CS217 – Algorithm Design and Analysis

Homework 3

Not Strong Enough

March 22, 2020

⌐ 1

Let $B_{i,j,k}$ be an indicator variable which is 1 if i is a common ancestor of j and k appear in the subtree $T(\pi)$ rooted at i. What is $\mathbb{E}[B_{i,j,k}]$? Give a succinct formula for this.

Solution. Suppose we are at node whose pivot is p. If $p < min(i,j,k)$ or $p > max(i,j,k)$, then i,j,k are all in the left subtree or all in the right subtree, which means we have to check the subtree. If $min(i,j,k) \leq p \leq max(i,j,k)$, we have to discuss some cases.

case 1.if $p = i$, then $B_{i,j,k} = 1$ because i is the ancestor of j and k, leading to $B_{i,j,k} = 1$.

case 2.if case 1 isn't satisfied and $p \neq j$ and $p \neq k$, then i,j,k are not in the same subtree. So $B_{i,j,k} = 0$.

case 3.if case 1 and 2 isn't satisfied , then j or k is the ancestor of i, so $B_{i,j,k} = 0$.

In conclusion,$\mathbb{E}[B_{i,j,k}] = \frac{1}{max(i,j,k)-min(i,j,k)+1}$ □

⌐ 2: L

t $C(\pi, k)$ be the number of comparisons made by QUICKSELECT when given π as input. Design a formula for $C(\pi, k)$ with the help of the indicator variables $A_{i,j}$ and $B_{i,j,k}$ (analogous to the formula $\sum_{i \neq j} A_{i,j}$) for the number of comparisons made by quicksort).

Solution. Observe that 2 number i,j will be compared if and only if $j \neq i$ and j and k are in the subtree of i.

So $C(\pi, k) = \sum_{i \neq j} B_{i,j,k}$. □