Team Notebook

October 18, 2023

Contents

1 Strings

1.1 Manacher

```
// odd[i]: length of longest palindrome centered at i
// even[i]: ...longest palindrome centered between i and i+1
void manacher(string &s,vector<int> &odd,vector<int> &even){
    string t = "$#";
    for(char c: s) t += c + string("#");
    t += "^";
    int n = t.size();
    vector<int> p(n);
    int l = 1, r = 1;
    repx(i, 1, n-1) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while(t[i - p[i]] == t[i + p[i]]) p[i]++;
        if(i + p[i] > r) l = i - p[i], r = i + p[i];
    }
    repx(i, 2, n-2) {
        if(i%2) even.push_back(p[i]-1);
        else odd.push_back(p[i]-1);
    }
}
```

1.2 aho-corasick

```
struct Vertex {
    int next[26], go[26];
    int p, link = -1, exit = -1, cnt = -1;
    vector<int> leaf;
    char pch;
    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        rep(i, 26) next[i] = -1, go[i] = -1;
    }
};
vector<Vertex> t(1);
void add(string &s, int id) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf.push_back(id);
}
int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0) t[v].link = 0;
        else t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1) t[v].go[c] = t[v].next[c];
        else t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}
int next_match(int v){ // Optional
    if(t[v].exit == -1){
        if(t[get_link(v)].leaf.size())t[v].exit=get_link(v);
        else t[v].exit = v==0 ? 0 : next_match(get_link(v));
    }
    return t[v].exit;
}
int cnt_matches(int v){ // Optional
    if(t[v].cnt == -1)
        t[v].cnt = v == 0 ? 0 : t[v].leaf.size() +
            cnt_matches(get_link(v));
    return t[v].cnt;
}
```

1.3 hash

```
struct Hash{
    const int K = 2;
    const ll MOD[K] = {999727999, 1070777777};
    const ll P = 1777771;
    vector<ll> h[K], p[K];
    Hash(string &s){
        int n = s.size();
        rep(k, K){
            h[k].resize(n+1, 0);
            p[k].resize(n+1, 1);
            repx(i, 1, n+1){
                h[k][i] = (h[k][i-1]*P + s[i-1]) % MOD[k];
                p[k][i] = (p[k][i-1]*P) % MOD[k];
            }
        }
    }
    vector<ll> get(int i, int j){
        vector<ll> r(K);
        rep(k, K){
            r[k] = (h[k][j] - h[k][i]*p[k][j-i]) % MOD[k];
            r[k] = (r[k] + MOD[k]) % MOD[k];
        } return r;
    }
};
```

1.4 hash2d

```
using Hash = pair<ll, int>;

struct Block {
    int x0, y0, x1, y1;
};

struct Hash2d {
    ll HMOD;
    int W, H;
    vector<int> h;
    vector<int> p;

    Hash2d() {}
    Hash2d(const string &s, int W_, int H_, ll HMOD_ =
         1000003931)
        : W(W_ + 1), H(H_ + 1), HMOD(HMOD_) {
        static const ll P =
            chrono::steady_clock::now().time_since_epoch().
                count() % (1 << 29);
        p.resize(W * H);
        p[0] = 1;
        rep(i, W * H - 1) p[i + 1] = p[i] * P % HMOD;
        h.assign(W * H, 0);
        repx(y, 1, H) repx(x, 1, W) {
            ll c = (ll)s[(y - 1) * (W - 1) + x - 1] * p[y * W
                 + x] % HMOD;
            h[y * W + x] = (HMOD + h[y * W + x - 1] + h[(y -
                1) * W + x] -
                        h[(y - 1) * W + x - 1] + c) %
                       HMOD;
        }
    }

    bool isout(Block s) {
        return s.x0 < 0 || s.x0 >= W || s.x1 < 0 || s.x1 >= W
             || s.y0 < 0 ||
               s.y0 >= H || s.y1 < 0 || s.y1 >= H;
    }
```

```
    Hash get(Block s) {
        return {(2 * HMOD + h[s.y1 * W + s.x1] - h[s.y1 * W +
              s.x0] -
                h[s.y0 * W + s.x1] + h[s.y0 * W + s.x0]) %
                   HMOD,
               s.y0 * W + s.x0};
    }

    bool cmp(Hash x0, Hash x1) {
        int d = x0.second - x1.second;
        ll &lo = d < 0 ? x0.first : x1.first;
        lo = lo * p[abs(d)] % HMOD;
        return x0.first == x1.first;
    }
};

struct Hash2dM {
    int N;
    vector<Hash2d> sub;

    Hash2dM() {}
    Hash2dM(const string &s, int W, int H, const vector<ll> &
         mods)
        : N(mods.size()), sub(N) {
        rep(i, N) sub[i] = Hash2d(s, W, H, mods[i]);
    }

    bool isout(Block s) { return sub[0].isout(s); }

    vector<Hash> get(Block s) {
        vector<Hash> hs(N);
        rep(i, N) hs[i] = sub[i].get(s);
        return hs;
    }

    bool cmp(const vector<Hash> &x0, const vector<Hash> &x1)
         {
        rep(i, N) if (!sub[i].cmp(x0[i], x1[i])) return false
             ;
        return true;
    }

    bool cmp(Block s0, Block s1) {
        rep(i, N) if (!sub[i].cmp(sub[i].get(s0), sub[i].get(
             s1))) return false;
        return true;
    }
};
```

```
const vector<ll> HMOD = {1000002649, 1000000933, 1000003787,
      1000002173};
```

1.5 palindromic-tree

```
struct Node {  // (*) = Optional
 int len;    // length of substring
 int to[26]; // insertion edge for all characters a-z
 int link;   // maximun palindromic suffix
 int i;      // (*) start index of current Node
    int cnt;    // (*) # of occurrences of this substring
    Node(int len, int link=0, int i=0, int cnt=1): len(len),
    link(link), i(i), cnt(cnt) {memset(to, 0, sizeof(to));}
};
struct EerTree {  // Palindromic Tree
    vector<Node> t; // tree (max size of tree is n+2)
    int last;      // current node
    EerTree(string &s) : last(0) {
        t.emplace_back(-1); t.emplace_back(0); // root 1 & 2
        rep(i, s.size()) add(i, s); // construct tree
        for(int i = t.size()-1; i > 1; i--)
            t[t[i].link].cnt += t[i].cnt;
    }
    void add(int i, string &s){
        int p=last, c=s[i]-'a';
        while(s[i-t[p].len-1] != s[i]) p = t[p].link;
        if(t[p].to[c]){ last = t[p].to[c]; t[last].cnt++; }
        else{
            int q = t[p].link;
            while(s[i-t[q].len-1] != s[i]) q = t[q].link;
            q = max(1, t[q].to[c]);
            last = t[p].to[c] = t.size();
            t.emplace_back(t[p].len + 2, q, i-t[p].len-1);
        }
    }
};
void main(){
 string s = "abcbab"; EerTree pt(s); // build EerTree
 repx(i, 2, pt.t.size()){// list all distinct palindromes
  repx(j,pt.t[i].i,pt.t[i].i+pt.t[i].len)cout << s[j];
  cout << " " << pt.t[i].cnt << endl;
 }
}
```

1.6 prefix-function

```
vector<int> prefix_function(string s) {
    int n = s.size();
    vector<int> pi(n);
    repx(i, 1, n) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
vector<vector<int>> aut;
void compute_automaton(string s) {
    s += '#';
    int n = s.size();
    vector<int> pi = prefix_function(s);
    aut.assign(n, vector<int>(26));
    rep(i, n) {
        rep(c, 26) {
            int j = i;
            while (j > 0 && 'a' + c != s[j])
                j = pi[j-1];
            if ('a' + c == s[j])
                j++;
            aut[i][c] = j;
        }
    }
}
// k = n - pi[n - 1]; if k divides n, then the string can be
// aprtitioned into blocks of length k otherwise there is no
// effective compression and the answer is n.
```

1.7 suffix-array

```
// build the suffix array
// suffixes are sorted, with each suffix represented by its
// starting position
vector<int> suffixarray(const string &s) {
    int N = s.size() + 1;//optional: include terminating NUL
    vector<int> p(N), p2(N), c(N), c2(N), cnt(256);
    rep(i, N) cnt[s[i]] += 1;
    repx(b, 1, 256) cnt[b] += cnt[b - 1];
    rep(i, N) p[--cnt[s[i]]] = i;
    repx(i, 1, N) c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i -
          1]]);
    for (int k = 1; k < N; k <<= 1) {
        int C = c[p[N - 1]] + 1;
        cnt.assign(C + 1, 0);
```

```
        for (int &pi : p) pi = (pi - k + N) % N;
        for (int cl : c) cnt[cl + 1] += 1;
        rep(i, C) cnt[i + 1] += cnt[i];
        rep(i, N) p2[cnt[c[p[i]]]++] = p[i];
        c2[p2[0]] = 0;
        repx(i, 1, N) c2[p2[i]] =
            c2[p2[i - 1]] + (c[p2[i]] != c[p2[i - 1]] ||
                             c[(p2[i] + k) % N] != c[(p2[i - 1]
                                   + k) % N]);
        swap(c, c2), swap(p, p2);
    }
    p.erase(p.begin()); // optional: erase terminating NUL
    return p;
}
// build the lcp
// `lcp[i]` represents the length of the longest common
// prefix between suffix i and suffix i+1 in the suffix
//array `p`. the last element of `lcp` is zero by convention
vector<int> makelcp(const string &s, const vector<int> &p) {
    int N = p.size(), k = 0;
    vector<int> r(N), lcp(N);
    rep(i, N) r[p[i]] = i;
    rep(i, N) {
        if (r[i] + 1 >= N) { k = 0; continue; }
        int j = p[r[i] + 1];
        while (i + k < N && j + k < N && s[i + k] == s[j + k
            ]) k += 1;
        lcp[r[i]] = k;
        if (k) k -= 1;
    }
    return lcp;
}
// lexicographically compare the suffixes starting from `i`
// and `j`, considering only up to `K` characters.
// `r` is the inverse suffix array, mapping suffix offsets
// to indices. requires an LCP sparse table.
int lcp_cmp(vector<int> &r, Sparse<int> &lcp, int i, int j,
     int K) {
    if (i == j) return 0;
    int ii = r[i], jj = r[j];
    int l = lcp.query(min(ii, jj), max(ii, jj));
    if (l >= K) return 0;
    return ii < jj ? -1 : 1;
}
```

1.8 suffix-automaton

```
struct SuffixAutomaton {
    vector<map<char,int>> edges;
    vector<int> link, len, cnt, paths, pos;
    vector<bool> terminal;
    int last; // idx of the eq. class of the whole string
    SuffixAutomaton(string s) :last(0) {
        edges.push_back({});
        link.push_back(-1);
        len.push_back(0);
        rep(i, s.size()) {
            edges.push_back({});
            len.push_back(i+1);
            link.push_back(0);
            int r = len.size() - 1, p = last;
            while(p >= 0 && !edges[p].count(s[i])) {
                edges[p][s[i]] = r;
                p = link[p];
            }
            if(p != -1) {
                int q = edges[p][s[i]];
                if(len[p] + 1 == len[q]) link[r] = q;
                else {
                    edges.push_back(edges[q]);
                    len.push_back(len[p] + 1);
                    link.push_back(link[q]);
                    int qq= link[q] = link[r] =len.size()-1;
                    while(p >= 0 && edges[p][s[i]] == q){
                        edges[p][s[i]] = qq;
                        p = link[p];
                    }
                }
            }
            last = r;
        } /* ------ Optional ------ */
        terminal.assign(len.size(), 0);
        for(int p = last; p > 0; p = link[p]) terminal[p]=1;
        cnt.assign(len.size(), -1); cnt_matches(0);
        //precompute # of paths (substr) starting from state
        paths.assign(len.size(), -1); cnt_paths(0);
        pos.assign(len.size(), -1); get_pos(0);
    }
    int cnt_matches(int state) {
        if(cnt[state] != -1) return cnt[state];
        int ans = terminal[state];
        for(auto edge : edges[state])
            ans += cnt_matches(edge.second);
        return cnt[state] = ans;
    }
    int cnt_paths(int state) {
        if(paths[state] != -1) return paths[state];
        int ans = state != 0;       // without repetitions
    //  int ans = state == 0 ? 0 : cnt[state]; // with rep.
        for(auto edge : edges[state])
            ans += cnt_paths(edge.second);
        return paths[state] = ans;
    }
    int get_pos(int state) { // gets first pos
        if(pos[state] != -1) return pos[state];
        int ans = 0;                    // max->first_pos
 // int ans = terminal[state] ? 0 : 1e9;// min->last_pos
        for(auto edge : edges[state])
            ans = max(ans, get_pos(edge.second)+1); //or min
        return pos[state] = ans;
    }
    string get_k_substring(int k) { // 0-indexed
        string ans; int state = 0;
        while(1){
            int curr = state != 0;   // without repetition
        //  int curr = state == 0 ? 0 : cnt[state]; // with
            if(curr > k) return ans;
            k -= curr;
            for(auto edge : edges[state]) {
                if(paths[edge.second] <= k) {
                    k -= paths[edge.second];
                } else {
                    ans += edge.first;
                    state = edge.second;
                    break;
                }
            }
        }
    }
};
```

