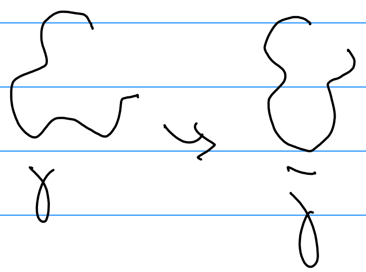
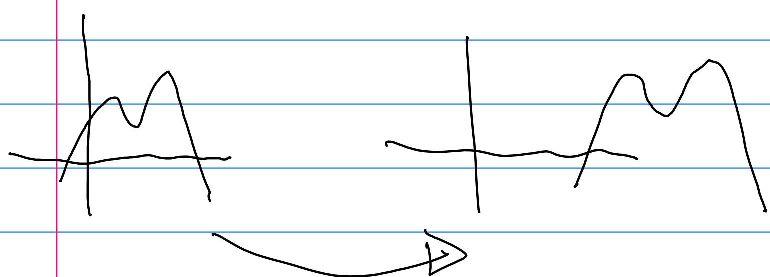
Dada la curvatura $\rightsquigarrow$ $\gamma = \gamma(s)$
$\kappa = \kappa(s)$ única salvo movimientos rígidos con curvatura κ.

→ Resuelve (en parte) el problema de reconocer cuándo 2 curvas coinciden salvo un movimiento rígido.



$\kappa = \kappa(s)$ $\qquad$ $\bar{\kappa} = \bar{\kappa}(\bar{s})$



κ y $\bar{\kappa}$ difieren en una traslación.

Objetivo: Reconocer si 2 curvas no necesariamente p.p.a. difieren en un mov. rígido.

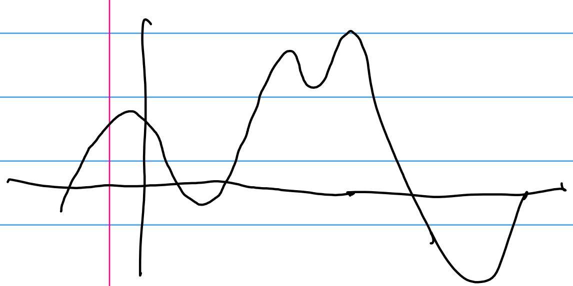
$$\kappa = \kappa(t) = \frac{\det(\gamma', \gamma'')}{\|\gamma'\|^3}$$ → permite ~~calcular~~ calcular la curv. si la curva no está p.p.a.

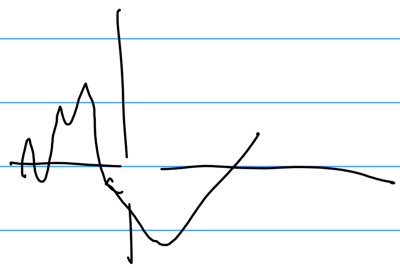
$$\kappa = \kappa(s(t))$$

$s(t) \equiv$ long. de arco; $\quad s(t) = \int_{t_0}^{t} \|\gamma'(\xi)\| \, d\xi$

$\gamma = \gamma(t) \qquad \tilde{\gamma} = \tilde{\gamma}(\tilde{t})$

$\kappa = \kappa(t) \qquad \tilde{\kappa} = \tilde{\kappa}(\tilde{t})$



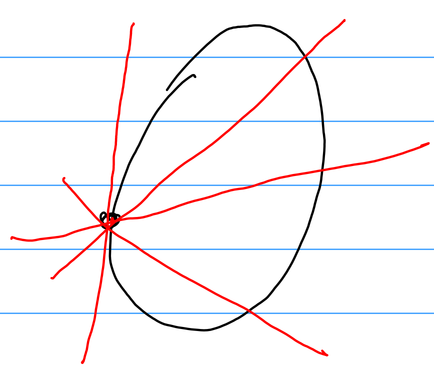


Ejemplo: Elipse

$$C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\begin{cases} x = a\cos t \\ y = b \operatorname{sen} t \end{cases}$ (no es p.p.a.)

Param. usando curvas racionales:



Dado un pto. $P_0 \in C$

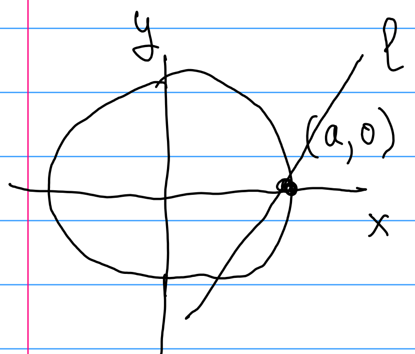
Tomamos el haz de rectas por p_0, $Haz(p_0)$

$\forall \ell \in Haz(p_0)$

$$\ell \cap C = \{p_0, p\}$$

$$Haz(p_0) \to C$$

$$\ell \longmapsto p$$



$\ell_0 : x - a = 0$

$\ell_1 : y = 0$

$\ell : y = t(x-a)$

$$\begin{cases} x^2/a^2 + y^2/b^2 = 1 \\ y = t(x-a) \end{cases} \qquad \frac{x^2}{a^2} + \frac{t^2(x-a)^2}{b^2} = 1$$

