

# Team Notebook

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# 1 Data Structures

## 1.1 dsu

```
struct Dsu {
    vector<int> p; Dsu(int N = 0) : p(N, -1) {}
    int get(int x) { return p[x] < 0 ? x : get(p[x]); }
    bool sameSet(int a, int b) { return get(a) == get(b); }
    int size(int x) { return -p[get(x)]; }
    vector<vector<int>> S;
    void unite(int x, int y) {
        if ((x = get(x)) == (y = get(y)))
            return S.push_back({-1});
        if (p[x] > p[y]) swap(x, y);
        S.push_back({x, y, p[x], p[y]});
        p[x] += p[y], p[y] = x;
    }
    void rollback() {
        auto a = S.back(); S.pop_back();
        if (a[0] != -1) p[a[0]] = a[2], p[a[1]] = a[3];
    }
};
```

## 1.2 fenwick tree

```
int ft[MAXN+1]; // add dimension for multi-d
void upd(int i0, int v){ // add v to i0th element
    for(int i=i0+1; i<=MAXN; i+=i&-i) ft[i]+=v; //+ fors
}
int get(int i0){ // get sum of range [0,i0]
    int r=0; // add fors
    for(int i=i0; i; i-=i&-i) r+=ft[i];
    return r;
}
int get_sum(int i0, int i1){ // sum of [i0,i1]
    return get(i1)-get(i0);
}
```

## 1.3 link cut tree

```
const int N_DEL = 0, N_VAL = 0; //delta, value
inline int mOp(int x, int y){return x+y;} //modify
inline int qOp(int lval, int rval){return lval + rval;} //
query
inline int dOnSeg(int d, int len){return d==N_DEL ? N_DEL :
d*len;}
//mostly generic
inline int joinD(int d1, int d2){
    if(d1==N_DEL) return d2; if(d2==N_DEL) return d1; return
mOp(d1, d2);}
inline int joinVD(int v, int d){return d==N_DEL ? v : mOp(v,
d);}
struct Node_t{
    int sz, nVal, tVal, d; bool rev;
```

```
Node_t *c[2], *p;
Node_t(int v) : sz(1), nVal(v), tVal(v), d(N_DEL), rev(0),
p(0){
    c[0]=c[1]=0;
}
bool isRoot(){return !p || (p->c[0] != this && p->c[1] !=
this);}
void push(){
    if(rev){
        rev=0; swap(c[0], c[1]); fore(x,0,2)if(c[x])c[x]-
>rev^=1;
    }
    nVal=joinVD(nVal, d); tVal=joinVD(tVal, dOnSeg(d, sz));
    fore(x,0,2)if(c[x])c[x]->d=joinD(c[x]->d, d);
    d=N_DEL;
}
void upd();
};
typedef Node_t* Node;
int getSize(Node r){return r ? r->sz : 0;}
int getPv(Node r){
    return r ? joinVD(r->tVal, dOnSeg(r->d, r->sz)) : N_VAL;}
void Node_t::upd(){
    tVal = qOp(qOp(getPv(c[0]), joinVD(nVal, d)),
    getPv(c[1]));
    sz = 1 + getSize(c[0]) + getSize(c[1]);
}
void conn(Node c, Node p, int il){if(c->p=p; if(il>=0)p-
>c[il]=c;}
void rotate(Node x){
    Node p = x->p, g = p->p;
    bool gCh=p->isRoot(), isl = x==p->c[0];
    conn(x->c[isl], p, isl); conn(p, x, !isl);
    conn(x, g, gCh?-1:(p==g->c[0])); p->upd();
}
void spa(Node x){//splay
    while(!x->isRoot()){
        Node p = x->p, g = p->p;
        if(!p->isRoot())g->push();
        p->push(); x->push();
        if(!p->isRoot())rotate((x==p->c[0])==(p==g->c[0])? p : x);
        rotate(x);
    }
    x->push(); x->upd();
}
Node exv(Node x){//expose
    Node last=0;
    for(Node y=x; y; y=y->p)spa(y), y->c[0]=last, y-
>upd(), last=y;
    spa(x);
    return last;
}
void mkR(Node x){exv(x); x->rev^=1;} //makeRoot
Node getR(Node x){exv(x); while(x->c[1])x=x-
>c[1]; spa(x); return x;}
```

```
Node lca(Node x, Node y){exv(x); return exv(y);}
bool connected(Node x, Node y){exv(x); exv(y); return x==y?
1:x->p!=0;}
void link(Node x, Node y){mkR(x); x->p=y;}
void cut(Node x, Node y){mkR(x); exv(y); y->c[1]->p=0; y-
>c[1]=0;}
Node father(Node x){
    exv(x); Node r=x->c[1];
    if(!r) return 0;
    while(r->c[0])r=r->c[0];
    return r;
}
void cut(Node x){ // cuts x from father keeping tree root
    exv(father(x)); x->p=0;}
int query(Node x, Node y){mkR(x); exv(y); return getPv(y);}
void modify(Node x, Node y, int d){mkR(x); exv(y); y-
>d=joinD(y->d, d);}
Node lift_rec(Node x, int t){
    if(!x) return 0;
    if(t==getSize(x->c[0])){spa(x); return x;}
    if(t<getSize(x->c[0])) return lift_rec(x->c[0], t);
    return lift_rec(x->c[1], t-getSize(x->c[0])-1);
}
Node lift(Node x, int t){ // t-th ancestor of x (lift(x,1)
is x's father)
    exv(x); return lift_rec(x, t);}
int depth(Node x){ // distance from x to its tree root
    exv(x); return getSize(x)-1;}
```

## 1.4 persistent segment tree lazy

```
template <class T>
struct Node {
    T x, lz;
    int l = -1, r = -1;
};

template <class T>
struct Pstl {
    int N;
    vector<Node<T>> a;
    vector<int> head;

    T qneut() { return 0; }
    T merge(T l, T r) { return l + r; }
    T uneut() { return 0; }
    T accum(T u, T x) { return u + x; }
    T apply(T x, T lz, int l, int r) { return x + (r - l) *
    lz; }

    int build(int vl, int vr) {
        if (vr - vl == 1) a.push_back({qneut(),
        uneut()}); // node construction
        else {
            int vm = (vl + vr) / 2, l = build(vl, vm), r =
```

```

build(vl, vr);
    a.push_back({merge(a[l].x, a[r].x), neut(), l,
r}); // query merge
    }
    return a.size() - 1;
}

T query(int l, int r, int v, int vl, int vr, T acc) {
    if (l >= vr || r <= vl) return
qneut(); // query neutral
    if (l <= vl && r >= vr) return apply(a[v].x, acc,
vl, vr); // update op
    acc = accum(acc,
a[v].lz); // update merge
    int vm = (vl + vr) / 2;
    return merge(query(l, r, a[v].l, vl, vm, acc),
query(l, r, a[v].r, vm, vr, acc)); // query merge
}

int update(int l, int r, T x, int v, int vl, int vr) {
    if (l >= vr || r <= vl || r <= l) return v;
    a.push_back(a[v]);
    v = a.size() - 1;
    if (l <= vl && r >= vr) {
        a[v].x = apply(a[v].x, x, vl, vr); // update op
        a[v].lz = accum(a[v].lz, x); // update
merge
    } else {
        int vm = (vl + vr) / 2;
        a[v].l = update(l, r, x, a[v].l, vl, vm);
        a[v].r = update(l, r, x, a[v].r, vm, vr);
        a[v].x = merge(a[a[v].l].x, a[a[v].r].x); //
query merge
    }
    return v;
}

Pstl() {}
Pstl(int N) : N(N) { head.push_back(build(0, N)); }

T query(int t, int l, int r) {
    return query(l, r, head[t], 0, N, neut()); //
update neutral
}

int update(int t, int l, int r, T x) {
    return head.push_back(update(l, r, x, head[t], 0,
N)), head.size() - 1;
}
};

```

## 1.5 persistent segment tree

```

// usage:
// Pst<Node<ll>> pst;
// pst = {N};

```

```

// int newtime = pst.update(time, index, value);
// Node<ll> result = pst.query(newtime, left, right);

template <class T>
struct Node {
    T x;
    int l = -1, r = -1;

    Node() : x(0) {}
    Node(T x) : x(x) {}
    Node(Node a, Node b, int l = -1, int r = -1) : x(a.x +
b.x), l(l), r(r) {}
};

template <class U>
struct Pst {
    int N;
    vector<U> a;
    vector<int> head;

    int build(int vl, int vr) {
        if (vr - vl == 1) a.push_back(U());
        else {
            int vm = (vl + vr) / 2, l = build(vl, vm),
r = build(vm, vr);
            a.push_back(U(a[l], a[r], l, r));
        }
        return a.size() - 1;
    }

    U query(int l, int r, int v, int vl, int vr) {
        if (l >= vr || r <= vl) return U();
        if (l <= vl && r >= vr) return a[v];
        int vm = (vl + vr) / 2;
        return U(query(l, r, a[v].l, vl, vm),
query(l, r, a[v].r, vm, vr));
    }

    int update(int i, U x, int v, int vl, int vr) {
        a.push_back(a[v]);
        v = a.size() - 1;
        if (vr - vl == 1) a[v] = x;
        else {
            int vm = (vl + vr) / 2;
            if (i < vm) a[v].l = update(i, x, a[v].l, vl,
vm);

            else a[v].r = update(i, x, a[v].r, vm, vr);
            a[v] = U(a[a[v].l], a[a[v].r], a[v].l, a[v].r);
        }
        return v;
    }

    Pst() {}
    Pst(int N) : N(N) { head.push_back(build(0, N)); }
}

```

```

U query(int t, int l, int r) {
    return query(l, r, head[t], 0, N);
}

int update(int t, int i, U x) {
    return head.push_back(update(i, x, head[t], 0, N)),
head.size() - 1;
}
};

```

## 1.6 rmq lineal

```

typedef int tf; // 0(n) construction, 0(1) query
struct rmq{
    int n; tf INF=1e9; //change sign of INF for MAX
    vector<unsigned int> mk; vector<tf> bk,v;
    rmq(){}
    tf op(tf a, tf b){return min(a,b);} //change for maximum
    int f(int x){return x>>5;}
    rmq(vector<tf> &vv):n(SZ(vv)),mk(n),bk(n,INF),v(vv){
        unsigned int lst=0;
        for(int i=0;i<SZ(v);i++,lst<=1){
            bk[f(i)]=op(bk[f(i)],v[i]);
            while(lst&&v[i-__builtin_ctz(lst)]>v[i])
lst^=lst&-lst; //MIN
            //while(lst&&v[i-__builtin_ctz(lst)]<v[i])
lst^=lst&-lst; //MAX
            mk[i]=++lst;
        }
        for(int k=1,top=f(n);(1<=k)<=top;k+=
+)fore(i,0,top)if(i+(1<=k)<=top)
            bk[top*k+i]=op(bk[top*(k-1)+i],
bk[top*(k-1)+i+(1<=k-1)]);
        }
        tf get(int st, int en){
            return v[en-31+__builtin_clz(mk[en]&((1ll<<en-
st+1)-1))];
        }
        tf query(int s, int e){ // [s,e]
            int b1=f(s),b2=f(e),top=f(n);
            if(b1==b2) return get(s,e);
            tf ans=op(get(s, (b1+1)*32-1), get(b2*32,e));
            s=(b1+1)*32; e=b2*32-1;
            if(s<=e){
                int k=31-__builtin_clz(f(e-s+1));
                ans=op(ans,op(bk[top*k+f(s)],bk[top*k+f(e)-
(1<=k)+1]));
            }
            return ans;
        }
    }
};

```

## 1.7 segment tree 2d

```

// #define MAXN 1024 #define op(a,b) (a+b) #define NEUT 0
int n,m; int a[MAXN][MAXN],st[2*MAXN][2*MAXN];

```

```

void build(){
    repx(i, 0, n) repx(j, 0, m) st[i+n][j+m] = a[i][j];
    repx(i, 0, n) for(int j = m-1; j; --j)
        st[i+n][j] = op(st[i+n][j<<1], st[i+n][j<<1|1]);
    for(int i = n-1; i; --i) repx(j, 0, 2*m)
        st[i][j] = op(st[i<<1][j], st[i<<1|1][j]);
}

void upd(int x, int y, int v){
    st[x+n][y+m]=v;
    for(int j = y+m; j > 1; j >= 1)
        st[x+n][j>>1] = op(st[x+n][j], st[x+n][j^1]);
    for(int i = x+n; i > 1; i >= 1) for(int j=y+m;j;j>=1)
        st[i>>1][j] = op(st[i][j], st[i^1][j]);
}

int query(int x0, int x1, int y0, int y1){
    int r=NEUT;
    for(int i0=x0+n, i1=x1+n; i0<i1; i0>=1, i1>=1){
        int t[4], q = 0;
        if(i0 & 1) t[q++] = i0++;
        if(i1 & 1) t[q++] = --i1;
        repx(k,0,q)
            for(int j0=y0+m, j1=y1+m; j0<j1; j0>=1,j1>=1){
                if(j0 & 1) r = op(r, st[t[k]][j0++]);
                if(j1 & 1) r = op(r, st[t[k]][--j1]);
            }
        return r;
    }
}

```

## 1.8 segment tree beats

```

struct Node {
    ll s, mx1, mx2, mxc, mn1, mn2, mnc, lz = 0;
    Node() : s(0), mx1(LLONG_MIN), mx2(LLONG_MIN), mxc(0),
mn1(LLONG_MAX), mn2(LLONG_MAX), mnc(0) {}
    Node(ll x) : s(x), mx1(x), mx2(LLONG_MIN), mxc(1),
mn1(x), mn2(LLONG_MAX), mnc(1) {}
    Node(const Node &a, const Node &b) {
        // add
        s = a.s + b.s;
        // min
        if (a.mx1 > b.mx1) mx1 = a.mx1, mxc = a.mxc, mx2 =
max(b.mx1, a.mx2);
        if (a.mx1 < b.mx1) mx1 = b.mx1, mxc = b.mxc, mx2 =
max(a.mx1, b.mx2);
        if (a.mx1 == b.mx1) mx1 = a.mx1, mxc = a.mxc +
b.mxc, mx2 = max(a.mx2, b.mx2);
        // max
        if (a.mn1 < b.mn1) mn1 = a.mn1, mnc = a.mnc, mn2 =
min(b.mn1, a.mn2);
        if (a.mn1 > b.mn1) mn1 = b.mn1, mnc = b.mnc, mn2 =
min(a.mn1, b.mn2);
        if (a.mn1 == b.mn1) mn1 = a.mn1, mnc = a.mnc +
b.mnc, mn2 = min(a.mn2, b.mn2);
    }
}

```

```

};

// 0 - indexed / inclusive - inclusive
template <class node>
struct STB {
    vector<node> st; int n;

    void build(int u, int i, int j, vector<node> &arr) {
        if (i == j) {
            st[u] = arr[i];
            return;
        }
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        build(l, i, m, arr), build(r, m + 1, j, arr);
        st[u] = node(st[l], st[r]);
    }

    void push_add(int u, int i, int j, ll v) {
        st[u].s += (j - i + 1) * v;
        st[u].mx1 += v, st[u].mn1 += v, st[u].lz += v;
        if (st[u].mx2 != LLONG_MIN) st[u].mx2 += v;
        if (st[u].mn2 != LLONG_MAX) st[u].mn2 += v;
    }

    void push_max(int u, ll v, bool l) { // for min op
        if (v >= st[u].mx1) return;
        st[u].s -= st[u].mx1 * st[u].mxc;
        st[u].mx1 = v;
        st[u].s += st[u].mx1 * st[u].mxc;
        if (l) st[u].mn1 = st[u].mx1;
        else if (v <= st[u].mn1) st[u].mn1 = v;
        else if (v < st[u].mn2) st[u].mn2 = v;
    }

    void push_min(int u, ll v, bool l) { // for max op
        if (v <= st[u].mn1) return;
        st[u].s -= st[u].mn1 * st[u].mnc;
        st[u].mn1 = v;
        st[u].s += st[u].mn1 * st[u].mnc;
        if (l) st[u].mx1 = st[u].mn1;
        else if (v >= st[u].mx1) st[u].mx1 = v;
        else if (v > st[u].mx2) st[u].mx2 = v;
    }

    void push(int u, int i, int j) {
        if (i == j) return;
        // add
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        push_add(l, i, m, st[u].lz);
        push_add(r, m + 1, j, st[u].lz);
        st[u].lz = 0;
        // min
        push_max(l, st[u].mx1, i == m);
        push_max(r, st[u].mx1, m + 1 == j);
        // max
        push_min(l, st[u].mn1, i == m);
        push_min(r, st[u].mn1, m + 1 == r);
    }

    node query(int a, int b, int u, int i, int j) {