1.9 z-function

```
// i-th element is equal to the greatest number of
// characters starting from the position i that coincide
// with the first characters of s
vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for(int i = 1; i < n; i++) {
        if(i < r) z[i] = min(r - i, z[i - l]);
        while(i + z[i] < n && s[z[i]] == s[i + z[i]])z[i]++;
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
```

```
    return z;
}
```

2 common

```
#pragma GCC optimize("Ofast")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt")
#pragma GCC target("avx,avx2,f16c,fma,sse3,ssse3,sse4.1,sse4
    .2")
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define repx(i, a, b) for (int i = a; i < b; i++)
#define rep(i, n) repx(i, 0, n)
#define invrepx(i, a, b) for (int i = b - 1; i >= a; i--)
#define invrep(i, n) invrepx(i, 0, n)
```

3 dp

3.1 convex-hull-trick

```
struct Line {
    mutable ll a, b, c;

    bool operator<(Line r) const { return a < r.a; }
    bool operator<(ll x) const { return c < x; }
};
// dynamically insert `a*x + b` lines and query for maximum
// at any x all operations have complexity O(log N)
struct LineContainer : multiset<Line, less<>> {
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator x, iterator y) {
        if (y == end()) return x->c = INF, 0;
        if (x->a == y->a) x->c = x->b > y->b ? INF : -INF;
        else x->c = div(y->b - x->b, x->a - y->a);
        return x->c >= y->c;
    }

    void add(ll a, ll b) {
        // a *= -1, b *= -1 // for min
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase
            (y));
        while ((y = x) != begin() && (--x)->c >= y->c) isect(
            x, erase(y));
    }

    ll query(ll x) {
        if (empty()) return -INF; // INF for min
        auto l = *lower_bound(x);
        return l.a * x + l.b;
        // return -l.a * x - l.b; // for min
    }
};
```

3.2 divide-and-conquer

```
// for every index i assign an optimal index j, such that
    cost(i, j) is
// minimal for every i. the property that if i2 >= i1 then
    j2 >= j1 is
// exploited (monotonic condition).
// calculate optimal index for all indices in range [l, r)
    knowing that
// the optimal index for every index in this range is within
    [optl, optr).
// time: O(N log N)
void calc(vector<int> &opt, int l, int r, int optl, int optr
    ) {
    if (l == r) return;
    int i = (l + r) / 2;
    ll optc = INF;
    int optj;
    repx(j, optl, optr) {
        ll c = i + j; // cost(i, j)
        if (c < optc) optc = c, optj = j;
    }
    opt[i] = optj;
    calc(opt, l, i, optl, optj + 1);
    calc(opt, i + 1, r, optj, optr);
}
```

4 geo2d

4.1 circle

```
struct C {
    P o; T r;

    C(P o, T r) : o(o), r(r) {}
    C() : C(P(), T()) {}

    // intersects the circle with a line, assuming they
        intersect
    // results are sorted with respect to the direction of
        the line
    pair<P, P> line_inter(L l) const {
        P c = l.closest_to(o);
        T c2 = (c - o).magsq();
        P e = sqrt(max(r * r - c2, T())) * l.d.unit();
        return {c - e, c + e};
    }

    // checks whether the given line collides with the circle
    // negative: 2 intersections
    // zero: 1 intersection
    // positive: 0 intersections
    T line_collide(L l) const {
        T c2 = (l.closest_to(o) - o).magsq();
        return c2 - r * r;
    }

    // calculates the two intersections between two circles
    // the circles must intersect in one or two points!
    pair<P, P> inter(C h) const {
        P d = h.o - o;
        T c = (r * r - h.r * h.r) / d.magsq();
        return h.line_inter({(1 + c) / 2 * d, d.rot()});
    }

    // check if the given circles intersect
    bool collide(C h) const {
        return (h.o - o).magsq() <= (h.r + r) * (h.r + r);
    }

    // get one of the two tangents that cross through the
        point
    // the point must not be inside the circle
    // a = -1: cw (relative to the circle) tangent
    // a = 1: ccw (relative to the circle) tangent
    P point_tangent(P p, T a) const {
        T c = r * r / p.magsq();
        return o + c * (p - o) - a * sqrt(c * (1 - c)) * (p -
            o).rot();
    }

    // get one of the 4 tangents between the two circles
```

```
    // a = 1: exterior tangents
    // a = -1: interior tangents (requires no area overlap)
    // b = 1: ccw tangent
    // b = -1: cw tangent
    // the line origin is on this circumference, and the
        direction
    // is a unit vector towards the other circle
    L tangent(C c, T a, T b) const {
        T dr = a * r - c.r;
        P d = c.o - o;
        P n = (d * dr + b * d.rot() * sqrt(d.magsq() - dr *
            dr)).unit();
        return {o + n * r, -b * n.rot()};
    }

    // find the circumcircle of the given **non-degenerate**
        triangle
    static C thru_points(P a, P b, P c) {
        L l((a + b) / 2, (b - a).rot());
        P p = l.intersection(L((a + c) / 2, (c - a).rot()));
        return {p, (p - a).mag()};
    }

    // find the two circles that go through the given point,
        are tangent
    // to the given line and have radius ‘r‘
    // the point-line distance must be at most ‘r‘!
    // the circles are sorted in the direction of the line
    static pair<C, C> thru_point_line_r(P a, L t, T r) {
        P d = t.d.rot().unit();
        if (d * (a - t.o) < 0) d = -d;
        auto p = C(a, r).line_inter({t.o + d * r, t.d});
        return {{p.first, r}, {p.second, r}};
    }

    // find the two circles that go through the given points
        and have
    // radius ‘r‘
    // the circles are sorted by angle with respect to the
        first point
    // the points must be at most at distance ‘r‘!
    static pair<C, C> thru_points_r(P a, P b, T r) {
        auto p = C(a, r).line_inter({(a + b) / 2, (b - a).rot
            ()});
        return {{p.first, r}, {p.second, r}};
    }
};
```

4.2 convex-hull

```
// ccw order, excludes collinear points by default
// adding collinears duplicates points
vector<P> chull(vector<P> p) {
    if (p.size() < 3) return p;
    vector<P> r; int k = 0;
    sort(p.begin(), p.end(), [](P a, P b) {
        return a.x != b.x ? a.x < b.x : a.y < b.y; });
    for (P q : p) { // lower hull
        while (k >= 2 && r[k - 1].left(r[k - 2], q) >= 0)
            r.pop_back(), k--; // >= to > to add collinears
        r.push_back(q), k++;
    }
    r.pop_back(), k--; int m = k;
    for (int i = p.size() - 1; i >= 0; --i) { // upper hull
        while (k >= m+2 && r[k-1].left(r[k-2], p[i]) >= 0)
            r.pop_back(), k--; // >= to > to add collinears
        r.push_back(p[i]), k++;
    }
    r.pop_back(); return r;
}
```

4.3 delaunay

```
typedef __int128_t lll; // if on a 64-bit platform

struct Q {
    Q *rot, *o; P p = {INF, INF}; bool mark;
    P &F() { return r()->p; }
    Q *&r() { return rot->rot; }
    Q *prev() { return rot->o->rot; }
    Q *next() { return r()->prev(); }
};

T cross(P a, P b, P c) { return (b - a) % (c - a); }

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.magsq(), A = a.magsq() - p2,
        B = b.magsq() - p2, C = c.magsq() - p2;
    return cross(p, a, b) * C + cross(p, b, c) * A + cross(p,
          c, a) * B > 0;
}

Q *makeEdge(Q *&H, P orig, P dest) {
    Q *r = H ? H : new Q{new Q{new Q{new Q{0}}}};
    H = r->o; r->r()->r() = r;
    repx(i, 0, 4) r = r->rot, r->p = {INF, INF},
        r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}

void splice(Q *a, Q *b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q *connect(Q *&H, Q *a, Q *b) {
    Q *q = makeEdge(H, a->F(), b->p);
    splice(q, a->next()); splice(q->r(), b); return q;
}

pair<Q *, Q *> rec(Q *&H, const vector<P> &s) {
    if (s.size() <= 3) {
        Q *a = makeEdge(H, s[0], s[1]), *b = makeEdge(H, s
            [1], s.back());
        if (s.size() == 2) return {a, a->r()}; splice(a->r(),
             b);
        auto side = cross(s[0], s[1], s[2]);
        Q *c = side ? connect(H, b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r()
            };
    }

#define J(e) e->F(), e->p
#define valid(e) (cross(e->F(), J(base)) > 0)
    Q *A, *B, *ra, *rb; int half = s.size() / 2;
    tie(ra, A) = rec(H, {s.begin(), s.end() - half});
    tie(B, rb) = rec(H, {s.begin() + s.size() - half, s.end()
        });
    while ((cross(B->p, J(A)) < 0 && (A = A->next())) ||
           (cross(A->p, J(B)) > 0 && (B = B->r()->o)));
    Q *base = connect(H, B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q *e = init->dir; \
    if (valid(e)) while (circ(e->dir->F(), J(base), e->F()))
        { \
          Q *t = e->dir; splice(e, e->prev()); \
          splice(e->r(), e->r()->prev()); e->o = H; H = e;
              e = t; \
        }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(J(RC), J(LC))))
            base = connect(H, RC, base->r());
        else base = connect(H, base->r(), LC->r());
```

```
    }
    return {ra, rb};
#undef J
#undef valid
#undef DEL
}

// there must be no duplicate points
// returns no triangles in the case of all collinear points
// produces counter-clockwise triangles ordered in triples
// maximizes the minimum angle across all triangulations
// the euclidean mst is a subset of these edges
// O(N log N)
vector<P> triangulate(vector<P> pts) {
    sort(pts.begin(), pts.end(), [](P a, P b) {
        return make_pair(a.x, a.y) < make_pair(b.x, b.y);
    });
    assert(unique(pts.begin(), pts.end()) == pts.end());
    if (pts.size() < 2) return {};
    Q *H = 0; Q *e = rec(H, pts).first;
    vector<Q *> q = {e}; int qi = 0;
    while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD                             \
    {                                   \
        Q *c = e;                       \
        do {                            \
            c->mark = 1; pts.push_back(c->p); \
            q.push_back(c->r()); c = c->next(); \
        } while (c != e);               \
    }
    ADD;
    pts.clear();
    while (qi < (int)q.size()) if (!(e = q[qi++])->mark) ADD;
    return pts;
#undef ADD
}
```

4.4 halfplane-intersect

```
// obtain the convex polygon that results from intersecting
    the given list
// of halfplanes, represented as lines that allow their left
    side
// assumes the halfplane intersection is bounded
vector<P> halfplane_intersect(vector<L> &H) {
    L bb(P(-INF, -INF), P(INF, 0));
    rep(k, 4) H.push_back(bb), bb.o = bb.o.rot(), bb.d = bb.d
        .rot();
    sort(begin(H), end(H), [](L a, L b) { return a.d.angcmp(b
        .d) < 0; });
    deque<L> q; int n = 0;
    rep(i, H.size()) {
        while (n >= 2 && H[i].side(q[n - 1].intersection(q[n
            - 2])) > 0)
            q.pop_back(), n--;
        while (n >= 2 && H[i].side(q[0].intersection(q[1])) >
            0)
            q.pop_front(), n--;
        if (n > 0 && H[i].parallel(q[n - 1])) {
            if (H[i].d * q[n - 1].d < 0) return {};
            if (H[i].side(q[n - 1].o) > 0) q.pop_back(), n--;
            else continue;
        }
        q.push_back(H[i]), n++;
    }

    while (n >= 3 && q[0].side(q[n - 1].intersection(q[n -
        2])) > 0)
        q.pop_back(), n--;
    while (n >= 3 && q[n - 1].side(q[0].intersection(q[1])) >
        0)
        q.pop_front(), n--;
    if (n < 3) return {};

    vector<P> ps(n);
    rep(i, n) ps[i] = q[i].intersection(q[(i + 1) % n]);
    return ps;
}
```

