

CS217 – Algorithm Design and Analysis

Homework 3

Not Strong Enough

March 23, 2020

┌ 1

Let $B_{i,j,k}$ be an indicator variable which is 1 if i is a common ancestor of j and k appear in the subtree $T(\pi)$ rooted at i . What is $\mathbb{E}[B_{i,j,k}]$? Give a succinct formula for this.

Solution. Suppose we are at the node whose pivot is p . If $p > \max\{i, j, k\}$ or $p < \min\{i, j, k\}$, then i, j, k are all in the left subtree or all in the right subtree, which means we have to check one of the subtrees.

If $\min\{i, j, k\} \leq p \leq \max\{i, j, k\}$, we have to discuss some cases:

1. If $p = i$, then $B_{i,j,k} = 1$ because i is the ancestor of j and k .
2. If case 1 isn't satisfied and $p \neq j$ and $p \neq k$, then i, j, k are not in the same subtree. So $B_{i,j,k} = 0$.
3. If case 1 and 2 isn't satisfied, then j or k is the ancestor of i . So $B_{i,j,k} = 0$.

In conclusion, $\mathbb{E}[B_{i,j,k}] = \frac{1}{\max\{i,j,k\} - \min\{i,j,k\} + 1}$.

□

▮ 2

Let $C(\pi, k)$ be the number of comparisons made by QUICKSELECT when given π as input. Design a formula for $C(\pi, k)$ with the help of the indicator variables $A_{i,j}$ and $B_{i,j,k}$ (analogous to the formula $\sum_{i \neq j} A_{i,j}$ for the number of comparisons made by quicksort).

Solution. Observe that two numbers i, j will be compared if and only if $j \neq i$ and j and k are in the subtree of i .

So $C(\pi, k) = \sum_{i \neq j} B_{i,j,k}$. □