```

```

        if (b < i || j < a) return node();
        if (a <= i && j <= b) return st[u];
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        return node(query(a, b, l, i, m), query(a, b, r, m +
1, j));
    }

    void update_add(int a, int b, ll v, int u, int i, int j)
    {
        if (b < i || j < a) return;
        if (a <= i && j <= b) {
            push_add(u, i, j, v);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_add(a, b, v, l, i, m);
        update_add(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    void update_min(int a, int b, ll v, int u, int i, int j)
    {
        if (b < i || j < a || v >= st[u].mx1) return;
        if (a <= i && j <= b && v > st[u].mx2) {
            push_max(u, v, i == j);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_min(a, b, v, l, i, m);
        update_min(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    void update_max(int a, int b, ll v, int u, int i, int j)
    {
        if (b < i || j < a || v <= st[u].mn1) return;
        if (a <= i && j <= b && v < st[u].mn2) {
            push_min(u, v, i == j);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_max(a, b, v, l, i, m);
        update_max(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    STB(vector<node> &v, int N) : n(N), st(N * 4 + 5)
    { build(0, 0, n - 1, v); }
    node query(int a, int b) { return query(a, b, 0, 0, n -
1); }
    void update_add(int a, int b, ll v) { update_add(a, b,
v, 0, 0, n - 1); }
    void update_min(int a, int b, ll v) { update_min(a, b,
v, 0, 0, n - 1); }
}

```

```
void update_max(int a, int b, ll v) { update_max(a, b,
v, 0, 0, n - 1); }
};
```

## 1.9 segment tree lazy

```
template <class T>
struct Stl {
    int n; vector<T> a, b;
    Stl(int n = 0) : n(n), a(4 * n, qneut()),
        b(4 * n, uneut()) {}

    T qneut() { return -2e9; }
    T uneut() { return 0; }
    T merge(T x, T y) { return max(x, y); }
    void upd(int v, T x, int l, int r)
        { a[v] += x, b[v] += x; }

    void push(int v, int vl, int vm, int vr) {
        upd(2 * v, b[v], vl, vm);
        upd(2 * v + 1, b[v], vm, vr);
        b[v] = uneut();
    }

    T query(int l, int r, int v=1, int vl=0, int vr=1e9) {
        vr = min(vr, n);
        if (l <= vl && r >= vr) return a[v];
        if (l >= vr || r <= vl) return qneut();
        int vm = (vl + vr) / 2;
        push(v, vl, vm, vr);
        return merge(query(l, r, 2 * v, vl, vm),
            query(l, r, 2 * v + 1, vm, vr));
    }

    void update(int l, int r, T x, int v = 1, int vl = 0,
        int vr = 1e9) {
        vr = min(vr, n);
        if (l >= vr || r <= vl || r <= l) return;
        if (l <= vl && r >= vr) upd(v, x, vl, vr);
        else {
            int vm = (vl + vr) / 2;
            push(v, vl, vm, vr);
            update(l, r, x, 2 * v, vl, vm);
            update(l, r, x, 2 * v + 1, vm, vr);
            a[v] = merge(a[2 * v], a[2 * v + 1]);
        }
    }
};
```

## 1.10 segment tree

```
struct St {
    ll neut() { return 0; }
    ll merge(ll x, ll y) { return x + y; }
```

```
int n; vector<ll> a;
St(int n = 0) : n(n), a(2 * n, neut()) {}

ll query(int l, int r) {
    ll x = neut(), y = neut();
    for (l += n, r += n; l < r; l /= 2, r /= 2) {
        if (l & 1) x = merge(x, a[l++]);
        if (r & 1) y = merge(a[--r], y);
    }
    return merge(x, y);
}

void update(int i, ll x) {
    for (a[i += n] = x; i /= 2;)
        a[i] = merge(a[2 * i], a[2 * i + 1]);
}
```

## 1.11 sparse table

```
template <class T>
struct Sparse {
    T op(T a, T b) { return max(a, b); }

    vector<vector<T>> st;
    Sparse() {}
    Sparse(vector<T> a) : st{a} {
        int N = st[0].size();
        int npot = N <= 1 ? 1 : 32 - __builtin_clz(N);
        st.resize(npot);
        rep(x(i, 1, npot) rep(j, N + 1 - (1 << i))
            st[i].push_back(
                op(st[i - 1][j], st[i - 1][j + (1 << (i - 1))])
            ); // query op
    }

    T query(int l, int r) { // range must be nonempty!
        int i = 31 - __builtin_clz(r - l);
        return op(st[i][l], st[i][r - (1 << i)]); // queryop
    }
};
```

## 1.12 treap implicit

```
mt19937 gen(chrono::high_resolution_clock::now()
    .time_since_epoch().count());
// 101 Implicit Treap //
struct Node {
    int p, sz = 0, v, acc, l = -1, r = -1;
    Node() : v(0), acc(0) {}
    Node(int x) : p(gen()), sz(1), v(x), acc(x) {}
    void recalc(const Node &a, const Node &b) {
        sz = a.sz + b.sz + 1; acc = v + a.acc + b.acc;
    }
};
```

```
template <class node>
struct Treap {
    vector<node> t;
    int n, r = -1;
    node get(int u) { return u != -1 ? t[u] : node(); }
    void recalc(int u)
        { t[u].recalc(get(t[u].l), get(t[u].r)); }
    int merge(int l, int r) {
        if (min(l, r) == -1) return l != -1 ? l : r;
        int ans = (t[l].p < t[r].p) ? l : r;
        if (ans == l) t[l].r = merge(t[l].r, r), recalc(l);
        if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
        return ans;
    }
    pii split(int u, int id) {
        if (u == -1) return {-1, -1};
        int szl = get(t[u].l).sz;
        if (szl >= id) {
            pii ans = split(t[u].l, id);
            t[u].l = ans.ss;
            recalc(u);
            return {ans.ff, u};
        }
        pii ans = split(t[u].r, id - szl - 1);
        t[u].r = ans.ff;
        recalc(u);
        return {u, ans.ss};
    }
    Treap(vi &v) : n(sz(v)) {
        for (int i=0; i<n; i++) t.eb(v[i], r = merge(r, i));
    }
};

// Complete Implicit Treap with Lazy propagation //
struct Node {
    int p, sz = 0, v, acc, l = -1, r = -1, par = -1, lzv=0;
    bool lz = false, f = false;
    Node() : v(0), acc(0) {}
    Node(int x) : p(gen()), sz(1), v(x), acc(x) {}
    void recalc(const Node &a, const Node &b) {
        sz = a.sz + b.sz + 1; acc = v + a.acc + b.acc;
    }
    void upd_lazy(int x) { lz = 1, lzv += x; }
    void lazy() { v+=lzv, acc += sz * lzv, lz = 0, lzv = 0; }
    void flip() { swap(l, r), f = 0; }
};

template <class node> struct Treap {
    vector<node> t;
    int n, r = -1;
    node get(int u) { return u != -1 ? t[u] : node(); }
    void recalc(int u) {
        int l = t[u].l, r = t[u].r;
        push(l), push(r), flip(l), flip(r);
        t[u].recalc(get(l), get(r));
    }
    void push(int u) {
```

```

    if (u == -1 || !t[u].lz) return;
    int l = t[u].l, r = t[u].r;
    if (l != -1) t[l].upd_lazy(t[u].lzv);
    if (r != -1) t[r].upd_lazy(t[u].lzv);
    t[u].lazy();
}
void flip(int u) {
    if (u == -1 || !t[u].f) return;
    int l = t[u].l, r = t[u].r;
    if (l != -1) t[l].f ^= 1;
    if (r != -1) t[r].f ^= 1;
    t[u].flip();
}
int merge(int l, int r) { // (*) = only if parent needed
    if (min(l, r) == -1) return l != -1 ? l : r;
    push(l), push(r), flip(l), flip(r);
    int ans = (t[l].p < t[r].p) ? l : r;
    if (ans == l) t[l].r = merge(t[l].r, r), recalc(l);
    if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
    if (t[ans].l != -1) t[t[ans].l].par = ans; // (*)
    if (t[ans].r != -1) t[t[ans].r].par = ans; // (*)
    return ans;
}
pii split(int u, int id) { // (*) = only if parent needed
    if (u == -1) return {-1, -1};
    push(u);
    flip(u);
    int szl = get(t[u].l).sz;
    if (szl >= id) {
        pii ans = split(t[u].l, id);
        if (ans.ss != -1) t[ans.ss].par = u; // (*)
        if (ans.ff != -1) t[ans.ff].par = -1; // (*)
        t[u].l = ans.ss;
        recalc(u);
        return {ans.ff, u};
    }
    pii ans = split(t[u].r, id - szl - 1);
    if (ans.ff != -1) t[ans.ff].par = u; // (*)
    if (ans.ss != -1) t[ans.ss].par = -1; // (*)
    t[u].r = ans.ff;
    recalc(u);
    return {u, ans.ss};
}
int update(int u, int l, int r, int v) {
    pii a = split(u, l), b = split(a.ss, r - l + 1);
    t[b.ff].upd_lazy(v);
    return merge(a.ff, merge(b.ff, b.ss));
}
void print(int u) {
    if (u == -1) return; push(u), flip(u); print(t[u].l);
    cout << t[u].v << ' '; print(t[u].r);
}
Treap(vi &v) : n(sz(v)) {
    for (int i=0; i<n; i++) t.eb(v[i], r = merge(r, i));
}

```

```

    }
};

```

## 1.13 treap

```

mt19937 rng(chrono::high_resolution_clock::now()
    .time_since_epoch().count());
struct item { // to allow duplicates, add random val to keys
    int p, k, n; item *l=0, *r=0;
    item(int k) : p(rng()), k(k), n(1) {}
};
int getn(item *x) { return x ? x->n : 0; }
void recalc(item *x) { if(x) x->n=getn(x->l)+1+getn(x->r); }
void split(item *x, int k, item *&l, item *&r) {
    if (!x) l = r = 0;
    else if (k < x->k) split(x->l, k, l, x->l), r = x;
    else split(x->r, k, x->r, r), l = x;
    recalc(x);
}
void merge(item *&x, item *l, item *r) {
    if (!l || !r) x = l ? l : r;
    else if (l->p > r->p) merge(l->r, l->r, r), x = l;
    else merge(r->l, l, r->l), x = r;
    recalc(x);
}
void insert(item *&x, item *y) {
    if (!x) x = y;
    else if (y->p > x->p) split(x, y->k, y->l, y->r), x = y;
    else insert(y->k < x->k ? x->l : x->r, y);
    recalc(x);
}
void erase(item *&x, int k) {
    if (x->k == k) merge(x, x->l, x->r);
    else erase(k < x->k ? x->l : x->r, k);
    recalc(x);
}

```

## 2 Geo2d

### 2.1 circle

```

struct C {
    P o; T r;
    // circle-line intersection, assuming it exists
    // points are sorted along the direction of the line
    pair<P, P> line_inter(L l) const {
        P c = l.closest_to(o); T c2 = (c - o).magsq();
        P e = l.d * sqrt(max(r*r - c2, T())) / l.d.magsq();
        return {c - e, c + e};
    }
    // check the type of line-circle collision
    // <0: 2 inters, =0: 1 inter, >0: 0 inters
    T line_collide(L l) const {
        T c2 = (l.closest_to(o) - o).magsq();
    }
}

```

```

    return c2 - r * r;
}
// calculates the two intersections between two circles
// the circles must intersect in one or two points!
pair<P, P> inter(C h) const {
    P d = h.o - o;
    T c = (r * r - h.r * h.r) / d.magsq();
    return h.line_inter({(1 + c) / 2 * d, d.rot()});
}
// check if the given circles intersect
bool collide(C h) const {
    return (h.o - o).magsq() <= (h.r + r) * (h.r + r);
}
// get one of the two tangents that go through the point
// the point must not be inside the circle
// a = -1: cw (relative to the circle) tangent
// a = 1: ccw (relative to the circle) tangent
P point_tangent(P p, T a) const {
    T c = r * r / p.magsq();
    return o + c*(p-o) - a*sqrt(c*(1-c))*(p-o).rot();
}
// get one of the 4 tangents between the two circles
// a = 1: exterior tangents
// a = -1: interior tangents (requires no area overlap)
// b = 1: ccw tangent
// b = -1: cw tangent
// the line origin is on this circumference, and the
// direction is a unit vector towards the other circle
L tangent(C c, T a, T b) const {
    T dr = a * r - c.r;
    P d = c.o - o;
    P n = (d*dr+b*d.rot()*sqrt(d.magsq()-dr*dr)).unit();
    return {o + n * r, -b * n.rot()};
}
// circumcircle of a **non-degenerate** triangle
static C thru_points(P a, P b, P c) {
    b = b - a, c = c - a;
    P p = (b*c.magsq() - c*b.magsq()).rot() / (b*c*2);
    return {a + p, p.mag()};
}
// find the two circles that go through the given point,
// are tangent to the given line and have radius `r`
// the point-line distance must be at most `r`!
// the circles are sorted in the direction of the line
static pair<C, C> thru_point_line_r(P a, L t, T r) {
    P d = t.d.rot().unit();
    if (d * (a - t.o) < 0) d = -d;
    auto p = C(a, r).line_inter({t.o + d * r, t.d});
    return {{p.first, r}, {p.second, r}};
}
// find the two circles that go through the given points
// and have radius `r`
// circles sorted by angle from the first point
// the points must be at most at distance `r`!
static pair<C, C> thru_points_r(P a, P b, T r) {
}

```



```

    auto p = C(a, r).line_inter({(a+b)/2, (b-a).rot()});
    return {{p.first, r}, {p.second, r}};
}
vector<P> linecol(L l){
    vector<P> s; P p=l.closest_to(o); double d=(p-
o).norm();
    if(d-EPS>r) return s;
    if(abs(d-r)<=EPS){s.pb(p); return s;}
    d=sqrt(r*r-d*d); s.pb(p+l.pq.unit()*d); s.pb(p-
l.pq.unit()*d);
    return s;
}
double intertriangle(P a,P b){ // intersection with oab
    if(abs((o-a)*(o-b))<=EPS) return 0.;
    vector<P> q={a},w=linecol(L{a,b-a});
    if(w.size()==2) for(auto p:w) if((a-p)*(b-p)<-EPS) q.pb(p);
    q.pb(b);
    if(q.size()==4&&(q[0]-q[1])*(q[2]-
q[1])>EPS) swap(q[1],q[2]);
    double s=0;
    for(i,0,q.size()-1){
        if(!has(q[i])||!has(q[i+1])) s+=r*r*(q[i]-
o).angle(q[i+1]-o)/2;
        else s+=abs((q[i]-o)*(q[i+1]-o)/2);
    }
    return s;
}
};

```

## 2.2 closest points

```

// sort by x
ll closest(vector<ii> &p) {
    int n = SZ(p);
    set<ii> s;
    ll best = 1e18;
    int j = 0;
    for(i, 0, n) {
        ll d = ceil(sqrt(best));
        while(p[i].fst - p[j].fst >= best)
            s.erase({p[j].snd, p[j].fst}), j++;
        auto it1=s.lower_bound({p[i].snd-d,p[i].fst});
        auto it2=s.upper_bound({p[i].snd+d,p[i].fst});
        for(auto it = it1; it != it2; ++it) {
            ll dx = p[i].fst - it->snd;
            ll dy = p[i].snd - it->fst;
            best = min(best, dx * dx + dy * dy);
        }
        s.insert({p[i].snd, p[i].fst});
    }
    return best;
}

```

## 2.3 convex hull

```

// ccw order, excludes collinear points by default
vector<P> chull(vector<P> p) {
    if (p.size() < 3) return p;
    vector<P> r; int m, k = 0;
    sort(p.begin(), p.end(), [](P a, P b) {
        return a.x != b.x ? a.x < b.x : a.y < b.y; });
    for (P q : p) { // lower hull
        while (k >= 2 && r[k-1].left(r[k-2], q) >= 0)
            r.pop_back(), k--; // >= to > to add collinears
        r.push_back(q), k++;
    }
    if (k == (int)p.size()) return r;
    r.pop_back(), k--; m = k;
    for (int i = p.size() - 1; i >= 0; --i) { // upper hull
        while (k >= m+2 && r[k-1].left(r[k-2], p[i]) >= 0)
            r.pop_back(), k--; // >= to > to add collinears
        r.push_back(p[i]), k++;
    }
    r.pop_back(); return r;
}