4.5 line

```
// a segment or an infinite line
// does not handle point segments correctly!
struct L {
    P o, d;
    L() : o(), d() {}
    L(P o, P d) : o(o), d(d) {}

    L(P ab, T c) : d(ab.rot()), o(ab * -c / ab.magsq()) {}
    pair<P, T> line_eq() { return {-d.rot(), d.rot() * o}; }

    // returns a number indicating which side of the line the
        point is in
    // negative: left, positive: right
    T side(P r) const { return (r - o) % d; }

    // returns the intersection coefficient
    // in the range [0, d % r.d]
    // if d % r.d is zero, the lines are parallel
    T inter(L r) const { return (r.o - o) % r.d; }

    // get the single intersection point
    // lines must not be parallel
    P intersection(L r) const { return o + d * inter(r) / (d
        % r.d); }

    // check if lines are parallel
    bool parallel(L r) const { return abs(d % r.d) <= EPS; }

    // check if segments intersect
    bool seg_collide(L r) const {
        T z = d % r.d;
        if (abs(z) <= EPS) {
            if (abs(side(r.o)) > EPS) return false;
            T s = (r.o - o) * d, e = s + r.d * d;
            if (s > e) swap(s, e);
            return s <= d * d + EPS && e >= -EPS;
        }
        T s = inter(r), t = -r.inter(*this);
        if (z < 0) s = -s, t = -t, z = -z;
        return s >= -EPS && s <= z + EPS && t >= -EPS && t <=
            z + EPS;
    }

    // full segment intersection
    // produces a point segment if the intersection is a
        point
    // however it **does not** handle point segments as input
        !
    bool seg_inter(L r, L *out) const {
        T z = d % r.d;
        if (abs(z) <= EPS) {
            if (abs(side(r.o)) > EPS) return false;
            if (r.d * d < 0) r = {r.o + r.d, -r.d};
            P s = o * d < r.o * d ? r.o : o;
            P e = (o + d) * d < (r.o + r.d) * d ? o + d : r.o
                + r.d;
            if (s * d > e * d) return false;
            return *out = {s, e - s}, true;
        }
        T s = inter(r), t = -r.inter(*this);
        if (z < 0) s = -s, t = -t, z = -z;
        if (s >= -EPS && s <= z + EPS && t >= -EPS && t <= z
            + EPS)
            return *out = {o + d * s / z, {0, 0}}, true;
        return false;
    }
```

```cpp
    // check if the given point is on the segment
    bool point_on_seg(P r) const {
        if (abs(side(r)) > EPS) return false;
        if ((r - o) * d < -EPS) return false;
        if ((r - o - d) * d > EPS) return false;
        return true;
    }

    // get the point in this line that is closest to a given
        point
    P closest_to(P r) const {
        P dr = d.rot(); return r + (o - r) * dr * dr / d.
            magsq();
    }
};
```

4.6 minkowski

```cpp
void reorder_polygon(vector<P> &ps) {
    int pos = 0;
    repx(i, 1, (int)ps.size()) {
        if (ps[i].y < ps[pos].y || (ps[i].y == ps[pos].y &&
            ps[i].x < ps[pos].x))
            pos = i;
    }
    rotate(ps.begin(), ps.begin() + pos, ps.end());
}
vector<P> minkowski(vector<P> ps, vector<P> qs) {
    // the first vertex must be the lowest
    reorder_polygon(ps); reorder_polygon(qs);
    ps.push_back(ps[0]); ps.push_back(ps[1]);
    qs.push_back(qs[0]); qs.push_back(qs[1]);
    vector<P> result; int i = 0, j = 0;
    while (i < ps.size() - 2 || j < qs.size() - 2) {
        result.push_back(ps[i] + qs[j]);
        auto z = (ps[i + 1] - ps[i]) % (qs[j + 1] - qs[j]);
        if (z >= 0 && i < ps.size() - 2) ++i;
        if (z <= 0 && j < qs.size() - 2) ++j;
    }
    return result;
}
```

4.7 point

```cpp
struct P {
    T x, y;
    P(T x, T y) : x(x), y(y) {}
    P() : P(0, 0) {}

    friend ostream &operator<<(ostream &s, const P &r) {
        return s << r.x << " " << r.y;
    }
    friend istream &operator>>(istream &s, P &r) { return s
        >> r.x >> r.y; }

    P operator+(P r) const { return {x + r.x, y + r.y}; }
    P operator-(P r) const { return {x - r.x, y - r.y}; }
    P operator*(T r) const { return {x * r, y * r}; }
    P operator/(T r) const { return {x / r, y / r}; }
    P operator-() const { return {-x, -y}; }
    friend P operator*(T l, P r) { return {l * r.x, l * r.y};
        }

    P rot() const { return {-y, x}; }
    T operator*(P r) const { return x * r.x + y * r.y; }
    T operator%(P r) const { return rot() * r; }
    T left(P a, P b) { return (b - a) % (*this - a); }

    T magsq() const { return x * x + y * y; }
    T mag() const { return sqrt(magsq()); }
    P unit() const { return *this / mag(); }

    bool half() const { return abs(y) <= EPS && x < -EPS || y
        < -EPS; }
    T angcmp(P r) const { // like strcmp(this, r)
        int h = (int)half() - r.half();
        return h ? h : r % *this;
    }
    T angcmp_rel(P a, P b) { // like strcmp(a, b)
        P z = *this;
        int h = z % a <= 0 && z * a < 0 || z % a < 0;
        h -= z % b <= 0 && z * b < 0 || z % b < 0;
        return h ? h : b % a;
    }

    bool operator==(P r) const { return abs(x - r.x) <= EPS
        && abs(y - r.y) <= EPS; }

    double angle() const { return atan2(y, x); }
    static P from_angle(double a) { return {cos(a), sin(a)};
        }
};
```

4.8 polygon

```cpp
// get TWICE the area of a simple polygon in ccw order
T area2(const vector<P> &p) {
    int n = p.size(); T a = 0;
    rep(i, n) a += (p[i] - p[0]) % (p[(i + 1) % n] - p[i]);
    return a;
}

// checks whether a point is inside a ccw simple polygon
// returns 1 if inside, 0 if on border, -1 if outside
int in_poly(const vector<P> &p, P q) {
    int w = 0;
    rep(i, p.size()) {
        P a = p[i], b = p[(i + 1) % p.size()];
        T k = (b - a) % (q - a);
        T u = a.y - q.y, v = b.y - q.y;
        if (k > 0 && u < 0 && v >= 0) w++;
        if (k < 0 && v < 0 && u >= 0) w--;
        if (k == 0 && (q - a) * (q - b) <= 0) return 0;
    }
    return w ? 1 : -1;
}

// check if point in ccw convex polygon, O(log n)
// + if inside, 0 if on border, - if outside
T in_convex(const vector<P> &p, P q) {
    int l = 1, h = p.size() - 2; assert(p.size() >= 3);
    while (l != h) { // collinear points are unsupported!
        int m = (l + h + 1) / 2;
        if (q.left(p[0], p[m]) >= 0) l = m;
        else h = m - 1;
    }
    T in = min(q.left(p[0], p[1]), q.left(p.back(), p[0]));
    return min(in, q.left(p[l], p[l + 1]));
}

int extremal(const vector<P> &p, P d) {
    int n = p.size(), l = 0, r = n - 1; assert(n);
    P e0 = (p[n - 1] - p[0]).rot();
    while (l < r) { // polygon must be convex
        int m = (l + r + 1) / 2;
        P e = (p[(m + n - 1) % n] - p[m]).rot();
        if (e0.angcmp_rel(d, e) < 0) r = m - 1;
        else l = m;
    }
    return l;
}

// square dist of most distant points of a ccw convex
// polygon with NO COLLINEAR POINTS
T callipers(const vector<P> &p) {
```

```
    int n = p.size();
    T r = 0;
    for (int i = 0, j = n < 2 ? 0 : 1; i < j; i++) {
        for (;; j = (j + 1) % n) {
            r = max(r, (p[i] - p[j]).magsq());
            if ((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p
                [j]) <= EPS) break;
        }
    }
    return r;
}

P centroid(const vector<P> &p) { // (barycenter)
    P r(0, 0); T t = 0; int n = p.size();
    rep(i, n) {
        r += (p[i] + p[(i+1)%n]) * (p[i] % p[(i+1)%n]);
        t += p[i] % p[(i+1)%n];
    }
    return r / t / 3;
}

// classify collision of a ray inside a ccw polygon vertex.
// ray is (o, d), vertex is b, previous vertex is a, next is
    c.
pair<bool, bool> inner_collide(P o, P d, P a, P b, P c) {
    T p = (a - o) % d;      // side of previous
    T n = (c - o) % d;      // side of next
    T v = (c - b) % (b - a); // is vertex convex?
    return {v > 0 ? n < 0 || (n == 0 && p < 0) : p > 0 || n <
        0,
        v > 0 ? p > 0 || (p == 0 && n > 0) : p > 0 || n <
            0};
}
```

4.9 sweep

```
#include "point.cpp"

// iterate over all pairs of points
// 'op' is called with all ordered pairs of different
    indices '(i, j)'
// additionally, the 'ps' vector is kept sorted by signed
    distance
// to the line formed by 'i' and 'j'
// for example, if the vector from 'i' to 'j' is pointing
    right,
// the 'ps' vector is sorted from smallest 'y' to largest 'y
    '
// note that, because the 'ps' vector is sorted by signed
    distance,
// 'j' is always equal to 'i + 1'
// this means that the amount of points to the left of the
    line is always 'N - i'
template <class OP>
void all_pair_points(vector<P> &ps, OP op) {
    int N = ps.size();
    sort(ps.begin(), ps.end(), [](P a, P b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    vector<pair<int, int>> ss;
    rep(i, N) rep(j, N) if (i != j) ss.push_back({i, j});
    stable_sort(ss.begin(), ss.end(), [&](auto a, auto b) {
        return (ps[a.second] - ps[a.first]).angle_lt(ps[b.
            second] - ps[b.first]);
    });
    vector<int> p(N); rep(i, N) p[i] = i;
    for (auto [i, j] : ss)
        { op(p[i], p[j]); swap(ps[p[i]], ps[p[j]]); swap(p[i
            ], p[j]); }
}
```

4.10 theorems

```
// Pick's theorem
//     Simple polygon with integer vertices:
//     A = I + B / 2 - 1
//     A: Area of the polygon
//     I: Integer points strictly inside the polygon
//     B: Integer points on the boundary of the polygon
```

5 graph

5.1 bellman-ford

```
struct Edge { int u, v; ll w; };

// find distance from source node to all nodes.
// supports negative edge weights.
// returns true if a negative cycle is detected.
//
// time: O(V E)
bool bellman_ford(int N, int s, vector<Edge> &E, vector<ll>
    &D, vector<int> &P) {
    P.assign(N, -1), D.assign(N, INF), D[s] = 0;
    rep(i, N - 1) {
        bool f = true;
        rep(ei, E.size()) {
            auto &e = E[ei];
            ll n = D[e.u] + e.w;
            if (D[e.u] < INF && n < D[e.v])
                D[e.v] = n, P[e.v] = ei, f = false;
        }
        if (f) return false;
    }
    return true;
}
```

5.2 blossom

```
vector<int> g[MAXN];int n,m,mt[MAXN],qh,qt,q[MAXN],ft[MAXN],
bs[MAXN];bool inq[MAXN],inb[MAXN],inp[MAXN];int lca(int root
,int x,int y){memset(inp,0,sizeof(inp));while(1){inp[x=bs[x]
]=true;if(x==root)break;x=ft[mt[x]];}while(1){if(inp[y=bs[y]
])return y;else y=ft[mt[y]];}}void mark(int z,int x){while(
bs[x]!=z){int y=mt[x];inb[bs[x]]=inb[bs[y]]=true;x=ft[y];if(
bs[x]!=z)ft[x]=y;}}void contr(int s,int x,int y){int z=lca(s
,x,y);memset(inb,0,sizeof(inb));mark(z,x);mark(z,y);if(bs[x]
!=z)ft[x]=y;if(bs[y]!=z)ft[y]=x;rep(x,n)if(inb[bs[x]]){bs[x]
=z;if(!inq[x])inq[q[++qt]=x]=true;}}int findp(int s){memset(
inq,0,sizeof(inq));memset(ft,-1,sizeof(ft));rep(i,n)bs[i]=i;
inq[q[qh=qt=0]=s]=true;while(qh<=qt){int x=q[qh++];for(int y
:g[x])if(bs[x]!=bs[y]&&mt[x]!=y){if(y==s||mt[y]>=0&&ft[mt[y]
]>=0)contr(s,x,y);else if(ft[y]<0){ft[y]=x;if(mt[y]<0)return
y;else if(!inq[mt[y]])inq[q[++qt]=mt[y]]=true;}}}return -1;}
int aug(int s,int t){int x=t,y,z;while(x>=0){y=ft[x];z=mt[y]
;mt[y]=x;mt[x]=y;x=z;}return t>=0;}int edmonds(){int r=0;
memset(mt,-1,sizeof(mt));rep(x,n)if(mt[x]<0)r+=aug(x,findp(x
));return r;}
```

