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We know that $\nu(G) = \tau(G)$ for all bipartite graphs (Kőnig's Theorem) and $\nu(G) \leq \tau(G)$ for all graphs (since every matched edge must be covered by a distinct vertex). Show that $\tau(G) \leq 2\nu(G)$ for all grpahs G.

Proof. Let M be a maximum matching of G. It follows that $|M| = \nu(G)$. Now we choose our vertex set V' to be all the matched vertices in G. So $|V'| = 2\nu(G)$.

Claim that V' is a vertex cover. To see that, assume there exists an edge (u, v) which is not covered by V'. It means that neither u nor v is matched. So we can add edge (u, v) to M, and thus M is not maximum, which leads to a contradiction.

So the size of minimum vertex cover $\tau(G) \leq |V'| = 2\nu(G)$. □