```

## 2.4 delaunay

```

typedef __int128_t lll; // if on a 64-bit platform

struct Q {
    Q *rot, *o; P p = {INF, INF}; bool mark;
    P &F() { return r()->p; }
    Q &r() { return rot->rot; }
    Q *prev() { return rot->o->rot; }
    Q *next() { return r()->prev(); }
};

T cross(P a, P b, P c) { return (b - a) % (c - a); }

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.magsq(), A = a.magsq() - p2,
        B = b.magsq() - p2, C = c.magsq() - p2;
    return cross(p, a, b) * C + cross(p, b, c) * A +
    cross(p, c, a) * B > 0;
}

Q *makeEdge(Q *H, P orig, P dest) {
    Q *r = H ? H : new Q{new Q{new Q{new Q{0}}}};
    H = r->o; r->r()->r() = r;
    repx(i, 0, 4) r = r->rot, r->p = {INF, INF},
        r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}

void splice(Q *a, Q *b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

```

```

Q *connect(Q *H, Q *a, Q *b) {
    Q *q = makeEdge(H, a->F(), b->p);
    splice(q, a->next()); splice(q->r(), b); return q;
}

pair<Q *, Q *> rec(Q *H, const vector<P> &s) {
    if (s.size() <= 3) {
        Q *a = makeEdge(H, s[0], s[1]), *b = makeEdge(H,
s[1], s.back());
        if (s.size() == 2) return {a, a->r()}; splice(a-
>r(), b);
        auto side = cross(s[0], s[1], s[2]);
        Q *c = side ? connect(H, b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b-
>r()};
    }

#define J(e) e->F(), e->p
#define valid(e) (cross(e->F(), J(base)) > 0)
    Q *A, *B, *ra, *rb; int half = s.size() / 2;
    tie(ra, A) = rec(H, {s.begin(), s.end() - half});
    tie(B, rb) = rec(H, {s.begin() + s.size() - half,
s.end()});
    while ((cross(B->p, J(A)) < 0 && (A = A->next())) ||
(cross(A->p, J(B)) > 0 && (B = B->r()->o)));
    Q *base = connect(H, B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q *e = init->dir; \
    if (valid(e)) while (circ(e->dir->F(), J(base), e->F()))
{ \
        Q *t = e->dir; splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); e->o = H; H = e;
e = t; \
}
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(J(RC), J(LC))))
            base = connect(H, RC, base->r());
        else base = connect(H, base->r(), LC->r());
    }
    return {ra, rb};
#undef J
#undef valid
#undef DEL
}

// there must be no duplicate points
// returns no triangles in the case of all collinear points
// produces counter-clockwise triangles ordered in triples
// maximizes the minimum angle across all triangulations
// the euclidean mst is a subset of these edges
// O(N log N)

```

```
vector<P> triangulate(vector<P> pts) {
    sort(pts.begin(), pts.end(), [](P a, P b) {
        return make_pair(a.x, a.y) < make_pair(b.x, b.y);
    });
    assert(unique(pts.begin(), pts.end()) == pts.end());
    if (pts.size() < 2) return {};
    Q *H = 0; Q *e = rec(H, pts).first;
    vector<Q *> q = {e}; int qi = 0;
    while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD
    {
        Q *c = e;
        do {
            c->mark = 1; pts.push_back(c->p); \
            q.push_back(c->r()); c = c->next(); \
        } while (c != e);
    }
    ADD;
    pts.clear();
    while (qi < (int)q.size()) if (!(e = q[qi++])>mark)
    ADD;
    return pts;
#undef ADD
}
```

## 2.5 halfplane intersect

```
// obtain the convex polygon that results from intersecting
// the given list
// of halfplanes, represented as lines that allow their left
// side
// assumes the halfplane intersection is bounded
vector<P> halfplane_intersect(vector<L> &H) {
    L bb(P(-INF, -INF), P(INF, 0));
    rep(k, 4) H.push_back(bb), bb.o = bb.o.rot(), bb.d =
    bb.d.rot();

    sort(begin(H), end(H), [](L a, L b) { return
    a.d.angcmp(b.d) < 0; });
    deque<L> q; int n = 0;
    rep(i, H.size()) {
        while (n >= 2 && H[i].side(q[n - 1].intersection(q[n
        - 2])) > 0)
            q.pop_back(), n--;
        while (n >= 2 && H[i].side(q[0].intersection(q[1]))
        > 0)
            q.pop_front(), n--;
        if (n > 0 && H[i].parallel(q[n - 1])) {
            if (H[i].d * q[n - 1].d < 0) return {};
            if (H[i].side(q[n - 1].o) > 0) q.pop_back(),
            n--;
            else continue;
        }
        q.push_back(H[i]), n++;
    }
}
```

```
while (n >= 3 && q[0].side(q[n - 1].intersection(q[n -
2])) > 0)
    q.pop_back(), n--;
while (n >= 3 && q[n - 1].side(q[0].intersection(q[1]))
> 0)
    q.pop_front(), n--;
if (n < 3) return {};

vector<P> ps(n);
rep(i, n) ps[i] = q[i].intersection(q[(i + 1) % n]);
return ps;
}
```

## 2.6 line

```
// a segment or an infinite line
// does not handle point segments correctly!
struct L {
    P o, d;

    static L from_eq(P ab, T c) {
        return L{ab.rot(), ab * -c / ab.magsq()};
    }
    pair<P, T> line_eq() { return {-d.rot(), d.rot() * o}; }

    // on which side of the line is the point
    // negative: left, positive: right
    T side(P r) const { return (r - o) % d; }

    // returns the intersection coefficient
    // in the range [0, d % r.d]
    // if d % r.d is zero, the lines are parallel
    T inter(L r) const { return (r.o - o) % r.d; }

    // get the single intersection point
    // lines must not be parallel
    P intersection(L r) const { return o + d * inter(r) / (d % r.d); }

    // check if lines are parallel
    bool parallel(L r) const { return abs(d % r.d) <= EPS; }

    // check if segments intersect
    bool seg_collide(L r) const {
        T z = d % r.d;
        if (abs(z) <= EPS) {
            if (abs(side(r.o)) > EPS) return false;
            T s = (r.o - o) * d, e = s + r.d * d;
            if (s > e) swap(s, e);
            return s <= d * d + EPS && e >= -EPS;
        }
        T s = inter(r), t = -r.inter(*this);
        if (z < 0) s = -s, t = -t, z = -z;
        return s >= -EPS && s <= z + EPS && t >= -EPS && t <= z + EPS;
    }
}
```

```
// full segment intersection
// makes a point segment if the intersection is a point
// however it does not handle point segments as input!
bool seg_inter(L r, L *out) const {
    T z = d % r.d;
    if (abs(z) <= EPS) {
        if (abs(side(r.o)) > EPS) return false;
        if (r.d * d < 0) r = {r.o + r.d, -r.d};
        P s = o * d < r.o * d ? r.o : o;
        P e = (o + d) * d < (r.o + r.d) * d ? o + d : r.o + r.d;
        if (s * d > e * d) return false;
        return *out = {s, e - s}, true;
    }
    T s = inter(r), t = -r.inter(*this);
    if (z < 0) s = -s, t = -t, z = -z;
    if (s >= -EPS && s <= z + EPS && t >= -EPS && t <= z + EPS)
        return *out = {o + d * s / z, {0, 0}}, true;
    return false;
}

// check if the given point is on the segment
bool point_on_seg(P r) const {
    if (abs(side(r)) > EPS) return false;
    if ((r - o) * d < -EPS) return false;
    if ((r - o - d) * d > EPS) return false;
    return true;
}

// point in this line that is closest to a given point
P closest_to(P r) const {
    P dr = d.rot(); return r + dr * ((o - r) * dr) / d.magsq();
}
};
```

## 2.7 minkowski

```
void reorder_polygon(vector<P> &ps) {
    int pos = 0;
    repx(i, 1, (int)ps.size()) {
        if (ps[i].y < ps[pos].y || (ps[i].y == ps[pos].y &&
        ps[i].x < ps[pos].x))
            pos = i;
    }
    rotate(ps.begin(), ps.begin() + pos, ps.end());
}

vector<P> minkowski(vector<P> ps, vector<P> qs) {
    // the first vertex must be the lowest
    reorder_polygon(ps); reorder_polygon(qs);
    ps.push_back(ps[0]); ps.push_back(ps[1]);
    qs.push_back(qs[0]); qs.push_back(qs[1]);
    vector<P> result; int i = 0, j = 0;
    while (i < ps.size() - 2 || j < qs.size() - 2) {
        result.push_back(ps[i] + qs[j]);
        auto z = (ps[i + 1] - ps[i]) % (qs[j + 1] - qs[j]);
    }
}
```



```

    if (z >= 0 && i < ps.size() - 2) ++i;
    if (z <= 0 && j < qs.size() - 2) ++j;
}
return result;
}

```

## 2.8 point

```

struct P {
    T x, y;
    P(T x, T y) : x(x), y(y) {}
    P() : P(0, 0) {}

    friend ostream &operator<<(ostream &s, const P &r) {
        return s << r.x << " " << r.y;
    }

    friend istream &operator>>(istream &s, P &r) { return s
>> r.x >> r.y; }

    P operator+(P r) const { return {x + r.x, y + r.y}; }
    P operator-(P r) const { return {x - r.x, y - r.y}; }
    P operator*(T r) const { return {x * r, y * r}; }
    P operator/(T r) const { return {x / r, y / r}; }
    P operator-() const { return {-x, -y}; }
    friend P operator*(T l, P r) { return {l * r.x, l *
r.y}; }

    P rot() const { return {-y, x}; }
    T operator*(P r) const { return x * r.x + y * r.y; }
    T operator%(P r) const { return rot() * r; }
    T left(P a, P b) { return (b - a) % (*this - a); }

    T magsq() const { return x * x + y * y; }
    T mag() const { return sqrt(magsq()); }
    P unit() const { return *this / mag(); }

    bool half() const { return abs(y) <= EPS && x < -EPS ||
y < -EPS; }
    T angcmp(P r) const { // like strcmp(this, r)
        int h = (int)half() - r.half();
        return h ? h : r % *this;
    }
    T angcmp_rel(P a, P b) { // like strcmp(a, b)
        P z = *this;
        int h = z % a <= 0 && z * a < 0 || z % a < 0;
        h -= z % b <= 0 && z * b < 0 || z % b < 0;
        return h ? h : b % a;
    }

    bool operator==(P r) const { return abs(x - r.x) <= EPS
&& abs(y - r.y) <= EPS; }

    double angle() const { return atan2(y, x); }
    static P from_angle(double a) { return {cos(a),

```

```

sin(a)); }
};

```

## 2.9 polygon

```

// get TWICE the area of a simple polygon in ccw order
T area2(const vector<P> &p) {
    int n = p.size(); T a = 0;
    rep(i, n) a += (p[i] - p[0]) % (p[(i + 1) % n] - p[i]);
    return a;
}

// checks whether a point is inside a ccw simple polygon
// returns 1 if inside, 0 if on border, -1 if outside
int in_poly(const vector<P> &p, P q) {
    int w = 0;
    rep(i, p.size()) {
        P a = p[i], b = p[(i + 1) % p.size()];
        T k = (b - a) % (q - a);
        T u = a.y - q.y, v = b.y - q.y;
        if (k > 0 && u < 0 && v >= 0) w++;
        if (k < 0 && v < 0 && u >= 0) w--;
        if (k == 0 && (q - a) * (q - b) <= 0) return 0;
    }
    return w ? 1 : -1;
}

// check if point in ccw convex polygon, 0(log n)
// + if inside, 0 if on border, - if outside
T in_convex(const vector<P> &p, P q) {
    int l = 1, h = p.size() - 2; assert(p.size() >= 3);
    while (l != h) { // collinear points are unsupported!
        int m = (l + h + 1) / 2;
        if (q.left(p[0], p[m]) >= 0) l = m;
        else h = m - 1;
    }
    T in = min(q.left(p[0], p[l]), q.left(p.back(), p[0]));
    return min(in, q.left(p[l], p[l + 1]));
}

int extremal(const vector<P> &p, P d) {
    int n = p.size(), l = 0, r = n - 1; assert(n);
    P e0 = (p[n - 1] - p[0]).rot();
    while (l < r) { // polygon must be convex
        int m = (l + r + 1) / 2;
        P e = (p[(m + n - 1) % n] - p[m]).rot();
        if (e0.angcmp_rel(d, e) < 0) r = m - 1;
        else l = m;
    }
    return l;
}

// square dist of most distant points of a ccw convex
// polygon with NO COLLINEAR POINTS
T callipers(const vector<P> &p) {

```

```

int n = p.size();
T r = 0;
for (int i = 0, j = n < 2 ? 0 : 1; i < j; i++) {
    for (; j = (j + 1) % n) {
        r = max(r, (p[i] - p[j]).magsq());
        if ((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] -
p[j]) <= EPS) break;
    }
    return r;
}

P centroid(const vector<P> &p) { // (barycenter)
    P r(0, 0); T t = 0; int n = p.size();
    rep(i, n) {
        r += (p[i] + p[(i+1)%n]) * (p[i] % p[(i+1)%n]);
        t += p[i] % p[(i+1)%n];
    }
    return r / t / 3;
}

// classify collision of a ray inside a ccw polygon vertex.
// ray is (o, d), vertex is b, previous vertex is a, next is
c.
pair<bool, bool> inner_collide(P o, P d, P a, P b, P c) {
    T p = (a - o) % d; // side of previous
    T n = (c - o) % d; // side of next
    T v = (c - b) % (b - a); // is vertex convex?
    return {v > 0 ? n < 0 || (n == 0 && p < 0) : p > 0 || n
< 0,
           v > 0 ? p > 0 || (p == 0 && n > 0) : p > 0 || n
< 0};
}

```

## 2.10 sweep

```

#include "point.cpp"

// iterate over all pairs of points
// `op` is called with all ordered pairs of different
indices `(i, j)`
// additionally, the `ps` vector is kept sorted by signed
distance
// to the line formed by `i` and `j`
// for example, if the vector from `i` to `j` is pointing
right,
// the `ps` vector is sorted from smallest `y` to largest
`y`
// note that, because the `ps` vector is sorted by signed
distance,
// `j` is always equal to `i + 1`
// this means that the amount of points to the left of the
line is always `N - i`
template <class OP>
void all_pair_points(vector<P> &ps, OP op) {

```

```

        B[u]=min(B[u],B[v]);
    }
    else if(D[v]<D[u])st.push(ne),B[u]=min(B[u],D[v]);
}
}

void doit(){
    memset(D,-1,sizeof(D));memset(art,0,sizeof(art));
    nbc=T=0; for(i,0,n)if(D[i]<0)dfs(i,-1),art[i]--;
}

```

## 2.11 theorems

```
// Pick's theorem
//   Simple polygon with integer vertices:
//    $A = I + B / 2 - 1$ 
//   A: Area of the polygon
//   I: Integer points strictly inside the polygon
//   B: Integer points on the boundary of the polygon
```

### 3 Graph

### 3.1 artic bridge biconn

```
vector<int> g[MAXN];int n;
struct edge {int u,v,comp;bool bridge;};
vector<edge> e;

void add_edge(int u, int v){
    g[u].pb(e.size());g[v].pb(e.size());
    e.pb((edge){u,v,-1,false});
}

int D[MAXN],B[MAXN],T;
int nbc; // number of biconnected components
int art[MAXN]; // articulation point iff !=0
stack<int> st; // only for biconnected
void dfs(int u,int pe){
    B[u]=D[u]=T++;
    for(int ne:g[u])if(ne!=pe){
        int v=e[ne].u^e[ne].v^u;
        if(D[v]<0){
            st.push(ne);dfs(v,ne);
            if(B[v]>D[u])e[ne].bridge = true; // bridge
            if(B[v]==D[u]){
                art[u]++; // articulation
                int last; // start biconnected
                do{last=st.top();st.pop();e[last].comp=nbc;}
                while(last!=ne);
                nbc++; // end biconnected
            }
        }
    }
}
```

