Probability, Week 1, exerciese 24

Not Strong Enough

March 7, 2020

1. chain:

For each chain $\mathfrak{C}$, since each two elements in $\mathfrak{C}$ are comparable, they cannot have same size. So we can have a function $f : \mathfrak{C} \to \mathbb{N} : f(C) =$ the size of C, which is an injection.

So there is $| \mathfrak{C} | \leq | \mathbb{N} | < | 2^{\mathbb{N}} |$.

2: antichain:

Construct an antichain $\mathfrak{A}$ by the following step:

Devide numbers by pairs: $(0,1); (2,3); \ldots (2n, 2n-1); \ldots$. Denote the pair $(2i, 2i+1)$ by p_i.

Each set $A \in \mathfrak{A}$ contains one and only one element in each pair, making them different with each other:

For each two set, they contians different number at least in one pair, so they are uncomparable. So $\mathfrak{A}$ is an antichain.

Now prove that $| \mathfrak{A} | = | 2^{\mathbb{N}} |$:

We construct such a function $f : \mathfrak{A} \to 2^{\mathbb{N}}$

$$f(A) = \{\text{if } A \text{ contains } 2i,\ i \text{ is in the set, otherwise(contains } 2i+1)\ i \text{ is not in the set}\}$$

It is apparently that f is a bijection between $\mathfrak{A}$ and $2^{\mathbb{N}}$.