

# 2301108 CALCULUS II

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# 1 Differential Equation

## 1.1 Seperable Equation

**Definition 1.1.1** (Seperable Equation). A differential equation is called seperable if it can be written in the form

$$\frac{dy}{dx} = g(y)h(x)$$

where  $g(y)$  is a function of  $y$  only and  $h(x)$  is a function of  $x$  only.

$$\frac{dy}{dx} = g(y)h(x)$$

$$\frac{dy}{g(y)} = h(x)dx$$

$$\int \frac{dy}{g(y)} = \int h(x)dx$$

**Definition 1.1.2** (Implicit Solution). An implicit solution to a differential equation is a solution that is not solved for  $y$  explicitly. Answer is in the form of  $F(x, y) = c$ .

**Definition 1.1.3** (Explicit Solution). An explicit solution to a differential equation is a solution that is solved for  $y$  explicitly. Answer is in the form of  $y = f(x)$ .

Sometimes there are initial condition of the equation.

**Theorem 1.1.1** (FTC 1). *If  $f$  is continuous on  $[a, b]$ , then the function  $f$  defined by*

$$\frac{d}{dx} \int_a^{u(x)} f(t)dt = f(u(x))u'(x)$$

*is continuous on  $[a, b]$  and differentiable in  $(a, b)$ , and  $g'(x) = f(x)$ .*

**Definition 1.1.4** (Integral Equation). An integral equation is an equation in which an unknown function appears under an integral sign.

We have to change the integral equation to a differential equation. Then, we can use FTC 1 to solve the differential equation.

## 1.2 Linear Equation

**Definition 1.2.1** (Linear Equation). A differential equation is called linear if it can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where  $P(x)$  and  $Q(x)$  are continuous functions of  $x$ .

The term “linear” refers to the fact that the unknown function  $y$  and its derivative  $\frac{dy}{dx}$  appear in the equation to the first power and are not multiplied together.

**Definition 1.2.2** (Integrating Factor). The integrating factor for the linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is the function  $I(x)$  defined by

$$I(x) = \exp\left(\int P(x)dx\right)$$

We can solve the linear differential equation by the formula.

$$y(x) = \frac{1}{I(x)}\left[\int I(x)Q(x)dx + C\right]$$

without any additional constraint  $c$  from integrating.

## 1.3 Bernoulli Equation

**Definition 1.3.1** (Bernoulli Equation). A differential equation is called Bernoulli if it can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

where  $P(x)$  and  $Q(x)$  are continuous functions of  $x$ .

As you can see, there is  $y^n$  in the equation which is not linear.

We have to change the Bernoulli equation to a linear equation by substitution.

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

divides both side by  $y^n$

$$\frac{dy}{dx}y^{-n} + P(x)y^{1-n} = Q(x)$$

Then we substitute  $u = y^{1-n}$

$$\begin{aligned}\frac{du}{dy} &= (1-n)y^{-n} \frac{dy}{dx} \\ \frac{du}{dx} \cdot \frac{1}{1-n} &= y^{-n} \frac{dy}{dx}\end{aligned}$$

Let  $u = y^{1-n}$

$$\begin{aligned}\frac{du}{dx} &= (1-n)y^{-n} \frac{dy}{dx} \\ \frac{du}{dx} \cdot \frac{1}{1-n} &= y^{-n} \frac{dy}{dx}\end{aligned}$$

Then, try to substitute the equation with  $u$ .

$$\begin{aligned}\frac{du}{dx} \cdot \frac{1}{1-n} + P(x)u &= Q(x) \\ \frac{du}{dx} + (1-n)P(x)u &= (1-n)Q(x)\end{aligned}$$

Now, we can solve the equation with the linear equation method. Since the equation is linear on  $u$ .

## 2 Sequence

### 2.1 Sequence

**Definition 2.1.1** (Sequence). A sequence is a function whose domain is  $\mathbb{N}$

We are considering behavior of a sequence as  $n$  becomes infinite.

$$\lim_{n \rightarrow \infty} a_n = \begin{cases} L \in \mathbb{R} & \text{convergent} \\ \pm\infty & \text{divergent} \\ \text{morethanonevalue} & \text{divergent} \end{cases}$$

**Definition 2.1.2** (Sequence Notation). The sequence  $\{a_1, a_2, a_3, \dots\}$  is denoted by  $\{a_n\}$  or  $\{a_n\}_{n=1}^{\infty}$

**Theorem 2.1.1** (Limit of Sequence). A sequence  $\{a_n\}$  converges to  $L$ .

$$\lim_{n \rightarrow \infty} a_n = L$$

or

$$a_n \rightarrow L \text{ as } n \rightarrow \infty$$

if for every  $\epsilon > 0$ , there exists  $N$  such that

$$\text{if } n > N$$

then

$$|a_n - L| < \epsilon$$

**Theorem 2.1.2.** If  $\lim_{x \rightarrow \infty} f(x) = L$  and  $f(n) = a_n$  for all  $n \in \mathbb{N}$ , then

$$\lim_{n \rightarrow \infty} a_n = L$$

Since our function is discrete, we can use the limit of the function to find the limit of the sequence.

