An Introduction to Digital Image Processing with MATLAB

Notes for SCM2511 Image Processing 1

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Contents

Chapter 1

Introduction

1.1 Images and pictures

As we mentioned in the preface, human beings are predominantly visual creatures: we rely heavily on our vision to make sense of the world around us. We not only look at things to identify and classify them, but we can scan for differences, and obtain an overall rough “feeling” for a scene with a quick glance.

Humans have evolved very precise visual skills: we can identify a face in an instant; we can differentiate colours; we can process a large amount of visual information very quickly.

However, the world is in constant motion: stare at something for long enough and it will change in some way. Even a large solid structure, like a building or a mountain, will change its appearance depending on the time of day (day or night); amount of sunlight (clear or cloudy), or various shadows falling upon it.

We are concerned with single images: snapshots, if you like, of a visual scene. Although image processing can deal with changing scenes, we shall not discuss it in any detail in this text.

For our purposes, an *image* is a single picture which represents something. It may be a picture of a person, of people or animals, or of an outdoor scene, or a microphotograph of an electronic component, or the result of medical imaging. Even if the picture is not immediately recognizable, it will not be just a random blur.

1.2 What is image processing?

Image processing involves changing the nature of an image in order to either

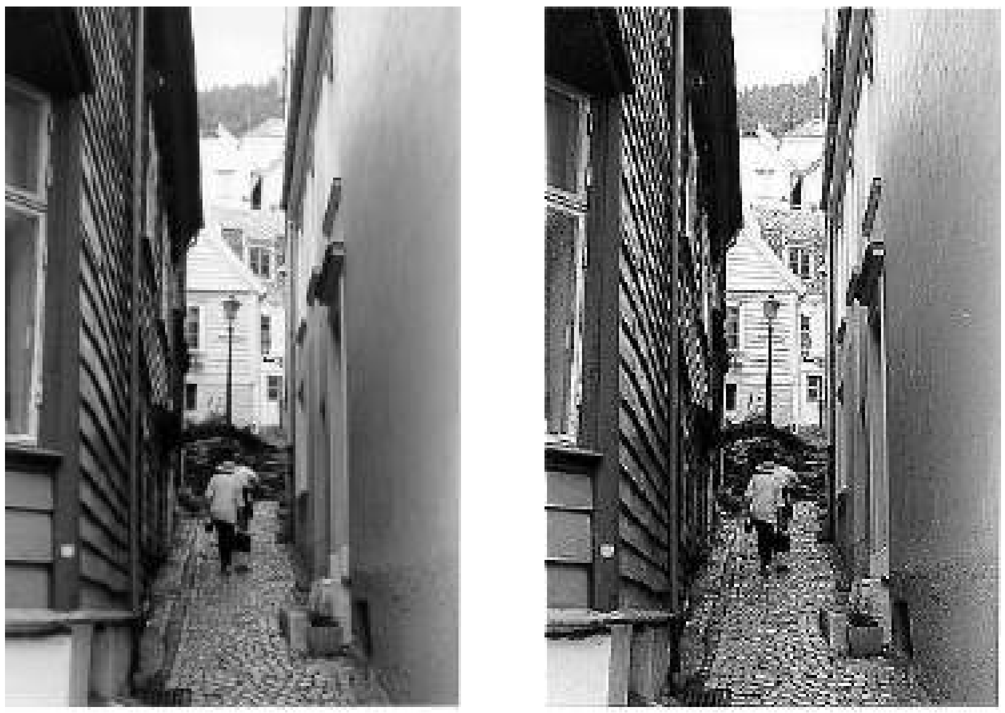
1. improve its pictorial information for human interpretation,

2. render it more suitable for autonomous machine perception.

We shall be concerned with *digital image processing*, which involves using a computer to change the nature of a *digital image* (see below). It is necessary to realize that these two aspects represent two separate but equally important aspects of image processing. A procedure which satisfies condition (1)—a procedure which makes an image “look better”—may be the very worst procedure for satisfying condition (2). Humans like their images to be sharp, clear and detailed; machines prefer their images to be simple and uncluttered.

Examples of (1) may include:

- Enhancing the edges of an image to make it appear sharper; an example is shown in figure 1.1. Note how the second image appears "cleaner"; it is a more pleasant image. Sharpening edges is a vital component of printing: in order for an image to appear "at its best" on the printed page; some sharpening is usually performed.



(a) The original image (b) Result after "sharperning"

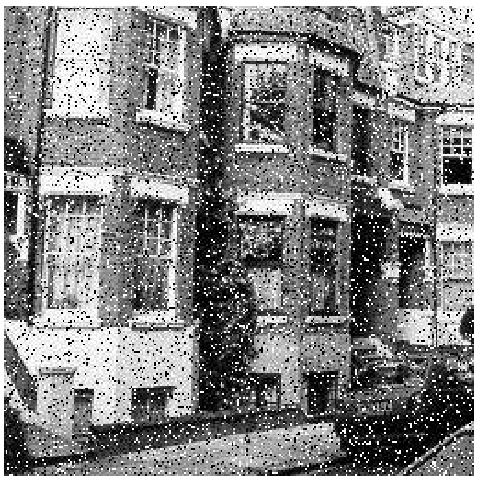
Figure 1.1: Image sharperning

- Removing "noise" from an image; noise being random errors in the image. An example is given in figure 1.2. Noise is a very common problem in data transmission: all sorts of electronic components may affect data passing through them, and the results may be undesirable. As we shall see in chapter 7 noise may take many different forms;each type of noise requiring a different method of removal.

- Removing motion blur from an image. An example is given in figure 1.3. Note that in the deblurred image (b) it is easy to read the numberplate, and to see the spokes on the wheels of the car, as well as other details not at all clear in the original image (a). Motion blur may occur when the shutter speed of the camera is too long for the speed of the object. In photographs of fast moving objects: athletes, vehicles for example, the problem of blur may be considerable.

Examples of (2) may include:

- Obtaining the edges of an image. This may be necessary for the measurement of objects in an image; an example is shown in figures 1.4. Once we have the edges we can measure their spread, and the area contained within them. We can also use edge detection algorithms as a first step in edge enhancement, as we saw above.


(a) The original image

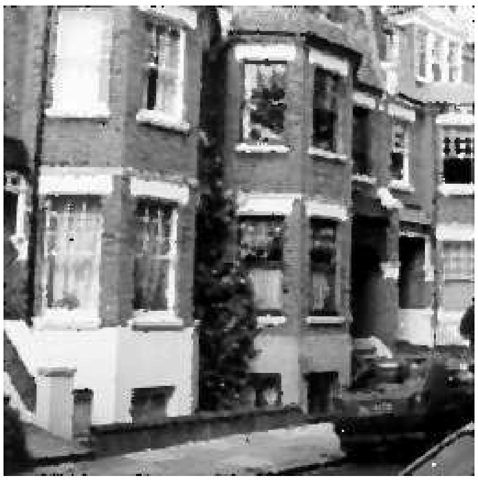
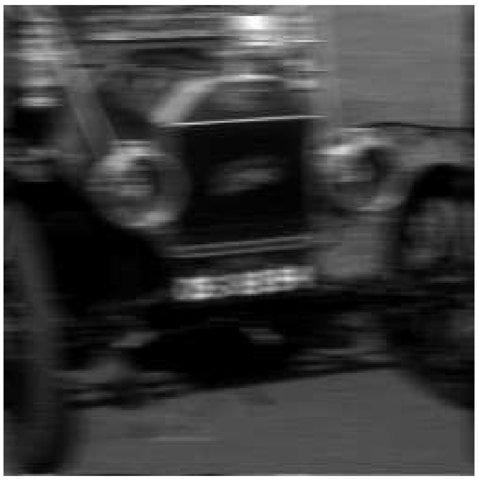
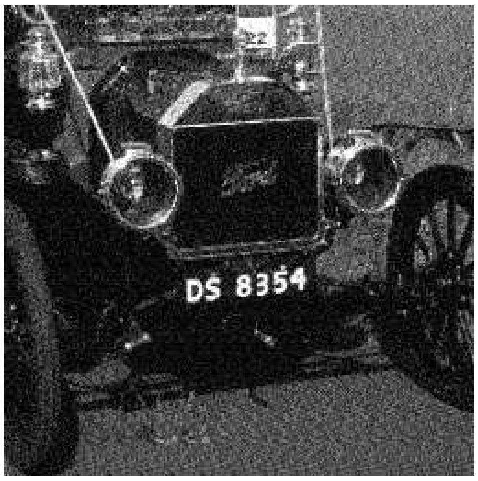

(b) After removing noise

Figure 1.2: Removing noise from an image


(a) The original image



(b) After removing the blur

Figure 1.3: Image deblurring

From the edge result, we see that it may be necessary to enhance the original image slightly, to make the edges clearer.

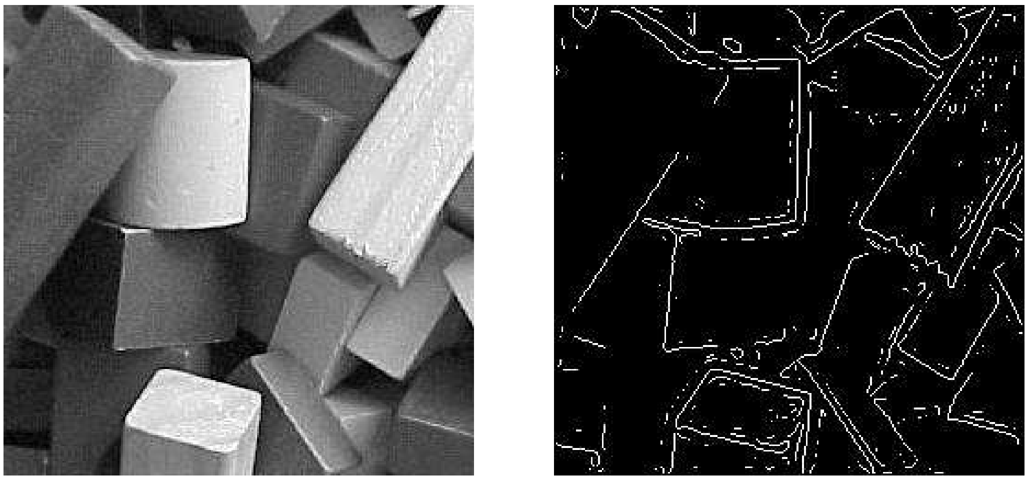

(a) The original image (b) Its edge image

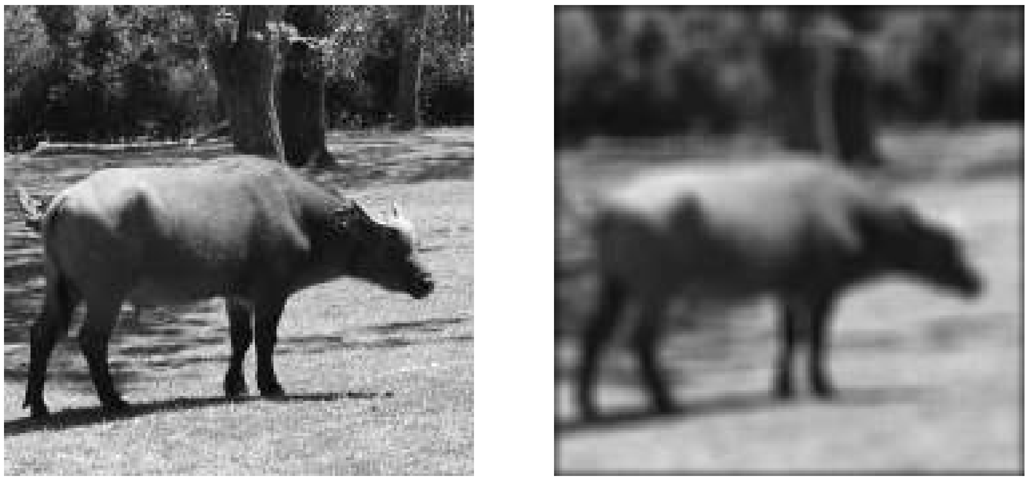
Figure 1.4: Finding edges in an image

- Removing detail from an image. For measurement or counting purposes, we may not be interested in all the detail in an image. For example, a machine inspected items on an assembly line, the only matters of interest may be shape, size or colour. For such cases, we might want to simplify the image. Figure 1.5 shows an example: in image (a) is a picture of an African buffalo, and image (b) shows a blurred version in which extraneous detail (like the logs of wood in the background) have been removed. Notice that in image (b) all the fine detail is gone; what remains is the coarse structure of the image. We could for example, measure ther size and shape of the animal without being "distracted" by unnecessary detail.

1.3 Images and digital images

Suppose we take an image, a photo, say. For the moment, lets make things easy and suppose the photo is black and white (that is, lots of shades of grey), so no colour. We may consider this image as being a two dimensional function, where the function values give the brightness of the image at any given point, as shown in figure 1.6. We may assume that in such an image brightness values can be any real numbers in the range 0.0 (black) to 1.0 (white). The ranges of x and y will clearly depend on the image, but they can take all real values between their minima and maxima.

A *digital image* differs from a photo in that the x, y, and $f(x,y)$ values are all *discrete*. Usually they take on only integer values, so the image shown in figure 1.6 will have x and y ranging from 1 to 256 each, and the brightness values also ranging from 0 (black) to 255 (white). A digital image can be considered as a large array of discrete dots, each of which has a brightness associated with it. These dots are called *picture elements*, or more simply *pixels*. The pixels surrounding a given pixel constitute its *neighbourhood*. A neighbourhood can be characterized by its shape in the same way as a matrix: we can speak of a 3×3 neighbourhood, or of a 5×7 neighbourhood. Except in very special circumstances, neighbourhoods have odd numbers of rows and columns; this ensures that the current pixel is in the centre of the neighbourhood. An example of a neighbourhood is



(a) The original image (b) Blurring to remove detail

Figure 1.5: Blurring an image

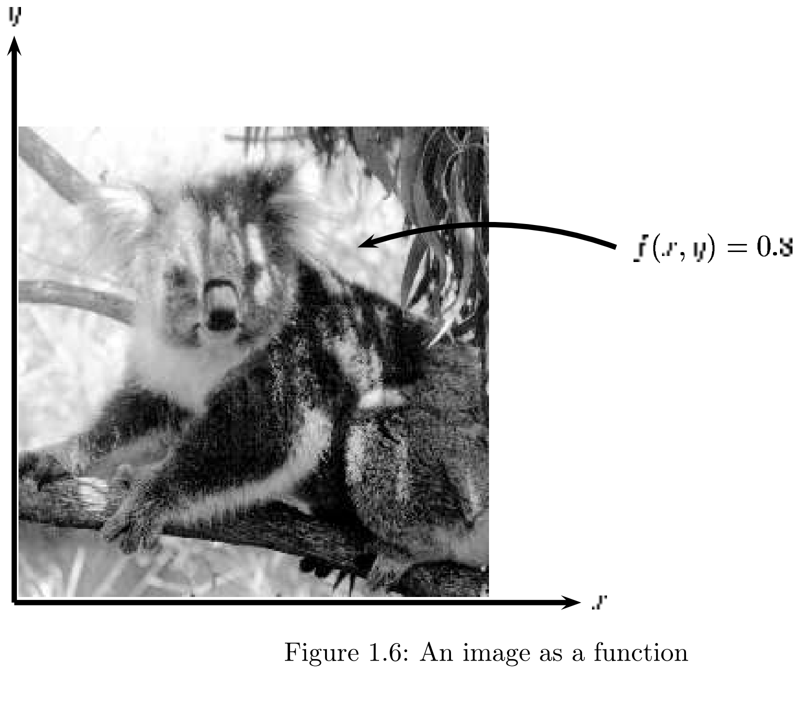
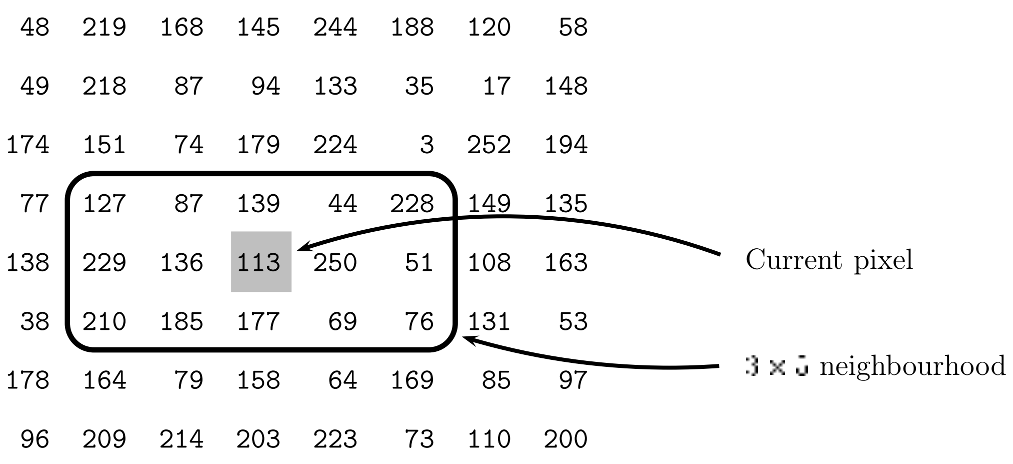


Figure 1.6: An image as a function

given in figure 1.7. If a neighbourhood has an even number of rows or columns (or both), it may be necessary to specify which pixel in the neighbourhood is the "current pixel".



48	219	168	145	244	188	120	58
49	218	87	94	133	35	17	148
174	151	74	179	224	3	252	194
77	127	87	139	44	228	149	135
138	229	136	113	250	51	108	163
38	210	185	177	69	76	131	53
178	164	79	158	64	169	85	97
96	209	214	203	223	73	110	200

Figure 1.7: Pixels, with a neighbourhood

1.4 Some applications

Image processing has an enormous range of applications; almost every area of science and technology can make use of image processing methods. Here is a short list just to give some indication of the range of image processing applications.

1. Medicine
   - Inspection and interpretation of images obtained from X-rays, MRI or CAT scans,
   - analysis of cell images, of chromosome karyotypes.
2. Agriculture
   - Satellite/aerial views of land, for example to determine how much land is being used for different purposes, or to investigate the suitability of different regions for different crops,
   - inspection of fruit and vegetables—distinguishing good and fresh produce from old.
3. Industry
   - Automatic inspection of items on a production line,
   - inspection of paper samples.
4. Law enforcement
   - Fingerprint analysis,
   - sharpening or de-blurring of speed-camera images.

1.5 Aspects of image processing

It is convenient to subdivide different image processing algorithms into broad subclasses. There are different algorithms for different tasks and problems, and often we would like to distinguish the nature of the task at hand.

Image enhancement. This refers to processing an image so that the result is more suitable for a particular application. Example include:

- sharpening or de-blurring an out of focus image,
- highlighting edges,
- improving image contrast, or brightening an image,
- removing noise.

Image restoration. This may be considered as reversing the damage done to an image by a known cause, for example:

- removing of blur caused by linear motion,
- removal of optical distortions,
- removing periodic interference.

Image segmentation. This involves subdividing an image into constituent parts, or isolating certain aspects of an image:

- finding lines, circles, or particular shapes in an image,
- in an aerial photograph, identifying cars, trees, buildings, or roads.

These classes are not disjoint; a given algorithm may be used for both image enhancement or for image restoration. However, we should be able to decide what it is that we are trying to do with our image: simply make it look better (enhancement), or removing damage (restoration).

1.6 An image processing task

We will look in some detail at a particular real-world task, and see how the above classes may be used to describe the various stages in performing this task. The job is to obtain, by an automatic process, the postcodes from envelopes. Here is how this may be accomplished:

Acquiring the image. First we need to produce a digital image from a paper envelope. This an be done using either a CCD camera, or a scanner.

Preprocessing. This is the step taken before the "major" image processing task. The problem here is to perform some basic tasks in order to render the resulting image more suitable for the job to follow. In this case it may involve enhancing the contrast, removing noise, or identifying regions likely to contain the postcode.

Segmentation. Here is where we actually "get" the postcode; in other words we extract from the image that part of it which contains just the postcode.

Representation and description. These terms refer to extracting the particular features which allow us to differentiate between objects. Here we will be looking for curves, holes and corners which allow us to distinguish the different digits which constitute a postcode.

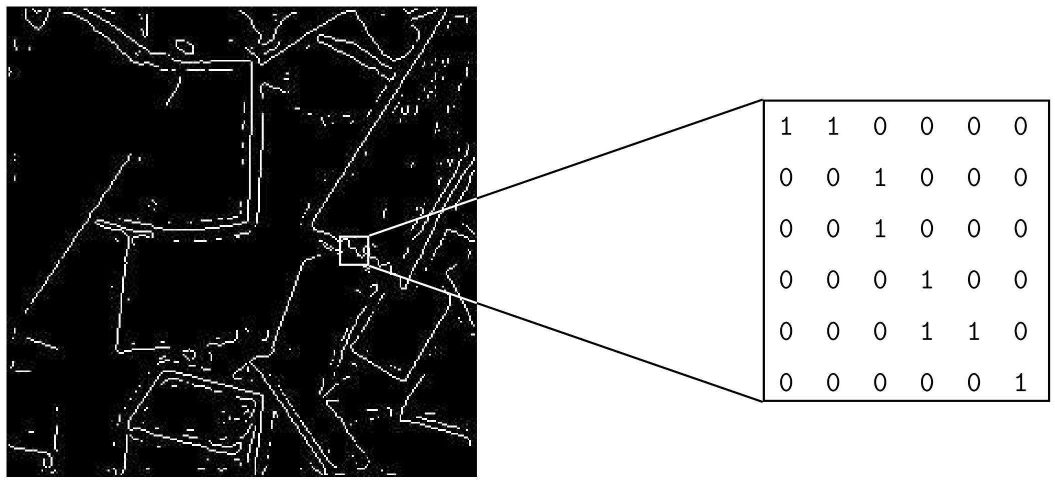
Recognition and interpretation. This means assigning labels to objects based on their descriptors (from the previous step), and assigning meanings to those labels. So we identify particular digits, and we interpret a string of four digits at the end of the address as the postcode.

1.7 Types of digital images

We shall consider four basic types of images:

Binary. Each pixel is just black or white. Since there are only two possible values for each pixel, we only need one bit per pixel. Such images can therefore be very efficient in terms of storage. Images for which a binary representation may be suitable include text (printed or handwriting), fingerprints, or architectural plans.

An example was the image shown in figure 1.4(b) above. In this image, we have only the two colours: white for the edges, and black for the background. See figure 1.8 below.

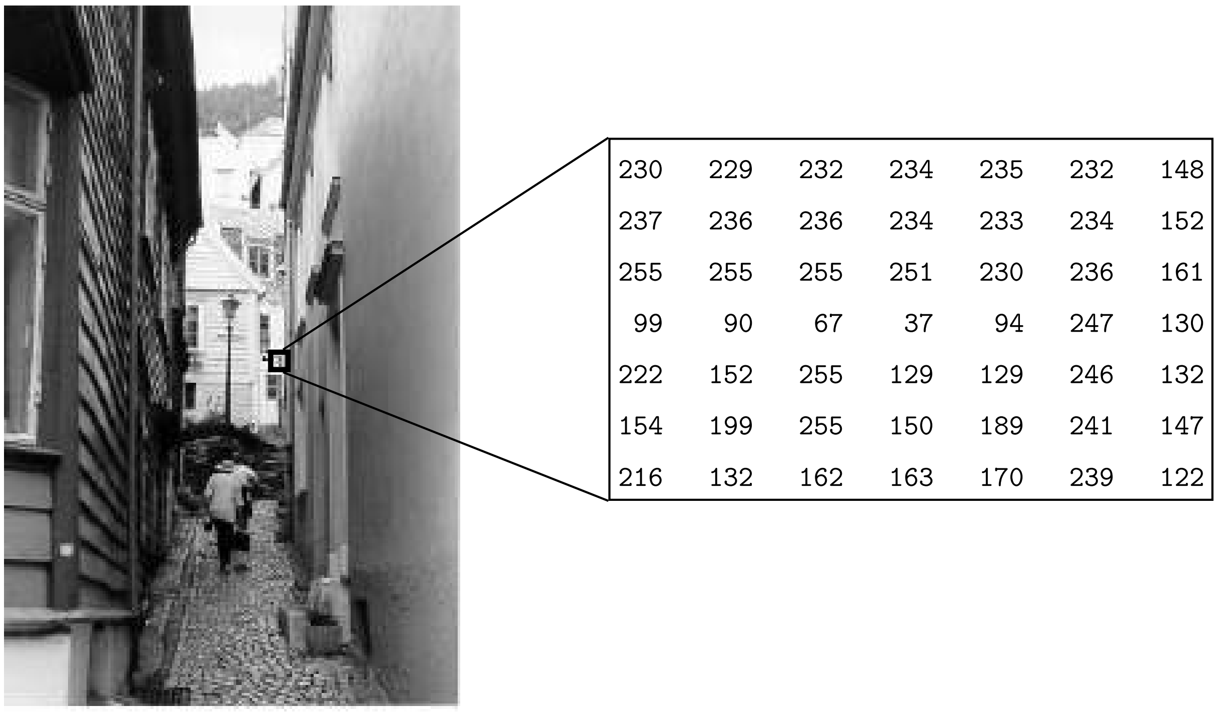


1	1	0	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	1	1	0
0	0	0	0	0	1

Figure 1.8: A binary image

Greyscale. Each pixel is a shade of grey, normally from 0 (black) to 255 (white). This range means that each pixel can be represented by eight bits, or exactly one byte. This is a very natural range for image file handling. Other greyscale ranges are used, but generally they are a power of 2. Such images arise in medicine (X-rays), images of printed works, and indeed 256 different grey levels is sufficient for the recognition of most natural objects.

An example is the street scene shown in figure 1.1 above, and in figure 1.9 below.



230	229	232	234	235	232	148
237	236	236	234	233	234	152
255	255	255	251	230	236	161
99	90	67	37	94	247	130
222	152	255	129	129	246	132
154	199	255	150	189	241	147
216	132	162	163	170	239	122

Figure 1.9: A greyscale image

True colour, or RGB. Here each pixel has a particular colour; that colour being described by the amount of red, green and blue in it. If each of these components has a range 0–255, this gives a total of $255^3 = 16,777,216$ different possible colours in the image. This is enough colours for any image. Since the total number of bits required for each pixel is 24, such images are also called 24-*bit colour images.*

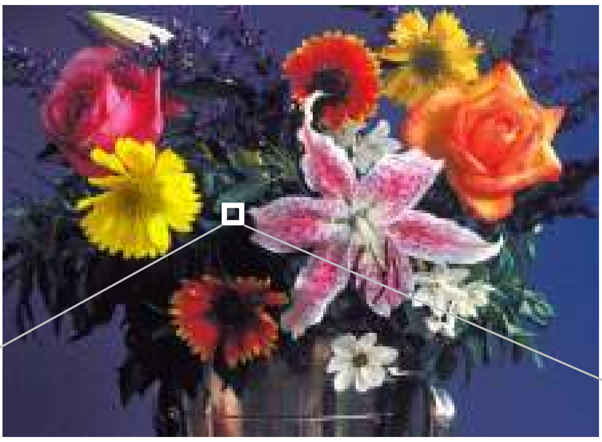
Such an image may be considered as consisting of a "stack" of three matrices; representing the red, green and blue values for each pixel. This measn that for every pixel there correspond three values.

We show an example in figure 1.10.

Indexed. Most colour images only have a small subset of the more than sixteen million possible colours. For convenience of storage and file handling, the image has an associated *colour map*, or *colour palette*, which is simply a list of all the colours used in that image. Each pixel has a value which does not give its colour (as for an RGB image), but an *index* to the colour in the map.

It is convenient if an image has 256 colours or less, for then the index values will only require one byte each to store. Some image file formats (for example, Compuserve GIF), allow only 256 colours or fewer in each image, for precisely this reason.

Figure 1.11 shows an example. In this image the indices, rather then being the grey values of the pixels, are simply indices into the colour map. Without the colour map, the image would be very dark and colourless. In the figure, for example, pixels labelled `5` correspond to



49	55	56	57	52	53
58	60	60	58	55	57
58	58	54	53	55	56
83	78	72	69	68	69
88	91	91	84	83	82
69	76	83	78	76	75
61	69	73	78	76	76

Red

64	76	82	79	78	78
93	93	91	91	86	86
88	82	88	90	88	89
125	119	113	108	111	110
137	136	132	128	126	120
105	108	114	114	118	113
96	103	112	108	111	107

Green

66	80	77	80	87	77
81	93	96	99	86	85
83	83	91	94	92	88
135	128	126	112	107	106
141	129	129	117	115	101
95	99	109	108	112	109
84	93	107	101	105	102

Blue

Figure 1.10: A true colour image

`0.2627 0.2588 0.2549`, which is a dark greyish colour.

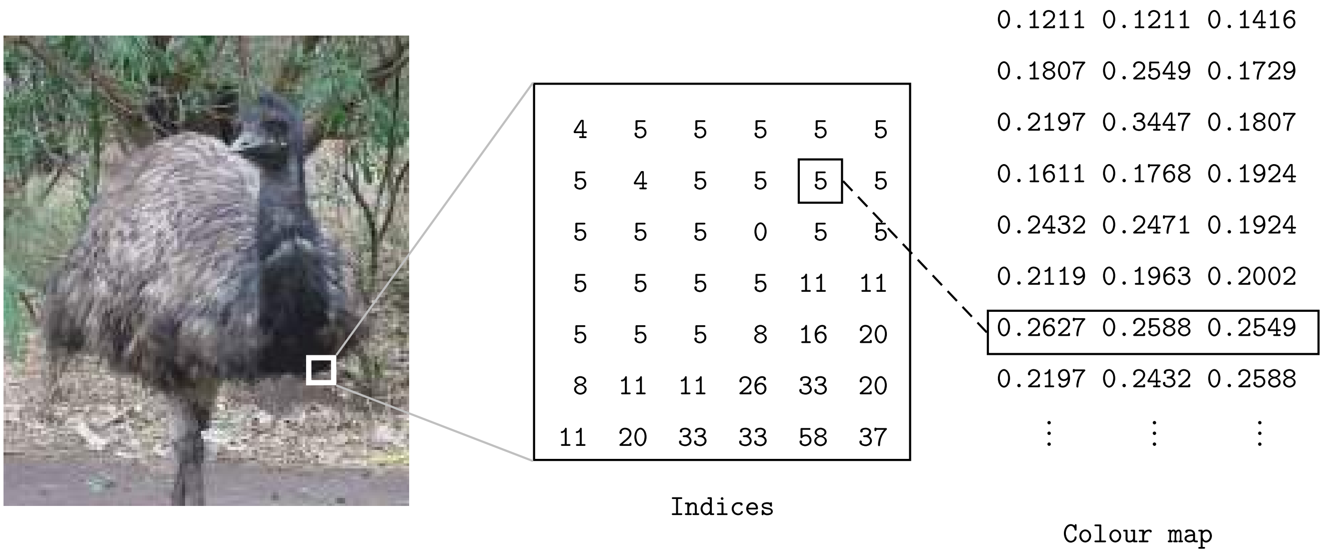


Figure 1.11: An indexed colour image

1.8 Image File Sizes

Image files tend to be large. We shall investigate the amount of information used in different image type of varying sizes. For example, suppose we consider a 512×512 binary image. The number of bits used in this image (assuming no compression, and neglecting, for the sake of discussion, any header information) is

$$\begin{aligned} 512 \times 512 \times 1 &= 262,144 \\ &= 32768 \text{ bytes} \\ &= 32.768 \text{ Kb} \\ &\approx 0.033\text{Mb}. \end{aligned}$$

(Here we use the convention that a kilobyte is one thousand bytes, and a megabyte is one million bytes.)

A greyscale image of the same size requires:

$$\begin{aligned} 512 \times 512 \times 1 &= 262,144 \text{ bytes} \\ &= 262.144 \text{ Kb} \\ &\approx 0.262 \text{ Mb}. \end{aligned}$$

If we now turn our attention to colour images, each pixel is associated with 3 bytes of colour information. A 512×512 image thus requires

$$521 \times 512 \times 3 = 786,432 \text{ bytes}$$

$$
\begin{aligned}
&= 786.43 \text{ Kb}\\
&\approx 0.786 \text{ Mb.}
\end{aligned}
$$

Many images are of course such larger than this; satellite images may be of the order of several thousand pixels in each direction.

"A picture is worth one thousand words"

Assuming a word to contain 10 ASCII characters (on average), and that each character requires 8 bits of storage, then 1000 words contain

$$1000 \times 10 \times 8 = 80,000 \text{ bits of information.}$$

This is roughly equivalent to the information in a

283×283	binary image
100×100	greyscale image
58×58	RGB colour image.

1.9 Image Acquisition

We will briefly discuss means for getting a picture "into" a computer.

CCD camera. Such a camera has, in place of the usual film, an array of *photosites*; these are silicon electronic devices whose voltage output is proportional to the intensity of light falling on them.

For a camera attached to a computer, information from the photosites is then output to a suitable storage medium. Generally this is done on hardware, as being much faster and more efficient than software, using a *frame-grabbing card*. This allows a large number of images to be captured in a very short time—in the order of one ten-thousandth of a second each. The images can then be copied onto a permanent storage device at some later time.

Digital still cameras use a range of devices, from floppy discs and CD's, to various specialized cards and "memory sticks". The information can then be downloaded from these devices to a computer hard disk.

Flat bed scanner. This works on a principle similar to the CCD camera. Instead of the entire image being captured at once on a large array, a single row of photosites is moved across the image, capturing it row-by-row as it moves. Since this is a much slower process than taking a picture with a camera, it is quite reasonable to allow all capture and storage to be processed by suitable software.

1.10 Image perception

Much of image processing is concerned with making an image appear "better" to human beings. We should therefore be aware of the limitations of the the human visual system. Image perception consists of two basic steps:

1. capturing the image with the eye,

2. recognising and interpreting the image with the *visual cortex* in the brain.

The combination and immense variability of these steps influences the ways in we perceive the world around us.

There are a number of things to bear in mind:

1. *Observed intensities* vary as to the background. A single block of grey will appear darker if placed on a white background than if it were placed on a black background. That is, we don't perceive grey scales "as they are", but rather as they differ from their surroundings. In figure 1.12 a grey square is shown on two different backgrounds. Notice how much darker the square appears when it is surrounded by a light grey. However, the two central squares have exactly the same intensity.