5.3 dinic

```
// time: O(E V^2)
//      O(E V^(2/3)) / O(E sqrt(E)) unit capacities
//      O(E sqrt(V))  (hopcroft-karp) unit networks
//unit network: c in {0,1} & forall v, indeg<=1 or outdeg<=1
//min-cut: nodes reachable from s in final residual graph
struct Dinic {
    struct Edge { int u, v; ll c, f = 0; };
    int N, s, t; vector<vector<int>> G;
    vector<Edge> E; vector<int> lvl, ptr;
    Dinic() {}
```

```
    Dinic(int N, int s, int t) : N(N), s(s), t(t), G(N) {}

    void add_edge(int u, int v, ll c) {
        G[u].push_back(E.size()); E.push_back({u, v, c});
        G[v].push_back(E.size()); E.push_back({v, u, 0});
    }

    ll push(int u, ll p) {
        if (u == t || p <= 0) return p;
        while (ptr[u] < G[u].size()) {
            int ei = G[u][ptr[u]++];
            Edge &e = E[ei];
            if (lvl[e.v] != lvl[u] + 1) continue;
            ll a = push(e.v, min(e.c - e.f, p));
            if (a <= 0) continue;
            e.f += a, E[ei ^ 1].f -= a; return a;
        }
        return 0;
    }

    ll maxflow() {
        ll f = 0;
        while (true) {
            lvl.assign(N, -1); queue<int> q;
            lvl[s] = 0; q.push(s);
            while (!q.empty()) {
                int u = q.front(); q.pop();
                for (int ei : G[u]) {
                    Edge &e = E[ei];
                    if (e.c-e.f<=0||lvl[e.v]!=-1) continue;
                    lvl[e.v] = lvl[u] + 1; q.push(e.v);
                }
            }
            if (lvl[t] == -1) break;
            ptr.assign(N,0);while(ll ff=push(s,INF))f += ff;
        }
        return f;
    }
};

/* Flujo con demandas (no necesariamente el maximo)
Agregar s' y t' nuevos source and sink
c'(s', v) = sum(d(u, v) for u in V) \forall arista (s', v)
c'(v, t') = sum(d(v, w) for w in V) \forall arista (v, t')
c'(u, v) = c(u, v) - d(u, v) \forall aristas antiguas
c'(t, s) = INF (el flujo por esta arista es el flujo real)*/
```

5.4 floyd-warshall

```
// calculate distances between every pair of nodes in O(V^3)
    time.
// works with negative edges, but not negative cycles.
void floyd(const vector<vector<pair<ll, int>>> &G, vector<
    vector<ll>> &D) {
    int N = G.size();
    D.assign(N, vector<ll>(N, INF));
    rep(u, N) D[u][u] = 0;
    rep(u, N) for (auto [w, v] : G[u]) D[u][v] = w;
    rep(k, N) rep(u, N) rep(v, N)
        D[u][v] = min(D[u][v], D[u][k] + D[k][v]);
}
```

5.5 heavy-light

```
struct Hld {
    vector<int> P, H, D, pos, top;

    Hld() {}
    void init(vector<vector<int>> &G) {
        int N = G.size();
        P.resize(N), H.resize(N), D.resize(N), pos.resize(N),
            top.resize(N);
        D[0] = -1, dfs(G, 0); int t = 0;
        rep(i, N) if (H[P[i]] != i) {
            int j = i;
            while (j != -1)
                { top[j] = i, pos[j] = t++; j = H[j]; }
        }
    }

    int dfs(vector<vector<int>> &G, int i) {
        int w = 1, mw = 0;
        D[i] = D[P[i]] + 1, H[i] = -1;
        for (int c : G[i]) {
            if (c == P[i]) continue;
            P[c] = i; int sw = dfs(G, c); w += sw;
            if (sw > mw) H[i] = c, mw = sw;
        }
        return w;
    }

    // visit the log N segments in the path from u to v
    template <class OP>
    void path(int u, int v, OP op) {
        while (top[u] != top[v]) {
            if (D[top[u]] > D[top[v]]) swap(u, v);
            op(pos[top[v]], pos[v] + 1); v = P[top[v]];
        }
        if (D[u] > D[v]) swap(u, v);
        op(pos[u], pos[v] + 1); // value on node
        // op(pos[u]+1, pos[v] + 1); // value on edge
    }

    // an alternative to `path` that considers order.
    // calls `op` with an `l <= r` inclusive-exclusive range,
        and a
    // boolean indicating if the query is forwards or
        backwards.
    template <class OP>
    void path(int u, int v, OP op) {
        int lu = u, lv = v;
        while (top[lu] != top[lv])
            if (D[top[lu]] > D[top[lv]]) lu = P[top[lu]];
            else lv = P[top[lv]];
        int lca = D[lu] > D[lv] ? lv : lu;

        while (top[u] != top[lca])
            op(pos[top[u]], pos[u] + 1, false), u = P[top[u
                ]];
        if (u != lca) op(pos[lca] + 1, pos[u] + 1, false);

        vector<int> stk;
        while (top[v] != top[lca])
            stk.push_back(v), v = P[top[v]];

        // op(pos[lca], pos[v] + 1, true); // value on node
        op(pos[lca] + 1, pos[v] + 1, true); // value on edge
        reverse(stk.begin(), stk.end());
        for (int w : stk) op(pos[top[w]], pos[w] + 1, true);
    }

    // commutative segment tree
    template <class T, class S>
    void update(S &seg, int i, T val) { seg.update(pos[i],
        val); }

    // commutative segment tree lazy
    template <class T, class S>
    void update(S &seg, int u, int v, T val) {
        path(u, v, [&](int l, int r) { seg.update(l, r, val);
            });
    }

    // commutative (lazy) segment tree
    template <class T, class S>
    T query(S &seg, int u, int v) {
```

```
        T ans = 0;
            // neutral element
        path(u, v, [&](int l, int r) { ans += seg.query(l, r)
            ; }); // query op
        return ans;
    }
};
```

5.6 hungarian

```
// find a maximum gain perfect matching in the given
    bipartite complete graph.
// input: gain matrix (G_{xy} = benefit of joining vertex x
    in set X with vertex
// y in set Y).
// output: maximum gain matching in members `xy[x]` and `yx[
    y]`.
// runtime: O(N^3)
struct Hungarian {
    int N, qi, root;
    vector<vector<ll>> gain;
    vector<int> xy, yx, p, q, slackx;
    vector<ll> lx, ly, slack;
    vector<bool> S, T;

    void add(int x, int px) {
        S[x] = true, p[x] = px;
        rep(y, N) if (lx[x] + ly[y] - gain[x][y] < slack[y])
            {
            slack[y] = lx[x] + ly[y] - gain[x][y], slackx[y]
                = x;
        }
    }

    void augment(int x, int y) {
        while (x != -2) {
            yx[y] = x; swap(xy[x], y); x = p[x];
        }
    }

    void improve() {
        S.assign(N, false), T.assign(N, false), p.assign(N,
            -1);
        qi = 0, q.clear();
        rep(x, N) if (xy[x] == -1) {
            q.push_back(root = x), p[x] = -2, S[x] = true;
            break;
        }
        rep(y, N) slack[y] = lx[root] + ly[y] - gain[root][y
            ], slackx[y] = root;

        while (true) {
            while (qi < q.size()) {
                int x = q[qi++];
                rep(y, N) if (lx[x] + ly[y] == gain[x][y] && !
                    T[y]) {
                    if (yx[y] == -1) return augment(x, y);
                    T[y] = true, q.push_back(yx[y]), add(yx[y
                        ], x);
                }
            }

            ll d = INF;
            rep(y, N) if (!T[y]) d = min(d, slack[y]);
            rep(x, N) if (S[x]) lx[x] -= d;
            rep(y, N) if (T[y]) ly[y] += d;
            rep(y, N) if (!T[y]) slack[y] -= d;

            rep(y, N) if (!T[y] && slack[y] == 0) {
                if (yx[y] == -1) return augment(slackx[y], y);
                T[y] = true;
                if (!S[yx[y]]) q.push_back(yx[y]), add(yx[y],
                    slackx[y]);
            }
        }
    }

    Hungarian(vector<vector<ll>> g)
        : N(g.size()), gain(g), xy(N, -1), yx(N, -1), lx(N, -
            INF),
        ly(N), slack(N), slackx(N) {
        rep(x, N) rep(y, N) lx[x] = max(lx[x], ly[y]);
        rep(i, N) improve();
    }
};
```

5.7 kuhn

```
// get a maximum cardinality matching in a bipartite graph.
// input: adjacency lists.
// output: matching (in `mt` member).
// runtime: O(V E)
struct Kuhn {
    int N, size;
    vector<vector<int>> G;
    vector<bool> seen;
    vector<int> mt;

    bool visit(int i) {
        if (seen[i]) return false;
        seen[i] = true;
        for (int to : G[i])
            if (mt[to] == -1 || visit(mt[to])) {
                mt[to] = i;
                return true;
            }
        return false;
    }

    Kuhn(vector<vector<int>> adj) : G(adj), N(G.size()), mt(N
        , -1) {
        rep(i, N) {
            seen.assign(N, false);
            size += visit(i);
        }
    }
};
```

5.8 lca

```
// calculates the lowest common ancestor for any two nodes
    in O(log N) time,
// with O(N log N) preprocessing
struct Lca {
    int N, K, t = 0;
    vector<vector<int>> U;
    vector<int> L, R;

    Lca() {}
    Lca(vector<vector<int>> &G) : N(G.size()), L(N), R(N) {
        K = N <= 1 ? 0 : 32 - __builtin_clz(N - 1);
        U.resize(K + 1, vector<int>(N));
        visit(G, 0, 0);
        rep(k, K) rep(u, N) U[k + 1][u] = U[k][U[k][u]];
    }

    void visit(vector<vector<int>> &G, int u, int p) {
        L[u] = t++, U[0][u] = p;
        for (int v : G[u]) if (v != p) visit(G, v, u);
        R[u] = t++;
    }

    bool is_anc(int up, int dn) {
        return L[up] <= L[dn] && R[dn] <= R[up];
    }
```

```
    int find(int u, int v) {
        if (is_anc(u, v)) return u;
        if (is_anc(v, u)) return v;
        for (int k = K; k >= 0;)
            if (is_anc(U[k][u], v)) k--;
            else u = U[k][u];
        return U[0][u];
    }
};
```

5.9 maxflow-mincost

```
// time: O(F V E)         F is the maximum flow
//      O(V E + F E log V) if bellman-ford is replaced by
     johnson
struct Flow {
    struct Edge {
        int u, v;
        ll c, w, f = 0;
    };

    int N, s, t;
    vector<vector<int>> G;
    vector<Edge> E;
    vector<ll> d, b;
    vector<int> p;

    Flow() {}
    Flow(int N, int s, int t) : N(N), s(s), t(t), G(N) {}

    void add_edge(int u, int v, ll c, ll w) {
        G[u].push_back(E.size());
        E.push_back({u, v, c, w});
        G[v].push_back(E.size());
        E.push_back({v, u, 0, -w});
    }

    // naive distances with bellman-ford: O(V E)
    void calcdists() {
        p.assign(N, -1), d.assign(N, INF), d[s] = 0;
        rep(i, N - 1) rep(ei, E.size()) {
            Edge &e = E[ei];
            ll n = d[e.u] + e.w;
            if (d[e.u] < INF && e.c - e.f > 0 && n < d[e.v])
                d[e.v] = n, p[e.v] = ei;
        }
    }

    // johnsons potentials: O(E log V)
    void calcdists() {
        if (b.empty()) {
            b.assign(N, 0);
            // code below only necessary if there are
                negative costs
            rep(i, N - 1) rep(ei, E.size()) {
                Edge &e = E[ei];
                if (e.f < e.c) b[e.v] = min(b[e.v], b[e.u] + e
                    .w);
            }
        }
        p.assign(N, -1), d.assign(N, INF), d[s] = 0;
        priority_queue<pair<ll, int>> q;
        q.push({0, s});
        while (!q.empty()) {
            auto [w, u] = q.top();
            q.pop();
            if (d[u] < -w + b[u]) continue;
            for (int ei : G[u]) {
                auto e = E[ei];
                ll n = d[u] + e.w;
                if (e.f < e.c && n < d[e.v]) {
                    d[e.v] = n, p[e.v] = ei;
                    q.push({b[e.v] - n, e.v});
                }
            }
        }
        b = d;
    }

    ll solve() {
        b.clear();
        ll ff = 0;
        while (true) {
            calcdists();
            if (p[t] == -1) break;

            ll f = INF;
            for (int cur = t; p[cur] != -1; cur = E[p[cur]].u
                )
                f = min(f, E[p[cur]].c - E[p[cur]].f);
            for (int cur = t; p[cur] != -1; cur = E[p[cur]].u
                )
                E[p[cur]].f += f, E[p[cur] ^ 1].f -= f;
            ff += f;
        }
        return ff;
    }
};
```

