Równanie MES

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1. Problem

$$\frac{-d}{dx}\left(E(x)\frac{du(x)}{dx}\right)=0$$

$$u(2)=0$$

$$\frac{du(0)}{dx}+u(0)=10$$

$$E(x)=\begin{cases}3, x\in[0,1)\\5, x\in(1,2]\end{cases}$$

Szukana funkcja $[0,2]\ni x \to u(x)\in\mathbb{R}$

2. Rozwiązanie

$$\frac{-d}{dx}\left(E(x)\frac{du(x)}{dx}\right)=0$$

$$-E(x)\frac{d^2u}{dx^2}=0$$

$$-E(x)\frac{d^2u}{dx^2}v=0 \qquad v(2)=0$$

$$-\int_0^2 E(x)\frac{d^2u}{dx^2}v=0$$

$$-\int_0^2 E(x)\frac{d^2u}{dx^2}v=-E(x)\frac{d^2u}{dx^2}v\Big|_0^2+\int_0^2 E(x)\frac{dv}{dx}\frac{du}{dx}dx=$$

$$=5\frac{du}{dx}(2)v(2)+3\frac{du}{dx}(0)v(0)+\int_0^2 E(x)\frac{dv}{dx}\frac{du}{dx}dx=$$

$$3(10-u(0))v(0)+\int_0^2 E(x)\frac{dv}{dx}\frac{du}{dx}dx$$

Wiec

$$B(u,v)=+\int_0^2 E(x)\frac{dv}{dx}\frac{du}{dx}dx \;-\; 3u(0)v(0)$$

$$L(u)=-30\,v(0)$$