Appendix: The Proof of Theorem 2

1 Introduction

This material gives the detailed proof of Theorem 2:

Theorem 2. *With given* $\mathbf{B}$ *and* $\mathbf{C}$*, if* $\rho > \max\{\rho_1, \rho_2, \rho_3\}$*:*

$$\rho_1 = 6N\tau\left(\|\mathbf{B}\|_F^4 + \|\mathbf{C}\|_F^4\right) / \left(\|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2\right)$$

$$\rho_2 + 2\|\mathbf{E}\|_F^2 = \frac{6}{\rho_2}\left(16N + N\tau\left(\|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2\right)\right)^2$$

$$\rho_3 = ||\mathbf{B}||_F^2 + ||\mathbf{C}||_F^2 + ||\mathcal{R}_p(\mathbf{E}) + \mathcal{R}_q(\mathbf{E})||_F^2$$

We can claim that:

- *The equality constraint on the auxiliary matrix is satisfied in the limit, i.e.,* $\lim_{t\to\infty}||\mathbf{H}^{(t)} - \mathbf{V}^{(t)}||_F^2 = 0$*.*
- *The sequence* $\{\mathbf{H}^{(t)}, \mathbf{V}^{(t)}, \mathbf{\Lambda}^{(t)}\}$ *generated by the NS-Alternating algorithm is bounded, and every limit point of the sequence is a KKT point of problem (6) of our paper.*

We first give the lemmas of our theorem and then present the derivation of these lemmas.

2 Lemmas

According to the study [Hong *et al.*, 2016], to give Theorem 2, we need to ensure that: (1) The size of the successive difference of the multipliers is bounded by that of the successive difference of the primal variables. (2) The augmented Lagrangian is decreasing and lower bounded.

Lemma 1. *We have a bounded successive difference of the multipliers, that is,*

$$\begin{aligned}\left\|\mathbf{\Lambda}^{(t+1)} - \mathbf{\Lambda}^{(t)}\right\|_F^2 \leq \quad & 3c_1 \cdot \left\|\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right\|_F^2 \\ & + 3c_2 \cdot \left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 \\ & + 3c_3 \cdot \left\|\mathbf{H}^{(t+1)}\left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2,\end{aligned}$$

where $c_i, i = \{1,2,3\}$ *are positive scalars:*

$$\begin{aligned} c_1 =& \left(16N + \tau N\left(\|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2\right)\right)^2 \\ c_2 =& \|\mathbf{B}\|_F^2 \cdot \left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{B}\right)^T - \mathbf{M}\right\|_F^2 \\ & + \|\mathbf{C}\|_F^2 \cdot \left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{C}\right)^T - \mathbf{S}\right\|_F^2 \\ c_3 =& N\tau \cdot \left(\|\mathbf{B}\|_F^4 \cdot \|\mathbf{C}\|_F^4\right) \end{aligned} \tag{1}$$

Lemma 2. *If the equations below are satisfied,*

$$\begin{aligned} \rho \ &> \max\{\rho_1, \rho_2\} \\ \rho_1 =& 6N\tau\left(\|\mathbf{B}\|_F^4 + \|\mathbf{C}\|_F^4\right) / \left(\|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2\right) \\ \rho_2 =& -2\|\mathbf{E}\|_F^2 + \frac{6}{\rho_2}\left(16N + N\tau\left(\|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2\right)\right)^2 \\ \beta^{(t)} >& -\rho + \frac{6}{\rho}\|\mathbf{B}\|_F^2 \cdot \left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T - \mathbf{M}\right\|_F^2 \\ & + \frac{6}{\rho}\|\mathbf{C}\|_F^2 \cdot \left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T - \mathbf{S}\right\|_F^2 \end{aligned}$$

we have positive scalars $c_i, i = \{1,2,3,4\}$ *so that:*

$$\begin{aligned} &\mathcal{L}\left(\mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)}\right) - \mathcal{L}\left(\mathbf{H}^{(t)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t)}\right) \\ < & -c_1\left\|\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right\|_F^2 - c_2\left\|(\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)})\mathbf{V}^{(t+1)^T}\right\|_F^2 \\ & -c_3\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 - c_4\left\|\mathbf{H}^{(t)}(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)})\right\|_F^2 \end{aligned}$$