*Remark.* We can use L'Hopital's rule to find the limit of the sequence.

If subsequences of  $a_n$  converge to different limits, then the sequence  $a_n$  diverges. For example

$$\begin{aligned}a_n &= (-1)^n \\a_n &= 1, -1, 1, -1, \dots\end{aligned}$$

Consider:  $a_{2k-1} = (-1)^{2k-1}; k \in \mathbb{N}$

$$\begin{aligned}a_{2k-1} &= (-1)^{2k-1} \\a_{2k-1} &= -1\end{aligned}$$

Consider:  $a_{2k} = (-1)^{2k}; k \in \mathbb{N}$

$$\begin{aligned}a_{2k} &= (-1)^{2k} \\a_{2k} &= 1\end{aligned}$$

Since  $\lim_{k \rightarrow \infty} a_{2k-1} = -1$  and  $\lim_{k \rightarrow \infty} a_{2k} = 1$ , the sequence  $a_n$  diverges.

**Theorem 2.1.3** (Squeeze theorem). *Let  $a_n \leq b_n \leq c_n$  for  $n \geq n_0$ . If*

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} c_n = L$$

*then*

$$\lim_{n \rightarrow \infty} b_n = L$$

**Theorem 2.1.4.** *If  $\lim_{n \rightarrow \infty} |a_n| = 0$ , then  $\lim_{n \rightarrow \infty} a_n = 0$*

*Proof.* From  $-|a_n| \leq a_n \leq |a_n|$ , we have

$$\lim_{n \rightarrow \infty} |a_n| = 0$$

by the squeeze theorem, we have  $\lim_{n \rightarrow \infty} a_n = 0$  □

**Theorem 2.1.5.** *If  $\lim_{n \rightarrow \infty} a_n = L$  and the function  $f$  is continuous at  $L$ , then*

$$\lim_{n \rightarrow \infty} f(a_n) = f(L)$$

*Remark.* If the function is continuous,

$$\lim_{n \rightarrow \infty} f(a_n) = f(\lim_{n \rightarrow \infty} a_n)$$

The sequence  $\{r^n\}$  is convergent if and only if  $-1 < r \leq 1$ .

$$\lim_{n \rightarrow \infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \\ \infty & \text{if } r > 1 \end{cases}$$

## 2.2 Mathematical Induction

**Definition 2.2.1** (Mathematical Induction). A proof technique used to prove a statement for all positive integers.

We can use mathematical induction to find the limit of a sequence. For example

$$a_n = \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{n!}$$

Consider:  $a_n \geq (\frac{3}{2})^n$  and  $\lim_{n \rightarrow \infty} (\frac{3}{2})^{n-1} = \infty$  so  $\lim_{n \rightarrow \infty} a_n = \infty$

*Proof.* Proof by Mathematical Induction

Let  $P(n)$ :  $a_n \geq (\frac{3}{2})^n$  for  $n \in \mathbb{N}$

**Base Case:**  $n = 1$

$$a_1 = \frac{1}{1!} = 1$$
$$(\frac{3}{2})^{1-1} = (\frac{3}{2})^0 = 1$$

Since  $a_1 \geq (\frac{3}{2})^0$ ,  $P(1)$  is true.

**Inductive Step:** Assume  $P(k)$  is true for  $k \in \mathbb{N}$

$$a_k \geq a_k \cdot \frac{2(k+1)-1}{k+1} = a_k \cdot \frac{2k+1}{k+1}$$
$$(\frac{3}{2})^k = (\frac{3}{2})^{k-1} \cdot \frac{3}{2}$$

Consider

$$a_k \cdot \frac{2k+1}{k+1} \geq (\frac{3}{2})^{k-1} \cdot \frac{3}{2}$$

So  $P(k) \rightarrow P(k+1)$  is true. Such that  $P(n)$  is true for all  $n \in \mathbb{N}$ . □

## 2.3 Monotonic and Bounded Sequences

Our goal is to determine if a sequence is convergent or divergent. We can tell that a sequence is "convergent" but may not know the absolute value of the limit.

