2301108 CALCULUS II

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Contents

1 Differential Equation

1.1 Seperable Equation

Definition 1.1.1 (Seperable Equation)**.** A differential equation is called seperable if it can be written in the form

$$\frac{dy}{dx} = g(y)h(x)$$

where $g(y)$ is a function of y only and $h(x)$ is a function of x only.

$$\begin{aligned} \frac{dy}{dx} &= g(y)h(x) \\ \frac{dy}{g(y)} &= h(x)dx \\ \int \frac{dy}{g(y)} &= \int h(x)dx \end{aligned}$$

Definition 1.1.2 (Implicit Solution)**.** An implicit solution to a differential equation is a solution that is not solved for y explicitly. Answer is in the form of $F(x, y) = c$.

Definition 1.1.3 (Explicit Solution)**.** An explicit solution to a differential equation is a solution that is solved for y explicitly. Answer is in the form of $y = f(x)$.

Sometimes there are initial condition of the equation.

Theorem 1.1.1 (FTC 1)**.** *If f is continuous on $[a, b]$, then the function f defined by*

$$\frac{d}{dx}\int_a^{u(x)} f(t)dt = f(u(x))u'(x)$$

is continuous on $[a, b]$ and differentiable in (a, b), and g'(x) = f(x).

Definition 1.1.4 (Integral Equation)**.** An integral equation is an equation in which an unknown function appears under an integral sign.

We have to change the integral equation to a differential equation. Then, we can use FTC 1 to solve the differential equation.

1.2 Linear Equation

Definition 1.2.1 (Linear Equation)**.** A differential equation is called linear if it can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x)$ and $Q(x)$ are continuous functions of x.

The term "linear" refers to the fact that the unknown function y and its derivative dy/dx appear in the equation to the first power and are not multiplied together.‘’

Definition 1.2.2 (Integrating Factor)**.** The integrating factor for the linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is the function $I(x)$ defined by

$$I(x) = \exp(\int P(x)dx)$$

We can solve the linear differential equation by the formula.

$$y(x) = \frac{1}{I(x)}[\int I(x)Q(x)dx + C]$$

without any additional constraint c from integrating.

1.3 Bernoulli Equation

Definition 1.3.1 (Bernoulli Equation)**.** A differential equation is called Bernoulli if it can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

where $P(x)$ and $Q(x)$ are continuous functions of x.

As you can see, there is y^n in the equation which is not linear.

We have to change the Bernoulli equation to a linear equation by substitution.

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

divides both side by y^n

$$\frac{dy}{dx}y^{-n} + P(x)y^{1-n} = Q(x)$$

Then we substitute $u = y^{1-n}$

$$\begin{aligned} \frac{du}{dy} &= (1-n)y^{-n}\frac{dy}{dx} \\ \frac{du}{dx} \cdot \frac{1}{1-n} &= y^{-n}\frac{dy}{dx} \end{aligned} \tag{1.1}$$

Let $u = y^{1-n}$

$$\begin{aligned} \frac{du}{dx} &= (1-n)y^{-n}\frac{dy}{dx} \\ \frac{du}{dx} \cdot \frac{1}{1-n} &= y^{-n}\frac{dy}{dx} \end{aligned} \tag{1.2}$$

Then, try to substitute the equation with u.

$$\begin{aligned} &\frac{du}{dx} \cdot \frac{1}{1-n} + P(x)u = Q(x) \\ &\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x) \end{aligned} \tag{1.3}$$

Now, we can solve the equation with the linear equation method. Since the equation is linear on u.

2 Sequence

Definition 2.0.1 (Sequence)**.** A sequence is a function whose domain is $\mathbb{N}$

We are considering behavior of a sequence as n becomes infinite.

$$\lim_{n\to\infty} a_n = \begin{cases} L \in \mathbb{R} & \text{convergent} \\ \pm\infty & \text{divergent} \\ \text{morethanonevalue} & \text{divergent} \end{cases}$$

Definition 2.0.2 (Sequence Notation)**.** The sequece $\{a_1, a_2, a_3, \dots\}$ is denoted by $\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$

Theorem 2.0.1 (Limit of Sequence)**.** *A sequence $\{a_n\}$ converges to L.*

$$\lim_{n\to\infty} a_n = L$$

or

$$a_n \to L \mathrm{as} n \to \infty$$

if for every $\epsilon > 0$, there exists N such that

$$\mathrm{if} n > N$$

then

$$|a_n - L| < \epsilon$$