

Team Notebook

Pontificia Universidad Católica de Chile - Bella y Sensual

1 Data Structures	2	3.5 dinic	11	5.9 linear diophantine	18
1.1 dsu	2	3.6 dominator tree	11	5.10 matrix	18
1.2 fenwick tree	2	3.7 eulerian	11	5.11 matroidisect	18
1.3 link cut tree	2	3.8 floyd warshall	11	5.12 mobius	19
1.4 persistent segment tree lazy	2	3.9 heavy light	12	5.13 multinv	19
1.5 persistent segment tree	3	3.10 hungarian	12	5.14 permtree	19
1.6 rmq lineal	3	3.11 kuhn	13	5.15 polar rho	19
1.7 segment tree 2d	3	3.12 lca	13	5.16 polynomials	20
1.8 segment tree beats	4	3.13 maxflow mincost	13	5.17 primes	20
1.9 segment tree lazy	5	3.14 parallel dfs	14	5.18 simplex	21
1.10 segment tree	5	3.15 push relabel	14	5.19 theorems and formulas	21
1.11 sparse table	5	3.16 strongly connected components	14	5.20 theorems	21
1.12 treap implicit	5	3.17 two sat	14	5.21 tonelli shanks	22
1.13 treap	6	4 Implementation	15	6 Strings	22
2 Geo2d	6	4.1 common template and bit tricks	15	6.1 aho corasick	22
2.1 circle	6	4.2 dp convex hull trick	15	6.2 debruijn sequence	22
2.2 closest points	7	4.3 dp divide and conquer	15	6.3 hash	22
2.3 convex hull	7	4.4 dynamic connectivity	15	6.4 manacher	22
2.4 delaunay	7	4.5 hash container	16	6.5 palindromic tree	23
2.5 halfplane intersect	8	4.6 mo	16	6.6 prefix function	23
2.6 line	8	4.7 ordered set	16	6.7 suffix array	23
2.7 minkowski	8	4.8 unordered map	16	6.8 suffix automaton	23
2.8 point	9	5 Math	16	6.9 z function	24
2.9 polygon	9	5.1 arithmetic	16		
2.10 sweep	9	5.2 berlekamp massey linear recurrence	16		
2.11 theorems	10	5.3 crt	16		
3 Graph	10	5.4 debrujinseq	17		
3.1 artic bridge biconn	10	5.5 discrete log	17		
3.2 bellman ford	10	5.6 fast hadamard transform	17		
3.3 blossom	10	5.7 fft	17		
3.4 chu liu minimum spanning arborescence ...	10	5.8 gauss	18		

1 Data Structures

1.1 dsu

```
struct Dsu {
    vector<int> p; Dsu(int N = 0) : p(N, -1) {}
    int get(int x) { return p[x] < 0 ? x : get(p[x]); }
    bool sameSet(int a, int b) { return get(a) == get(b); }
    int size(int x) { return -p[get(x)]; }
    vector<vector<int>> S;
    void unite(int x, int y) {
        if ((x = get(x)) == (y = get(y)))
            return S.push_back({-1});
        if (p[x] > p[y]) swap(x, y);
        S.push_back({x, y, p[x], p[y]});
        p[x] += p[y], p[y] = x;
    }
    void rollback() {
        auto a = S.back(); S.pop_back();
        if (a[0] != -1) p[a[0]] = a[2], p[a[1]] = a[3];
    }
};
```

1.2 fenwick tree

```
int ft[MAXN+1]; // add dimension for multi-d
void upd(int i0, int v){ // add v to i0th element
    for(int i=i0+1; i<=MAXN; i+=i&-i) ft[i]+=v; // fors
}
int get(int i0){ // get sum of range [0,i0]
    int r=0; // add fors
    for(int i=i0; i>=1; i-=i&-i) r+=ft[i];
    return r;
}
int get_sum(int i0, int i1){ // sum of [i0,i1]
    return get(i1)-get(i0);
}
```

1.3 link cut tree

```
const int N_DEL = 0, N_VAL = 0; //delta, value
inline int mOp(int x, int y){return x+y;} //modify
inline int qOp(int lval, int rval){return lval + rval;} //
query
inline int d0nSeg(int d, int len){return d==N_DEL ? N_DEL :
d*len;}
//mostly generic
inline int joinD(int d1, int d2){
    if(d1==N_DEL) return d2; if(d2==N_DEL) return d1; return
mOp(d1, d2);}
inline int joinVD(int v, int d){return d==N_DEL ? v : mOp(v,
d);}
struct Node_t{
    int sz, nVal, tVal, d; bool rev;
```

```
Node_t *c[2], *p;
Node_t(int v) : sz(1), nVal(v), tVal(v), d(N_DEL), rev(0),
p(0){
    c[0]=c[1]=0;
}
bool isRoot(){return !p || (p->c[0] != this && p->c[1] !=
this);}
void push(){
    if(rev){
        rev=0; swap(c[0], c[1]); fore(x,0,2)if(c[x])c[x]-
>rev^=1;
    }
    nVal=joinVD(nVal, d); tVal=joinVD(tVal, d0nSeg(d, sz));
    fore(x,0,2)if(c[x])c[x]->d=joinD(c[x]->d, d);
    d=N_DEL;
}
void upd();
};
typedef Node_t* Node;
int getSize(Node r){return r ? r->sz : 0;}
int getPv(Node r){
    return r ? joinVD(r->tVal, d0nSeg(r->d, r->sz)) : N_VAL;}
void Node_t::upd(){
    tVal = qOp(qOp(getPv(c[0]), joinVD(nVal, d)),
getPv(c[1]));
    sz = 1 + getSize(c[0]) + getSize(c[1]);
}
void conn(Node c, Node p, int il){if(c->p=p; if(il>=0)p-
>c[il]=c;}
void rotate(Node x){
    Node p = x->p, g = p->p;
    bool gCh=p->isRoot(), isl = x==p->c[0];
    conn(x->c[isl], p, isl); conn(p, x, !isl);
    conn(x, g, gCh?-1: (p==g->c[0])); p->upd();
}
void spa(Node x){//splay
    while(!x->isRoot()){
        Node p = x->p, g = p->p;
        if(!p->isRoot())g->push();
        p->push(); x->push();
        if(!p->isRoot())rotate((x==p->c[0])==(p==g->c[0])? p : x);
        rotate(x);
    }
    x->push(); x->upd();
}
Node exv(Node x){//expose
    Node last=0;
    for(Node y=x; y; y=y->p)spa(y), y->c[0]=last, y-
>upd(), last=y;
    spa(x);
    return last;
}
void mkR(Node x){exv(x); x->rev^=1;} //makeRoot
Node getR(Node x){exv(x); while(x->c[1])x=x-
>c[1]; spa(x); return x;}
```

```
Node lca(Node x, Node y){exv(x); return exv(y);}
bool connected(Node x, Node y){exv(x); exv(y); return x==y?
1:x->p!=0;}
void link(Node x, Node y){mkR(x); x->p=y;}
void cut(Node x, Node y){mkR(x); exv(y); y->c[1]->p=0; y-
>c[1]=0;}
Node father(Node x){
    exv(x); Node r=x->c[1];
    if(!r) return 0;
    while(r->c[0])r=r->c[0];
    return r;
}
void cut(Node x){ // cuts x from father keeping tree root
    exv(father(x)); x->p=0;}
int query(Node x, Node y){mkR(x); exv(y); return getPv(y);}
void modify(Node x, Node y, int d){mkR(x); exv(y); y-
>d=joinD(y->d, d);}
Node lift_rec(Node x, int t){
    if(!x) return 0;
    if(t==getSize(x->c[0])){spa(x); return x;}
    if(t<getSize(x->c[0])) return lift_rec(x->c[0], t);
    return lift_rec(x->c[1], t-getSize(x->c[0])-1);
}
Node lift(Node x, int t){ // t-th ancestor of x (lift(x,1)
is x's father)
    exv(x); return lift_rec(x, t);}
int depth(Node x){ // distance from x to its tree root
    exv(x); return getSize(x)-1;}
```

1.4 persistent segment tree lazy

```
template <class T>
struct Node {
    T x, lz;
    int l = -1, r = -1;
};

template <class T>
struct Pstl {
    int N;
    vector<Node<T>> a;
    vector<int> head;

    T qneut() { return 0; }
    T merge(T l, T r) { return l + r; }
    T uneut() { return 0; }
    T accum(T u, T x) { return u + x; }
    T apply(T x, T lz, int l, int r) { return x + (r - l) *
lz; }

    int build(int vl, int vr) {
        if (vr - vl == 1) a.push_back({qneut(),
uneut()}); // node construction
        else {
            int vm = (vl + vr) / 2, l = build(vl, vm), r =
```

```

build(vl, vr);
    a.push_back({merge(a[l].x, a[r].x), uneut(), l,
r}); // query merge
    }
    return a.size() - 1;
}

T query(int l, int r, int v, int vl, int vr, T acc) {
    if (l >= vr || r <= vl) return
qneut(); // query neutral
    if (l <= vl && r >= vr) return apply(a[v].x, acc,
vl, vr); // update op
    acc = accum(acc,
a[v].lz); // update merge
    int vm = (vl + vr) / 2;
    return merge(query(l, r, a[v].l, vl, vm, acc),
query(l, r, a[v].r, vm, vr, acc)); // query merge
}

int update(int l, int r, T x, int v, int vl, int vr) {
    if (l >= vr || r <= vl || r <= l) return v;
    a.push_back(a[v]);
    v = a.size() - 1;
    if (l <= vl && r >= vr) {
        a[v].x = apply(a[v].x, x, vl, vr); // update op
        a[v].lz = accum(a[v].lz, x); // update
merge
    } else {
        int vm = (vl + vr) / 2;
        a[v].l = update(l, r, x, a[v].l, vl, vm);
        a[v].r = update(l, r, x, a[v].r, vm, vr);
        a[v].x = merge(a[a[v].l].x, a[a[v].r].x); //
query merge
    }
    return v;
}

Pstl() {}
Pstl(int N) : N(N) { head.push_back(build(0, N)); }

T query(int t, int l, int r) {
    return query(l, r, head[t], 0, N, uneut()); //
update neutral
}

int update(int t, int l, int r, T x) {
    return head.push_back(update(l, r, x, head[t], 0,
N)), head.size() - 1;
}
};

```

1.5 persistent segment tree

```

// usage:
// Pst<Node<ll>> pst;
// pst = {N};

```

```

// int newtime = pst.update(time, index, value);
// Node<ll> result = pst.query(newtime, left, right);

template <class T>
struct Node {
    T x;
    int l = -1, r = -1;

    Node() : x(0) {}
    Node(T x) : x(x) {}
    Node(Node a, Node b, int l = -1, int r = -1) : x(a.x +
b.x), l(l), r(r) {}
};

template <class U>
struct Pst {
    int N;
    vector<U> a;
    vector<int> head;

    int build(int vl, int vr) {
        if (vr - vl == 1) a.push_back(U());
        else {
            int vm = (vl + vr) / 2, l = build(vl, vm),
r = build(vm, vr);
            a.push_back(U(a[l], a[r], l, r));
        }
        return a.size() - 1;
    }

    U query(int l, int r, int v, int vl, int vr) {
        if (l >= vr || r <= vl) return U();
        if (l <= vl && r >= vr) return a[v];
        int vm = (vl + vr) / 2;
        return U(query(l, r, a[v].l, vl, vm),
query(l, r, a[v].r, vm, vr));
    }

    int update(int i, U x, int v, int vl, int vr) {
        a.push_back(a[v]);
        v = a.size() - 1;
        if (vr - vl == 1) a[v] = x;
        else {
            int vm = (vl + vr) / 2;
            if (i < vm) a[v].l = update(i, x, a[v].l, vl,
vm);

            else a[v].r = update(i, x, a[v].r, vm, vr);
            a[v] = U(a[a[v].l], a[a[v].r], a[v].l, a[v].r);
        }
        return v;
    }

    Pst() {}
    Pst(int N) : N(N) { head.push_back(build(0, N)); }
}

```

```

U query(int t, int l, int r) {
    return query(l, r, head[t], 0, N);
}

int update(int t, int i, U x) {
    return head.push_back(update(i, x, head[t], 0, N)),
head.size() - 1;
}
};

```

1.6 rmq lineal

```

typedef int tf; // 0(n) construction, 0(1) query
struct rmq{
    int n; tf INF=1e9; //change sign of INF for MAX
    vector<unsigned int> mk; vector<tf> bk,v;
    rmq(){}
    tf op(tf a, tf b){return min(a,b);} //change for maximum
    int f(int x){return x>>5;}
    rmq(vector<tf> &vv):n(SZ(vv)),mk(n),bk(n,INF),v(vv){
        unsigned int lst=0;
        for(int i=0;i<SZ(v);i++,lst<=1){
            bk[f(i)]=op(bk[f(i)],v[i]);
            while(lst&&v[i-__builtin_ctz(lst)]>v[i])
lst^=lst&-lst; //MIN
            //while(lst&&v[i-__builtin_ctz(lst)]<v[i])
lst^=lst&-lst; //MAX
            mk[i]=++lst;
        }
        for(int k=1,top=f(n);(1<=k)<=top;k+=
+)fore(i,0,top)if(i+(1<=k)<=top)
            bk[top*k+i]=op(bk[top*(k-1)+i],
bk[top*(k-1)+i+(1<=k-1)]);
    }
    tf get(int st, int en){
        return v[en-31+__builtin_clz(mk[en]&((1ll<<en-
st+1)-1))];
    }
    tf query(int s, int e){ // [s,e]
        int b1=f(s),b2=f(e),top=f(n);
        if(b1==b2) return get(s,e);
        tf ans=op(get(s, (b1+1)*32-1), get(b2*32,e));
        s=(b1+1)*32; e=b2*32-1;
        if(s<=e){
            int k=31-__builtin_clz(f(e-s+1));
            ans=op(ans,op(bk[top*k+f(s)],bk[top*k+f(e)-
(1<=k)+1]));
        }
        return ans;
    }
};

```

1.7 segment tree 2d

```

// #define MAXN 1024 #define op(a,b) (a+b) #define NEUT 0
int n,m; int a[MAXN][MAXN],st[2*MAXN][2*MAXN];

```

```

void build(){
    repx(i, 0, n) repx(j, 0, m) st[i+n][j+m] = a[i][j];
    repx(i, 0, n) for(int j = m-1; j; --j)
        st[i+n][j] = op(st[i+n][j<<1], st[i+n][j<<1|1]);
    for(int i = n-1; i; --i) repx(j, 0, 2*m)
        st[i][j] = op(st[i<<1][j], st[i<<1|1][j]);
}

void upd(int x, int y, int v){
    st[x+n][y+m]=v;
    for(int j = y+m; j > 1; j >= 1)
        st[x+n][j>>1] = op(st[x+n][j], st[x+n][j^1]);
    for(int i = x+n; i > 1; i >= 1) for(int j=y+m;j;j>>=1)
        st[i>>1][j] = op(st[i][j], st[i^1][j]);
}

int query(int x0, int x1, int y0, int y1){
    int r=NEUT;
    for(int i0=x0+n, i1=x1+n; i0<i1; i0>=1, i1>=1){
        int t[4], q = 0;
        if(i0 & 1) t[q++] = i0++;
        if(i1 & 1) t[q++] = --i1;
        repx(k,0,q)
            for(int j0=y0+m, j1=y1+m; j0<j1; j0>=1,j1>=1){
                if(j0 & 1) r = op(r, st[t[k]][j0++]);
                if(j1 & 1) r = op(r, st[t[k]][--j1]);
            }
        return r;
    }
}

```

1.8 segment tree beats

```

struct Node {
    ll s, mx1, mx2, mxc, mn1, mn2, mnc, lz = 0;
    Node() : s(0), mx1(LLONG_MIN), mx2(LLONG_MIN), mxc(0),
mn1(LLONG_MAX), mn2(LLONG_MAX), mnc(0) {}
    Node(ll x) : s(x), mx1(x), mx2(LLONG_MIN), mxc(1),
mn1(x), mn2(LLONG_MAX), mnc(1) {}
    Node(const Node &a, const Node &b) {
        // add
        s = a.s + b.s;
        // min
        if (a.mx1 > b.mx1) mx1 = a.mx1, mxc = a.mxc, mx2 =
max(b.mx1, a.mx2);
        if (a.mx1 < b.mx1) mx1 = b.mx1, mxc = b.mxc, mx2 =
max(a.mx1, b.mx2);
        if (a.mx1 == b.mx1) mx1 = a.mx1, mxc = a.mxc +
b.mxc, mx2 = max(a.mx2, b.mx2);
        // max
        if (a.mn1 < b.mn1) mn1 = a.mn1, mnc = a.mnc, mn2 =
min(b.mn1, a.mn2);
        if (a.mn1 > b.mn1) mn1 = b.mn1, mnc = b.mnc, mn2 =
min(a.mn1, b.mn2);
        if (a.mn1 == b.mn1) mn1 = a.mn1, mnc = a.mnc +
b.mnc, mn2 = min(a.mn2, b.mn2);
    }
}