**Definition 2.3.1** (Increasing Sequence). A sequence  $\{a_n\}$  is increasing if  $a_n < a_{n+1}$  for all  $n \geq 1$ .

**Definition 2.3.2** (Decreasing Sequence). A sequence  $\{a_n\}$  is decreasing if  $a_n > a_{n+1}$  for all  $n \geq 1$ .

**Definition 2.3.3** (Monotonic Sequence). A sequence  $\{a_n\}$  is monotonic if it is either increasing or decreasing.

*Remark.* In this case, increasing and decreasing sequences are defined by "strictly" increasing and decreasing in order.

**Definition 2.3.4** (Bounded Sequence). A sequence  $\{a_n\}$  is bounded above if there is a number  $M$  such that

$$a_n \leq M \text{ for } n \geq 1$$

A sequence  $\{a_n\}$  is bounded below if there is a number  $m$  such that

$$a_n \geq m \text{ for } n \geq 1$$

If a sequence is bounded above and below, then it is called a bounded sequence.

**Theorem 2.3.1** (Monotonic Sequence Theorem). *Every bounded, monotonic sequence is convergent.*

*Remark.* Techniques to check that the sequence is increasing or decreasing:

- $a_{n+1} - a_n$ 
  - If  $a_{n+1} - a_n > 0$ , then the sequence is increasing
  - If  $a_{n+1} - a_n < 0$ , then the sequence is decreasing
- $\frac{a_{n+1}}{a_n}$ 
  - If  $\frac{a_{n+1}}{a_n} > 1$ , then the sequence is increasing
  - If  $\frac{a_{n+1}}{a_n} < 1$ , then the sequence is decreasing
- Find  $f'(n)$ 
  - If  $f'(n) > 0$ , then the sequence is increasing
  - If  $f'(n) < 0$ , then the sequence is decreasing

Example: Prove that the following sequence is convergent

$$a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} \text{ for } n \in \mathbb{N}$$



*Proof.* Since there are factorials in each term, we cannot use the derivative to determine if the sequence is increasing or decreasing.

1. Determine if the sequence is increasing or decreasing

$$\begin{aligned} a_{n+1} - a_n &= \left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{(n+1)!}\right) - \left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}\right) \\ &= \frac{1}{(n+1)!} \end{aligned}$$

Since  $\frac{1}{(n+1)!} > 0$ , the sequence is increasing.

2. Determine if the sequence is bounded

$$a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} \leq 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{n-1}} + \cdots$$

$$\sum_{i=1}^n \frac{1}{i!} \leq \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^{i-1} \leq \frac{1}{1 - \frac{1}{2}} = 2$$

So  $1 \leq a_n \leq 2$ . The sequence is bounded.

By theorem:  $\{a_n\}$  is monotonic and bounded, so it is convergent. □

**Theorem 2.3.2.** *If there are 2 or more subsequences of  $\{a_n\}$  which converges to different values then  $\{a_n\}$  is divergent.*

For example:  $\lim_{n \rightarrow \infty} |a_n| \neq 0$  and there is  $(-1)^n$  in each term.

$$\cos(n\pi) = \begin{cases} a_{2k} = 1 & \text{if } n = 2k \\ a_{2k-1} = -1 & \text{if } n = 2k - 1 \end{cases}$$

A sequence  $\{a_n\}$  is given by  $a_1 = \sqrt{2}$ ,  $a_{n+1} = \sqrt{2 + a_n}$  for  $n \geq 1$ .

(a) Find  $\lim_{n \rightarrow \infty} a_n$

Let  $L = \lim_{n \rightarrow \infty} a_n$

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \sqrt{2 + a_n} \\ L &= \sqrt{2 + L} \\ L^2 &= 2 + L \\ L^2 - L - 2 &= 0 \\ (L - 2)(L + 1) &= 0 \\ L &= 2, -1 \end{aligned}$$

Since under the square root,  $a_n \geq 0$ , so  $L = 2$ .

(b) Prove that  $\{a_n\}$  is increasing and bounded above by 3. Then use the Monotonic Sequence Theorem to show that  $\{a_n\}$  is convergent.