5.10 push-relabel

```
#include "../common.h"

const ll INF = 1e18;

// maximum flow algorithm.
// to run, use `maxflow()`.
//
// time: O(V^2 sqrt(E)) <= O(V^3)
// memory: O(V^2)
struct PushRelabel {
    vector<vector<ll>> cap, flow;
    vector<ll> excess;
    vector<int> height;

    PushRelabel() {}
    void resize(int N) { cap.assign(N, vector<ll>(N)); }

    // push as much excess flow as possible from u to v.
    void push(int u, int v) {
        ll f = min(excess[u], cap[u][v] - flow[u][v]);
        flow[u][v] += f;
        flow[v][u] -= f;
        excess[v] += f;
        excess[u] -= f;
    }

    // relabel the height of a vertex so that excess flow may
        be pushed.
    void relabel(int u) {
        int d = INT32_MAX;
        rep(v, cap.size()) if (cap[u][v] - flow[u][v] > 0) d
            =
            min(d, height[v]);
        if (d < INF) height[u] = d + 1;
    }

    // get the maximum flow on the network specified by `cap`
        with source `s`
    // and sink `t`.
    // node-to-node flows are output to the `flow` member.
    ll maxflow(int s, int t) {
        int N = cap.size(), M;
        flow.assign(N, vector<ll>(N));
        height.assign(N, 0), height[s] = N;
        excess.assign(N, 0), excess[s] = INF;
        rep(i, N) if (i != s) push(s, i);

        vector<int> q;
```

```
        while (true) {
            // find the highest vertices with excess
            q.clear(), M = 0;
            rep(i, N) {
                if (excess[i] <= 0 || i == s || i == t)
                     continue;
                if (height[i] > M) q.clear(), M = height[i];
                if (height[i] >= M) q.push_back(i);
            }
            if (q.empty()) break;
            // process vertices
            for (int u : q) {
                bool relab = true;
                rep(v, N) {
                    if (excess[u] <= 0) break;
                    if (cap[u][v] - flow[u][v] > 0 && height[u
                         ] > height[v])
                        push(u, v), relab = false;
                }
                if (relab) {
                    relabel(u);
                    break;
                }
            }
        }

        ll f = 0; rep(i, N) f += flow[i][t]; return f;
    }
};
```

5.11 strongly-connected-components

```
// compute strongly connected components.
// time: O(V + E), memory: O(V)
//
// after building:
//  comp = map from vertex to component (components are
     toposorted, root first, leaf last)
//  N = number of components
//  G = condensation graph (component DAG)
//
// byproducts:
//  vgi = transposed graph
//  order = reverse topological sort (leaf first, root last)
//
// others:
//  vn = number of vertices
//  vg = original vertex graph
struct Scc {
    int vn, N;
    vector<int> order, comp;
    vector<vector<int>> vg, vgi, G;

    void toposort(int u) {
        if (comp[u]) return;
        comp[u] = -1;
        for (int v : vg[u]) toposort(v);
        order.push_back(u);
    }

    bool carve(int u) {
        if (comp[u] != -1) return false;
        comp[u] = N;
        for (int v : vgi[u]) {
            carve(v);
            if (comp[v] != N) G[comp[v]].push_back(N);
        }
        return true;
    }

    Scc() {}
    Scc(vector<vector<int>> &g) : vn(g.size()), vg(g), comp(
         vn), vgi(vn), G(vn), N(0) {
        rep(u, vn) toposort(u);
        rep(u, vn) for (int v : vg[u]) vgi[v].push_back(u);
        invrep(i, vn) N += carve(order[i]);
    }
};
```

5.12 two-sat

```
// calculate the solvability of a system of logical
     equations, where every equation is of the form `a or b
     `.
// `neg`: get negation of `u`
// `then`: `u` implies `v`
// `any`: `u` or `v`
// `set`: `u` is true
//
// after `solve` (O(V+E)) returns true, `sol` contains one
     possible solution.
// determining all solutions is O(V*E) hard (requires
     computing reachability in a DAG).
struct TwoSat {
    int N; vector<vector<int>> G;
    Scc scc; vector<bool> sol;
    TwoSat(int n) : N(n), G(2 * n), sol(n) {}
    TwoSat() {}

    int neg(int u) { return (u + N) % (2 * N); }
    void then(int u, int v) { G[u].push_back(v), G[neg(v)].
         push_back(neg(u)); }
    void any(int u, int v) { then(neg(u), v); }
    void set(int u) { G[neg(u)].push_back(u); }

    bool solve() {
        scc = Scc(G);
        rep(u, N) if (scc.comp[u] == scc.comp[neg(u)]) return
             false;
        rep(u, N) sol[u] = (scc.comp[u] > scc.comp[neg(u)]);
        return true;
    }
};
```

6 implementation

6.1 SegmentTreeBeats

```
struct Node {
    ll s, mx1, mx2, mxc, mn1, mn2, mnc, lz = 0;
    Node() : s(0), mx1(LLONG_MIN), mx2(LLONG_MIN), mxc(0),
         mn1(LLONG_MAX), mn2(LLONG_MAX), mnc(0) {}
    Node(ll x) : s(x), mx1(x), mx2(LLONG_MIN), mxc(1), mn1(x)
         , mn2(LLONG_MAX), mnc(1) {}
    Node(const Node &a, const Node &b) {
        // add
        s = a.s + b.s;
        // min
        if (a.mx1 > b.mx1) mx1 = a.mx1, mxc = a.mxc, mx2 =
             max(b.mx1, a.mx2);
        if (a.mx1 < b.mx1) mx1 = b.mx1, mxc = b.mxc, mx2 =
             max(a.mx1, b.mx2);
        if (a.mx1 == b.mx1) mx1 = a.mx1, mxc = a.mxc + b.mxc,
              mx2 = max(a.mx2, b.mx2);
        // max
        if (a.mn1 < b.mn1) mn1 = a.mn1, mnc = a.mnc, mn2 =
             min(b.mn1, a.mn2);
        if (a.mn1 > b.mn1) mn1 = b.mn1, mnc = b.mnc, mn2 =
             min(a.mn1, b.mn2);
        if (a.mn1 == b.mn1) mn1 = a.mn1, mnc = a.mnc + b.mnc,
              mn2 = min(a.mn2, b.mn2);
    }
};

// 0 - indexed / inclusive - inclusive
template <class node>
```

```
struct STB {
    vector<node> st; int n;

    void build(int u, int i, int j, vector<node> &arr) {
        if (i == j) {
            st[u] = arr[i];
            return;
        }
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        build(l, i, m, arr), build(r, m + 1, j, arr);
        st[u] = node(st[l], st[r]);
    }
    void push_add(int u, int i, int j, ll v) {
        st[u].s += (j - i + 1) * v;
        st[u].mx1 += v, st[u].mn1 += v, st[u].lz += v;
        if (st[u].mx2 != LLONG_MIN) st[u].mx2 += v;
        if (st[u].mn2 != LLONG_MAX) st[u].mn2 += v;
    }
    void push_max(int u, ll v, bool l) { // for min op
        if (v >= st[u].mx1) return;
        st[u].s -= st[u].mx1 * st[u].mxc;
        st[u].mx1 = v;
        st[u].s += st[u].mx1 * st[u].mxc;
        if (l) st[u].mn1 = st[u].mx1;
        else if (v <= st[u].mn1) st[u].mn1 = v;
        else if (v < st[u].mn2) st[u].mn2 = v;
    }
    void push_min(int u, ll v, bool l) { // for max op
        if (v <= st[u].mn1) return;
        st[u].s -= st[u].mn1 * st[u].mnc;
        st[u].mn1 = v;
        st[u].s += st[u].mn1 * st[u].mnc;
        if (l) st[u].mx1 = st[u].mn1;
        else if (v >= st[u].mx1) st[u].mx1 = v;
        else if (v > st[u].mx2) st[u].mx2 = v;
    }
    void push(int u, int i, int j) {
        if (i == j) return;
        // add
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        push_add(l, i, m, st[u].lz);
        push_add(r, m + 1, j, st[u].lz);
        st[u].lz = 0;
        // min
        push_max(l, st[u].mx1, i == m);
        push_max(r, st[u].mx1, m + 1 == j);
        // max
        push_min(l, st[u].mn1, i == m);
        push_min(r, st[u].mn1, m + 1 == r);
    }
    node query(int a, int b, int u, int i, int j) {
        if (b < i || j < a) return node();
        if (a <= i && j <= b) return st[u];
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        return node(query(a, b, l, i, m), query(a, b, r, m +
            1, j));
    }
    void update_add(int a, int b, ll v, int u, int i, int j)
        {
        if (b < i || j < a) return;
        if (a <= i && j <= b) {
            push_add(u, i, j, v);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_add(a, b, v, l, i, m);
        update_add(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }
    void update_min(int a, int b, ll v, int u, int i, int j)
        {
        if (b < i || j < a || v >= st[u].mx1) return;
        if (a <= i && j <= b && v > st[u].mx2) {
            push_max(u, v, i == j);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_min(a, b, v, l, i, m);
        update_min(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }
    void update_max(int a, int b, ll v, int u, int i, int j)
        {
        if (b < i || j < a || v <= st[u].mn1) return;
        if (a <= i && j <= b && v < st[u].mn2) {
            push_min(u, v, i == j);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_max(a, b, v, l, i, m);
        update_max(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    STB(vector<node> &v, int N) : n(N), st(N * 4 + 5) { build
        (0, 0, n - 1, v); }
    node query(int a, int b) { return query(a, b, 0, 0, n -
        1); }
    void update_add(int a, int b, ll v) { update_add(a, b, v,
        0, 0, n - 1); }
    void update_min(int a, int b, ll v) { update_min(a, b, v,
        0, 0, n - 1); }
    void update_max(int a, int b, ll v) { update_max(a, b, v,
        0, 0, n - 1); }
};
```

