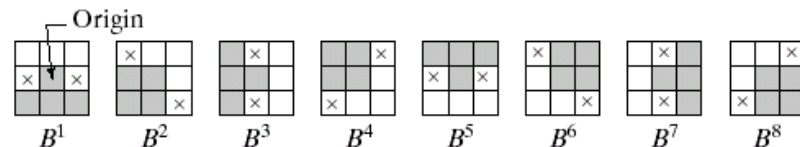
**FIGURE 9.20** Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

# ... 3. Applications of Morphological Operations

## 3.7. Thinning

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

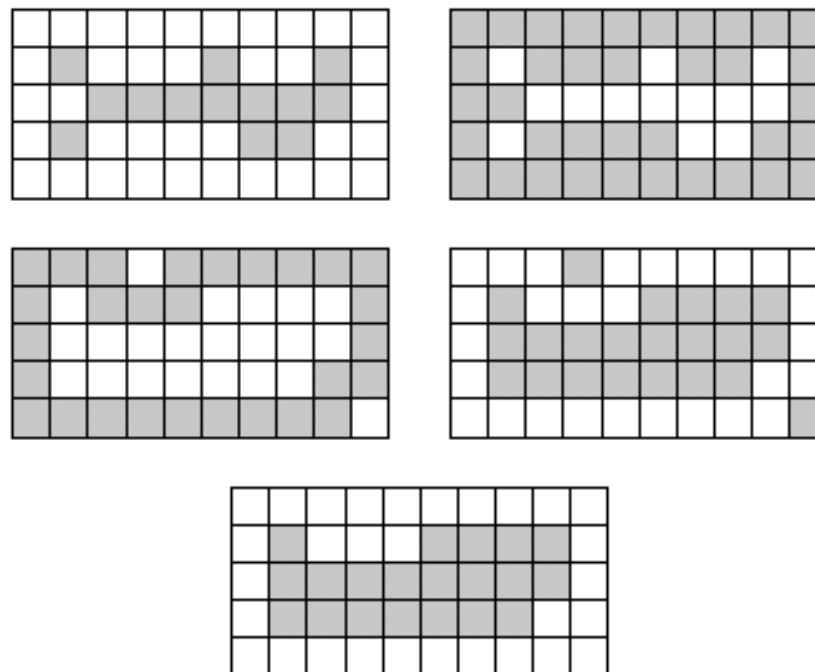


**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set  $A$ . (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to  $m$ -connectivity.

## ... 3. Applications of Morphological Operations

### 3.8. Thickening

$$A \odot B = A \cup (A \otimes B)$$



a b  
c d  
e

**FIGURE 9.22** (a) Set  $A$ . (b) Complement of  $A$ . (c) Result of thinning the complement of  $A$ . (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

## ... 3. Applications of Morphological Operations

### 3.9. Skeletons

$$S(A) = \bigcup_{k=0}^K S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

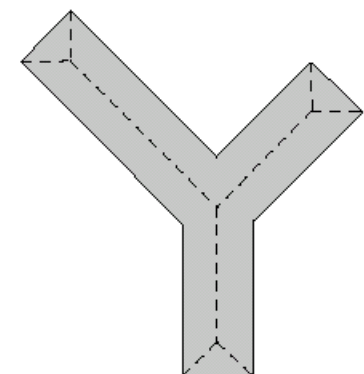
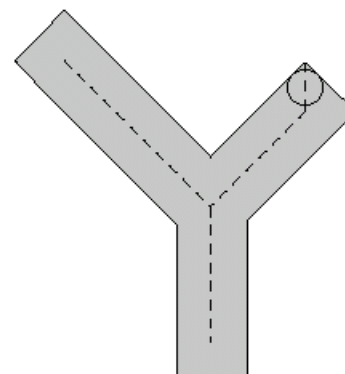
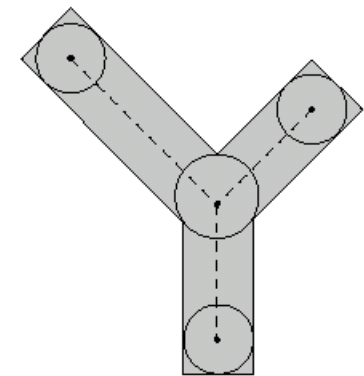
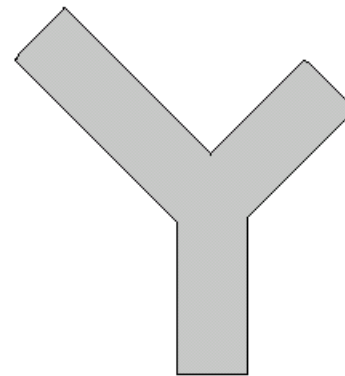
$$K = \max \{k \mid (A - kB) \neq \Phi\}$$