Lemma 3. *If the equation below is satisfied,*

$$\rho \geq \|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2 + 2\|\mathcal{R}_p(\mathbf{E}) + \mathcal{R}_q(\mathbf{E})\|_F^2$$

we have lower bound of 0*, that is,*

$$\mathcal{L}\left(\mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)}\right) \geq 0$$

3 Derivations

For the ease of derivation, we introduce elementary matrices, $\mathbf{E}_1$ and $\mathbf{E}_2$, to characterize the rotating operator $\mathcal{R}_p$ and $\mathcal{R}_q$ respectively.

Derivation of Lemma 1:

We first give the optimal condition of $\mathbf{\Lambda}$ as follows:

$$\begin{aligned} &\left(\mathbf{H}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right) \cdot \mathbf{V}^{(t+1)}\mathbf{B}^T \\ &+ \left(\mathbf{H}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right) \cdot \mathbf{V}^{(t+1)}\mathbf{C}^T \\ &+ \rho\left(\mathbf{H} - \mathbf{V}^{(t+1)} - \mathbf{\Lambda}^{(t)}/\rho\right) \\ =& -2\left(\mathbf{E}_1^T\mathbf{E}_1 - \mathbf{E}_1^T\mathbf{E}_2 - \mathbf{E}_2^T\mathbf{E}_1 + \mathbf{E}_2^T\mathbf{E}_2\right)\mathbf{H} \end{aligned} \tag{2}$$

Together with the updating rule of $\mathbf{\Lambda}$, we obtain:

$$\begin{aligned} \mathbf{\Lambda}^{(t+1)} =& \left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right) \cdot \mathbf{V}^{(t+1)}\mathbf{B}^T \\ &+ \left(\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right) \cdot \mathbf{V}^{(t+1)}\mathbf{C}^T \\ &+ 2\left(\mathbf{E}_1^T\mathbf{E}_1 - \mathbf{E}_1^T\mathbf{E}_2 - \mathbf{E}_2^T\mathbf{E}_1 + \mathbf{E}_2^T\mathbf{E}_2\right)\mathbf{H}^{(t+1)} \end{aligned} \tag{3}$$

Then, the successive difference of $\mathbf{\Lambda}$ is given below:

$$
\begin{aligned}
&\mathbf{\Lambda}^{(t+1)}-\mathbf{\Lambda}^{(t)}\\
=&\left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T}-\mathbf{M}\right)\cdot\mathbf{V}^{(t+1)}\mathbf{B}^T\\
&+\left(\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T}-\mathbf{S}\right)\cdot\mathbf{V}^{(t+1)}\mathbf{C}^T\\
&+2\left(\mathbf{E}_1^T\mathbf{E}_1-\mathbf{E}_1^T\mathbf{E}_2-\mathbf{E}_2^T\mathbf{E}_1+\mathbf{E}_2^T\mathbf{E}_2\right)\left(\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right)\\
&+\left(\mathbf{H}^{(t)}\mathbf{B}\mathbf{V}^{(t)^T}-\mathbf{M}\right)\cdot\mathbf{V}^{(t)}\mathbf{B}^T\\
&+\left(\mathbf{H}^{(t)}\mathbf{C}\mathbf{V}^{(t)^T}-\mathbf{S}\right)\cdot\mathbf{V}^{(t)}\mathbf{C}^T\\
=&\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T}\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{H}^{(t)}\mathbf{B}\mathbf{V}^{(t)^T}\mathbf{V}^{(t)}\mathbf{B}^T\\
&+\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T}\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{H}^{(T)}\mathbf{C}\mathbf{V}^{(t)^T}\mathbf{V}^{(t)}\mathbf{C}^T\\
&-\mathbf{M}\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)-\mathbf{S}\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)\\
&+2\left(\mathbf{E}_1^T\mathbf{E}_1-\mathbf{E}_1^T\mathbf{E}_2-\mathbf{E}_2^T\mathbf{E}_1+\mathbf{E}_2^T\mathbf{E}_2\right)\left(\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right)\\
=&\left(\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right)\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)\\
&+\mathbf{H}^{(t)}\left[\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)-\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)\right]\\
&+\left(\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right)\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)\\
&+\mathbf{H}^{(t)}\left[\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)-\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)\right]\\
&-\mathbf{M}\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)-\mathbf{S}\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)\\
&+2\left(\mathbf{E}_1^T\mathbf{E}_1-\mathbf{E}_1^T\mathbf{E}_2-\mathbf{E}_2^T\mathbf{E}_1+\mathbf{E}_2^T\mathbf{E}_2\right)\left(\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right)\\
=&-\mathbf{M}\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)-\mathbf{S}\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)\\
&+\left[\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right]\left[\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)\right]\\
&+\left[\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right]\left[\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)\right]\\
&+2\left[\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right]\left[\left(\mathbf{E}_1^T\mathbf{E}_1-\mathbf{E}_1^T\mathbf{E}_2-\mathbf{E}_2^T\mathbf{E}_1+\mathbf{E}_2^T\mathbf{E}_2\right)\right]\\
&+\mathbf{H}^{(t)}(\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)\\
&+\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right))\\
&+\mathbf{H}^{(t)}(\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)\\
&+\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right))\\
=&\left(\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T-\mathbf{M}\right)\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)\\
&-\left(\left(\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T-\mathbf{M}\right)-\mathbf{S}\right)\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)\\
&+\left[\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right]\left[\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)\right]\\
&+\left[\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right]\left[\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)\right]\\
&+2\left[\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right]\left(\mathbf{E}_1^T\mathbf{E}_1-\mathbf{E}_1^T\mathbf{E}_2-\mathbf{E}_2^T\mathbf{E}_1+\mathbf{E}_2^T\mathbf{E}_2\right)\\
&+\mathbf{H}^{(t)}\left[\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)\right]\\
&+\mathbf{H}^{(t)}\left[\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)\right]
\end{aligned}
\tag{4}
$$

Using triangle inequality, we obtain:

$$
\begin{aligned}
&||\mathbf{\Lambda}^{(t+1)}-\mathbf{\Lambda}^{(t)}||_F\\
\le&||\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}||_F||\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{B}^T\right)\\
&+2\left(\mathbf{E}_1^T\mathbf{E}_1-\mathbf{E}_1^T\mathbf{E}_2-\mathbf{E}_2^T\mathbf{E}_1+\mathbf{E}_2^T\mathbf{E}_2\right)\\
&+\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)^T\left(\mathbf{V}^{(t+1)}\mathbf{C}^T\right)||_F\\
&+||\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T-\mathbf{M}||_F||\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T||_F\\
&+||\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T-\mathbf{S}||_F||\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T||_F\\
&+||\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)^T||_F||\mathbf{V}^{(t+1)}\mathbf{B}^T||_F\\
&+||\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)^T||_F||\mathbf{V}^{(t+1)}\mathbf{C}^T||_F
\end{aligned}
\tag{5}
$$

Given $||\mathbf{V}||_F\le\sqrt{N\tau}$, finally, we obtain:

$$
\begin{aligned}
&\left\|\mathbf{\Lambda}^{(t+1)}-\mathbf{\Lambda}^{(t)}\right\|_F^2\\
\le&3\left(16N+\tau N\left(\|\mathbf{B}\|_F^2+\|\mathbf{C}\|_F^2\right)\right)^2\left\|\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right\|_F^2\\
&+3\left\|\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right\|_F^2\cdot\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T-\mathbf{M}\right\|_F^2\\
&+3\left\|\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right\|_F^2\cdot\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T-\mathbf{S}\right\|_F^2\\
&+3N\tau\|\mathbf{B}\|_F^2\cdot\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)}\mathbf{B}^T-\mathbf{V}^{(t)}\mathbf{B}^T\right)^T\right\|_F^2\\
&+3N\tau\|\mathbf{C}\|_F^2\cdot\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)}\mathbf{C}^T-\mathbf{V}^{(t)}\mathbf{C}^T\right)^T\right\|_F^2\\
\le&3\left(16N+\tau N\left(\|\mathbf{B}\|_F^2+\|\mathbf{C}\|_F^2\right)\right)^2\left\|\mathbf{H}^{(t+1)}-\mathbf{H}^{(t)}\right\|_F^2\\
&+3(\|\mathbf{B}\|_F^2\cdot\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{B}^T\right)^T-\mathbf{M}\right\|_F^2\\
&\quad+3\|\mathbf{C}\|_F^2\cdot\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t)}\mathbf{C}^T\right)^T-\mathbf{S}\right\|_F^2)\\
&\cdot\left\|\mathbf{V}^{(t+1)}-\mathbf{V}^{(t)}\right\|_F^2\\
&+3N\tau\left(\|\mathbf{B}\|_F^4+\|\mathbf{C}\|_F^4\right)\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)}-\mathbf{V}^{(t)}\right)\right\|_F^2
\end{aligned}
\tag{6}
$$

Derivation of Lemma 2:

First, we let

$$
\begin{aligned}
A&\triangleq\mathcal{L}\left(\mathbf{H}^{(t)},\mathbf{V}^{(t+1)},\mathbf{\Lambda}^{(t)}\right)-\mathcal{L}\left(\mathbf{H}^{(t)},\mathbf{V}^{(t)},\mathbf{\Lambda}^{(t)}\right)\\
B&\triangleq\mathcal{L}\left(\mathbf{H}^{(t+1)},\mathbf{V}^{(t+1)},\mathbf{\Lambda}^{(t)}\right)-\mathcal{L}\left(\mathbf{H}^{(t)},\mathbf{V}^{(t+1)},\mathbf{\Lambda}^{(t)}\right)\\
C&\triangleq\mathcal{L}\left(\mathbf{H}^{(t+1)},\mathbf{V}^{(t+1)},\mathbf{\Lambda}^{(t+1)}\right)-\mathcal{L}\left(\mathbf{H}^{(t+1)},\mathbf{V}^{(t+1)},\mathbf{\Lambda}^{(t)}\right)\\
\hat{A}&=\hat{\mathcal{L}}\left(\mathbf{H}^{(t)},\mathbf{V}^{(t+1)},\mathbf{\Lambda}^{(t)}\right)-\mathcal{L}\left(\mathbf{H}^{(t)},\mathbf{V}^{(t)},\mathbf{\Lambda}^{(t)}\right)
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
&\hat{\mathcal{L}}\left(\mathbf{H}^{(t)},\mathbf{V}^{(t)},\mathbf{\Lambda}^{(t)}\right)\\
\triangleq&\ \frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{B}\mathbf{V}^T-\mathbf{M}\right\|_F^2+\frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{C}\mathbf{V}^T-\mathbf{S}\right\|_F^2\\
&+\frac{\rho}{2}\left\|\mathbf{H}^{(t)}-\mathbf{V}+\mathbf{\Lambda}^{(t)}/\rho\right\|_F^2+\|\mathbf{E}_1\mathbf{H}-\mathbf{E}_2\mathbf{H}\|_F^2\\
&+\frac{\beta^{(t)}}{2}\left\|\mathbf{V}-\mathbf{V}^{(t)}\right\|_F^2
\end{aligned}
\tag{8}
$$

Then, we have

$$
\begin{aligned}
&\mathcal{L}\left(\mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)}\right) - \mathcal{L}\left(\mathbf{H}^{(t)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t)}\right) \\
=&A + B + C \leq \hat{A} + B + C
\end{aligned} \tag{9}
$$

