CS217 – Algorithm Design and Analysis

Homework 3

Not Strong Enough

April 3, 2020

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Let T be a minimum spanning tree of G, and let $c \in \mathbb{R}$. Show that T_c and G_c have exactly the same connected components. (That is, two vertices $u, v \in V$ are connected in T_c if and only if they are connected in G_c). You are encouraged to draw pictures to illustrate your proof.

Solution. Since T_c is a subset of G_c, if $u, v \in V$ are connected in T_c, then u, v are connected in G_c.

Now let's assume $u, v \in V$ are not connected in T_c, but are connected in G_c. We call the edge (u, v) x, then $w(x) \leq c$. Also, the path in T from u to v contains an edge whose weight is greater than c. We call it y, then $w(y) > c$. If we add x into T, this forms a cycle which contains both x and y. After that, removing y from T still keeps the connectivity. We name $T' = T + x - y$. Now T' becomes a tree, because it has $|V| - 1$ edges and it's connected. The total weight of T' is $w(T) + w(x) - w(y) < w(T)$, which violates that T is an MST. So u, v are not connected in G_c if they are not connected in T_c.

In conclusion, two vertices $u, v \in V$ are connected in T_c if and only if they are connected in G_c. □