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Suppose G is connected, and no two edges of G have the same weight. Show that G has exactly one minimum spanning tree

Proof. Suppose there are two different minimum spanning trees, named T and T'.

Now we remove an edge e in T, and the graph becomes two connected parts. If in T' the two parts are connected by e too, then remove another edge in T until the one in T' is different from the one in T, and we name the one in T' be e' in this case.

There should be such a pair (e, e'): otherwise, each edge in T is the same as that in T', which is a contradiction.

Now consider such pair(e, e'). Without loss of generality, assume that $w(e) < w(e')$. Then if we replace e' in T' by e, the total weight of T' is less, so T' is not a minimum spanning tree. $\square$