

┌ 1

Let $G = (V, c)$ be a flow network. Prove that flow is “transitive” in the following sense: if r, s, t are vertices, and there is an $r - s$ -flow of value k and an $s - t$ -flow of value k , then there is an $r - t$ -flow of value k . ┐

Proof. Note that there is an $r - s$ -flow of value k means that the value of the maximum $r - s$ -flow is at least k , which also means that the value of the minimum $r - s$ -cut is at least k . Similarly, the value of the minimum $s - t$ -cut is also at least k .

Now consider an $r - t$ -cut. It is either an $r - s$ -cut (if s is not in the cut) or an $s - t$ -cut (if s is in the cut). So the capacity of the minimum $r - t$ -cut is at least k . It follows that the value of the maximum $r - t$ -flow is at least k , and thus there is an $r - t$ -flow of value k . \square