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Suppose G is connected, and no two edges of G have the same weight. Show that G has exactly one minimum spanning tree

Proof. Suppose there are two different minimum spanning trees, named T and T' .

Now we remove an edge e in T , and the graph becomes two connected parts. If in T' the two parts are connected by e too, then remove another edge in T until the one in T' is different from the one in T .

There should be such a pair: otherwise, each edge in T is the same as that in T' , which is a contradiction.

Now consider such pair. Without loss of generality, assume that $w(e) < w(e')$. Then if we replace e' in T' by e , the total weight of T' is less, so T' is not a minimum spanning tree. \square