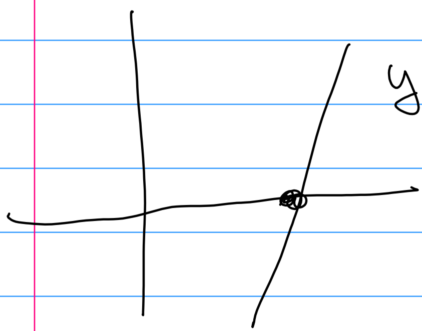
$$x^2\left(\frac{1}{a^2} + \frac{t^2}{b^2}\right) - \frac{2t^2 a x}{b^2} + \frac{t^2 a^2}{b^2} = 1.$$

$a = 2, \ b = 3$

$\gamma(t) = \left(\frac{8t^2 - 18}{4t^2 + 9}, \ -\frac{36t}{4t^2+9}\right)$ param. racional.

→ es rápida de calcular.

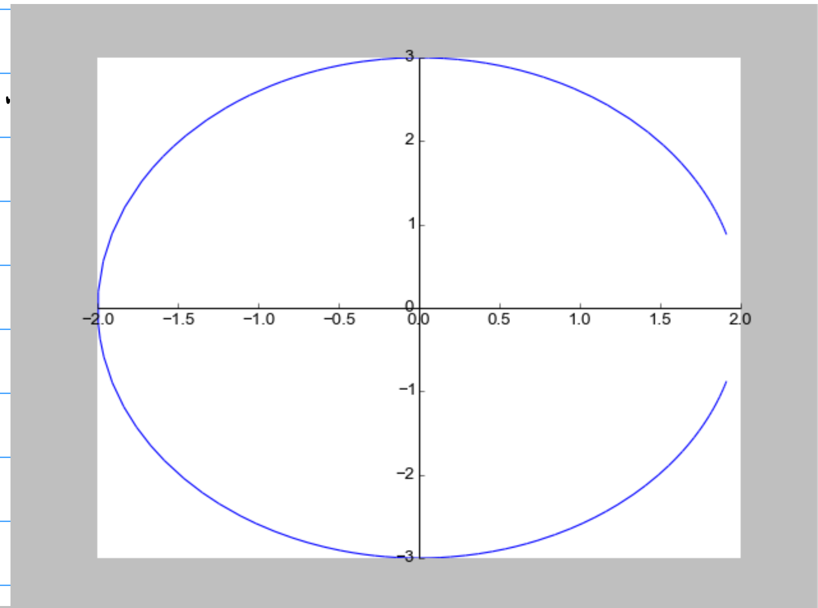
$$\gamma(t) = (\underset{x}{8t^2-18} : \underset{y}{-36t} : \underset{z}{4t^2+9}) \text{ param. en } \mathbb{P}^2,$$

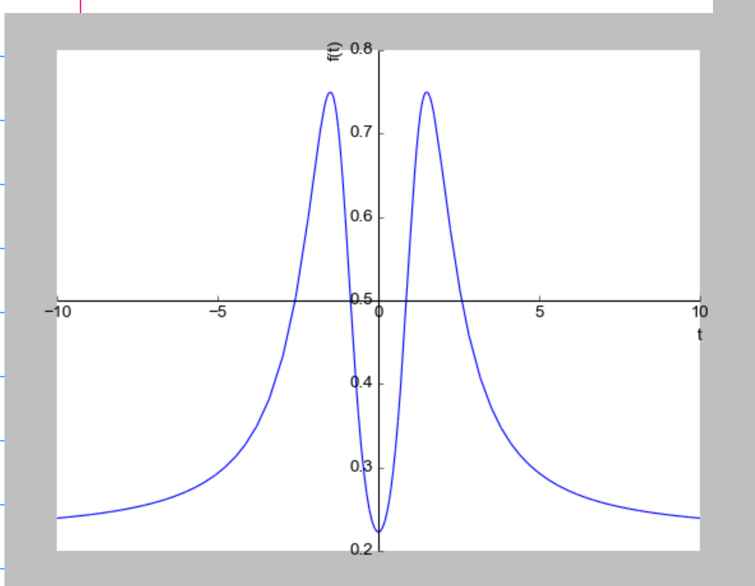


$y = t(x-2)$

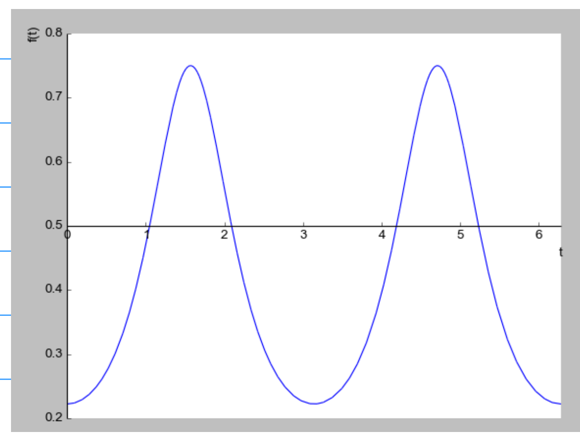
↑

pendiente.





Curv. de la elipse como curv. racional.



Curv. de la elipse como curva trig.