### 3.2 bellman ford

```
struct Edge { int u, v; ll w; };

// find distance from source node to all nodes.
// supports negative edge weights.
// returns true if a negative cycle is detected.
//
// time:  $O(V E)$ 
bool bellman_ford(int N, int s, vector<Edge> &E, vector<ll>
&D, vector<int> &P) {
    P.assign(N, -1), D.assign(N, INF), D[s] = 0;
    rep(i, N - 1) {
        bool f = true;
        rep(ei, E.size()) {
            auto &e = E[ei];
            ll n = D[e.u] + e.w;
            if (D[e.u] < INF && n < D[e.v])
                D[e.v] = n, P[e.v] = ei, f = false;
        }
        if (f) return false;
    }
    return true;
}
```

### 3.3 blossom

```
vector<int> g[MAXN];int n,m,mt[MAXN],qh,qt,q[MAXN],ft[MAXN],
bs[MAXN];bool inq[MAXN],inb[MAXN],inp[MAXN];int lca(int root
,int x,int y){memset(inp,0,sizeof(inp));while(1){inp[x=bs[x]
]=true;if(x==root)break;x=ft[x];}}while(1){if(inp[y=bs[y]
])return y;else y=ft[y];}}void mark(int z,int x){while(
bs[x]!=z){int y=mt[x];inb[bs[x]]=inb[bs[y]]=true;x=ft[y];if(
bs[x]!=z)ft[x]=y;}}void contr(int s,int x,int y){int z=lca(s
,x,y);memset(inb,0,sizeof(inb));mark(z,x);mark(z,y);if(bs[x]
!=z)ft[x]=y;if(bs[y]!=z)ft[y]=x;rep(x,n)if(inb[bs[x]]){bs[x]
=z;if(!inq[x])inq[q[+qt]=x]=true;}}int findp(int s){memset(
inq,0,sizeof(inq));memset(ft,-1,sizeof(ft));rep(i,n)bs[i]=i;
inq[q[qh=qt=0]=s]=true;while(qh<=qt){int x=q[qh++];for(int y
:g[x])if(bs[x]!=bs[y]&&mt[x]!=y){if(y==s||mt[y]>=0&&ft[mt[y]
]>=0)contr(s,x,y);else if(ft[y]<0){ft[y]=x;if(mt[y]<0)return
y;else if(!inq[mt[y]])inq[q[+qt]=mt[y]]=true;}}}return -1;}
int aug(int s,int t){int x=t,y,z;while(x!=y){y=ft[x];z=mt[y]
;mt[y]=x;mt[x]=y;x=z;}}return t>=0;int edmonds(){int r=0;
```

```
memset(mt, -1, sizeof(mt)); rep(x, n) if(mt[x] < 0) r += aug(x, findp(x)); return r; }
```

### 3.4 chu liu minimum spanning arborescence

```

//O(n*m) minimum spanning tree in directed graph
//returns -1 if not possible
//included i-th edge if take[i]!=0
typedef int tw; tw INF=1ll<<30;
struct edge{int u,v,id;tw len;};
struct ChuLiu{
    int n; vector<edge> e;
    vector<int> inc,dec,take,pre,num,id,vis;
    vector<tw> inw;
    void add_edge(int x, int y, tw w){
        inc.pb(0); dec.pb(0); take.pb(0);
        e.pb({x,y,SZ(e),w});
    }
    ChuLiu(int n):n(n),pre(n),num(n),id(n),vis(n),inw(n){}
    tw doit(int root){
        auto e2=e;
        tw ans=0; int eg=SZ(e)-1,pos=SZ(e)-1;
        while(1){
            fore(i,0,n) inw[i]=INF,id[i]=vis[i]=-1;
            for(auto ed:e2) if(ed.len<inw[ed.v]){
                inw[ed.v]=ed.len; pre[ed.v]=ed.u;
                num[ed.v]=ed.id;
            }
            inw[root]=0;
            fore(i,0,n) if(inw[i]==INF) return -1;
            int tot=-1;
            fore(i,0,n){
                ans+=inw[i];
                if(i!=root)take[num[i]]++;
                int j=i;
                while(vis[j]!=i&&j!
=root&&id[j]<0)vis[j]=i,j=pre[j];
                if(j!=root&&id[j]<0){
                    id[j]=++tot;
                    for(int k=pre[j];k!=j;k=pre[k])
id[k]=tot;
                }
            }
            if(tot<0)break;
            fore(i,0,n) if(id[i]<0)id[i]=++tot;
            n=tot+1; int j=0;
            fore(i,0,SZ(e2)){
                int v=e2[i].v;
                e2[j].v=id[e2[i].v];
                e2[j].u=id[e2[i].u];
                if(e2[j].v!=e2[j].u){
                    e2[j].len=e2[i].len-inw[v];
                    inc.pb(e2[i].id);

```

```

        dec.pb(num[v]);
        take.pb(0);
        e2[j++].id=++pos;
    }
    }
    e2.resize(j);
    root=id[root];
}
while(pos>eg){
    if(take[pos]>0) take[inc[pos]]++,
    take[dec[pos]]--;
    pos--;
}
return ans;
}
};

```

### 3.5 dinic

```

// time: O(E V^2)
// O(E V^(2/3)) / O(E sqrt(E)) unit capacities
// O(E sqrt(V)) (hopcroft-karp) unit networks
// unit network: c in {0,1} & forall v, indeg<=1 or outdeg<=1
// min-cut: nodes reachable from s in final residual graph
struct Dinic {
    struct Edge { int u, v; ll c, f = 0; };
    int N, s, t; vector<vector<int>> G;
    vector<Edge> E; vector<int> lvl, ptr;
    Dinic() {}
    Dinic(int N, int s, int t) : N(N), s(s), t(t), G(N) {}

    void add_edge(int u, int v, ll c) {
        G[u].push_back(E.size()); E.push_back({u, v, c});
        G[v].push_back(E.size()); E.push_back({v, u, 0});
    }

    ll push(int u, ll p) {
        if (u == t || p <= 0) return p;
        while (ptr[u] < G[u].size()) {
            int ei = G[u][ptr[u]++];
            Edge &e = E[ei];
            if (lvl[e.v] != lvl[u] + 1) continue;
            ll a = push(e.v, min(e.c - e.f, p));
            if (a <= 0) continue;
            e.f += a, E[ei ^ 1].f -= a; return a;
        }
        return 0;
    }

    ll maxflow() {
        ll f = 0;
        while (true) {
            lvl.assign(N, -1); queue<int> q;
            lvl[s] = 0; q.push(s);
            while (!q.empty()) {

```

```

                int u = q.front(); q.pop();
                for (int ei : G[u]) {
                    Edge &e = E[ei];
                    if (e.c - e.f <= 0 || lvl[e.v] != -1) continue;
                    lvl[e.v] = lvl[u] + 1; q.push(e.v);
                }
            }
            if (lvl[t] == -1) break;
            ptr.assign(N, 0); while (ll ff = push(s, INF)) f += ff;
        }
        return f;
    }
};

/* Flujo con demandas (no necesariamente el maximo)
Agregar s' y t' nuevos source and sink
c'(s', v) = sum(d(u, v) for u in V) \forall arista (s', v)
c'(v, t') = sum(d(v, w) for w in V) \forall arista (v, t')
c'(u, v) = c(u, v) - d(u, v) \forall aristas antiguas
c'(t, s) = INF (el flujo por esta arista es el flujo real)*/

```

### 3.6 dominator tree

```

// idom[i]=parent of i in dominator tree with root=rt, or -1
// if not exists
int
n, rnk[MXN], pre[MXN], anc[MXN], idom[MXN], semi[MXN], low[MXN];
vector<int> g[MXN], rev[MXN], dom[MXN], ord;
void dfspre(int pos) {
    rnk[pos] = SZ(ord); ord.pb(pos);
    for (auto x : g[pos]) {
        rev[x].pb(pos);
        if (rnk[x] == n) pre[x] = pos, dfspre(x);
    }
}
int eval(int v) {
    if (anc[v] < n && anc[anc[v]] < n) {
        int x = eval(anc[v]);
        if (rnk[semi[low[v]]] > rnk[semi[x]]) low[v] = x;
        anc[v] = anc[anc[v]];
    }
    return low[v];
}
void dominators(int rt) {
    for (i, 0, n) {
        dom[i].clear(); rev[i].clear();
        rnk[i] = pre[i] = anc[i] = idom[i] = n;
        semi[i] = low[i] = i;
    }
    ord.clear(); dfspre(rt);
    for (int i = SZ(ord) - 1; i--;) {
        int w = ord[i];
        for (int v : rev[w]) {
            int u = eval(v);
            if (rnk[semi[w]] > rnk[semi[u]]) semi[w] = semi[u];

```

```

        }
        dom[semi[w]].pb(w); anc[w] = pre[w];
        for (int v : dom[pre[w]]) {
            int u = eval(v);
            idom[v] = (rnk[pre[w]] > rnk[semi[u]] ? u : pre[w]);
        }
        dom[pre[w]].clear();
    }
    for (int w : ord) if (w != rt && idom[w] != semi[w])
        idom[w] = idom[idom[w]];
    fore(i, 0, n) if (idom[i] == n) idom[i] = -1;
}
}

```

### 3.7 eulerian

```

// path/tour for directed graphs. uncomment for undirected.
struct Euler {
    struct Edge { int v, rev; };
    vector<vector<Edge>> G; vector<Edge> P;
    Euler(int N = 0) : G(N) {}
    void add_edge(int u, int v) {
        G[u].push_back({v, (int)G[v].size()});
        // G[v].push_back({u, (int)G[u].size() - 1});
    }

    void go(int u) {
        while (G[u].size()) {
            Edge e = G[u].back(); G[u].pop_back();
            // if (e.v == -1) continue;
            // G[e.v][e.rev].v = -1;
            go(e.v); P.push_back(e);
        }

        // works ONLY if the vertex degrees are eulerian! check!
        vector<Edge> get_path(int u) {
            return P.clear(), go(u), reverse(P.begin(), P.end()), P;
        }
    };
}

```

### 3.8 floyd warshall

```

// calculate distances between every pair of nodes in O(V^3)
// time.
// works with negative edges, but not negative cycles.
void floyd(const vector<vector<pair<ll, int>>> &G,
vector<vector<ll>> &D) {
    int N = G.size();
    D.assign(N, vector<ll>(N, INF));
    rep(u, N) D[u][u] = 0;
    rep(u, N) for (auto [w, v] : G[u]) D[u][v] = w;
    rep(k, N) rep(u, N) rep(v, N)
        D[u][v] = min(D[u][v], D[u][k] + D[k][v]);
}

```

### 3.9 gomory hu

```
// O(N * F) (F = max flow)
vector<pair<ii,ll>> gomoryHu(int N, vector<pair<ii,ll>>&ed)
{
    vector<int> par(N); Dinic D(N, 0, 0);
    for(auto [e, w]: ed){
        auto [u, v] = e;
        D.G[u].push_back(D.E.size());D.E.push_back({u,v,w});
        D.G[v].push_back(D.E.size());D.E.push_back({v,u,w});
    }
    vector<pair<ii,ll>> ans;
    repx(i,1,N){
        for(int j = 0; j<int(D.E.size()); j = j + 2){
            D.E[j].c = D.E[j^1].c = ((D.E[j].c - D.E[j].f)
                + (D.E[j^1].c - D.E[j^1].f)) / 2;
            D.E[j].f = D.E[j^1].f = 0;
        }
        D.s = i; D.t = par[i];
        ans.push_back({i, par[i]}, D.maxflow());
        repx(j,i+1,N)
            if (par[j] == par[i] && D.lvl[j] >= 0) par[j]=i;
    }
    return ans;
}
```

### 3.10 heavy light

```
struct Hld {
    vector<int> P, H, D, pos, top;

    Hld() {}
    void init(vector<vector<int>> &G) {
        int N = G.size();
        P.resize(N), H.resize(N), D.resize(N),
        pos.resize(N),
        top.resize(N);
        D[0] = -1, dfs(G, 0); int t = 0;
        rep(i, N) if (H[P[i]] != i) {
            int j = i;
            while (j != -1)
                { top[j] = i, pos[j] = t++; j = H[j]; }
        }

        int dfs(vector<vector<int>> &G, int i) {
            int w = 1, mw = 0;
            D[i] = D[P[i]] + 1, H[i] = -1;
            for (int c : G[i]) {
                if (c == P[i]) continue;
                P[c] = i; int sw = dfs(G, c); w += sw;
                if (sw > mw) H[i] = c, mw = sw;
            }
            return w;
        }
    }
}
```

```
// visit the log N segments in the path from u to v
template <class OP>
void path(int u, int v, OP op) {
    while (top[u] != top[v]) {
        if (D[top[u]] > D[top[v]]) swap(u, v);
        op(pos[top[v]], pos[v] + 1); v = P[top[v]];
    }
    if (D[u] > D[v]) swap(u, v);
    op(pos[u], pos[v] + 1); // value on node
    // op(pos[u]+1, pos[v] + 1); // value on edge
}

// an alternative to `path` that considers order.
// calls `op` with an `l <= r` inclusive-exclusive
// range, and a
// boolean indicating if the query is forwards or
// backwards.
template <class OP>
void path(int u, int v, OP op) {
    int lu = u, lv = v;
    while (top[lu] != top[lv])
        if (D[top[lu]] > D[top[lv]]) lu = P[top[lu]];
        else lv = P[top[lv]];
    int lca = D[lu] > D[lv] ? lv : lu;

    while (top[u] != top[lca])
        op(pos[top[u]], pos[u] + 1, false), u =
        P[top[u]];
    if (u != lca) op(pos[lca] + 1, pos[u] + 1, false);

    vector<int> stk;
    while (top[v] != top[lca])
        stk.push_back(v), v = P[top[v]];

    // op(pos[lca], pos[v] + 1, true); // value on node
    op(pos[lca] + 1, pos[v] + 1, true); // value on edge
    reverse(stk.begin(), stk.end());
    for (int w : stk) op(pos[top[w]], pos[w] + 1, true);
}

// commutative segment tree
template <class T, class S>
void update(S &seg, int i, T val) { seg.update(pos[i],
val); }

// commutative segment tree lazy
template <class T, class S>
void update(S &seg, int u, int v, T val) {
    path(u, v, [&](int l, int r) { seg.update(l, r,
val); });
}

// commutative (lazy) segment tree
template <class T, class S>
```

```
T query(S &seg, int u, int v) {
    T ans =
    0;
    neutral element
    path(u, v, [&](int l, int r) { ans += seg.query(l,
r); }); // query op
    return ans;
}
};
```

### 3.11 hungarian

```
// find a maximum gain perfect matching in the given
// bipartite complete graph.
// input: gain matrix (G_{xy} = benefit of joining vertex x
// in set X with vertex
// y in set Y).
// output: maximum gain matching in members `xy[x]` and
// `yx[y]`.
// runtime: O(N^3)
struct Hungarian {
    int N, qi, root;
    vector<vector<ll>> gain;
    vector<int> xy, yx, p, q, slackx;
    vector<ll> lx, ly, slack;
    vector<bool> S, T;

    void add(int x, int px) {
        S[x] = true, p[x] = px;
        rep(y, N) if (lx[x] + ly[y] - gain[x][y] < slack[y])
        {
            slack[y] = lx[x] + ly[y] - gain[x][y], slackx[y]
        }
    }

    void augment(int x, int y) {
        while (x != -2) {
            yx[y] = x; swap(xy[x], y); x = p[x];
        }
    }

    void improve() {
        S.assign(N, false), T.assign(N, false), p.assign(N,
-1);
        qi = 0, q.clear();
        rep(x, N) if (xy[x] == -1) {
            q.push_back(root = x), p[x] = -2, S[x] = true;
            break;
        }
        rep(y, N) slack[y] = lx[root] + ly[y] - gain[root]
[y], slackx[y] = root;

        while (true) {
            while (qi < q.size()) {
```

```

        int x = q[qi++];
        rep(y, N) if (lx[x] + ly[y] == gain[x][y]
&& !T[y]) {
            if (yx[y] == -1) return augment(x, y);
            T[y] = true, q.push_back(yx[y]),
add(yx[y], x);
        }
    }

    ll d = INF;
    rep(y, N) if (!T[y]) d = min(d, slack[y]);
    rep(x, N) if (S[x]) lx[x] -= d;
    rep(y, N) if (T[y]) ly[y] += d;
    rep(y, N) if (!T[y]) slack[y] -= d;

    rep(y, N) if (!T[y] && slack[y] == 0) {
        if (yx[y] == -1) return augment(slackx[y],
y);
        T[y] = true;
        if (!S[yx[y]]) q.push_back(yx[y]),
add(yx[y], slackx[y]);
    }
}

Hungarian(vector<vector<ll>> g)
: N(g.size()), gain(g), xy(N, -1), yx(N, -1), lx(N,
-INF),
ly(N), slack(N), slackx(N) {
    rep(x, N) rep(y, N) lx[x] = max(lx[x], ly[y]);
    rep(i, N) improve();
}
};