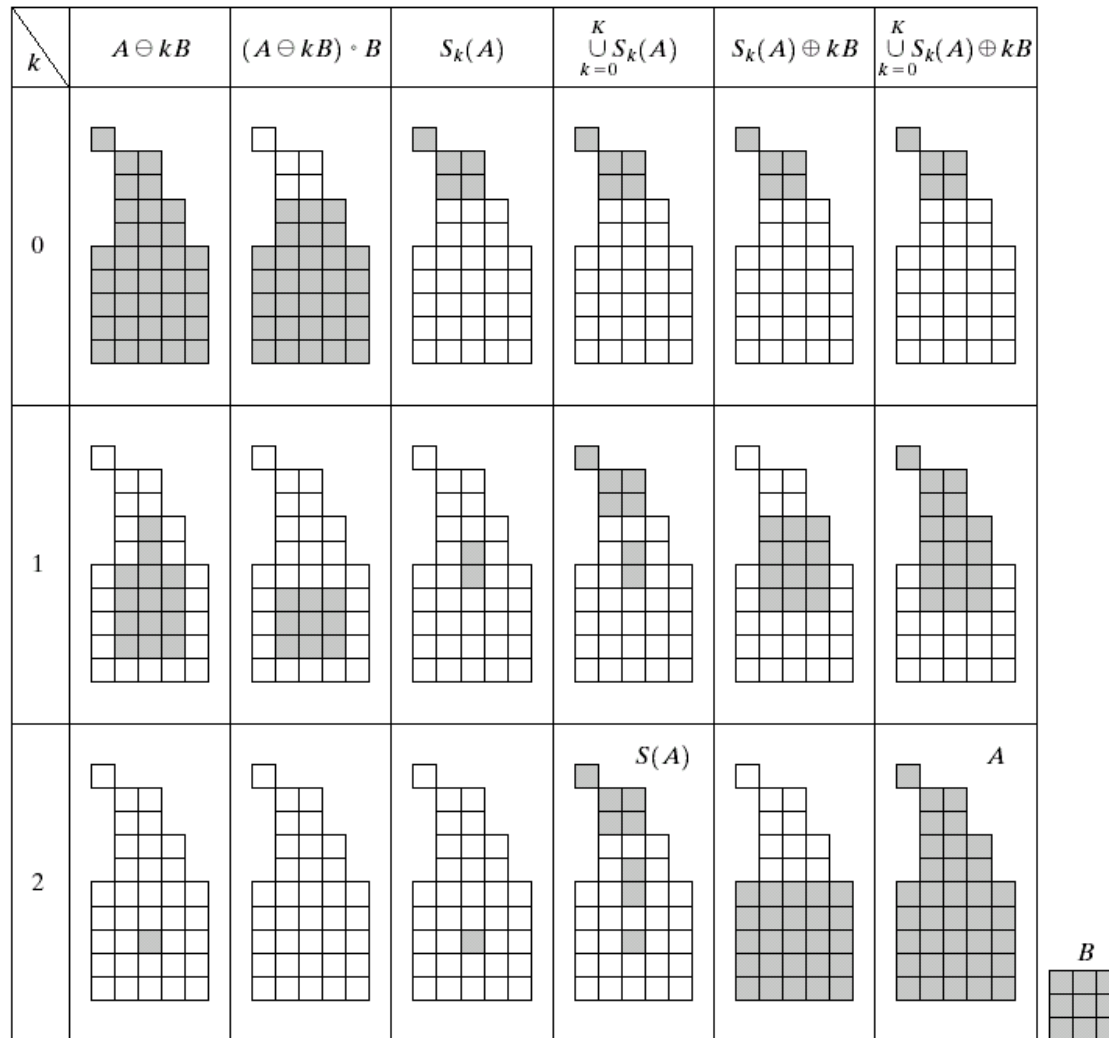
$$A = \bigcup_{k=0}^K (S_k(A) \oplus kB)$$

a	b
c	d

**FIGURE 9.23**

- (a) Set  $A$ .  
 (b) Various positions of maximum disks with centers on the skeleton of  $A$ .  
 (c) Another maximum disk on a different segment of the skeleton of  $A$ .  
 (d) Complete skeleton.





**FIGURE 9.24** Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

# ... 3. Applications of Morphological Operations

## 3.10. Pruning

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^8 (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