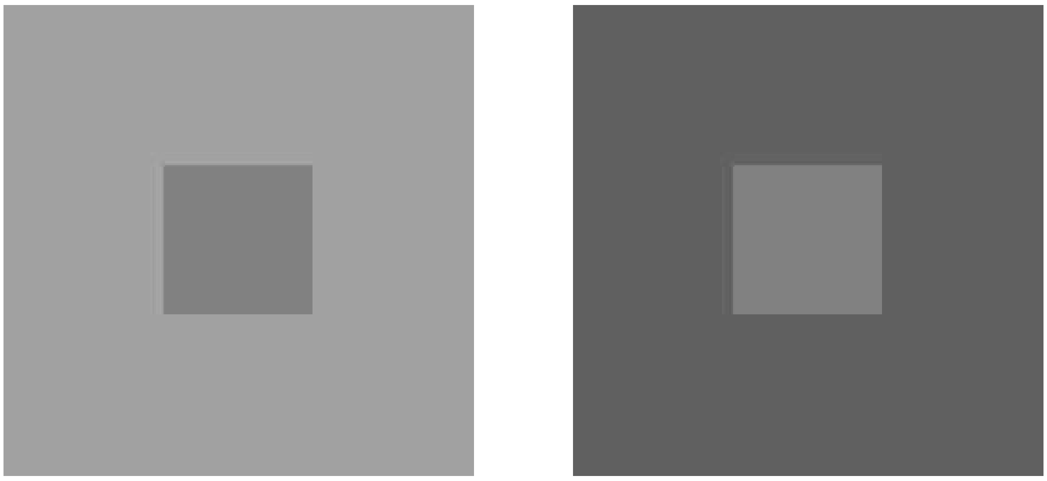


Figure 1.12: A grey square on different backgrounds

2. We may observe non-existent intensities as bars in continuously varying grey levels. See for example figure 1.13. This image varies continuously from light to dark as we travel from left to right. However, it is impossible for our eyes not to see a few horizontal edges in this image.

3. Our visual system tends to undershoot or overshoot around the boundary of regions of different intensities. For example, suppose we had a light grey blob on a dark grey background. As our eye travels from the dark background to the light region, the boundary of the region appears lighter than the rest of it. Conversely, going in the other direction, the boundary of the background appears *darker* than the rest of it.

Figure 1.13: Continuously varying intensities

Chapter 2

Basic use of Matlab

2.1 Introduction

Matlab is a data analysis and visualization tool which has been designed with powerful support for matrices and matrix operations. As well as this, Matlab has excellent graphics capabilities, and its own powerful programming language. One of the reasons that Matlab has become such an important tool is through the use of sets of Matlab programs designed to support a particular task. These sets of programs are called *toolboxes*, and the particular toolbox of interest to us is the *image processing toolbox*.

Rather than give a description of all of Matlab's capabilities, we shall restrict ourselves to just those aspects concerned with handling of images. We shall introduce functions, commands and techniques as required. A Matlab *function* is a keyword which accepts various parameters, and produces some sort of output: for example a matrix, a string, a graph or figure. Examples of such functions are `sin`, `imread`, `imclose`. There are *many*functions in Matlab, and as we shall see, it is very easy (and sometimes necessary) to write our own. A *command* is a particular use of a function. Examples of commands might be

```
>> sin(pi/3)
>> c=imread('cameraman.tif');
>> a=imclose(b);
```

As we shall see, we can combine functions and commands, or put multiple commnds on a single input line.

Matlab's standard data type is the matrix—all data are considered to be matrices of some sort. Images, of course, are matrices whose elements are the grey values (or possibly the RGB values) of its pixels. Single values are considered by Matlab to be 1×1 matrices, while a string is merely a $1 \times n$ matrix of characters; n being the string's length.

In this chapter we will look at the more generic Matlab commands, and discuss images in further chapters.

When you start up Matlab, you have a blank window called the "Command Window" in which you enter commands. This is shown in figure 2.1. Given the vast number of Matlab's functions, and the different parameters they can take, a command line style interface is in fact much more efficient than a complex sequence of pull-down menus.

The prompt consists of two right arrows:

```
>>
```

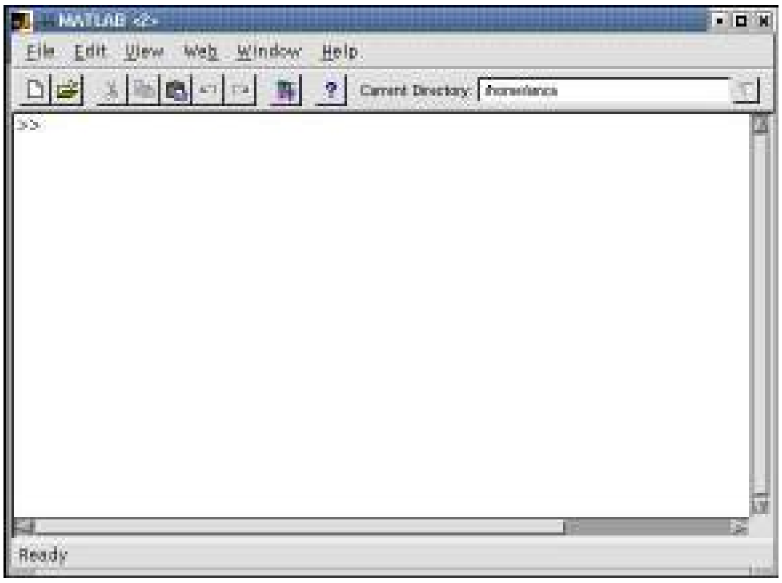


Figure 2.1: The MATLAB command window ready for action

2.2 Basic use of MATLAB

If you have never used MATLAB before, we will experiment with some simple calculations. We first note that MATLAB is *command line driven*; all commands are entered by typing them after the prompt symbol. Let's start off with a mathematical classic:

```
>> 2+2
```

What this means is that you type in

```
2
```

at the prompt, and then press your $\boxed{\text{Enter}}$ key. This sends the command to the MATLAB kernel. What you should now see is

```
ans =

     4
```

Good, huh?

MATLAB of course can be used as a calculator; it understands the standard arithmetic operations of addition (as we have just seen), subtraction, multiplication, division and exponentiation. Try these:

```
>> 3*4

>> 7-3

>> 11/7
>> 2^5
```

The results should not surprise you. Note that for the output of `11/7` the result was only given to a few decimal places. In fact MATLAB does all its calculations internally to *double precision*. However, the default display format is to use only eight decimal places. We can change this by using the `format` function. For example:

```
>> format long
>> 11/7

ans =

   1.57142857142857
```

Entering the command `format` by itself returns to the default format.

MATLAB has all the elementary mathematical functions built in:

```
>> sqrt(2)

ans =

    1.4142

>> sin(pi/8)

ans =

    0.3827

>> log(10)

ans =

    2.3026

>> log10(2)

ans =

    0.3010
```

The trigonometric functions all take radian arguments; and `pi` is a built-in constant. The functions `log` and `log10` are the natural logarithm and logarithms to base 10.

2.3 Variables and the workspace

When using any sort of computer system, we need to store things with appropriate names. In the context of MATLAB, we use *variables* to store values. Here are some examples:

```
>> a=5^(7/2)

a =

  279.5085

>> b=sin(pi/9)-cos(pi/9)

b =

   -0.5977
```

Note that although `a` and `b` are displayed using the `short` format, MATLAB in fact stores their full values. We can see this with:

```
>> format long;a,format

a =

     2.795084971874737e+02
```

We can now use these new variables in further calculations:

```
>> log(a^2)/log(5)

ans =

     7

>> >> atan(1/b)

ans =

   -1.0321
```

2.3.1 The workspace

If you are using a windowed version of MATLAB, you may find a `Workspace` item in the `View` menu. This lists all your currently defined variables, their numeric data types, and their sizes in bytes. To open the workspace window, use the `View` menu, and choose `Workspace`. The same information can be obtained using the `whos` function:

```
>> whos
  Name      Size         Bytes  Class

  a         1x1              8  double array
  ans       1x1              8  double array
  b         1x1              8  double array
```

```
Grand total is 3 elements using 24 bytes
```

Note also that `ans` is variable: it is automatically created by MATLAB to store the result of the last calculation. A listing of the variable names only is obtained using `who`:

```
>> who

Your variables are:

a    ans  b
```

The numeric date type `double` is `Matlab`'s standard for numbers; such numbers are stored as double-precision 8-byte values.

Other data types will be discussed below.

2.4 Dealing with matrices

MATLAB has an enormous number of commands for generating and manipulating matrices. Since a greyscale image is an matrix, we can use some of these commands to investigate aspects of the image.

We can enter a small matrix by listing its elements row by row, using spaces or commas as delimiters for the elements in each row, and using semicolons to separate the rows. Thus the matrix

$$a = \begin{bmatrix} 4 & -2 & -4 & 7 \\ 1 & 5 & -3 & 2 \\ 6 & -8 & -5 & -6 \\ -7 & 3 & 0 & 1 \end{bmatrix}$$

can be entered as

```
>> a=[4 -2 -4 7;1 5 -3 2;6 -8 -5 -6;-7 3 0 1]
```

2.4.1 Matrix elements

Matrix elements can be obtained by using the standard row, column indexing scheme. So for our image matrix `a` above, the command

```
>> a(2,3)

ans =

    -3
```

returns the element of the matrix in row 2 and column 3.

MATLAB also allows matrix elements to be obtained using a single number; this number being the position when the matrix is written out as a single column. Thus in a 4×4 matrix as above, the order of elements is

$$\begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix}.$$

So the element `a(2,3)` can also be obtained as `a(10)`:

```
>> a(10)

ans =

    -3
```

In general, for a matrix M with r rows and c columns, element $m(i, j)$ corresponds to $m(i+r(j-1))$. Using the single indexing allows us to extract multiple values from a matrix:

```
>> a([1 6 11 16])

ans =

     4     5    -5     1
```

To obtain a row of values, or a block of values, we use MATLAB's *colon* operator (`:`). This generates a vector of values; the command

```
 a:b
```

where `a` and `b` are integers, lists all integers from `a` to `b`. The more general version of this command:

```
i:b
```

lists all values from `a` by increment `i` up to `b`. So for example:

```
>> 2:3:16
```

generates the list

```
ans =

      2  5  8  11  14
```

Applied to our matrix `a`, for example:

```
>> a(2,1:3)

ans =

     6    -8     -5
```

lists all values in row 2 which are between columns 1 and 3 inclusive.

Similarly

```
>> a(2:4,3)

ans =

    -3
    -5
     0
```

lists all the values in column 3 which are between rows 2 to 4 inclusive. And we can choose a block of values such as

```
>> a(2:3,3:4)

ans =

    -3     2
    -5    -6
```

which lists the 2 by 2 block of values which lie between rows 2 to 3 and columns 3 to 4.

The colon operator by itself lists *all* the elements along that particular row or column. So, for example, all of row 3 can be obtained with:

```
>> a(3,:)

ans =

     6    -8    -5    -6
```

and all of column 2 with

```
>> a(:,2)

ans =

    -2
     5
    -8
     3
```

Finally, the colon on its own lists all the matrix elements as a single column:

```
a(:)
```

shows all the 16 elements of `a`.

2.4.2 Matrix operations

All the standard operations are supported. We can add, subtract, multiply and invert matrices, and take matrix powers. For example, with the matrix `a` from above, and a new matrix `b` defined by

```
>> b=[2 4 -7 -4;5 6 3 -2;1 -8 -5 -3;0 -6 7 -1]
```

we can have, for example:

```
>> 2*a-3*b

ans =

     2   -16    13    26
   -13    -8   -15    10
     9     8     5    -3
   -14    24   -21     1
```

As an example of matrix powers:

```
>> a^3*b^4

ans =

     103788     2039686     1466688      618345
     964142     2619886     2780222      345543
   -2058056    -2327582      721254     1444095
    1561358     3909734    -3643012    -1482253
```

Inversion is performed using the `inv` function:

```
>> inv(a)

ans =

  -0.0125    0.0552   -0.0231   -0.1619
  -0.0651    0.1456   -0.0352   -0.0466
  -0.0406   -0.1060   -0.1039   -0.1274
   0.1082   -0.0505   -0.0562    0.0064
```

A transpose is obtained by using the apostrophe:

```
>> a'

ans =

    4     1     6    -7
   -2     5    -8     3
   -4    -3    -5     0
    7     2    -6     1
```

As well as these standard arithmetic operations, MATLAB supports some geometric operations on matrices; `flipud` and `fliplr` flip a matrix up/down and left/right respectively, and `rot90` rotates a matrix by 90 degrees:

```
>> flipud(a)

ans =

    -7     3     0     1
     6    -8    -5    -6
     1     5    -3     2
     4    -2    -4     7

>> fliplr(a)

ans =

     7    -4    -2     4
     2    -3     5     1
    -6    -5    -8     6
     1     0     3    -7
>> rot90(a)

ans =

     7     2    -6     1
    -4    -3    -5     0
    -2     5    -8     3
     4     1     6    -7
```

The **reshape** function produces a matrix with elements taken column by column from the given matrix. Thus:

```
>> c=[1 2 3 4 5;6 7 8 9 10;11 12 13 14 15;16 17 18 19 20]

c =

     1     2     3     4     5
     6     7     8     9    10
    11    12    13    14    15
    16    17    18    19    20

>> reshape(c,2,10)

ans =

     1    11     2    12     3    13     4    14     5    15
     6    16     7    17     8    18     9    19    10    20

>> reshape(c,5,4)
```

```
ans =

     1     7    13    19
     6    12    18     5
    11    17     4    10
    16     3     9    15
     2     8    14    20
```

`Reshape` produces an error if the product of the two values is not equal to the number of elements of the matrix. Note that we could have produced the original matrix above with

```
>> c=reshape([1:20],5,4])'
```

All these commands work equally well on vectors. In fact, MATLAB makes no distinction between matrices and vectors; a vector merely being a matrix with number of rows or columns equal to 1.

The dot operators

A very distinctive class of operators in MATLAB are those which use dots; these operate in an element-wise fashion. For example, the command

```
b
```

performs the usual matrix multiplication of `a` and `b`. But the corresponding dot operator:

```
*b
```

produces the matrix whose elements are the products of the corresponding elements of `a` and `b`. That is, if

```
a.*b
```

then $c(i,j) = a(i,j) \times b(i,j)$:

```
>> a.*b

ans =

     8    -8    28   -28
     5    30    -9    -4
     6    64    25    18
     0   -18     0    -1
```

We have dot division, and dot powers. The command `a.^2` produces a matrix each element of which is a square of the corresponding elements of `a`:

```
>> a.^2

ans =

    16     4    16    49
     1    25     9     4
    36    64    25    36
    49     9     0     1
```

Similarly we can produce a matrix of reciprocals by writing `1./a`:

```
>> 1./a

ans =

    0.2500   -0.5000   -0.2500    0.1429
    1.0000    0.2000   -0.3333    0.5000
    0.1667   -0.1250   -0.2000   -0.1667
   -0.1429    0.3333       Inf    1.0000
```

The value `Inf` is MATLAB's version of infinity; it is returned for the operation $1/0$.

Operators on matrices

Many functions in MATLAB, when applied to a matrix, work by applying the function to each element in turn. Such functions are the trigonometric and exponential functions, and logarithms. The use of functions in this way means that in MATLAB many iterations and repetitions can be done with *vectorization* rather than by using loops. We will explore this below.

2.4.3 Constructing matrices

We have seen that we can construct matrices by listing all their elements. However, this can be tedious if the matrix is large, or if it can be generated by a function of its indices.

Two special matrices are the matrix consisting of all zeros, and the matrix consisting of all ones. These are generated by the `zero` and `ones` functions respectively. Each function can be used in several different ways:

`zeros(n)`	if `n` is a number, will produce a zeros matrix of size $n \times n$
`zeros(m,n)`	if `m` and `n` are numbers, will produce a zeros matrix of size $m \times n$
`zeros(m,n,p,...)`	where `m`, `n`, `p` and so on are numbers, will produce an $m \times n \times p \times \cdots$ multidimensional array of zeros
`zeros(a)`	where `a` is a matrix, will produce a matrix of zeros of the same size as `a`.

Matrices of random numbers can be produced using the `rand` and `randn` functions. They differ in that the numbers produced by `rand` are taken from a uniform distribution on the interval $[0, 1]$, and those produced by `randn` are take from a normal distribution with mean zero and standard deviation one. For creating matrices the syntax of each is the same as the first three options of `zeros` above. The `rand` and `randn` functions on their own produce single numbers taken from the appropriate distribution.

We can construct random integer matrices by multiplying the results of `rand` or `randn` by an integer and then using the `floor` function to take the integer part of the result:

```
>> floor(10*rand(3))

ans =
     8     4     6
     8     8     8
     5     8     6

>> floor(100*randn(3,5))

ans =

  -134   -70  -160   -40    71
    71    85  -145    68   129
   162   125    57    81    66
```

The `floor` function will be automatically applied to every element in the matrix.

Suppose we wish to create a matrix every element of which is a function of one of its indices. For example, the 10×10 matrix A for which $A_{ij} = i + j - 1$. In most programming languages, such a task would be performed using nested loops. We can use nested loops in `Matlab`, but it is easier here to use dot operators. We can first construct two matrices: one containing all the row indices, and one containing all the column indices:

```
>> rows=(1:10)'*ones(1,10)

rows =

     1     1     1     1     1     1     1     1     1     1
     2     2     2     2     2     2     2     2     2     2
     3     3     3     3     3     3     3     3     3     3
     4     4     4     4     4     4     4     4     4     4
     5     5     5     5     5     5     5     5     5     5
     6     6     6     6     6     6     6     6     6     6
     7     7     7     7     7     7     7     7     7     7
     8     8     8     8     8     8     8     8     8     8
     9     9     9     9     9     9     9     9     9     9
    10    10    10    10    10    10    10    10    10    10

>> cols=ones(10,1)*(1:10)

cols =

     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
```

Now we can construct our matrix using `rows` and `cols`:

```
>> A=rows+cols-1

A =

     1     2     3     4     5     6     7     8     9    10
     2     3     4     5     6     7     8     9    10    11
     3     4     5     6     7     8     9    10    11    12
     4     5     6     7     8     9    10    11    12    13
     5     6     7     8     9    10    11    12    13    14
     6     7     8     9    10    11    12    13    14    15
     7     8     9    10    11    12    13    14    15    16
     8     9    10    11    12    13    14    15    16    17
     9    10    11    12    13    14    15    16    17    18
    10    11    12    13    14    15    16    17    18    19
```

The construction of `rows` and `cols` can be done automatically with the `meshgrid` function:

```
[cols,rows]=meshgrid(1:10,1:10)
```

will produce the two index matrices above.

The size of our matrix `a` can be obtained by using the `size` function:

```
>> size(a)

ans =

     4     4
```

which returns the number of rows and columns of `a`.

2.4.4 Vectorization

Vectorization, in MATLAB, refers to an operation carried out over an entire matrix or vector. We have seen examples of this already, in our construction of the 10×10 matrix .1 above, and in our use of the dot operators. In most programming languages, applying an operation to elements of a list or array will require the use of a loop, or a sequence of nested loops. Vectorization in MATLAB allows us to dispense with loops in almost all instances, and is a very efficient replacement for them.

For example, suppose we wish to calculate the sine values of all the integer radians one to one million. We can do this with a `for` loop:

```
>> for i=1:10^6,sin(i);end
```

and we can measure the time of the operation with `Matlab`'s `tic`, `toc` timer: `tic` starts a stop watch timer, and `toc` stops it and prints out the elapsed time in seconds. Thus, on my computer:

```
>> tic,for i=1:10^6,sin(i);end,toc

elapsed_time =
   27.4969
```

We can perform the same calculation with:

```
>> i=1:10^6;sin(i);
```

and print out the elapsed time with:

```
>> tic,i=1:10^6;sin(i);toc

elapsed_time =

    1.3522
```

Note that the second command applies the sine function to `all` the elements of the vector `1:10^6`, whereas with the `for` loop, sine is only applied to each element of the loop in turn.

As another example, we can easily generate the first 10 square nunmbers with:

```
>> [1:10].^2

ans =

1 4 9 16 25 36 49 64 81 100
```

What happens here is that `[1:10]` generates a vector consisting of the numbers 1 to 10; and the dot operator `.^2` squares each element in turn.

Vectorization can also be used with logical operators; we can obtain all positive elements of the matrix `a` above with:

```
>> a>0

ans =

     1     0     0     1
     1     1     0     1
     1     0     0     0
     0     1     0     1
```

The result consists of 1's only in the places where the elements are positive.

MATLAB is designed to perform vectorized commands very quickly, and whenever possible such a command should be used instead of a `for` loop.

2.5 Plots

MATLAB has outstanding graphics capabilities. But we shall just look at some simple plots. The idea is straightforward: we create two vectors `x` and `y` of the same size, then the command

```
plot(x,y)
```

will plot `y` against `x`. If `y` has been created from `x` by using a vectorized function $f(x)$ the plot will show the graph of $y = f(x)$. Here's a simple example:

```
>> x=[0:0.1:2*pi];
>> plot(x,sin(x))
```

and the result is shown in figure 2.2. The `plot` function can be used to produce many different plots.

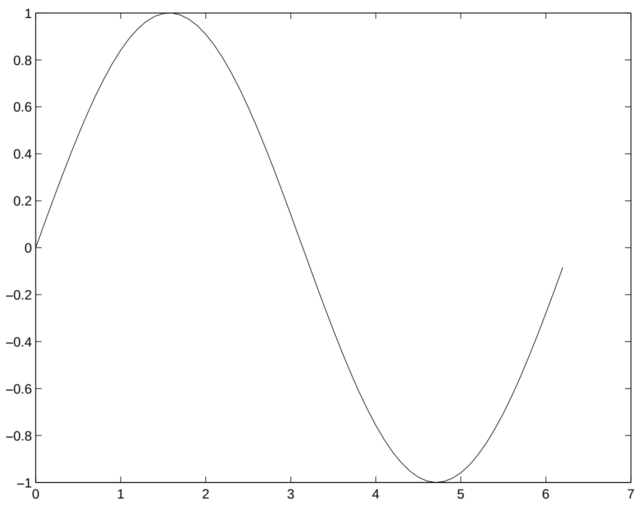


Figure 2.2: A simple plot in MATLAB

We can, for example, plot two functions simultaneously with different colours or plot symbols. For example:

```
>> plot(x,sin(x),'.',x,cos(x),'o')
```

produces the graph shown in figure 2.3.

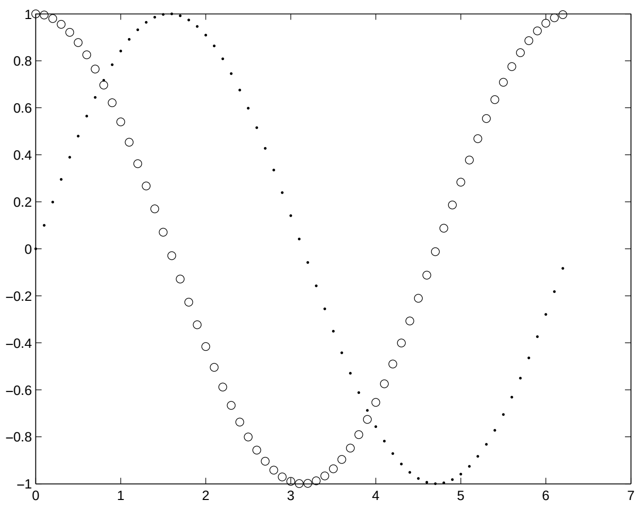


Figure 2.3: A different plot in MATLAB

2.6 Help in Matlab

Matlab comes with a vast amount of online help and information. So much, in fact, that it's quite easy to use Matlab without a manual. To obtain information on a particular command, you can use `help`. For example:

```
>> help for

 FOR    Repeat statements a specific number of times.
    The general form of a FOR statement is:

       FOR variable = expr, statement, ..., statement END

    The columns of the expression are stored one at a time in
    the variable and then the following statements, up to the
    END, are executed. The expression is often of the form X:Y,
    in which case its columns are simply scalars. Some examples
    (assume N has already been assigned a value).

         FOR I = 1:N,
             FOR J = 1:N,
                 A(I,J) = 1/(I+J-1);
             END
         END

    FOR S = 1.0: -0.1: 0.0, END steps S with increments of -0.1
    FOR E = EYE(N), ... END  sets E to the unit N-vectors.

    Long loops are more memory efficient when the colon expression appears
    in the FOR statement since the index vector is never created.

    The BREAK statement can be used to terminate the loop prematurely.

    See also IF, WHILE, SWITCH, BREAK, END.
```

If there is too much information, it may scroll past you too fast to see. In such case you can turn on the `Matlab` pager with the command

```
>> more on
```

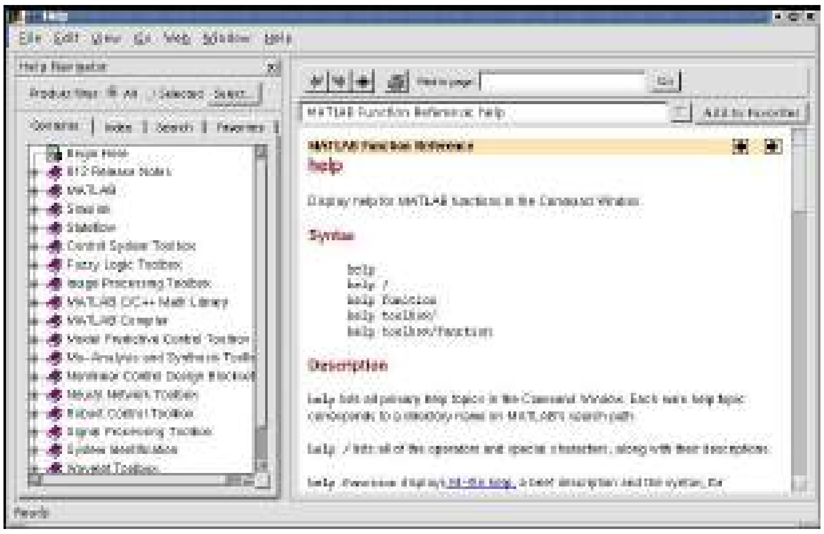
For more help on `help`, try:

```
>> help help
```

Better formatted help can be obtained with the `doc` function, which opens up a help browser which interprets HTML-formatted help files. The result of the command

```
>> doc help
```

is the window shown in figure 2.4.


File Edit View Go Web Window Help
Help Navigator
Product filter: All Selected Select...
Contents Index Search Favorites
Begin Here
R12 Release Notes
MATLAB
Simulink
Stateflow
Control System Toolbox
Fuzzy Logic Toolbox
Image Processing Toolbox
MATLAB C/C++ Math Library
MATLAB Compiler
Model Predictive Control Toolbox
Mu-Analysis and Synthesis Toolbox
Nonlinear Control Design Blockset
Neural Network Toolbox
Robust Control Toolbox
Signal Processing Toolbox
System Identification
Wavelet Toolbox
Find in page:
Go
MATLAB Function Reference: help
Add to Favorites
Ready

MATLAB Function Reference

help

Display help for MATLAB functions in the Command Window

Syntax

```
help
help /
help function
help toolbox/
help toolbox/function
```

Description

help lists all primary help topics in the Command Window. Each main help topic corresponds to a directory name on MATLAB's search path.

help / lists all of the operators and special characters, along with their descriptions.

help function displays M-file help, a brief description and the syntax for

Figure 2.4: The MATLAB help browser

You can find more about the `doc` function with any of

```
>> doc doc
>> help doc
```

If you want to find help on a particular topic, but don't know the function to use, the `lookfor` function is extremely helpful. The command

```
lookfor topic
```

lists all commands for which the first line of the help text contains the string `topic`. For example, suppose we want to find out if MATLAB supports the exponential function

$$e^x.$$

Here's how:

```
>> lookfor exponential
EXP    Exponential.
EXPINT Exponential integral function.
EXPM   Matrix exponential.
EXPM1  Matrix exponential via Pade approximation.
EXPM2  Matrix exponential via Taylor series.
EXPM3  Matrix exponential via eigenvalues and eigenvectors.
BLKEXP Defines a function that returns the exponential of the input.
```

and we now know that the function is implemented using `exp`. We could have used

```
>> lookfor exp
```

and this would have returned many more functions.

Note that MATLAB convention is to use uppercase names for functions in help texts, even though the function itself is called in lowercase.

Exercises

1. Perform the following calculations in MATLAB:

   $$132 + 45, \quad 235 \times 645, \quad 12.45/17.56, \quad \sin(\pi/6), \quad e^{0.5}, \quad \sqrt{2}$$

2. Now enter `format long` and repeat the above calculations.

3. Read the help file for `format`, and experiment with some of the other settings.

4. Enter the following variables: $a = 123456$, $b = 3^{1/4}$, $c = \cos(\pi/8)$. Now calculate:

   $$(a + b)/c, \quad 2a - 3b, \quad c^2 - \sqrt{a - b}, \quad a/(3b + 4c), \quad \exp(a^{1/4} - b^{10})$$

5. Find the MATLAB functions for the inverse trigonometric functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Then calculate:

   $$\sin^{-1}(0.5), \quad \cos^{-1}(\sqrt{3}/2), \quad \tan^{-1}(2)$$

   Convert your answers from radians to degrees.

6. Using vectorization and the colon operator, use a single command each to generate:

   (a) the first 15 cubes,

   (b) the values $\sin(n\pi/16)$ for n from 1 to 16,

   (c) the values $\sqrt{n}$ for n from 10 to 20.

7. Enter the following matrices:

   $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & -1 \\ -3 & -4 & 5 \\ 2 & 3 & -4 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -2 & 1 \\ -3 & 5 & 2 \\ 1 & 1 & -7 \end{bmatrix}$$

   Now calculate:

   $$2A - 3B, \quad A^T, \quad AB - BA, \quad BC^{-1}, \quad (AB)^T, \quad B^T A^T, \quad A^2 + B^3$$

8. Use the `det` function to determine the determinant of each of the matrices in the previous question. What happens if you try to find the inverse of matrix A?

9. Plot the function $\tan(x)$ with the following commands:

```
>> x=[0:0.1:10];
>> plot(x,tan(x))
>> figure,plot(x,tan(x)),axis([0,10,-10,10])
```

   What does the `axis` function do? Read the help file for it. Experiment with changing the last two numbers in `axis` for the above command.

Chapter 3

Images and Matlab

We have seen in the previous chapter that matrices can be handled very efficiently in Matlab. Images may be considered as matrices whose elements are the pixel values of the image. In this chapter we shall investigate how the matrix capabilities of Matlab allow us to investigate images and their properties.

3.1 Greyscale images

Suppose you are sitting at your computer and have started Matlab. You will have a Matlab *command window* open, and in it the Matlab prompt

```
>>
```

ready to receive commands. Type in the command

```
>> w=imread('wombats.tif');
```

This takes the grey values of all the pixels in the greyscale image `wombats.tif` and puts them all into a matrix `w`. This matrix `w` is now a Matlab *variable*, and so we can perform various matrix operations on it. In general the `imread` function reads the pixel values from an image file, and returns a matrix of all the pixel values.

Two things to note about this command:

1. It ends in a *semicolon*; this has the effect of not displaying the results of the command to the screen. As the result of this particular command is a matrix of size 256×256, or with 65536 elements, we don't really want all its values displayed.

2. The name `wombats.tif` is given in single quote marks. Without them, Matlab would assume that `wombats.tif` was the name of a variable, rather than the name of a file.

Now we can display this matrix as a greyscale image:

```
>> figure,imshow(w),pixval on
```

This is really three commands on the one line. Matlab allows many commands to be entered on the same line; using commas to separate the different commands. The three commands we are using here are:

`figure`, which creates a *figure* on the screen. A figure is a window in which a graphics object can be placed. Objects may include images, or various types of graphs.

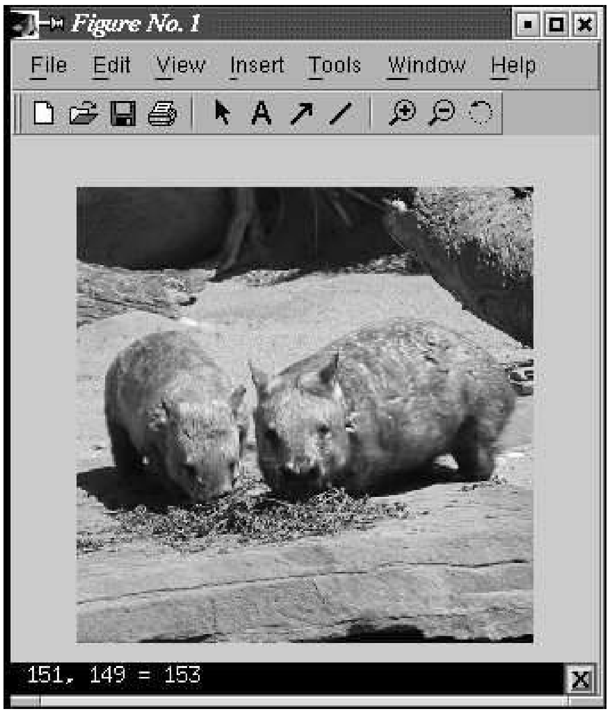
`imshow(g)`, which displays the matrix `g` as an image.

`pixval on`, which turns on the pixel values in our figure. This is a display of the grey values of the pixels in the image. They appear at the bottom of the figure in the form

$$c \times r = p$$

where c is the column value of the given pixel; r its row value, and p its grey value. Since `wombats.tif` is an 8-bit greyscale image, the pixel values appear as integers in the range 0–255.

This is shown in figure 3.1.



```
151, 149 = 153
```

Figure 3.1: The wombats image with `pixval on`

If there are no figures open, then an `imshow` command, or any other command which generates a graphics object, will open a new figure for displaying the object. However, it is good practice to use the `figure` command whenever you wish to create a new figure.

We could display this image directly, without saving its grey values to a matrix, with the command

```
imshow('wombats.tif')
```

However, it is better to use a matrix, seeing as these are handled very efficiently in MATLAB.

3.2 RGB Images

Matlab handles 24-bit RGB images in much the same way as greyscale. We can save the colour values to a matrix and view the result:

```
>> a=imread('autumn.tif');
>> figure,imshow(a),pixval on
```

Note now that the pixel values now consist of a list of three values, giving the red, green and blue components of the colour of the given pixel.

An important difference between this type of image and a greyscale image can be seen by the command

```
>> size(a)
```

which returns *three* values: the number of rows, columns, and "pages" of `a`, which is a three-dimensional matrix, also called a *multidimensional array*. Matlab can handle arrays of any dimension, and `a` is an example. We can think of `a` as being a stack of three matrices, each of the same size.

To obtain any of the RGB values at a given location, we use similar indexing methods to above. For example

```
>> a(100,200,2)
```

returns the second colour value (green) at the pixel in row 100 and column 200. If we want all the colour values at that point, we can use

```
>> a(100,200,1:3)
```

However, Matlab allows a convenient shortcut for listing all values along a particular dimension; just using a colon on its own:

```
>> a(100,200,:)
```

A useful function for obtaining RGB values is `impixel`; the command

```
>> impixel(a,200,100)
```

returns the red, green, and blue values of the pixel at column 200, row 100. Notice that the order of indexing is the same as that which is provided by the `pixval on` command. This is opposite to the row, column order for matrix indexing. This command also applies to greyscale images:

```
>> impixel(g,100,200)
```

will return three values, but since `g` is a single two-dimensional matrix, all three values will be the same.

3.3 Indexed colour images

The command

```
>> figure,imshow('emu.tif'),pixval on
```

produces a nice colour image of an emu. However, the pixel values, rather than being three integers as they were for the RGB image above, are three fractions between 0 and 1. What is going on here?

If we try saving to a matrix first and then displaying the result:

```
>> em=imread('emu.tif');
>> figure,imshow(em),pixval on
```

we obtain a dark, barely distinguishable image, with single integer grey values, indicating that `em` is being interpreted as a single greyscale image.

In fact the image `emu.tif` is an example of an *indexed image*, consisting of two matrices: a *colour map*, and an *index* to the colour map. Assigning the image to a single matrix picks up only the index; we need to obtain the colour map as well:

```
>> [em,emap]=imread('emu.tif');
>> figure,imshow(em,emap),pixval on
```

Matlab stores the RGB values of an indexed image as values of type `double`, with values between 0 and 1.

