1

Let $G = (V, c)$ be a flow network. Prove that flow is "transitive" in the following sense: if r, s, t are vertices, and there is an r-s-flow of value k and an s-t-flow of value k, then there is an r-t-flow of value k.

Proof. Note that there is an r-s-flow of value k means that the value of the maximum r-s-flow is at least k, which also means that the value of the minimum r-s-cut is at least k. Similarly, the value of the minimum s-t-cut is also at least k.

Now consider an r-t-cut. It is either an r-s-cut (if s is not in the cut) or an s-t-cut (if s is in the cut). So the capacity of the minimum r-t-cut is at least k. It follows that the value of the maximum r-t-flow is at least k, and thus there is an r-t-flow of value k. □