```

```

};

// 0 - indexed / inclusive - inclusive
template <class node>
struct STB {
    vector<node> st; int n;

    void build(int u, int i, int j, vector<node> &arr) {
        if (i == j) {
            st[u] = arr[i];
            return;
        }
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        build(l, i, m, arr), build(r, m + 1, j, arr);
        st[u] = node(st[l], st[r]);
    }

    void push_add(int u, int i, int j, ll v) {
        st[u].s += (j - i + 1) * v;
        st[u].mx1 += v, st[u].mn1 += v, st[u].lz += v;
        if (st[u].mx2 != LLONG_MIN) st[u].mx2 += v;
        if (st[u].mn2 != LLONG_MAX) st[u].mn2 += v;
    }

    void push_max(int u, ll v, bool l) { // for min op
        if (v >= st[u].mx1) return;
        st[u].s -= st[u].mx1 * st[u].mxc;
        st[u].mx1 = v;
        st[u].s += st[u].mx1 * st[u].mxc;
        if (l) st[u].mn1 = st[u].mx1;
        else if (v <= st[u].mn1) st[u].mn1 = v;
        else if (v < st[u].mn2) st[u].mn2 = v;
    }

    void push_min(int u, ll v, bool l) { // for max op
        if (v <= st[u].mn1) return;
        st[u].s -= st[u].mn1 * st[u].mnc;
        st[u].mn1 = v;
        st[u].s += st[u].mn1 * st[u].mnc;
        if (l) st[u].mx1 = st[u].mn1;
        else if (v >= st[u].mx1) st[u].mx1 = v;
        else if (v > st[u].mx2) st[u].mx2 = v;
    }

    void push(int u, int i, int j) {
        if (i == j) return;
        // add
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        push_add(l, i, m, st[u].lz);
        push_add(r, m + 1, j, st[u].lz);
        st[u].lz = 0;
        // min
        push_max(l, st[u].mx1, i == m);
        push_max(r, st[u].mx1, m + 1 == j);
        // max
        push_min(l, st[u].mn1, i == m);
        push_min(r, st[u].mn1, m + 1 == r);
    }

    node query(int a, int b, int u, int i, int j) {

```

```

        if (b < i || j < a) return node();
        if (a <= i && j <= b) return st[u];
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        return node(query(a, b, l, i, m), query(a, b, r, m +
1, j));
    }

    void update_add(int a, int b, ll v, int u, int i, int j)
    {
        if (b < i || j < a) return;
        if (a <= i && j <= b) {
            push_add(u, i, j, v);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_add(a, b, v, l, i, m);
        update_add(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    void update_min(int a, int b, ll v, int u, int i, int j)
    {
        if (b < i || j < a || v >= st[u].mx1) return;
        if (a <= i && j <= b && v > st[u].mx2) {
            push_max(u, v, i == j);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_min(a, b, v, l, i, m);
        update_min(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    void update_max(int a, int b, ll v, int u, int i, int j)
    {
        if (b < i || j < a || v <= st[u].mn1) return;
        if (a <= i && j <= b && v < st[u].mn2) {
            push_min(u, v, i == j);
            return;
        }
        push(u, i, j);
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        update_max(a, b, v, l, i, m);
        update_max(a, b, v, r, m + 1, j);
        st[u] = node(st[l], st[r]);
    }

    STB(vector<node> &v, int N) : n(N), st(N * 4 + 5)
    { build(0, 0, n - 1, v); }
    node query(int a, int b) { return query(a, b, 0, 0, n -
1); }
    void update_add(int a, int b, ll v) { update_add(a, b,
v, 0, 0, n - 1); }
    void update_min(int a, int b, ll v) { update_min(a, b,
v, 0, 0, n - 1); }
}

```

```
void update_max(int a, int b, ll v) { update_max(a, b,
v, 0, 0, n - 1); }
};
```

1.9 segment tree lazy

```
template <class T>
struct Stl {
    int n; vector<T> a, b;
    Stl(int n = 0) : n(n), a(4 * n, qneut()),
        b(4 * n, uneut()) {}

    T qneut() { return -2e9; }
    T uneut() { return 0; }
    T merge(T x, T y) { return max(x, y); }
    void upd(int v, T x, int l, int r)
        { a[v] += x, b[v] += x; }

    void push(int v, int vl, int vm, int vr) {
        upd(2 * v, b[v], vl, vm);
        upd(2 * v + 1, b[v], vm, vr);
        b[v] = uneut();
    }

    T query(int l, int r, int v=1, int vl=0, int vr=1e9) {
        vr = min(vr, n);
        if (l <= vl && r >= vr) return a[v];
        if (l >= vr || r <= vl) return qneut();
        int vm = (vl + vr) / 2;
        push(v, vl, vm, vr);
        return merge(query(l, r, 2 * v, vl, vm),
            query(l, r, 2 * v + 1, vm, vr));
    }

    void update(int l, int r, T x, int v = 1, int vl = 0,
        int vr = 1e9) {
        vr = min(vr, n);
        if (l >= vr || r <= vl || r <= l) return;
        if (l <= vl && r >= vr) upd(v, x, vl, vr);
        else {
            int vm = (vl + vr) / 2;
            push(v, vl, vm, vr);
            update(l, r, x, 2 * v, vl, vm);
            update(l, r, x, 2 * v + 1, vm, vr);
            a[v] = merge(a[2 * v], a[2 * v + 1]);
        }
    }
};
```

1.10 segment tree

```
struct St {
    ll neut() { return 0; }
    ll merge(ll x, ll y) { return x + y; }
```

```
int n; vector<ll> a;
St(int n = 0) : n(n), a(2 * n, neut()) {}

ll query(int l, int r) {
    ll x = neut(), y = neut();
    for (l += n, r += n; l < r; l /= 2, r /= 2) {
        if (l & 1) x = merge(x, a[l++]);
        if (r & 1) y = merge(a[--r], y);
    }
    return merge(x, y);
}

void update(int i, ll x) {
    for (a[i += n] = x; i /= 2;)
        a[i] = merge(a[2 * i], a[2 * i + 1]);
}
```

1.11 sparse table

```
template <class T>
struct Sparse {
    T op(T a, T b) { return max(a, b); }

    vector<vector<T>> st;
    Sparse() {}
    Sparse(vector<T> a) : st{a} {
        int N = st[0].size();
        int npot = N <= 1 ? 1 : 32 - __builtin_clz(N);
        st.resize(npot);
        rep(x(i, 1, npot) rep(j, N + 1 - (1 << i))
            st[i].push_back(
                op(st[i - 1][j], st[i - 1][j + (1 << (i - 1))])
            ); // query op
    }

    T query(int l, int r) { // range must be nonempty!
        int i = 31 - __builtin_clz(r - l);
        return op(st[i][l], st[i][r - (1 << i)]); // queryop
    }
};
```

1.12 treap implicit

```
mt19937 gen(chrono::high_resolution_clock::now()
    .time_since_epoch().count());
// 101 Implicit Treap //
struct Node {
    int p, sz = 0, v, acc, l = -1, r = -1;
    Node() : v(0), acc(0) {}
    Node(int x) : p(gen()), sz(1), v(x), acc(x) {}
    void recalc(const Node &a, const Node &b) {
        sz = a.sz + b.sz + 1; acc = v + a.acc + b.acc;
    }
};
```

```
template <class node>
struct Treap {
    vector<node> t;
    int n, r = -1;
    node get(int u) { return u != -1 ? t[u] : node(); }
    void recalc(int u)
        { t[u].recalc(get(t[u].l), get(t[u].r)); }
    int merge(int l, int r) {
        if (min(l, r) == -1) return l != -1 ? l : r;
        int ans = (t[l].p < t[r].p) ? l : r;
        if (ans == l) t[l].r = merge(t[l].r, r), recalc(l);
        if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
        return ans;
    }
    pii split(int u, int id) {
        if (u == -1) return {-1, -1};
        int szl = get(t[u].l).sz;
        if (szl >= id) {
            pii ans = split(t[u].l, id);
            t[u].l = ans.ss;
            recalc(u);
            return {ans.ff, u};
        }
        pii ans = split(t[u].r, id - szl - 1);
        t[u].r = ans.ff;
        recalc(u);
        return {u, ans.ss};
    }
    Treap(vi &v) : n(sz(v)) {
        for (int i=0; i<n; i++) t.eb(v[i], r = merge(r, i));
    }
};

// Complete Implicit Treap with Lazy propagation //
struct Node {
    int p, sz = 0, v, acc, l = -1, r = -1, par = -1, lzv=0;
    bool lz = false, f = false;
    Node() : v(0), acc(0) {}
    Node(int x) : p(gen()), sz(1), v(x), acc(x) {}
    void recalc(const Node &a, const Node &b) {
        sz = a.sz + b.sz + 1; acc = v + a.acc + b.acc;
    }
    void upd_lazy(int x) { lz = 1, lzv += x; }
    void lazy() { v+=lzv, acc += sz * lzv, lz = 0, lzv = 0; }
    void flip() { swap(l, r), f = 0; }
};

template <class node> struct Treap {
    vector<node> t;
    int n, r = -1;
    node get(int u) { return u != -1 ? t[u] : node(); }
    void recalc(int u) {
        int l = t[u].l, r = t[u].r;
        push(l), push(r), flip(l), flip(r);
        t[u].recalc(get(l), get(r));
    }
    void push(int u) {
```

```

    if (u == -1 || !t[u].lz) return;
    int l = t[u].l, r = t[u].r;
    if (l != -1) t[l].upd_lazy(t[u].lzv);
    if (r != -1) t[r].upd_lazy(t[u].lzv);
    t[u].lazy();
}
void flip(int u) {
    if (u == -1 || !t[u].f) return;
    int l = t[u].l, r = t[u].r;
    if (l != -1) t[l].f ^= 1;
    if (r != -1) t[r].f ^= 1;
    t[u].flip();
}
int merge(int l, int r) { // (*) = only if parent needed
    if (min(l, r) == -1) return l != -1 ? l : r;
    push(l), push(r), flip(l), flip(r);
    int ans = (t[l].p < t[r].p) ? l : r;
    if (ans == l) t[l].r = merge(t[l].r, r), recalc(l);
    if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
    if (t[ans].l != -1) t[t[ans].l].par = ans; // (*)
    if (t[ans].r != -1) t[t[ans].r].par = ans; // (*)
    return ans;
}
pii split(int u, int id) { // (*) = only if parent needed
    if (u == -1) return {-1, -1};
    push(u);
    flip(u);
    int szl = get(t[u].l).sz;
    if (szl >= id) {
        pii ans = split(t[u].l, id);
        if (ans.ss != -1) t[ans.ss].par = u; // (*)
        if (ans.ff != -1) t[ans.ff].par = -1; // (*)
        t[u].l = ans.ss;
        recalc(u);
        return {ans.ff, u};
    }
    pii ans = split(t[u].r, id - szl - 1);
    if (ans.ff != -1) t[ans.ff].par = u; // (*)
    if (ans.ss != -1) t[ans.ss].par = -1; // (*)
    t[u].r = ans.ff;
    recalc(u);
    return {u, ans.ss};
}
int update(int u, int l, int r, int v) {
    pii a = split(u, l), b = split(a.ss, r - l + 1);
    t[b.ff].upd_lazy(v);
    return merge(a.ff, merge(b.ff, b.ss));
}
void print(int u) {
    if (u == -1) return; push(u), flip(u); print(t[u].l);
    cout << t[u].v << ' '; print(t[u].r);
}
Treap(vi &v) : n(sz(v)) {
    for (int i=0; i<n; i++) t.eb(v[i], r = merge(r, i));
}

```

```

    }
};

1.13 treap

typedef struct item *pitem;
struct item {
    int pr, key, cnt; pitem l, r;
    item(int key):key(key), pr(rand()), cnt(1), l(0), r(0) {}
};
int cnt(pitem t){return t? t->cnt:0;}
void upd_cnt(pitem t){if(t) t->cnt = cnt(t->l) + cnt(t->r) + 1;}
void split(pitem t, int key, pitem& l, pitem& r) { // l: <= key, r: > key
    if(!t) l=r=0;
    else if(key < t->key) split(t->l, key, l, t->l), r=t;
    else split(t->r, key, t->r, r), l=t;
    upd_cnt(t);
}
void insert(pitem& t, pitem it) {
    if(!t) t=it;
    else if(it->pr > t->pr) split(t, it->key, it->l, it->r), t=it;
    else insert(it->key < t->key ? t->l : t->r, it);
    upd_cnt(t);
}
void merge(pitem& t, pitem l, pitem r) {
    if(!l || !r) t=l?l:r;
    else if(l->pr > r->pr) merge(l->r, l->r, r), t=l;
    else merge(r->l, l, r->l), t=r;
    upd_cnt(t);
}
void erase(pitem& t, int key) {
    if(t->key == key) merge(t, t->l, t->r);
    else erase(key < t->key ? t->l : t->r, key);
    upd_cnt(t);
}
}

```

2 Geo2d

2.1 circle

```

struct C {
    P o; T r;
    // circle-line intersection, assuming it exists
    // points are sorted along the direction of the line
    pair<P, P> line_inter(L l) const {
        P c = l.closest_to(o); T c2 = (c - o).magsq();
        P e = l.d * sqrt(max(r*r - c2, T()) / l.d.magsq());
        return {c - e, c + e};
    }
    // check the type of line-circle collision
    // <0: 2 inters, =0: 1 inter, >0: 0 inters
    T line_collide(L l) const {
        T c2 = (l.closest_to(o) - o).magsq();
    }
}

```

```

    return c2 - r * r;
}
// calculates the two intersections between two circles
// the circles must intersect in one or two points!
pair<P, P> inter(C h) const {
    P d = h.o - o;
    T c = (r * r - h.r * h.r) / d.magsq();
    return h.line_inter({(1 + c) / 2 * d, d.rot()});
}
// check if the given circles intersect
bool collide(C h) const {
    return (h.o - o).magsq() <= (h.r + r) * (h.r + r);
}
// get one of the two tangents that go through the point
// the point must not be inside the circle
// a = -1: cw (relative to the circle) tangent
// a = 1: ccw (relative to the circle) tangent
P point_tangent(P p, T a) const {
    T c = r * r / p.magsq();
    return o + c*(p-o) - a*sqrt(c*(1-c))*(p-o).rot();
}
// get one of the 4 tangents between the two circles
// a = 1: exterior tangents
// a = -1: interior tangents (requires no area overlap)
// b = 1: ccw tangent
// b = -1: cw tangent
// the line origin is on this circumference, and the
// direction is a unit vector towards the other circle
L tangent(C c, T a, T b) const {
    T dr = a * r - c.r;
    P d = c.o - o;
    P n = (d*dr+b*d.rot()*sqrt(d.magsq()-dr*dr)).unit();
    return {o + n * r, -b * n.rot()};
}
// circumcircle of a **non-degenerate** triangle
static C thru_points(P a, P b, P c) {
    b = b - a, c = c - a;
    P p = (b*c.magsq() - c*b.magsq()).rot() / (b*c*2);
    return {a + p, p.mag()};
}
// find the two circles that go through the given point,
// are tangent to the given line and have radius `r`
// the point-line distance must be at most `r`!
// the circles are sorted in the direction of the line
static pair<C, C> thru_point_line_r(P a, L t, T r) {
    P d = t.d.rot().unit();
    if (d * (a - t.o) < 0) d = -d;
    auto p = C(a, r).line_inter({t.o + d * r, t.d});
    return {{p.first, r}, {p.second, r}};
}
// find the two circles that go through the given points
// and have radius `r`
// circles sorted by angle from the first point
// the points must be at most at distance `r`!
static pair<C, C> thru_points_r(P a, P b, T r) {
}