Information about your image

A great deal of information can be obtained with the `imfinfo` function. For example, suppose we take our indexed image `emu.tif` from above.

```
>> imfinfo('emu.tif')

ans =

                     Filename: 'emu.tif'
                  FileModDate: '26-Nov-2002 14:23:01'
                     FileSize: 119804
                       Format: 'tif'
                FormatVersion: []
                        Width: 331
                       Height: 384
                     BitDepth: 8
                    ColorType: 'indexed'
              FormatSignature: [73 73 42 0]
                    ByteOrder: 'little-endian'
               NewSubfileType: 0
                BitsPerSample: 8
                  Compression: 'PackBits'
    PhotometricInterpretation: 'RGB Palette'
                 StripOffsets: [16x1 double]
              SamplesPerPixel: 1
                 RowsPerStrip: 24
              StripByteCounts: [16x1 double]
                  XResolution: 72
                  YResolution: 72
               ResolutionUnit: 'Inch'
                     Colormap: [256x3 double]
          PlanarConfiguration: 'Chunky'
                    TileWidth: []
                   TileLength: []
                  TileOffsets: []
               TileByteCounts: []
                  Orientation: 1
                    FillOrder: 1
             GrayResponseUnit: 0.0100
               MaxSampleValue: 255
               MinSampleValue: 0
                 Thresholding: 1
```

Mushc of this information is not useful to us; but we can see the size of the image in pixels, the size of the file (in bytes), the number of bits per pixel (this is given by `BitDepth`), and the colour type (in this case “indexed”).

For comparison, let’s look at the output of a true colour file (showing only the first few lines of the output):

```
>> imfinfo('flowers.tif')

ans =

                    Filename: [1x57 char]
                 FileModDate: '26-Oct-1996 02:11:09'
                    FileSize: 543962
                      Format: 'tif'
               FormatVersion: []
                       Width: 500
                      Height: 362
                    BitDepth: 24
                   ColorType: 'truecolor'
```

Now we shall test this function on a binary image:

```
>> imfinfo('text.tif')

ans =

                    Filename: [1x54 char]
                 FileModDate: '26-Oct-1996 02:12:23'
                    FileSize: 3474
                      Format: 'tif'
               FormatVersion: []
                       Width: 256
                      Height: 256
                    BitDepth: 1
                   ColorType: 'grayscale'
```

What is going on here? We have a binary image, and yet the colour type is given as "grayscale". The fact is that MATLAB does not distinguish between greyscale and binary images: a binary image is just a special case of a greyscale image which has only two intensities. However, we can see that `text.tif` is a binary image since the number of bits per pixel is only one.

3.4 Data types and conversions

Elements in MATLAB matrices may have a number of different numeric data types; the most common are listed in table 3.1. There are others; see the help for `datatypes`. These data types are also

Data type	**Description**	**Range**
`int8`	8-bit integer	−128 — 127
`uint8`	8-bit unsigned integer	0 — 255
`int16`	16-bit integer	−32768 — 32767
`uint16`	16-bit unsigned integer	0 — 65535
`double`	Double precision real number	Machine specific

Table 3.1: Data types in MATLAB

functions, we can convert from one type to another. For example:

```
>>  a=23;
>> b=uint8(a);
>> b

b =

    23

>> whos a b
  Name      Size           Bytes  Class

  a         1x1                8  double array
  b         1x1                1  uint8 array
```

Even though the variables `a` and `b` have the same numeric value, they are of different data types. An important consideration (of which we shall more) is that arithmetic operations are not permitted with the data types `int8`, `int16`, `uint8` and `uint16`.

A greyscale image may consist of pixels whose values are of data type `uint8`. These images are thus reasonably efficient in terms of storage space, since each pixel requires only one byte. However, arithmetic operations are not permitted on this data type; a `uint8` image must be converted to `double` before any arithmetic is attempted.

We can convert images from one image type to another. Table 3.2 lists all of MATLAB's functions for converting between different image types. Note that the `gray2rgb` function, does not create a colour image, but an image all of whose pixel colours were the same as before. This is done by

Function	**Use**	**Format**
`ind2gray`	Indexed to Greyscale	`y=ind2gray(x,map);`
`gray2ind`	Greyscale to indexed	`[y,map]=gray2ind(x);`
`rgb2gray`	RGB to greyscale	`y=rgb2gray(x);`
`gray2rgb`	Greyscale to RGB	`y=gray2rgb(x);`
`rgb2ind`	RGB to indexed	`[y,map]=rgb2ind;`
`ind2rgb`	Indexed to RGB	`y=ind2rgb(x,map);`

Table 3.2: Converting images in Matlab

simply replicating the grey values of each pixel: greys in an RGB image are obtained by equality of the red, green and blue values.

Exercises

1. Type in the command

   ```
   >> help imdemos
   ```

   This will give you a list of, amongst other things, all the sample TIFF images which come with the Image Processing Toolbox. Make a list of these sample images, and for each image

   (a) determine its type (binary, greyscale, true colour or indexed colour),

   (b) determine its size (in pixels)

   (c) give a brief description of the picture (what it looks like; what it seems to be a pixture of)

2. Pick a greyscale image, say `cameraman.tif` or `wombats.tif`. Using the `imwrite` function, write it to files of type JPEG, PNG and BMP.

   What are the sizes of those files?

3. Repeat the above question with

   (a) a binary image,

   (b) an indexed colour image,

   (c) a true colour image.

Chapter 4

Image Display

4.1 Introduction

We have touched briefly in chapter 2 on image display. In this chapter, we investigate this matter in more detail. We look more deeply at the use of the `imshow` function, and how spatial resolution and quantization can affect the display and appearance of an image. In particular, we look at image quality, and how that may be affected by various image attributes. Quality is of course a highly subjective matter: no two people will agree precisely as to the quality of different images. However, for human vision in general, images are preferred to be sharp and detailed. This is a consequence of two properties of an image: its spatial resolution, and its quantization.

4.2 The `imshow` function

Greyscale images

We have seen that if `x` is a matrix of type `uint8`, then the command

```
imshow(x)
```

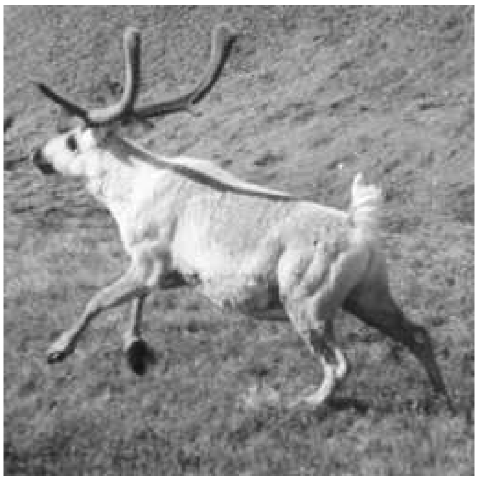
will display `x` as an image. This is reasonable, since the data type `uint8` restricts values to be integers between 0 and 255. However, not all image matrices come so nicely bundled up into this data type, and lots of MATLAB image processing commands produces output matrices which are of type `double`. We have two choices with a matrix of this type:

1. convert to type `uint8` and then display,

2. display the matrix directly.

The second option is possible because `imshow` will display a matrix of type `double` as a greyscale image as long as the matrix elements are between 0 and 1. Suppose we take an image and convert it to type `double`:

```
>> c=imread('caribou.tif');
>> cd=double(c);
>> imshow(c),figure,imshow(cd)
```



(a) The original image (b) After conversion to type `double`

Figure 4.1: An attempt at data type conversion

The results are shown in figure 4.1.

However, as you can see, figure 4.1(b) doesn't look much like the original picture at all! This is because for a matrix of type `double`, the `imshow` function expects the values to be between 0 and 1, where 0 is displayed as black, and 1 is displayed as white. A value x with $0 \le x \le 1$ is displayed as grey scale $\lfloor 255x \rfloor$. Conversely, values greater than 1 will be displayed as 1 (white) and values less than 0 will be displayed as zero (black). In the caribou image, every pixel has value greater than or equal to 1 (in fact the minimum value is 21), so that every pixel will be displayed as white. To display the matrix `cd`, we need to scale it to the range 0—1. This is easily done simply by dividing all values by 255:

```
>> imshow(cd/255)
```

and the result will be the caribou image as shown in figure 4.1(a).

We can vary the display by changing the scaling of the matrix. Results of the commands:

```
>> imshow(cd/512)
>> imshow(cd/128)
```

are shown in figures 4.2.

Dividing by 512 darkens the image, as all matrix values are now between 0 and 0.5, so that the brightest pixel in the image is a mid-grey. Dividing by 128 means that the range is 0—2, and all pixels in the range 1—2 will be displayed as white. Thus the image has an over-exposed, washed-out appearance.

The display of the result of a command whose output is a matrix of type `double` can be greatly affected by a judicious choice of a scaling factor.

We can convert the original image to `double` more properly using the function `im2double`. This applies correct scaling so that the output values are between 0 and 1. So the commands

```
>> cd=im2double(c);
>> imshow(cd)
```

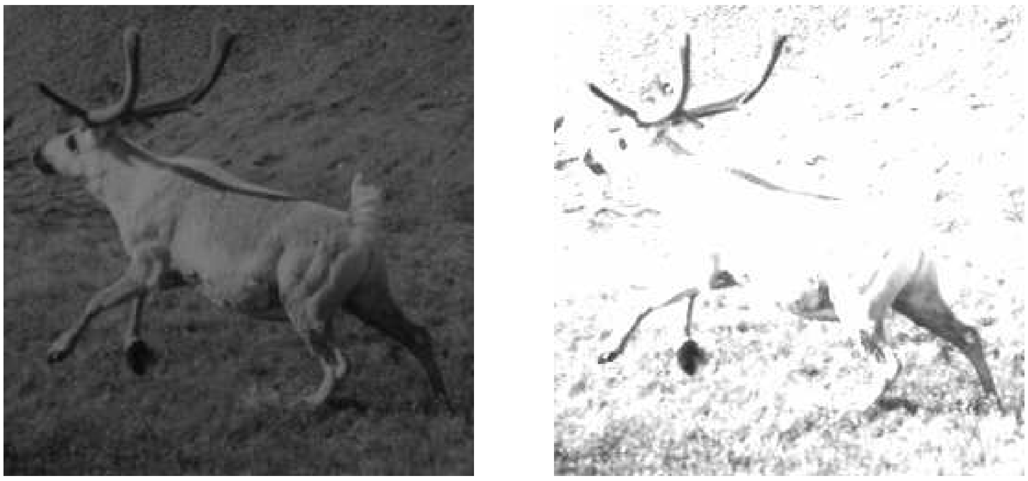

(a) The matrix cd divided by 512 (b) The matrix cd divided by 128

Figure 4.2: Scaling by dividing an image matrix by a scalar

will produce a correct image. It is important to make the distinction between the two functions `double` and `im2double`: `double` changes the data type but does not change the numeric values; `im2double` changes both the numeric data type and the values. The exception of course is if the original image is of type `double`, in which case `im2double` does nothing. Although the command `double` is not of much use for direct image display, it can be very useful for image arithmetic. We have seen examples of this above with scaling.

Corresponding to the functions `double` and `im2double` are the functions `uint8` and `im2uint8`. If we take our image `cd` of type `double`, properly scaled so that all elements are between 0 and 1, we can convert it back to an image of type `uint8` in two ways:

```
>> c2=uint8(255*cd);
>> c3=im2uint8(cd);
```

Use of `im2uint8` is to be preferred; it takes other data types as input, and always returns a correct result.

Binary images

Recall that a binary image will have only two values: 0 and 1. MATLAB does not have a `binary` data type as such, but it does have a `logical` flag, where `uint8` values as 0 and 1 can be interpreted as logical data. The logical flag will be set by the use of relational operations such as `==`, `<` or `>` or any other operations which provide a yes/no answer. For example, suppose we take the caribou matrix and create a new matrix with

```
>> cl=c>120;
```

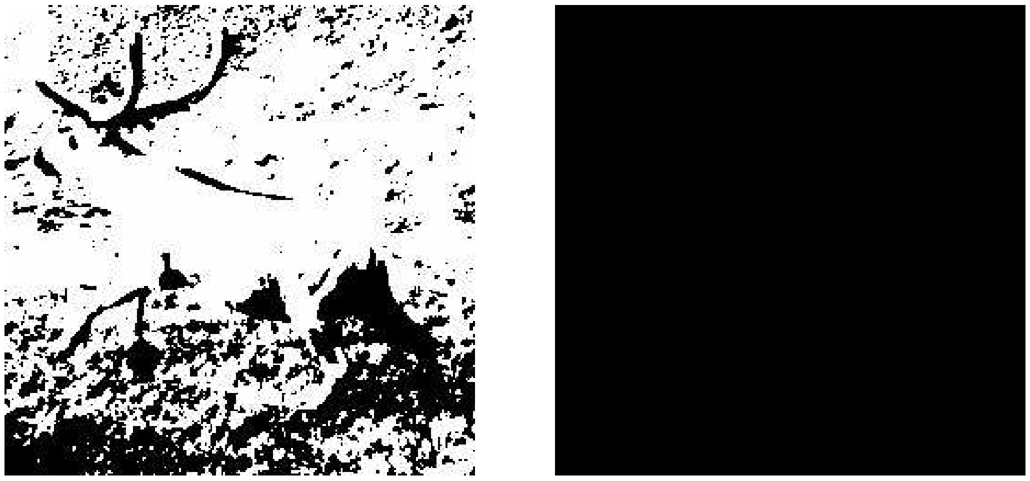
(we will see more of this type of operation in chapter 5.) If we now check all of our variables with `whos`, the output will include the line:

```
  cl      256x256        65536  uint8 array (logical)
```

This means that the command

```
>> imshow(cl)
```

will display the matrix as a binary image; the result is shown in figure 4.3.



(a) The caribou image turned binary (b) After conversion to type `uint8`

Figure 4.3: Making the image binary

Suppose we remove the logical flag from `cl`; this can be done by a simple command:

```
>> cl = +cl;
```

Now the output of `whos` will include the line:

```
  cl      256x256         65536  uint8 array
```

If we now try to display this matrix with `imshow`, we obtain the result shown in figure 4.3(b). A very disappointing image! But this is to be expected; in a matrix of type `uint8`, white is 255, 0 is black, and 1 is a very dark grey–indistinguishable from black.

To get back to a viewable image, we can either turn the logical flag back on, and the view the result:

```
>> imshow(logical(cl))
```

or simply convert to type `double`:

```
>> imshow(double(cl))
```

Both these commands will produce the image seen in figure 4.3.

4.3 Bit planes

Greyscale images can be transformed into a sequence of binary images by breaking them up into their *bit-planes*. If we consider the grey value of each pixel of an 8-bit image as an 8-bit binary word, then the 0th bit plane consists of the last bit of each grey value. Since this bit has the least effect in terms of the magnitude of the value, it is called the *least significant bit*, and the plane consisting of those bits the *least significant bit plane*. Similarly the 7th bit plane consists of the first bit in each value. This bit has the greatest effect in terms of the magnitude of the value, so it is called the *most significant bit*, and the plane consisting of those bits the *most significant bit plane*.

If we take a greyscale image, we start by making it a matrix of type `double`; this means we can perform arithmetic on the values.

```
>> c=imread('cameraman.tif');
>> cd=double(c);
```

We now isolate the bit planes by simply dividing the matrix `cd` by successive powers of 2, throwing away the remainder, and seeing if the final bit is 0 or 1. We can do this with the `mod` function.

```
>> c0=mod(cd,2);
>> c1=mod(floor(cd/2),2);
>> c2=mod(floor(cd/4),2);
>> c3=mod(floor(cd/8),2);
>> c4=mod(floor(cd/16),2);
>> c5=mod(floor(cd/32),2);
>> c6=mod(floor(cd/64),2);
>> c7=mod(floor(cd/128),2);
```

These are all shown in figure 4.4. Note that the least significant bit plane, `c0`, is to all intents and purposes a random array, and that we see more of the image appearing. The most significant bit plane, `c7`, is actually a *threshold* of the image at level 127:

```
>> ct=c>127;
>> all(c7(:)==ct(:))

ans =

     1
```

We shall discuss thresholding in chapter 5.

We can recover and display the original image with

```
>> cc=2*(2*(2*(2*(2*(2*(2*c7+c6)+c5)+c4)+c3)+c2)+c1)+c0;
>> imshow(uint8(cc))
```

4.4 Spatial Resolution

Spatial resolution is the density of pixels over the image: the greater the spatial resolution, the more pixels are used to display the image. We can experiment with spatial resolution with MATLAB's `imresize` function. Suppose we have an 256×256 8-bit greyscale image saved to the matrix `x`. Then the command

```
imresize(x,1/2);
```

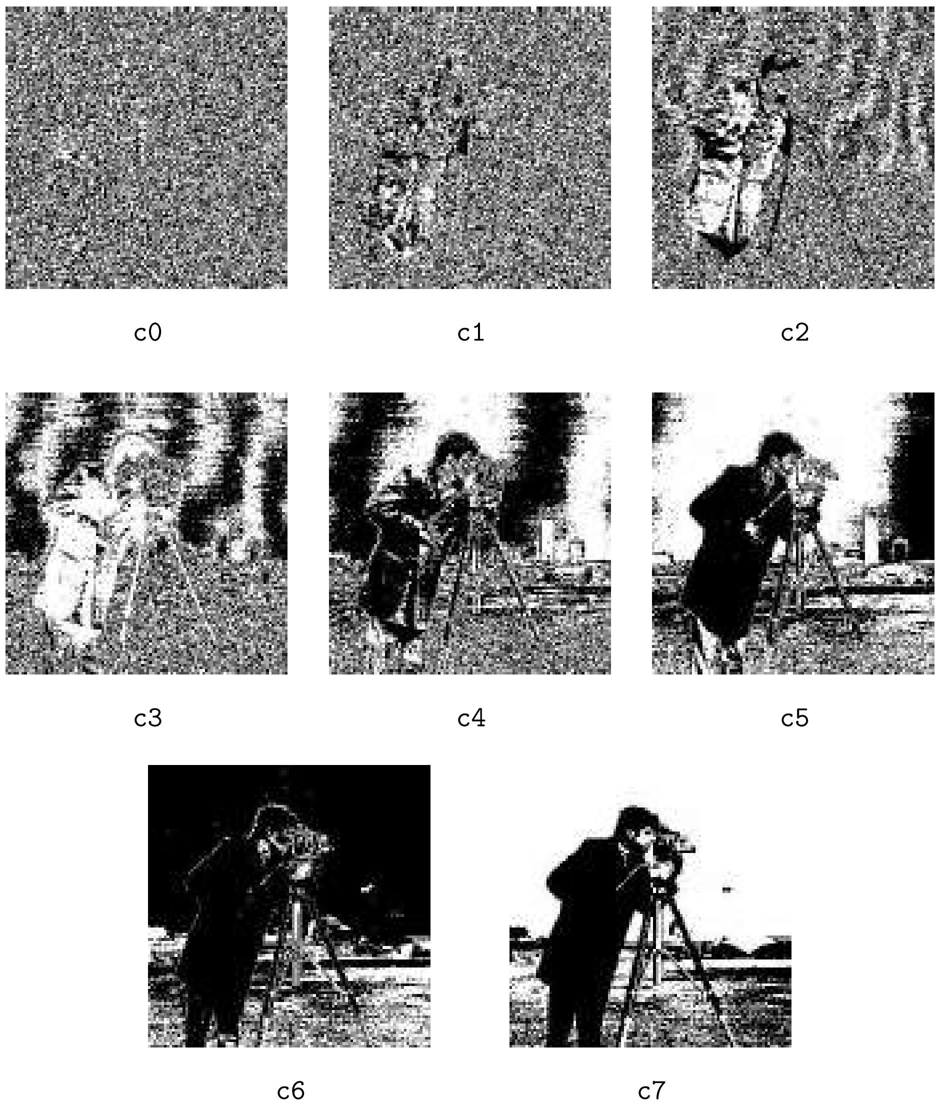


Figure 4.4: The bit planes of an 8-bit greyscale image

will halve the size of the image. It does this by taking out every other row and every other column, thus leaving only those matrix elements whose row and column indices are even:

$$\begin{array}{ccccccc}
x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & \cdots \\
x_{21} & \boxed{x_{22}} & x_{23} & \boxed{x_{24}} & x_{25} & \boxed{x_{26}} & \cdots \\
x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & \cdots \\
x_{41} & \boxed{x_{42}} & x_{43} & \boxed{x_{44}} & x_{45} & \boxed{x_{46}} & \cdots \\
x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & \cdots \\
x_{61} & \boxed{x_{62}} & x_{63} & \boxed{x_{64}} & x_{65} & \boxed{x_{66}} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}
\longrightarrow \texttt{imresize(x,1/2)} \longrightarrow
\begin{array}{cccc}
x_{22} & x_{24} & x_{26} & \cdots \\
x_{42} & x_{44} & x_{46} & \cdots \\
x_{62} & x_{64} & x_{66} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{array}$$

If we apply `imresize` to the result with the parameter `2` rather than `1/2`, all the pixels are repeated to produce an image with the same size as the original, but with half the resolution in each direction:

$$\begin{array}{cccc}
\boxed{\begin{matrix} x_{22} & x_{22} \\ x_{22} & x_{22} \end{matrix}} & \boxed{\begin{matrix} x_{24} & x_{24} \\ x_{24} & x_{24} \end{matrix}} & \boxed{\begin{matrix} x_{26} & x_{26} \\ x_{26} & x_{26} \end{matrix}} & \cdots \\
\boxed{\begin{matrix} x_{42} & x_{42} \\ x_{42} & x_{42} \end{matrix}} & \boxed{\begin{matrix} x_{44} & x_{44} \\ x_{44} & x_{44} \end{matrix}} & \boxed{\begin{matrix} x_{46} & x_{46} \\ x_{46} & x_{46} \end{matrix}} & \cdots \\
\boxed{\begin{matrix} x_{62} & x_{62} \\ x_{62} & x_{62} \end{matrix}} & \boxed{\begin{matrix} x_{64} & x_{64} \\ x_{64} & x_{64} \end{matrix}} & \boxed{\begin{matrix} x_{66} & x_{66} \\ x_{66} & x_{66} \end{matrix}} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{array}$$

The effective resolution of this new image is only 128×128. We can do all this in one line:

```
x2=imresize(imresize(x,1/2),2);
```

By changing the parameters of `imresize`, we can change the effective resolution of the image to smaller amounts:

Command	Effective resolution
`imresize(imresize(x,1/4),4);`	64×64
`imresize(imresize(x,1/8),8);`	32×32
`imresize(imresize(x,1/16),16);`	16×16
`imresize(imresize(x,1/32),32);`	8×8

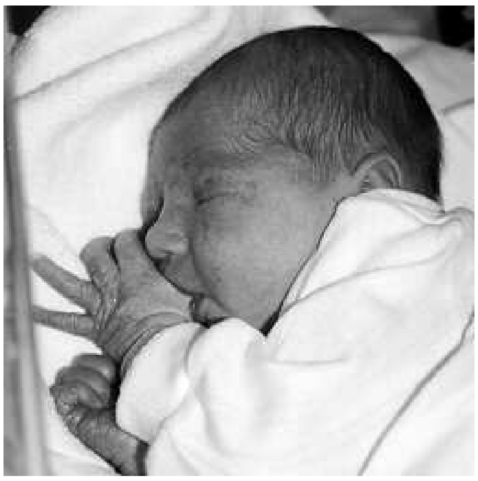
To see the effects of these commands, suppose we apply them to the image `newborn.tif`:

```
x=imread('newborn.tif');
```

The effects of increasing blockiness or *pixelization* become quite pronounced as the resolution decreases; even at 128×128 resolution fine detail, such as the edges of the baby's fingers, are less clear, and at 64×64 all edges are now quite blocky. At 32×32 the image is barely recognizable, and at 16×16 and 8×8 the image becomes unrecognizable.

Exercises

1. Open the greyscale image `cameraman.tif` and view it. What data type is it?
2. Enter the following commands:


(a) The original image

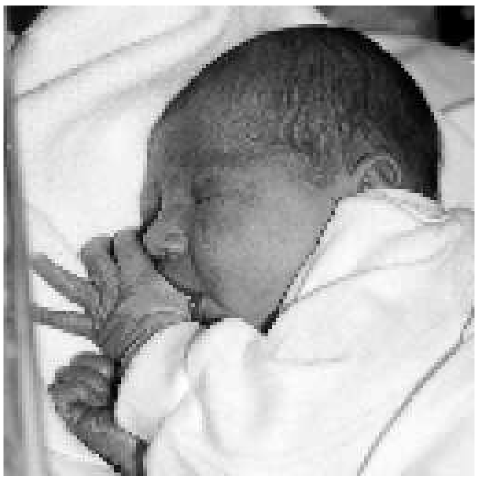
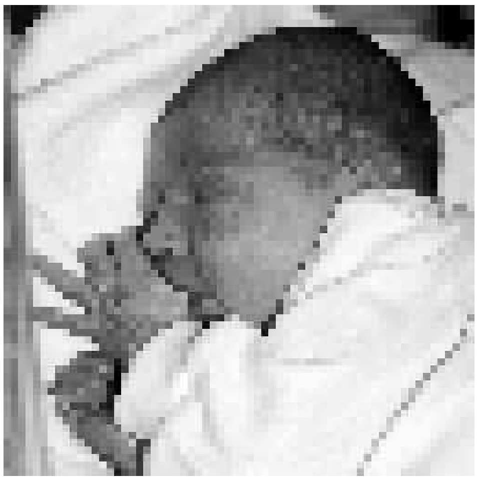

(b) at 128×128 resolution

Figure 4.5: Reducing resolution of an image


(a) At 64×64 resolution

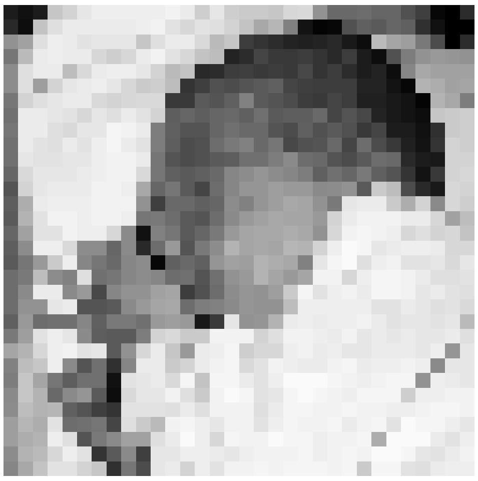
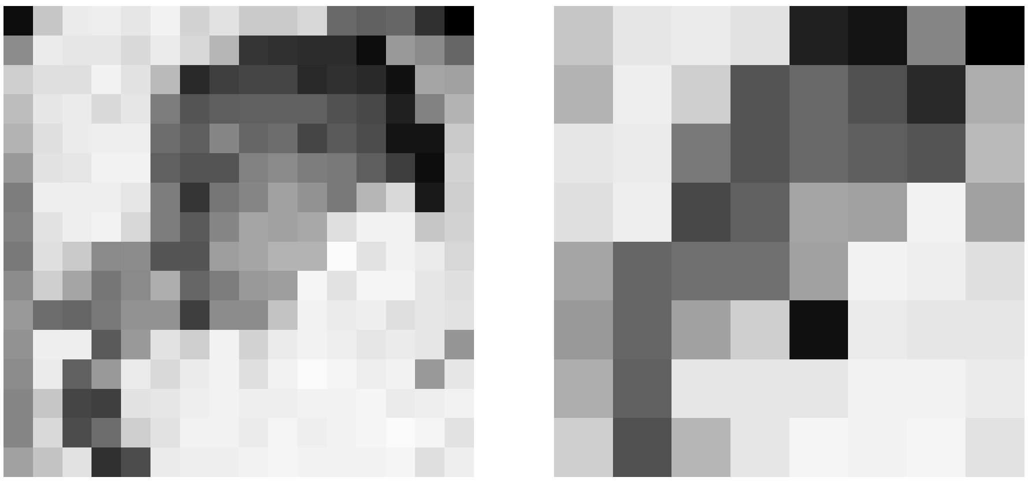

(b) At 32×32 resolution

Figure 4.6: Further reducing the resolution of an image



(a) At 16×16 resolution (b) at 8×8 resolution

Figure 4.7: Even more reducing the resolution of an image

```
>> em,map]=imread(’emu.tif’);
>> e=ind2gray(em,map);
```

These will produce a greyscale image of type `double`. View this image.

3. Enter the command

```
>> e2=im2uint8(e);
```

and view the output.

What does the function `im2uint8` do? What affect does it have on

(a) the appearance of the image?

(b) the elements of the image matrix?

4. What happens if you apply `im2uint8` to the cameraman image?

5. Experiment with reducing spatial resolution of the following images:

(a) `cameraman.tif`

(b) The greyscale emu image

(c) `blocks.tif`

(d) `buffalo.tif`

In each case note the point at which the image becomes unrecognizable.

Chapter 5

Point Processing

5.1 Introduction

Any image processing operation transforms the grey values of the pixels. However, image processing operations may be divided into into three classes based on the information required to perform the transformation. From the most complex to the simplest, they are:

1. **Transforms.** We require a knowledge of all the grey levels in the entire image to transform the image. In other words, the entire image is processed as a single large block. This may be illustrated by the diagram shown in figure 5.1.

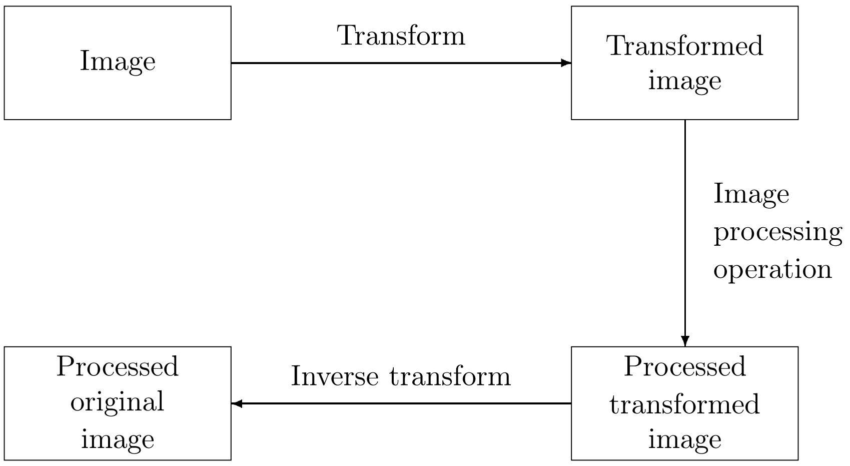


Figure 5.1: Schema for transform processing

2. **Spatial filters.** To change the grey level of a given pixel we need only know the value of the grey levels in a small neighbourhood of pixels around the given pixel.

3. **Point operations.** A pixel's grey value is changed without any knowledge of its surrounds.

Although point operations are the simplest, they contain some of the most powerful and widely used of all image processing operations. They are especially useful in image *pre-processing*, where an image is required to be modified before the main job is attempted.

5.2 Arithmetic operations

These operations act by applying a simple function

$$y = f(x)$$

to each grey value in the image. Thus $f(x)$ is a function which maps the range $0 \ldots 255$ onto itself. Simple functions include adding or subtract a constant value to each pixel:

$$y = x \pm C$$

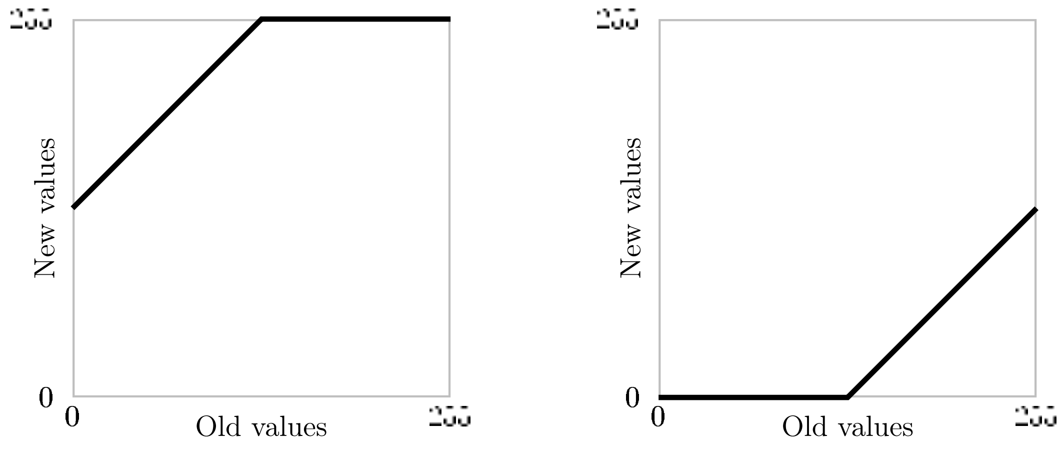
or multiplying each pixel by a constant:

$$y = Cx.$$

In each case we may have to fiddle the output slightly in order to ensure that the results are integers in the $0 \ldots 255$ range. We can do this by first rounding the result (if necessary) to obtain an integer, and then "clipping" the values by setting:

$$y \leftarrow \begin{cases} 255 & \text{if } y > 255, \\ 0 & \text{if } y < 0. \end{cases}$$

We can obtain an understanding of how these operations affect an image by looking at the graph of old grey values against new values. Figure 5.2 shows the result of adding or subtracting 128 from each pixel in the image. Notice that when we add 128, all grey values of 127 or greater will



Adding 128 to each pixel Subtracting 128 from each pixel

Figure 5.2: Adding and subtracting a constant

be mapped to 255. And when we subtract 128, all grey values of 128 or less will be mapped to 0. By looking at these graphs, we see that in general adding a constant will lighten an image, and subtracting a constant will darken it.