```

### 3.12 kuhn

```

// get a maximum cardinality matching in a bipartite graph.
// input: adjacency lists.
// output: matching (in `mt` member).
// runtime: O(V E)
struct Kuhn {
    vector<vector<int>> G;
    int N, size;
    vector<bool> seen;
    vector<int> mt;

    bool visit(int i) {
        if (seen[i]) return false;
        seen[i] = true;
        for (int to : G[i])
            if (mt[to] == -1 || visit(mt[to])) {
                mt[to] = i;
                return true;
            }
        return false;
    }
};

```

```

}

Kuhn(vector<vector<int>> adj) : G(adj), N(G.size()),
mt(N, -1) {
    rep(i, N) {
        seen.assign(N, false);
        size += visit(i);
    }
}
};

```

### 3.13 lca

```

// calculates the lowest common ancestor for any two nodes
in O(log N) time,
// with O(N log N) preprocessing
struct Lca {
    int N, K, t = 0;
    vector<vector<int>> U;
    vector<int> L, R;

    Lca() {}
    Lca(vector<vector<int>> &G) : N(G.size()), L(N), R(N) {
        K = N <= 1 ? 0 : 32 - __builtin_clz(N - 1);
        U.resize(K + 1, vector<int>(N));
        visit(G, 0, 0);
        rep(k, K) rep(u, N) U[k + 1][u] = U[k][U[k][u]];
    }

    void visit(vector<vector<int>> &G, int u, int p) {
        L[u] = t++, U[0][u] = p;
        for (int v : G[u]) if (v != p) visit(G, v, u);
        R[u] = t++;
    }

    bool is_anc(int up, int dn) {
        return L[up] <= L[dn] && R[dn] <= R[up];
    }

    int find(int u, int v) {
        if (is_anc(u, v)) return u;
        if (is_anc(v, u)) return v;
        for (int k = K; k >= 0; k--)
            if (is_anc(U[k][u], v)) k--;
        else u = U[k][u];
        return U[0][u];
    }
};

```

### 3.14 maxflow mincost

```

// time: O(F V E)          F is the maximum flow
//          O(V E + F E log V) if bellman-ford is replaced by
johnson
struct Flow {

```

```

struct Edge {
    int u, v;
    ll c, w, f = 0;
};

int N, s, t;
vector<vector<int>> G;
vector<Edge> E;
vector<ll> d, b;
vector<int> p;

Flow() {}
Flow(int N, int s, int t) : N(N), s(s), t(t), G(N) {}

void add_edge(int u, int v, ll c, ll w) {
    G[u].push_back(E.size());
    E.push_back({u, v, c, w});
    G[v].push_back(E.size());
    E.push_back({v, u, 0, -w});
}

// naive distances with bellman-ford: O(V E)
void calcdists() {
    p.assign(N, -1), d.assign(N, INF), d[s] = 0;
    rep(i, N - 1) rep(ei, E.size()) {
        Edge &e = E[ei];
        ll n = d[e.u] + e.w;
        if (d[e.u] < INF && e.c - e.f > 0 && n < d[e.v])
d[e.v] = n, p[e.v] = ei;
    }

    // johnsons potentials: O(E log V)
    void calcdists() {
        if (b.empty()) {
            b.assign(N, 0);
            // code below only necessary if there are
negative costs
            rep(i, N - 1) rep(ei, E.size()) {
                Edge &e = E[ei];
                if (e.f < e.c) b[e.v] = min(b[e.v], b[e.u] +
e.w);
            }
        }
        p.assign(N, -1), d.assign(N, INF), d[s] = 0;
        priority_queue<pair<ll, int>> q;
        q.push({0, s});
        while (!q.empty()) {
            auto [w, u] = q.top();
            q.pop();
            if (d[u] < -w + b[u]) continue;
            for (int ei : G[u]) {
                auto e = E[ei];
                ll n = d[u] + e.w;
                if (e.f < e.c && n < d[e.v]) {

```

```

        d[e.v] = n, p[e.v] = ei;
        q.push({b[e.v] - n, e.v});
    }
}
b = d;
}

ll solve() {
    b.clear();
    ll ff = 0;
    while (true) {
        calcdists();
        if (p[t] == -1) break;

        ll f = INF;
        for (int cur = t; p[cur] != -1; cur = E[p[cur]].u)
            f = min(f, E[p[cur]].c - E[p[cur]].f);
        for (int cur = t; p[cur] != -1; cur = E[p[cur]].u)
            E[p[cur]].f += f, E[p[cur]^1].f -= f;
        ff += f;
    }
    return ff;
}
};

```

### 3.15 parallel dfs

```

struct Tree {
    int n, z[2];
    vector<vector<int>> g;
    vector<int> ex, ey, p, w, f, v[2];
    Tree(int n): g(n), w(n), f(n) {}
    void add_edge(int x, int y) {
        p.pb(g[x].size()); g[x].pb(ex.size());
        ex.pb(x); ey.pb(y);
        p.pb(g[y].size()); g[y].pb(ex.size());
        ex.pb(y); ey.pb(x);
    }
    bool go(int k) { // returns 1 if it finds new node
        int& x = z[k];
        while (x >= 0 &&
            (w[x] == g[x].size() || w[x] == g[x].size() - 1
            && (g[x].back() ^ 1) == f[x]))
            x = f[x] >= 0 ? ex[f[x]] : -1;
        if (x < 0) return false;
        if ((g[x][w[x]] ^ 1) == f[x]) w[x]++;
        int e = g[x][w[x]], y = ey[e]; f[y] = e;
        w[x]++; w[y] = 0; x = y; v[k].pb(x);
        return true;
    }
    vector<int> erase_edge(int e) {
        e *= 2; // erases eth edge, returns smaller comp

```

```

    int x = ex[e], y = ey[e]; p[g[x].back()] = p[e];
    g[x][p[e]] = g[x].back(); g[x].pop_back();
    p[g[y].back()] = p[e ^ 1]; g[y][p[e ^ 1]] = g[y].back();
    g[y].pop_back();
    f[x] = f[y] - 1; w[x] = w[y] = 0; z[0] = x; z[1] = y;
    v[0] = {x}; v[1] = {y};
    bool d0 = true, d1 = true; while (d0 && d1) d0 = go(0), d1 = go(1);
    return v[1 - d1];
}
};

```

### 3.16 push relabel

```

#include "../common.h"

const ll INF = 1e18;

// maximum flow algorithm.
// to run, use `maxflow()`.
//
// time:  $O(V^2 \sqrt{E}) \leq O(V^3)$ 
// memory:  $O(V^2)$ 
struct PushRelabel {
    vector<vector<ll>> cap, flow;
    vector<ll> excess;
    vector<int> height;

    PushRelabel() {}
    void resize(int N) { cap.assign(N, vector<ll>(N)); }

    // push as much excess flow as possible from u to v.
    void push(int u, int v) {
        ll f = min(excess[u], cap[u][v] - flow[u][v]);
        flow[u][v] += f;
        flow[v][u] -= f;
        excess[v] += f;
        excess[u] -= f;
    }

    // relabel the height of a vertex so that excess flow
    // may be pushed.
    void relabel(int u) {
        int d = INT32_MAX;
        rep(v, cap.size()) if (cap[u][v] - flow[u][v] > 0) d =
            min(d, height[v]);
        if (d < INF) height[u] = d + 1;
    }

    // get the maximum flow on the network specified by
    // `cap` with source `s`
    // and sink `t`.
    // node-to-node flows are output to the `flow` member.
    ll maxflow(int s, int t) {
        int N = cap.size(), M;

```

```

        flow.assign(N, vector<ll>(N));
        height.assign(N, 0), height[s] = N;
        excess.assign(N, 0), excess[t] = INF;
        rep(i, N) if (i != s) push(s, i);

        vector<int> q;
        while (true) {
            // find the highest vertices with excess
            q.clear(), M = 0;
            rep(i, N) {
                if (excess[i] <= 0 || i == s || i == t)
                    continue;
                if (height[i] > M) q.clear(), M = height[i];
                if (height[i] >= M) q.push_back(i);
            }
            if (q.empty()) break;
            // process vertices
            for (int u : q) {
                bool relab = true;
                rep(v, N) {
                    if (excess[u] <= 0) break;
                    if (cap[u][v] - flow[u][v] > 0 &&
                        height[u] > height[v])
                        push(u, v), relab = false;
                }
                if (relab) {
                    relabel(u);
                    break;
                }
            }

            ll f = 0; rep(i, N) f += flow[i][t]; return f;
        }
    };
}

```

### 3.17 strongly connected components

```

/* time:  $O(V + E)$ , memory:  $O(V)$ 
after building:
    comp = map from vertex to component
           (components are toposorted, root first, leaf last)
    N = number of components
    G = condensation graph (component DAG)
byproducts:
    vgi = transposed graph
    order = reverse topological sort (leaf first, root last)
others:
    vn = number of vertices
    vg = original vertex graph */
struct Scc {
    int vn, N;
    vector<int> order, comp;
    vector<vector<int>> vg, vgi, G;
    void toposort(int u) {

```



```

    if (comp[u]) return;
    comp[u] = -1;
    for (int v : vg[u]) toposort(v);
    order.push_back(u);
}

bool carve(int u) {
    if (comp[u] != -1) return false;
    comp[u] = N;
    for (int v : vgi[u]) {
        carve(v);
        if (comp[v] != N) G[comp[v]].push_back(N);
    }
    return true;
}

Scc() {}
Scc(vector<vector<int>> &g)
: vn(g.size()), vg(g), comp(vn), vgi(vn), G(vn), N(0) {
    rep(u, vn) toposort(u);
    rep(u, vn) for (int v : vg[u]) vgi[v].push_back(u);
    invrep(i, vn) N += carve(order[i]);
}
};

```

### 3.18 two sat

```

// calculate the solvability of a system of logical
// equations, where every equation is of the form `a or b`.
// `neg`: get negation of `u`
// `then`: `u` implies `v`
// `any`: `u` or `v`
// `set`: `u` is true
//
// after `solve` (O(V+E)) returns true, `sol` contains one
// possible solution.
// determining all solutions is O(V*E) hard (requires
// computing reachability in a DAG).
struct TwoSat {
    int N; vector<vector<int>> G;
    Scc scc; vector<bool> sol;
    TwoSat(int n) : N(n), G(2 * n), sol(n) {}
    TwoSat() {}

    int neg(int u) { return (u + N) % (2 * N); }
    void then(int u, int v) { G[u].push_back(v),
    G[neg(v)].push_back(neg(u)); }
    void any(int u, int v) { then(neg(u), v); }
    void set(int u) { G[neg(u)].push_back(u); }

    bool solve() {
        scc = Scc(G);
        rep(u, N) if (scc.comp[u] == scc.comp[neg(u)])
        return false;
        rep(u, N) sol[u] = (scc.comp[u] > scc.comp[neg(u)]);
        return true;
    }
};

```

```

}
};

```

## 4 Implementation

### 4.1 common template and bit tricks

```

#pragma GCC optimize("Ofast")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt")
#pragma GCC
target("avx,avx2,f16c,fma,sse3,ssse3,sse4.1,sse4.2")
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define repx(i, a, b) for (int i = a; i < b; i++)
#define rep(i, n) repx(i, 0, n)
#define invrepx(i, a, b) for (int i = b - 1; i >= a; i--)
#define invrep(i, n) invrepx(i, 0, n)
// Command to check time and memory usage:
//      /usr/bin/time -v ./tmp
// See "Maximum resident set size" for max memory used
// Commands for interactive checker:
//      mkfifo fifo
//      (./solution < fifo) | (./interactor > fifo)
// Does not work on the Windows file system, i.e., /mnt/c/
// The special fifo file must be used, otherwise the
// solution will not wait for input and will read EOF
y = x & (x-1) // Turn off rightmost lbit
y = x & (-x) // Isolate rightmost lbit
y = x | (x-1) // Right propagate rightmost lbit(fill in 1s)
y = x | (x+1) // Turn on rightmost 0bit
y = ~x & (x+1) // Isolate rightmost 0bit
// If x is of long type, use __builtin_popcountll(x)
// If x is of long long type, use __builtin_popcountll(x)
// 1. Counts the number of one's(set bits) in an integer.
__builtin_popcount(x)
// 2. Checks the Parity of a number. Returns true(1) if the
// number has odd number of set bits, else it returns
// false(0) for even number of set bits.
__builtin_parity(x)
// 3. Counts the leading number of zeros of the integer.
__builtin_clz(x)
// 4. Counts the trailing number of zeros of the integer.
__builtin_ctz(x)
// 5. Returns 1 + the index of the least significant 1-bit.
__builtin_ffs(x) // If x == 0, returns 0.
// Iterate over non empty subsets of bitmask
for(int s=m;s=(s-1)&m) // Decreasing order
for(int s=0;s=s-m&m;) // Increasing order

```

### 4.2 dp convex hull trick

```

struct Line {
    mutable ll a, b, c;
};

```

```

bool operator<(Line r) const { return a < r.a; }
bool operator<(ll x) const { return c < x; }
};
// dynamically insert `a*x + b` lines and query for maximum
// at any x all operations have complexity O(log N)
struct LineContainer : multiset<Line, less<>> {
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator x, iterator y) {
        if (y == end()) return x->c = INF, 0;
        if (x->a == y->a) x->c = x->b > y->b ? INF : -INF;
        else x->c = div(y->b - x->b, x->a - y->a);
        return x->c >= y->c;
    }

    void add(ll a, ll b) {
        // a *= -1, b *= -1 // for min
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
        while ((y = x) != begin() && (--x)->c >= y->c)
        isect(x, erase(y));
    }

    ll query(ll x) {
        if (empty()) return -INF; // INF for min
        auto l = *lower_bound(x);
        return l.a * x + l.b;
        // return -l.a * x - l.b; // for min
    }
};

```

### 4.3 dp divide and conquer

```

// for every index i assign an optimal index j, such that
// cost(i, j) is minimal for every i. the property that if
// i2 >= i1 then j2 >= j1 is exploited (monotonic condition)
// calculate optimal index for all indices in range [l, r)
// knowing that the optimal index for every index in this
// range is within [optl, optr).
// time: O(N log N)
void calc(vector<int> &opt, int l, int r, int optl, int optr) {
    if (l == r) return;
    int i = (l + r) / 2;
    ll optc = INF;
    int optj;
    repx(j, optl, optr) {
        ll c = i + j; // cost(i, j)
        if (c < optc) optc = c, optj = j;
    }
    opt[i] = optj;
}

```

```

    calc(opt, l, i, optl, optj + 1);
    calc(opt, i + 1, r, optj, optr);
}

```

## 4.4 dynamic connectivity

```

struct DC {
    int n; Dsu D;
    vector<vector<pair<int, int>>> t;
    DC(int N) : n(N), D(N), t(2 * N) {}
    // add edge p to all times in interval [l, r]
    void upd(int l, int r, pair<int, int> p) {
        for (l += n, r += n; l < r; l >= 1, r >= 1) {
            if (l & 1) t[l++].push_back(p);
            if (r & 1) t[--r].push_back(p);
        }
    }
    void process(int u = 1) { // process all queries
        for (auto &e : t[u]) D.unite(e.first, e.second);
        if (u >= n) {
            // do stuff with D at time u - n
        } else process(2 * u), process(2 * u + 1);
        for (auto &e : t[u]) D.rollback();
    }
};