```



```

    auto p = C(a, r).line_inter({(a+b)/2, (b-a).rot()});
    return {{p.first, r}, {p.second, r}};
}
vector<P> linecol(L l){
    vector<P> s; P p=l.closest_to(o); double d=(p-
o).norm();
    if(d-EPS>r) return s;
    if(abs(d-r)<=EPS){s.pb(p); return s;}
    d=sqrt(r*r-d*d); s.pb(p+l.pq.unit()*d); s.pb(p-
l.pq.unit()*d);
    return s;
}
double intertriangle(P a,P b){ // intersection with oab
    if(abs((o-a)*(o-b))<=EPS) return 0.;
    vector<P> q={a},w=linecol(L{a,b-a});
    if(w.size()==2) for(auto p:w) if((a-p)*(b-p)<-EPS) q.pb(p);
    q.pb(b);
    if(q.size()==4&&(q[0]-q[1])*(q[2]-
q[1])>EPS) swap(q[1],q[2]);
    double s=0;
    for(i,0,q.size()-1){
        if(!has(q[i])||!has(q[i+1])) s+=r*r*(q[i]-
o).angle(q[i+1]-o)/2;
        else s+=abs((q[i]-o)*(q[i+1]-o)/2);
    }
    return s;
}
};

```

2.2 closest points

```

// sort by x
ll closest(vector<ii> &p) {
    int n = SZ(p);
    set<ii> s;
    ll best = 1e18;
    int j = 0;
    for(i, 0, n) {
        ll d = ceil(sqrt(best));
        while(p[i].fst - p[j].fst >= best)
            s.erase({p[j].snd, p[j].fst}), j++;
        auto it1=s.lower_bound({p[i].snd-d,p[i].fst});
        auto it2=s.upper_bound({p[i].snd+d,p[i].fst});
        for(auto it = it1; it != it2; ++it) {
            ll dx = p[i].fst - it->snd;
            ll dy = p[i].snd - it->fst;
            best = min(best, dx * dx + dy * dy);
        }
        s.insert({p[i].snd, p[i].fst});
    }
    return best;
}

```

2.3 convex hull

```

// ccw order, excludes collinear points by default
vector<P> chull(vector<P> p) {
    if (p.size() < 3) return p;
    vector<P> r; int m, k = 0;
    sort(p.begin(), p.end(), [](P a, P b) {
        return a.x != b.x ? a.x < b.x : a.y < b.y; });
    for (P q : p) { // lower hull
        while (k >= 2 && r[k-1].left(r[k-2], q) >= 0)
            r.pop_back(), k--; // >= to > to add collinears
        r.push_back(q), k++;
    }
    if (k == (int)p.size()) return r;
    r.pop_back(), k--; m = k;
    for (int i = p.size() - 1; i >= 0; --i) { // upper hull
        while (k >= m+2 && r[k-1].left(r[k-2], p[i]) >= 0)
            r.pop_back(), k--; // >= to > to add collinears
        r.push_back(p[i]), k++;
    }
    r.pop_back(); return r;
}

```

2.4 delaunay

```

typedef __int128_t lll; // if on a 64-bit platform

struct Q {
    Q *rot, *o; P p = {INF, INF}; bool mark;
    P &F() { return r()->p; }
    Q *R() { return rot->rot; }
    Q *prev() { return rot->o->rot; }
    Q *next() { return r()->prev(); }
};

T cross(P a, P b, P c) { return (b - a) % (c - a); }

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.magsq(), A = a.magsq() - p2,
        B = b.magsq() - p2, C = c.magsq() - p2;
    return cross(p, a, b) * C + cross(p, b, c) * A +
    cross(p, c, a) * B > 0;
}

Q *makeEdge(Q *H, P orig, P dest) {
    Q *r = H ? H : new Q{new Q{new Q{new Q{0}}}};
    H = r->o; r->r()->r() = r;
    repx(i, 0, 4) r = r->rot, r->p = {INF, INF},
        r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}

void splice(Q *a, Q *b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

```

```

Q *connect(Q *H, Q *a, Q *b) {
    Q *q = makeEdge(H, a->F(), b->p);
    splice(q, a->next()); splice(q->r(), b); return q;
}

pair<Q *, Q *> rec(Q *H, const vector<P> &s) {
    if (s.size() <= 3) {
        Q *a = makeEdge(H, s[0], s[1]), *b = makeEdge(H,
s[1], s.back());
        if (s.size() == 2) return {a, a->r()}; splice(a-
>r(), b);
        auto side = cross(s[0], s[1], s[2]);
        Q *c = side ? connect(H, b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b-
>r()};
    }

#define J(e) e->F(), e->p
#define valid(e) (cross(e->F(), J(base)) > 0)
    Q *A, *B, *ra, *rb; int half = s.size() / 2;
    tie(ra, A) = rec(H, {s.begin(), s.end() - half});
    tie(B, rb) = rec(H, {s.begin() + s.size() - half,
s.end()});
    while ((cross(B->p, J(A)) < 0 && (A = A->next())) ||
(cross(A->p, J(B)) > 0 && (B = B->r()->o)));
    Q *base = connect(H, B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q *e = init->dir; \
    if (valid(e)) while (circ(e->dir->F(), J(base), e->F()))
{ \
        Q *t = e->dir; splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); e->o = H; H = e;
e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(J(RC), J(LC))))
            base = connect(H, RC, base->r());
        else base = connect(H, base->r(), LC->r());
    }
    return {ra, rb};
#undef J
#undef valid
#undef DEL
}

// there must be no duplicate points
// returns no triangles in the case of all collinear points
// produces counter-clockwise triangles ordered in triples
// maximizes the minimum angle across all triangulations
// the euclidean mst is a subset of these edges
// O(N log N)

```

```
vector<P> triangulate(vector<P> pts) {
    sort(pts.begin(), pts.end(), [](P a, P b) {
        return make_pair(a.x, a.y) < make_pair(b.x, b.y);
    });
    assert(unique(pts.begin(), pts.end()) == pts.end());
    if (pts.size() < 2) return {};
    Q *H = 0; Q *e = rec(H, pts).first;
    vector<Q *> q = {e}; int qi = 0;
    while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD
    {
        Q *c = e;
        do {
            c->mark = 1; pts.push_back(c->p); \
            q.push_back(c->r()); c = c->next(); \
        } while (c != e);
    }
    ADD;
    pts.clear();
    while (qi < (int)q.size()) if (!(e = q[qi++])->mark)
    ADD;
    return pts;
#undef ADD
}
```

2.5 halfplane intersect

```
// obtain the convex polygon that results from intersecting
the given list
// of halfplanes, represented as lines that allow their left
side
// assumes the halfplane intersection is bounded
vector<P> halfplane_intersect(vector<L> &H) {
    L bb(P(-INF, -INF), P(INF, 0));
    rep(k, 4) H.push_back(bb), bb.o = bb.o.rot(), bb.d =
bb.d.rot();

    sort(begin(H), end(H), [](L a, L b) { return
a.d.angcmp(b.d) < 0; });
    deque<L> q; int n = 0;
    rep(i, H.size()) {
        while (n >= 2 && H[i].side(q[n - 1].intersection(q[n
- 2])) > 0)
            q.pop_back(), n--;
        while (n >= 2 && H[i].side(q[0].intersection(q[1]))
> 0)
            q.pop_front(), n--;
        if (n > 0 && H[i].parallel(q[n - 1])) {
            if (H[i].d * q[n - 1].d < 0) return {};
            if (H[i].side(q[n - 1].o) > 0) q.pop_back(),
n--;
            else continue;
        }
        q.push_back(H[i]), n++;
    }
}
```

```
while (n >= 3 && q[0].side(q[n - 1].intersection(q[n -
2])) > 0)
    q.pop_back(), n--;
while (n >= 3 && q[n - 1].side(q[0].intersection(q[1]))
> 0)
    q.pop_front(), n--;
if (n < 3) return {};

vector<P> ps(n);
rep(i, n) ps[i] = q[i].intersection(q[(i + 1) % n]);
return ps;
}
```

2.6 line

```
// a segment or an infinite line
// does not handle point segments correctly!
struct L {
    P o, d;

    static L from_eq(P ab, T c) {
        return L{ab.rot(), ab * -c / ab.magsq()};
    }
    pair<P, T> line_eq() { return {-d.rot(), d.rot() * o}; }

    // on which side of the line is the point
    // negative: left, positive: right
    T side(P r) const { return (r - o) % d; }

    // returns the intersection coefficient
    // in the range [0, d % r.d]
    // if d % r.d is zero, the lines are parallel
    T inter(L r) const { return (r.o - o) % r.d; }

    // get the single intersection point
    // lines must not be parallel
    P intersection(L r) const { return o + d * inter(r) / (d % r.d); }

    // check if lines are parallel
    bool parallel(L r) const { return abs(d % r.d) <= EPS; }

    // check if segments intersect
    bool seg_collide(L r) const {
        T z = d % r.d;
        if (abs(z) <= EPS) {
            if (abs(side(r.o)) > EPS) return false;
            T s = (r.o - o) * d, e = s + r.d * d;
            if (s > e) swap(s, e);
            return s <= d * d + EPS && e >= -EPS;
        }
        T s = inter(r), t = -r.inter(*this);
        if (z < 0) s = -s, t = -t, z = -z;
        return s >= -EPS && s <= z + EPS && t >= -EPS && t <= z + EPS;
    }
}
```

```
// full segment intersection
// makes a point segment if the intersection is a point
// however it does not handle point segments as input!
bool seg_inter(L r, L *out) const {
    T z = d % r.d;
    if (abs(z) <= EPS) {
        if (abs(side(r.o)) > EPS) return false;
        if (r.d * d < 0) r = {r.o + r.d, -r.d};
        P s = o * d < r.o * d ? r.o : o;
        P e = (o + d) * d < (r.o + r.d) * d ? o + d : r.o + r.d;
        if (s * d > e * d) return false;
        return *out = {s, e - s}, true;
    }
    T s = inter(r), t = -r.inter(*this);
    if (z < 0) s = -s, t = -t, z = -z;
    if (s >= -EPS && s <= z + EPS && t >= -EPS && t <= z + EPS)
        return *out = {o + d * s / z, {0, 0}}, true;
    return false;
}

// check if the given point is on the segment
bool point_on_seg(P r) const {
    if (abs(side(r)) > EPS) return false;
    if ((r - o) * d < -EPS) return false;
    if ((r - o - d) * d > EPS) return false;
    return true;
}

// point in this line that is closest to a given point
P closest_to(P r) const {
    P dr = d.rot(); return r + dr * ((o - r) * dr) / d.magsq();
}
};
```

2.7 minkowski

```
void reorder_polygon(vector<P> &ps) {
    int pos = 0;
    repx(i, 1, (int)ps.size()) {
        if (ps[i].y < ps[pos].y || (ps[i].y == ps[pos].y &&
ps[i].x < ps[pos].x))
            pos = i;
    }
    rotate(ps.begin(), ps.begin() + pos, ps.end());
}

vector<P> minkowski(vector<P> ps, vector<P> qs) {
    // the first vertex must be the lowest
    reorder_polygon(ps); reorder_polygon(qs);
    ps.push_back(ps[0]); ps.push_back(ps[1]);
    qs.push_back(qs[0]); qs.push_back(qs[1]);
    vector<P> result; int i = 0, j = 0;
    while (i < ps.size() - 2 || j < qs.size() - 2) {
        result.push_back(ps[i] + qs[j]);
        auto z = (ps[i + 1] - ps[i]) % (qs[j + 1] - qs[j]);
    }
}
```



```

    if (z >= 0 && i < ps.size() - 2) ++i;
    if (z <= 0 && j < qs.size() - 2) ++j;
}
return result;
}

```

2.8 point

```

struct P {
    T x, y;
    P(T x, T y) : x(x), y(y) {}
    P() : P(0, 0) {}

    friend ostream &operator<<(ostream &s, const P &r) {
        return s << r.x << " " << r.y;
    }
    friend istream &operator>>(istream &s, P &r) { return s
>> r.x >> r.y; }

    P operator+(P r) const { return {x + r.x, y + r.y}; }
    P operator-(P r) const { return {x - r.x, y - r.y}; }
    P operator*(T r) const { return {x * r, y * r}; }
    P operator/(T r) const { return {x / r, y / r}; }
    P operator-() const { return {-x, -y}; }
    friend P operator*(T l, P r) { return {l * r.x, l *
r.y}; }

    P rot() const { return {-y, x}; }
    T operator*(P r) const { return x * r.x + y * r.y; }
    T operator%(P r) const { return rot() * r; }
    T left(P a, P b) { return (b - a) % (*this - a); }

    T magsq() const { return x * x + y * y; }
    T mag() const { return sqrt(magsq()); }
    P unit() const { return *this / mag(); }

    bool half() const { return abs(y) <= EPS && x < -EPS ||
y < -EPS; }
    T angcmp(P r) const { // like strcmp(this, r)
        int h = (int)half() - r.half();
        return h ? h : r % *this;
    }
    T angcmp_rel(P a, P b) { // like strcmp(a, b)
        P z = *this;
        int h = z % a <= 0 && z * a < 0 || z % a < 0;
        h -= z % b <= 0 && z * b < 0 || z % b < 0;
        return h ? h : b % a;
    }

    bool operator==(P r) const { return abs(x - r.x) <= EPS
&& abs(y - r.y) <= EPS; }

    double angle() const { return atan2(y, x); }
    static P from_angle(double a) { return {cos(a),

```

```

sin(a)); }
};

```

2.9 polygon

```

// get TWICE the area of a simple polygon in ccw order
T area2(const vector<P> &p) {
    int n = p.size(); T a = 0;
    rep(i, n) a += (p[i] - p[0]) % (p[(i + 1) % n] - p[i]);
    return a;
}

// checks whether a point is inside a ccw simple polygon
// returns 1 if inside, 0 if on border, -1 if outside
int in_poly(const vector<P> &p, P q) {
    int w = 0;
    rep(i, p.size()) {
        P a = p[i], b = p[(i + 1) % p.size()];
        T k = (b - a) % (q - a);
        T u = a.y - q.y, v = b.y - q.y;
        if (k > 0 && u < 0 && v >= 0) w++;
        if (k < 0 && v < 0 && u >= 0) w--;
        if (k == 0 && (q - a) * (q - b) <= 0) return 0;
    }
    return w ? 1 : -1;
}

// check if point in ccw convex polygon, 0(log n)
// + if inside, 0 if on border, - if outside
T in_convex(const vector<P> &p, P q) {
    int l = 1, h = p.size() - 2; assert(p.size() >= 3);
    while (l != h) { // collinear points are unsupported!
        int m = (l + h + 1) / 2;
        if (q.left(p[0], p[m]) >= 0) l = m;
        else h = m - 1;
    }
    T in = min(q.left(p[0], p[l]), q.left(p.back(), p[0]));
    return min(in, q.left(p[l], p[l + 1]));
}

int extremal(const vector<P> &p, P d) {
    int n = p.size(), l = 0, r = n - 1; assert(n);
    P e0 = (p[n - 1] - p[0]).rot();
    while (l < r) { // polygon must be convex
        int m = (l + r + 1) / 2;
        P e = (p[(m + n - 1) % n] - p[m]).rot();
        if (e0.angcmp_rel(d, e) < 0) r = m - 1;
        else l = m;
    }
    return l;
}

// square dist of most distant points of a ccw convex
// polygon with NO COLLINEAR POINTS
T callipers(const vector<P> &p) {

```

```

int n = p.size();
T r = 0;
for (int i = 0, j = n < 2 ? 0 : 1; i < j; i++) {
    for (; j = (j + 1) % n) {
        r = max(r, (p[i] - p[j]).magsq());
        if ((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] -
p[j]) <= EPS) break;
    }
    return r;
}

P centroid(const vector<P> &p) { // (barycenter)
    P r(0, 0); T t = 0; int n = p.size();
    rep(i, n) {
        r += (p[i] + p[(i+1)%n]) * (p[i] % p[(i+1)%n]);
        t += p[i] % p[(i+1)%n];
    }
    return r / t / 3;
}

// classify collision of a ray inside a ccw polygon vertex.
// ray is (o, d), vertex is b, previous vertex is a, next is
c.
pair<bool, bool> inner_collide(P o, P d, P a, P b, P c) {
    T p = (a - o) % d; // side of previous
    T n = (c - o) % d; // side of next
    T v = (c - b) % (b - a); // is vertex convex?
    return {v > 0 ? n < 0 || (n == 0 && p < 0) : p > 0 || n
< 0,
           v > 0 ? p > 0 || (p == 0 && n > 0) : p > 0 || n
< 0};
}