We can test this on the "blocks" image `blocks.tif`, which we have seen in figure 1.4. We start by reading the image in:

```
>> b=imread('blocks.tif');
>> whos b
  Name      Size           Bytes  Class

  b       256x256          65536  uint8 array
```

The point of the second command was to find the numeric data type of b; it is uint8. The unit8 data type is used for data *storage* only; we can't perform arithmetic operations. If we try, we just get an error message:

```
>> b1=b+128
??? Error using ==> +
Function '+' not defined for variables of class 'uint8'.
```

We can get round this in two ways. We can first turn b into a matrix of type double, add the 128, and then turn back to uint8 for display:

```
>> b1=uint8(double(b)+128);
```

A second, and more elegant way, is to use the MATLAB function imadd which is designed precisely to do this:

```
>> b1=imadd(b,128);
```

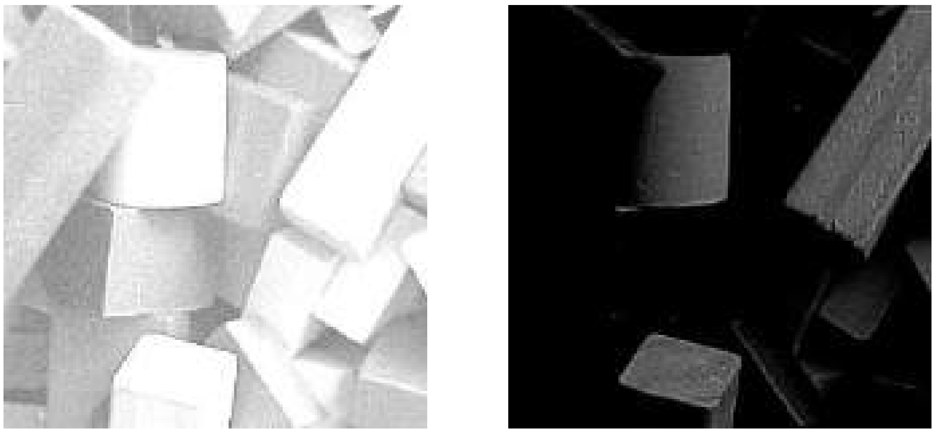
Subtraction is similar; we can transform out matrix in and out of double, or use the imsubtract function:

```
>> b2=imsubtract(b,128);
```

And now we can view them:

```
>> imshow(b1),figure,imshow(b2)
```

and the results are seen in figure 5.3.



b1: Adding 128 b2: Subtracting 128

Figure 5.3: Arithmetic operations on an image: adding or subtracting a constant

We can also perform lightening or darkening of an image by multiplication; figure 5.4 shows some examples of functions which will have these effects. To implement these functions, we use the immultiply function. Table 5.1 shows the particular commands required to implement the functions of figure 5.4. All these images can be viewed with imshow; they are shown in figure 5.5. Compare the results of darkening b2 and b3. Note that b3, although darker than the original, is

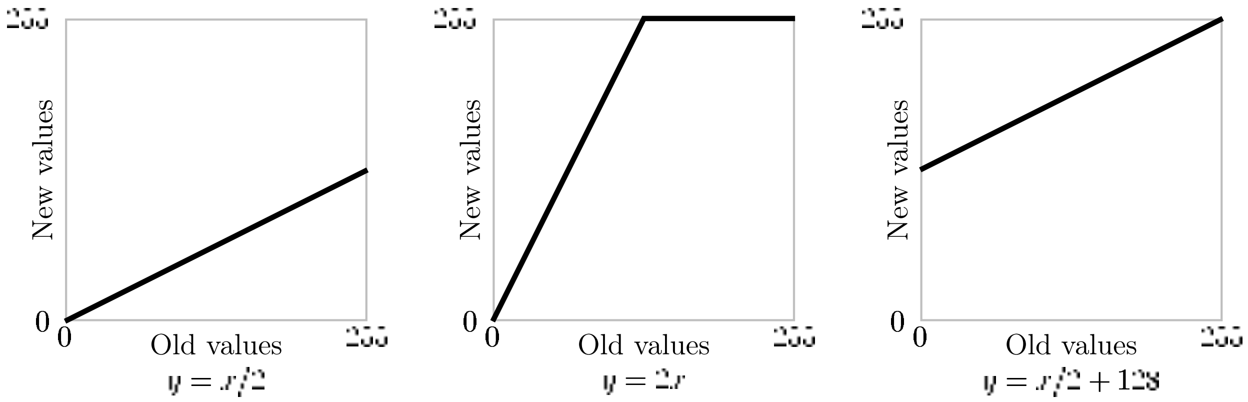


Figure 5.4: Using multiplication and division

$y = x/2$	`b3=immultiply(b,0.5);` or `b3=imdivide(b,2)`
$y = 2x$	`b4=immultiply(b,2);`
$y = x/2 + 128$	`b5=imadd(immultiply(b,0.5),128);` or `b5=imadd(imdivide(b,2),128);`

Table 5.1: Implementing pixel multiplication by MATLAB commands

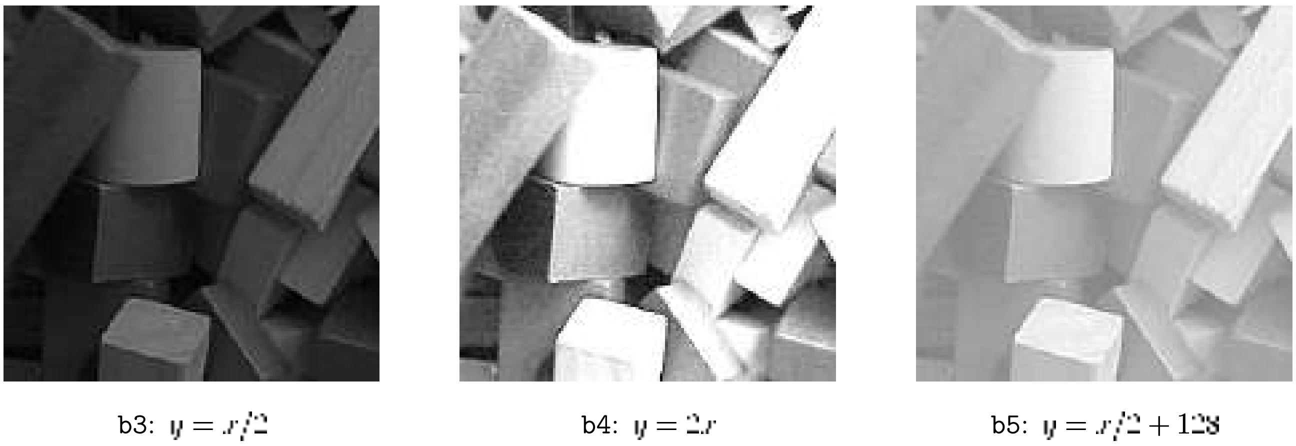


Figure 5.5: Arithmetic operations on an image: multiplication and division

still quite clear, whereas a lot of information has been lost by the subtraction process, as can be seen in image b2. This is because in image b2 all pixels with grey values 128 or less have become zero.

A similar loss of information has occurred in the images b1 and b4. Note in particular the edges of the light coloured block in the bottom centre; in both b1 and b4 the right hand edge has disappeared. However, the edge is quite visible in image b5.

Complements

The *complement* of a greyscale image is its photographic negative. If an image matrix m is of type double and so its grey values are in the range 0.0 to 1.0, we can obtain its negative with the command

```
>> 1-m
```

If the image is binary, we can use

```
>> ~m
```

If the image is of type uint8, the best approach is the imcomplement function. Figure 5.6 shows the complement function $y = 255 - x$, and the result of the commands

```
>> bc=imcomplement(b);
>> imshow(bc)
```

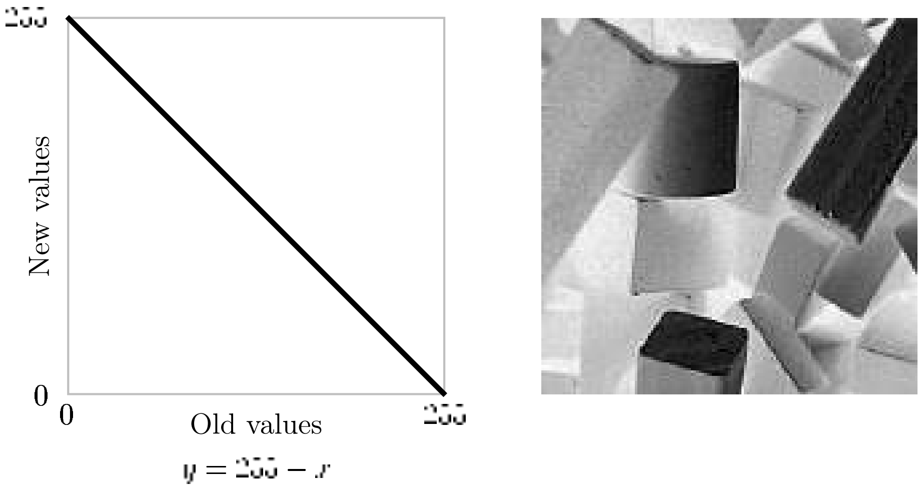


Figure 5.6: Image complementation

Interesting special effects can be obtained by complementing only *part* of the image; for example by taking the complement of pixels of grey value 128 or less, and leaving other pixels untouched. Or we could take the complement of pixels which are 128 or greater, and leave other pixels untouched. Figure 5.7 shows these functions. The effect of these functions is called *solarization*.

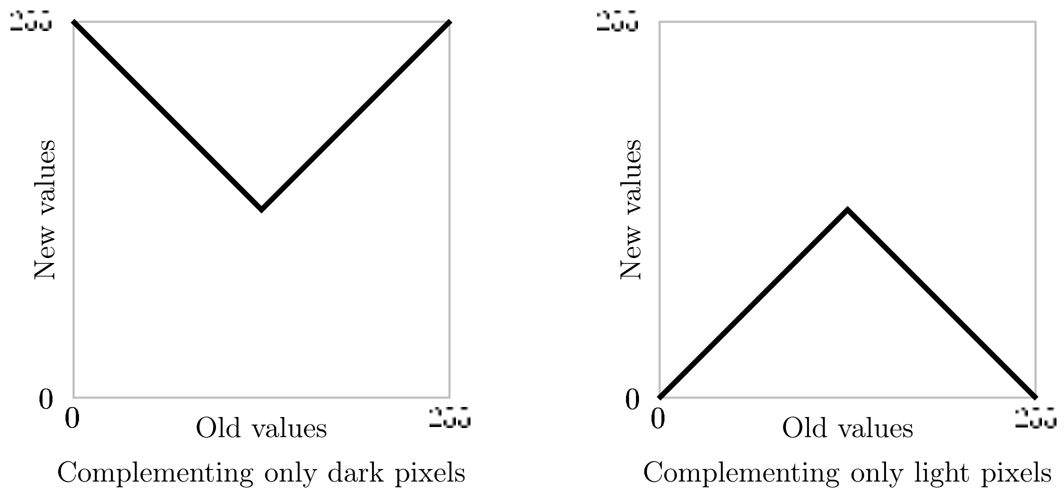


Figure 5.7: Part complementation

5.3 Histograms

Given a greyscale image, its *histogram* consists of the histogram of its grey levels; that is, a graph indicating the number of times each grey level occurs in the image. We can infer a great deal about the appearance of an image from its histogram, as the following examples indicate:

- In a dark image, the grey levels (and hence the histogram) would be clustered at the lower end:
- In a uniformly bright image, the grey levels would be clustered at the upper end:
- In a well contrasted image, the grey levels would be well spread out over much of the range:

We can view the histogram of an image in MATLAB by using the `imhist` function:

```
>> p=imread('pout.tif');
>> imshow(p),figure,imhist(p),axis tight
```

(the `axis tight` command ensures the axes of the histogram are automatically scaled to fit all the values in). The result is shown in figure 5.8. Since the grey values are all clustered together in the centre of the histogram, we would expect the image to be poorly contrasted, as indeed it is.

Given a poorly contrasted image, we would like to enhance its contrast, by spreading out its histogram. There are two ways of doing this.

5.3.1 Histogram stretching (Contrast stretching)

Suppose we have an image with the histogram shown in figure 5.9, associated with a table of the numbers n_i of grey values:

Grey level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	70	110	45	70	35	0	0	0	0	0	15

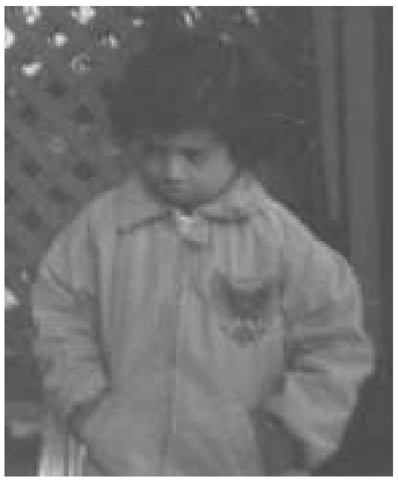
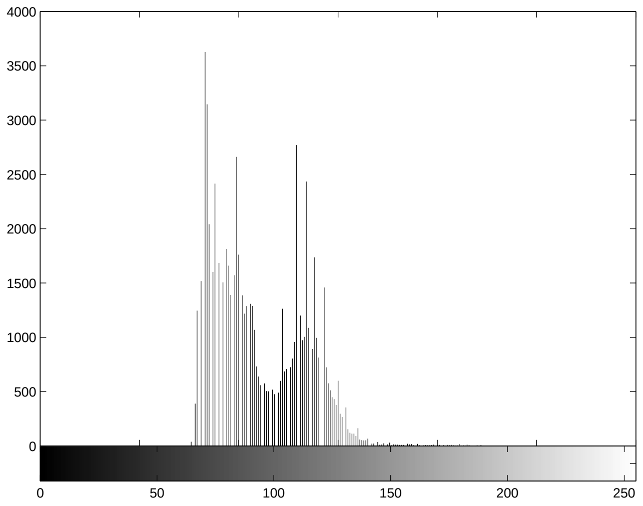



Figure 5.8: The image `pout.tif` and its histogram

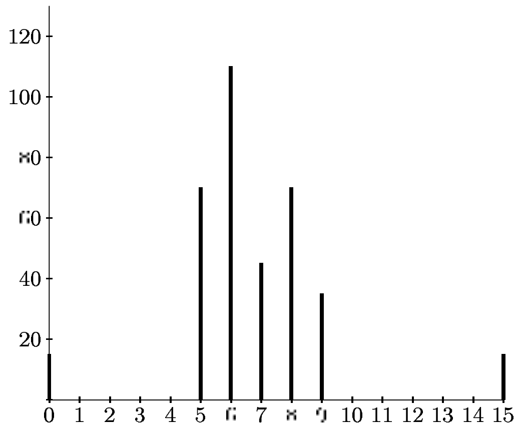
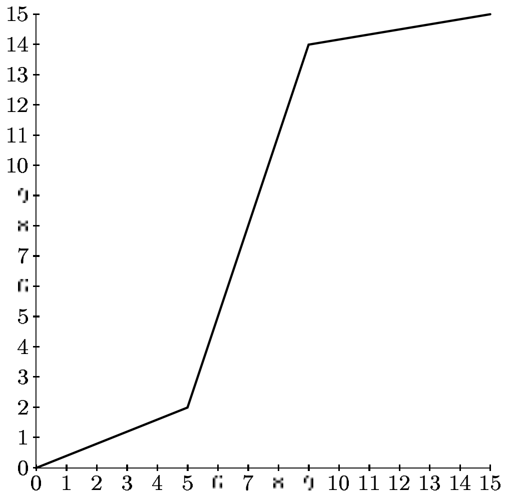



Figure 5.9: A histogram of a poorly contrasted image, and a stretching function

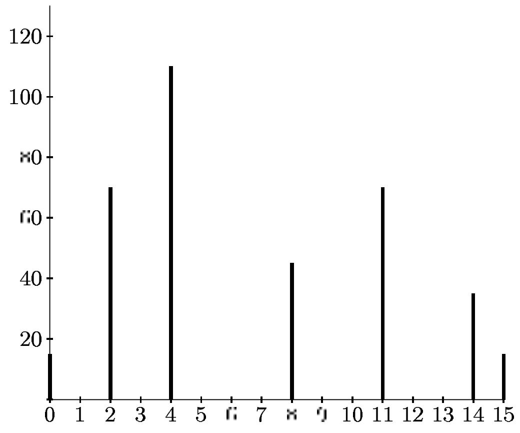
(with $n = 360$, as before.) We can stretch the grey levels in the centre of the range out by applying the piecewise linear function shown at the right in figure 5.9. This function has the effect of stretching the grey levels 5–9 to grey levels 2–14 according to the equation:

$$j = \frac{14 - 2}{9 - 5}(i - 5) + 2$$

where i is the original grey level and j its result after the transformation. Grey levels outside this range are either left alone (as in this case) or transformed according to the linear functions at the ends of the graph above. This yields:

i	5	6	7	8	9
j	2	5	8	11	14

and the corresponding histogram:



which indicates an image with greater contrast than the original.

Use of `imadjust`

To perform histogram stretching in MATLAB the `imadjust` function may be used. In its simplest incarnation, the command

```
imadjust(im,[a,b],[c,d])
```

stretches the image according to the function shown in figure 5.10. Since `imadjust` is designed to

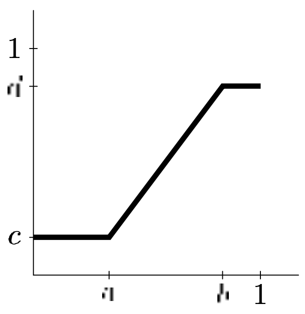


Figure 5.10: The stretching function given by `imadjust`

work equally well on images of type `double`, `uint8` or `uint16` the values of a, b, c and d must be between 0 and 1; the function automatically converts the image (if needed) to be of type `double`.

Note that `imadjust` does not work quite in the same way as shown in figure 5.9. Pixel values less than a are all converted to c, and pixel values greater than b are all converted to d. If either of `[a,b]` or `[c,d]` are chosen to be `[0,1]`, the abbreviation `[]` may be used. Thus, for example, the command

```
>> imadjust(im,[],[])
```

does nothing, and the command

```
>> imadjust(im,[],[1,0])
```

inverts the grey values of the image, to produce a result similar to a photographic negative.

The `imadjust` function has one other optional parameter: the *gamma* value, which describes the shape of the function between the coordinates (a, c) and (b, d). If `gamma` is equal to 1, which is the default, then a linear mapping is used, as shown above in figure 5.10. However, values less than one produce a function which is concave downward, as shown on the left in figure 5.11, and values greater than one produce a figure which is concave upward, as shown on the right in figure 5.11.

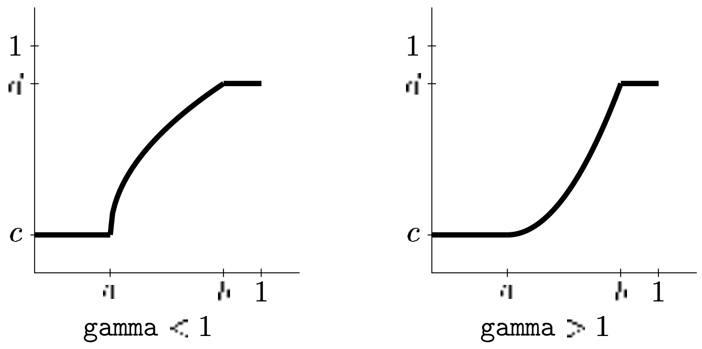


Figure 5.11: The `imadjust` function with `gamma` not equal to 1

The function used is a slight variation on the standard line between two points:

$$y = \left(\frac{x-a}{b-a}\right)^{\gamma}(d-c)+c.$$

Use of the gamma value alone can be enough to substantially change the appearance of the image. For example:

```
>> t=imread('tire.tif');
>> th=imadjust(t,[],[],0.5);
>> imshow(t),figure,imshow(th)
```

produces the result shown in figure 5.12.

We may view the `imadjust` stretching function with the `plot` function. For example,

```
>> plot(t,th,'.'),axis tight
```

produces the plot shown in figure 5.13. Since `p` and `ph` are matrices which contain the original values and the values after the `imadjust` function, the `plot` function simply plots them, using dots to do it.

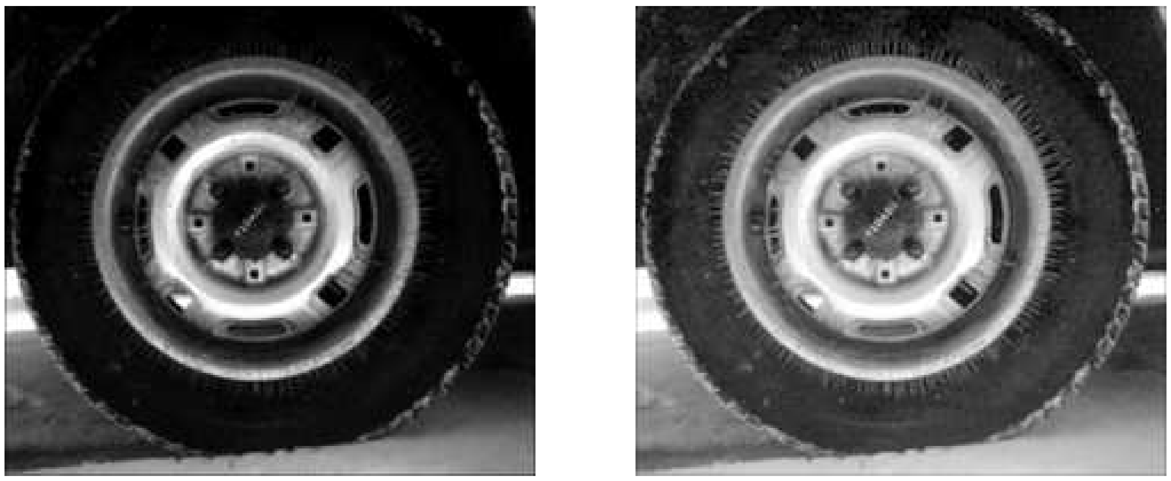


Figure 5.12: The tire image and after adjustment with the `gamma` value

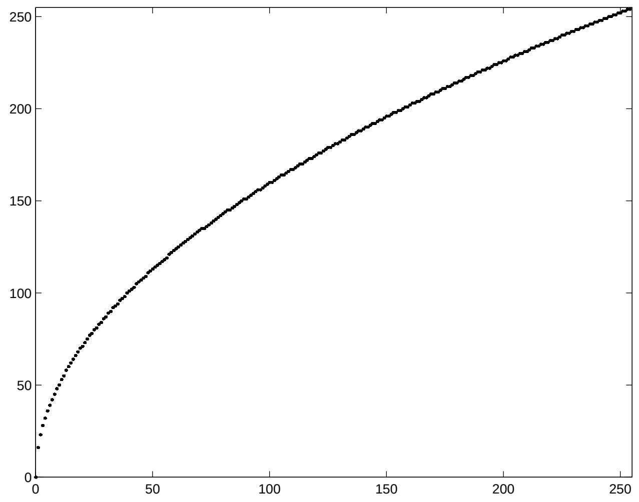


Figure 5.13: The function used in figure 5.12

A piecewise linear stretching function

We can easily write our own function to perform piecewise linear stretching as shown in figure 5.14. To do this, we will make use of the `find` function, to find the pixel values in the image between a_i and a_{i+1}. Since the line between the coordinates (a_i, b_i) and (a_{i+1}, b_{i+1}) has the equation

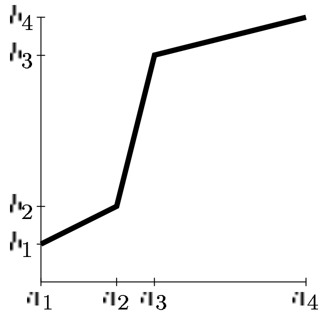


Figure 5.14: A piecewise linear stretching function

$$y = \frac{b_{i+1} - b_i}{a_{i+1} - a_i}(x - a_i) + b_i$$

the heart of our function will be the lines

```
pix=find(im >= a(i) & im < a(i+1));
out(pix)=(im(pix)-a(i))*(b(i+1)-b(i))/(a(i+1)-a(i))+b(i);
```

where `im` is the input image and `out` is the output image. A simple procedure which takes as inputs images of type `uint8` or `double` is shown in figure 5.15. As an example of the use of this function:

```
>> th=histpwl(t,[0 .25 .5 .75 1],[0 .75 .25 .5 1]);
>> imshow(th)
>> figure,plot(t,th,'.'),axis tight
```

produces the figures shown in figure 5.16.

5.3.2 Histogram equalization

The trouble with any of the above methods of histogram stretching is that they require user input. Sometimes a better approach is provided by *histogram equalization*, which is an entirely automatic procedure.

Suppose our image has L different grey levels $0, 1, 2, \ldots L-1$, and that grey level i occurs n_i times in the image. Suppose also that the total number of pixels in the image is n (so that $n_0 + n_1 + n_2 + \cdots + n_{L-1} = n$. To transform the grey levels to obtain a better contrasted image, we change grey level i to

$$\left(\frac{n_0 + n_1 + \cdots + n_i}{n}\right)(L-1).$$

and this number is rounded to the nearest integer.

```
function out = histpwl(im,a,b)
%
% HISTPWL(IM,A,B) applies a piecewise linear transformation to the pixel values
% of image IM, where A and B are vectors containing the x and y coordinates
% of the ends of the line segments.  IM can be of type UINT8 or DOUBLE,
% and the values in A and B must be between 0 and 1.
%
% For example:
%
%   histpwl(x,[0,1],[1,0])
%
% simply inverts the pixel values.
%
classChanged = 0;
if ~isa(im, 'double'),
    classChanged = 1;
    im = im2double(im);
end

if length(a) ~= length (b)
  error('Vectors A and B must be of equal size');
end

N=length(a);
out=zeros(size(im));

for i=1:N-1
  pix=find(im>=a(i) & im<a(i+1));
  out(pix)=(im(pix)-a(i))*(b(i+1)-b(i))/(a(i+1)-a(i))+b(i);
end

pix=find(im==a(N));
out(pix)=b(N);

if classChanged==1
  out = uint8(255*out);
end
```

Figure 5.15: A Matlab function for applying a piecewise linear stretching function

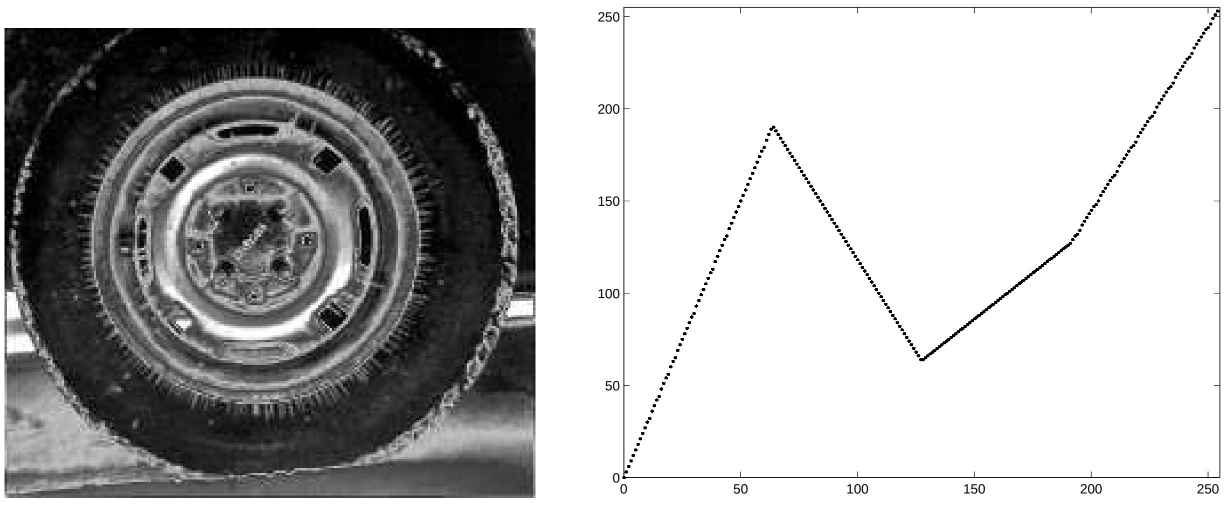


Figure 5.16: The tire image and after adjustment with the `gamma` value

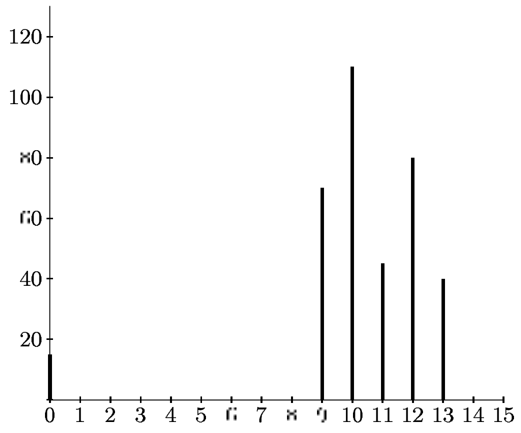


Figure 5.17: Another histogram indicating poor contrast

An example

Suppose a 4-bit greyscale image has the histogram shown in figure 5.17. associated with a table of the numbers n_i of grey values:

Grey level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	0	0	0	0	70	110	45	80	40	0	0

(with $n = 360$.) We would expect this image to be uniformly bright, with a few dark dots on it. To equalize this histogram, we form running totals of the n_i, and multiply each by $15/360 = 1/24$:

Grey level i	n_i	Σn_i	$(1/24)\Sigma n_i$	Rounded value
0	15	15	0.63	1
1	0	15	0.63	1
2	0	15	0.63	1
3	0	15	0.63	1
4	0	15	0.63	1
5	0	15	0.63	1
6	0	15	0.63	1
7	0	15	0.63	1
8	0	15	0.63	1
9	70	85	3.55	4
10	110	195	8.13	8
11	45	240	10	10
12	80	320	13.33	13
13	40	360	15	15
14	0	360	15	15
15	0	360	15	15

We now have the following transformation of grey values, obtained by reading off the first and last columns in the above table:

Original grey level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Final grey level j	1	1	1	1	1	1	1	1	1	4	8	10	13	15	15	15

and the histogram of the j values is shown in figure 5.18. This is far more spread out than the original histogram, and so the resulting image should exhibit greater contrast.

To apply histogram equalization in MATLAB, use the `histeq` function; for example:

```
>> p=imread('pout.tif');
>> ph=histeq(p);
>> imshow(ph),figure,imhist(ph),axis tight
```

applies histogram equalization to the `pout` image, and produces the resulting histogram. These results are shown in figure 5.19. Notice the far greater spread of the histogram. This corresponds to the greater increase of contrast in the image.

We give one more example, that of a very dark image. We can obtain a dark image by taking the index values only of an indexed colour image.

```
>> c=imread('cat.tif');
```

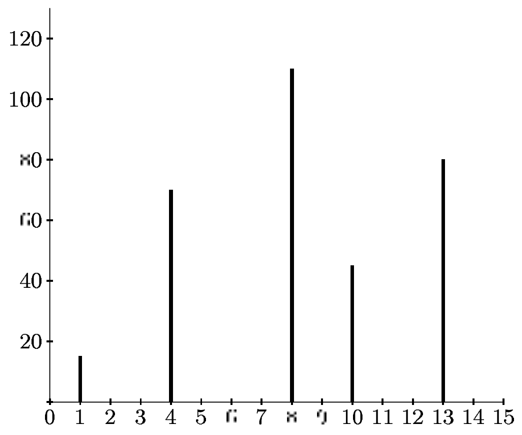


Figure 5.18: The histogram of figure 5.17 after equalization

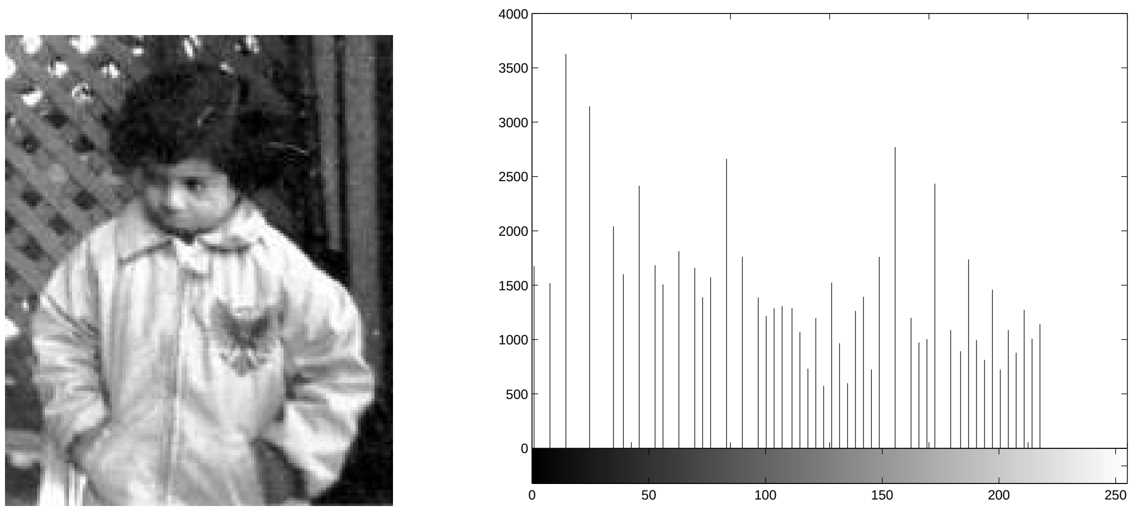


Figure 5.19: The histogram of figure 5.8 after equalization

We now have an index matrix c; in this case consisting of values between 0 and 63, and without the colour map. Since the index matrix contains only low values it will appear very dark when displayed. We can display this matrix and its histogram with the usual commands:

```
>> imshow(c),figure,imhist(c),axis tight
```

and the results are shown in figure 5.20.

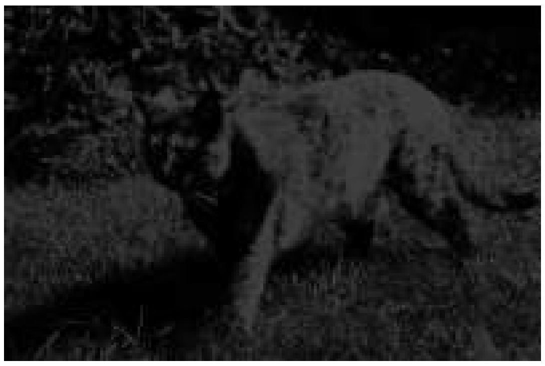
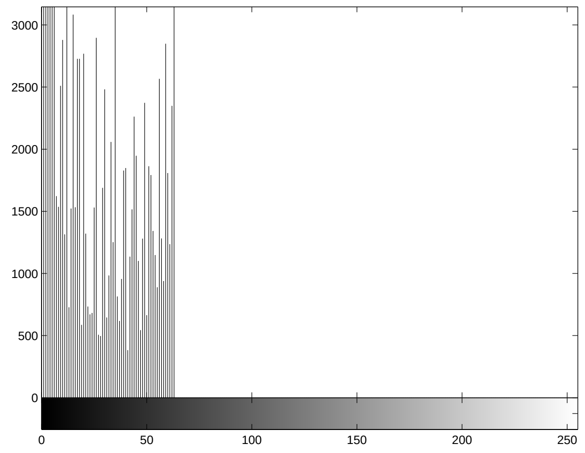



Figure 5.20: The index values from cat.tif and its histogram

As you see, the very dark image has a corresponding histogram heavily clustered at the lower end of the scale.

But we can apply histogram equalization to this image, and display the results:

```
>> ch=histeq(c);
>> imshow(ch),figure,imhist(ch),axis tight
```

and the results are shown in figure 5.21.