Specifically,

$$
\begin{aligned}
\hat{A} =&\frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right\|_F^2 - \frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{B}\mathbf{V}^{(t)} - \mathbf{M}\right\|_F^2 \\
&+ \frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right\|_F^2 - \frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{C}\mathbf{V}^{(t)} - \mathbf{S}\right\|_F^2 \\
&+ \frac{\rho}{2}\left\|\mathbf{H}^{(t)} - \mathbf{V}^{(t+1)} + \frac{\mathbf{\Lambda}^{(t)}}{\rho}\right\|_F^2 - \frac{\rho}{2}\left\|\mathbf{H}^{(t)} - \mathbf{V}^{(t)} + \frac{\mathbf{\Lambda}^{(t)}}{\rho}\right\|_F^2 \\
&+ \frac{\beta^{(t)}}{2}\left\|\mathbf{V} - \mathbf{V}^{(t)}\right\|_F^2 \\
\overset{(a)}{\leq}& \left\langle \mathbf{V}^{(t+1)}\mathbf{B}^T\mathbf{H}^{(t)^T}\mathbf{H}^{(t)}\mathbf{B} - \mathbf{B}^T\mathbf{H}^{(t)^T}\mathbf{M}, \mathbf{V}^{(t+1)} - \mathbf{V}^{(t+1)} \right\rangle \\
&+ \left\langle \mathbf{V}^{(t+1)}\mathbf{C}^T\mathbf{H}^{(t)^T}\mathbf{H}^{(t)}\mathbf{C} - \mathbf{C}^T\mathbf{H}^{(t)^T}\mathbf{S}, \mathbf{V}^{(t+1)} - \mathbf{V}^{(t+1)} \right\rangle \\
&- \frac{1}{2}\left\|\mathbf{B}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2 \\
&- \frac{1}{2}\left\|\mathbf{C}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2 \\
&+ \rho\left\langle \mathbf{H}^{(t)} - \mathbf{V}^{(t+1)} + \frac{\mathbf{\Lambda}^{(t)}}{\rho}, \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \right\rangle \\
&- \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 + \frac{\beta^{(t)}}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 \\
\overset{(b)}{\leq}& - \frac{1}{2}\left\|\mathbf{B}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2 \\
&- \frac{1}{2}\left\|\mathbf{C}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2 \\
&- \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 + \frac{\beta^{(t)}}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2
\end{aligned} \tag{10}
$$

where (a) is the Taylor expansion and (b) is optimal condition. Similarly,

$$
\begin{aligned}
B =&\frac{1}{2}\left\|\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right\|_F^2 - \frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right\|_F^2 \\
&+ \frac{1}{2}\left\|\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right\|_F^2 - \frac{1}{2}\left\|\mathbf{H}^{(t)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right\|_F^2 \\
&+ \left\|\mathbf{E}_1\mathbf{H}^{(t+1)} - \mathbf{E}_2\mathbf{H}^{(t+1)}\right\|_F^2 - \left\|\mathbf{E}_1\mathbf{H}^{(t)} - \mathbf{E}_2\mathbf{H}^{(t)}\right\|_F^2 \\
&+ \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\mathbf{\Lambda}^{(t)}}{\rho}\right\|_F^2 - \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{H}^{(t)} + \frac{\mathbf{\Lambda}^{(t)}}{\rho}\right\|_F^2 \\
\leq& - \frac{1}{2}\left(\left\|\mathbf{B}\right\|_F^2 + \left\|\mathbf{C}\right\|_F^2\right)\left\|\left(\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right)\mathbf{V}^{(t+1)^T}\right\|_F^2 \\
&- \left(\frac{\rho}{2} + \left\|\mathbf{E}_1 - \mathbf{E}_2\right\|_F^2\right)\left\|\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right\|_F^2
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
C =&\frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\mathbf{\Lambda}^{(t+1)}}{\rho}\right\|_F^2 - \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\mathbf{\Lambda}^{(t)}}{\rho}\right\|_F^2 \\
=&\left\langle \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)} - \mathbf{\Lambda}^{(t+1)} \right\rangle \\
\overset{(a)}{=}&\frac{1}{\rho}\left\|\mathbf{\Lambda}^{(t+1)} - \mathbf{\Lambda}^{(t)}\right\|_F^2
\end{aligned} \tag{12}
$$

Then, we need to incorporate the result of Lemma 1 into C.