```

2.10 sweep

```

#include "point.cpp"

// iterate over all pairs of points
// `op` is called with all ordered pairs of different
indices `i, j`
// additionally, the `ps` vector is kept sorted by signed
distance
// to the line formed by `i` and `j`
// for example, if the vector from `i` to `j` is pointing
right,
// the `ps` vector is sorted from smallest `y` to largest
`y`
// note that, because the `ps` vector is sorted by signed
distance,
// `j` is always equal to `i + 1`
// this means that the amount of points to the left of the
line is always `N - i`
template <class OP>
void all_pair_points(vector<P> &ps, OP op) {

```

```

        B[u]=min(B[u],B[v]);
    }
    else if(D[v]<D[u])st.push(ne),B[u]=min(B[u],D[v]);
}
}

void doit(){
    memset(D,-1,sizeof(D));memset(art,0,sizeof(art));
    nbc=T=0; for(i,0,n)if(D[i]<0)dfs(i,-1),art[i]--;
}

```

2.11 theorems

```
// Pick's theorem
//   Simple polygon with integer vertices:
//    $A = I + B / 2 - 1$ 
//   A: Area of the polygon
//   I: Integer points strictly inside the polygon
//   B: Integer points on the boundary of the polygon
```

3 Graph

3.1 artic bridge biconn

```
vector<int> g[MAXN];int n;
struct edge {int u,v,comp;bool bridge;};
vector<edge> e;
void add_edge(int u, int v){
    g[u].pb(e.size());g[v].pb(e.size());
    e.pb((edge){u,v,-1,false});
}
int D[MAXN],B[MAXN],T;
int nbc; // number of biconnected components
int art[MAXN]; // articulation point iff !=0
stack<int> st; // only for biconnected
void dfs(int u,int pe){
    B[u]=D[u]=T++;
    for(int ne:g[u])if(ne!=pe){
        int v=e[ne].u^e[ne].v^u;
        if(D[v]<0){
            st.push(ne);dfs(v,ne);
            if(B[v]>D[u])e[ne].bridge = true; // bridge
            if(B[v]==D[u]){
                art[u]++; // articulation
                int last; // start biconnected
                do{last=st.top();st.pop();e[last].comp=nbc;}
                while(last!=ne);
                nbc++; // end biconnected
            }
        }
    }
}
```

3.2 bellman ford

```

struct Edge { int u, v; ll w; };

// find distance from source node to all nodes.
// supports negative edge weights.
// returns true if a negative cycle is detected.
//
// time:  $O(V E)$ 
bool bellman_ford(int N, int s, vector<Edge> &E, vector<ll>
&D, vector<int> &P) {
    P.assign(N, -1), D.assign(N, INF), D[s] = 0;
    rep(i, N - 1) {
        bool f = true;
        rep(ei, E.size()) {
            auto &e = E[ei];
            ll n = D[e.u] + e.w;
            if (D[e.u] < INF && n < D[e.v])
                D[e.v] = n, P[e.v] = ei, f = false;
        }
        if (f) return false;
    }
    return true;
}

```

3.3 blossom

```
vector<int> g[MAXN];int n,m,mt[MAXN],qh,qt,q[MAXN],ft[MAXN],
bs[MAXN];bool inq[MAXN],inb[MAXN],inp[MAXN];int lca(int root
,int x,int y){memset(inp,0,sizeof(inp));while(1){inp[x]=bs[x]
]=true;if(x==root)break;x=ft[mt[x]];}while(1){if(inp[y]=bs[y]
])return y;else y=ft[mt[y]];}void mark(int z,int x){while(
bs[x]!=z){int y=mt[x];inb[bs[x]]=inb[bs[y]]=true;x=ft[y];if(
bs[x]!=z)ft[x]=y;}}void contr(int s,int x,int y){int z=lca(s
,x,y);memset(inb,0,sizeof(inb));mark(z,x);mark(z,y);if(bs[x]
!=z)ft[x]=y;if(bs[y]!=z)ft[y]=x;rep(x,n)if(inb[bs[x]]){bs[x]
=z;if(!inq[inq][q[+qt]=x]=true;}}int findp(int x){memset(
inq,0,sizeof(inq));memset(ft,-1,sizeof(ft));rep(i,n)bs[i]=i;
inq[q[qh=qt=0]=s]=true;while(qh<=qt){int x=q[qh++];for(int y
:g[x])if(bs[x]!=bs[y]&&mt[x]!=y){if(y==s||mt[y]>=0&&ft[mt[y]
]>=0)contr(s,x,y);else if(ft[y]<0){ft[y]=x;if(mt[y]<0)return
y;else if(!inq[mt[y]])inq[q[+qt]=mt[y]]=true;}}return -1;}
int aug(int s,int t){int x=t,y,z;while(x>=0){y=ft[x];z=mt[y]
;mt[y]=x;mt[x]=y;x=z;return t>=0;}int edmonds(){int r=0;
```

```
memset(mt, -1, sizeof(mt)); rep(x, n) if(mt[x] < 0) r += aug(x, findp(x)); return r; }
```

3.4 chu liu minimum spanning arborescence

```

//O(n*m) minimum spanning tree in directed graph
//returns -1 if not possible
//included i-th edge if take[i]!=0
typedef int tw; tw INF=1ll<<30;
struct edge{int u,v,id;tw len;};
struct ChuLiu{
    int n; vector<edge> e;
    vector<int> inc,dec,take,pre,num,id,vis;
    vector<tw> inw;
    void add_edge(int x, int y, tw w){
        inc.pb(0); dec.pb(0); take.pb(0);
        e.pb({x,y,SZ(e),w});
    }
    ChuLiu(int n):n(n),pre(n),num(n),id(n),vis(n),inw(n){}
    tw doit(int root){
        auto e2=e;
        tw ans=0; int eg=SZ(e)-1,pos=SZ(e)-1;
        while(1){
            fore(i,0,n) inw[i]=INF,id[i]=vis[i]=-1;
            for(auto ed:e2) if(ed.len<inw[ed.v]){
                inw[ed.v]=ed.len; pre[ed.v]=ed.u;
                num[ed.v]=ed.id;
            }
            inw[root]=0;
            fore(i,0,n) if(inw[i]==INF) return -1;
            int tot=-1;
            fore(i,0,n){
                ans+=inw[i];
                if(i!=root)take[num[i]]++;
                int j=i;
                while(vis[j]!=i&&j!
=root&&id[j]<0)vis[j]=i,j=pre[j];
                if(j!=root&&id[j]<0){
                    id[j]=++tot;
                    for(int k=pre[j];k!=j;k=pre[k])
id[k]=tot;
                }
            }
            if(tot<0)break;
            fore(i,0,n) if(id[i]<0)id[i]=++tot;
            n=tot+1; int j=0;
            fore(i,0,SZ(e2)){
                int v=e2[i].v;
                e2[j].v=id[e2[i].v];
                e2[j].u=id[e2[i].u];
                if(e2[j].v!=e2[j].u){
                    e2[j].len=e2[i].len-inw[v];
                    inc.pb(e2[i].id);

```

```

        dec.pb(num[v]);
        take.pb(0);
        e2[j++].id=++pos;
    }
    }
    e2.resize(j);
    root=id[root];
}
while(pos>eg){
    if(take[pos]>0) take[inc[pos]]++,
    take[dec[pos]]--;
    pos--;
}
return ans;
}
};

```

3.5 dinic

```

// time: O(E V^2)
//      O(E V^(2/3)) / O(E sqrt(E))    unit capacities
//      O(E sqrt(V))    (hopcroft-karp) unit networks
//unit network: c in {0,1} & forall v, indeg<=1 or outdeg<=1
//min-cut: nodes reachable from s in final residual graph
struct Dinic {
    struct Edge { int u, v; ll c, f = 0; };
    int N, s, t; vector<vector<int>> G;
    vector<Edge> E; vector<int> lvl, ptr;
    Dinic() {}
    Dinic(int N, int s, int t) : N(N), s(s), t(t), G(N) {}

    void add_edge(int u, int v, ll c) {
        G[u].push_back(E.size()); E.push_back({u, v, c});
        G[v].push_back(E.size()); E.push_back({v, u, 0});
    }

    ll push(int u, ll p) {
        if (u == t || p <= 0) return p;
        while (ptr[u] < G[u].size()) {
            int ei = G[u][ptr[u]++];
            Edge &e = E[ei];
            if (lvl[e.v] != lvl[u] + 1) continue;
            ll a = push(e.v, min(e.c - e.f, p));
            if (a <= 0) continue;
            e.f += a, E[ei ^ 1].f -= a; return a;
        }
        return 0;
    }

    ll maxflow() {
        ll f = 0;
        while (true) {
            lvl.assign(N, -1); queue<int> q;
            lvl[s] = 0; q.push(s);
            while (!q.empty()) {

```

```

                int u = q.front(); q.pop();
                for (int ei : G[u]) {
                    Edge &e = E[ei];
                    if (e.c - e.f <= 0 || lvl[e.v] != -1) continue;
                    lvl[e.v] = lvl[u] + 1; q.push(e.v);
                }
            }
            if (lvl[t] == -1) break;
            ptr.assign(N, 0); while (ll ff = push(s, INF)) f += ff;
        }
        return f;
    }
};

/* Flujo con demandas (no necesariamente el maximo)
Agregar s' y t' nuevos source and sink
c'(s', v) = sum(d(u, v) for u in V) \forall arista (s', v)
c'(v, t') = sum(d(v, w) for w in V) \forall arista (v, t')
c'(u, v) = c(u, v) - d(u, v) \forall aristas antiguas
c'(t, s) = INF (el flujo por esta arista es el flujo real)*/

```

3.6 dominator tree

```

//idom[i]=parent of i in dominator tree with root=rt, or -1
//if not exists
int
n, rnk[MAXN], pre[MAXN], anc[MAXN], idom[MAXN], semi[MAXN], low[MAXN];
vector<int> g[MAXN], rev[MAXN], dom[MAXN], ord;
void dfspre(int pos){
    rnk[pos]=SZ(ord); ord.pb(pos);
    for(auto x:g[pos]){
        rev[x].pb(pos);
        if(rnk[x]==n) pre[x]=pos, dfspre(x);
    }
}
int eval(int v){
    if(anc[v]<n&&anc[anc[v]]<n){
        int x=eval(anc[v]);
        if(rnk[semi[low[v]]]>rnk[semi[x]]) low[v]=x;
        anc[v]=anc[anc[v]];
    }
    return low[v];
}
void dominators(int rt){
    fore(i,0,n){
        dom[i].clear(); rev[i].clear();
        rnk[i]=pre[i]=anc[i]=idom[i]=n;
        semi[i]=low[i]=i;
    }
    ord.clear(); dfspre(rt);
    for(int i=SZ(ord)-1;i--){
        int w=ord[i];
        for(int v:rev[w]){
            int u=eval(v);
            if(rnk[semi[w]]>rnk[semi[u]]) semi[w]=semi[u];

```

```

        }
        dom[semi[w]].pb(w); anc[w]=pre[w];
        for(int v:dom[pre[w]]){
            int u=eval(v);
            idom[v]=(rnk[pre[w]]>rnk[semi[u]]?u:pre[w]);
        }
        dom[pre[w]].clear();
    }
    for(int w:ord) if(w!=rt&&idom[w]!=semi[w])
        idom[w]=idom[idom[w]];
    fore(i,0,n) if(idom[i]==n) idom[i]=-1;
}
}

```

3.7 eulerian

```

// path/tour for directed graphs. uncomment for undirected.
struct Euler {
    struct Edge { int v, rev; };
    vector<vector<Edge>> G; vector<Edge> P;
    Euler(int N = 0) : G(N) {}
    void add_edge(int u, int v) {
        G[u].push_back({v, (int)G[v].size()});
        // G[v].push_back({u, (int)G[u].size() - 1});
    }

    void go(int u) {
        while (G[u].size()) {
            Edge e = G[u].back(); G[u].pop_back();
            // if (e.v == -1) continue;
            // G[e.v][e.rev].v = -1;
            go(e.v); P.push_back(e);
        }

        // works ONLY if the vertex degrees are eulerian! check!
        vector<Edge> get_path(int u) {
            return P.clear(), go(u), reverse(P.begin(), P.end()), P;
        }
    }
};

```

3.8 floyd warshall

```

// calculate distances between every pair of nodes in O(V^3)
// time.
// works with negative edges, but not negative cycles.
void floyd(const vector<vector<pair<ll, int>>> &G,
vector<vector<ll>> &D) {
    int N = G.size();
    D.assign(N, vector<ll>(N, INF));
    rep(u, N) D[u][u] = 0;
    rep(u, N) for (auto [w, v] : G[u]) D[u][v] = w;
    rep(k, N) rep(u, N) rep(v, N)
        D[u][v] = min(D[u][v], D[u][k] + D[k][v]);
}

```

3.9 heavy light

```

struct Hld {
    vector<int> P, H, D, pos, top;

    Hld() {}
    void init(vector<vector<int>> &G) {
        int N = G.size();
        P.resize(N), H.resize(N), D.resize(N),
        pos.resize(N),
        top.resize(N);
        D[0] = -1, dfs(G, 0); int t = 0;
        rep(i, N) if (H[P[i]] != i) {
            int j = i;
            while (j != -1)
                { top[j] = i, pos[j] = t++; j = H[j]; }
        }

        int dfs(vector<vector<int>> &G, int i) {
            int w = 1, mw = 0;
            D[i] = D[P[i]] + 1, H[i] = -1;
            for (int c : G[i]) {
                if (c == P[i]) continue;
                P[c] = i; int sw = dfs(G, c); w += sw;
                if (sw > mw) H[i] = c, mw = sw;
            }
            return w;
        }

        // visit the log N segments in the path from u to v
        template <class OP>
        void path(int u, int v, OP op) {
            while (top[u] != top[v]) {
                if (D[top[u]] > D[top[v]]) swap(u, v);
                op(pos[top[v]], pos[v] + 1); v = P[top[v]];
            }
            if (D[u] > D[v]) swap(u, v);
            op(pos[u], pos[v] + 1); // value on node
            // op(pos[u]+1, pos[v] + 1); // value on edge
        }

        // an alternative to `path` that considers order.
        // calls `op` with an `l` <= `r` inclusive-exclusive
        // range, and a
        // boolean indicating if the query is forwards or
        // backwards.
        template <class OP>
        void path(int u, int v, OP op) {
            int lu = u, lv = v;
            while (top[lu] != top[lv])
                if (D[top[lu]] > D[top[lv]]) lu = P[top[lu]];
            else lv = P[top[lv]];
            int lca = D[lu] > D[lv] ? lv : lu;

```

```

        while (top[u] != top[lca])
            op(pos[top[u]], pos[u] + 1, false), u =
            P[top[u]];
        if (u != lca) op(pos[lca] + 1, pos[u] + 1, false);

        vector<int> stk;
        while (top[v] != top[lca])
            stk.push_back(v), v = P[top[v]];

        // op(pos[lca], pos[v] + 1, true); // value on node
        op(pos[lca] + 1, pos[v] + 1, true); // value on edge
        reverse(stk.begin(), stk.end());
        for (int w : stk) op(pos[top[w]], pos[w] + 1, true);
    }

    // commutative segment tree
    template <class T, class S>
    void update(S &seg, int i, T val) { seg.update(pos[i],
    val); }

    // commutative segment tree lazy
    template <class T, class S>
    void update(S &seg, int u, int v, T val) {
        path(u, v, [&](int l, int r) { seg.update(l, r,
    val); });
    }

    // commutative (lazy) segment tree
    template <class T, class S>
    T query(S &seg, int u, int v) {
        T ans =
        0; //
        neutral element
        path(u, v, [&](int l, int r) { ans += seg.query(l,
    r); }); // query op
        return ans;
    }
};

```

3.10 hungarian

```

// find a maximum gain perfect matching in the given
// bipartite complete graph.
// input: gain matrix (G_{xy} = benefit of joining vertex x
// in set X with vertex
// y in set Y).
// output: maximum gain matching in members `xy[x]` and
// `yx[y]`.
// runtime: O(N^3)
struct Hungarian {
    int N, qi, root;
    vector<vector<ll>> gain;
    vector<int> xy, yx, p, q, slackx;
    vector<ll> lx, ly, slack;
    vector<bool> S, T;