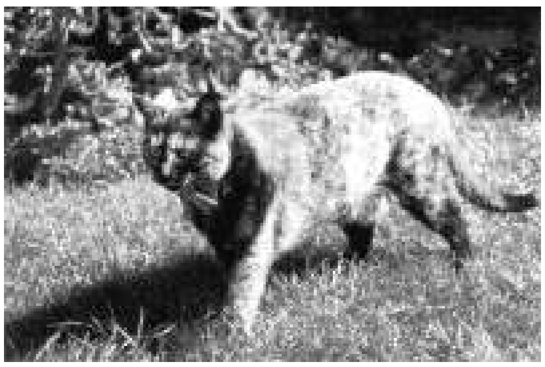
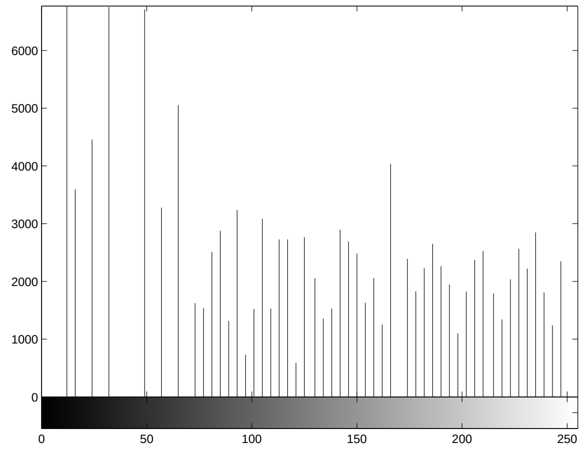



Figure 5.21: The image from 5.20 equalized and its histogram

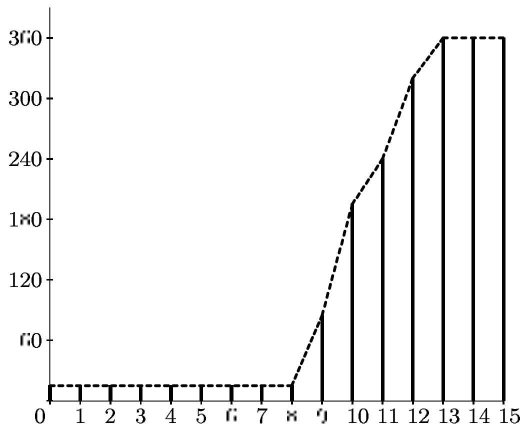


Figure 5.22: The cumulative histogram

Why it works

Consider the histogram in figure 5.17. To apply histogram stretching, we would need to stretch out the values between grey levels 9 and 13. Thus, we would need to apply a piecewise function similar to that shown in figure 5.9.

Let's consider the cumulative histogram, which is shown in figure 5.22. The dashed line is simply joining the top of the histogram bars. However, it can be interpreted as an appropriate histogram stretching function. To do this, we need to scale the y values so that they are between 0 and 15, rather than 0 and 360. But this is precisely the method described in section 5.3.2.

5.4 Thresholding

5.4.1 Single thresholding

A greyscale image is turned into a binary (black and white) image by first choosing a grey level T in the original image, and then turning every pixel black or white according to whether its grey value is greater than or less than T:

$$\text{A pixel becomes } \begin{cases} \text{white if its grey level is } > T, \\ \text{black if its grey level is } \leq T. \end{cases}$$

Thresholding is a vital part of image *segmentation*, where we wish to isolate objects from the background. It is also an important component of robot vision.

Thresholding can be done very simply in MATLAB. Suppose we have an 8 bit image, stored as the variable `X`. Then the command

```
X>T
```

will perform the thresholding. We can view the result with `imshow`. For example, the commands

```
>> r=imread('rice.tif');
>> imshow(r),figure,imshow(r>110)
```

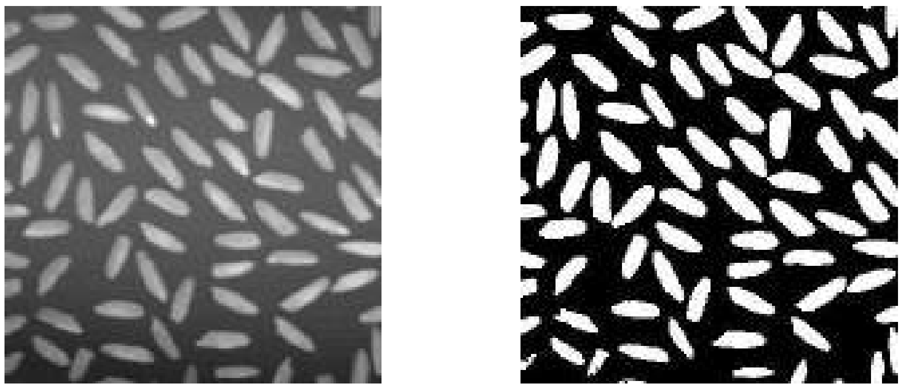


Figure 5.23: Thresholded image of rice grains

will produce the images shown in figure 5.23. The resulting image can then be further processed to find the number, or average size of the grains.

To see how this works, recall that in Matlab, an operation on a single number, when applied to a matrix, is interpreted as being applied simultaneously to all elements of the matrix; this is vectorization, which we have seen in chapter 2. The command `X>T` will thus return 1 (for true) for all those pixels for which the grey values are greater than `T`, and 0 (for false) for all those pixels for which the grey values are less than or equal to `T`. We thus end up with a matrix of 0's and 1's, which can be viewed as a binary image.

The rice image shown above has light grains on a dark background; an image with dark objects over a light background may be treated the same::

```
>> b=imread('bacteria.tif');
>> imshow(b),figure,imshow(b>100)
```

will produce the images shown in figure 5.24.

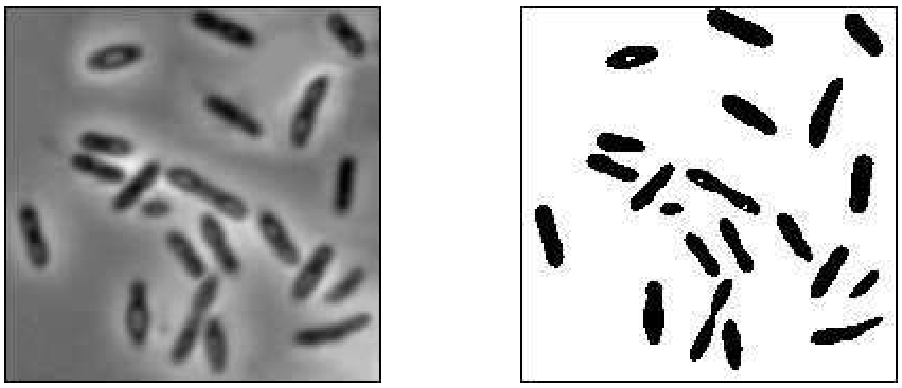


Figure 5.24: Thresholded image of bacteria

As well as the above method, Matlab has the `im2bw` function, which thresholds an image *of any data type*, using the general syntax

```
im2bw(image,level)
```

where `level` is a value between 0 and 1 (inclusive), indicating the fraction of grey values to be turned white. This command will work on greyscale, coloured and indexed images of data type `uint8`, uint16 or `double`. For example, the thresholded rice and bacteria images above could be obtained using

```
>> im2bw(r,0.43);
>> im2bw(b,0.39);
```

The `im2bw` function automatically scales the value `level` to a grey value appropriate to the image type, and then performs a thresholding by our first method.

As well as isolating objects from the background, thresholding provides a very simple way of showing hidden aspects of an image. For example, the image `paper.tif` appears all white, as nearly all the grey values are very high. However, thresholding at a high level produces an image of far greater interest. We can use the commands

```
>> p=imread('paper1.tif');
>> imshow(p),figure,imshow(p>241)
```

to provide the images shown in figure 5.25.

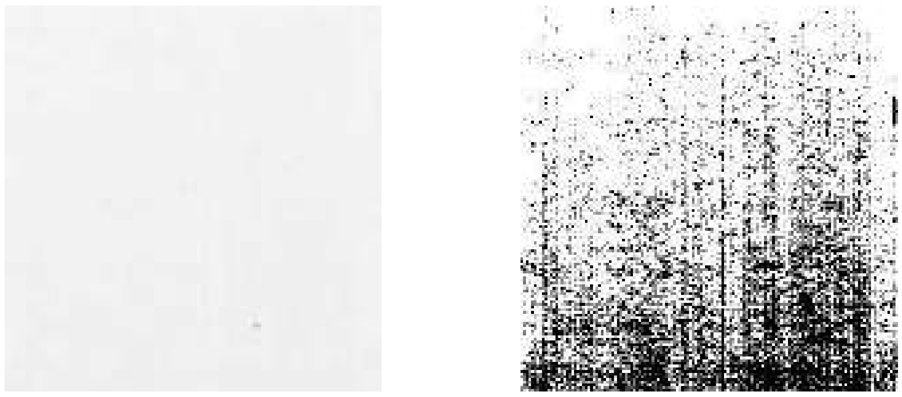


Figure 5.25: The paper image and result after thresholding

5.4.2 Double thresholding

Here we choose two values T_1 and T_2 and apply a thresholding operation as:

$$\text{A pixel becomes } \begin{cases} \text{white if its grey level is between } T_1 \text{ and } T_2, \\ \text{black if its grey level is otherwise.} \end{cases}$$

We can implement this by a simple variation on the above method:

```
X>T1 & X<T2
```

Since the ampersand acts as a logical "and", the result will only produce a one where both inequalities are satisfied. Consider the following sequence of commands, which start by producing an 8-bit grey version of the indexed image `spine.tif`:

```
>> [x,map]=imread('spine.tif');
>> s=uint8(256*ind2gray(x,map));
>> imshow(s),figure,imshow(s>115 & s<125)
```

The output is shown in figure 5.26. Note how double thresholding brings out subtle features of the

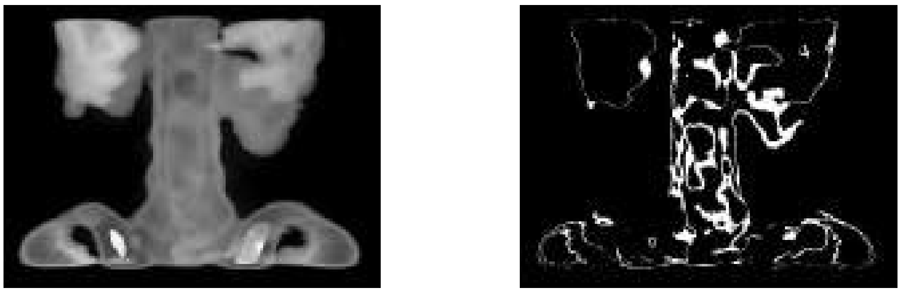


Figure 5.26: The image `spine.tif` an the result after double thresholding

spine which single thresholding would be unable to do. We can obtain similar results using `im2bw`:

```
imshow(im2bw(x,map,0.45)&~{}im2bw(x,map,0.5))}
```

but this is somewhat slower because of all the extra computation involved when dealing with an indexed image.

5.5 Applications of thresholding

We have seen that thresholding can be useful in the following situations:

1. When we want to remove unnecessary detail from an image, to concentrate on essentials. Examples of this were given in the rice and bacteria images: by removing all grey level information, the rice and bacteria were reduced to binary blobs. But this information may be all we need to investigate sizes, shapes, or numbers of blobs.

2. To bring out hidden detail. This was illustrated with paper and spine images. In both, the detail was obscured because of the similarity of the grey levels involved.

But thresholding can be vital for other purposes. We list a few more:

3. When we want to remove a varying background from text or a drawing. We can simulate a varying background by taking the image `text.tif` and placing it on a random background. This can be easily implemented with some simple MATLAB commands:

```
>> r=rand(256)*128+127;
>> t=imread('text.tif');
>> tr=uint8(r.*double(not(t));
>> imshow(tr)
```

The first command simply uses the `rand` function (which produces matrices of uniformly generated random numbers between 0.0 and 1.0), and scales the result so the random numbers are between 127 and 255. We then read in the text image, which shows white text on a dark background.

The third command does several things at once: `not(t)` reverses the text image so as to have black text on a white background; `double` changes the numeric type so that the matrix can be used with arithmetic operations; finally the result is multiplied into the random matrix, and the whole thing converted to `uint8` for display. The result is shown on the left in figure 5.27.

If we threshold this image and display the result with

```
>> imshow(tr>100)
```

the result is shown on the right in figure 5.27, and the background has been completely removed.

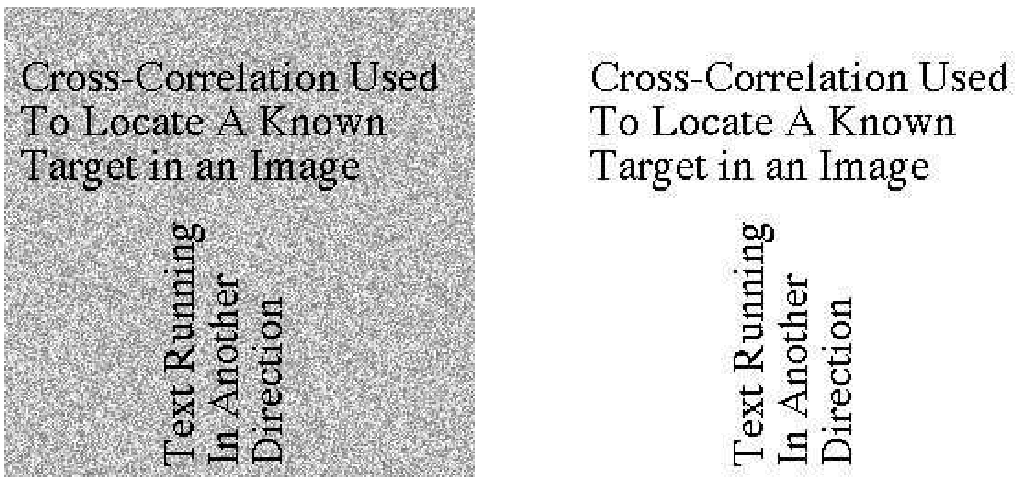


Figure 5.27: Text on a varying background, and thresholding

Exercises

Thresholding

1. Consider the following 3×3 image:

```
3   148   117   148   145   178   132   174
2   176   174   110   185   155   118   165
0   100   124   113   193   136   146   108
0   155   170   106   158   130   178   170
9   196   138   113   108   127   144   139
6   188   143   183   137   162   105   169
9   122   156   119   188   179   100   151
8   176   137   114   135   123   134   183
```

Threshold it at

(a) level 100

(b) level 150

2. What happens to the results of thresholding as the threshold level is increased?

3. Can you can create a small image which produces an "X" shape when thresholded at level 100, and a cross shape "+" when thresholded at 150?

   If not, why not?

4. Superimpose the image `text.tif` onto the image `cameraman.tif`. You can do this with:

```
>> t=imread('text.tif');}
>> c=imread('cameraman.tif');}
>> m=uint8(double(c)+255*double(t));}
```

   Can you threshold this new image `m` to isolate the text?

5. Try the same problem as above, but define `m` as:

```
>> m=uint8(double(c).*double(~t));
```

Histogram Equalization

6. Write informal code to calculate a histogram $h[f]$ of the grey values of an image $f[row][col]$.

7. The following table gives the number of pixels at each of the grey levels 0–7 in an image with those grey values only:

0	1	2	3	4	5	6	7
3244	3899	4559	2573	1428	530	101	50

   Draw the histogram corresponding to these grey levels, and then perform a histogram equalization and draw the resulting histogram.

8. The following tables give the number of pixels at each of the grey levels 0–15 in an image with those grey values only. In each case draw the histogram corresponding to these grey levels, and then perform a histogram equalization and draw the resulting histogram.

(a)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
20	40	60	75	80	75	65	55	50	45	40	35	30	25	20	30

(b)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	40	80	45	110	70	0	0	0	0	0	0	0	0	15

9. The following small image has grey values in the range 0 to 19. Compute the grey level histogram and the mapping that will equalize this histogram. Produce an 8×8 grid containing the grey values for the new histogram-equalized image.

```
12   6   5  13  14  14  16  15
11  10   8   5   8  11  14  14
 9   8   3   4   7  12  18  19
10   7   4   2  10  12  13  17
16   9  13  13  16  19  19  17
12  10  14  15  18  18  16  14
11   8  10  12  14  13  14  15
 8   6   3   7   9  11  12  12
```

10. Is the histogram equalization operation idempotent? That is, is performing histogram equalization *twice* the same as doing it just once?

11. Apply histogram equalization to the indices of the image `emu.tif`.

12. Create a dark image with

```
>> c=imread('cameraman.tif');
>> [x,map]=gray2ind(c);
```

The matrix `x`, when viewed, will appear as a very dark version of the cameraman image. Apply histogram equalization to it, and compare the result with the original image.

13. Using `p` and `ph` from section 5.5, enter the command

```
>> figure,plot(p,ph,'.'),grid on
```

What are you seeing here?

14. Experiment with some other greyscale images.

Chapter 6

Spatial Filtering

6.1 Introduction

We have seen in chapter 5 that an image can be modified by applying a particular function to each pixel value. Spatial filtering may be considered as an extension of this, where we apply a function to a neighbourhood of each pixel.

The idea is to move a "mask": a rectangle (usually with sides of odd length) or other shape over the given image. As we do this, we create a new image whose pixels have grey values calculated from the grey values under the mask, as shown in figure 6.1. The combination of mask and function

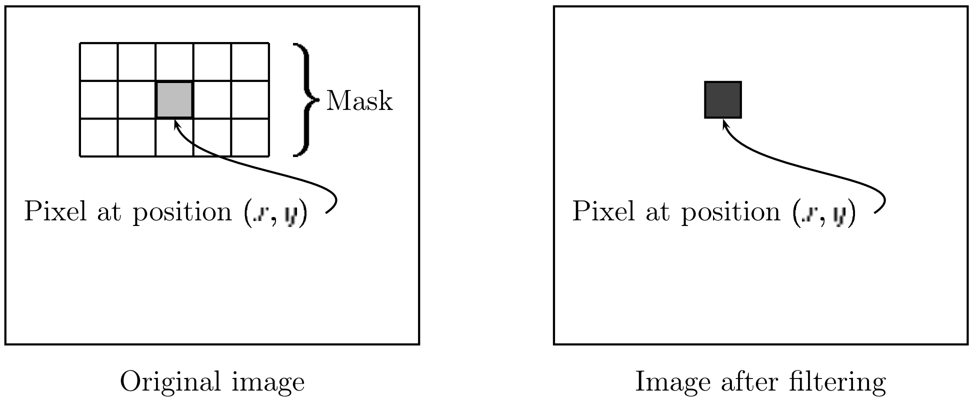


Figure 6.1: Using a spatial mask on an image

is called a *filter*. If the function by which the new grey value is calculated is a linear function of all the grey values in the mask, then the filter is called a *linear filter*.

We can implement a linear filter by multiplying all elements in the mask by corresponding elements in the neighbourhood, and adding up all these products. Suppose we have a 3×5 mask as illustrated in figure 6.1. Suppose that the mask values are given by:

$m(-1,-2)$	$m(-1,-1)$	$m(-1,0)$	$m(-1,1)$	$m(-1,2)$
$m(0,-2)$	$m(0,-1)$	$m(0,0)$	$m(0,1)$	$m(0,2)$
$m(1,-2)$	$m(1,-1)$	$m(1,0)$	$m(1,1)$	$m(1,2)$

and that corresponding pixel values are

$p(i-1,j-2)$	$p(i-1,j-1)$	$p(i-1,j)$	$p(i-1,j+1)$	$p(i-1,j+2)$
$p(i,j-2)$	$p(i,j-1)$	$p(i,j)$	$p(i,j+1)$	$p(i,j+2)$
$p(i+1,j-2)$	$p(i+1,j-1)$	$p(i+1,j)$	$p(i+1,j+1)$	$p(i+1,j+2)$

We now multiply and add:

$$\sum_{s=-1}^{1}\sum_{t=-2}^{2} m(s,t)p(i+s,j+t).$$

A diagram illustrating the process for performing spatial filtering is given in figure 6.2.

We see that spatial filtering requires three steps:

1. position the mask over the current pixel,
2. form all products of filter elements with the corresponding elements of the neighbourhood,
3. add up all the products.

This must be repeated for every pixel in the image.

An example: One important linear filter is to use a 3×3 mask and take the average of all nine values within the mask. This value becomes the grey value of the corresponding pixel in the new

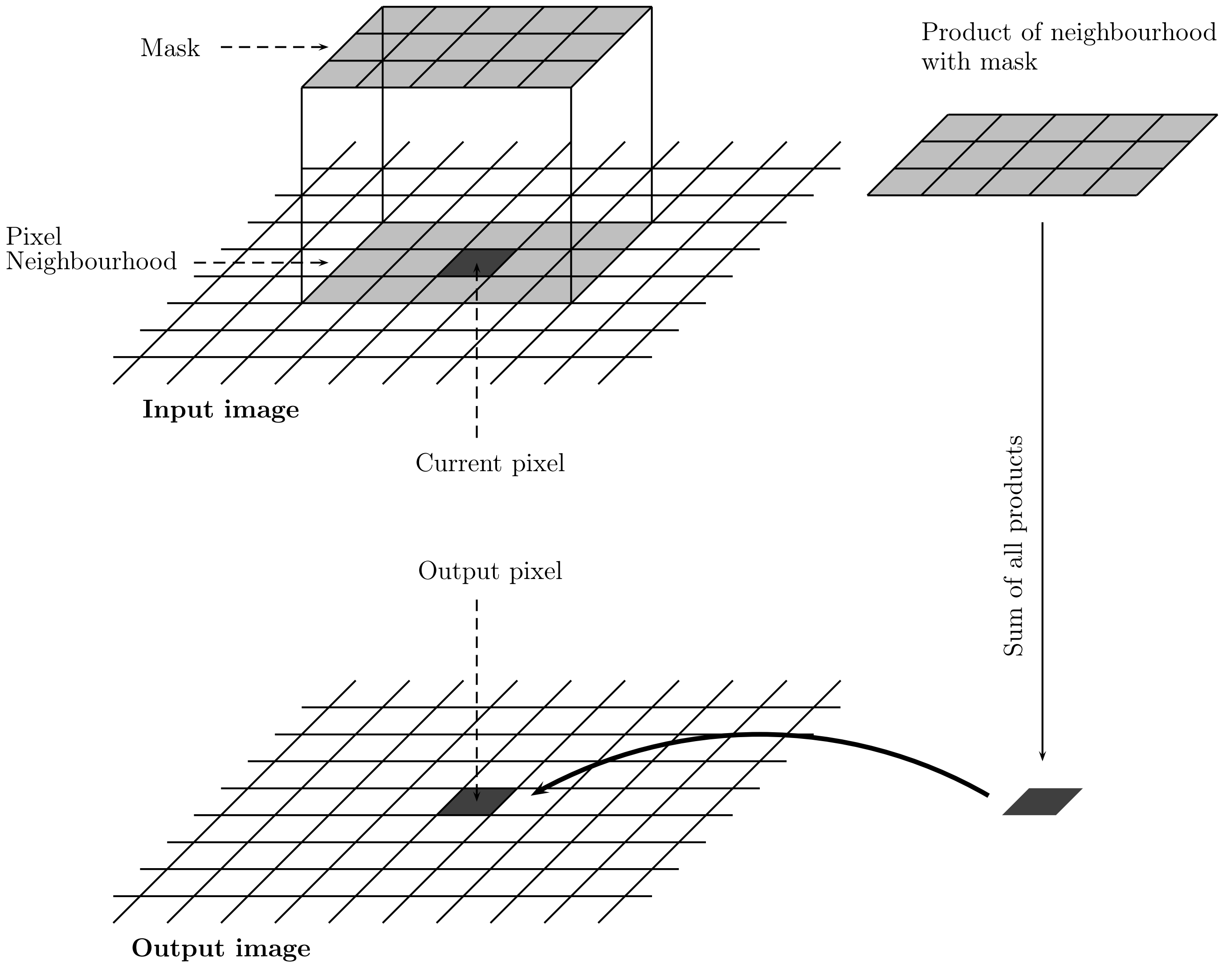
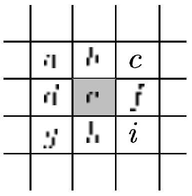


Figure 6.2: Performing spatial filtering

image. We may describe this operation as follows:



$$\longrightarrow \frac{1}{9}(a+b+c+d+e+f+g+h+i)$$

where e is grey value of the current pixel in the original image, and the average is the grey value of the corresponding pixel in the new image.

To apply this to an image, consider the 5×5 "image" obtained by:

```
>> x=uint8(10*magic(5))

x =

   170   240    10    80   150
   230    50    70   140   160
    40    60   130   200   220
   100   120   190   210    30
   110   180   250    20    90
```

We may regard this array as being made of nine overlapping 3×3 neighbourhoods. The output of our working will thus consist only of nine values. We shall see later how to obtain 25 values in the output.

Consider the top left 3×3 neighbourhood of our image **x**:

```
┌──────────────┐
│170  240   10 │  80  150
│230   50   70 │ 140  160
│ 40   60  130 │ 200  220
└──────────────┘
 100  120  190   210   30
 110  180  250    20   90
```

Now we take the average of all these values:

```
>> mean2(x(1:3,1:3))

ans =

  111.1111
```

which we can round to 111. Now we can move to the second neighbourhood:

$$\begin{array}{r|rrr|r}
\cline{2-4}
170 & 240 & 10 & 80 & 150 \\
230 & 50 & 70 & 140 & 160 \\
40 & 60 & 130 & 200 & 220 \\
\cline{2-4}
\multicolumn{1}{r}{100} & 120 & 190 & \multicolumn{1}{r}{210} & 30 \\
\multicolumn{1}{r}{110} & 180 & 250 & \multicolumn{1}{r}{20} & 90
\end{array}$$

and take its average:

```
>> mean2(x(1:3,2:4))

ans =

  108.8889
```

and we can round this either down to 108, or to the nearest integer 109. If we continue in this manner, we will build up the following output:

```
  111.1111  108.8889  128.8889
  110.0000  130.0000  150.0000
  131.1111  151.1111  148.8889
```

This array is the result of filtering **x** with the 3×3 averaging filter.

6.2 Notation

It is convenient to describe a linear filter simply in terms of the coefficients of all the grey values of pixels within the mask. This can be written as a matrix.

The averaging filter above, for example, could have its output written as

$$\frac{1}{9}a + \frac{1}{9}b + \frac{1}{9}c + \frac{1}{9}d + \frac{1}{9}e + \frac{1}{9}f + \frac{1}{9}g + \frac{1}{9}h + \frac{1}{9}i$$

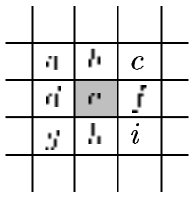
and so we can describe this filter by the matrix

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

An example: The filter

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

would operate on grey values as



$$\to a - 2b + c - 2d + 4e - 2f + g - 2h + i$$

Edges of the image

There is an obvious problem in applying a filter—what happens at the edge of the image, where the mask partly falls outside the image? In such a case, as illustrated in figure 6.3 there will be a lack of grey values to use in the filter function.

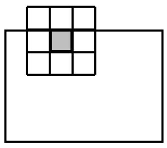


Figure 6.3: A mask at the edge of an image

There are a number of different approaches to dealing with this problem:

Ignore the edges. That is, we only apply the mask to those pixels in the image for with the mask will lie fully within the image. This means all pixels except for the edges, and results in an output image which is smaller than the original. If the mask is very large, we may lose a significant amount of information by this method.

We applied this method in our example above.

"Pad" with zeros. We assume that all necessary values outside the image are zero. This gives us all values to work with, and will return an output image of the same size as the original, but may have the effect of introducing unwanted artifacts (for example, edges) around the image.

6.3 Filtering in Matlab

The `filter2` function does the job of linear filtering for us; its use is

```
filter2(filter,image,shape)
```

and the result is a matrix of data type `double`. The parameter `shape` is optional, it describes the method for dealing with the edges:

- `filter2(filter,image,'same')` is the default; it produces a matrix of equal size to the original `image` matrix. It uses zero padding:

```
>> a=ones(3,3)/9

a =
     0.1111    0.1111    0.1111
     0.1111    0.1111    0.1111
     0.1111    0.1111    0.1111

>> filter2(a,x,'same')

ans =

   76.6667   85.5556   65.5556   67.7778   58.8889
   87.7778  111.1111  108.8889  128.8889  105.5556
   66.6667  110.0000  130.0000  150.0000  106.6667
   67.7778  131.1111  151.1111  148.8889   85.5556
   56.6667  105.5556  107.7778   87.7778   38.8889
```

- `filter2(filter,image,'valid')` applies the mask only to "inside" pixels. We can see that the result will always be smaller than the original:

  ```
  >> filter2(a,x,'valid')

  ans =

    111.1111  108.8889  128.8889
    110.0000  130.0000  150.0000
    131.1111  151.1111  148.8889
  ```

  We can obtain the result of `'same'` above by padding with zeros and using `'valid'`:

  ```
  >> x2=zeros(7,7);
  >> x2(2:6,2:6)=x

  x2 =

       0     0     0     0     0     0     0
       0   170   240    10    80   150     0
       0   230    50    70   140   160     0
       0    40    60   130   200   220     0
       0   100   120   190   210    30     0
       0   110   180   250    20    90     0
       0     0     0     0     0     0     0

  >> filter2(a,x2,'valid')
  ```

- `filter2(filter,image,'full')` returns a result *larger* than the original; it does this by padding with zero, and applying the filter at all places on and around the image where the mask intersects the image matrix.

```
>> filter2(a,x,'full')

ans =

   18.8889   45.5556   46.6667   36.6667   26.6667   25.5556   16.6667
   44.4444   76.6667   85.5556   65.5556   67.7778   58.8889   34.4444
   48.8889   87.7778  111.1111  108.8889  128.8889  105.5556   58.8889
   41.1111   66.6667  110.0000  130.0000  150.0000  106.6667   45.5556
   27.7778   67.7778  131.1111  151.1111  148.8889   85.5556   37.7778
   23.3333   56.6667  105.5556  107.7778   87.7778   38.8889   13.3333
   12.2222   32.2222   60.0000   50.0000   40.0000   12.2222   10.0000
```

The shape parameter, being optional, can be omitted; in which case the default value is `'same'`.

There is no single "best" approach; the method must be dictated by the problem at hand; by the filter being used, and by the result required.

We can create our filters by hand, or by using the `fspecial` function; this has many options which makes for easy creation of many different filters. We shall use the `average` option, which produces averaging filters of given size; thus

```
fspecial('average',[5,7])
```

will return an averaging filter of size 5×7; more simply

```
fspecial('average',11)
```

will return an averaging filter of size 11×11. If we leave out the final number or vector, the 3×3 averaging filter is returned.

For example, suppose we apply the 3×3 averaging filter to an image as follows:

```
>> c=imread('cameraman.tif');
>> f1=fspecial('average');
>> cf1=filter2(f1,c);
```

We now have a matrix of data type double. To display this, we can do any of the following:

- transform it to a matrix of type `uint8`, for use with `imshow`,
- divide its values by 255 to obtain a matrix with values in the 0.1–1.0 range, for use with `imshow`,
- use `mat2gray` to scale the result for display. We shall discuss the use of this function later.

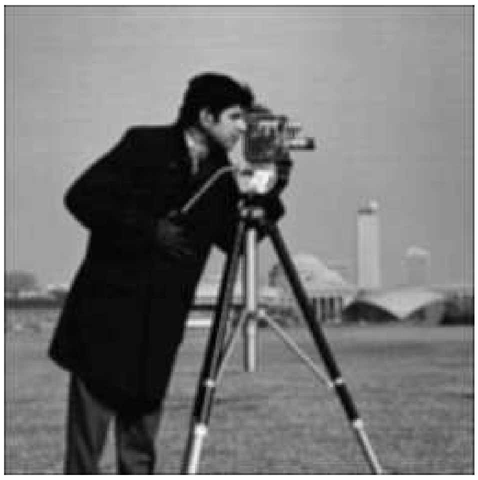
Using the second method:

```
>> figure,imshow(c),figure,imshow(cf1/255)
```

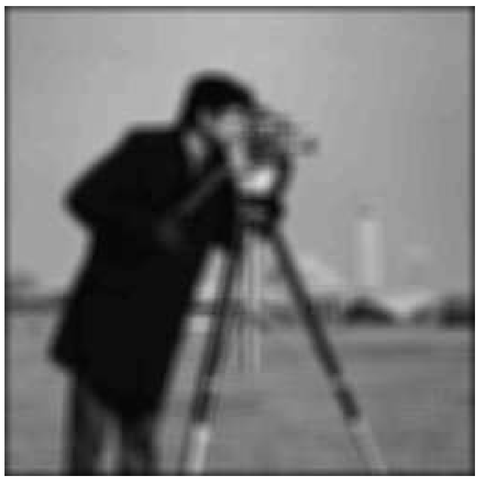
will produce the images shown in figures 6.4(a) and 6.4(b).

The averaging filter blurs the image; the edges in particular are less distinct than in the original. The image can be further blurred by using an averaging filter of larger size. This is shown in

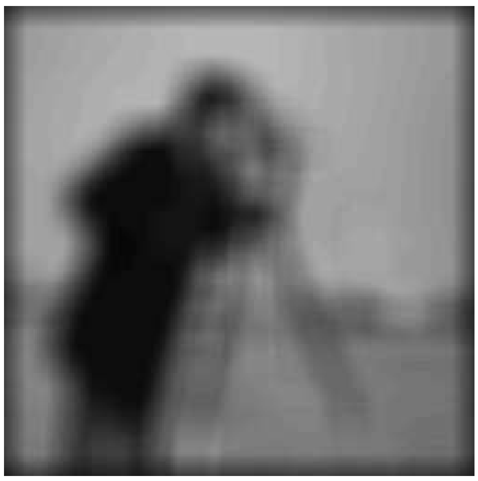
(a) Original image



(b) Average filtering



(c) Using a 9×9 filter



(d) Using a 25×25 filter

Figure 6.4: Average filtering

figure 6.4(c), where a 9×9 averaging filter has been used, and in figure 6.4(d), where a 25×25 averaging filter has been used.

Notice how the zero padding used at the edges has resulted in a dark border appearing around the image. This is especially noticeable when a large filter is being used. If this is an unwanted artifact of the filtering; if for example it changes the average brightness of the image, then it may be more appropriate to use the `'valid'` shape option.

The resulting image after these filters may appear to be much "worse" than the original. However, applying a blurring filter to reduce detail in an image may the perfect operation for autonomous machine recognition, or if we are only concentrating on the "gross" aspects of the image: numbers of objects; amount of dark and light areas. In such cases, too much detail may obscure the outcome.

6.4 Frequencies; low and high pass filters

It will be convenient to have some standard terminology by which we can discuss the effects a filter will have on an image, and to be able to choose the most appropriate filter for a given image processing task. One important aspect of an image which enables us to do this is the notion of *frequencies*. Fundamentally, the frequencies of an image are the amount by which grey values change with distance. *High frequency components* are characterized by large changes in grey values over small distances; example of high frequency components are edges and noise. *Low frequency components*, on the other hand, are parts of the image characterized by little change in the grey values. These may include backgrounds, skin textures. We then say that a filter is a

high pass filter if it "passes over" the high frequency components, and reduces or eliminates low frequency components,

low pass filter if it "passes over" the low frequency components, and reduces or eliminates high frequency components,

For example, the 3×3 averaging filter is low pass filter, as it tends to blur edges. The filter

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

is a high pass filter.

We note that the sum of the coefficients (that is, the sum of all e elements in the matrix), in the high pass filter is zero. This means that in a low frequency part of an image, where the grey values are similar, the result of using this filter is that the corresponding grey values in the new image will be close to zero. To see this, consider a 4×4 block of similar values pixels, and apply the above high pass filter to the central four:

150	152	148	149
147	152	151	150
152	148	149	151
151	149	150	148

$\longrightarrow$

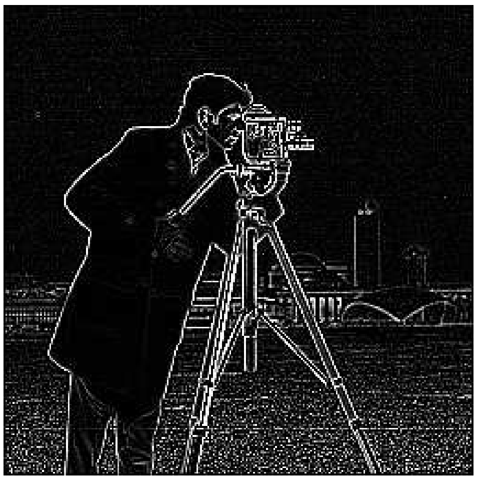
11	6
−13	−5

The resulting values are close to zero, which is the expected result of applying a high pass filter to a low frequency component. We shall see how to deal with negative values below.

