CAPM and Multi-factor Models

The objective of this homework is to test if some anomalies observed in U.S. stock markets can be explained by the 5-factor of Fama and French (2015). For this purpose, we consider portfolios based on some well-known fundamental corporate variables, such as size or book-to-market ratio, operating profitability, and investment. We use a long sample of data from July 1963 to December 2018. The XL file contains monthly percent returns for the portfolios (folders Size-BM, Size-OP, and Size-Inv) and the 5 Fama-French factors (folder 5FFF). The risk-free rate is the last column of this last folder.

1. Characteristics of the fundamental portfolios

We begin with portfolios based on (size, book-to-market ratio), (size, operating profitability), and (size, investment). For each pair of fundamental variables, we have a (5x5) matrix of portfolios.

a) For each portfolio, compute the average excess return (average return less risk-free rate) and the Sharpe ratio. You should have 2 matrices of dimension (5x5) for each pair of fundamental variables.

b) Comments each of these tables. The goal is to identify how fundamental variables affect the performance. For example, if you work on the pair of fundamental variables (*x*,*y*), one could observe : regardless of y, as fundamental variable x increases the average excess return increases. This is more pronounced when y is high.

c) Based on your results, what do you think is the pair of fundamentals most likely to generate portfolios with a positive alpha in a CAPM. Explain your reasoning in a few lines.

d) Based on your results, what do you think is the pair of fundamentals most likely to generate portfolios with a positive alpha in a 3-factor model. Explain your reasoning in a few lines.

2. Characteristics of the Fama-French factors

a) For each of the 5 factors, compute the average excess return, the Sharpe ratio for each factor and report the correlation matrix between the factors.

b) Comment your results.

c) If you were to add a factor to the 3-factor model, would you choose RMW or CMA? Why?

3. Test of the CAPM, 3-factor and 5-factor models with (size , B/M) portfolios

a) CAPM

i. We start with the test of the CAPM using the time series approach. We use the (size, book-to-market ratio) portfolios for the moment. For a given portfolio, consider the OLS regression

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + e_{i,t} \qquad t=1,...T, \ i=1,..., N$$

Run the regression for each portfolio (25 in total), but report the α_i and β_i parameters and their t-stats only for the (high book-to-market ratio, size) and (low book-to-market ratio, size) portfolios (10 portfolios in total). What do you observe?

ii. We test the null hypothesis $H_0^M: \alpha_1 = \cdots = \alpha_N = 0$, against the alternative hypothesis $H_a^M: \exists i$ such that $\alpha_i \neq 0$, $i=1, ..., N$. So, we test that the alphas for the 25 portfolios built on the fundamental pair (size, book-to-market ratio) are jointly equal to zero. Perform the test of H_0^M using OLS regressions and the Wald test procedure. Describe how you proceed. Present the results of the tests and comment.

b) 3-Factor model

i. We now test of the 3-factor model <u>using the same approach</u>:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + e_{i,t}$$

As for the CAPM, report the $\alpha_i, \beta_i, s_i, h_i$ parameters for the 10 portfolios given by (high book-to-market ratio, size) and (low book-to-market ratio, size). Comment your results.

ii. What is your conclusion of the test of the null hypothesis is $H_0^{3FF}: \alpha_1 = \cdots = \alpha_N = 0$, against the alternative hypothesis $H_a^{3FF}: \exists i$ such that $\alpha_i \neq 0$, $i=1,...,N$?

c) 5-Factor model

i. Finally, we test of the 5-factor model <u>using the same approach</u>:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{i,t}$$

As before report your results and comment.

ii. Again comment your results and test of the null hypothesis is $H_0^{5FF}: \alpha_1 = \cdots = \alpha_N = 0$, against the alternative hypothesis $H_a^{5FF}: \exists i$ such that $\alpha_i \neq 0$, $i=1,...,N$. What do you conclude?

4. Test of the CAPM, 3-factor, and 5-factor models with other fundamental portfolios

We now run the same analysis (redo question 3) for the other portfolios based on the pair of fundamentals (size, operating profitability).

Give your analysis of the results based on these portfolios.