*Proof.* Let  $P(n)$ :  $a_{n+1} > a_n$  and  $a_n \leq 3$

**Base Case:**  $n = 1$

$$a_1 = \sqrt{2} \text{ and } a_2 = \sqrt{2 + \sqrt{2}}$$

Since  $\sqrt{2} < \sqrt{2 + \sqrt{2}}$  and  $\sqrt{2} \leq 3$ ,  $P(1)$  is true.

**Inductive Step:** Assume  $P(k)$  is true for  $k \in \mathbb{N}$

$$a_{k+1} > a_k \text{ and } a_k \leq 3$$

$$a_{k+2} = \sqrt{2 + a_{k+1}}$$

Since  $a_{k+1} > a_k$ , we have

$$\sqrt{2 + a_{k+1}} > \sqrt{2 + a_k}$$

From  $a_k \leq 3$ , we have

$$a_{k+1} = \sqrt{2 + a_k}$$

$$\sqrt{2 + a_k} \leq \sqrt{2 + 3}$$

$$a_{k+1} \leq \sqrt{5}$$

$$< \sqrt{9}$$

$$< 3$$

So  $P(k) \rightarrow P(k+1)$  is true. Such that  $P(n)$  is true for all  $n \in \mathbb{N}$ . □

## 3 Series

### 3.1 Series

**Definition 3.1.1** (Series). Series is the sum of the terms of a sequence.

$$a_1 + a_2 + a_3 + \cdots + a_n$$

**Definition 3.1.2** (Infinite Series). An infinite series is the sum of the terms of an infinite sequence.

$$a_1 + a_2 + a_3 + \cdots + a_n + \dots$$

In this chapter we will try testing the convergence of the series.

**Definition 3.1.3.** Given a series  $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \dots$ , let  $s_n$  denote its  $n$ th partial sum:

$$s_n = \sum_{i=1}^n a_i = a_1 + a_2 + \cdots + a_n$$

**Lemma 3.1.1.**

$$\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n a_i$$

Sometimes  $s_n$  is expressing in telescoping sum form. For example

$$\begin{aligned} \sum_{n=1}^{\infty} \ln \frac{n}{n+1} &= \sum_{n=1}^{\infty} \ln n - \ln(n+1) \\ &= \lim_{n \rightarrow \infty} (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + \cdots + (\ln n - \ln(n+1)) \\ &= \lim_{n \rightarrow \infty} \ln 1 - \ln(n+1) \\ &= \lim_{n \rightarrow \infty} \ln \frac{1}{n+1} \\ &= \ln 0 \\ &= -\infty \end{aligned}$$

Such that,  $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$  is divergent.

$$\begin{aligned}
\sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)}) &= \sum_{n=1}^{\infty} (\sqrt[n]{e} - \sqrt[n+1]{e}) \\
&= \lim_{n \rightarrow \infty} (e - \sqrt{e}) + (\sqrt{e} - \sqrt[3]{e}) + (\sqrt[3]{e} - \sqrt[4]{e}) + \cdots + (\sqrt[n]{e} - \sqrt[n+1]{e}) \\
&= \lim_{n \rightarrow \infty} e - \sqrt[n+1]{e} \\
&= e - e^0 \\
&= e - 1
\end{aligned}$$

Such that,  $\sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$  converges to  $e - 1$ .

## 3.2 Geometric Series

**Definition 3.2.1.**

$$a + ar + ar^2 + \cdots + ar^{n-1} + \cdots = \sum_{n=1}^{\infty} ar^{n-1} \text{ where } a \neq 0$$

is defined as a geometric series which  $a$  is the first term and  $r$  is the common ratio.

$$r = \frac{a_{n+1}}{a_n}, \quad n \geq 1$$

**Lemma 3.2.1.** *If  $|r| \neq 1$ , we have*

$$\begin{aligned}
s_n &= a + ar + ar^2 + \cdots + ar^{n-1} \\
r \cdot s_n &= ar + ar^2 + ar^3 + \cdots + ar^n \\
s_n - r \cdot s_n &= a - ar^n \\
s_n &= \frac{a(1 - r^n)}{1 - r}
\end{aligned}$$

**Lemma 3.2.2.** *The geometric series*

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$$

*is convergent if  $|r| < 1$  and its sum is*

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1 - r}$$

*If  $|r| \geq 1$ , the series is divergent.*

*Proof.* If  $|r| < 1$ , we have

$$\begin{aligned}\lim_{n \rightarrow \infty} s_n &= \lim_{n \rightarrow \infty} \frac{a(1 - r^n)}{1 - r} \\&= \frac{a}{1 - r} - \frac{a}{1 - r} \cdot \lim_{n \rightarrow \infty} r^n \\&= \frac{a}{1 - r} - \frac{a}{1 - r} \cdot 0 \\&= \frac{a}{1 - r}\end{aligned}$$