Finally,

$$
\begin{aligned}
&\mathcal{L}\left(\mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)}\right) - \mathcal{L}\left(\mathbf{H}^{(t)}, \mathbf{V}^{(t)}, \mathbf{\Lambda}^{(t)}\right) \\
\leq& \hat{A} + B + C \\
\leq& - \frac{1}{2}\left(\left\|\mathbf{B}\right\|_F^2 + \left\|\mathbf{C}\right\|_F^2\right)\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2 \\
&- \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 - \frac{\beta^{(t)}}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 \\
&- \frac{1}{2}\left(\left\|\mathbf{B}\right\|_F^2 + \left\|\mathbf{C}\right\|_F^2\right)\left\|\left(\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right)\mathbf{V}^{(t+1)^T}\right\|_F^2 \\
&- \left(\frac{\rho}{2} + \left\|\mathbf{E}_1 - \mathbf{E}_2\right\|_F^2\right)\left\|\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right\|_F^2 \\
&+ \frac{3}{\rho}\left(16N + N\tau\left(\left\|\mathbf{B}\right\|_F^2 + \left\|\mathbf{C}\right\|_F^2\right)\right)^2\left\|\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right\|_F^2 \\
&+ \frac{3}{\rho}\left\|\mathbf{B}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t)} \cdot \mathbf{B}^T\right)^T - \mathbf{M}\right\|_F^2 \cdot \left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 \\
&+ \frac{3}{\rho}\left\|\mathbf{C}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t)} \cdot \mathbf{C}^T\right)^T - \mathbf{S}\right\|_F^2 \cdot \left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 \\
&+ \frac{3}{\rho}N\tau\left(\left\|\mathbf{B}\right\|_F^4 + \left\|\mathbf{C}\right\|_F^4\right)\left\|\mathbf{H}^{(t)}\left(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right)\right\|_F^2 \\
<& - c_1\left\|\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)}\right\|_F^2 - c_2\left\|(\mathbf{H}^{(t+1)} - \mathbf{H}^{(t)})\mathbf{V}^{(t+1)^T}\right\|_F^2 \\
&- c_3\left\|\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)}\right\|_F^2 - c_4\left\|\mathbf{H}^{(t)}(\mathbf{V}^{(t+1)} - \mathbf{V}^{(t)})\right\|_F^2
\end{aligned} \tag{13}
$$

where

$$
\begin{aligned}
c_1 =&\frac{\rho}{2} + \left\|\mathbf{E}_1 - \mathbf{E}_2\right\|_F^2 - \frac{3}{\rho}\left(16N + N\tau\left(\left\|\mathbf{B}\right\|_F^2 + \left\|\mathbf{C}\right\|_F^2\right)\right)^2 \\
c_2 =&\frac{\rho}{2} + \frac{\beta^{(t)}}{2} - \frac{3}{\rho}\left\|\mathbf{B}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t)} \cdot \mathbf{B}^T\right)^T - \mathbf{M}\right\|_F^2 \\
&- \frac{3}{\rho}\left\|\mathbf{C}\right\|_F^2 \cdot \left\|\mathbf{H}^{(t)} \cdot \left(\mathbf{V}^{(t)} \cdot \mathbf{C}^T\right)^T - \mathbf{S}\right\|_F^2 \\
c_3 =&\frac{1}{2}\left\|\mathbf{B}\right\|_F^2 + \frac{1}{2}\left\|\mathbf{C}\right\|_F^2 - \frac{3}{\rho}N\tau\left(\left\|\mathbf{B}\right\|_F^4 + \left\|\mathbf{C}\right\|_F^4\right) \\
c_4 =&\frac{1}{2}\left(\left\|\mathbf{B}\right\|_F^2 + \left\|\mathbf{C}\right\|_F^2\right)
\end{aligned} \tag{14}
$$

Let $c_i > 0, i = \{1, 2, 3, 4\}$, we have Lemma 2.