```

```

    void add(int x, int px) {
        S[x] = true, p[x] = px;
        rep(y, N) if (lx[x] + ly[y] - gain[x][y] < slack[y])
        {
            slack[y] = lx[x] + ly[y] - gain[x][y], slackx[y]
        = x;
        }
    }

    void augment(int x, int y) {
        while (x != -2) {
            yx[y] = x; swap(xy[x], y); x = p[x];
        }
    }

    void improve() {
        S.assign(N, false), T.assign(N, false), p.assign(N,
    -1);
        qi = 0, q.clear();
        rep(x, N) if (xy[x] == -1) {
            q.push_back(root = x), p[x] = -2, S[x] = true;
            break;
        }
        rep(y, N) slack[y] = lx[root] + ly[y] - gain[root]
    [y], slackx[y] = root;

        while (true) {
            while (qi < q.size()) {
                int x = q[qi++];
                rep(y, N) if (lx[x] + ly[y] == gain[x][y]
    && !T[y]) {
                    if (yx[y] == -1) return augment(x, y);
                    T[y] = true, q.push_back(yx[y]),
                add(yx[y], x);
                }
            }

            ll d = INF;
            rep(y, N) if (!T[y]) d = min(d, slack[y]);
            rep(x, N) if (S[x]) lx[x] -= d;
            rep(y, N) if (T[y]) ly[y] += d;
            rep(y, N) if (!T[y]) slack[y] -= d;

            rep(y, N) if (!T[y] && slack[y] == 0) {
                if (yx[y] == -1) return augment(slackx[y],
    y);
                T[y] = true;
                if (!S[yx[y]]) q.push_back(yx[y]),
                add(yx[y], slackx[y]);
            }
        }

        Hungarian(vector<vector<ll>> g)
    }

```

```

: N(g.size()), gain(g), xy(N, -1), yx(N, -1), lx(N,
-INF),
ly(N), slack(N), slackx(N) {
rep(x, N) rep(y, N) lx[x] = max(lx[x], ly[y]);
rep(i, N) improve();
}
};

```

3.11 kuhn

```

// get a maximum cardinality matching in a bipartite graph.
// input: adjacency lists.
// output: matching (in `mt` member).
// runtime: O(V E)
struct Kuhn {
vector<vector<int>> G;
int N, size;
vector<bool> seen;
vector<int> mt;

bool visit(int i) {
if (seen[i]) return false;
seen[i] = true;
for (int to : G[i])
if (mt[to] == -1 || visit(mt[to])) {
mt[to] = i;
return true;
}
return false;
}

Kuhn(vector<vector<int>> adj) : G(adj), N(G.size()),
mt(N, -1) {
rep(i, N) {
seen.assign(N, false);
size += visit(i);
}
}
};

```

3.12 lca

```

// calculates the lowest common ancestor for any two nodes
in O(log N) time,
// with O(N log N) preprocessing
struct Lca {
int N, K, t = 0;
vector<vector<int>> U;
vector<int> L, R;

Lca() {}
Lca(vector<vector<int>> &G) : N(G.size()), L(N), R(N) {
K = N <= 1 ? 0 : 32 - __builtin_clz(N - 1);
U.resize(K + 1, vector<int>(N));
visit(G, 0, 0);
}

```

```

rep(k, K) rep(u, N) U[k + 1][u] = U[k][U[k][u]];
}

void visit(vector<vector<int>> &G, int u, int p) {
L[u] = t++, U[0][u] = p;
for (int v : G[u]) if (v != p) visit(G, v, u);
R[u] = t++;
}

bool is_anc(int up, int dn) {
return L[up] <= L[dn] && R[dn] <= R[up];
}

int find(int u, int v) {
if (is_anc(u, v)) return u;
if (is_anc(v, u)) return v;
for (int k = K; k >= 0; k--)
if (is_anc(U[k][u], v)) k--;
else u = U[k][u];
return U[0][u];
}
};

```

3.13 maxflow mincost

```

// time: O(F V E) F is the maximum flow
// O(V E + F E log V) if bellman-ford is replaced by
johnson
struct Flow {
struct Edge {
int u, v;
ll c, w, f = 0;
};

int N, s, t;
vector<vector<int>> G;
vector<Edge> E;
vector<ll> d, b;
vector<int> p;

Flow() {}
Flow(int N, int s, int t) : N(N), s(s), t(t), G(N) {}

void add_edge(int u, int v, ll c, ll w) {
G[u].push_back(E.size());
E.push_back({u, v, c, w});
G[v].push_back(E.size());
E.push_back({v, u, 0, -w});
}

// naive distances with bellman-ford: O(V E)
void calcdists() {
p.assign(N, -1), d.assign(N, INF), d[s] = 0;
rep(i, N - 1) rep(ei, E.size()) {
Edge &e = E[ei];

```

```

ll n = d[e.u] + e.w;
if (d[e.u] < INF && e.c - e.f > 0 && n < d[e.v])
d[e.v] = n, p[e.v] = ei;
}

// johnsons potentials: O(E log V)
void calcdists() {
if (b.empty()) {
b.assign(N, 0);
// code below only necessary if there are
negative costs
rep(i, N - 1) rep(ei, E.size()) {
Edge &e = E[ei];
if (e.f < e.c) b[e.v] = min(b[e.v], b[e.u] +
e.w);
}
}
p.assign(N, -1), d.assign(N, INF), d[s] = 0;
priority_queue<pair<ll, int>> q;
q.push({0, s});
while (!q.empty()) {
auto [w, u] = q.top();
q.pop();
if (d[u] < -w + b[u]) continue;
for (int ei : G[u]) {
auto e = E[ei];
ll n = d[u] + e.w;
if (e.f < e.c && n < d[e.v]) {
d[e.v] = n, p[e.v] = ei;
q.push({b[e.v] - n, e.v});
}
}
}
b = d;

ll solve() {
b.clear();
ll ff = 0;
while (true) {
calcdists();
if (p[t] == -1) break;

ll f = INF;
for (int cur = t; p[cur] != -1; cur =
E[p[cur]].u)
f = min(f, E[p[cur]].c - E[p[cur]].f);
for (int cur = t; p[cur] != -1; cur =
E[p[cur]].u)
E[p[cur]].f += f, E[p[cur] ^ 1].f -= f;
ff += f;
}
return ff;
}

```

```

}
};

```

3.14 parallel dfs

```

struct Tree {
    int n, z[2];
    vector<vector<int>> g;
    vector<int> ex, ey, p, w, f, v[2];
    Tree(int n): g(n), w(n), f(n) {}
    void add_edge(int x, int y) {
        p.pb(g[x].size()); g[x].pb(ex.size());
        ex.pb(x); ey.pb(y);
        p.pb(g[y].size()); g[y].pb(ex.size());
        ex.pb(y); ey.pb(x);
    }
    bool go(int k) { // returns 1 if it finds new node
        int& x = z[k];
        while (x >= 0 &&
            (w[x] == g[x].size() || w[x] == g[x].size() - 1
            && (g[x].back() ^ 1) == f[x]))
            x = f[x] >= 0 ? ex[f[x]] - 1;
        if (x < 0) return false;
        if ((g[x][w[x]] ^ 1) == f[x]) w[x]++;
        int e = g[x][w[x]], y = ey[e]; f[y] = e;
        w[x]++; w[y] = 0; x = y; v[k].pb(x);
        return true;
    }
    vector<int> erase_edge(int e) {
        e *= 2; // erases eth edge, returns smaller comp
        int x = ex[e], y = ey[e]; p[g[x].back()] = p[e];
        g[x][p[e]] = g[x].back(); g[x].pop_back();
        p[g[y].back()] = p[e ^ 1]; g[y][p[e ^ 1]] = g[y].back();
        g[y].pop_back();
        f[x] = f[y] - 1; w[x] = w[y] = 0; z[0] = x; z[1] = y;
        v[0] = {x}; v[1] = {y};
        bool d0 = true, d1 = true; while (d0 && d1) d0 = go(0), d1 = go(1);
        return v[1 - d1];
    }
};

```

3.15 push relabel

```

#include "../common.h"

const ll INF = 1e18;

// maximum flow algorithm.
// to run, use `maxflow()`.
//
// time:  $O(V^2 \sqrt{E}) \leq O(V^3)$ 
// memory:  $O(V^2)$ 
struct PushRelabel {
    vector<vector<ll>> cap, flow;
    vector<ll> excess;

```

```

    vector<int> height;

    PushRelabel() {}
    void resize(int N) { cap.assign(N, vector<ll>(N)); }

    // push as much excess flow as possible from u to v.
    void push(int u, int v) {
        ll f = min(excess[u], cap[u][v] - flow[u][v]);
        flow[u][v] += f;
        flow[v][u] -= f;
        excess[v] += f;
        excess[u] -= f;
    }

    // relabel the height of a vertex so that excess flow
    may be pushed.
    void relabel(int u) {
        int d = INT32_MAX;
        rep(v, cap.size()) if (cap[u][v] - flow[u][v] > 0) d =
        = min(d, height[v]);
        if (d < INF) height[u] = d + 1;
    }

    // get the maximum flow on the network specified by
    `cap` with source `s`
    // and sink `t`.
    // node-to-node flows are output to the `flow` member.
    ll maxflow(int s, int t) {
        int N = cap.size(), M;
        flow.assign(N, vector<ll>(N));
        height.assign(N, 0), height[s] = N;
        excess.assign(N, 0), excess[s] = INF;
        rep(i, N) if (i != s) push(s, i);

        vector<int> q;
        while (true) {
            // find the highest vertices with excess
            q.clear(), M = 0;
            rep(i, N) {
                if (excess[i] <= 0 || i == s || i == t)
                    continue;
                if (height[i] > M) q.clear(), M = height[i];
                if (height[i] >= M) q.push_back(i);
            }
            if (q.empty()) break;
            // process vertices
            for (int u : q) {
                bool relab = true;
                rep(v, N) {
                    if (excess[u] <= 0) break;
                    if (cap[u][v] - flow[u][v] > 0 &&
                        height[u] > height[v])
                        push(u, v), relab = false;
                }
            }

```

```

                if (relab) {
                    relabel(u);
                    break;
                }
            }
            ll f = 0; rep(i, N) f += flow[i][t]; return f;
        }
    };

```

3.16 strongly connected components

```

/* time:  $O(V + E)$ , memory:  $O(V)$ 
after building:
    comp = map from vertex to component
            (components are toposorted, root first, leaf last)
    N = number of components
    G = condensation graph (component DAG)
byproducts:
    vgi = transposed graph
    order = reverse topological sort (leaf first, root last)
others:
    vn = number of vertices
    vg = original vertex graph */
struct Scc {
    int vn, N;
    vector<int> order, comp;
    vector<vector<int>> vg, vgi, G;
    void toposort(int u) {
        if (comp[u]) return;
        comp[u] = -1;
        for (int v : vg[u]) toposort(v);
        order.push_back(u);
    }
    bool carve(int u) {
        if (comp[u] != -1) return false;
        comp[u] = N;
        for (int v : vgi[u]) {
            carve(v);
            if (comp[v] != N) G[comp[v]].push_back(N);
        }
        return true;
    }
    Scc() {}
    Scc(vector<vector<int>> &g)
        : vn(g.size()), vg(g), comp(vn), vgi(vn), G(vn), N(0) {
        rep(u, vn) toposort(u);
        rep(u, vn) for (int v : vg[u]) vgi[v].push_back(u);
        invrep(i, vn) N += carve(order[i]);
    }
};

```

3.17 two sat


```
// calculate the solvability of a system of logical
// equations, where every equation is of the form `a or b`.
// `neg`: get negation of `u`
// `then`: `u` implies `v`
// `any`: `u` or `v`
// `set`: `u` is true
//
// after `solve` (O(V+E)) returns true, `sol` contains one
// possible solution.
// determining all solutions is O(V*E) hard (requires
// computing reachability in a DAG).
struct TwoSat {
    int N; vector<vector<int>> G;
    Scc scc; vector<bool> sol;
    TwoSat(int n) : N(n), G(2 * n), sol(n) {}
    TwoSat() {}

    int neg(int u) { return (u + N) % (2 * N); }
    void then(int u, int v) { G[u].push_back(v),
G[neg(v)].push_back(neg(u)); }
    void any(int u, int v) { then(neg(u), v); }
    void set(int u) { G[neg(u)].push_back(u); }

    bool solve() {
        scc = Scc(G);
        rep(u, N) if (scc.comp[u] == scc.comp[neg(u)])
return false;
        rep(u, N) sol[u] = (scc.comp[u] > scc.comp[neg(u)]);
        return true;
    }
};
```

4 Implementation

4.1 common template and bit tricks

```
#pragma GCC optimize("Ofast")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt")
#pragma GCC
target("avx,avx2,f16c,fma,sse3,ssse3,sse4.1,sse4.2")
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define repx(i, a, b) for (int i = a; i < b; i++)
#define invrep(i, n) repx(i, 0, n)
#define invrepx(i, a, b) for (int i = b - 1; i >= a; i--)
#define invrep(i, n) invrepx(i, 0, n)
// Command to check time and memory usage:
//      /usr/bin/time -v ./tmp
// See "Maximum resident set size" for max memory used
// Commands for interactive checker:
//      mkfifo fifo
//      (./solution < fifo) | (./interactor > fifo)
// Does not work on the Windows file system, i.e., /mnt/c/
```

```
// The special fifo file must be used, otherwise the
// solution will not wait for input and will read EOF
y = x & (x-1) // Turn off rightmost lbit
y = x & (-x) // Isolate rightmost lbit
y = x | (x-1) // Right propagate rightmost lbit(fill in 1s)
y = x | (x+1) // Turn on rightmost 0bit
y = ~x & (x+1) // Isolate rightmost 0bit
// If x is of long type, use __builtin_popcountll(x)
// If x is of long long type, use __builtin_popcountll(x)
// 1. Counts the number of one's(set bits) in an integer.
__builtin_popcount(x)
// 2. Checks the Parity of a number. Returns true(1) if the
// number has odd number of set bits, else it returns
// false(0) for even number of set bits.
__builtin_parity(x)
// 3. Counts the leading number of zeros of the integer.
__builtin_clz(x)
// 4. Counts the trailing number of zeros of the integer.
__builtin_ctz(x)
// 5. Returns 1 + the index of the least significant 1-bit.
__builtin_ffs(x) // If x == 0, returns 0.
// Iterate over non empty subsets of bitmask
for(int s=m;s=(s-1)&m) // Decreasing order
for(int s=0;s=s-m&m;) // Increasing order
```

4.2 dp convex hull trick

```
struct Line {
    mutable ll a, b, c;

    bool operator<(Line r) const { return a < r.a; }
    bool operator<(ll x) const { return c < x; }
};
// dynamically insert `a*x + b` lines and query for maximum
// at any x all operations have complexity O(log N)
struct LineContainer : multiset<Line, less<>> {
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator x, iterator y) {
        if (y == end()) return x->c = INF, 0;
        if (x->a == y->a) x->c = x->b > y->b ? INF : -INF;
        else x->c = div(y->b - x->b, x->a - y->a);
        return x->c >= y->c;
    }

    void add(ll a, ll b) {
        // a *= -1, b *= -1 // for min
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
erase(y));
        while ((y = x) != begin() && (--x)->c >= y->c)
isect(x, erase(y));
    }
};
```

```
}

ll query(ll x) {
    if (empty()) return -INF; // INF for min
    auto l = *lower_bound(x);
    return l.a * x + l.b;
    // return -l.a * x - l.b; // for min
}
};
```

4.3 dp divide and conquer

```
// for every index i assign an optimal index j, such that
// cost(i, j) is minimal for every i. the property that if
// i2 >= i1 then j2 >= j1 is exploited (monotonic condition)
// calculate optimal index for all indices in range [l, r)
// knowing that the optimal index for every index in this
// range is within [optl, opttr).
// time: O(N log N)
void calc(vector<int> &opt, int l, int r, int optl, int opttr){
    if (l == r) return;
    int i = (l + r) / 2;
    ll optc = INF;
    int optj;
    repx(j, optl, opttr) {
        ll c = i + j; // cost(i, j)
        if (c < optc) optc = c, optj = j;
    }
    opt[i] = optj;
    calc(opt, l, i, optl, optj + 1);
    calc(opt, i + 1, r, optj + 1, opttr);
}
```