High pass filters are of particular value in edge detection and edge enhancement (of which we shall see more in chapter 8). But we can provide a sneak preview, using the cameraman image.

```
>> f=fspecial('laplacian')

f =

    0.1667    0.6667    0.1667
    0.6667   -3.3333    0.6667
    0.1667    0.6667    0.1667

>> cf=filter2(f,c);
>> imshow(cf/100)
>> f1=fspecial('log')

f1 =

    0.0448    0.0468    0.0564    0.0468    0.0448
    0.0468    0.3167    0.7146    0.3167    0.0468
    0.0564    0.7146   -4.9048    0.7146    0.0564
    0.0468    0.3167    0.7146    0.3167    0.0468
    0.0448    0.0468    0.0564    0.0468    0.0448

>> cf1=filter2(f1,c);
>> figure,imshow(cf1/100)
```

The images are shown in figure 6.5. Image (a) is the result of the Laplacian filter; image (b) shows the result of the Laplacian of Gaussian ("log") filter.


(a) Laplacian filter

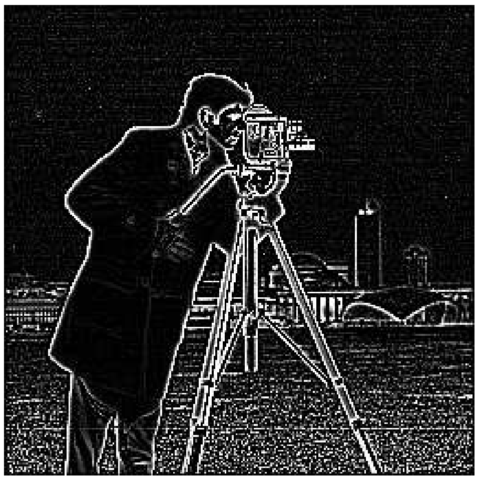

(b) Laplacian of Gaussian ("log") filtering

Figure 6.5: High pass filtering

In each case, the sum of all the filter elements is zero.

Values outside the range 0–255

We have seen that for image display, we would like the grey values of the pixels to lie between 0 and 255. However, the result of applying a linear filter may be values which lie outside this range. We may consider ways of dealing with values outside of this "displayable" range.

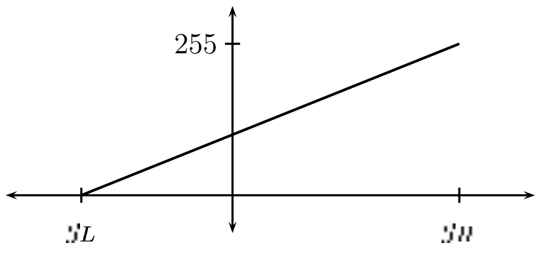
Make negative values positive. This will certainly deal with negative values, but not with values greater than 255. Hence, this can only be used in specific circumstances; for example, when there are only a few negative values, and when these values are themselves close to zero.

Clip values. We apply the following thresholding type operation to the grey values x produced by the filter to obtain a displayable value y:

$$y = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 255 \\ 255 & \text{if } x > 255 \end{cases}$$

This will produce an image with all pixel values in the required range, but is not suitable if there are many grey values outside the 0–255 range; in particular, if the grey values are equally spread over a larger range. In such a case this operation will tend to destroy the results of the filter.

Scaling transformation. Suppose the lowest grey value produced by the filter if g_L and the highest value is g_H. We can transform all values in the range g_L–g_H to the range 0–255 by the linear transformation illustrated below:



Since the gradient of the line is $255/(g_H - g_L)$ we can write the equation of the line as

$$y = 255\frac{x - g_L}{g_H - g_L}$$

and applying this transformation to all grey levels x produced by the filter will result (after any necessary rounding) in an image which can be displayed.

As an example, let's apply the high pass filter given in section 6.4 to the cameraman image:

```
>> f2=[1 -2 1;-2 4 -2;1 -2 1];
>> cf2=filter2(f2,c);
```

Now the maximum and minimum values of the matrix `cf2` are 593 and -541 respectively. The `mat2gray` function automatically scales the matrix elements to displayable values; for any matrix M, it applies a linear transformation to to its elements, with the lowest value mapping to 0.0, and the highest value mapping to 1.0. This means the output of `mat2gray` is always of type `double`. The function also requires that the input type is `double`.

```
>> figure,imshow(mat2gray(cf2));
```

To do this by hand, so to speak, applying the linear transformation above, we can use:

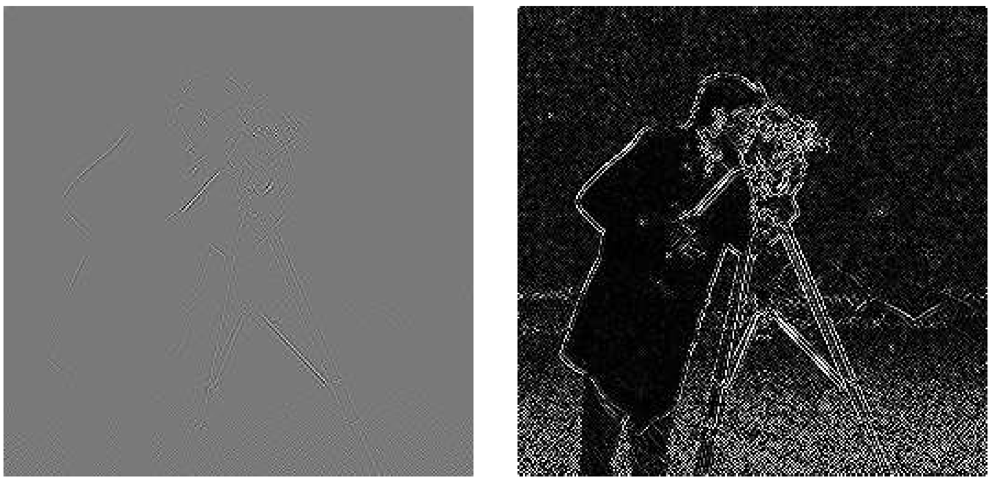
```
>> maxcf2=max(cf2(:));
>> mincf2=min(cf2(:));
>> cf2g=(cf2-mincf2)/(maxcf2-mncf2);
```

The result will be a matrix of type `double`, with entries in the range 0.0–1.0. This can be be viewed with `imshow`. We can make it a `uint8` image by multiplying by 255 first. The result can be seen in figure 6.6.

We can generally obtain a better result by dividing the result of the filtering by a constant before displaying it:

```
>> figure,imshow(cf2/60)
```

and this is also shown in figure 6.6.



Using `mat2gray` Dividing by a constant

Figure 6.6: Using a high pass filter and displaying the result

High pass filters are often used for edge detection. These can be seen quite clearly in the right hand image of figure 6.6.

6.5 Gaussian filters

We have seen some examples of linear filters so far: the averaging filter, and a high pass filter. The `fspecial` function can produce many different filters for use with the `filter2` function; we shall look at a particularly important filter here.

Gaussian filters are a class of low-pass filters, all based on the Gaussian probability distribution function

$$f(x) = e^{-\frac{x^2}{2\sigma^2}}$$

where σ is the standard deviation: a large value of σ produces to a flatter curve, and a small value leads to a "pointier" curve. Figure 6.7 shows examples of such one dimensional gaussians.

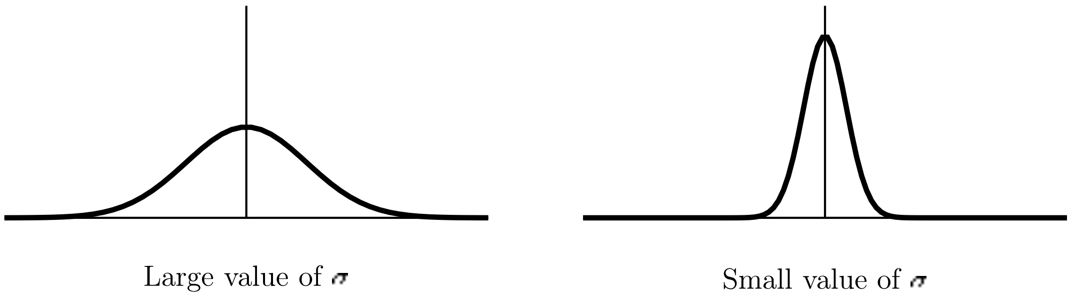


Figure 6.7: One dimensional gaussians

A two dimensional Gaussian function is given by

$$f(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

The command `fspecial('gaussian')` produces a discrete version of this function. We can draw pictures of this with the `surf` function, and to ensure a nice smooth result, we shall create a large filter (size 50×50) with different standard deviations.

```
>> a=50;s=3;
>> g=fspecial('gaussian',[a a],s);
>> surf(1:a,1:a,g)
>> s=9;
>> g2=fspecial('gaussian',[a a],s);
>> figure,surf(1:a,1:a,g2)
```

The surfaces are shown in figure 6.8.

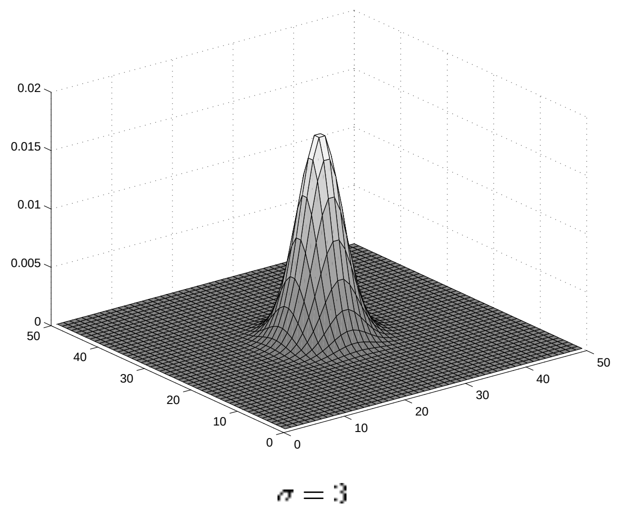
Gaussian filters have a blurring effect which looks very similar to that produced by neighbourhood averaging. Let's experiment with the cameraman image, and some different gaussian filters.

```
>> g1=fspecial('gaussian',[5,5]);
>> g1=fspecial('gaussian',[5,5],2);
>> g1=fspecial('gaussian',[11,11],1);
>> g1=fspecial('gaussian',[11,11],5);
```

The final parameter is the standard deviation; which if not given defaults to 0.5. The second parameter (which is also optional), gives the size of the filter; the default is 3×3. If the filter is to be square, as in all the above examples, we can just give a single number in each case.

Now we can apply the filter to the cameraman image matrix `c` and vier the result.

```
>> imshow(filter2(g1,c)/256)
>> figure,imshow(filter2(g2,c)/256)
>> figure,imshow(filter2(g3,c)/256)
>> figure,imshow(filter2(g4,c)/256)
```



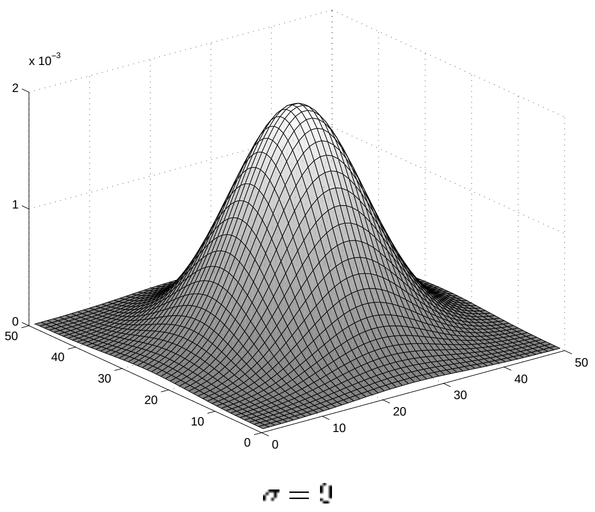


Figure 6.8: Two dimensional gaussians

.

and the results are shown in figure 6.9. We see that to obtain a spread out blurring effect, we need a large standard deviation. In fact, if we let the standard deviation grow large without bound, we obtain the averaging filters as limiting values. For example:

```
>> fspecial('gaussian',3,100)

ans =

    0.1111    0.1111    0.1111
    0.1111    0.1111    0.1111
    0.1111    0.1111    0.1111
```

and we have the 3×3 averaging filter.

Although the results of Gaussian blurring and averaging look similar, the Gaussian filter has some elegant mathematical properties which make it particularly suitable for blurring.

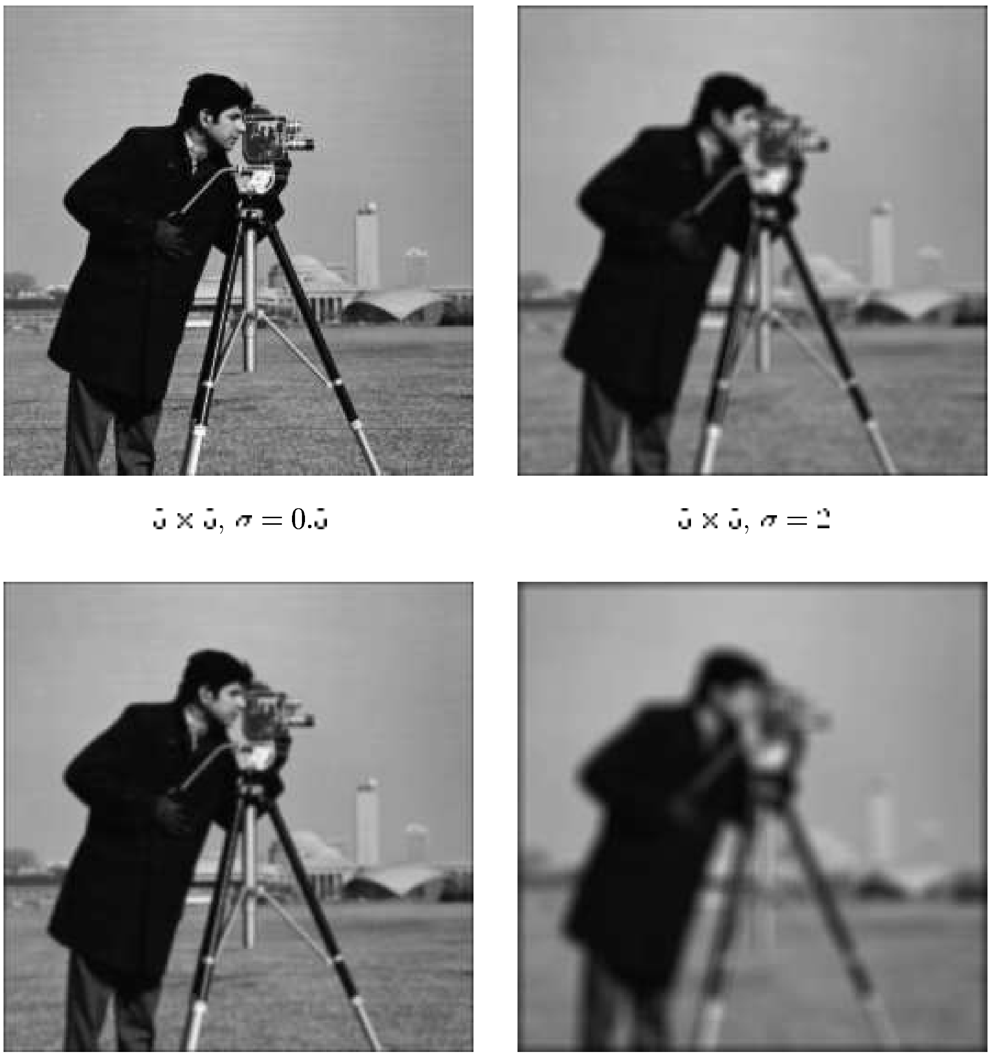
Other filters will be discussed in future chapters; also check the documentation for `fspecial` for other filters.

6.6 Non-linear filters

Linear filters, as we have seen in the previous sections, are easy to describe, and can be applied very quickly and efficiently by Matlab. However, non-linear filters can also be used, if less efficiently. The function to use is `nlfilter`, which applies a filter to an image according to a pre-defined function. If the function is not already defined, we have to create an m-file which defines it.

Here are some examples; first to define a filter which takes the maximum value in a 3×3 neighbourhood:

```
    cmax=nlfilter(c,[3,3],'max(x(:))');
```



5×5, $\sigma = 0.5$ 5×5, $\sigma = 2$

11×11, $\sigma = 1$ 11×11, $\sigma = 5$

Figure 6.9: Effects of different gaussian filters on an image

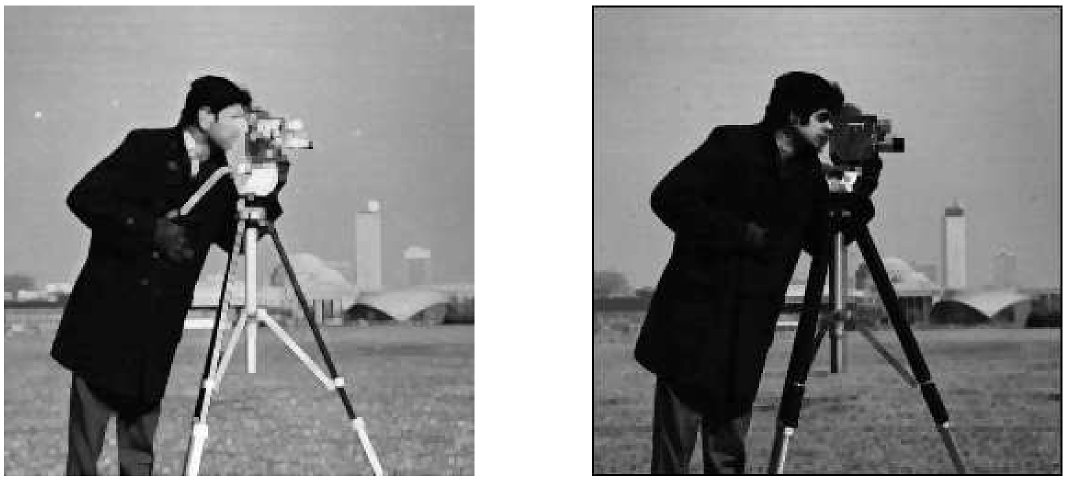
.

The `nlfilter` function requires three arguments: the image matrix, the size of the filter, and the function to be applied. The function must be a matrix function which returns a scalar value. The result of this operation is shown in figure 6.10(a).

A corresponding filter which takes the *minimum* value can be just as easily applied:

```
cmin=nlfilter(c,[3,3],'min(x(:))');
```

and the result is shown in figure 6.10(b).



(a) Using a maximum filter (b) Using a minimum filter

Figure 6.10: Using non-linear filters

Both these filters can in fact be more efficiently implemented by using the MATLAB function `ordfilt2`—see its help page!

Note that in each case the image has lost some sharpness, and has been brightened by the maximum filter, and darkened by the minimum filter. The `nlfilter` function is very slow; in general there is little call for non-linear filters except for a few which are defined by their own commands. We shall investigate these in later chapters.

Exercises

1. The array below represents a small greyscale image. Compute the images that result when the image is convolved with each of the masks (a) to (h) shown. At the edge of the image use a restricted mask. (In other words, pad the image with zeroes.)

```
20  20  20  10  10  10  10  10  10
20  20  20  20  20  20  20  20  10
20  20  20  10  10  10  10  20  10
20  20  10  10  10  10  10  20  10
20  10  10  10  10  10  10  20  10
10  10  10  10  20  10  10  20  10
10  10  10  10  10  10  10  10  10
```

```
20  10  20  20  10  10  10  20  20
20  10  10  20  10  10  20  10  20
```

```
     -1  -1   0          0  -1  -1         -1  -1  -1         -1   2  -1
(a)  -1   0   1     (b)  1   0  -1    (c)   2   2   2    (d)  -1   2  -1
      0   1   1          1   1   0         -1  -1  -1         -1   2  -1

     -1  -1  -1          1   1   1         -1   0   1          0  -1   0
(e)  -1   8  -1     (f)  1   1   1    (g)  -1   0   1    (h)  -1   4  -1
     -1  -1  -1          1   1   1         -1   0   1          0  -1   0
```

2. Check your answers to the previous question with MATLAB.

3. Describe what each of the masks in the previous question might be used for. If you can't do this, wait until question 5 below.

4. Devise a 3×3 mask for an "identity filter"; which causes no change in the image.

5. Obtain a greyscale image of a monkey (a *mandrill*) with the following commands:

```
>> load('mandrill.mat');
>> m=im2uint8(ind2gray(X,map));
```

   Apply all the filters listed in question 1 to this image. Can you now see what each filter does?

6. Apply larger and larger averaging filters to this image. What is the smallest sized filter for which the whiskers cannot be seen?

7. Repeat the previous question with Gaussian filters with the following parameters:

Size	Standard deviation		
[3,3]	0.5	1	2
[7,7]	1	3	6
[11,11]	1	4	8
[21,21]	1	5	10

   At what values do the whiskers disappear?

8. Can you see any observable differnce in the results of average filtering and of using a Gaussian filter?

9. Read through the help page of the `fspecial` function, and apply some of the other filters to the cameraman image, and to the mandrill image.

10. Apply different laplacian filters to the mandrill and cameraman images. Which produces the best edge image?

11. MATLAB also has an `imfilter` function, which if `x` is an image matrix (of any type), and `f` is a filter, has the syntax

```
imfilter(x,f);
```

   It differs from `filter2` in the different parameters it takes (read its help file), and in that the output is always of the same class as the original image.

   (a) Use `imfilter` on the mandrill image with the filters listed in question 1.
   (b) Apply different sized averaging filters to the mandrill image using `imfilter`.
   (c) Apply different laplacian filters to the mandrill image using `imfilter`. Compare the results with those obtained with `filter2`. Which do you think gives the best results?

12. Display the difference between the `cmax` and `cmin` images obtained in section 6.6. You can do this with

    ```
    >> imshow(imsubtract(cmax,cmin))
    ```

    What are you seeing here? Can you account for the output of these commands?

Chapter 7

Noise

7.1 Introduction

We may define *noise* to be any degradation in the image signal, caused by external disturbance. If an image is being sent electronically from one place to another, via satellite or wireless transmission, or through networked cable, we may expect errors to occur in the image signal. These errors will appear on the image output in different ways depending on the type of disturbance in the signal. Usually we know what type of errors to expect, and hence the type of noise on the image; hence we can choose the most appropriate method for reducing the effects. Cleaning an image corrupted by noise is thus an important area of *image restoration*.

In this chapter we will investigate some of the standard noise forms, and the different methods of eliminating or reducing their effects on the image.

7.2 Types of noise

We will look at four different noise types, and how they appear on an image.

Salt and pepper noise

Also called *impulse noise*, *shot noise*, or *binary noise*. This degradation can be caused by sharp, sudden disturbances in the image signal; its appearance is randomly scattered white or black (or both) pixels over the image.

To demonstrate its appearance, we will first generate a grey-scale image, starting with a colour image:

```
>> tw=imread('twins.tif');
>> t=rgb2gray(tw);
```

To add noise, we use the MATLAB function `imnoise`, which takes a number of different parameters. To add salt and pepper noise:

```
>> t_sp=imnoise(t,'salt & pepper');
```

The amount of noise added defaults to 10%; to add more or less noise we include an optional parameter, being a value between 0 and 1 indicating the fraction of pixels to be corrupted. Thus, for example

```
>> imnoise(t,'salt & pepper',0.2);
```

would produce an image with 20% of its pixels corrupted by salt and pepper noise.

The twins image is shown in figure 7.1(a) and the image with noise is shown in figure 7.1(b).

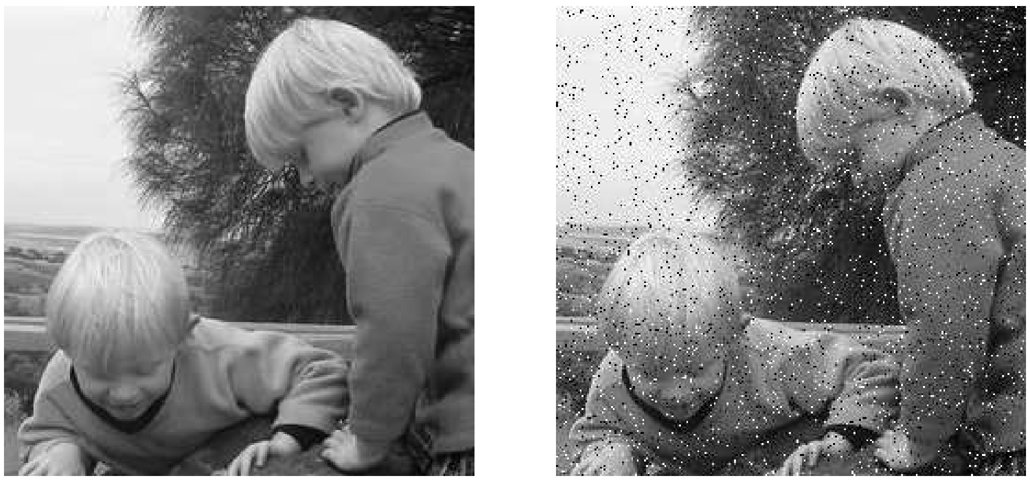

(a) Original image (b) With added salt & pepper noise

Figure 7.1: Noise on an image

Gaussian noise

Gaussian noise is an idealized form of *white noise*, which is caused by random fluctuations in the signal. We can observe white noise by watching a television which is slightly mistuned to a particular channel. Gaussian noise is white noise which is normally distributed. If the image is represented as I, and the Gaussian noise by N, then we can model a noisy image by simply adding the two:

$$I + N.$$

Here we may assume that I is a matrix whose elements are the pixel values of our image, and N is a matrix whose elements are normally distributed. It can be shown that this is an appropriate model for noise. The effect can again be demonstrated by the `imnoise` function:

```
>> t_ga=inoise(t,'gaussian');
```

As with salt and pepper noise, the "`gaussian`" parameter also can take optional values, giving the mean and variance of the noise. The default values are 0 and 0.01, and the result is shown in figure 7.2(a).

Speckle noise

Whereas Gaussian noise can be modelled by random values *added* to an image; *speckle noise* (or more simply just *speckle*) can be modelled by random values *multiplied* by pixel values, hence it is also called *multiplicative noise.* Speckle noise is a major problem in some radar applications. As above, `imnoise` can do speckle:

```
>> t_spk=imnoise(t,'speckle');
```

and the result is shown in figure 7.2(b). In MATLAB, speckle noise is implemented as

$$I(1+\mathcal{N})$$

where I is the image matrix, and $\mathcal{N}$ consists of normally distributed values with mean 0. An optional parameter gives the variance of $\mathcal{N}$; its default value is 0.04.

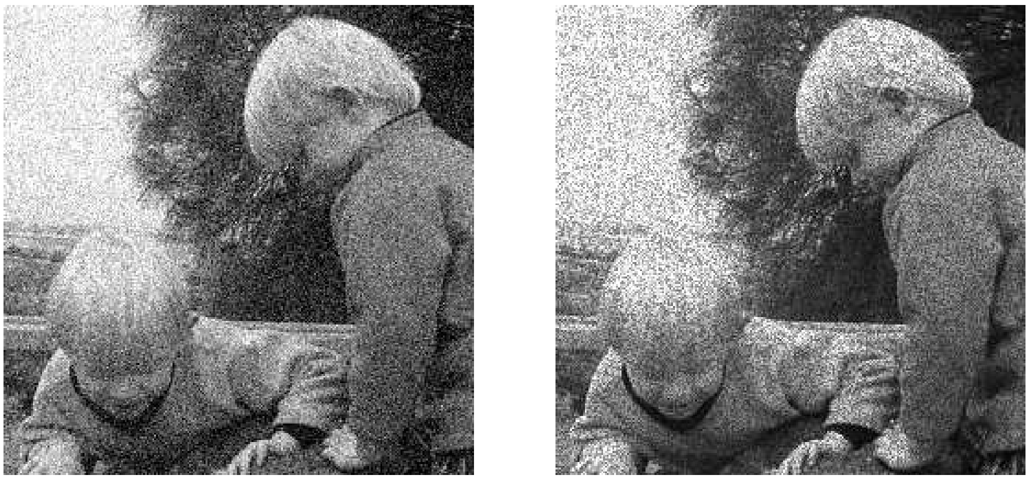

(a) Gaussian noise (b) Speckle noise

Figure 7.2: The twins image corrupted by Gaussian and speckle noise

Although Gaussian noise and speckle noise appear superficially similar, they are formed by two totally different methods, and, as we shall see, require different approaches for their removal.

Periodic noise

If the image signal is subject to a periodic, rather than a random disturbance, we might obtain an image corrupted by *periodic noise*. The effect is of bars over the image. The function `imnoise` does not have a periodic option, but it is quite easy to create our own, by adding a periodic matrix (using a trigonometric function), to our image:

```
>> s=size(t);
>> [x,y]=meshgrid(1:s(1),1:s(2));
>> p=sin(x/3+y/5)+1;
>> t_pn=(im2double(t)+p/2)/2;
```

and the resulting image is shown in figure 7.3.

Salt and pepper noise, Gaussian noise and speckle noise can all be cleaned by using spatial filtering techniques. Periodic noise, however, requires image transforms for best results, and so we will leave the discussion on cleaning up periodic noise until a later chapter.

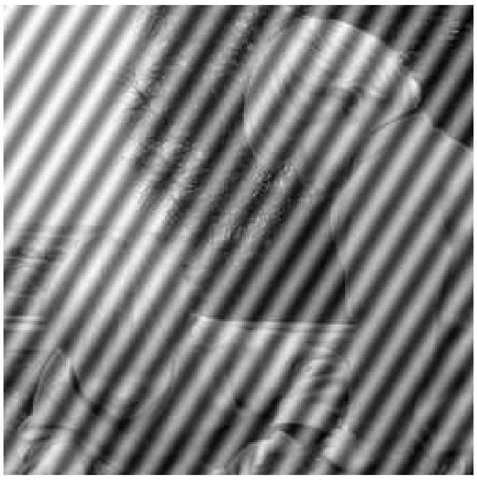


Figure 7.3: The twins image corrupted by periodic noise

7.3 Cleaning salt and pepper noise

Low pass filtering

Given that pixels corrupted by salt and pepper noise are high frequency components of an image, we should expect a low-pass filter should reduce them. So we might try filtering with an average filter:

```
>> a3=fspecial('average');
>> t_sp_a3=filter2(a3,t_sp);
```

and the result is shown in figure 7.4(a). Notice, however, that the noise is not so much removed as "smeared" over the image; the result is not noticeably "better" than the noisy image. The effect is even more pronounced if we use a larger averaging filter:

```
>> a7=fspecial('average',[7,7]);
>> t_sp_a7=filter2(a7,t_sp);
```

and the result is shown in figure 7.4(b).

Median filtering

Median filtering seems almost tailor-made for removal of salt and pepper noise. Recall that the *median* of a set is the middle value when they are sorted. If there are an even number of values, the median is the mean of the middle two. A median filter is an example of a non-linear spatial filter; using a 3×3 mask, the output value is the median of the values in the mask. For example:

$$\begin{array}{|c|c|c|}\hline 50 & 65 & 52 \\ \hline 63 & 255 & 58 \\ \hline 61 & 60 & 57 \\ \hline \end{array} \to \quad 50 \quad 52 \quad 57 \quad 58 \quad \boxed{60} \quad 61 \quad 63 \quad 65 \quad 255 \quad \to 60$$

We see that very large or very small values—noisy values—will end up at the top or bottom of the sorted list. Thus the median will in general replace a noisy value with one closer to its surroundings.

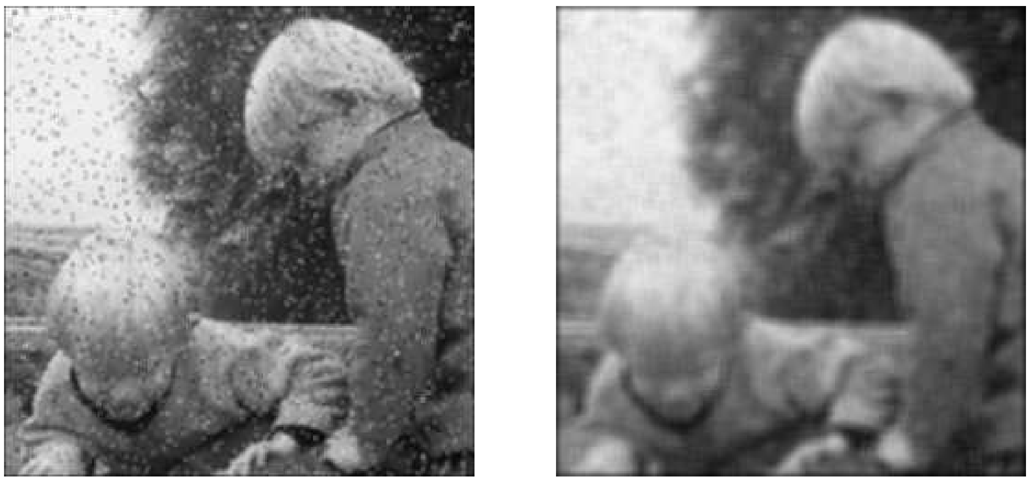

(a) 3×3 averaging (b) 7×7 averaging

Figure 7.4: Attempting to clean salt & pepper noise with average filtering

In MATLAB, median filtering is implemented by the `medfilt2` function:

```
>> t_sp_m3=medfilt2(t_sp);
```

and the result is shown in figure 7.5. The result is a vast improvement on using averaging filters. As

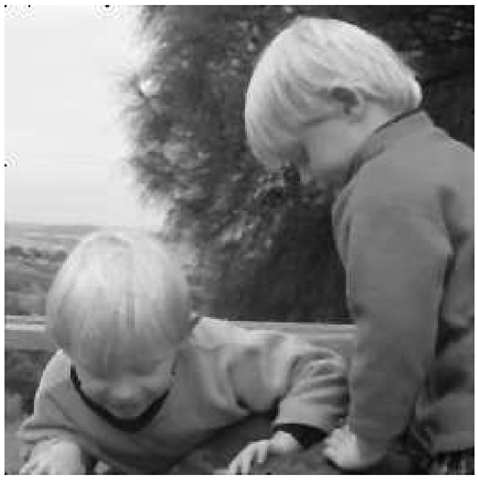


Figure 7.5: Cleaning salt and pepper noise with a median filter

with most functions, `medfilt2` takes an optional parameter; in this case a 2 element vector giving the size of the mask to be used.