```

## 4.5 hash container

```

namespace{ //add (#define tmp1 template)(#define ty typename)
    tmp1<ty T> size_t mk_h(const T& v){return hash<T>()(v);}
    void h_cmb(size_t& h, const size_t& v)
    { h ^= v + 0x9e3779b9 + (h << 6) + (h >> 2); }
    tmp1<ty T> struct h_ct{size_t operator()(const T& v) const{
        size_t h=0;for(const auto& e:v){h_cmb(h,mk_h(e));}return h;
    }};
}namespace std{ //support for pair<T,U>, vector<T> & map<T,U>
    tmp1<ty T, ty U> struct hash<pair<T, U>>{
        size_t operator()(const pair<T,U>& v) const
        {size_t h=mk_h(v.first);h_cmb(h, mk_h(v.second));return h;}
    };
    tmp1<ty... T>struct hash<vector<T...>>:h_ct<vector<T...>>{};
    tmp1<ty... T>struct hash<map<T...>>:h_ct<map<T...>>{}; }

```

## 4.6 mo

```

struct Query { int l, r, idx; };

// answer segment queries using only `add(i)`, `remove(i)`
// and `get()`
// functions.
//
// complexity: O((N + Q) * sqrt(N) * F)
// N = length of the full segment
// Q = amount of queries
// F = complexity of the `add`, `remove` functions

```

```

template <class A, class R, class G, class T>
void mo(vector<Query> &queries, vector<T> &ans, A add, R
remove, G get) {
    int Q = queries.size(), B = (int)sqrt(Q);
    sort(queries.begin(), queries.end(), [&](Query &a, Query
&b) {
        return make_pair(a.l / B, a.r) < make_pair(b.l / B,
b.r);
    });
    ans.resize(Q);

    int l = 0, r = 0;
    for (auto &q : queries) {
        while (r < q.r) add(r), r++;
        while (l > q.l) l--, add(l);
        while (r > q.r) r--, remove(r);
        while (l < q.l) remove(l), l++;
        ans[q.idx] = get();
    }
}

```

## 4.7 ordered set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
// find_by_order(i) -> iterator to ith element
// order_of_key(k) -> position (int) of lower_bound of k

```

## 4.8 unordered map

```

static mt19937 rng(chrono::steady_clock::now()
.time_since_epoch().count());
#define rnd(a,b) (uniform_int_distribution<ll>(a,b)(rng))

struct Hash {
    size_t operator()(const ll &x) const {
        const uint64_t RAND = chrono::steady_clock::now()
.time_since_epoch().count();
        uint64_t z = x + RAND + 0x9e3779b97f4a7c15;
        z = (z ^ (z >> 30)) * 0xbf58476d1ce4e5b9;
        z = (z ^ (z >> 27)) * 0x94d049bb133111eb;
        return z ^ (z >> 31);
    }
};

template<class T, class U>using umap=unordered_map<T,U,Hash>;
template<class T> using uset = unordered_set<T, Hash>;

```

# 5 Math

## 5.1 arithmetic

```

inline int floor_log2(int n)
{ return n <= 1 ? 0 : 31 - __builtin_clz(n); }
inline int ceil_log2(int n)
{ return n <= 1 ? 0 : 32 - __builtin_clz(n - 1); }
inline ll floordiv(ll a, ll b) {return a/b-((a^b)<0&&a%b);}
inline ll ceildiv(ll a, ll b) {return a/b+((a^b)>=0&&a%b);}

```

## 5.2 berlekamp massey linear recurrence

```

vector<int> BM(vector<int> x) {
    vector<int> ls, cur;
    int lf, ld;
    rep(i, x.size()) {
        ll t = 0;
        rep(j, cur.size()) t = (t+x[i-j-1]*(ll)cur[j])%MOD;
        if ((t - x[i]) % MOD == 0) continue;
        if (!cur.size()) {
            cur.resize(i + 1); lf = i; ld = (t-x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * bin_exp(ld, MOD - 2) % MOD;
        vector<int> c(i - lf - 1); c.push_back(k);
        rep(j, ls.size()) c.push_back((-ls[j] * k % MOD));
        if (c.size() < cur.size()) c.resize(cur.size());
        rep(j, cur.size()) c[j] = (c[j] + cur[j]) % MOD;
        if (i - lf + ls.size() >= cur.size())
            ls = cur, lf = i, ld = (t - x[i]) % MOD;
        cur = c;
    }
    rep(i, cur.size()) cur[i] = (cur[i] % MOD + MOD) % MOD;
    return cur;
}

// Linear Recurrence
ll MOD = 998244353;
ll LOG = 60;
struct LinearRec{
    typedef vector<int> vi;
    int n; vi terms, trans; vector<vi> bin;
    vi add(vi &a, vi &b){
        vi res(n*2+1);
        rep(i,n+1) rep(j,n+1)
            res[i+j]=(res[i+j]*1LL+(ll)a[i]*b[j])%MOD;
        for(int i=2*n; i>n; --i){
            rep(j,n)
                res[i-1-j]=(res[i-1-j]*1LL+(ll)res[i]*trans[j])%MOD;
            res[i]=0;
        }
        res.erase(res.begin()+n+1,res.end());
        return res;
    }
    LinearRec(vi &terms, vi &trans):terms(terms),trans(trans){

```

```

n=trans.size();vi a(n+1);a[l]=1;
bin.push_back(a);
repx(i,1,LOG)bin.push_back(add(bin[i-1],bin[i-1]));
}
int calc(ll k){
vi a(n+1);a[0]=1;
rep(i,LOG)if((k>>i)&1)a=add(a,bin[i]);
int ret=0;
rep(i,n)ret=((ll)ret+(ll)a[i+1]*terms[i])%MOD;
ret = ret%MOD + MOD;
return ret%MOD;
}
};

```

### 5.3 crt

```

pair<ll, ll> solve_crt(const vector<pair<ll, ll>> &eqs) {
ll a0 = eqs[0].first, p0 = eqs[0].second;
repx(i, 1, eqs.size()) {
ll a1 = eqs[i].first, p1 = eqs[i].second;
ll k1, k0;
ll d = ext_gcd(p1, p0, k1, k0);
a0 -= a1;
if (a0 % d != 0) return {-1, -1};
p0 = p0 / d * p1;
a0 = a0 / d * k1 % p0 * p1 % p0 + a1;
a0 = (a0 % p0 + p0) % p0;
}
return {a0, p0};
}

```

### 5.4 debrujinseq

```

vector<int> deBruijnSeq(int k, int n) { /// Recursive FKM
if (k == 1) return {0};
vector<int> seq, aux(n+1);
function<void(int,int)> gen = [&](int t, int p) {
if (t > n) { // +lyndon word of len p
if (n%p == 0)repx(i,1,p+1)seq.push_back(aux[i]);
} else {
aux[t] = aux[t-p]; gen(t+1,p);
while (++aux[t] < k) gen(t+1,t);
}
};
gen(1,1); return seq;
}

```

### 5.5 discrete log

```

// discrete logarithm log_a(b).
// solve b ^ x = a (mod M) for the smallest x.
// returns -1 if no solution is found.
//
// time: O(sqrt(M))
ll dlog(ll a, ll b, ll M) {

```

```

ll k = 1, s = 0;
while (true) {
ll g = __gcd(b, M);
if (g <= 1) break;
if (a == k) return s;
if (a % g != 0) return -1;
a /= g, M /= g, s += 1, k = b / g * k % M;
}
ll N = sqrt(M) + 1;

umap<ll, ll> r;
rep(q, N + 1) {
r[a] = q;
a = a * b % M;
}

ll bN = binexp(b, N, M), bNp = k;
repx(p, 1, N + 1) {
bNp = bNp * bN % M;
if (r.count(bNp)) return N * p - r[bNp] + s;
}
return -1;
}

```

### 5.6 fast hadamard transform

```

ll c1[MAXN+9],c2[MAXN+9];//MAXN must be power of 2!
void fht(ll* p, int n, bool inv){
for(int l=1;2*l<=n;l*=2)for(int
i=0;i<n;i+=2*l)fore(j,0,l){
ll u=p[i+j],v=p[i+l+j];
if(!inv)p[i+j]=u+v,p[i+l+j]=u-v; // XOR
else p[i+j]=(u+v)/2,p[i+l+j]=(u-v)/2;
//if(!inv)p[i+j]=v,p[i+l+j]=u+v; // AND
//else p[i+j]=-u+v,p[i+l+j]=u;
//if(!inv)p[i+j]=u+v,p[i+l+j]=u; // OR
//else p[i+j]=v,p[i+l+j]=u-v;
}
}
// like polynomial multiplication, but XORing exponents
// instead of adding them (also ANDing, ORing)
vector<ll> multiply(vector<ll>& p1, vector<ll>& p2){
int n=1<<(32-__builtin_clz(max(SZ(p1),SZ(p2))-1));
fore(i,0,n)c1[i]=0,c2[i]=0;
fore(i,0,SZ(p1))c1[i]=p1[i];
fore(i,0,SZ(p2))c2[i]=p2[i];
fht(c1,n,false);fht(c2,n,false);
fore(i,0,n)c1[i]*=c2[i];
fht(c1,n,true);
return vector<ll>(c1,c1+n);
}

```

### 5.7 fft

```

using cd = complex<double>;
const double PI = acos(-1);
// compute the DFT of a power-of-two-length sequence.
// if `inv` is true, computes the inverse DFT.
void fft(vector<cd> &a, bool inv) {
int N = a.size(), k = 0, b;
assert(N == 1 << __builtin_ctz(N));
repx(i, 1, N) {
for (b = N >> 1; k & b; k ^= b, b >>= 1;
if (i < (k ^ b)) swap(a[i], a[k]));
}
for (int l = 2; l <= N; l <= 1) {
double ang = 2 * PI / l * (inv ? -1 : 1);
cd w1(cos(ang), sin(ang));
for (int i = 0; i < N; i += l) {
cd w = 1;
rep(j, l / 2) {
cd u = a[i + j], v = a[i + j + l / 2] * w;
a[i + j] = u + v;
a[i + j + l / 2] = u - v;
w *= w1;
}
}
if (inv) rep(i, N) a[i] /= N;
}
const ll MOD = 998244353, ROOT = 15311432;
// const ll MOD = 2130706433, ROOT = 1791270792;
// const ll MOD = 9223372036737335297ll, ROOT =
5320774565496356983ll;
void find_root_of_unity(ll M) {
ll c = M - 1, k = 0;
while (c % 2 == 0) c /= 2, k += 1;
// find proper divisors of M - 1
vector<ll> divs;
for (ll d = 1; d < c; d++) {
if (d * d > c) break;
if (c % d == 0) rep(i, k + 1) divs.push_back(d <<
i);
}
rep(i, k) divs.push_back(c << i);
// find any primitive root of M
ll G = -1;
repx(g, 2, M) {
bool ok = true;
for (ll d : divs) ok &= (binexp(g, d, M) != 1);
if (ok) {
G = g;
break;
}
}
assert(G != -1);
ll w = binexp(G, c, M);
cerr << "M = c * 2^k + 1" << endl;
cerr << " M = " << M << endl;

```

```

cerr << " c = " << c << endl;
cerr << " k = " << k << endl;
cerr << " w^(2^k) == 1" << endl;
cerr << " w = g^((M-1)/2^k) = g^c" << endl;
cerr << " g = " << G << endl;
cerr << " w = " << w << endl;
}
// compute the DFT of a power-of-two-length sequence, modulo
a special prime
// number with an Nth root of unity, where N is the length
of the sequence.
void ntt(vector<ll> &a, bool inv) {
    vector<ll> wn;
    for (ll p = ROOT; p != 1; p = p * p % MOD)
wn.push_back(p);
    int N = a.size(), k = 0, b;
    assert(N == 1 << __builtin_ctz(N) && N <= 1 <<
wn.size());
    rep(i, N) a[i] = (a[i] % MOD + MOD) % MOD;
    repx(i, 1, N) {
        for (b = N >> 1; k & b; k ^= b, b >>= 1;
            if (i < (k ^= b)) swap(a[i], a[k]));
    }
    for (int l = 2; l <= N; l <= 1) {
        ll wL = wn[wn.size() - __builtin_ctz(l)];
        if (inv) wL = multinv(wL, MOD);
        for (int i = 0; i < N; i += l) {
            ll w = 1;
            repx(j, 0, l / 2) {
                ll u = a[i + j], v = a[i + j + l / 2] * w %
MOD;
                a[i + j] = (u + v) % MOD;
                a[i + j + l / 2] = (u - v + MOD) % MOD;
                w = w * wL % MOD;
            }
        }
        ll q = multinv(N, MOD);
        if (inv) rep(i, N) a[i] = a[i] * q % MOD;
    }
}
void convolve(vector<cd> &a, vector<cd> b, int n) {
    n = 1 << (32 - __builtin_clz(2 * n - 1));
    a.resize(n), b.resize(n);
    fft(a, false), fft(b, false);
    rep(i, n) a[i] *= b[i];
    fft(a, true);
}

```

## 5.8 gauss

```

const double EPS = 1e-9;
// solve a system of equations.
// complexity: O(min(N, M) * N * M)
// `a` is a list of rows
// the last value in each row is the result of the equation

```

```

// return values:
// 0 -> no solutions
// 1 -> unique solution, stored in `ans`
// -1 -> infinitely many solutions, one of which is stored
in `ans`
// UNTESTED
int gauss(vector<vector<double>> a, vector<double> &ans) {
    int N = a.size(), M = a[0].size() - 1;
    vector<int> where(M, -1);
    for (int j = 0, i = 0; j < M && i < N; j++) {
        int sel = i;
        repx(k, i, N) if (abs(a[k][j]) > abs(a[sel][j])) sel
= k;
        if (abs(a[sel][j]) < EPS) continue;
        repx(k, j, M + 1) swap(a[sel][k], a[i][k]);
        where[j] = i;
        rep(k, N) if (k != i) {
            double c = a[k][j] / a[i][j];
            repx(l, j, M + 1) a[k][l] -= a[i][l] * c;
        }
        i++;
    }
    ans.assign(M, 0);
    rep(i, M) if (where[i] != -1) ans[i] = a[where[i]][M] /
a[where[i]][i];
    rep(i, N) {
        double sum = 0;
        rep(j, M) sum += ans[j] * a[i][j];
        if (abs(sum - a[i][M]) > EPS) return 0;
    }
    rep(i, M) if (where[i] == -1) return -1;
    return 1;
}

```

## 5.9 linear diophantine

```

ii extendedEuclid(ll a, ll b){
    ll x, y; //a*x + b*y = gcd(a,b)
    if (b == 0) return {1, 0};
    auto p = extendedEuclid(b, a%b);
    x = p.second;
    y = p.first - (a/b)*x;
    if(a*x + b*y == -__gcd(a,b)) x=-x, y=-y;
    return {x, y};
}
pair<ii, ii> diophantine(ll a, ll b, ll r){
    //a*x+b*y=r where r is multiple of gcd(a,b);
    ll d = __gcd(a, b);
    a/=d; b/=d; r/=d;
    auto p = extendedEuclid(a, b);
    p.first*=r; p.second*=r;
    assert(a*p.first + b*p.second == r);
    return {p, {-b, a}}; //solutions: p+t*ans.second
}

```

## 5.10 matrix

```

typedef vector<vector<double>> Mat;
Mat matmul(Mat l, Mat r) {
    int n = l.N, m = r.M, p = l.M; assert(l.M == r.N);
    Mat a(n, vector<double>(m)); // neutral
    rep(i, n) rep(j, m)
        rep(k, p) a[i][j] = a[i][k] + l[i][k] * r[k][j];
    return a;
}
double reduce(vector<vector<double>> &A) {
    int n = A.size(), m = A[0].size();
    int i = 0, j = 0; double r = 1.;
    while (i < n && j < m) {
        int l = i;
        repx(k, i+1, n) if(abs(A[k][j]) > abs(A[l][j])) l=k;
        if (abs(A[l][j]) < EPS) { j++; r = 0.; continue; }
        if (l != i) { r = -r; swap(A[i], A[l]); }
        r *= A[l][j];
        for (int k = m - 1; k >= j; k--) A[i][k] /= A[i][j];
        repx(k, 0, n) {
            if (k == i) continue;
            for(int l=m-1;l>=j;l--)A[k][l]-=A[k][j]*A[i][l];
        }
        i++, j++;
    }
    return r; // returns determinant
}

```

## 5.11 matroidisect