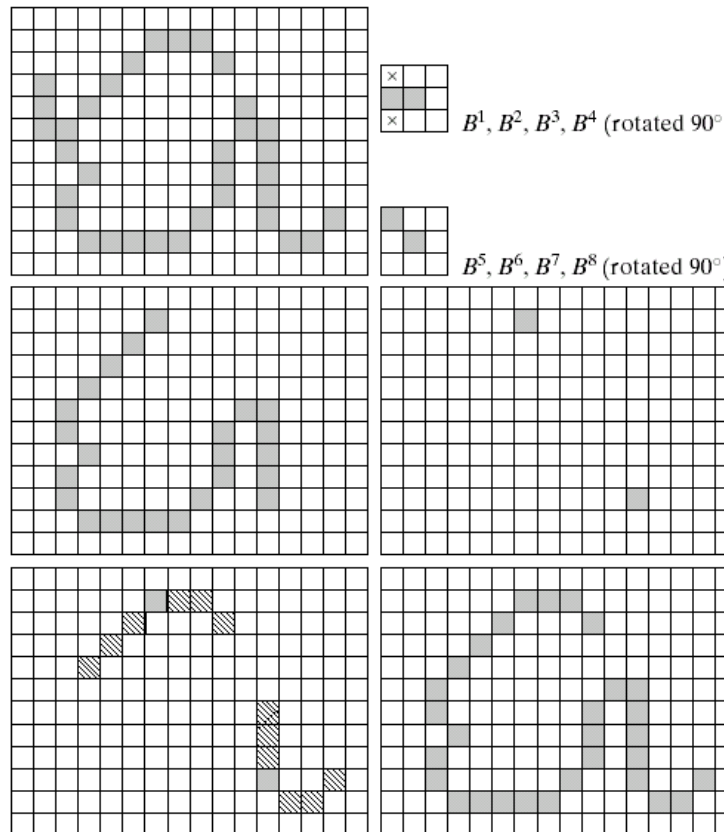
$$X_4 = X_1 \cup X_3$$

$H = 3 \times 3$  structuring element of 1's

a b  
 c  
 d e  
 f g

**FIGURE 9.25**

(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.



# Summary of Morphological Operations

**TABLE 9.2**  
Summary of  
morphological  
operations and  
their properties.

Operation		Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation		$(A)_z = \{w   w = a + z, \text{ for } a \in A\}$	Translates the origin of $A$ to point $z$ .
Reflection		$\hat{B} = \{w   w = -b, \text{ for } b \in B\}$	Reflects all elements of $B$ about the origin of this set.
Complement		$A^c = \{w   w \notin A\}$	Set of points not in $A$ .
Difference		$A - B = \{w   w \in A, w \notin B\}$ $= A \cap B^c$	Set of points that belong to $A$ but not to $B$ .
Dilation		$A \oplus B = \{z   (\hat{B})_z \cap A \neq \emptyset\}$	“Expands” the boundary of $A$ . (I)
Erosion		$A \ominus B = \{z   (B)_z \subseteq A\}$	“Contracts” the boundary of $A$ . (I)
Opening		$A \circ B = (A \ominus B) \oplus B$	Smooths contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing		$A \bullet B = (A \oplus B) \ominus B$	Smooths contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)



## ... Summary of Morphological Operations

Hit-or-miss  
transform

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) \\ = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

The set of points  
(coordinates) at which,  
simultaneously,  $B_1$  found  
a match (“hit”) in  $A$  and  
 $B_2$  found a match in  $A^c$ .

Boundary  
extraction

$$\beta(A) = A - (A \ominus B)$$

Set of points on the  
boundary of  
set  $A$ . (I)

Region filling

$$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p \text{ and } k = 1, 2, 3, \dots$$

Fills a region in  $A$ , given a  
point  $p$  in the region. (II)

Connected  
components

$$X_k = (X_{k-1} \oplus B) \cap A; X_0 = p \text{ and } k = 1, 2, 3, \dots$$

Finds a connected  
component  $Y$  in  $A$ , given  
a point  $p$  in  $Y$ . (I)

Convex hull

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A; i = 1, 2, 3, 4; \\ k = 1, 2, 3, \dots; X_0^i = A; \text{ and } \\ D^i = X_{\text{conv}}^i.$$

Finds the convex hull  $C(A)$   
of set  $A$ , where “conv”  
indicates convergence  
in the sense that  
 $X_k^i = X_{k-1}^i$ . (III)

## ... Summary of Morphological Operations

Operation	Equation	Comments
		(The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\dots ((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$	<p>Thins set <math>A</math>. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)</p>
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\dots (A \odot B^1) \odot B^2 \dots) \odot B^n)$	<p>Thickens set <math>A</math>. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.</p>

**TABLE 9.2**  
Summary of morphological results and their properties.  
(continued)

## ... Summary of Morphological Operations

Skeletons

$$S(A) = \bigcup_{k=0}^K S_k(A)$$

$$S_k(A) = \bigcup_{k=0}^K \{ (A \ominus kB) - [(A \ominus kB) \circ B] \}$$

Reconstruction of  $A$ :

$$A = \bigcup_{k=0}^K (S_k(A) \oplus kB)$$

Finds the skeleton  $S(A)$  of set  $A$ . The last equation indicates that  $A$  can be reconstructed from its skeleton subsets  $S_k(A)$ . In all three equations,  $K$  is the value of the iterative step after which the set  $A$  erodes to the empty set. The notation  $(A \ominus kB)$  denotes the  $k$ th iteration of successive erosion of  $A$  by  $B$ . (I)

Pruning

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^8 (X_1 \otimes B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$

$X_4$  is the result of pruning set  $A$ . The number of times that the first equation is applied to obtain  $X_1$  must be specified. Structuring elements  $V$  are used for the first two equations. In the third equation  $H$  denotes structuring element  $I$ .