4.4 dynamic connectivity

```
struct DC {
    int n; Dsu D;
    vector<vector<pair<int, int>>> t;
    DC(int N) : n(N), D(N), t(2 * N) {}
    // add edge p to all times in interval [l, r]
    void upd(int l, int r, pair<int, int> p) {
        for (l += n, r += n; l < r; l >= 1, r >= 1) {
            if (l & 1) t[l++].push_back(p);
            if (r & 1) t[--r].push_back(p);
        }
    }
    void process(int u = 1) { // process all queries
        for (auto &e : t[u]) D.unite(e.first, e.second);
        if (u >= n) {
            // do stuff with D at time u - n
        } else process(2 * u), process(2 * u + 1);
        for (auto &e : t[u]) D.rollback();
    }
};
```

4.5 hash container

```
namespace{//add (#define tpl template)(#define ty typename)
    tpl<ty T> size_t mk_h(const T& v){return hash<T>()(v);}
    void h_cmb(size_t& h, const size_t& v)
    { h ^= v + 0x9e3779b9 + (h << 6) + (h >> 2); }
    tpl<ty T> struct h_ct{size_t operator()(const T& v) const{
        size_t h=0;for(const auto& e:v){h_cmb(h,mk_h(e));}return h;
    }};
}namespace std{//support for pair<T,U>, vector<T> & map<T,U>
    tpl<ty T, ty U> struct hashpair<T, U>{
        size_t operator()(const pair<T,U>& v) const
        {size_t h=mk_h(v.first);h_cmb(h, mk_h(v.second));return h;}
    };
    tpl<ty... T>struct hash<vector<T...>>:h_ct<vector<T...>>{};
    tpl<ty... T>struct hash<map<T...>>:h_ct<map<T...>>{}; }
```

4.6 mo

```
struct Query { int l, r, idx; };

// answer segment queries using only `add(i)`, `remove(i)`
// and `get()`
// functions.
//
// complexity: O((N + Q) * sqrt(N) * F)
// N = length of the full segment
// Q = amount of queries
// F = complexity of the `add`, `remove` functions
template <class A, class R, class G, class T>
void mo(vector<Query> &queries, vector<T> &ans, A add, R
remove, G get) {
    int Q = queries.size(), B = (int)sqrt(Q);
    sort(queries.begin(), queries.end(), [&](Query &a, Query
&b) {
        return make_pair(a.l / B, a.r) < make_pair(b.l / B,
b.r);
    });
    ans.resize(Q);

    int l = 0, r = 0;
    for (auto &q : queries) {
        while (r < q.r) add(r), r++;
        while (l > q.l) l--, add(l);
        while (r > q.r) r--, remove(r);
        while (l < q.l) remove(l), l++;
        ans[q.idx] = get();
    }
}
```

4.7 ordered set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
```

```
typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> ordered_set;
// find_by_order(i) -> iterator to ith element
// order_of_key(k) -> position (int) of lower_bound of k
```

4.8 unordered map

```
static mt19937 rng(chrono::steady_clock::now()
    .time_since_epoch().count());
#define rnd(a,b) (uniform_int_distribution<ll>(a,b)(rng))

struct Hash {
    size_t operator()(const ll &x) const {
        const uint64_t RAND = chrono::steady_clock::now()
            .time_since_epoch().count();
        uint64_t z = x + RAND + 0x9e3779b97f4a7c15;
        z = (z ^ (z >> 30)) * 0xbf58476d1ce4e5b9;
        z = (z ^ (z >> 27)) * 0x94d049bb133111eb;
        return z ^ (z >> 31);
    }
};
template<class T, class U>using umap=unordered_map<T,U,Hash>;
template<class T> using uset = unordered_set<T, Hash>;
```

5 Math

5.1 arithmetic

```
inline int floor_log2(int n)
{ return n <= 1 ? 0 : 31 - __builtin_clz(n); }
inline int ceil_log2(int n)
{ return n <= 1 ? 0 : 32 - __builtin_clz(n - 1); }
inline ll floordiv(ll a, ll b) {return a/b-((a^b)<0&&a%b);}
inline ll ceildiv(ll a, ll b) {return a/b+((a^b)>=0&&a%b);}
```

5.2 berlekamp massey linear

recurrence

```
vector<int> BM(vector<int> x) {
    vector<int> ls, cur;
    int lf, ld;
    rep(i, x.size()) {
        ll t = 0;
        rep(j, cur.size()) t = (t+x[i-j-1]*(ll)cur[j])%MOD;
        if ((t - x[i]) % MOD == 0) continue;
        if (!cur.size()) {
            cur.resize(i + 1); lf = i; ld = (t-x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * bin_exp(ld, MOD - 2) % MOD;
        vector<int> c(i - lf - 1); c.push_back(k);
        rep(j, ls.size()) c.push_back((-ls[j] * k % MOD);
```

```
if (c.size() < cur.size()) c.resize(cur.size());
rep(j, cur.size()) c[j] = (c[j] + cur[j]) % MOD;
if (i - lf + ls.size() >= cur.size())
    ls = cur, lf = i, ld = (t - x[i]) % MOD;
    cur = c;
}
rep(i, cur.size()) cur[i] = (cur[i] % MOD + MOD) % MOD;
return cur;
}
// Linear Recurrence
ll MOD = 998244353;
ll LOG = 60;
struct LinearRec{
    typedef vector<int> vi;
    int n; vi terms, trans; vector<vi> bin;
    vi add(vi &a, vi &b){
        vi res(n*2+1);
        rep(i,n+1) rep(j,n+1)
            res[i+j]=(res[i+j]*1LL+(ll)a[i]*b[j])%MOD;
        for(int i=2*n; i>n; --i){
            rep(j,n)
                res[i-1-j]=(res[i-1-j]*1LL+(ll)res[i]*trans[j])%MOD;
            res[i]=0;
        }
        res.erase(res.begin()+n+1,res.end());
        return res;
    }
    LinearRec(vi &terms, vi &trans):terms(terms),trans(trans){
        n=trans.size();vi a(n+1);a[1]=1;
        bin.push_back(a);
        repx(i,1,LOG)bin.push_back(add(bin[i-1],bin[i-1]));
    }
    int calc(ll k){
        vi a(n+1);a[0]=1;
        rep(i,LOG)if((k>>i)&1)a=add(a,bin[i]);
        int ret=0;
        rep(i,n)ret=((ll)ret+(ll)a[i+1]*terms[i])%MOD;
        ret = ret%MOD + MOD;
        return ret%MOD;
    }
};
```

5.3 crt

```
pair<ll, ll> solve_crt(const vector<pair<ll, ll>> &eqs) {
    ll a0 = eqs[0].first, p0 = eqs[0].second;
    repx(i, 1, eqs.size()) {
        ll a1 = eqs[i].first, p1 = eqs[i].second;
        ll k1, k0;
        ll d = ext_gcd(p1, p0, k1, k0);
        a0 -= a1;
        if (a0 % d != 0) return {-1, -1};
        p0 = p0 / d * p1;
        a0 = a0 / d * k1 % p0 * p1 % p0 + a1;
        a0 = (a0 % p0 + p0) % p0;
```

```

}
return {a0, p0};
}

```

5.4 debruijseq

```

vector<int> deBruijnSeq(int k, int n) { /// Recursive FKM
    if (k == 1) return {0};
    vector<int> seq, aux(n+1);
    function<void(int,int)> gen = [&](int t, int p) {
        if (t > n) { // +lyndon word of len p
            if (n%p == 0) repx(i,1,p+1) seq.push_back(aux[i]);
        } else {
            aux[t] = aux[t-p]; gen(t+1,p);
            while (++aux[t] < k) gen(t+1,t);
        }
    };
    gen(1,1); return seq;
}

```

5.5 discrete log

```

// discrete logarithm log_a(b).
// solve b ^ x = a (mod M) for the smallest x.
// returns -1 if no solution is found.
//
// time: O(sqrt(M))
ll dlog(ll a, ll b, ll M) {
    ll k = 1, s = 0;
    while (true) {
        ll g = __gcd(b, M);
        if (g <= 1) break;
        if (a == k) return s;
        if (a % g != 0) return -1;
        a /= g, M /= g, s += 1, k = b / g * k % M;
    }
    ll N = sqrt(M) + 1;

    umap<ll, ll> r;
    rep(q, N + 1) {
        r[a] = q;
        a = a * b % M;
    }

    ll bN = binexp(b, N, M), bNp = k;
    repx(p, 1, N + 1) {
        bNp = bNp * bN % M;
        if (r.count(bNp)) return N * p - r[bNp] + s;
    }
    return -1;
}

```

5.6 fast hadamard transform

```

ll c1[MAXN+9], c2[MAXN+9]; // MAXN must be power of 2!
void fht(ll* p, int n, bool inv) {
    for(int l=1; 2*l<=n; l*=2) for(int
    i=0; i<n; i+=2*l) fore(j, 0, l) {
        ll u=p[i+j], v=p[i+l+j];
        if(!inv) p[i+j]=u+v, p[i+l+j]=u-v; // XOR
        else p[i+j]=(u+v)/2, p[i+l+j]=(u-v)/2;
        //if(!inv) p[i+j]=v, p[i+l+j]=u+v; // AND
        //else p[i+j]=-u+v, p[i+l+j]=u;
        //if(!inv) p[i+j]=u+v, p[i+l+j]=u; // OR
        //else p[i+j]=v, p[i+l+j]=u-v;
    }
}
// like polynomial multiplication, but XORing exponents
// instead of adding them (also ANDing, ORing)
vector<ll> multiply(vector<ll>& p1, vector<ll>& p2) {
    int n=1<<(32-__builtin_clz(max(SZ(p1), SZ(p2))-1));
    fore(i, 0, n) c1[i]=0, c2[i]=0;
    fore(i, 0, SZ(p1)) c1[i]=p1[i];
    fore(i, 0, SZ(p2)) c2[i]=p2[i];
    fht(c1, n, false); fht(c2, n, false);
    fore(i, 0, n) c1[i]*=c2[i];
    fht(c1, n, true);
    return vector<ll>(c1, c1+n);
}

```

5.7 fft

```

using cd = complex<double>;
const double PI = acos(-1);
// compute the DFT of a power-of-two-length sequence.
// if `inv` is true, computes the inverse DFT.
void fft(vector<cd> &a, bool inv) {
    int N = a.size(), k = 0, b;
    assert(N == 1 << __builtin_ctz(N));
    repx(i, 1, N) {
        for (b = N >> 1; k & b; k ^= b, b >>= 1;
            if (i < (k ^ b)) swap(a[i], a[k]);
        }
    }
    for (int l = 2; l <= N; l <= 1) {
        double ang = 2 * PI / l * (inv ? -1 : 1);
        cd wL(cos(ang), sin(ang));
        for (int i = 0; i < N; i += l) {
            cd w = 1;
            rep(j, l / 2) {
                cd u = a[i + j], v = a[i + j + l / 2] * w;
                a[i + j] = u + v;
                a[i + j + l / 2] = u - v;
                w *= wL;
            }
        }
    }
    if (inv) rep(i, N) a[i] /= N;
}
const ll MOD = 998244353, ROOT = 15311432;

```

```

// const ll MOD = 2130706433, ROOT = 1791270792;
// const ll MOD = 9223372036737335297ll, ROOT =
5320774565496356983ll;
void find_root_of_unity(ll M) {
    ll c = M - 1, k = 0;
    while (c % 2 == 0) c /= 2, k += 1;
    // find proper divisors of M - 1
    vector<ll> divs;
    for (ll d = 1; d < c; d++) {
        if (d * d > c) break;
        if (c % d == 0) rep(i, k + 1) divs.push_back(d <<
i);
    }
    rep(i, k) divs.push_back(c << i);
    // find any primitive root of M
    ll G = -1;
    repx(g, 2, M) {
        bool ok = true;
        for (ll d : divs) ok &= (binexp(g, d, M) != 1);
        if (ok) {
            G = g;
            break;
        }
    }
    assert(G != -1);
    ll w = binexp(G, c, M);
    cerr << "M = c * 2^k + 1" << endl;
    cerr << " M = " << M << endl;
    cerr << " c = " << c << endl;
    cerr << " k = " << k << endl;
    cerr << " w^(2^k) == 1" << endl;
    cerr << " w = g^((M-1)/2^k) = g^c" << endl;
    cerr << " g = " << G << endl;
    cerr << " w = " << w << endl;
}
// compute the DFT of a power-of-two-length sequence, modulo
a special prime
// number with an Nth root of unity, where N is the length
of the sequence.
void ntt(vector<ll> &a, bool inv) {
    vector<ll> wn;
    for (ll p = ROOT; p != 1; p = p * p % MOD)
wn.push_back(p);
    int N = a.size(), k = 0, b;
    assert(N == 1 << __builtin_ctz(N) && N <= 1 <<
wn.size());
    rep(i, N) a[i] = (a[i] % MOD + MOD) % MOD;
    repx(i, 1, N) {
        for (b = N >> 1; k & b; k ^= b, b >>= 1;
            if (i < (k ^ b)) swap(a[i], a[k]);
        }
    }
    for (int l = 2; l <= N; l <= 1) {
        ll wL = wn[wn.size() - __builtin_ctz(l)];
        if (inv) wL = multinv(wL, MOD);
        for (int i = 0; i < N; i += l) {

```

```

    ll w = 1;
    repx(j, 0, l / 2) {
        ll u = a[i + j], v = a[i + j + l / 2] * w %
MOD;

        a[i + j] = (u + v) % MOD;
        a[i + j + l / 2] = (u - v + MOD) % MOD;
        w = w * wl % MOD;
    }
}
ll q = multinv(N, MOD);
if (inv) rep(i, N) a[i] = a[i] * q % MOD;
}
void convolve(vector<cd> &a, vector<cd> b, int n) {
    n = 1 << (32 - __builtin_clz(2 * n - 1));
    a.resize(n), b.resize(n);
    fft(a, false), fft(b, false);
    rep(i, n) a[i] *= b[i];
    fft(a, true);
}

```

5.8 gauss

```

const double EPS = 1e-9;
// solve a system of equations.
// complexity: O(min(N, M) * N * M)
// `a` is a list of rows
// the last value in each row is the result of the equation
// return values:
// 0 -> no solutions
// 1 -> unique solution, stored in `ans`
// -1 -> infinitely many solutions, one of which is stored
in `ans`
// UNTESTED
int gauss(vector<vector<double>> a, vector<double> &ans) {
    int N = a.size(), M = a[0].size() - 1;
    vector<int> where(M, -1);
    for (int j = 0, i = 0; j < M && i < N; j++) {
        int sel = i;
        repx(k, i, N) if (abs(a[k][j]) > abs(a[sel][j])) sel
= k;

        if (abs(a[sel][j]) < EPS) continue;
        repx(k, j, M + 1) swap(a[sel][k], a[i][k]);
        where[j] = i;
        rep(k, N) if (k != i) {
            double c = a[k][j] / a[i][j];
            repx(l, j, M + 1) a[k][l] -= a[i][l] * c;
        }
        i++;
    }
    ans.assign(M, 0);
    rep(i, M) if (where[i] != -1) ans[i] = a[where[i]][M] /
a[where[i]][i];
    rep(i, N) {
        double sum = 0;

```

```

        rep(j, M) sum += ans[j] * a[i][j];
        if (abs(sum - a[i][M]) > EPS) return 0;
    }
    rep(i, M) if (where[i] == -1) return -1;
    return 1;
}

```

5.9 linear diophantine

```

ii extendedEuclid(ll a, ll b){
    ll x, y; //a*x + b*y = gcd(a,b)
    if (b == 0) return {1, 0};
    auto p = extendedEuclid(b, a%b);
    x = p.second;
    y = p.first - (a/b)*x;
    if(a*x + b*y == -__gcd(a,b)) x=-x, y=-y;
    return {x, y};
}
pair<ii, ii> diophantine(ll a, ll b, ll r){
    //a*x+b*y=r where r is multiple of gcd(a,b);
    ll d = __gcd(a, b);
    a/=d; b/=d; r/=d;
    auto p = extendedEuclid(a, b);
    p.first*=r; p.second*=r;
    assert(a*p.first + b*p.second == r);
    return {p, {-b, a}}; //solutions: p+t*ans.second
}

```

5.10 matrix