```

//O(G*I^{1.5}) calls to oracle
//G ground set, I independent set
//MatroidIsect<Gmat,Cmat> M(ed.size(),Gmat(ed),Cmat(col));
struct Gmat { // graphic matroid
    int V = 0; vector<ii> ed; Dsu D;
    Gmat(vector<ii> _ed):ed(_ed){
        map<int,int> m;
        for(auto &t: ed) m[t.first] = m[t.second] = 0;
        for(auto &t: m) t.second = V++;
        for(auto &t: ed)
            t.first = m[t.first], t.second = m[t.second];
    }
    void clear() { D.p = vector<int>(V, -1); }
    void ins(int i) { D.unite(ed[i].first, ed[i].second); }
    bool indep(int i)
    { return !D.sameSet(ed[i].first, ed[i].second); }
};
struct Cmat { // colorful matroid
    int C = 0; vector<int> col; vector<bool> used;
    Cmat(vector<int> col):col(col)
    {for(auto &t: col) C = max(C, t+1);}
}

```

```

void clear() { used.assign(C, 0); }
void ins(int i) { used[col[i]] = 1; }
bool indep(int i) { return !used[col[i]]; }
};
template<class M1, class M2> struct MatroidIsect {
    int n; vector<bool> iset; M1 m1; M2 m2;
    bool augment() {
        vector<int> pre(n+1,-1); queue<int> q({n});
        while(q.size()){
            int x = q.front(); q.pop();
            if (iset[x]) {
                m1.clear();
                rep(i,n) if (iset[i] && i != x) m1.ins(i);
                rep(i,n)
                    if(!iset[i]&& pre[i]==-1 && m1.indep(i))
                        pre[i] = x, q.push(i);
            } else {
                auto backE = [&]() { // back edge
                    m2.clear();
                    rep(c,2) rep(i,n) if((x==i||
iset[i])&&(pre[i]==-1)==c){
                        if (!m2.indep(i)) return c?
pre[i]=x,q.push(i),i:-1;
                        m2.ins(i); }
                    return n;
                };
                for (int y; (y = backE()) != -1; if(y==n) {
                    for(; x != n; x = pre[x]) iset[x]=!iset[x];
                    return 1; }
            }
        }
        return 0;
    }
    MatroidIsect(int n, M1 m1, M2 m2):n(n), m1(m1), m2(m2) {
        iset.assign(n+1,0); iset[n] = 1;
        m1.clear(); m2.clear(); // greedily add to basis
        invrep(i,n) if (m1.indep(i) && m2.indep(i))
            iset[i] = 1, m1.ins(i), m2.ins(i);
        while (augment());
    }
};

```

## 5.12 mobius

```

short mu[MAXN] = {0,1};
void mobius(){
    repx(i,1,MAXN) if(mu[i]) for(int j=i+i; j<MAXN; j+=i) mu[j] -= mu[i];
}

```

## 5.13 multinv

```

// a * x + b * y == gcd(a, b)
ll ext_gcd(ll a, ll b, ll &x, ll &y) {
    if (b == 0) { x = 1, y = 0; return a; }

```

```

    ll d = ext_gcd(b, a % b, y, x); y -= a / b * x; return d;
}

// inverse exists if and only if a and M are coprime
// if M is prime: multinv(a, M) = (a**(M-2)) % M
ll multinv(ll a, ll M)
{ ll x, y; ext_gcd(a, M, x, y); return x; }

// all modular inverses from 1 to inv.size()-1
void multinv_all(vector<ll> &inv) {
    inv[1] = 1;
    repx(i, 2, inv.size())
        inv[i] = MOD - (MOD / i) * inv[MOD % i] % MOD;
}

```

## 5.14 permtree

```

struct PermTree {
    vector<int> P; int n; // N = number of nodes in perm tree
    int N = 0, root; vector<vector<int>> child;
    vector<ii> inter, range; vector<int> typ, loc; // inter =
index range in perm
    void init(vector<int> _P) {
        P = _P; n = P.size(); Stl L(n);
        vector<int> mn{-1}, mx{-1}, st;
        rep(i, n){
            if(i) L.update(0, i, -1);

            while (mn.back() != -1 && P[mn.back()] > P[i]) {
                int t = mn.back(); mn.pop_back();
                L.update(mn.back()+1,t+1,P[t]);
            }
            L.update(mn.back()+1,i+1,-P[i]); mn.push_back(i);

            while (mx.back() != -1 && P[mx.back()] < P[i]) {
                int t = mx.back(); mx.pop_back();
                L.update(mx.back()+1,t+1,-P[t]);
            }
            L.update(mx.back()+1,i+1,P[i]); mx.push_back(i);

            int cur = N++; loc.push_back(cur);
            inter.push_back({i,i}); range.push_back({P[i],P[i]});
            typ.push_back(0); child.emplace_back();
            auto add = [](ii a, ii b) -> ii { return {min(a.first,
b.first), max(a.second,b.second)};};
            auto adj = [&](int x, int y) { ii a = range[x], b =
range[y];
                return a.second+1 == b.first || b.second+1 ==
a.first;};
            while(st.size()){
                if(adj(st.back(), cur)){
                    if(child[st.back()].size() &&
adj(child[st.back()].back(),cur)){
                        inter[st.back()] =
add(inter[st.back()],inter[cur]);

```

```

                        range[st.back()] =
add(range[st.back()],range[cur]);
                        child[st.back()].push_back(cur); cur =
st.back(); st.pop_back();
                    } else { // make new join node
                        int CUR = N++;

                        inter.push_back(add(inter[cur],inter[st.back()]));

                        range.push_back(add(range[cur],range[st.back()]));
                        typ.push_back(range[st.back()] < range[cur] ?
1 : 2);
                        child.push_back({st.back(),cur}); cur = CUR;
                        st.pop_back();
                    }
                    continue;
                }
                if (L.query(0, inter[cur].first) != 0) break;
                int CUR = N++;
                inter.push_back(inter[cur]);
                range.push_back(range[cur]); typ.push_back(0);
                child.push_back({cur});
                auto len = [](ii p) { return p.second-p.first;};
                do{
                    inter[CUR] = add(inter[CUR],inter[st.back()]);
                    range[CUR] = add(range[CUR],range[st.back()]);
                    child[CUR].push_back(st.back()); st.pop_back();
                } while (len(inter.back()) != len(range.back()));
                reverse(child[CUR].begin(), child[CUR].end()); cur =
CUR;
            }
            st.push_back(cur);
        }
        root = st.back();
    }
};

```

## 5.15 polar rho

```

ll mulmod(ll a, ll b, ll m) {
    ll r=a*b-(ll)((long double)a*b/m+.5)*m;
    return r<0?r+m:r;
}
bool is_prime_prob(ll n, int a){
    if(n==a) return true;
    ll s=0,d=n-1;
    while(d%2==0) s++,d/=2;
    ll x=expmod(a,d,n);
    if((x==1)|| (x+1==n)) return true;
    fore(_,0,s-1){
        x=mulmod(x,x,n);
        if(x==1) return false;
        if(x+1==n) return true;
    }
    return false;
}

```



```

}
bool rabin(ll n){ // true iff n is prime
    if(n==1)return false;
    int ar[]={2,3,5,7,11,13,17,19,23};
    for(i,0,9)if(!is_prime_prob(n,ar[i]))return false;
    return true;
}
ll rho(ll n){
    if(!(n&1))return 2;
    ll x=2,y=2,d=1;
    ll c=rand()%n+1;
    while(d==1){
        x=(mulmod(x,x,n)+c)%n;
        for(it,0,2) y=(mulmod(y,y,n)+c)%n;
        if(x==y)d=__gcd(x-y,n);
        else d=__gcd(y-x,n);
    }
    return d==n?rho(n):d;
}
void fact(ll n, map<ll,int>& f){ //O (lg n)^3
    if(n==1)return;
    if(rabin(n)){f[n]++;return;}
    ll q=rho(n);fact(q,f);fact(n/q,f);
}
// optimized version: replace rho and fact with the
// following:
const int MAXP=1e6+1; // sieve size
int sv[MAXP]; // sieve
ll add(ll a, ll b, ll m){return (a+b)<m?a+b-m;}
ll rho(ll n){
    static ll s[MAXP];
    while(1){
        ll x=rand()%n,y=x,c=rand()%n;
        ll *px=s,*py=s,v=0,p=1;
        while(1){
            *py++=y=add(mulmod(y,y,n),c,n);
            *py++=y=add(mulmod(y,y,n),c,n);
            if((x=*px++)==y)break;
            ll t=p; p=mulmod(p,abs(y-x),n);
            if(!p)return __gcd(t,n);
            if(++v==26){
                if((p=__gcd(p,n))>1&&p<n)return p;
                v=0;
            }
        }
        if(v&&(p=__gcd(p,n))>1&&p<n)return p;
    }
}
void init_sv(){ for(i,2,MAXP)if(!sv[i])for(ll
j=i;j<MAXP;j+=i)sv[j]=i; }
void fact(ll n,map<ll,int>&f){//call init_sv first!
    for(auto& p:f)while(n%p.fst==0)p.snd++,n/=p.fst;
    if(n<MAXP)while(n>1)f[sv[n]]++,n/=sv[n];
    else if(rabin(n))f[n]++;
}

```

```

else {ll q=rho(n);fact(q,f);fact(n/q,f);}
}

```

## 5.16 polynomials

```

typedef int tp; // type of polynomial
template<class T=tp>
struct poly { // poly<T> : 1 variable, poly<poly<T>>: 2
variables, etc.
    vector<T> c;
    T& operator[](int k){return c[k];}
    poly(vector<T>& c):c(c){}
    poly(initializer_list<T> c):c(c){}
    poly(int k):c(k){}
    poly(){}
    poly operator+(poly<T> o);
    poly operator*(tp k);
    poly operator*(poly o);
    poly operator-(poly<T> o){return *this+(o*-1);}
    T operator()(tp v){
        T sum(0);
        for(int i=c.size()-1;i>=0;--i)sum=sum*v+c[i];
        return sum;
    }
};
// example: p(x,y)=2*x^2+3*x*y-y+4
// poly<poly<T>> p={{4,-1},{0,3},{2}}
// printf("%d\n",p(2)(3)) // 27 (p(2,3))
set<tp> roots(poly<T> p){ // only for integer polynomials
    set<tp> r;
    while(!p.c.empty()&&!p.c.back())p.c.pop_back();
    if(!p(0))r.insert(0);
    if(p.c.empty())return r;
    tp a0=0,an=abs(p[p.c.size()-1]);
    for(int k=0;!a0;a0=abs(p[k+1]));
    vector<tp> ps,qs;
    for(i,1,sqrt(a0)+1)if(a0%i==0)ps.pb(i),ps.pb(a0/i);
    for(i,1,sqrt(an)+1)if(an%i==0)qs.pb(i),qs.pb(an/i);
    for(auto pt:ps)for(auto qt:qs)if(pt*qt==0){
        tp x=pt/qt;
        if(!p(x))r.insert(x);
        if(!p(-x))r.insert(-x);
    }
    return r;
}
pair<poly<T>,tp> ruffini(poly<T> p, tp r){ // returns pair
(result,rem)
    int n=p.c.size()-1;
    vector<tp> b(n);
    b[n-1]=p[n];
    for(int k=n-2;k>=0;--k)b[k]=p[k+1]+r*b[k+1];
    return {poly<T>(b),p[0]+r*b[0]};
}
// only for double polynomials
pair<poly<T>,poly<T>> polydiv(poly<T> p, poly<T> q){ // returns

```

```

pair (result,rem)
    int n=p.c.size()-q.c.size()+1;
    vector<tp> b(n);
    for(int k=n-1;k>=0;--k){
        b[k]=p.c.back()/q.c.back();
        for(i,0,q.c.size())p[i+k]-=b[k]*q[i];
        p.c.pop_back();
    }
    while(!p.c.empty()&&abs(p.c.back())<EPS)p.c.pop_back();
    return {poly<T>(b),p};
}
// only for double polynomials
poly<T> interpolate(vector<tp> x, vector<tp> y){
    poly<T> q={1},S={0};
    for(tp a:x)q=poly<T>({-a,1})*q;
    for(i,0,x.size()){
        poly<T> Li=ruffini(q,x[i]).fst;
        Li=Li*(1.0/Li(x[i])); // change for int polynomials
        S=S+Li*y[i];
    }
    return S;
}
}

```

## 5.17 primes

```

// counts the divisors of a positive integer in O(sqrt(n))
ll count_divisors(ll x) {
    ll divs = 1, i = 2;
    for (ll divs = 1, i = 2; x > 1; i++) {
        if (i * i > x) { divs *= 2; break; }
        for (ll d = divs; x % i == 0; x /= i) divs += d;
    }
    return divs;
}
// gets the prime factorization of a number in O(sqrt(n))
vector<pair<ll, int>> factorize(ll x) {
    vector<pair<ll, int>> f;
    for (ll k = 2; x > 1; k++) {
        if (k * k > x) { f.push_back({x, 1}); break; }
        int n = 0;
        while (x % k == 0) x /= k, n++;
        if (n > 0) f.push_back({k, n});
    }
    return f;
}
// iterate over all divisors of a number.
// divisor count upper bound: n^(1.07 / ln ln n)
template <class OP>
void divisors(ll x, OP op) {
    auto facts = factorize(x);
    vector<int> f(facts.size());
    while (true) {
        ll y = 1;
        rep(i, f.size()) rep(j, f[i]) y *= facts[i].first;
        op(y);
    }
}

```



```

int i;
for (i = 0; i < f.size(); i++) {
    f[i] += 1;
    if (f[i] <= facts[i].second) break;
    f[i] = 0;
}
if (i == f.size()) break;
}

// computes euler totative function phi(x), counting the
// amount of integers in [1, x] that are coprime with x.
// time: O(sqrt(x))
ll phi(ll x) {
    ll phi = 1, k = 2;
    for (; x > 1; k++) {
        if (k * k > x) { phi *= x - 1; break; }
        ll k1 = 1, k0 = 0;
        while (x % k == 0) x /= k, k0 = k1, k1 *= k;
        phi *= k1 - k0;
    }
    return phi;
}

// test-prime.cpp
// change to __int128 if checking numbers over 10^9
bool isprime(ll n) {
    if (n < 2 || n % 4 != 1) return n - 2 < 2;
    ll A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    ll s = __builtin_ctzll(n - 1), d = n >> s;
    for (int a : A) {
        ll p = binexp(a, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--) p = p *
p % n;
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}

```

## 5.18 simplex

/\* Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^T x$  otherwise. The input vector is set to an optimal  $x$  (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that  $x = 0$  is viable. Usage:

```

vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vvd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: O(NM * #pivots), where a pivot may be e.g. an edge relaxation. O(2^n) in the general case.*/
typedef double T; //long double, Rational, double + mod<P>...

```

```

typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1 / .0;
#define MP make_pair
#define ltj(X) \
    if (s == -1 || MP(X[j], N[j]) < MP(X[s], N[s])) s = j
struct LPSolver {
    int m, n; vector<int> N, B; vvd D;
    LPSolver(const vvd &A, const vd &b, const vd &c) :
    m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i, m) rep(j, n) D[i][j] = A[i][j];
        rep(i, m) {
            B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i];
        }
        rep(j, n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }
    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i, m + 2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            repx(j, 0, n + 2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j, n + 2) if (j != s) D[r][j] *= inv;
        rep(i, m + 2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }
    bool simplex(int phase) {
        int x = m + phase - 1;
        for (;;) {
            int s = -1;
            rep(j, n + 1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i, m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || MP(D[i][n + 1] / D[i][s],
B[i]) < MP(D[r][n + 1] / D[r][s], B[r])) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    T solve(vd &x) {
        int r = 0;
        repx(i, 1, m) if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m + 1][n + 1] < -eps)
return -inf;
        }
        rep(i, m) if (B[i] == -1) {
            int s = 0;
            repx(j, 1, n + 1) ltj(D[i]);

```

