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For a weighted graph G, let $m_c(G) := |\{e \in E(G) \mid w(e) \leq c\}|$, i.e., the number of edges of weight at most c (so G_c has $m_c(G)$ edges). Let T, T' be two minimum spanning trees of G. Show that $m_c(T) = m_c(T')$.

Proof. For a graph $G = (V, E)$, from the definition of $m_c(G)$ we have $m_c(G) = |\{e \in E(G) \mid w(e) \leq c\}| = |E(G_c)|$. Since T and T' are two minimum spanning trees of G, by the conclusion of exersice 1, we know that T_c and G_c have exactly the same connected components, and T'_c and G_c also have the same connected components. It follows that T_c and T'_c have the same connected components.

Note that T_c and T'_c are two *forests*. We know that for a forest with n nodes and k connected components, there are exactly $n - k$ edges in the forest. Hence T_c and T'_c having the same connected components means that the number of edges in T_c and T'_c are also the same, i.e., $m_c(T) = m_c(T')$. $\square$