

# CS217 – Algorithm Design and Analysis

## Homework 1

### Not Strong Enough

March 7, 2020

┌ 1

A person writes two distinct integers on two cards, one per card, and puts them on the table face down. Pick either of the two, look at it, and then guess whether the other number is larger or smaller. Suppose that you have a good random number generator. Prove that you have a strategy to make a correct guess with probability strictly greater than  $\frac{1}{2}$ .

Let the two distinct integers on cards be  $p, q \in \mathbb{Z}$ .

The strategy exists if we can generate a random number on  $\mathbb{Z}$ . Let this random number be  $n$ . Our strategy is as follow: If  $n > p$ , we make the guess that the other number is greater.

Suppose  $p < q$ , there will be six situations:

Seen number	$p$	$q$
$n < p$	Wrong	Correct
$p < n < q$	Correct	Correct
$q < n$	Correct	Wrong

And we have the same probability to see  $p$  and  $q$ . Thus the probability to guess correctly is

$$p = \frac{1}{2} + \frac{P(p < n < q)}{2}$$

, which is strictly greater than  $\frac{1}{2}$ .

For example, if we can get a random angle  $\theta \in [0, 2\pi)$ , and generate by  $n = \cot \frac{\theta}{2}$ . Then

$$p = \frac{1}{2} + \frac{\cot^{-1} q - \cot^{-1} p}{2\pi}.$$

And if we only have a coin, we can toss it for  $k$  times to generate a random angle  $k \in [0, 2\pi]$  by  $k = \frac{\text{integer represented by the sequence}}{2^k} \cdot 2\pi$ . Then apply the procedure above.