If we corrupt more pixels with noise:

```
>> t_sp2=imnoise(t,'salt & pepper',0.2);
```

then `medfilt2` still does a remarkably good job, as shown in figure 7.6. To remove noise completely,

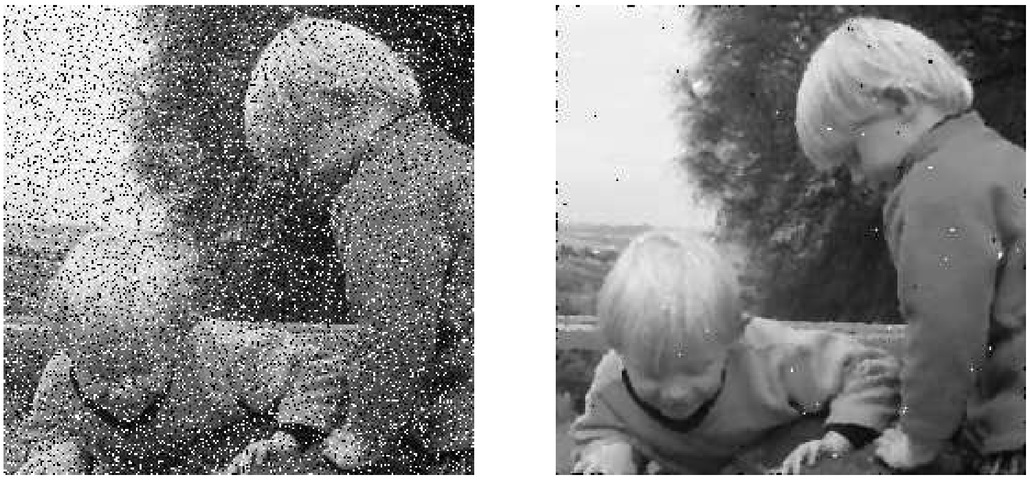

(a) 20% salt & pepper noise (b) After median fitering

Figure 7.6: Using a 3×3 median filter on more noise

we can either try a second application of the 3×3 median filter, the result of which is shown in figure 7.7(a) or try a 5×5 median filter on the original noisy image:

```
>> t_sp2_m5=medfilt2(t_sp2,[5,5]);
```

the result of which is shown in figure 7.7(b).

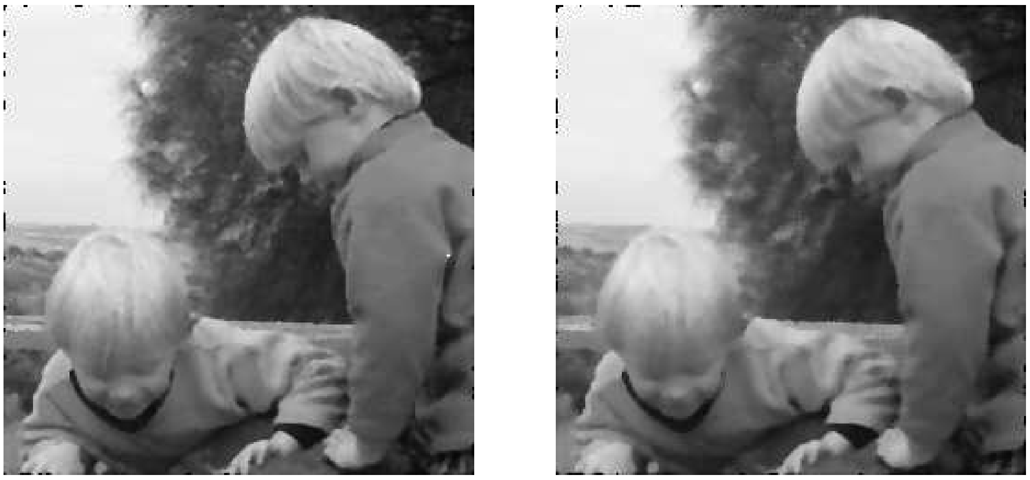
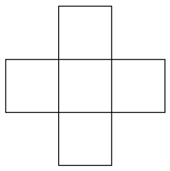

(a) Using `medfilt2` twice (b) using a 5×5 median filter

Figure 7.7: Cleaning 20% salt & pepper noise with median filtering

Rank-order filtering

Median filtering is a special case of a more general process called *rank-order filtering*. Rather than take the median of a set, we order the set and take the n-th value, for some predetermined value of n.

Thus median filtering using a 3×3 mask is equivalent to rank-order filtering with $n = 5$. Similarly, median filtering using a 5×5 mask is equivalent to rank-order filtering with $n = 13$. MATLAB implements rank-order filtering with the `ordfilt2` function; in fact the procedure for `medfilt2` is really just a wrapper for a procedure which calls `ordfilt2`. There is only one reason for using rank-order filtering instead of median filtering, and that is that it allows us to choose the median of non-rectangular masks. For example, if we decided to use as a mask a 3×3 cross shape:



then the median would be the *third* of these values when sorted. The command to do this is

```
>> ordfilt2(t_sp,3,[0 1 0;1 1 1;0 1 0]);
```

In general, the second argument of `ordfilt2` gives the value of the ordered set to take, and the third element gives the *domain*; the non-zero values of which specify the mask. If we wish to use a cross with size and width 5 (so containing nine elements), we can use:

```
>> ordfilt2(t_sp,5,[0 0 1 0 0;0 0 1 0 0;1 1 1 1 1;0 0 1 0 0;0 0 1 0 0])
```

An outlier method

Applying the median filter can in general be a slow operation: each pixel requires the sorting of at least nine values[1]. To overcome this difficulty, Pratt [16] has proposed the use of cleaning salt and pepper noise by treating noisy pixels as *outliers*; that is, pixels whose grey values are significantly different from those of their neighbours. This leads to the following approach for noise cleaning:

1. Choose a threshold value D.
2. For a given pixel, compare its value p with the mean m of the values of its eight neighbours.
3. If $|p - m| > D$, then classify the pixel as noisy, otherwise not.
4. If the pixel is noisy, replace its value with m; otherwise leave its value unchanged.

There is no MATLAB function for doing this, but it is very easy to write one. First, we can calculate the average of a pixel's eight neighbours by convolving with the linear filter

$$\frac{1}{8}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.125 & 0.125 & 0.125 \\ 0.125 & 0 & 0.125 \\ 0.125 & 0.125 & 0.125 \end{bmatrix}$$

We can then produce a matrix r consisting of 1's at only those places where the difference of the original and the filter are greater than D; that is, where pixels are noisy. Then $1 - r$ will consist of ones at only those places where pixels are not noisy. Multiplying r by the filter replaces noisy values with averages; multiplying $1 - r$ with original values gives the rest of the output.

[1]In fact, this is not the case with MATLAB, which uses a highly optimized method. Nonetheless, we introduce a different method to show that there are other ways of cleaning salt and pepper noise.

A Matlab function for implementing this is shown in figure 7.8. An immediate problem with the outlier method is that is it not completely automatic—the threshold D must be chosen. An appropriate way to use the outlier method is to apply it with several different thresholds, and choose the value which provides the best results. Suppose we attempt to use the outlier method to clean

```
function res=outlier(im,d)
% OUTLIER(IMAGE,D) removes salt and pepper noise using an outlier method.
% This is done by using the following algorithm:
%
% For each pixel in the image, if the difference between its grey value
% and the average of its eight neighbours is greater than D, it is
% classified as noisy, and its grey value is changed to that of the
% average of its neighbours.
%
% IMAGE can be of type UINT8 or DOUBLE; the output is of type
% UINT8.  The threshold value D must be chosen to be between 0 and 1.

f=[0.125 0.125 0.125; 0.125 0 0.125; 0.125 0.125 0.125];
imd=im2double(im);
imf=filter2(f,imd);
r=abs(imd-imf)-d>0;
res=im2uint8(r.*imf+(1-r).*imd);
```

Figure 7.8: A Matlab function for cleaning salt and pepper noise using an outlier method

the noise from figure 7.1(b); that is, the twins image with 10% salt and pepper noise. Choosing $D = 0.2$ gives the image in figure 7.9(a). This is not as good a result as using a median filter: the affect of the noise has been lessened, but there are still noise "artifacts" over the image. In this case we have chosen a threshold which is too small. If we choose $D = 0.4$, we obtain the image in figure 7.9(b), which still has smoe noise artifacts, although in different places. We can see that a lower values of D tends to remove noise from dark areas, and a higher value of D tends to remove noise from light areas. A mid-way valaue, round about $D = 0.3$ does in fact produce an acceptable result, although not quite as good as median filtering.

Clearly using an appropriate value of D is essential for cleaning salt and pepper noise by this method. If D is too small, then too many "non-noisy" pixels will be classified as noisy, and their values changed to the average of their neighbours. This will result in a blurring effect, similar to that obtained by using an averaging filter. If D is chosen to be too large, then not enough noisy pixels will be classified as noisy, and there will be little change in the output.

The outlier method is not particularly suitable for cleaning large amounts of noise; for such situations the median filter is to be preferred. The outlier method may thus be considered as a "quick and dirty" method for cleaning salt and pepper noise when the median filter proves too slow.

7.4 Cleaning Gaussian noise

Image averaging

It may sometimes happen that instead of just *one* image corrupted with Gaussian noise, we have many different copies of it. An example is satellite imaging; if a satellite passes over the same spot many times, we will obtain many different images of the same place. Another example is in

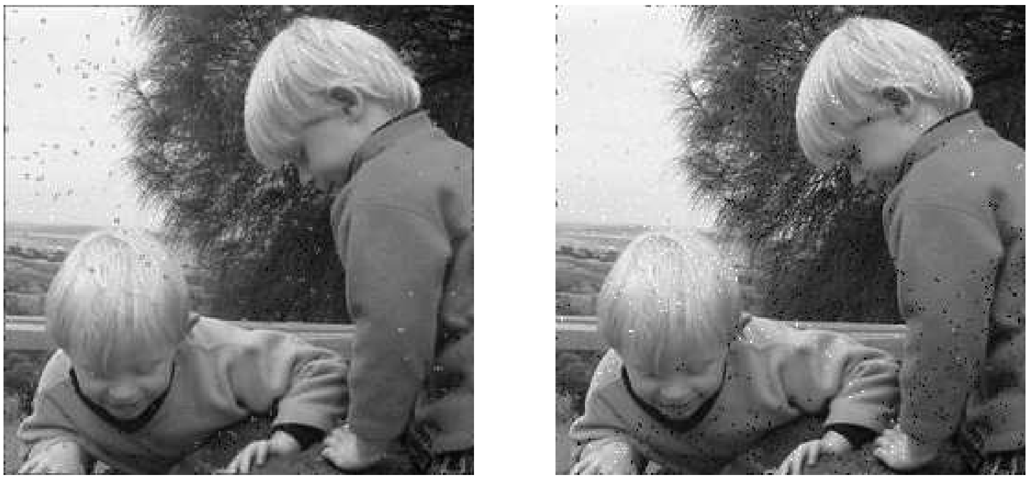

(a) $D = 0.2$ (b) $D = 0.4$

Figure 7.9: Applying the outlier method to 10% salt and pepper noise

microscopy: we might take many different images of the same object. In such a case a very simple approach to cleaning Gaussian noise is to simply take the average—the mean—of all the images. T

To see why this works, suppose we have 100 copies of our image, each with noise; then the i-th noisy image will be:

$$M + N_i$$

where M is the matrix of original values, and N_i is a matrix of normally distributed random values with mean 0. We can find the mean M' of these images by the usual add and divide method:

$$\begin{aligned} M' &= \frac{1}{100}\sum_{i=1}^{100}(M + N_i) \\ &= \frac{1}{100}\sum_{i=1}^{100} M + \frac{1}{100}\sum_{i=1}^{100} N_i \\ &= M + \frac{1}{100}\sum_{i=1}^{100} N_i \end{aligned}$$

Since N_i is normally distributed with mean 0, it can be readily shown that the mean of all the N_i's will be close to zero—the greater the number of N_i's; the closer to zero. Thus

$$M' \approx M$$

and the approximation is closer for larger number of images $M + N_i$.

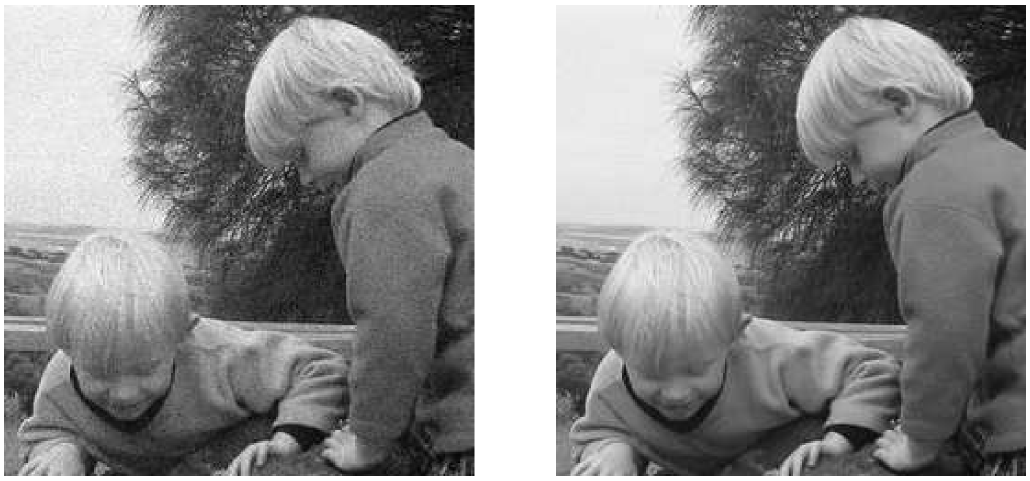
We can demonstrate this with the twins image. We first need to create different versions with Gaussian noise, and then take the average of them. We shall create 10 versions. One way is to create an empty three-dimensional array of depth 10, and fill each "level" with a noisy image:

```
>> s=size(t);
>> t_ga10=zeros(s(1),s(2),10);
>> for i=1:10 t_ga10(:,:,i)=imnoise(t,'gaussian'); end
```

Note here that the "`gaussian`" option of `imnoise` calls the random number generator `randn`, which creates normally distributed random numbers. Each time `randn` is called, it creates a *different* sequence of numbers. So we may be sure that all levels in our three-dimensional array do indeed contain different images. Now we can take the average:

```
>> t_ga10_av=mean(t_ga10,3);
```

The optional parameter 3 here indicates that we are taking the mean along the *third* dimension of our array. The result is shown in figure 7.10(a). This is not quite clear, but is a vast improvement on the noisy image of figure 7.2(a). An even better result is obtained by taking the average of 100 images; this can be done by replacing 10 with 100 in the commands above, and the result is shown in figure 7.10(b). Note that this method only works if the Gaussian noise has mean 0.



(a) 10 images (b) 100 images

Figure 7.10: Image averaging to remove Gaussian noise

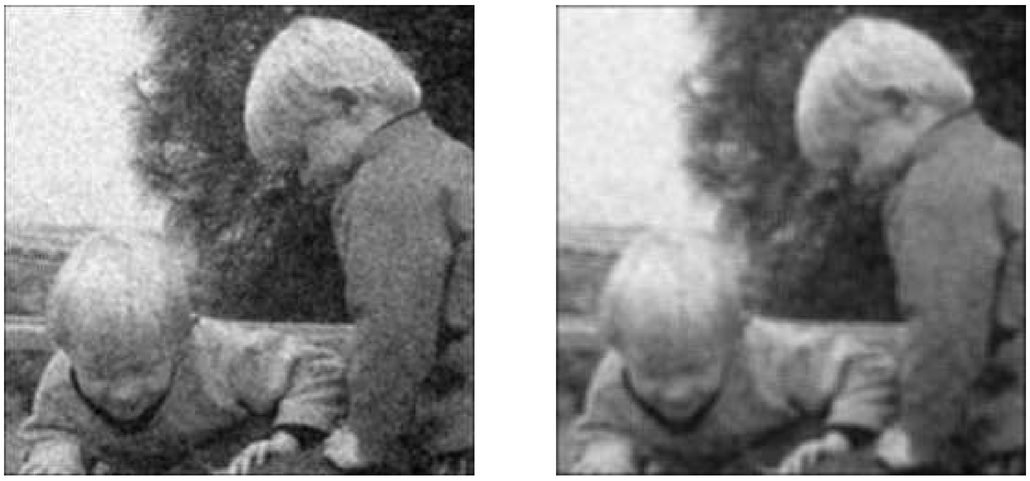
Average filtering

If the Gaussian noise has mean 0, then we would expect that an average filter would average the noise to 0. The larger the size of the filter mask, the closer to zero. Unfortunately, averaging tends to blur an image, as we have seen in chapter 6. However, if we are prepared to trade off blurring for noise reduction, then we can reduce noise significantly by this method.

Suppose we take the 3×3 and 5×5 averaging filters, and apply them to the noisy image `t_ga`.

```
>> a3=fspecial('average');
>> a5=fspecial('average',[5,5]);
>> tg3=filter2(a3,t_ga);
>> tg5=filter2(a5,t_ga);
```

The results are shown in figure 7.11. The results are not really particularly pleasing; although there has been some noise reduction, the "smeary" nature of the resulting images is unattractive.



(a) 3×3 averaging (b) 5×5 averaging

Figure 7.11: Using averaging filtering to remove Gaussian noise

Wiener filtering

Before we describe this method, we shall discuss a more general question: given a degraded image M' of some original image M and a restored version R, what measure can we use to say whether our restoration has done a good job? Clearly we would like R to be as close as possible to the "correct" image M. One way of measuring the closeness of R to M is by adding the squares of all differences:

$$\sum(m_{i,j} - r_{i,j})^2$$

where the sum is taken over all pixels of R and M (which we assume to be of the same size). This sum can be taken as a measure of the closeness of R to M. If we can minimize this value, we may be sure that our procedure has done as good a job as possible. Filters which operate on this principle of *least squares* are called *Wiener filters*. They come in many guises; we shall look at the particular filter which is designed to reduce Gaussian noise.

This filter is a (non-linear) spatial filter; we move a mask across the noisy image, pixel by pixel, and as we move, we create an output image the grey values of whose pixels are based on the values under the mask.

Since we are dealing with additive noise, our noisy image M' can be written as

$$M' = M + N$$

where M is the original correct image, and N is the noise; which we assume to be normally distributed with mean 0. However, within our mask, the mean may not be zero; suppose the mean is m_f, and the variance in the mask is σ_f^2. Suppose also that the variance of the noise over the entire image is known to be σ_g^2. Then the output value can be calculated as

$$m_f + \frac{\sigma_f^2}{\sigma_f^2 + \sigma_g^2}(g - m_f)$$

where g is the current value of the pixel in the noisy image. See Lim [13] for details. In practice, we calculate m_f by simply taking the mean of all grey values under the mask, and σ_f^2 by calculating the variance of all grey values under the mask. We may not necessarily know the value σ_g^2, so the Matlab function wiener2 which implements Wiener filtering uses a slight variant of the above equation:

$$m_f + \frac{\max\{0, \sigma_f^2 - n\}}{\max\{\sigma_f^2, n\}}(g - m_f)$$

where n is the computed noise variance, and is calculated by taking the mean of all values of σ_f^2 over the entire image. This can be very efficiently calculated in Matlab.

Suppose we take the noisy image shown in figure 7.2(a), and attempt to clean this image with Wiener filtering. We will use the wiener2 function, which can take an optional parameter indicating the size of the mask to be used. The default size is 3×3. We shall create four images:

```
>> t1=wiener2(t_ga);
>> t2=wiener2(t_ga,[5,5]);
>> t3=wiener2(t_ga,[7,7]);
>> t4=wiener2(t_ga,[9,9]);
```

and these are shown in figure 7.12. Being a low pass filter, Wiener filtering does tend to blur edges and high frequency components of the image. But it does a far better job than using a low pass blurring filter.

We can achieve very good results for noise where the variance is not as high as that in our current image.

```
>> t2=imnoise(t,'gaussian',0,0.005);
>> imshow(t2)
>> t2w=wiener2(t2,[7,7]);
>> figure,imshow(t2w)
```

The image and its appearance after Wiener filtering as shown in figure 7.13. The result is a great improvement over the original noisy image.

Exercises

1. The arrays below represent small greyscale images. Compute the 4×4 image that would result in each case if the middle 16 pixels were transformed using a 3×3 median filter:

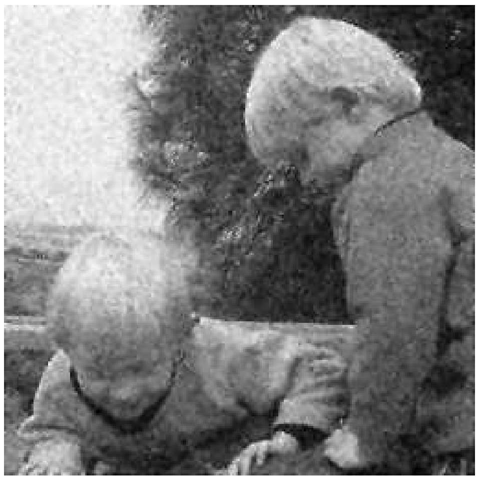
8	17	4	10	15	12
10	12	15	7	3	10
15	10	50	5	3	12
4	8	11	4	1	8
16	7	4	3	0	7
16	24	19	3	20	10

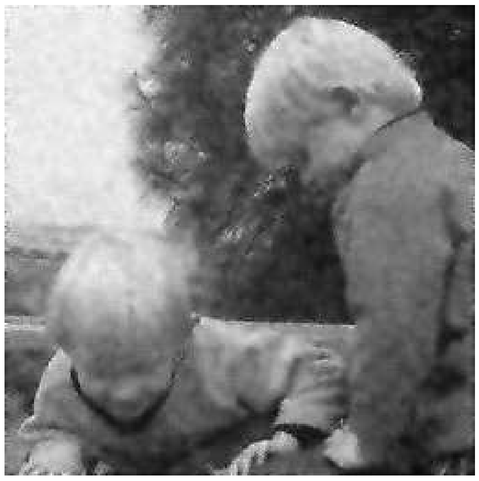
1	1	2	5	3	1
3	20	5	6	4	6
4	6	4	20	2	2
4	3	3	5	1	5
6	5	20	2	20	2
6	3	1	4	1	2

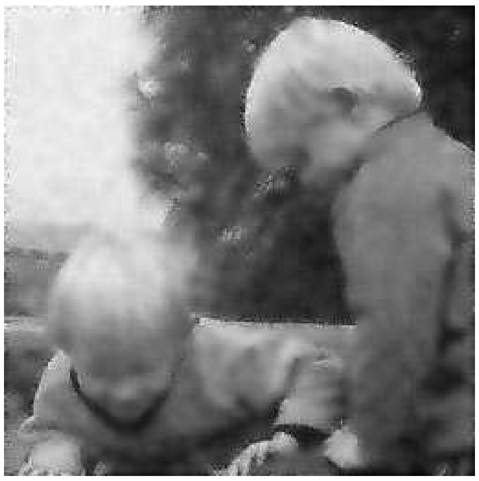
7	8	11	12	13	9
8	14	0	9	7	10
11	23	10	14	1	8
14	7	11	8	9	11
13	13	18	10	7	12
9	11	14	12	13	10

2. Using the same images as in question 1, transform them by using a 3×3 averaging filter.

3. Use the outlier method to find noisy pixels in each of the images given in question 1. What are the reasonable values to use for the difference between the grey value of a pixel and the average of its eight 8-neighbours?


(a) 3×3 filtering


(b) 5×5 filtering


(a) 7×7 filtering

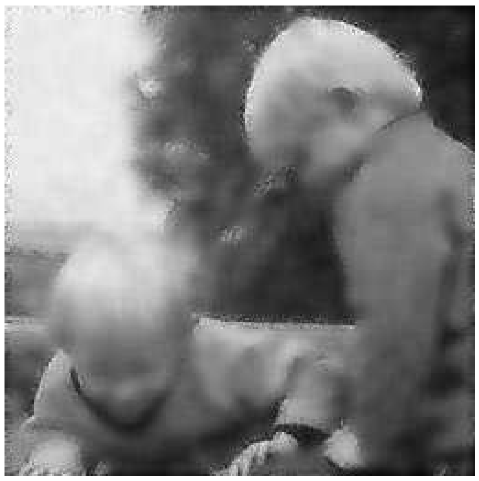

(b) 9×9 filtering

Figure 7.12: Examples of Wiener filtering to remove Gaussian noise

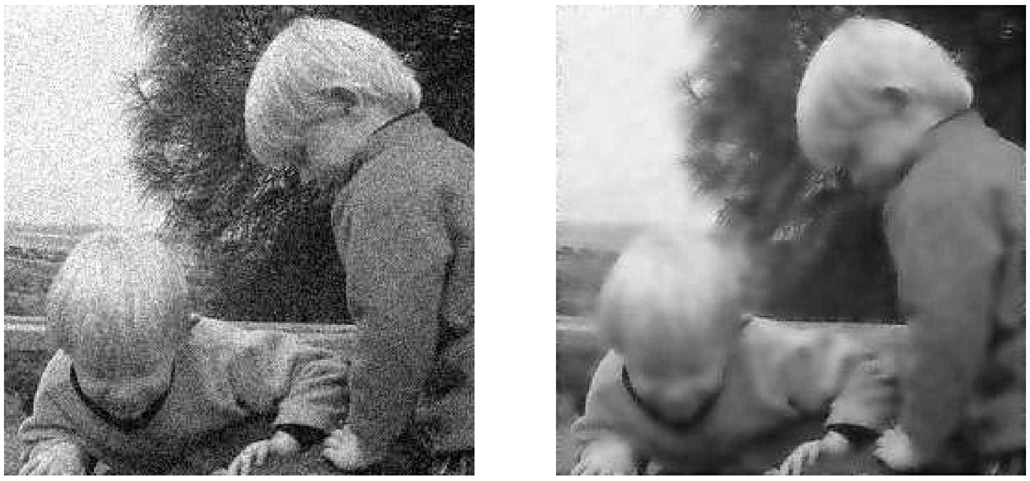


Figure 7.13: Using Wiener filtering to remove Gaussian noise with low variance

4. Pratt [16] has proposed a "pseudo-median" filter, in order to overcome some of the speed disadvantages of the median filter. For example, given a five element sequence $\{a, b, c, d, e\}$, its pseudo-median is defined as

$$\begin{aligned}\text{psmed}(a, b, c, d, e) &= \tfrac{1}{2}\max\Big[\min(a, b, c) + \min(b, c, d) + \min(c, d, e)\Big] \\ &+ \tfrac{1}{2}\min\Big[\max(a, b, c) + \max(b, c, d) + \max(c, d, e)\Big]\end{aligned}$$

   So for a sequence of length 5, we take the maxima and minima of all subsequences of length three. In general, for an odd-length sequence L of length $2n + 1$, we take the maxima and minima of all subsequences of length $n + 1$.

   We can apply the pseudo-median to 3×3 neighbourhoods of an image, or cross-shaped neighbourhoods containing 5 pixels, or any other neighbourhood with an odd number of pixels.

   Apply the pseudo-median to the images in question 1, using 3×3 neighbourhoods of each pixel.

5. Write a MATLAB function to implement the pseudo-median, and apply it to the images above with the `nlfilter` function. Does it produce a good result?

6. Produce a grey subimage of the colour image `flowers.tif` by

```
>> f=imread('flowers.tif');
>> fg=rgb2gray(f);
>> f=im2uint8(f(30:285,60:315));
```

   Add 5% salt & pepper noise to the image. Attempt to remove the noise with

   (a) average filtering,
   (b) median filtering,
   (c) the outlier method,

   (d) pseudo-median filtering.

   Which method gives the best results?

7. Repeat the above question but with 10%, and then with 20% noise.

8. For 20% noise, compare the results with a 5×5 median filter, and two applications of a 3×3 median filter.

9. Add Gaussian noise to the greyscale flowers image with the following parameters:

   (a) mean 0, variance 0.01 (the default),
   (b) mean 0, variance 0.02,
   (c) mean 0, variance 0.05,
   (d) mean 0, variance 0.1.

   In each case, attempt to remove the noise with average filtering and with Wiener filtering.

   Can you produce satisfactory results with the last two noisy images?

Chapter 8

Edges

8.1 Introduction

Edges contain some of the most useful information in an image. We may use edges to measure the size of objects in an image; to isolate particular objects from their background; to recognize or classify objects. There are a large number of edge-finding algorithms in existence, and we shall look at some of the more straightforward of them. The general MATLAB command for finding edges is

```
edge(image,'method',parameters...)
```

where the parameters available depend on the method used. In this chapter, we shall show how to create edge images using basic filtering methods, and discuss the MATLAB edge function.

An *edge* may be loosely defined as a line of pixels showing an *observable* difference. For example, consider the two ·I blocks of pixels shown in figure 8.1.

51	52	53	59
54	52	53	62
50	52	53	68
55	52	53	55

50	53	150	160
51	53	150	170
52	53	151	190
51	53	152	155

Figure 8.1: Blocks of pixels

In the right hand block, there is a clear difference between the grey values in the second and third columns. This would be easily discernable in an image—the human eye can pick out grey differences of this magnitude with relative ease. Our aim is to develop methods which will enable us to pick out the edges of an image.

8.2 Differences and edges

8.2.1 Fundamental definitions

Suppose we follow the grey values of pixels in a line crossing an edge, as illustrated in figure 8.2.
If we consider the grey values along this line, and plot their values, we will have something similar to that shown in figure 8.3 (a) or (b).
If we now plot the *differences* between each grey value and its predecessor from the ramp edge, we would obtain a graph similar to that shown in figure 8.4.

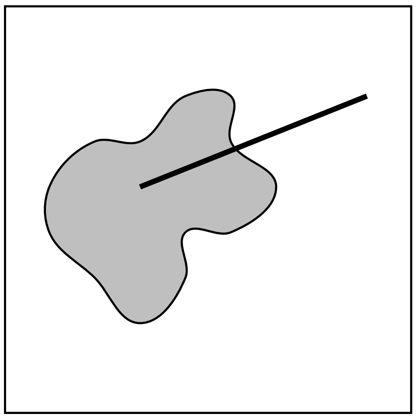


Figure 8.2: A line crossing an edge

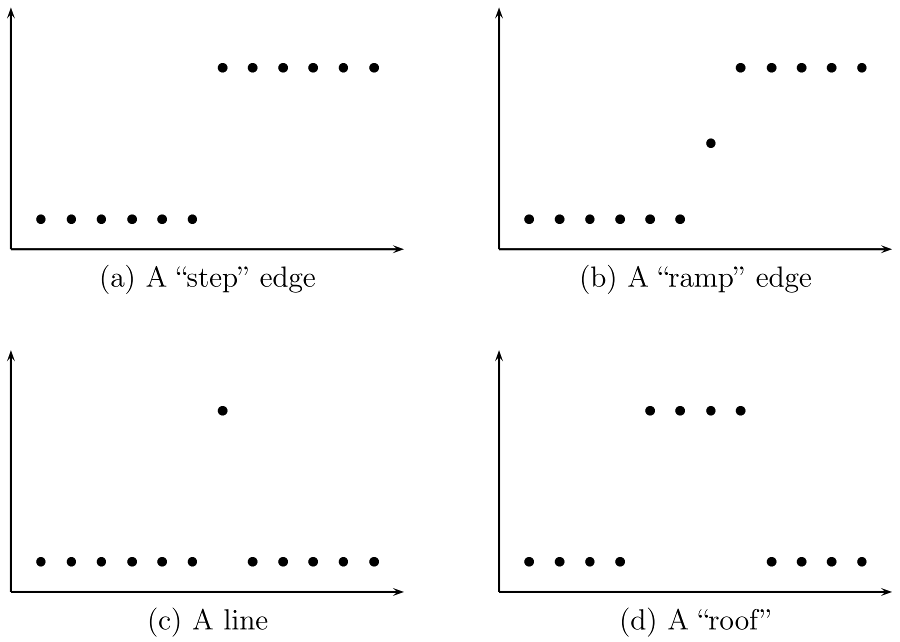


Figure 8.3: Grey values across edges

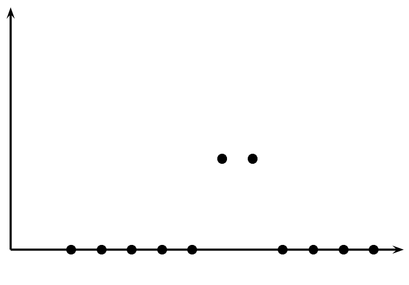


Figure 8.4: Differences of the edge function

To see where this graph comes from, suppose that the values of the "ramp" edge in figure 8.3(b) are, from left to right:

$$20,\ 20,\ 20,\ 20,\ 20,\ 20,\ 100,\ 180,\ 180,\ 180,\ 180,\ 180.$$

If we form the differences, by subtracting each value from its successor, we obtain:

$$0,\ 0,\ 0,\ 0,\ 0,\ 80,\ 80,\ 0,\ 0,\ 0,\ 0$$

and it is these values which are plotted in figure 8.4.

It appears that the difference tends to enhance edges, and reduce other components. So if we could "difference" the image, we would obtain the effect we want.

We can define the difference in three separate ways:

- the *forward difference*: $\Delta f(x) = f(x+1) - f(x)$,
- the *backward difference*: $\nabla f(x) = f(x) - f(x-1)$,
- the *central difference*: $\delta f(x) = f(x+1) - f(x-1)$.

However, an image is a function of two variables, so we can generalize these definitions to include both the x and y values:

$$\begin{array}{rclcrcl}
\Delta_x f(x,y) & = & f(x+1,y) - f(x,y) & & \Delta_y f(x,y) & = & f(x,y+1) - f(x,y) \\
\nabla_x f(x,y) & = & f(x,y) - f(x-1,y) & & \nabla_y f(x,y) & = & f(x,y) - f(x,y-1) \\
\delta_x f(x,y) & = & f(x+1,y) - f(x-1,y) & & \delta_y f(x,y) & = & f(x,y+1) - f(x,y-1)
\end{array}$$

where the subscripts in each case indicate the direction of the difference.

8.2.2 Some difference filters

To see how we might use δ_x to determine edges in the x direction, consider the function values around a point (x,y) (and using the convention that x values increase to the right, y values to the bottom):

$f(x-1,y-1)$	$f(x,y-1)$	$f(x+1,y-1)$
$f(x-1,y)$	$f(x,y)$	$f(x+1,y)$
$f(x-1,y+1)$	$f(x,y+1)$	$f(x+1,y+1)$

To find the filter which returns the value δ_x, we just compare the coefficients of the function's values in δ_x with their position in the array:

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This filter thus will find vertical edges in an image and produce a reasonably bright result. However, the edges in the result can be a bit "jerky"; this can be overcome by smoothing the result in the opposite direction; by using the filter

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Both filters can be applied at once, using the combined filter:

$$P_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

This filter, and its companion for finding horizontal edges:

$$P_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

are the *Prewitt* filters for edge detection.

We are now in the position of having to apply two filters to an image. The trick here is to apply each filter independently, and form the output image from a function of the filter values produced. So if p_x and p_y were the grey values produced by applying P_x and P_y to an image, then the output grey value p can be chosen by any of these methods:

1. $p = \max\{|p_x|, |p_y|\}$,
2. $p = |p_x| + |p_y|$,
3. $p = \sqrt{p_x^2 + p_y^2}$.