6.2 Treap

```
mt19937 gen(chrono::high_resolution_clock::now().
    time_since_epoch().count());

//  101 Implicit Treap //

struct Node {
    int p, sz = 0, v, acc, l = -1, r = -1;
    Node() : v(0), acc(0) {}
    Node(int x) : p(gen()), sz(1), v(x), acc(x) {}
    void recalc(const Node &a, const Node &b) {
        sz = a.sz + b.sz + 1;
        acc = v + a.acc + b.acc;
    }
};

template <class node>
struct Treap {
    vector<node> t;
    int n, r = -1;

    node get(int u) { return u != -1 ? t[u] : node(); }
    void recalc(int u) { t[u].recalc(get(t[u].l), get(t[u].r)
        ); }
    int merge(int l, int r) {
        if (min(l, r) == -1) return l != -1 ? l : r;
        int ans = (t[l].p < t[r].p) ? l : r;
        if (ans == l) t[l].r = merge(t[l].r, r), recalc(l);
        if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
        return ans;
    }
    pii split(int u, int id) {
        if (u == -1) return {-1, -1};
        int szl = get(t[u].l).sz;
        if (szl >= id) {
            pii ans = split(t[u].l, id);
            t[u].l = ans.ss;
            recalc(u);
```

```
            return {ans.ff, u};
        }
        pii ans = split(t[u].r, id - szl - 1);
        t[u].r = ans.ff;
        recalc(u);
        return {u, ans.ss};
    }

    Treap(vi &v) : n(sz(v)) {
        for (int i = 0; i < n; i++) t.eb(v[i]), r = merge(r,
            i);
    }
};

// Complete Implicit Treap with Lazy propagation //

struct Node {
    int p, sz = 0, v, acc, l = -1, r = -1, par = -1, lzv = 0;
    bool lz = false, f = false;
    Node() : v(0), acc(0) {}
    Node(int x) : p(gen()), sz(1), v(x), acc(x) {}
    void recalc(const Node &a, const Node &b) {
        sz = a.sz + b.sz + 1;
        acc = v + a.acc + b.acc;
    }
    void upd_lazy(int x) { lz = 1, lzv += x; }
    void lazy() { v += lzv, acc += sz * lzv, lz = 0, lzv = 0;
        }
    void flip() { swap(l, r), f = 0; }
};

template <class node>
struct Treap {
    vector<node> t;
    int n, r = -1;

    node get(int u) { return u != -1 ? t[u] : node(); }
    void recalc(int u) {
        int l = t[u].l, r = t[u].r;
        push(l), push(r), flip(l), flip(r);
        t[u].recalc(get(l), get(r));
    }
    void push(int u) {
        if (u == -1 || !t[u].lz) return;
        int l = t[u].l, r = t[u].r;
        if (l != -1) t[l].upd_lazy(t[u].lzv);
        if (r != -1) t[r].upd_lazy(t[u].lzv);
        t[u].lazy();
    }
    void flip(int u) {
        if (u == -1 || !t[u].f) return;
        int l = t[u].l, r = t[u].r;
        if (l != -1) t[l].f ^= 1;
        if (r != -1) t[r].f ^= 1;
        t[u].flip();
    }
    int merge(int l, int r) {
        if (min(l, r) == -1) return l != -1 ? l : r;
        push(l), push(r), flip(l), flip(r);
        int ans = (t[l].p < t[r].p) ? l : r;
        if (ans == l) t[l].r = merge(t[l].r, r), recalc(l);
        if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
        if (t[ans].l != -1) t[t[ans].l].par = ans; // only if
            parent needed
        if (t[ans].r != -1) t[t[ans].r].par = ans; // only if
            parent needed
        return ans;
    }
    pii split(int u, int id) {
        if (u == -1) return {-1, -1};
        push(u);
        flip(u);
        int szl = get(t[u].l).sz;
        if (szl >= id) {
            pii ans = split(t[u].l, id);
            if (ans.ss != -1) t[ans.ss].par = u; // only if
                parent needed
            if (ans.ff != -1) t[ans.ff].par = -1; // only if
                parent needed
            t[u].l = ans.ss;
            recalc(u);
            return {ans.ff, u};
        }
        pii ans = split(t[u].r, id - szl - 1);
        if (ans.ff != -1) t[ans.ff].par = u; // only if
            parent needed
        if (ans.ss != -1) t[ans.ss].par = -1; // only if
            parent needed
        t[u].r = ans.ff;
        recalc(u);
        return {u, ans.ss};
    }
    int update(int u, int l, int r, int v) {
        pii a = split(u, l), b = split(a.ss, r - l + 1);
        t[b.ff].upd_lazy(v);
        return merge(a.ff, merge(b.ff, b.ss));
    }
    void print(int u) {
        if (u == -1) return;
        push(u), flip(u);
        print(t[u].l);
        cout << t[u].v << ' ';
        print(t[u].r);
    }

    Treap(vi &v) : n(sz(v)) {
        for (int i = 0; i < n; i++) t.eb(v[i]), r = merge(r,
            i);
    }
};
```

6.3 bit-tricks

```
y = x & (x-1) // Turn off rightmost 1bit
y = x & (-x)  // Isolate rightmost 1bit
y = x | (x-1) // Right propagate rightmost 1bit(fill in 1s)
y = x | (x+1) // Turn on rightmost 0bit
y = ~x & (x+1) // Isolate rightmost 0bit
// If x is of long type, use __builtin_popcountl(x)
// If x is of long long type, use __builtin_popcountll(x)
// 1. Counts the number of ones(set bits) in an integer.
__builtin_popcount(x)
// 2. Checks the Parity of a number. Returns true(1) if the
// number has odd number of set bits, else it returns
// false(0) for even number of set bits.
 __builtin_parity(x)
// 3. Counts the leading number of zeros of the integer.
__builtin_clz(x)
// 4. Counts the trailing number of zeros of the integer.
__builtin_ctz(x)
// 5. Returns 1 + the index of the least significant 1-bit.
__builtin_ffs(x) // If x == 0, returns 0.
// Iterate over non empty subsets of bitmask
for(int s=m;s;s=(s-1)&m) // Decreasing order
for (int s=0;s=s-m&m;) // Increasing order
```

6.4 dsu

```
struct Dsu {
    vector<int> p; Dsu() {} Dsu(int N) : p(N, -1) {}
    int get(int x) { return p[x] < 0 ? x : get(p[x]); }
    bool sameSet(int a, int b) { return get(a) == get(b); }
    int size(int x) { return -p[get(x)]; }
    vector<vector<int>> S;
    void unite(int x, int y) {
        if ((x = get(x)) == (y = get(y))) { S.push_back({-1})
            ; return; }
```

```
        if (p[x] > p[y]) swap(x, y);
        S.push_back({x, y, p[x], p[y]});
        p[x] += p[y], p[y] = x;
    }
    void rollback() {
        auto a = S.back(); S.pop_back();
        if (a[0] != -1) p[a[0]] = a[2], p[a[1]] = a[3];
    }
};
```

6.5 dynamic-connectivity

```
struct DC {
    int n; Dsu D;
    vector<vector<pair<int, int>>> t;
    DC(int N) : n(N), D(N), t(2 * N) {}
    // add edge p to all times in interval [l, r]
    void upd(int l, int r, pair<int, int> p) {
        for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
            if (l & 1) t[l++].push_back(p);
            if (r & 1) t[--r].push_back(p);
        }
    }
    void process(int u = 1) { // process all queries
        for (auto &e : t[u]) D.unite(e.first, e.second);
        if (u >= n) {
            // do stuff with D at time u - n
        } else process(2 * u), process(2 * u + 1);
        for (auto &e : t[u]) D.rollback();
    }
};
```

6.6 hash-container

```
namespace{//add (#define tmpl template)(#define ty typename)
  tmpl<ty T> size_t mk_h(const T& v){return hash<T>()(v);}
  void h_cmb(size_t& h, const size_t& v)
  { h ^= v + 0x9e3779b9 + (h << 6) + (h >> 2); }
  tmpl<ty T> struct h_ct{size_t operator()(const T& v)const{
 size_t h=0;for(const auto& e:v){h_cmb(h,mk_h(e));}return h;
  }};
}namespace std{//support for pair<T,U>, vector<T> & map<T,U>
  tmpl<ty T, ty U> struct hash<pair<T, U>>{
    size_t operator()(const pair<T,U>& v) const
 {size_t h=mk_h(v.first);h_cmb(h, mk_h(v.second));return h;}
  };
tmpl<ty... T>struct hash<vector<T...>>:h_ct<vector<T...>>{};
tmpl<ty... T>struct hash<map<T...>>:h_ct<map<T...>>{}; }
```

6.7 mo

```
struct Query { int l, r, idx; };

// answer segment queries using only `add(i)`, `remove(i)`
    and `get()`
// functions.
//
// complexity: O((N + Q) * sqrt(N) * F)
//   N = length of the full segment
//   Q = amount of queries
//   F = complexity of the `add`, `remove` functions
template <class A, class R, class G, class T>
void mo(vector<Query> &queries, vector<T> &ans, A add, R
    remove, G get) {
    int Q = queries.size(), B = (int)sqrt(Q);
    sort(queries.begin(), queries.end(), [&](Query &a, Query
        &b) {
        return make_pair(a.l / B, a.r) < make_pair(b.l / B, b
            .r);
    });
    ans.resize(Q);

    int l = 0, r = 0;
    for (auto &q : queries) {
        while (r < q.r) add(r), r++;
        while (l > q.l) l--, add(l);
        while (r > q.r) r--, remove(r);
        while (l < q.l) remove(l), l++;
        ans[q.idx] = get();
    }
}
```

6.8 ordered-set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update> ordered_set;
// find_by_order(i) -> iterator to ith element
// order_of_key(k) -> position (int) of lower_bound of k
```

6.9 persistent-segment-tree-lazy

```
template <class T>
struct Node {
    T x, lz;
    int l = -1, r = -1;
};

template <class T>
struct Pstl {
    int N;
    vector<Node<T>> a;
    vector<int> head;

    T qneut() { return 0; }
    T merge(T l, T r) { return l + r; }
    T uneut() { return 0; }
    T accum(T u, T x) { return u + x; }
    T apply(T x, T lz, int l, int r) { return x + (r - l) *
        lz; }

    int build(int vl, int vr) {
        if (vr - vl == 1) a.push_back({qneut(), uneut()}); //
             node construction
        else {
            int vm = (vl + vr) / 2, l = build(vl, vm), r =
                build(vm, vr);
            a.push_back({merge(a[l].x, a[r].x), uneut(), l, r
                }); // query merge
        }
        return a.size() - 1;
    }

    T query(int l, int r, int v, int vl, int vr, T acc) {
        if (l >= vr || r <= vl) return qneut();
             // query neutral
        if (l <= vl && r >= vr) return apply(a[v].x, acc, vl,
             vr); // update op
        acc = accum(acc, a[v].lz);
             // update merge
        int vm = (vl + vr) / 2;
        return merge(query(l, r, a[v].l, vl, vm, acc), query(
             l, r, a[v].r, vm, vr, acc)); // query merge
    }

    int update(int l, int r, T x, int v, int vl, int vr) {
        if (l >= vr || r <= vl || r <= l) return v;
        a.push_back(a[v]);
        v = a.size() - 1;
        if (l <= vl && r >= vr) {
```

```
            a[v].x = apply(a[v].x, x, vl, vr); // update op
            a[v].lz = accum(a[v].lz, x);    // update merge
        } else {
            int vm = (vl + vr) / 2;
            a[v].l = update(l, r, x, a[v].l, vl, vm);
            a[v].r = update(l, r, x, a[v].r, vm, vr);
            a[v].x = merge(a[a[v].l].x, a[a[v].r].x); //
                query merge
        }
        return v;
    }

    Pstl() {}
    Pstl(int N) : N(N) { head.push_back(build(0, N)); }

    T query(int t, int l, int r) {
        return query(l, r, head[t], 0, N, uneut()); // update
            neutral
    }
    int update(int t, int l, int r, T x) {
        return head.push_back(update(l, r, x, head[t], 0, N))
            , head.size() - 1;
    }
};
```

6.10 persistent-segment-tree

```
// usage:
// Pst<Node<ll>> pst;
// pst = {N};
// int newtime = pst.update(time, index, value);
// Node<ll> result = pst.query(newtime, left, right);

template <class T>
struct Node {
    T x;
    int l = -1, r = -1;

    Node() : x(0) {}
    Node(T x) : x(x) {}
    Node(Node a, Node b, int l = -1, int r = -1) : x(a.x + b.
        x), l(l), r(r) {}
};

template <class U>
struct Pst {
    int N;
    vector<U> a;
    vector<int> head;

    int build(int vl, int vr) {
        if (vr - vl == 1) a.push_back(U());
        else {
            int vm = (vl + vr) / 2, l = build(vl, vm),
                r = build(vm, vr);
            a.push_back(U(a[l], a[r], l, r));
        }
        return a.size() - 1;
    }

    U query(int l, int r, int v, int vl, int vr) {
        if (l >= vr || r <= vl) return U();
        if (l <= vl && r >= vr) return a[v];
        int vm = (vl + vr) / 2;
        return U(query(l, r, a[v].l, vl, vm),
                 query(l, r, a[v].r, vm, vr));
    }

    int update(int i, U x, int v, int vl, int vr) {
        a.push_back(a[v]);
        v = a.size() - 1;
        if (vr - vl == 1) a[v] = x;
        else {
            int vm = (vl + vr) / 2;
            if (i < vm) a[v].l = update(i, x, a[v].l, vl, vm)
                ;
            else a[v].r = update(i, x, a[v].r, vm, vr);
            a[v] = U(a[a[v].l], a[a[v].r], a[v].l, a[v].r);
        }
        return v;
    }

    Pst() {}
    Pst(int N) : N(N) { head.push_back(build(0, N)); }

    U query(int t, int l, int r) {
        return query(l, r, head[t], 0, N);
    }
    int update(int t, int i, U x) {
        return head.push_back(update(i, x, head[t], 0, N)),
            head.size() - 1;
    }
};
```

6.11 segment-tree-lazy

```
template <class T>
struct Stl {
    int n;
    vector<T> a, b;