If  $|r| \geq 1$ , we have  $\lim_{n \rightarrow \infty} r^n = \infty$ , such that the series is divergent. □

## 4 Convergence Test

### 4.1 Test for Divergence

**Theorem 4.1.1** (Test for Divergence). *If the series  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\lim_{n \rightarrow \infty} a_n = 0$*

**Corollary 4.1.1.** *If  $\lim_{n \rightarrow \infty} a_n$  does not exist or  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then the series  $\sum_{n=1}^{\infty} a_n$  is divergent.*

**Lemma 4.1.1.** *If  $\sum a_n$  diverges and  $b_n$  converges, then  $\sum(a_n + b_n)$  will be divergent.*

For example

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{2^n + 4^n}{e^n} &= \sum_{n=1}^{\infty} \frac{2^n}{e^n} + \sum_{n=1}^{\infty} \frac{4^n}{e^n} \\ &= \sum_{n=1}^{\infty} \left(\frac{2}{e}\right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{e}\right)^n\end{aligned}$$

Since

- $\frac{2}{e} < 1$  so  $\sum_{n=1}^{\infty} \left(\frac{2}{e}\right)^n$  is convergent
- $\frac{4}{e} > 1$  so  $\sum_{n=1}^{\infty} \left(\frac{4}{e}\right)^n$  is divergent

Therefore,  $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{e^n}$  is divergent.

### 4.2 Integral Test

**Theorem 4.2.1** (Integral Test). *Let  $f(x)$  be a continuous, positive, and decreasing function on  $[1, \infty)$ . Then the series  $\sum_{n=1}^{\infty} f(n)$  is convergent if and only if the improper integral  $\int_1^{\infty} f(x) dx$  is convergent.*

- If  $\int_1^{\infty} f(x) dx$  is convergent, then  $\sum_{n=1}^{\infty} a_n$  is convergent
- If  $\int_1^{\infty} f(x) dx$  is divergent, then  $\sum_{n=1}^{\infty} a_n$  is divergent

**Lemma 4.2.1** (P-series). *The series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is convergent if  $p > 1$  and divergent if  $p \leq 1$ .*

For example

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{n}{n^4 + 1} \\ f(x) &= \frac{x}{x^4 + 1} \\ F(x) &= \int \frac{x}{x^4 + 1} dx \\ &= \lim_{t \rightarrow \infty} \int_1^t \frac{x}{x^4 + 1} dx \end{aligned}$$

Let  $u = x^2$ , then  $du = 2x dx$

$$\begin{aligned} \lim_{t \rightarrow \infty} \int_1^t \frac{x}{x^4 + 1} dx &= \lim_{t \rightarrow \infty} \int_{x=1}^{x=t} \frac{x}{u^2 + 1} \cdot \frac{du}{2x} \\ &= \lim_{t \rightarrow \infty} \frac{1}{2} \int_1^{\infty} \frac{1}{u^2 + 1} du \\ &= \lim_{t \rightarrow \infty} \frac{1}{2} \arctan u \Big|_1^t \\ &= \lim_{t \rightarrow \infty} \frac{1}{2} \arctan t - \frac{1}{2} \arctan 1 \\ &= \frac{\pi}{4} - \frac{\pi}{8} \\ &= \frac{\pi}{8} \end{aligned}$$

$$\begin{aligned} \sum_{n=1}^{\infty} n(1 + n^2)^p \\ f(x) &= x(1 + x^2)^p \\ F(x) &= \int_1^{\infty} x(1 + x^2)^p dx \\ &= \lim_{t \rightarrow \infty} \int_1^t x(1 + x^2)^p dx \end{aligned}$$

Let  $u = 1 + x^2$ , then  $du = 2x dx$

$$\begin{aligned} \lim_{t \rightarrow \infty} \int_1^t x(1 + x^2)^p dx &= \lim_{t \rightarrow \infty} \int_2^{\infty} x \cdot u^p \frac{du}{2x} \\ &= \lim_{t \rightarrow \infty} \frac{1}{2} \int_2^{\infty} u^p du \\ &= \lim_{t \rightarrow \infty} \frac{1}{2} \frac{u^{p+1}}{p+1} \Big|_2^{\infty} \end{aligned}$$

Since  $p \neq -1$ , then  $p + 1 > 0$ . Therefore, the integral is divergent.