Derivation of Lemma 3:

$$
\begin{aligned}
&\mathcal{L}\left(\mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)}\right) \\
=&\frac{1}{2}\left\|\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right\|_F^2 + \frac{1}{2}\left\|\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right\|_F^2 + \\
&\left\|\mathbf{E}_1\mathbf{H}^{(t+1)} - \mathbf{E}_2\mathbf{H}^{(t+1)}\right\|_F^2 + \frac{\rho}{2}\left\|\mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\mathbf{\Lambda}^{(t+1)}}{\rho}\right\|_F^2 \\
=&\frac{1}{2}\left\|\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right\|_F^2 + \frac{1}{2}\left\|\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right\|_F^2 + \\
&\left\|\mathbf{E}_1\mathbf{H}^{(t+1)} - \mathbf{E}_2\mathbf{H}^{(t+1)}\right\|_F^2 + \frac{\rho}{2}\left\|\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right\|_F^2 \\
&\left\langle \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}, \left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right)\mathbf{V}^{(t+1)}\mathbf{B}^T \right\rangle + \\
&\left\langle \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}, \left(\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right)\mathbf{V}^{(t+1)}\mathbf{C}^T \right\rangle + \\
&2\left\langle \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}, \left(\mathbf{E}_1^T\mathbf{E}_1 - \mathbf{E}_1^T\mathbf{E}_2 - \mathbf{E}_2^T\mathbf{E}_1 + \mathbf{E}_2^T\mathbf{E}_2\right)\mathbf{H}^{(t+1)} \right\rangle
\end{aligned} \tag{15}
$$

First, we note that,

$$
\begin{aligned}
0 \leq & \left\| \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \mathbf{B}\mathbf{V}^{(t+1)^T} + \left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right) \right\|_F^2 \\
= & \left\| \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \mathbf{B}\mathbf{V}^{(t+1)^T} \right\|_F^2 + \left\| \mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M} \right\|_F^2 \\
& + 2 \left\langle \mathbf{B}\mathbf{V}^{(t+1)^T} \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right), \mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M} \right\rangle \\
= & \left\| \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \mathbf{B}\mathbf{V}^{(t+1)^T} \right\|_F^2 + \left\| \mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M} \right\|_F^2 \\
& + 2 \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right), \left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right) \mathbf{V}^{(t+1)}\mathbf{B}^T \right\rangle,
\end{aligned} \tag{16}
$$

where we have

$$
\left\langle \mathbf{Y}^T \left(\mathbf{X} - \mathbf{Y}\right), \mathbf{X}\mathbf{Y} - \mathbf{Z} \right\rangle = \left\langle \mathbf{X} - \mathbf{Y}, (\mathbf{X}\mathbf{Y} - \mathbf{Z})\mathbf{Y} \right\rangle. \tag{17}
$$

That is,

$$
\begin{aligned}
& \left\| \mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M} \right\|_F^2 + \\
& + 2 \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right), \left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right) \mathbf{B}\mathbf{V}^{(t+1)} \right\rangle \\
\leq & - \left\| \left(\mathbf{H}^{(t+1)} - \mathbf{V}_2^{(t+1)}\right) \mathbf{B}\mathbf{V}^{(t+1)^T} \right\|_F^2 \\
\leq & - \left\|\mathbf{B}\right\|_F^2 N\tau \cdot \left\| \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)} \right\|_F^2.
\end{aligned} \tag{18}
$$