    T qneut() { return -2e9; }
    T uneut() { return 0; }
    T merge(T x, T y) { return max(x, y); }
    void upd(int v, T x, int l, int r)
        { a[v] += x, b[v] += x; }

    Stl(int n = 0) : n(n), a(4 * n, qneut()),
        b(4 * n, uneut()) {}

    void push(int v, int vl, int vm, int vr) {
        upd(2 * v, b[v], vl, vm);
        upd(2 * v + 1, b[v], vm, vr);
        b[v] = uneut();
    }

    T query(int l, int r, int v=1, int vl=0, int vr=1e9) {
        vr = min(vr, n);
        if (l <= vl && r >= vr) return a[v];
        if (l >= vr || r <= vl) return qneut();
        int vm = (vl + vr) / 2;
        push(v, vl, vm, vr);
        return merge(query(l, r, 2 * v, vl, vm),
            query(l, r, 2 * v + 1, vm, vr));
    }

    void update(int l, int r, T x, int v = 1, int vl = 0,
        int vr = 1e9) {
        vr = min(vr, n);
        if (l >= vr || r <= vl || r <= l) return;
        if (l <= vl && r >= vr) upd(v, x, vl, vr);
        else {
            int vm = (vl + vr) / 2;
            push(v, vl, vm, vr);
            update(l, r, x, 2 * v, vl, vm);
            update(l, r, x, 2 * v + 1, vm, vr);
            a[v] = merge(a[2 * v], a[2 * v + 1]);
        }
    }
};
```

6.12 segment-tree

```
struct St {
    ll neut() { return 0; }
    ll merge(ll x, ll y) { return x + y; }
```

```
    int n; vector<ll> a;
    St(int n = 0) : n(n), a(2 * n, neut()) {}

    ll query(int l, int r) {
        ll x = neut(), y = neut();
        for (l += n, r += n; l < r; l /= 2, r /= 2) {
            if (l & 1) x = merge(x, a[l++]);
            if (r & 1) y = merge(a[--r], y);
        }
        return merge(x, y);
    }

    void update(int i, ll x) {
        for (a[i += n] = x; i /= 2;)
            a[i] = merge(a[2 * i], a[2 * i + 1]);
    }
};
```

6.13 sparse-table

```
template <class T>
struct Sparse {
    T op(T a, T b) { return max(a, b); }

    vector<vector<T>> st;
    Sparse() {}
    Sparse(vector<T> a) : st{a} {
        int N = st[0].size();
        int npot = N <= 1 ? 1 : 32 - __builtin_clz(N);
        st.resize(npot);
        repx(i, 1, npot) rep(j, N + 1 - (1 << i))
        st[i].push_back(
            op(st[i - 1][j], st[i - 1][j + (1 << (i - 1))])
        ); // query op
    }

    T query(int l, int r) { // range must be nonempty!
        int i = 31 - __builtin_clz(r - l);
        return op(st[i][l], st[i][r - (1 << i)]); // queryop
    }
};
```

6.14 unordered-map

```
static mt19937 rng(chrono::steady_clock::now().
    time_since_epoch().count());
#define rnd(a,b) (uniform_int_distribution<ll>(a,b)(rng))

struct Hash {
    size_t operator()(const ll &x) const {
        const uint64_t RAND = chrono::steady_clock::now()
            .time_since_epoch().count();
        uint64_t z = x + RAND + 0x9e3779b97f4a7c15;
        z = (z ^ (z >> 30)) * 0xbf58476d1ce4e5b9;
        z = (z ^ (z >> 27)) * 0x94d049bb133111eb;
        return z ^ (z >> 31);
    }
};
template<class T,class U>using umap=unordered_map<T,U,Hash>;
template<class T> using uset = unordered_set<T, Hash>;
```

7 imprimible

8 math

8.1 arithmetic

```
inline int floor_log2(int n) { return n <= 1 ? 0 : 31 -
    __builtin_clz(n); }
inline int ceil_log2(int n) { return n <= 1 ? 0 : 32 -
    __builtin_clz(n - 1); }
inline ll floordiv(ll a, ll b) {
    return a / b - ((a ^ b) < 0 && a % b);
}
inline ll ceildiv(ll a, ll b) {
    return a / b + ((a ^ b) >= 0 && a % b);
}

ll binexp(ll a, ll e) {
    ll res = 1; // neutral element
    while (e) {
        if (e & 1) res = res * a; // multiplication
        a = a * a;              // multiplication
        e >>= 1;
    }
    return res;
}
```

8.2 crt

```
pair<ll, ll> solve_crt(const vector<pair<ll, ll>> &eqs) {
    ll a0 = eqs[0].first, p0 = eqs[0].second;
    repx(i, 1, eqs.size()) {
        ll a1 = eqs[i].first, p1 = eqs[i].second;
        ll k1, k0;
        ll d = ext_gcd(p1, p0, k1, k0);
        a0 -= a1;
        if (a0 % d != 0) return {-1, -1};
        p0 = p0 / d * p1;
        a0 = a0 / d * k1 % p0 * p1 % p0 + a1;
        a0 = (a0 % p0 + p0) % p0;
    }
    return {a0, p0};
}
```

8.3 discrete-log

```
// discrete logarithm log_a(b).
// solve b ^ x = a (mod M) for the smallest x.
// returns -1 if no solution is found.
//
// time: O(sqrt(M))
ll dlog(ll a, ll b, ll M) {
    ll k = 1, s = 0;
    while (true) {
        ll g = __gcd(b, M);
        if (g <= 1) break;
        if (a == k) return s;
        if (a % g != 0) return -1;
        a /= g, M /= g, s += 1, k = b / g * k % M;
    }
    ll N = sqrt(M) + 1;

    umap<ll, ll> r;
    rep(q, N + 1) {
        r[a] = q;
        a = a * b % M;
    }

    ll bN = binexp(b, N, M), bNp = k;
    repx(p, 1, N + 1) {
        bNp = bNp * bN % M;
        if (r.count(bNp)) return N * p - r[bNp] + s;
    }
    return -1;
}
```

8.4 fft

```
using cd = complex<double>;
const double PI = acos(-1);

// compute the DFT of a power-of-two-length sequence.
// if `inv` is true, computes the inverse DFT.
void fft(vector<cd> &a, bool inv) {
    int N = a.size(), k = 0, b;
    assert(N == 1 << __builtin_ctz(N));

    repx(i, 1, N) {
        for (b = N >> 1; k & b;) k ^= b, b >>= 1;
        if (i < (k ^= b)) swap(a[i], a[k]);
    }

    for (int l = 2; l <= N; l <<= 1) {
        double ang = 2 * PI / l * (inv ? -1 : 1);
        cd wl(cos(ang), sin(ang));
        for (int i = 0; i < N; i += l) {
            cd w = 1;
            rep(j, l / 2) {
                cd u = a[i + j], v = a[i + j + l / 2] * w;
                a[i + j] = u + v;
                a[i + j + l / 2] = u - v;
                w *= wl;
            }
        }
    }

    if (inv) rep(i, N) a[i] /= N;
}

const ll MOD = 998244353, ROOT = 15311432;
// const ll MOD = 2130706433, ROOT = 1791270792;
// const ll MOD = 922337203673733529711, ROOT =
     532077456549635698311;

void find_root_of_unity(ll M) {
    ll c = M - 1, k = 0;
    while (c % 2 == 0) c /= 2, k += 1;

    // find proper divisors of M - 1
    vector<ll> divs;
    for (ll d = 1; d < c; d++) {
        if (d * d > c) break;
        if (c % d == 0) rep(i, k + 1) divs.push_back(d << i);
    }
    rep(i, k) divs.push_back(c << i);

    // find any primitive root of M
    ll G = -1;
    repx(g, 2, M) {
        bool ok = true;
        for (ll d : divs) ok &= (binexp(g, d, M) != 1);
        if (ok) {
            G = g;
            break;
        }
    }
    assert(G != -1);

    ll w = binexp(G, c, M);
    cerr << "M = c * 2^k + 1" << endl;
    cerr << " M = " << M << endl;
    cerr << " c = " << c << endl;
    cerr << " k = " << k << endl;
    cerr << " w^(2^k) == 1" << endl;
    cerr << "   w = g^((M-1)/2^k) = g^c" << endl;
    cerr << "   g = " << G << endl;
    cerr << "   w = " << w << endl;
}

// compute the DFT of a power-of-two-length sequence, modulo
      a special prime
// number with an Nth root of unity, where N is the length
     of the sequence.
void ntt(vector<ll> &a, bool inv) {
    vector<ll> wn;
    for (ll p = ROOT; p != 1; p = p * p % MOD) wn.push_back(p
         );

    int N = a.size(), k = 0, b;
    assert(N == 1 << __builtin_ctz(N) && N <= 1 << wn.size())
         ;
    rep(i, N) a[i] = (a[i] % MOD + MOD) % MOD;

    repx(i, 1, N) {
        for (b = N >> 1; k & b;) k ^= b, b >>= 1;
        if (i < (k ^= b)) swap(a[i], a[k]);
    }

    for (int l = 2; l <= N; l <<= 1) {
        ll wl = wn[wn.size() - __builtin_ctz(l)];
        if (inv) wl = multinv(wl, MOD);

        for (int i = 0; i < N; i += l) {
            ll w = 1;
            repx(j, 0, l / 2) {
                ll u = a[i + j], v = a[i + j + l / 2] * w %
                     MOD;
                a[i + j] = (u + v) % MOD;
                a[i + j + l / 2] = (u - v + MOD) % MOD;
                w = w * wl % MOD;
            }
        }
    }

    ll q = multinv(N, MOD);
    if (inv) rep(i, N) a[i] = a[i] * q % MOD;
}

void convolve(vector<cd> &a, vector<cd> b, int n) {
    n = 1 << (32 - __builtin_clz(2 * n - 1));
    a.resize(n), b.resize(n);
    fft(a, false), fft(b, false);
    rep(i, n) a[i] *= b[i];
    fft(a, true);
}
```

8.5 gauss

```
const double EPS = 1e-9;

// solve a system of equations.
// complexity: O(min(N, M) * N * M)
//
// `a` is a list of rows
// the last value in each row is the result of the equation
// return values:
//  0 -> no solutions
//  1 -> unique solution, stored in `ans`
// -1 -> infinitely many solutions, one of which is stored
      in `ans`
// UNTESTED
int gauss(vector<vector<double>> a, vector<double> &ans) {
    int N = a.size(), M = a[0].size() - 1;

    vector<int> where(M, -1);
    for (int j = 0, i = 0; j < M && i < N; j++) {
        int sel = i;
        repx(k, i, N) if (abs(a[k][j]) > abs(a[sel][j])) sel
             = k;
        if (abs(a[sel][j]) < EPS) continue;
        repx(k, j, M + 1) swap(a[sel][k], a[i][k]);
        where[j] = i;

        rep(k, N) if (k != i) {
            double c = a[k][j] / a[i][j];
            repx(l, j, M + 1) a[k][l] -= a[i][l] * c;
        }
```

```
        i++;
    }

    ans.assign(M, 0);
    rep(i, M) if (where[i] != -1) ans[i] = a[where[i]][M] / a
        [where[i]][i];
    rep(i, N) {
        double sum = 0;
        rep(j, M) sum += ans[j] * a[i][j];
        if (abs(sum - a[i][M]) > EPS) return 0;
    }

    rep(i, M) if (where[i] == -1) return -1;
    return 1;
}
```

8.6 matrix

```
typedef vector<vector<double>> Mat;
Mat matmul(Mat l, Mat r) {
    int n = l.N, m = r.M, p = l.M; assert(l.M == r.N);
    Mat a(n, vector<double>(m)); // neutral
    rep(i, n) rep(j, m)
        rep(k, p) a[i][j] = a[i][j] + l[i][k] * r[k][j];
    return a;
}

double reduce(vector<vector<double>> &A) {
    int n = A.size(), m = A[0].size();
    int i = 0, j = 0; double r = 1.;
    while (i < n && j < m) {
        int l = i;
        repx(k, i+1, n) if(abs(A[k][j]) > abs(A[l][j])) l=k;
        if (abs(A[l][j]) < EPS) { j++; r = 0.; continue; }
        if (l != i) { r = -r; swap(A[i], A[l]); }
        r *= A[i][j];
        for (int k = m - 1; k >= j; k--) A[i][k] /= A[i][j];
        repx(k, 0, n) {
            if (k == i) continue;
            for(int l=m-1;l>=j;l--)A[k][l]-=A[k][j]*A[i][l];
        }
        i++, j++;
    }
    return r; // returns determinant
}
```

8.7 mobius