```

typedef vector<vector<double>> Mat;
Mat matmul(Mat l, Mat r) {
    int n = l.N, m = r.M, p = l.M; assert(l.M == r.N);
    Mat a(n, vector<double>(m)); // neutral
    rep(i, n) rep(j, m)
        rep(k, p) a[i][j] = a[i][j] + l[i][k] * r[k][j];
    return a;
}

double reduce(vector<vector<double>> &A) {
    int n = A.size(), m = A[0].size();
    int i = 0, j = 0; double r = 1.;
    while (i < n && j < m) {
        int l = i;
        repx(k, i+1, n) if(abs(A[k][j]) > abs(A[l][j])) l=k;
        if (abs(A[l][j]) < EPS) { j++; r = 0.; continue; }
        if (l != i) { r = -r; swap(A[i], A[l]); }
        r *= A[i][j];
        for (int k = m - 1; k >= j; k--) A[i][k] /= A[i][j];
        repx(k, 0, n) {
            if (k == i) continue;
            for(int l=m-1;l>=j;l--)A[k][l]-=A[k][j]*A[i][l];
        }
        i++, j++;
    }
}

```

```

        return r; // returns determinant
    }
}

```

5.11 matroidisect

```

//O(G*I^{1.5}) calls to oracle
//G ground set, I independent set
//MatroidIsect<Gmat,Cmat> M(ed.size(),Gmat(ed),Cmat(col));

```

```

struct Gmat { // graphic matroid
    int V = 0; vector<ii> ed; Dsu D;
    Gmat(vector<ii> _ed):ed(_ed){
        map<int,int> m; for(auto &t: ed) m[t.first] =
m[t.second] = 0;
        for(auto &t: m) t.second = V++;
        for(auto &t: ed) t.first = m[t.first], t.second =
m[t.second];
    }
    void clear() { D.p = vector<int>(V, -1); }
    void ins(int i) { D.unite(ed[i].first, ed[i].second); }
    bool indep(int i) { return !D.sameSet(ed[i].first,
ed[i].second); }
};

```

```

struct Cmat { // colorful matroid
    int C = 0; vector<int> col; vector<bool> used;
    Cmat(vector<int> col):col(col) {for(auto &t: col) C =
max(C, t+1);}
    void clear() { used.assign(C, 0); }
    void ins(int i) { used[col[i]] = 1; }
    bool indep(int i) { return !used[col[i]]; }
};

template<class M1, class M2> struct MatroidIsect {
    int n; vector<bool> iset; M1 m1; M2 m2;
    bool augment() {
        vector<int> pre(n+1,-1); queue<int> q({n});
        while(q.size()){
            int x = q.front(); q.pop();
            if (iset[x]) {
                m1.clear(); rep(i,n) if (iset[i] && i != x)
m1.ins(i);
                rep(i,n) if (!iset[i] && pre[i] == -1 &&
m1.indep(i))
                    pre[i] = x, q.push(i);
            } else {
                auto backE = [&]() { // back edge
                    m2.clear();
                    rep(c,2)rep(i,n)if((x==i||
iset[i])&&(pre[i]==-1)==c){
                        if (!m2.indep(i))return c?
pre[i]=x,q.push(i),i:-1;
                        m2.ins(i); }
                    return n;
                };
                for (int y; (y = backE()) != -1;) if (y == n) {

```

```

    for(; x != n; x = pre[x]) iset[x] = !iset[x];
    return 1; }
}
return 0;
}
MatroidIsect(int n, M1 m1, M2 m2):n(n), m1(m1), m2(m2) {
    iset.assign(n+1,0); iset[n] = 1;
    m1.clear(); m2.clear(); // greedily add to basis
    invrep(i,n) if (m1.indep(i) && m2.indep(i))
        iset[i] = 1, m1.ins(i), m2.ins(i);
    while (augment());
}
};

```

5.12 mobius

```

short mu[MAXN] = {0,1};
void mobius(){
    repx(i,1,MAXN)if(mu[i])for(int j=i+i;j<MAXN;j+=i)mu[j]-
    =mu[i];
}

```

5.13 multinv

```

// a * x + b * y == gcd(a, b)
ll ext_gcd(ll a, ll b, ll &x, ll &y) {
    if (b == 0) { x = 1, y = 0; return a; }
    ll d = ext_gcd(b, a % b, y, x); y -= a / b * x; return d;
}

// inverse exists if and only if a and M are coprime
// if M is prime: multinv(a, M) = (a**(M-2)) % M
ll multinv(ll a, ll M)
{ ll x, y; ext_gcd(a, M, x, y); return x; }

// all modular inverses from 1 to inv.size()-1
void multinv_all(vector<ll> &inv) {
    inv[1] = 1;
    repx(i, 2, inv.size())
        inv[i] = MOD - (MOD / i) * inv[MOD % i] % MOD;
}

```

5.14 permtree

```

struct PermTree {
    vector<int> P; int n; // N = number of nodes in perm tree
    int N = 0, root; vector<vector<int>> child;
    vector<ii> inter, range; vector<int> typ, loc; // inter =
    index range in perm
    void init(vector<int> _P) {
        P = _P; n = P.size(); Stl L(n);
        vector<int> mn{-1}, mx{-1}, st;
        rep(i, n){
            if(i) L.update(0, i, -1);

```

```

        while (mn.back() != -1 && P[mn.back()] > P[i]) {
            int t = mn.back(); mn.pop_back();
            L.update(mn.back()+1,t+1,P[t]);
        }
        L.update(mn.back()+1,i+1,-P[i]); mn.push_back(i);

        while (mx.back() != -1 && P[mx.back()] < P[i]) {
            int t = mx.back(); mx.pop_back();
            L.update(mx.back()+1,t+1,-P[t]);
        }
        L.update(mx.back()+1,i+1,P[i]); mx.push_back(i);

        int cur = N++; loc.push_back(cur);
        inter.push_back({i,i}); range.push_back({P[i],P[i]});
        typ.push_back(0); child.emplace_back();
        auto add = [](ii a, ii b) -> ii { return {min(a.first,
        b.first), max(a.second,b.second)};};
        auto adj = [&](int x, int y) { ii a = range[x], b =
        range[y];
            return a.second+1 == b.first || b.second+1 ==
        a.first;};
        while(st.size()){
            if(adj(st.back(), cur)){
                if(child[st.back()].size() &&
        adj(child[st.back()].back(),cur)){
                    inter[st.back()] =
        add(inter[st.back()],inter[cur]);
                    range[st.back()] =
        add(range[st.back()],range[cur]);
                    child[st.back()].push_back(cur); cur =
        st.back(); st.pop_back();
                } else { // make new join node
                    int CUR = N++;

        inter.push_back(add(inter[cur],inter[st.back()]));
        range.push_back(add(range[cur],range[st.back()]));
        typ.push_back(range[st.back()] < range[cur] ?
        1 : 2);
                    child.push_back({st.back(),cur}); cur = CUR;
        st.pop_back();
                }
                continue;
            }
            if (L.query(0, inter[cur].first) != 0) break;
            int CUR = N++;
            inter.push_back(inter[cur]);
            range.push_back(range[cur]); typ.push_back(0);
            child.push_back({cur});
            auto len = [](ii p) { return p.second-p.first;};
            do{
                inter[CUR] = add(inter[CUR],inter[st.back()]);
                range[CUR] = add(range[CUR],range[st.back()]);
                child[CUR].push_back(st.back()); st.pop_back();

```

```

        } while (len(inter.back()) != len(range.back()));
        reverse(child[CUR].begin(), child[CUR].end()); cur =
        CUR;
        }
        st.push_back(cur);
        }
        root = st.back();
        }
    };
};

```

5.15 polar rho

```

ll mulmod(ll a, ll b, ll m) {
    ll r=a*b-(ll)((long double)a*b/m+.5)*m;
    return r<0?r+m:r;
}
bool is_prime_prob(ll n, int a){
    if(n==a)return true;
    ll s=0,d=n-1;
    while(d%2==0)s++,d/=2;
    ll x=expmod(a,d,n);
    if((x==1)|| (x+1==n))return true;
    fore(_,0,s-1){
        x=mulmod(x,x,n);
        if(x==1)return false;
        if(x+1==n)return true;
    }
    return false;
}
bool rabin(ll n){ // true iff n is prime
    if(n==1)return false;
    int ar[]={2,3,5,7,11,13,17,19,23};
    fore(i,0,9)if(!is_prime_prob(n,ar[i]))return false;
    return true;
}
ll rho(ll n){
    if(!(n&1))return 2;
    ll x=2,y=2,d=1;
    ll c=rand()%n+1;
    while(d==1){
        x=(mulmod(x,x,n)+c)%n;
        fore(it,0,2) y=(mulmod(y,y,n)+c)%n;
        if(x>=y)d=__gcd(x-y,n);
        else d=__gcd(y-x,n);
    }
    return d==n?rho(n):d;
}
void fact(ll n, map<ll,int>& f){ //0 (lg n)^3
    if(n==1)return;
    if(rabin(n)){f[n]++;return;}
    ll q=rho(n);fact(q,f);fact(n/q,f);
}
// optimized version: replace rho and fact with the
following:
const int MAXP=1e6+1; // sieve size

```



```

int sv[MAXP]; // sieve
ll add(ll a, ll b, ll m){return (a+=b)<m?a:a-m;}
ll rho(ll n){
    static ll s[MAXP];
    while(1){
        ll x=rand()%n,y=x,c=rand()%n;
        ll *px=s,*py=s,v=0,p=1;
        while(1){
            *py+=y=add(mulmod(y,y,n),c,n);
            *py+=y=add(mulmod(y,y,n),c,n);
            if((x=*px++)==y)break;
            ll t=p; p=mulmod(p,abs(y-x),n);
            if(!p)return __gcd(t,n);
            if(++v==26){
                if((p=__gcd(p,n))>1&&p<n)return p;
                v=0;
            }
        }
        if(v&&(p=__gcd(p,n))>1&&p<n)return p;
    }
}
void init_sv(){ for(i,2,MAXP)if(!sv[i])for(ll
j=i;j<MAXP;j+=i)sv[j]=i; }
void fact(ll n,map<ll,int>&f){//call init_sv first!
    for(auto&& p:f)while(n%p.fst==0)p.snd++,n/=p.fst;
    if(n<MAXP)while(n>1)f[sv[n]]++,n/=sv[n];
    else if(rabin(n))f[n]++;
    else {ll q=rho(n);fact(q,f);fact(n/q,f);}
}

```

5.16 polynomials

```

typedef int tp; // type of polynomial
template<class T=tp>
struct poly { // poly<T> : 1 variable, poly<poly<T>>: 2
variables, etc.
    vector<T> c;
    T& operator[](int k){return c[k];}
    poly(vector<T>& c):c(c){}
    poly(initializer_list<T> c):c(c){}
    poly(int k):c(k){}
    poly(){}
    poly operator+(poly<T> o);
    poly operator*(tp k);
    poly operator*(poly o);
    poly operator-(poly<T> o){return *this+(o*-1);}
    T operator()(tp v){
        T sum(0);
        for(int i=c.size()-1;i>=0;--i)sum=sum*v+c[i];
        return sum;
    }
};
// example: p(x,y)=2*x^2+3*x*y-y+4
// poly<poly<T>> p={{4,-1},{0,3},{2}}
// printf("%d\n",p(2)(3)) // 27 (p(2,3))

```

```

set<tp> roots(poly<T> p){ // only for integer polynomials
    set<tp> r;
    while(!p.c.empty()&&!p.c.back())p.c.pop_back();
    if(!p(0))r.insert(0);
    if(p.c.empty())return r;
    tp a0=0,an=abs(p.c.size()-1);
    for(int k=0;!a0;a0=abs(p[k++]));
    vector<tp> ps,qs;
    fore(i,1,sqrt(a0)+1)if(a0%i==0)ps.pb(i),ps.pb(a0/i);
    fore(i,1,sqrt(an)+1)if(an%i==0)qs.pb(i),qs.pb(an/i);
    for(auto pt:ps)for(auto qt:qs)if(pt*qt==0){
        tp x=pt/qt;
        if(!p(x))r.insert(x);
        if(!p(-x))r.insert(-x);
    }
    return r;
}
pair<poly<T>,tp> ruffini(poly<T> p, tp r){ // returns pair
(result,rem)
    int n=p.c.size()-1;
    vector<tp> b(n);
    b[n-1]=p[n];
    for(int k=n-2;k>=0;--k)b[k]=p[k+1]+r*b[k+1];
    return {poly<T>(b),p[0]+r*b[0]};
}
// only for double polynomials
pair<poly<T>,poly<T>> polydiv(poly<T> p, poly<T> q){ // returns
pair (result,rem)
    int n=p.c.size()-q.c.size()+1;
    vector<tp> b(n);
    for(int k=n-1;k>=0;--k){
        b[k]=p.c.back()/q.c.back();
        fore(i,0,q.c.size())p[i+k]-=b[k]*q[i];
        p.c.pop_back();
    }
    while(!p.c.empty()&&abs(p.c.back())<EPS)p.c.pop_back();
    return {poly<T>(b),p};
}
// only for double polynomials
poly<T> interpolate(vector<tp> x, vector<tp> y){
    poly<T> q={1},S={0};
    for(tp a:x)q=poly<T>({-a,1})*q;
    fore(i,0,x.size()){
        poly<T> Li=ruffini(q,x[i]).fst;
        Li=Li*(1.0/Li(x[i])); // change for int polynomials
        S=S+Li*y[i];
    }
    return S;
}

```

5.17 primes

```

// counts the divisors of a positive integer in O(sqrt(n))
ll count_divisors(ll x){
    ll divs = 1, i = 2;

```

```

    for (ll divs = 1, i = 2; x > 1; i++) {
        if (i * i > x) { divs *= 2; break; }
        for (ll d = divs; x % i == 0; x /= i) divs *= d;
    }
    return divs;
}
// gets the prime factorization of a number in O(sqrt(n))
vector<pair<ll, int>> factorize(ll x) {
    vector<pair<ll, int>> f;
    for (ll k = 2; x > 1; k++) {
        if (k * k > x) { f.push_back({x, 1}); break; }
        int n = 0;
        while (x % k == 0) x /= k, n++;
        if (n > 0) f.push_back({k, n});
    }
    return f;
}
// iterate over all divisors of a number.
// divisor count upper bound: n^(1.07 / ln ln n)
template <class OP>
void divisors(ll x, OP op) {
    auto facts = factorize(x);
    vector<int> f(facts.size());
    while (true) {
        ll y = 1;
        rep(i, f.size()) rep(j, f[i]) y *= facts[i].first;
        op(y);

        int i;
        for (i = 0; i < f.size(); i++) {
            f[i] += 1;
            if (f[i] <= facts[i].second) break;
            f[i] = 0;
        }
        if (i == f.size()) break;
    }
}
// computes euler totative function phi(x), counting the
// amount of integers in [1, x] that are coprime with x.
// time: O(sqrt(x))
ll phi(ll x) {
    ll phi = 1, k = 2;
    for (; x > 1; k++) {
        if (k * k > x) { phi *= x - 1; break; }
        ll k1 = 1, k0 = 0;
        while (x % k == 0) x /= k, k0 = k1, k1 *= k;
        phi *= k1 - k0;
    }
    return phi;
}
// test-prime.cpp
// change to __int128 if checking numbers over 10^9
bool isprime(ll n) {
    if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
    ll A[] = {2,325,9375,28178,450775,9780504,1795265022};

```



```

ll s = __builtin_ctzll(n - 1), d = n >> s;
for (int a : A) {
    ll p = binexp(a, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--) p = p *
p % n;
    if (p != n - 1 && i != s) return 0;
}
return 1;
}

```

5.18 simplex

```

/* Solves a general linear maximization problem: maximize
$c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if
there is no solution, inf if there are arbitrarily good
solutions, or the maximum value of $c^T x$ otherwise. The
input vector is set to an optimal $x$ (or in the unbounded
case, an arbitrary solution fulfilling the constraints).
Numerical stability is not guaranteed. For better
performance, define variables such that $x = 0$ is viable.
Usage:
vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: O(NM * \#pivots), where a pivot may be e.g. an edge
relaxation. O(2^n) in the general case.*/
typedef double T;//long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1 / .0;
#define MP make_pair
#define ltj(X) \
    if (s == -1 || MP(X[j], N[j]) < MP(X[s], N[s])) s = j
struct LPSolver {
    int m, n; vector<int> N, B; vvd D;
    LPSolver(const vvd &A, const vd &b, const vd &c) :
m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, vd(n+2)){
        rep(i, m) rep(j, n) D[i][j] = A[i][j];
        rep(i, m) {
            B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i];
        }
        rep(j, n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }
    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i, m + 2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            repx(j, 0, n + 2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j, n + 2) if (j != s) D[r][j] *= inv;
        rep(i, m + 2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

```