```

        pivot(i, s);
    }
    bool ok = simplex(1);
    x = vd(n);
    rep(i, m) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return ok ? D[m][n + 1] : inf;
};

```

## 5.19 theorems and formulas

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

$$\sum_{i=0}^k \binom{n+i}{i} = \binom{n+k+1}{k}$$

$\left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right]$  = perm of  $n$  elements with  $k$  cycles

$$\left[ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right] = n \left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right] + \left[ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right]$$

$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$  = partitions of an  $n$ -element set into  $k$  parts

$$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} = \frac{1}{k!} \sum_{i=0}^k (-1)^i \binom{k}{i} (k-i)^n$$

$$\left\{ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right\}$$

Integers  $d_1 \geq \dots \geq d_n \geq 0$  can be the degree sequence of a finite simple graph on  $n$  vertices  $\Leftrightarrow d_1 + \dots + d_n$  is even and for every  $k$  in  $1 \leq k \leq n$

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

$$a^n = a^{\varphi(m) + n \bmod \varphi(m)} \pmod{m} \text{ if } n > \lg(m)$$

Misere Nim: if  $\exists a_i > 1$  then normal nim; else the condition is reversed.

Derangements: Num of permutations of  $n = 0, 1, 2, \dots$  elements without fixed points is  $1, 0, 1, 2, 9, 44, 265, 1854, 14833, \dots$  Recurrence:  $D_n = (n-1)(D_{n-1} + D_{n-2}) = n * D_{n-1} + (-1)^n$

Collary: number of permutations with exactly  $k$  fixed points  $\binom{n}{k} D_{n-k}$

Eulerian numbers:  $E(n, k)$  is the number of permutations with exactly  $k$  descents ( $i : \pi_i <$

$\pi_{i+1})$ , ascents ( $\pi_i > \pi_{i+1}$ ) / excedances ( $\pi > i$ ) /  $k + 1$  weak excedances ( $\pi \geq i$ ).

$E_{n,k} = (k + 1)E_{n-1,k} + (n - k)E_{n-1,k-1}$

Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g\left(\frac{n}{d}\right)$$

Other useful formulas/forms

$$\sum_{d|n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{1 \leq m \leq n} f\left(\left\lfloor \frac{n}{m} \right\rfloor\right) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g\left(\left\lfloor \frac{n}{m} \right\rfloor\right)$$

$$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right)g(d)$$

Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p\left(n - \frac{k(3k-1)}{2}\right)$$

$$p(n) \approx \frac{0.145}{n} \cdot e^{2.56\sqrt{n}}$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\approx 2e5$	$\approx 2e8$

Bell numbers

Total number of partitions of  $n$  distinct elements.

$$B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$$

For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$$

Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$

# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$

# with degrees  $d_i$ :  $\frac{(n-2)!}{(d_1-1)! \dots (d_n-1)!}$

Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with  $n + 1$  leaves (0 or 2 children).
- ordered trees with  $n + 1$  vertices.
- ways a convex polygon with  $n + 2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

Narayana Numbers

$$N(n, k) = \frac{1}{n} \binom{n}{k} \binom{n}{k-1}$$

5.20 theorems

Burnside lemma  
Tomemos imagenes  $x$  en  $X$  y operaciones ( $g: X \rightarrow X$ ) en  $G$ . Si  $\#g$  es la cantidad de imagenes que son puntos fijos de  $g$ , entonces la cantidad de objetos es  $\frac{(\sum_{g \in G} \#g)}{|G|}$ . Es requisito que  $G$  tenga la operacion identidad, que toda operacion tenga inversa y que todo par de operaciones tenga su combinacion.

Rational root theorem  
Las raices racionales de un polinomio de orden  $n$  con coeficientes enteros  $A[i]$  son de la forma  $p/q$ , donde  $p$  y  $q$  son coprimos,  $p$  es divisor de  $A[0]$  y  $q$  es divisor de  $A[n]$ . Notar que si  $A[0] = 0$ , cero es raiz, se puede dividir el

polinomio por  $x$  y aplica nuevamente el teorema.

Petersens theorem  
Every cubic and bridgeless graph has a perfect matching.

Number of divisors for powers of 10  
(0,1) (1,4) (2,12) (3,32) (4,64) (5,128) (6,240) (7,448) (8,768) (9,1344) (10,2304) (11,4032) (12,6720) (13,10752) (14,17280) (15,26880) (16,41472) (17,64512) (18,103680)

Kirchoff Theorem: Sea  $A$  la matriz de adyacencia del multi-grafo ( $A[u][v]$  indica la cantidad de aristas entre  $u$  y  $v$ ) Sea  $D$  una matriz diagonal tal que  $D[v][v]$  es igual al grado de  $v$  (considerando auto aristas y multi aristas). Sea  $L = A - D$ . Todos los cofactores de  $L$  son iguales y equivalen a la cantidad de Spanning Trees del grafo. Un cofactor  $(i,j)$  de  $L$  es la multiplicación de  $(-1)^{i+j}$  con el determinant de la matriz al quitar la fila  $i$  y la columna  $j$

Prufer Code: Dado un árbol con los nodos indexados: busca la hoja de menor índice, bórrala y anota el índice del nodo al que estaba conectado. Repite el paso anterior  $n-2$  veces. Lo anterior muestra una biyección entre los arreglos de tamaño  $n-2$  con elementos en  $[1, n]$  y los árboles de  $n$  nodos, por lo que hay  $n^{n-2}$  spanning trees en un grafo completo. Corolario: Si tenemos  $k$  componentes de tamaños  $s_1, s_2, \dots, s_k$  entonces podemos hacerlos conexos agregando  $k-1$  aristas entre nodos de  $s_1 s_2 \dots s_k n^{k-2}$  formas

Combinatoria  
Catalan:  $C_{n+1} = \sum (C_i * C_{n-i})$  for  $i \in [0, n]$   
Catalan:  $C_n = \frac{1}{n+1} \binom{2n}{n}$   
Sea  $C_n^k$  las formas de poner  $n+k$  pares de paréntesis, con los primeros  $k$  paréntesis abiertos (esto es, hay  $2n + 2k$  caracteres), se tiene que  $C_n^k = \frac{(2n+k-1)!}{(n-k)! (n+k)!} * C_{n-1}^k$   
Sea  $D_n$  el número de permutaciones sin puntos fijos, entoces  $D_n = (n-1)(D_{n-1} + D_{n-2})$ ,  $D_0 = 1$ ,  $D_1 = 0$

5.21 tonelli shanks

```
ll legendre(ll a, ll p) {
    if (a % p == 0) return 0; if (p == 2) return 1;
    return binexp(a, (p - 1) / 2, p);
}
// sqrt(n) mod p (p must be a prime)
// rnd(a, b) return a random number in [a, b]
ll tonelli_shanks(ll n, ll p) {
    if (n == 0) return 0;
    if (legendre(n, p) != 1) return -1; // no existe
    if (p == 2) return 1;
    ll s = __builtin_ctzll(p - 1);
    ll q = (p - 1LL) >> s, z = rnd(1, p - 1);
    if (s == 1) return binexp(n, (p + 1) / 4LL, p);
    while (legendre(z, p) != p - 1) z = rnd(1, p - 1);
```

```

ll c = binexp(z, q, p), r = binexp(n, (q + 1) / 2, p);
ll t = binexp(n, q, p), m = s;
while (t != 1) {
    ll i = 1, ts = (t * t) % p;
    while (ts != 1) i++, ts = (ts * ts) % p;
    ll b = c;
    repx(_, 0, m - i - 1) b = (b * b) % p;
    r = r*b%p; c = b*b%p; t = t*c%p; m = i;
}
return r;
}

```

## 6 Strings

### 6.1 aho corasick

```

struct Vertex {
    int next[26], go[26];
    int p, link = -1, exit = -1, cnt = -1;
    vector<int> leaf;
    char pch;
    Vertex(int p=-1, char ch=' ') : p(p), pch(ch) {
        rep(i, 26) next[i] = -1, go[i] = -1;
    }
};
vector<Vertex> t(1);
void add(string &s, int id) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf.push_back(id);
}
int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0) t[v].link = 0;
        else t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1) t[v].go[c] = t[v].next[c];
        else t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}

```

```

int next_match(int v){ // Optional
    if(t[v].exit == -1){
        if(t[get_link(v)].leaf.size())t[v].exit=get_link(v);
        else t[v].exit = v==0 ? 0 : next_match(get_link(v));
    }
    return t[v].exit;
}
int cnt_matches(int v){ // Optional
    if(t[v].cnt == -1)
        t[v].cnt = v == 0 ? 0 : t[v].leaf.size() +
        cnt_matches(get_link(v));
    return t[v].cnt;
}

```

### 6.2 debruijn sequence

```

// Given alphabet [0,k) constructs a cyclic string of length
// k^n that contains every length n string as substr.
vector<int> deBruijnSeq(int k, int n) { // Recursive FKM
    if (k == 1) return {0};
    vector<int> seq, aux(n+1);
    function<void(int,int)> gen = [&](int t, int p) {
        if (t > n) { // +lyndon word of len p
            if (n%p == 0) repx(i, 1, p+1) seq.pb(aux[i]);
        } else {
            aux[t] = aux[t-p]; gen(t+1, p);
            while (++aux[t] < k) gen(t+1, t);
        }
    };
    gen(1, 1); return seq;
}

```

### 6.3 hash

```

const int K = 2;
struct Hash{
    const ll MOD[K] = {999727999, 1070777777};
    const ll P = 1777771;
    vector<ll> h[K], p[K];
    Hash(string &s){
        int n = s.size();
        rep(k, K){
            h[k].resize(n+1, 0);
            p[k].resize(n+1, 1);
            repx(i, 1, n+1){
                h[k][i] = (h[k][i-1]*P + s[i-1]) % MOD[k];
                p[k][i] = (p[k][i-1]*P) % MOD[k];
            }
        }
    }
    vector<ll> get(int i, int j){
        vector<ll> r(K);
        rep(k, K){
            r[k] = (h[k][j] - h[k][i]*p[k][j-i]) % MOD[k];
            r[k] = (r[k] + MOD[k]) % MOD[k];
        }
    }
}

```

```

    } return r;
}
};

```

### 6.4 manacher

```

// odd[i]: length of longest palindrome centered at i
// even[i]: ...longest palindrome centered between i and i+1
void manacher(string &s, vector<int> &odd, vector<int> &even){
    string t = "$#";
    for(char c: s) t += c + string("#");
    t += "^";
    int n = t.size();
    vector<int> p(n);
    int l = 1, r = 1;
    repx(i, 1, n-1) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while(t[i - p[i]] == t[i + p[i]]) p[i]++;
        if(i + p[i] > r) l = i - p[i], r = i + p[i];
    }
    repx(i, 2, n-2) {
        if(i%2) even.push_back(p[i]-1);
        else odd.push_back(p[i]-1);
    }
}

```

### 6.5 palindromic tree

```

struct Node { // (*) = Optional
    int len; // length of substring
    int to[26]; // insertion edge for all characters a-z
    int link; // maximun palindromic suffix
    int i; // (*) start index of current Node
    int cnt; // (*) # of occurrences of this substring
    Node(int len, int link=0, int i=0, int cnt=1): len(len),
    link(link), i(i), cnt(cnt) {memset(to, 0, sizeof(to));}
}; struct EerTree { // Palindromic Tree
    vector<Node> t; // tree (max size of tree is n+2)
    int last; // current node
    EerTree(string &s) : last(0) {
        t.emplace_back(-1); t.emplace_back(0); // root 1 & 2
        rep(i, s.size()) add(i, s); // construct tree
        for(int i = t.size()-1; i > 1; i--)
            t[t[i].link].cnt += t[i].cnt;
    }
    void add(int i, string &s){ // vangrind warning:
        int p=last, c=s[i]-'a'; // i-t[p].len-1 = -1
        while(s[i-t[p].len-1] != s[i]) p = t[p].link;
        if(t[p].to[c]){ last = t[p].to[c]; t[last].cnt++; }
        else{
            int q = t[p].link;
            while(s[i-t[q].len-1] != s[i]) q = t[q].link;
            q = max(1, t[q].to[c]);
            last = t[p].to[c] = t.size();
            t.emplace_back(t[p].len + 2, q, i-t[p].len-1);
        }
    }
}

```

```

    }
}
};
void main(){
    string s = "abcbab"; EerTree pt(s); // build EerTree
    repx(i, 2, pt.t.size()){// list all distinct palindromes
        repj(x,pt.t[i].i,pt.t[i].i+pt.t[i].len)cout << s[j];
        cout << " " << pt.t[i].cnt << endl;
    }
}
}

```

## 6.6 prefix function

```

vector<int> prefix_function(string s) {
    int n = s.size();
    vector<int> pi(n);
    repx(i, 1, n) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}

vector<vector<int>> aut;
void compute_automaton(string s) {
    s += '#';
    int n = s.size();
    vector<int> pi = prefix_function(s);
    aut.assign(n, vector<int>(26));
    rep(i, n) {
        rep(c, 26) {
            int j = i;
            while (j > 0 && 'a' + c != s[j])
                j = pi[j-1];
            if ('a' + c == s[j])
                j++;
            aut[i][c] = j;
        }
    }
}

// k = n - pi[n - 1]; if k divides n, then the string can be
// apportioned into blocks of length k otherwise there is no
// effective compression and the answer is n.

```

## 6.7 suffix array

```

// build the suffix array
// suffixes are sorted, with each suffix represented by its
// starting position
vector<int> suffixarray(const string &s) {
    int N = s.size() + 1; // optional: include terminating NUL
    vector<int> p(N), p2(N), c(N), c2(N), cnt(256);

```

```

    rep(i, N) cnt[s[i]] += 1;
    repx(b, 1, 256) cnt[b] += cnt[b - 1];
    rep(i, N) p[--cnt[s[i]]] = i;
    repx(i, 1, N) c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);
    for (int k = 1; k < N; k <= 1) {
        int C = c[p[N - 1]] + 1;
        cnt.assign(C + 1, 0);
        for (int &pi : p) pi = (pi - k + N) % N;
        for (int cl : c) cnt[cl + 1] += 1;
        rep(i, C) cnt[i + 1] += cnt[i];
        rep(i, N) p2[cnt[c[p[i]]]++] = p[i];
        c2[p2[0]] = 0;
        repx(i, 1, N) c2[p2[i]] =
            c2[p2[i - 1]] + (c[p2[i]] != c[p2[i - 1]] ||
                c[(p2[i] + k) % N] != c[(p2[i - 1] + k) % N]);
        swap(c, c2), swap(p, p2);
    }
    p.erase(p.begin()); // optional: erase terminating NUL
    return p;
}

// build the lcp
// `lcp[i]` represents the length of the longest common
// prefix between suffix i and suffix i+1 in the suffix
// array `p`. the last element of `lcp` is zero by convention
vector<int> makelcp(const string &s, const vector<int> &p) {
    int N = p.size(), k = 0;
    vector<int> r(N), lcp(N);
    rep(i, N) r[p[i]] = i;
    rep(i, N) {
        if (r[i] + 1 >= N) { k = 0; continue; }
        int j = p[r[i] + 1];
        while (i + k < N && j + k < N && s[i + k] == s[j + k]) k += 1;
        lcp[r[i]] = k;
        if (k) k -= 1;
    }
    return lcp;
}

// lexicographically compare the suffixes starting from `i`
// and `j`, considering only up to `K` characters.
// `r` is the inverse suffix array, mapping suffix offsets
// to indices. requires an LCP sparse table.
int lcp_cmp(vector<int> &r, Sparse<int> &lcp, int i, int j, int K) {
    if (i == j) return 0;
    int ii = r[i], jj = r[j];
    int l = lcp.query(min(ii, jj), max(ii, jj));
    if (l >= K) return 0;
    return ii < jj ? -1 : 1;
}

```

## 6.8 suffix automaton

```

struct State {int len, link; map<char,int> next; };
State st[2*MAXN]; int sz, last; // clear next!!
void sa_init(){ last=st[0].len=0; sz=1; st[0].link=-1; }
void sa_extend(char c){ // total build O(n log alphabet_size)
    int k = sz++, p; st[k].len = st[last].len + 1;
    for(p=last; p!=-1 && !st[p].next[c]; p=st[p].link)
        st[p].next[c] = k;
    if(p == -1) st[k].link = 0;
    else {
        int q = st[p].next[c];
        if(st[p].len + 1 == st[q].len) st[k].link = q;
        else {
            int w = sz++; st[w].len = st[p].len + 1;
            st[w].next=st[q].next; st[w].link=st[q].link;
            for(; p!=-1 && st[p].next[c]==q; p=st[p].link)
                st[p].next[c] = w;
            st[q].link=st[k].link = w;
        }
    }
    last = k;
} // # states <= 2n-1 && transitions <= 3n-4 (for n > 2)
// Follow link from `last` to 0, nodes on path are terminal
// # matches = # paths from state to a terminal node
// # substrings = # paths from 0 to any node
// # substrings = sum of (len - len(link)) for all nodes

```

## 6.9 z function

```

// i-th element is equal to the greatest number of
// characters starting from the position i that coincide
// with the first characters of s
vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for(int i = 1; i < n; i++) {
        if(i < r) z[i] = min(r - i, z[i - l]);
        while(i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}

```