```
short mu[MAXN] = {0,1};
void mobius(){
 repx(i,1,MAXN)if(mu[i])for(int j=i+i;j<MAXN;j+=i)mu[j]-=mu[
    i];
}
```

8.8 mod

```
ll binexp(ll a, ll e, ll M) {
    assert(e >= 0);
    ll res = 1 % M;
    while (e) {
        if (e & 1) res = res * a % M;
        a = a * a % M;
        e >>= 1;
    }
    return res;
}

ll multinv(ll a, ll M) { return binexp(a, M - 2, M); }

// calculate gcd(a, b).
// also, calculate x and y such that:
// a * x + b * y == gcd(a, b)
//
// time: O(log min(a, b))
// (ignoring complexity of arithmetic)
ll ext_gcd(ll a, ll b, ll &x, ll &y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    ll d = ext_gcd(b, a % b, y, x);
    y -= a / b * x;
    return d;
}

// compute inverse with any M.
// a and M must be coprime for inverse to exist!
ll multinv_euc(ll a, ll M) {
    ll x, y;
    ext_gcd(a, M, x, y);
    return x;
}

// multiply two big numbers (~10^18) under a large modulo,
    without resorting to
// bigints.
ll bigmul(ll x, ll y, ll M) {
    ll z = 0;
    while (y) {
        if (y & 1) z = (z + x) % M;
        x = (x << 1) % M, y >>= 1;
    }
    return z;
}

// all modular inverses from 1 to inv.size()-1
void multinv_all(vector<ll> &inv) {
    inv[1] = 1;
    repx(i, 2, inv.size())
        inv[i] = MOD - (MOD / i) * inv[MOD % i] % MOD;
}

// change to __int128 if checking numbers over 10^9
bool isprime(ll n) {
    if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
    ll A[] = {2,325,9375,28178,450775,9780504,1795265022};
    ll s = __builtin_ctzll(n - 1), d = n >> s;
    for (int a : A) {
        ll p = binexp(a, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--) p = p *
            p % n;
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}

struct Mod {
    int a;
    static const int M = 1e9 + 7;

    Mod(ll aa) : a((aa % M + M) % M) {}

    Mod operator+(Mod rhs) const { return (a + rhs.a) % M; }
    Mod operator-(Mod rhs) const { return (a - rhs.a + M) % M
        ; }
    Mod operator-() const { return Mod(0) - *this; }
    Mod operator*(Mod rhs) const { return (ll)a * rhs.a % M;
        }

    Mod operator+=(Mod rhs) { return *this = *this + rhs; }
    Mod operator-=(Mod rhs) { return *this = *this - rhs; }
    Mod operator*=(Mod rhs) { return *this = *this * rhs; }

    Mod bigmul(ll big) const { return ::bigmul(a, big, M); }

    Mod binexp(ll e) const { return ::binexp(a, e, M); }
```

```
    // Mod multinv() const { return ::multinv(a, M); } //
        prime M
    Mod multinv() const { return ::multinv_euc(a, M); } //
        possibly composite M
};

// dynamic modulus
struct DMod {
    int a, M;

    DMod(ll aa, ll m) : M(m), a((aa % m + m) % m) {}

    DMod operator+(DMod rhs) const { return {(a + rhs.a) % M,
         M}; }
    DMod operator-(DMod rhs) const { return {(a - rhs.a + M)
        % M, M}; }
    DMod operator-() const { return DMod(0, M) - *this; }
    DMod operator*(DMod rhs) const { return {(ll)a * rhs.a %
        M, M}; }

    DMod operator+=(DMod rhs) { return *this = *this + rhs; }
    DMod operator-=(DMod rhs) { return *this = *this - rhs; }
    DMod operator*=(DMod rhs) { return *this = *this * rhs; }

    DMod bigmul(ll big) const { return {::bigmul(a, big, M),
        M}; }

    DMod binexp(ll e) const { return {::binexp(a, e, M), M};
        }
    DMod multinv() const { return {::multinv(a, M), M}; } //
        prime M
    // DMod multinv() const { return {::multinv_euc(a, M), M
        }; } // possibly composite M
};
```

8.9 primes

```
// counts the divisors of a positive integer in O(sqrt(n))
ll count_divisors(ll x) {
    ll divs = 1, i = 2;
    for (ll divs = 1, i = 2; x > 1; i++) {
        if (i * i > x) {
            divs *= 2;
            break;
        }
        for (ll d = divs; x % i == 0; x /= i) divs += d;
    }
    return divs;
}

// gets the prime factorization of a number in O(sqrt(n))
vector<pair<ll, int>> factorize(ll x) {
    vector<pair<ll, int>> f;
    for (ll k = 2; x > 1; k++) {
        if (k * k > x) {
            f.push_back({x, 1});
            break;
        }
        int n = 0;
        while (x % k == 0) x /= k, n++;
        if (n > 0) f.push_back({k, n});
    }
    return f;
}

// iterate over all divisors of a number.
//
// divisor count upper bound: n^(1.07 / ln ln n)
template <class OP>
void divisors(ll x, OP op) {
    auto facts = factorize(x);
    vector<int> f(facts.size());
    while (true) {
        ll y = 1;
        rep(i, f.size()) rep(j, f[i]) y *= facts[i].first;
        op(y);

        int i;
        for (i = 0; i < f.size(); i++) {
            f[i] += 1;
            if (f[i] <= facts[i].second) break;
            f[i] = 0;
        }
        if (i == f.size()) break;
    }
}

// computes euler totative function phi(x), counting the
    amount of integers in
// [1, x] that are coprime with x.
//
// time: O(sqrt(x))
ll phi(ll x) {
    ll phi = 1, k = 2;
    for (; x > 1; k++) {
        if (k * k > x) {
            phi *= x - 1;
            break;
        }
        ll k1 = 1, k0 = 0;
        while (x % k == 0) x /= k, k0 = k1, k1 *= k;
        phi *= k1 - k0;
    }
    return phi;
}

// isprime is in mod.cpp
```

8.10 simplex

```
// Solves a general linear maximization problem: maximize $c
    ^T x$ subject to $Ax \le b$, $x \ge 0$.
// Returns -inf if there is no solution, inf if there are
    arbitrarily good solutions, or the maximum value of $c^
    T x$ otherwise.
// The input vector is set to an optimal $x$ (or in the
    unbounded case, an arbitrary solution fulfilling the
    constraints).
// Numerical stability is not guaranteed. For better
    performance, define variables such that $x = 0$ is
    viable.
// Usage:
// vvd A = {{1,-1}, {-1,1}, {-1,-2}};
// vd b = {1,1,-4}, c = {-1,-1}, x;
// T val = LPSolver(A, b, c).solve(x);
// Time: O(NM * \#pivots), where a pivot may be e.g. an edge
     relaxation. O(2^n) in the general case.

#include "../common.h"

typedef double T; // long double, Rational, double + mod<P
    >...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1 / .0;
#define MP make_pair
#define ltj(X) \
    if (s == -1 || MP(X[j], N[j]) < MP(X[s], N[s])) s = j

struct LPSolver {
    int m, n;
    vector<int> N, B;
    vvd D;

    LPSolver(const vvd &A, const vd &b, const vd &c) : m(b.
        size()), n(c.size()), N(n + 1), B(m), D(m + 2, vd(n
        + 2)) {
```

```
    rep(i, m) rep(j, n) D[i][j] = A[i][j];
    rep(i, m) {
        B[i] = n + i;
        D[i][n] = -1;
        D[i][n + 1] = b[i];
    }
    rep(j, n) {
        N[j] = j;
        D[m][j] = -c[j];
    }
    N[n] = -1;
    D[m + 1][n] = 1;
}

void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i, m + 2) if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv;
        repx(j, 0, n + 2) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
    }
    rep(j, n + 2) if (j != s) D[r][j] *= inv;
    rep(i, m + 2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
}

bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
        int s = -1;
        rep(j, n + 1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i, m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || MP(D[i][n + 1] / D[i][s], B[i])
                < MP(D[r][n + 1] / D[r][s], B[r])) r = i
                ;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}

T solve(vd &x) {
    int r = 0;
    repx(i, 1, m) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -eps) {
        pivot(r, n);
        if (!simplex(2) || D[m + 1][n + 1] < -eps) return
            -inf;
        rep(i, m) if (B[i] == -1) {
            int s = 0;
            repx(j, 1, n + 1) ltj(D[i]);
            pivot(i, s);
        }
    }
    bool ok = simplex(1);
    x = vd(n);
    rep(i, m) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return ok ? D[m][n + 1] : inf;
  }
};
```

8.11 theorems

```
Burnside lemma
Tomemos imagenes x en X y operaciones (g: X -> X) en G.
Si #g es la cantidad de imagenes que son puntos fijos de g,
entonces la cantidad de objetos es `(sum_{g in G} #g) / |G|`
Es requisito que G tenga la operacion identidad, que toda
operacion tenga inversa y que todo par de operaciones tenga
su combinacion.

Rational root theorem
Las raices racionales de un polinomio de orden n con
coeficientes enteros A[i] son de la forma p / q, donde p y q
son coprimos, p es divisor de A[0] y q es divisor de A[n].
Notar que si A[0] = 0, cero es raiz, se puede dividir el
polinomio por x y aplica nuevamente el teorema.

Petersens theorem
Every cubic and bridgeless graph has a perfect matching.

Number of divisors for powers of 10
(0,1) (1,4) (2,12) (3,32) (4,64) (5,128) (6,240) (7,448)
(8,768) (9,1344) (10,2304) (11,4032) (12,6720) (13,10752)
(14,17280) (15,26880) (16,41472) (17,64512) (18,103680)

Kirchoff Theorem: Sea A la matriz de adyacencia del multi-
grafo (A[u][v] indica la cantidad de aristas entre u y v)
Sea D una matriz diagonal tal que D[v][v] es igual al grado
de v (considerando auto aristas y multi aristas). Sea
L = A - D. Todos los cofactores de L son iguales y equivalen
a la cantidad de Spanning Trees del grafo. Un cofactor (i,j)
de L es la multiplicacin de (-1)^{i + j} con el determinant
de la matriz al quitar la fila i y la columna j

Prufer Code: Dado un rbol con los nodos indexados: busca la
hoja de menor ndice, brrala y anota el ndice del nodo al
que estaba conectado. Repite el paso anterior n-2 veces. Lo
anterior muestra una biyeccin entre los arreglos de tamao
n-2 con elementos en [1, n] y los rboles de n nodos, por lo
que hay n^{n-2} spanning trees en un grafo completo.
Corolario: Si tenemos k componentes de tamaos s1,s2,...,sk
entonces podemos hacerlos conexos agregando k-1 aristas
entre nodos de s1*s2*...*sk*n^{k-2} formas

Combinatoria
Catalan: C_{n+1} = sum(C_i*C_{n-i} for i \in [0, n])
Catalan: C_n = \frac{1}{n+1}*\binom{2n, n}
Sea C_n^k las formas de poner n+k pares de parntesis, con
los primeros k parntesis abiertos (esto es, hay 2n + 2k
 carcteres), se tiene que
C_n^k = (2n+k-1)*(2n+k)/(n*(n+k+1)) * C_{n-1}^k
Sea D_n el nmero de permutaciones sin puntos fijos, entoces
D_n = (n-1)*(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0
```

8.12 tonelli-shanks

```
ll legendre(ll a, ll p) {
    if (a % p == 0) return 0; if (p == 2) return 1;
    return binexp(a, (p - 1) / 2, p);
}
// sqrt(n) mod p (p must be a prime)
// rnd(a, b) return a random number in [a, b]
ll tonelli_shanks(ll n, ll p) {
    if (n == 0) return 0;
    if (legendre(n, p) != 1) return -1; // no existe
    if (p == 2) return 1;
    ll s = __builtin_ctzll(p - 1);
    ll q = (p - 1LL) >> s, z = rnd(1, p - 1);
    if (s == 1) return binexp(n, (p + 1) / 4LL, p);
    while (legendre(z, p) != p - 1) z = rnd(1, p - 1);
    ll c = binexp(z, q, p), r = binexp(n, (q + 1) / 2, p);
    ll t = binexp(n, q, p), m = s;
    while (t != 1) {
        ll i = 1, ts = (t * t) % p;
        while (ts != 1) i++, ts = (ts * ts) % p;
        ll b = c;
        repx(_, 0, m - i - 1) b = (b * b) % p;
        r = r*b%p; c = b*b%p; t = t*c%p; m = i;
    }
    return r;
}
```