```

}
bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
        int s = -1;
        rep(j, n + 1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i, m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || MP(D[i][n + 1] / D[i][s],
B[i]) < MP(D[r][n + 1] / D[r][s], B[r])) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}
T solve(vd &x) {
    int r = 0;
    repx(i, 1, m) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -eps) {
        pivot(r, n);
        if (!simplex(2) || D[m + 1][n + 1] < -eps)
return -inf;
        rep(i, m) if (B[i] == -1) {
            int s = 0;
            repx(j, 1, n + 1) ltj(D[i]);
            pivot(i, s);
        }
    }
    bool ok = simplex(1);
    x = vd(n);
    rep(i, m) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return ok ? D[m][n + 1] : inf;
}
};

```

5.19 theorems and formulas

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

$$\sum_{i=0}^k \binom{n+i}{i} = \binom{n+k+1}{k}$$

$$\begin{bmatrix} n \\ k \end{bmatrix} = \text{perm of } n \text{ elements with } k \text{ cycles}$$

$$\begin{bmatrix} n+1 \\ k \end{bmatrix} = n \begin{bmatrix} n \\ k \end{bmatrix} + \begin{bmatrix} n \\ k-1 \end{bmatrix}$$

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \text{partitions of an } n\text{-element set into } k \text{ parts}$$

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \frac{1}{k!} \sum_{i=0}^k (-1)^i \binom{k}{i} (k-i)^n$$

$$\left\{ \begin{matrix} n+1 \\ k \end{matrix} \right\} = k \left\{ \begin{matrix} n \\ k \end{matrix} \right\} + \left\{ \begin{matrix} n \\ k-1 \end{matrix} \right\}$$

Integers $d_1 \geq \dots \geq d_n \geq 0$ can be the degree sequence of a finite simple graph on n vertices $\Leftrightarrow d_1 + \dots + d_n$ is even and for every k in $1 \leq k \leq n$

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

$$a^n = a^{\varphi(m) + n \bmod \varphi(m)} \pmod{m} \text{ if } n > \lg(m)$$

Misere Nim: if $\exists a_i > 1$ then normal nim; else the condition is reversed.

Derangements: Num of permutations of $n = 0, 1, 2, \dots$ elements without fixed points is $1, 0, 1, 2, 9, 44, 265, 1854, 14833, \dots$ Recurrence: $D_n = (n-1)(D_{n-1} + D_{n-2}) = n * D_{n-1} + (-1)^n$

Collary: number of permutations with exactly k fixed points $\binom{n}{k} D_{n-k}$

Eulerian numbers: $E(n, k)$ is the number of permutations with exactly k descents ($i : \pi_i < \pi_{i+1}$), ascents ($\pi_i > \pi_{i+1}$) / excedances ($\pi > i$) / $k + 1$ weak excedances ($\pi \geq i$).

$$E_{n,k} = (k+1)E_{n-1,k} + (n-k)E_{n-1,k-1}$$

5.20 theorems

Burnside lemma

Tomemos imagenes x en X y operaciones $(g: X \rightarrow X)$ en G . Si $\#g$ es la cantidad de imagenes que son puntos fijos de g , entonces la cantidad de objetos es $\frac{\sum_{g \in G} \#g}{|G|}$. Es requisito que G tenga la operacion identidad, que toda operacion tenga inversa y que todo par de operaciones tenga su combinacion.

Rational root theorem

Las raices racionales de un polinomio de orden n con coeficientes enteros $A[i]$ son de la forma p/q , donde p y q son coprimos, p es divisor de $A[0]$ y q es divisor de $A[n]$. Notar que si $A[0] = 0$, cero es raiz, se puede dividir el polinomio por x y aplica nuevamente el teorema.

Petersens theorem

Every cubic and bridgeless graph has a perfect matching.

Number of divisors for powers of 10

(0,1) (1,4) (2,12) (3,32) (4,64) (5,128) (6,240) (7,448) (8,768) (9,1344) (10,2304) (11,4032) (12,6720) (13,10752)

(14,17280) (15,26880) (16,41472) (17,64512) (18,103680)

Kirchoff Theorem: Sea A la matriz de adyacencia del multi-grafo (A[u][v] indica la cantidad de aristas entre u y v) Sea D una matriz diagonal tal que D[v][v] es igual al grado de v (considerando auto aristas y multi aristas). Sea $L = A - D$. Todos los cofactores de L son iguales y equivalen a la cantidad de Spanning Trees del grafo. Un cofactor (i,j) de L es la multiplicación de $(-1)^{i+j}$ con el determinant de la matriz al quitar la fila i y la columna j

Prufer Code: Dado un árbol con los nodos indexados: busca la hoja de menor índice, bórrala y anota el índice del nodo al que estaba conectado. Repite el paso anterior n-2 veces. Lo anterior muestra una biyección entre los arreglos de tamaño n-2 con elementos en [1, n] y los árboles de n nodos, por lo que hay n^{n-2} spanning trees en un grafo completo. Corolario: Si tenemos k componentes de tamaños s_1, s_2, \dots, s_k entonces podemos hacerlos conexos agregando k-1 aristas entre nodos de $s_1 s_2 \dots s_k n^{k-2}$ formas

Combinatoria

Catalan: $C_{n+1} = \sum (C_i * C_{n-i})$ for $i \in [0, n]$

Catalan: $C_n = \frac{1}{n+1} \binom{2n}{n}$

Sea C_n^k las formas de poner n+k pares de paréntesis, con los primeros k paréntesis abiertos (esto es, hay $2n + 2k$ caracteres), se tiene que

$C_n^k = (2n+k-1) * (2n+k) / (n * (n+k+1)) * C_{n-1}^k$

Sea D_n el número de permutaciones sin puntos fijos, entoces $D_n = (n-1) * (D_{n-1} + D_{n-2})$, $D_0 = 1$, $D_1 = 0$

5.21 tonelli shanks

```
ll legendre(ll a, ll p) {
    if (a % p == 0) return 0; if (p == 2) return 1;
    return binexp(a, (p - 1) / 2, p);
}
// sqrt(n) mod p (p must be a prime)
// rnd(a, b) return a random number in [a, b]
ll tonelli_shanks(ll n, ll p) {
    if (n == 0) return 0;
    if (legendre(n, p) != 1) return -1; // no existe
    if (p == 2) return 1;
    ll s = __builtin_ctzll(p - 1);
    ll q = (p - 1LL) >> s, z = rnd(1, p - 1);
    if (s == 1) return binexp(n, (p + 1) / 4LL, p);
    while (legendre(z, p) != p - 1) z = rnd(1, p - 1);
    ll c = binexp(z, q, p), r = binexp(n, (q + 1) / 2, p);
    ll t = binexp(n, q, p), m = s;
    while (t != 1) {
        ll i = 1, ts = (t * t) % p;
        while (ts != 1) i++, ts = (ts * ts) % p;
        ll b = c;
        repx(_, 0, m - i - 1) b = (b * b) % p;
        r = r * b % p; c = b * b % p; t = t * c % p; m = i;
    }
```

```
}
return r;
}
```

6 Strings

6.1 aho corasick

```
struct Vertex {
    int next[26], go[26];
    int p, link = -1, exit = -1, cnt = -1;
    vector<int> leaf;
    char pch;
    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        rep(i, 26) next[i] = -1, go[i] = -1;
    }
};
vector<Vertex> t(1);
void add(string &s, int id) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf.push_back(id);
}
int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0) t[v].link = 0;
        else t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1) t[v].go[c] = t[v].next[c];
        else t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}
int next_match(int v){ // Optional
    if(t[v].exit == -1){
        if(t[get_link(v)].leaf.size())t[v].exit=get_link(v);
        else t[v].exit = v==0 ? 0 : next_match(get_link(v));
    }
    return t[v].exit;
}
int cnt_matches(int v){ // Optional
```

```
if(t[v].cnt == -1)
    t[v].cnt = v == 0 ? 0 : t[v].leaf.size() +
cnt_matches(get_link(v));
return t[v].cnt;
}
```

6.2 debruijn sequence

```
// Given alphabet [0,k) constructs a cyclic string of length
// k^n that contains every length n string as substr.
vector<int> deBruijnSeq(int k, int n) { // Recursive FKM
    if (k == 1) return {0};
    vector<int> seq, aux(n+1);
    function<void(int,int)> gen = [&](int t, int p) {
        if (t > n) { // +lyndon word of len p
            if (n%p == 0) repx(i, 1, p+1) seq.pb(aux[i]);
        } else {
            aux[t] = aux[t-p]; gen(t+1, p);
            while (++aux[t] < k) gen(t+1, t);
        }
    };
    gen(1, 1); return seq;
}
```

6.3 hash

```
const int K = 2;
struct Hash{
    const ll MOD[K] = {999727999, 1070777777};
    const ll P = 1777771;
    vector<ll> h[K], p[K];
    Hash(string &s){
        int n = s.size();
        rep(k, K){
            h[k].resize(n+1, 0);
            p[k].resize(n+1, 1);
            repx(i, 1, n+1){
                h[k][i] = (h[k][i-1]*P + s[i-1]) % MOD[k];
                p[k][i] = (p[k][i-1]*P) % MOD[k];
            }
        }
    }
    vector<ll> get(int i, int j){
        vector<ll> r(K);
        rep(k, K){
            r[k] = (h[k][j] - h[k][i]*p[k][j-i]) % MOD[k];
            r[k] = (r[k] + MOD[k]) % MOD[k];
        } return r;
    }
};
```

6.4 manacher

```
// odd[i]: length of longest palindrome centered at i
// even[i]: ...longest palindrome centered between i and i+1
```

```
void manacher(string &s,vector<int> &odd,vector<int> &even){
    string t = "$#";
    for(char c: s) t += c + string("#");
    t += "^";
    int n = t.size();
    vector<int> p(n);
    int l = 1, r = 1;
    repx(i, 1, n-1) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while(t[i - p[i]] == t[i + p[i]]) p[i]++;
        if(i + p[i] > r) l = i - p[i], r = i + p[i];
    }
    repx(i, 2, n-2) {
        if(i%2) even.push_back(p[i]-1);
        else odd.push_back(p[i]-1);
    }
}
```

6.5 palindromic tree

```
struct Node { // (*) = Optional
    int len; // length of substring
    int to[26]; // insertion edge for all characters a-z
    int link; // maximun palindromic suffix
    int i; // (*) start index of current Node
    int cnt; // (*) # of occurrences of this substring
    Node(int len, int link=0, int i=0, int cnt=1): len(len),
    link(link), i(i), cnt(cnt) {memset(to, 0, sizeof(to));}
}; struct EerTree { // Palindromic Tree
    vector<Node> t; // tree (max size of tree is n+2)
    int last; // current node
    EerTree(string &s) : last(0) {
        t.emplace_back(-1); t.emplace_back(0); // root 1 & 2
        rep(i, s.size()) add(i, s); // construct tree
        for(int i = t.size()-1; i > 1; i--)
            t[t[i].link].cnt += t[i].cnt;
    }
    void add(int i, string &s){ // vangrind warning:
        int p=last, c=s[i]-'a'; // i-t[p].len-1 = -1
        while(s[i-t[p].len-1] != s[i]) p = t[p].link;
        if(t[p].to[c]){ last = t[p].to[c]; t[last].cnt++; }
        else{
            int q = t[p].link;
            while(s[i-t[q].len-1] != s[i]) q = t[q].link;
            q = max(1, t[q].to[c]);
            last = t[p].to[c] = t.size();
            t.emplace_back(t[p].len + 2, q, i-t[p].len-1);
        }
    }
};
void main(){
    string s = "abcbab"; EerTree pt(s); // build EerTree
    repx(i, 2, pt.t.size()){// list all distinct palindromes
        repx(j,pt.t[i].i,pt.t[i].i+pt.t[i].len)cout << s[j];
        cout << " " << pt.t[i].cnt << endl;
    }
```

```
}
}
```

6.6 prefix function

```
vector<int> prefix_function(string s) {
    int n = s.size();
    vector<int> pi(n);
    repx(i, 1, n) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
vector<vector<int>> aut;
void compute_automaton(string s) {
    s += '#';
    int n = s.size();
    vector<int> pi = prefix_function(s);
    aut.assign(n, vector<int>(26));
    repx(i, n) {
        repx(c, 26) {
            int j = i;
            while (j > 0 && 'a' + c != s[j])
                j = pi[j-1];
            if ('a' + c == s[j])
                j++;
            aut[i][c] = j;
        }
    }
}
// k = n - pi[n - 1]; if k divides n, then the string can be
// aprtitioned into blocks of length k otherwise there is no
// effective compression and the answer is n.
```

6.7 suffix array

```
// build the suffix array
// suffixes are sorted, with each suffix represented by its
// starting position
vector<int> suffixarray(const string &s) {
    int N = s.size() + 1;//optional: include terminating NUL
    vector<int> p(N), p2(N), c(N), c2(N), cnt(256);
    repx(i, N) cnt[s[i]] += 1;
    repx(b, 1, 256) cnt[b] += cnt[b - 1];
    repx(i, N) p[--cnt[s[i]]] = i;
    repx(i, 1, N) c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);
    for (int k = 1; k < N; k <= 1) {
        int C = c[p[N - 1]] + 1;
        cnt.assign(C + 1, 0);
```

```
        for (int &pi : p) pi = (pi - k + N) % N;
        for (int cl : c) cnt[cl + 1] += 1;
        repx(i, C) cnt[i + 1] += cnt[i];
        repx(i, N) p2[cnt[c[p[i]]]++] = p[i];
        c2[p2[0]] = 0;
        repx(i, 1, N) c2[p2[i]] =
            c2[p2[i - 1]] + (c[p2[i]] != c[p2[i - 1]] ||
                c[(p2[i] + k) % N] != c[(p2[i - 1] + k) % N]);
        swap(c, c2), swap(p, p2);
    }
    p.erase(p.begin()); // optional: erase terminating NUL
    return p;
}
// build the lcp
// `lcp[i]` represents the length of the longest common
// prefix between suffix i and suffix i+1 in the suffix
// array `p`. the last element of `lcp` is zero by convention
vector<int> makelcp(const string &s, const vector<int> &p) {
    int N = p.size(), k = 0;
    vector<int> r(N), lcp(N);
    repx(i, N) r[p[i]] = i;
    repx(i, N) {
        if (r[i] + 1 >= N) { k = 0; continue; }
        int j = p[r[i] + 1];
        while (i + k < N && j + k < N && s[i + k] == s[j + k]) k += 1;
        lcp[r[i]] = k;
        if (k) k -= 1;
    }
    return lcp;
}
// lexicographically compare the suffixes starting from `i`
// and `j`, considering only up to `K` characters.
// `r` is the inverse suffix array, mapping suffix offsets
// to indices. requires an LCP sparse table.
int lcp_cmp(vector<int> &r, Sparse<int> &lcp, int i, int j,
    int K) {
    if (i == j) return 0;
    int ii = r[i], jj = r[j];
    int l = lcp.query(min(ii, jj), max(ii, jj));
    if (l >= K) return 0;
    return ii < jj ? -1 : 1;
}
```

6.8 suffix automaton

```
struct State {int len, link; map<char,int> next; };
State st[2*MAXN]; int sz, last; // clear next!!
void sa_init(){ last=st[0].len=0; sz=1; st[0].link=-1; }
void sa_extend(char c){// total build O(n log alphabet_size)
    int k = sz++, p; st[k].len = st[last].len + 1;
    for(p=last; p!=-1 && !st[p].next[c]; p=st[p].link)
        st[p].next[c] = k;
    if(p == -1) st[k].link = 0;
```

```

else {
    int q = st[p].next[c];
    if(st[p].len + 1 == st[q].len) st[k].link = q;
    else {
        int w = sz++; st[w].len = st[p].len + 1;
        st[w].next=st[q].next; st[w].link=st[q].link;
        for(; p!=-1 && st[p].next[c]==q; p=st[p].link)
            st[p].next[c] = w;
        st[q].link=st[k].link = w;
    }
}
last = k;
} // # states <= 2n-1 && transitions <= 3n-4 (for n > 2)
// Follow link from `last` to 0, nodes on path are terminal
// # matches = # paths from state to a terminal node
// # substrings = # paths from 0 to any node
// # substrings = sum of (len - len(link)) for all nodes

```

6.9 z function

```

// i-th element is equal to the greatest number of
// characters starting from the position i that coincide
// with the first characters of s
vector<int> z_function(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for(int i = 1; i < n; i++) {
        if(i < r) z[i] = min(r - i, z[i - l]);
        while(i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}

```