Assignment 3

Your First Rodeo

Deadline: Nov 3rd, 11:55pm.
Perfect score: 85.

Assignment Instructions:

Teams: Assignments should be completed by teams of up to three students. You can work on this assignment individually if you prefer. No additional credit will be given for students that complete an assignment individually. Make sure to write the name and RUID of every member of your group on your submitted report.

Submission Rules: Submit your reports electronically as a PDF document through Canvas (`canvas.rutgers.edu`). Each team of students should submit only a single copy of their solutions and indicate all team members on their submission. Failure to follow these rules will result in lower grade for this assignment.

Precision: Try to be precise. Have in mind that you are trying to convince a very skeptical reader (and computer scientists are the worst kind...) that your answers are correct.

Collusion, Plagiarism, etc.: Each team must prepare its solutions independently from other teams, i.e., without using common notes, code or worksheets with other students or trying to solve problems in collaboration with other teams. You must indicate any external sources you have used in the preparation of your solution. Do not plagiarize online sources and in general make sure you do not violate any of the academic standards of the department or the university. Failure to follow these rules may result in failure in the course.

1 Your First Rodeo

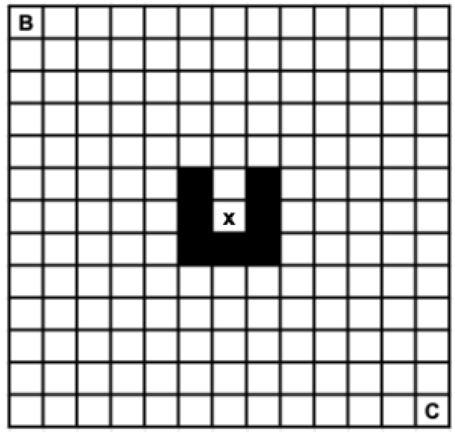

Figure 1: A square corral containing a bull (B), a robotic rodeo bullfighter (C), and a target (x).

A bull is running loose in the above square corral. The robotic rodeo bullfighter is meant to pen the bull, by getting it to chase them, and leading it to the square x so that it can be closed in [1]. The rules are these:

- The bull can only move up/down/left/right (limited by the walls/obstacles).
- The robot can move in any of the eight immediate neighboring directions (limited by the walls/obstacles).
- Each round, the robot moves, then the bull moves.
- If the robot is outside the 5x5 square surrounding the bull, the bull moves uniformly at random in its available directions.
- If the robot is within the 5x5 square surrounding the bull, the bull will charge with equal probability in any direction that doesn't take it farther (manhattan distance) from the robot.
- The bull cannot enter the same square as the robot, and vice versa. The bull will skip moving if it has to. (Shy bull.)
- The game is over when the bull walks onto the x.

Answer the following questions with math, code, or computation:

1. Intuitively, if the robot moves strategically, argue that the game can't last forever (with probability 1). **(2 points)**
2. How many possible states can the game be in? **(3 points)**
3. Let $T^*(\text{pos}_B, \text{pos}_C)$ be the minimal number of rounds remaining expected to corral the bull on the x when the robot is at position pos_C and the bull is at position pos_B. Express T^* of a given pair of positions in terms of T^* of the feasible 'next' positions. What are the cases to consider? Are there any positions that you know the value of T^* for? **(30 points)**
4. Describe an algorithm for computing T^*. **(10 points)**
5. Implement that algorithm, and compute T^* for the pictured configuration. Can the robot pen the bull in finite (expected) time? If so, what is the expected time? **(40 points)**
6. **(Bonus)** The rules are the same, but now the bull wants to kill the robot, and will move into the robot's cell and crush it if it can. The robot wants to survive and pen the bull. If everyone moves optimally, who (if anyone) wins? **(15 points)**
7. **(Bonus)** What would a good heuristic estimate for T^* be, as a function of pos_B and pos_C. Why is this useful, practically? **(10 points)**

[1] This assignment has been created by Dr. Charles Wes Cowan, Rutgers University.