Combinatorics and Graph Theory

Not Strong Enough

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1 Set Theory

1.0.1: Definition

- $[n] = \{1, \cdots, n\}$
- $A \subseteq [n]$, then a, the eigenvector of A, $\in \mathbb{F}_2^n$
- $A \times B := \{(a, b) : a \in A, b \in B\}$, actually the ordered pair (a, b) can be represented by $\{a, \{a, b\}\}$ without having to introduce new symbols.

- e.g. $A = \{1, 3, 4\} \subseteq [n]$, $a = (1, 0, 1, 1, 1)$.
  $A \triangle B \leftrightarrow a + b$, $+$ is under $\mathbb{F}_2^n$.

1.0.2: Cantor-Bernstein-Schröder Theorem

For set A, B if there exists injections $f : A \to B$ and $g : B \to A$, then there is a bijection $h : A \to B$.

- With this theorem, we can show that $\mathbb{N}^k \sim \mathbb{N}$ by construction two injections.
  $f_1 : \mathbb{N}^k \to \mathbb{N}, (a_1, a_2, \cdots, a_n) \mapsto \prod_{i=1}^k p_i^{a_i}$, where p_i are different prime numbers.
  $f_2 : \mathbb{N}^k \to \mathbb{N}, (a_1, a_2, \cdots, a_n) \mapsto$ "$a_1, a_2, \cdots, a_n$" in 13 base with $',' = 11$.
- Furthermore,

  1.0.3

  If any element of set A can be represented by a **finite** string in a **countable** alphabet Σ, then A is **countable** .

- And we can only put at most countable disjoint "plate"s in $\mathbb{R}^2$. But there can be uncountable "ring"s.
  What about "8"s, that are two closed curve with exactly one shared point? And "y"s, that are three curves with shared starting point?

1.0.4: Definition

Let $A, B \subseteq U$, $A + B := \{a +^U b : a \in A, b \in B\}$.

- $U = \mathbb{R}$, then $|A + B| \geq s + t - 1$
  Under which U and $+^U$ can this hold? $\mathbb{Z}_5$ is a counter-example.
- ### 1.0.5: The Cauchy-Davenport Theorem

  $A, B \subseteq \mathbb{Z}_p, |A + B| \geq \min(p, s + t - 1)$

  Induction on $|B|$.

1.0.6: Definition

$A^B := \{f : B \to A\}$

- $f : B \to A \leftrightarrow A^{|B|}$
  e.g. $[10]^{[3]} \leftrightarrow [10]^3$, $f \in [10]^{[3]} \leftrightarrow (f(1), f(2), f(3)) \in [10]^3$.
  - $u + v \leftrightarrow f + g$
  - (Cauchy-Schwarz) $\langle i, v\rangle^2 \leq \langle u, u\rangle \langle v, v\rangle \leftrightarrow (\int fg\mathrm{d}x)^2 \leq \int f^2\mathrm{d}x + \int g^2\mathrm{d}x$
- $f \in A^B$ devides B into $|A|$ parts.

1.0.7: Definition

A is a set, $r \in \mathbb{Z}$, $\binom{A}{r} := \{B \in A : |B| = r\} \subseteq 2^A$

- e.g. $\binom{A}{0} = \{\varnothing\}, \binom{A}{-1} = \varnothing$.
- $n \in \mathbb{N}$, $r \in \mathbb{Z}$, $\binom{n}{r} := |\binom{[n]}{r}|$
  $\binom{n}{r} = \frac{(n)_r}{r!} = \frac{n!}{r!(n-r)!}$, should $r \geq 0$ and $r \leq n$?