Therefore, we obtain

$$
\begin{aligned}
& \frac{1}{2} \left\| \mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M} \right\|_F^2 \\
& + \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right), \left(\mathbf{H}^{(t+1)}\mathbf{B}\mathbf{V}^{(t+1)^T} - \mathbf{M}\right) \mathbf{B}\mathbf{V}^{(t+1)} \right\rangle \\
\leq & - \frac{1}{2} \left\|\mathbf{B}\right\|_F^2 N\tau \cdot \left\| \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)} \right\|_F^2.
\end{aligned} \tag{19}
$$

Second, we similarly derive the inequality as follows:

$$
\begin{aligned}
& \frac{1}{2} \left\| \mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S} \right\|_F^2 \\
& + \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right), \left(\mathbf{H}^{(t+1)}\mathbf{C}\mathbf{V}^{(t+1)^T} - \mathbf{S}\right) \mathbf{C}\mathbf{V}^{(t+1)} \right\rangle \\
\leq & - \frac{1}{2} \left\|\mathbf{C}\right\|_F^2 N\tau \cdot \left\| \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)} \right\|_F^2
\end{aligned} \tag{20}
$$

Third, we first derive equations as follows:

$$
\begin{aligned}
& 2 \left\langle \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}, \left(\mathbf{E}_1 - \mathbf{E}_2\right)^T \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\rangle \\
& + \left\| \mathbf{E}_1\mathbf{H}^{(t+1)} - \mathbf{E}_2\mathbf{H}^{(t+1)} \right\|_F^2 \\
= & 2 \left\langle \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}, \left(\mathbf{E}_1 - \mathbf{E}_2\right)^T \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\rangle \\
& + \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\|_F^2 \\
= & 2 \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \left(\mathbf{E}_1 - \mathbf{E}_2\right), \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\rangle \\
& + \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\|_F^2
\end{aligned} \tag{21}
$$

Note that,

$$
\begin{aligned}
0 \leq & \left\| \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \left(\mathbf{E}_1 - \mathbf{E}_2\right) - \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\|_F^2 \\
= & \left\| \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \left(\mathbf{E}_1 - \mathbf{E}_2\right) \right\|_F^2 + \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\|_F^2 \\
& + 2 \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \left(\mathbf{E}_1 - \mathbf{E}_2\right), \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\rangle
\end{aligned} \tag{22}
$$

That is,

$$
\begin{aligned}
& \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\|_F^2 \\
& + 2 \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \left(\mathbf{E}_1 - \mathbf{E}_2\right), \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\rangle \\
\geq & - \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \right\|_F^2 \cdot \left\| \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)} \right\|_F^2
\end{aligned} \tag{23}
$$

Therefore, we obtain

$$
\begin{aligned}
& 2 \left\langle \left(\mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)}\right) \left(\mathbf{E}_1 - \mathbf{E}_2\right), \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\rangle \\
& + \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \mathbf{H}^{(t+1)} \right\|_F^2 \\
\geq & - \left\| \left(\mathbf{E}_1 - \mathbf{E}_2\right) \right\|_F^2 \cdot \left\| \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)} \right\|_F^2
\end{aligned} \tag{24}
$$

Finally, we obtain:

$$
\mathcal{L}\left(\mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{\Lambda}^{(t+1)}\right) = c \cdot \left\| \mathbf{H}^{(t+1)} - \mathbf{V}^{(t+1)} \right\|_F^2 \tag{25}
$$

$$
c = \frac{\rho}{2} - \frac{1}{2} \left\|\mathbf{B}\right\|_F^2 N\tau - \frac{1}{2} \left\|\mathbf{C}\right\|_F^2 N\tau - \left\|\mathbf{E}_1 - \mathbf{E}_2\right\|_F^2 \tag{26}
$$

Let $c > 0$, we have Lemma 3.

Note that, we use $\mathbf{X}$, instead of $\tilde{\mathbf{X}}$, to denote the block diagonal matrix in this material.

References

[Hong *et al.*, 2016] Mingyi Hong, Zhi-Quan Luo, and Meisam Razaviyayn. Convergence analysis of alternating direction method of multipliers for a family of nonconvex problems. *SIAM Journal on Optimization*, 26(1):337–364, 2016.