For example, let us take the image of the integrated circuit shown in figure 8.5, which can be read into Matlab with

```
>> ic=imread('ic.tif');
```

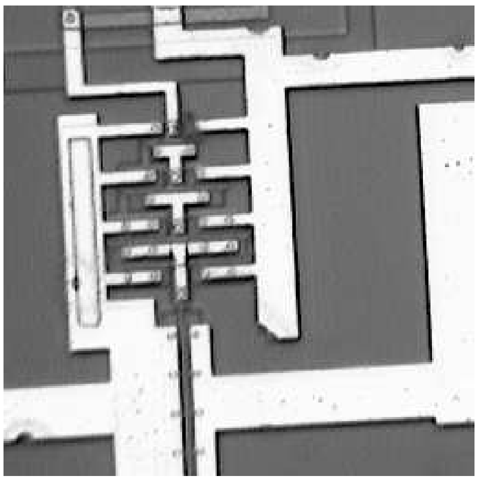


Figure 8.5: An integrated circuit

Applying each of P_x and P_y individually provides the results shown in figure 8.6 Figure 8.6(a) was produced with the following Matlab commands:

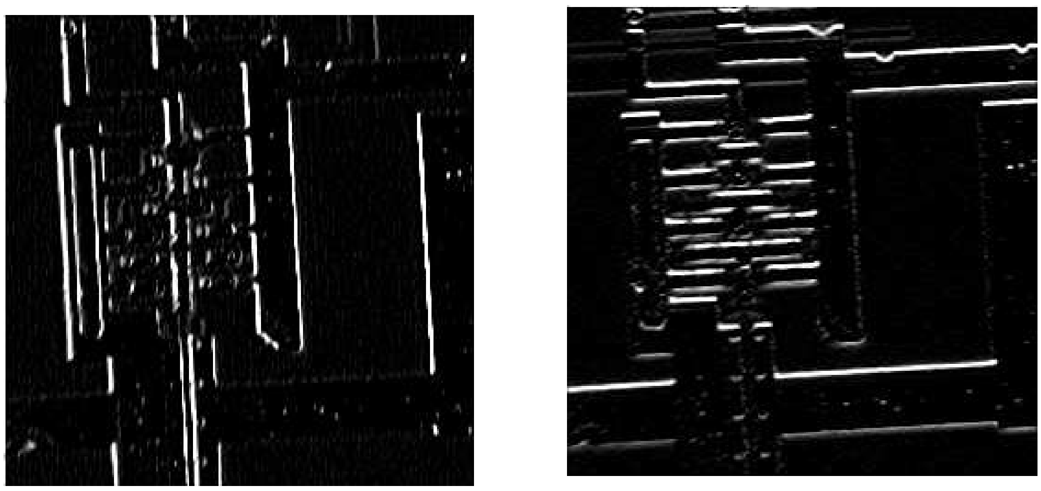

(a) (b)

Figure 8.6: The circuit after filtering with the Prewitt filters

```
>> px=[-1 0 1;-1 0 1;-1 0 1];
>> icx=filter2(px,ic);
>> figure,imshow(icx/255)
```

and figure 8.6(b) with

```
>> py=px';
>> icy=filter2(py,ic);
>> figure,imshow(icy/255)
```

Note that the filter P_x highlights vertical edges, and P_y horizontal edges. We can create a figure containing all the edges with:

```
>> edge_p=sqrt(icx.^2+icy.^2);
>> figure,imshow(edge_p/255)
```

and the result is shown in figure 8.7(a). This is a grey-scale image; a binary image containing edges only can be produced by thresholding. Figure 8.7(b) shows the result after the command

```
>> edge_t=im2bw(edge_p/255,0.3);
```

We can obtain edges by the Prewitt filters directly by using the command

```
>> edge_p=edge(ic,'prewitt');
```

and the `edge` function takes care of all the filtering, and of choosing a suitable threshold level; see its help text for more information. The result is shown in figure 8.8. Note that figures 8.7(b) and 8.8 seem different to each other. This is because the `edge` function does some extra processing over and above taking the square root of the sum of the squares of the filters.

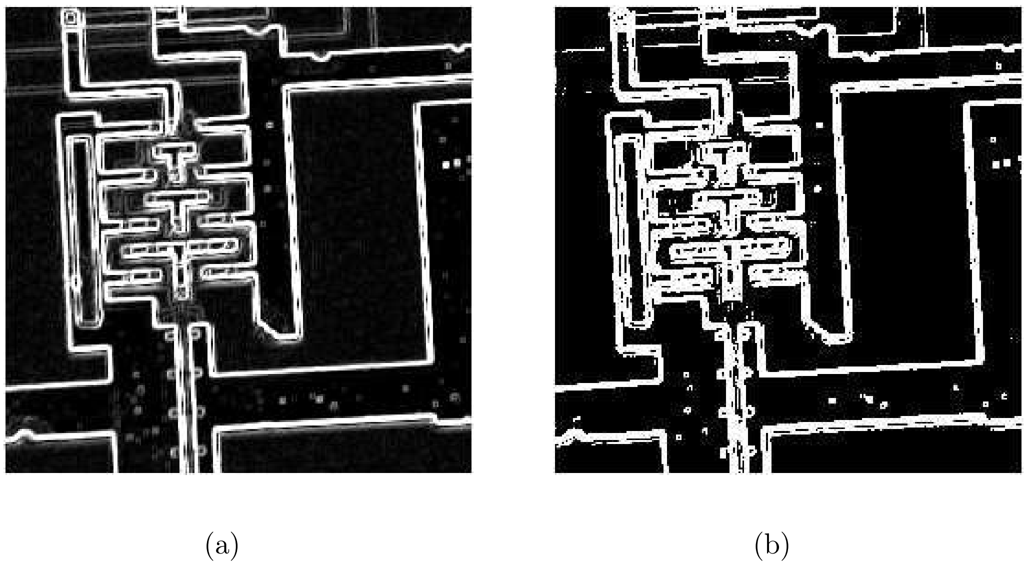

(a) (b)

Figure 8.7: All the edges of the circuit

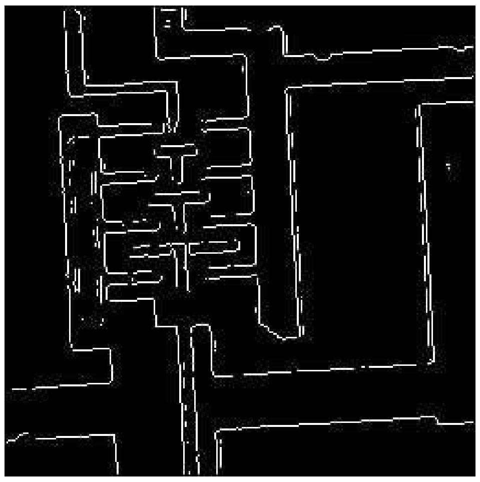


Figure 8.8: The `prewitt` option of `edge`

Slightly different edge finding filters are the *Roberts cross-gradient filters*:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the *Sobel filters*:

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}.$$

The *Sobel filters* are similar to the Prewitt filters, in that they apply a smoothing filter in the opposite direction to the central difference filter. In the Sobel filters, the smoothing takes the form

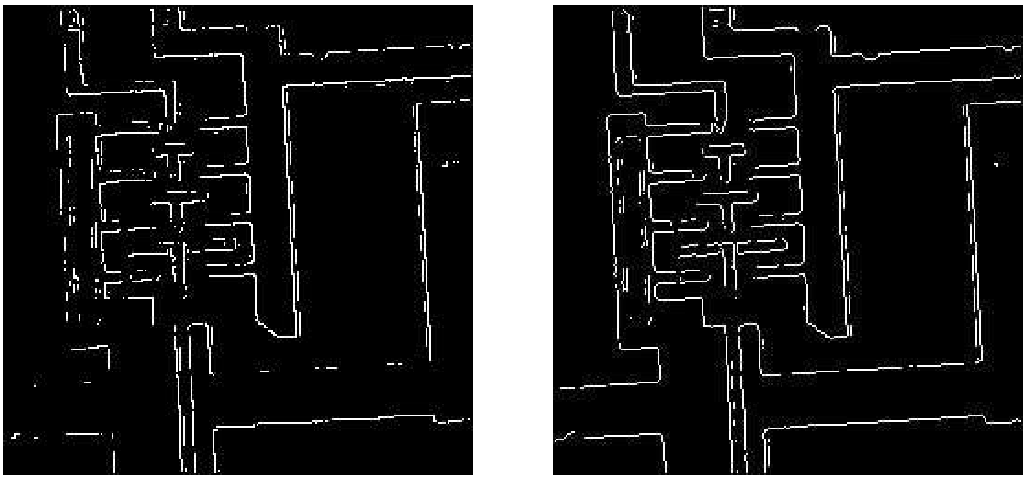
$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

which gives slightly more prominence to the central pixel. Figure 8.9 shows the respective results of the MATLAB commands

```
>> edge_r=edge(ic,'roberts');
>> figure,imshow(edge_r)
```

and

```
>> edge_s=edge(ic,'sobel');
>> figure,imshow(edge_s)
```



(a) Roberts edge detection (b) Sobel edge detection

Figure 8.9: Results of the Roberts and Sobel filters

The appearance of each of these can be changed by specifying a threshold level.

Of the three filters, the Sobel filters are probably the best; they provide good edges, and they perform reasonably well in the presence of noise.

8.3 Second differences

8.3.1 The Laplacian

Another class of edge-detection method is obtained by considering the difference of differences; these are called *second differences.*

To calculate a (central) second difference, take the *backward* difference of a *forward* difference:

$$\begin{aligned}
\partial_x^2 &= \nabla\Delta f(x,y)\\
&= \nabla(f(x+1,y)-f(x,y))\\
&= (f(x+1,y)-f(x,y))-(f(x,y)-f(x-1,y))\\
&= f(x+1,y)-2f(x,y)+f(x-1,y)
\end{aligned}$$

This can be implemented by the filter

$$\begin{bmatrix} 0 & 0 & 0\\ 1 & -2 & 1\\ 0 & 0 & 0 \end{bmatrix}.$$

The corresponding filter for second differences ∂_y^2 in the y direction is

$$\begin{bmatrix} 0 & 1 & 0\\ 0 & -2 & 0\\ 0 & 1 & 0 \end{bmatrix}.$$

The sum of these two is written

$$\nabla^2 = \partial_x^2 + \partial_y^2$$

and is implemented by the filter

$$\begin{bmatrix} 0 & 1 & 0\\ 1 & -4 & 1\\ 0 & 1 & 0 \end{bmatrix}.$$

This is known as a *discrete Laplacian.*

To see how the second difference affects an edge, take the difference of the pixel values as plotted in figure 8.4; the results are shown in figure 8.10.

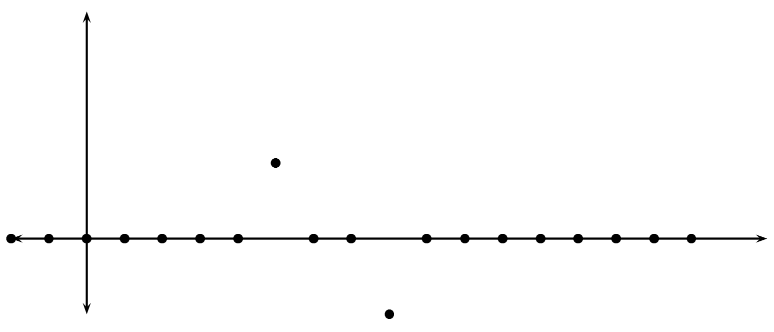


Figure 8.10: Second differences of the edge function

We see that the Laplacian (after taking an absolute value, or squaring) gives double edges. It is also extremely sensitive to noise. However, the Laplacian does have the advantage of detecting edges in *all* directions equally well. To see an example, suppose we enter the MATLAB commands:

```
>> l=fspecial('laplacian',0);
>> ic_l=filter2(l,ic);
>> figure,imshow(mat2gray(ic_l))
```

the result of which is shown in figure 8.11.

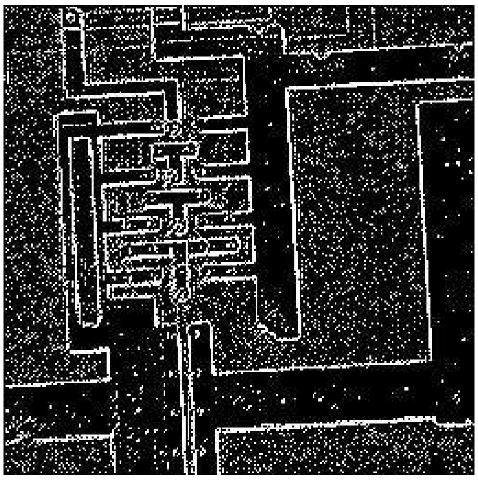


Figure 8.11: Result after filtering with a discrete laplacian

Although the result is adequate, it is very messy when compared to the results of the Prewitt and Sobel methods discussed earlier. Other Laplacian masks can be used; some are:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}.$$

In MATLAB, Laplacians of all sorts can be generated using the `fspecial` function, in the form

```
fspecial('laplacian',ALPHA)
```

which produces the Laplacian

$$\frac{1}{\alpha+1}\begin{bmatrix} \alpha & 1-\alpha & \alpha \\ 1-\alpha & -4 & 1-\alpha \\ \alpha & 1-\alpha & \alpha \end{bmatrix}.$$

If the parameter `ALPHA` (which is optional) is omitted, it is assumed to be 0.2. The value 0 gives the Laplacian developed earlier.

8.3.2 Zero crossings

A more appropriate use for the Laplacian is to find the *position* of edges by locating *zero crossings*. If we look at figure 8.10, we see that the position of the edge is given by the place where the value of the filter takes on a zero value. In general, these are places where the result of the filter changes sign. For example, consider the the simple image given in figure 8.12(a), and the result after filtering with a Laplacian mask in figure 8.12(b).

50	50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50	50
50	50	200	200	200	200	200	200	50	50
50	50	200	200	200	200	200	200	50	50
50	50	200	200	200	200	200	200	50	50
50	50	200	200	200	200	200	200	50	50
50	50	50	50	200	200	200	200	50	50
50	50	50	50	200	200	200	200	50	50
50	50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50	50

(a) A simple image

−100	−50	−50	−50	−50	−50	−50	−50	−50	−100
−50	**0**	150	150	150	150	150	150	**0**	−50
−50	150	**−300**	**−150**	**−150**	**−150**	**−150**	**−300**	150	−50
−50	150	**−150**	0	0	0	0	**−150**	150	−50
−50	150	**−150**	0	0	0	0	**−150**	150	−50
−50	150	**−300**	**−150**	0	0	0	**−150**	150	−50
−50	**0**	150	300	**−150**	0	0	**−150**	150	−50
−50	0	0	150	**−300**	**−150**	**−150**	**−300**	150	−50
−50	0	0	**0**	150	150	150	150	**0**	−50
−100	−50	−50	−50	−50	−50	−50	−50	−50	−100

(b) After laplace filtering

Figure 8.12: Locating zero crossings in an image

We define the *zero crossings* in such a filtered image to be pixels which satisfy either of the following:

1. they have a negative grey value and are next to (by four-adjacency) a pixel whose grey value is positive,
2. they have a value of zero, and are between negative and positive valued pixels.

To give an indication of the way zero-crossings work, look at the edge plots and their second differences in figure 8.13.

In each case the zero-crossing is circled. The important point is to note that across any edge there can be only *one* zero-crossing. Thus an image formed from zero-crossings has the potential to be very neat.

In figure 8.12(b) the zero crossings are shaded. We now have a further method of edge detection: take the zero-crossings after a laplace filtering. This is implemented in MATLAB with the `zerocross` option of `edge`, which takes the zero crossings after filtering with a given filter:

```
>> l=fspecial('laplace',0);
>> icz=edge(ic,'zerocross',l);
>> imshow(icz)
```

The result is shown in figure 8.14(a). We see that this is not in fact a very good result—far too many grey level changes have been interpreted as edges by this method. To eliminate them, we may first smooth the image with a Gaussian filter. This leads to the following sequence of steps for edge detection; the *Marr-Hildreth* method:

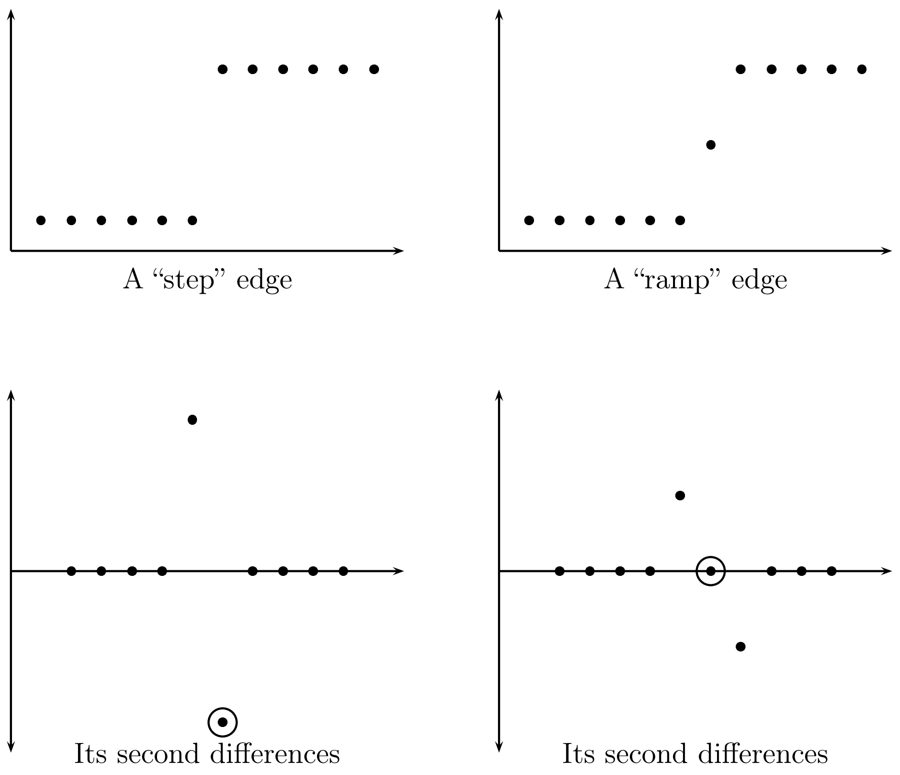


Figure 8.13: Edges and second differences

1. smooth the image with a Gaussian filter,
2. convolve the result with a laplacian,
3. find the zero crossings.

This method was designed to provide a edge detection method to be as close as possible to biological vision. The first two steps can be combined into one, to produce a "Laplacian of Gaussian" or "LoG" filter. These filters can be created with the `fspecial` function. If no extra parameters are provided to the `zerocross` edge option, then the filter is chosen to be the LoG filter found by

```
>> fspecial('log',13,2)
```

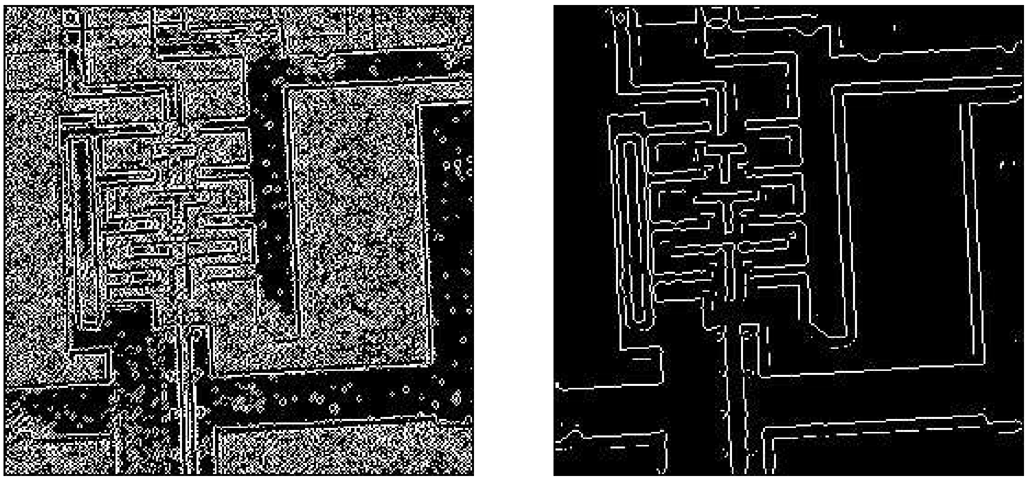
This means that the following command:

```
>> edge(ic,'log');
```

produces exactly the same result as the commands:

```
>> log=fspecial('log',13,2);
>> edge(ic,'zerocross',log);
```

In fact the `LoG` and `zerocross` options implement the same edge finding method; the difference being that the `zerocross` option allows you to specify your own filter. The result after applying an LoG filter and finding its zero crossings is given in figure 8.14(b).



(a) Zeros crossings (b) Using an LoG filter first

Figure 8.14: Edge detection using zero crossings

8.4 Edge enhancement

So far we have seen how to isolate edges in an image. A related operation is to make edges in an image slightly sharper and crisper, which generally results in an image more pleasing to the human eye. The operation is variously called "edge enhancement", "edge crispening", or "unsharp masking". This last term comes from the printing industry.

Unsharp masking

The idea of unsharp masking is to subtract a scaled "unsharp" version of the image from the original. In practice, we can achieve this affect by subtracting a scaled blurred image from the original. The schema for unsharp masking is shown in figure 8.15.

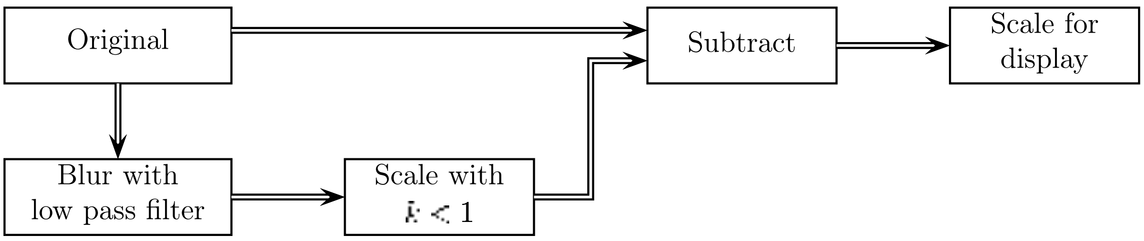
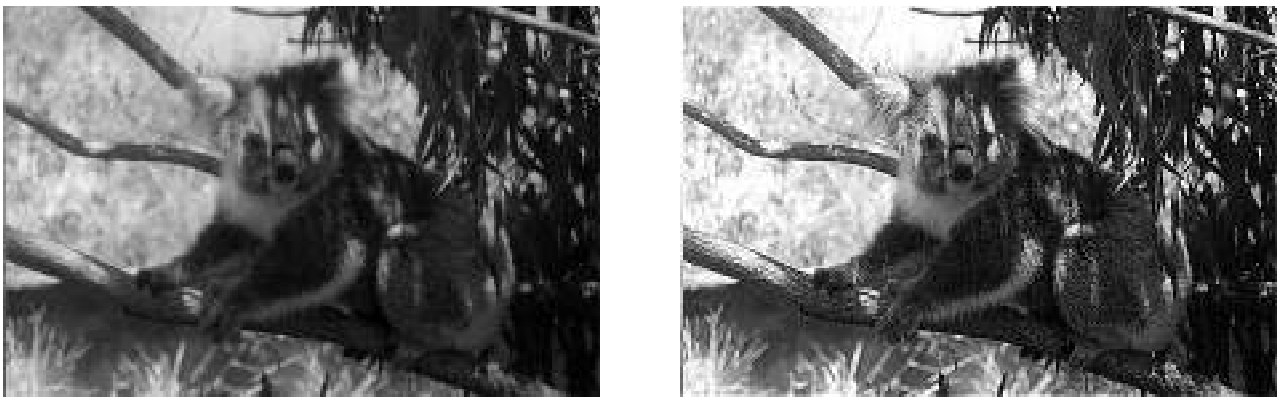


Figure 8.15: Schema for unsharp masking

Suppose we have an image `x` of type `uint8`. The we can apply unsharp masking by the following sequence of commands:

```
>> f=fspecial('average');
>> xf=filter2(f,x);
>> xu=double(x)-xf/1.5
>> imshow(xu/70)
```

The last command scales the result so that `imshow` displays an appropriate image; the value may need to be adjusted according to the input image. Suppose that `x` is the image shown in figure 8.16(a), then the result of unsharp masking is given in figure 8.16(b). The result appears to be a better image than the original; the edges are crisper and more clearly defined.



(a) Original image (b) The image after unsharp masking

Figure 8.16: An example of unsharp masking

To see why this works, we may consider the function of grey values as we travel across an edge, as shown in figure 8.17.

As a scaled blur is subtracted from the original, the result is that the edge is enhanced, as shown in graph (c) of figure 8.17.

We can in fact perform the filtering and subtracting operation in one command, using the linearity of the filter, and that the 3×3 filter

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is the “identity filter”.

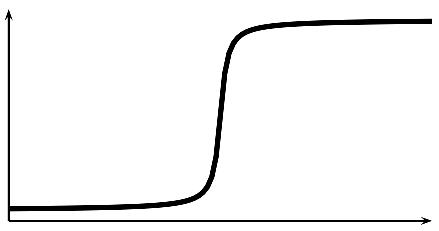
Hence unsharp masking can be implemented by a filter of the form

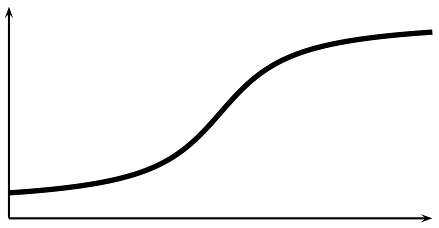
$$f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{k}\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

where k is a constant chosen to provide the best result. Alternatively, the unsharp masking filter may be defined as

$$f = k\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

so that we are in effect subtracting a blur from a scaled version of the original; the scaling factor may also be split between the identity and blurring filters.


(a) Pixel values over an edge


(b) The edge blurred

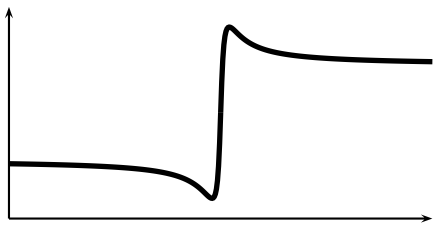

(c) (a) $- k$(b)

Figure 8.17: Unsharp masking

The `unsharp` option of `fspecial` produces such filters; the filter created has the form

$$\frac{1}{\alpha+1}\begin{bmatrix} -\alpha & \alpha-1 & -\alpha \\ \alpha-1 & \alpha+5 & \alpha-1 \\ -\alpha & \alpha-1 & -\alpha \end{bmatrix}$$

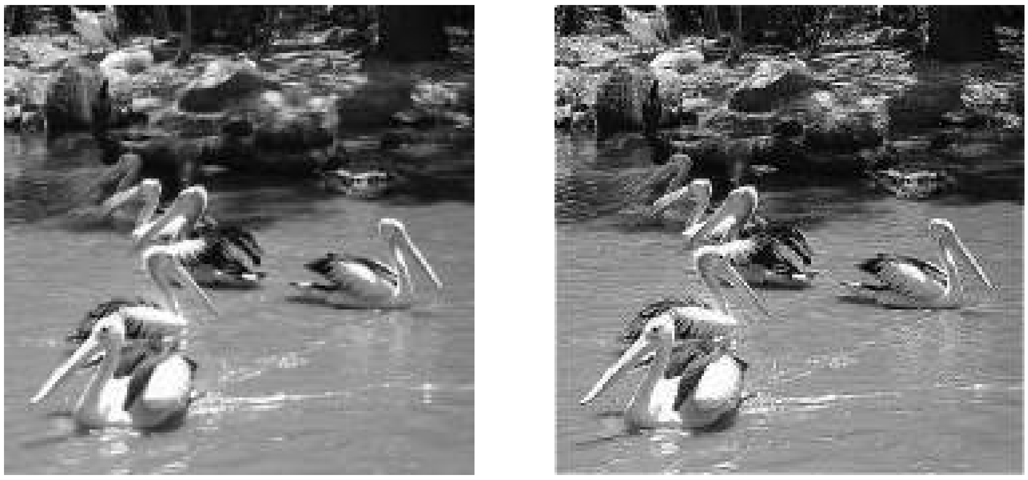
where α is an optional parameter which defaults to 0.2. If $\alpha = 0.5$ the filter is

$$\frac{1}{3}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 11 & -1 \\ -1 & -1 & -1 \end{bmatrix} = 4\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - 3\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

Figure 8.18 was created using the MATLAB commands

```
>> p=imread('pelicans.tif');
>> u=fspecial('unsharp',0.5);
>> pu=filter2(u,p);
>> imshow(p),figure,imshow(pu/255)
```

Figure 8.18(b), appears much sharper and "cleaner" than the original. Notice in particular the rocks and trees in the background, and the ripples on the water.



(a) The original (b) After unsharp masking

Figure 8.18: Edge enhancement with unsharp masking

Although we have used averaging filters above, we can in fact use any low pass filter for unsharp masking.

High boost filtering

Allied to unsharp masking filters are the *high boost* filters, which are obtained by

$$\text{high boost} = A(\text{original}) - (\text{low pass}).$$

where A is an "amplification factor". If $A = 1$, then the high boost filter becomes an ordinary high pass filter. If we take as the low pass filter the 3×3 averaging filter, then a high boost filter will have the form

$$\frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & z & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

where $z > 8$. If we put $z = 11$, we obtain a filtering very similar to the unsharp filter above, except for a scaling factor. Thus the commands:

```
>> f=[-1 -1 -1;-1 11 -1;-1 -1 -1]/9;
>> xf=filter2(x,f);
>> imshow(xf/80)
```

will produce an image similar to that in figure 8.16. The value 80 was obtained by trial and error to produce an image with similar intensity to the original.

We can also write the high boost formula above as

$$\begin{aligned} \text{high boost} &= A(\text{original}) - (\text{low pass}) \\ &= A(\text{original}) - ((\text{original}) - (\text{high pass})) \\ &= (A - 1)(\text{original}) + (\text{high pass}). \end{aligned}$$

Best results for high boost filtering are obtained if we multiply the equation by a factor w so that the filter values sum to 1; this requires

$$wA - w = 1$$

or

$$w = \frac{1}{A - 1}.$$

So a general unsharp masking formula is

$$\frac{A}{A-1}(\text{original}) - \frac{1}{A-1}(\text{low pass}).$$

Another version of this formula is

$$\frac{A}{2A-1}(\text{original}) - \frac{1-A}{2A-1}(\text{low pass})$$

where for best results A is taken so that

$$\frac{3}{5} \le A \le \frac{5}{6}.$$

If we take $A = 3/5$, the formula becomes

$$\frac{3/5}{2(3/5)-1}(\text{original}) - \frac{1-(3/5)}{2(3/5)-1}(\text{low pass}) = 3(\text{original}) - 2(\text{low pass})$$

If we take $A = 5/6$ we obtain

$$\frac{5}{4}(\text{original}) - \frac{1}{4}(\text{low pass})$$

Using the identity and averaging filters, we can obtain high boost filters by:

```
>> id=[0 0 0;0 1 0;0 0 0];
>> f=fspecial('average');
>> hb1=3*id-2*f

hb1 =

   -0.2222   -0.2222   -0.2222
   -0.2222    2.7778   -0.2222
   -0.2222   -0.2222   -0.2222

>> hb2=1.25*id-0.25*f

hb2 =

   -0.0278   -0.0278   -0.0278
   -0.0278    1.2222   -0.0278
   -0.0278   -0.0278   -0.0278
```

If each of the filters `hb1` and `hb2` are applied to an image with `filter2`, the result will have enhanced edges. The images in figure 8.19 show these results; figure 8.19(a) was obtained with

```
>> x1=filter2(hb1,x);
>> imshow(x1/255)
```

and figure 8.19(b) similarly.

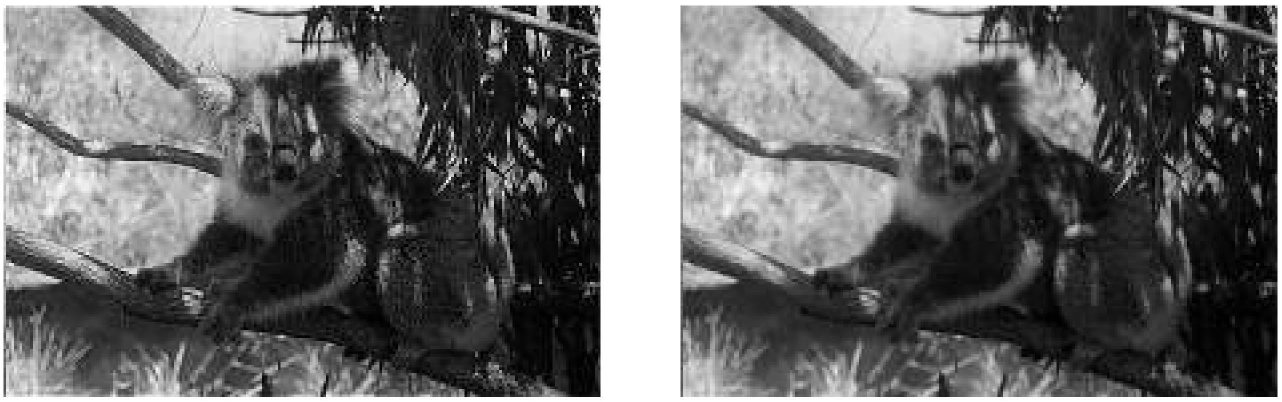

(a) High boost filtering with `hb1` (b) High boost filtering with `hb2`

Figure 8.19: High boost filtering

Of the two filters, `hb1` appears to produce the best result; `hb2` produces an image not very much crisper than the original.

8.5 Final Remarks

As we have seen there are a great many different edge detection algorithms; we have only looked at those which are implemented in the `edge` function of the MATLAB image Processing Toolbox. There are, however, a vast number of further algorithms and methods in existence. Most of them can be programmed as MATLAB programs with little effort.

As to which method is the best; like so many image processing tasks, this is highly subjective. The choice of an suitable edge detector will depend very much on the nature of the image, the amount (and type) of noise, and the use for which the edges are to be put.

Exercises

1. Enter the following matrix into MATLAB:

```
201  195  203  203  199  200  204  190  198  203
201  204  209  197  210  202  205  195  202  199
205  198   46   60   53   37   50   51  194  205
208  203   54   50   51   50   55   48  193  194
200  193   50   56   42   53   55   49  196  211
200  198  203   49   51   60   51  205  207  198
205  196  202   53   52   34   46  202  199  193
199  202  194   47   51   55   48  191  190  197
194  206  198  212  195  196  204  204  199  200
201  189  203  200  191  196  207  203  193  204
```

   and use `imfilter` to apply each of the Roberts, Prewitt, Sobel, Laplacian, and zero-crossing edge-finding methods to the image. In the case of applying two filters (such as with Roberts, Prewitt, or Sobel) apply each filter separately, and join the results.

   Apply thresholding if necessary to obtain a binary image showing only the edges.

   Which method seems to produce the best results?

2. Now with the same matrix as above, use the `edge` function with all possible parameters.

   Which method seems to produce the best results?

3. Open up the image `cameraman.tif` in MATLAB, and apply each of the following edge finding techniques in turn:

   (a) Roberts
   (b) Prewitt
   (c) Sobel
   (d) Laplacian
   (e) Zero-crossings of a laplacian
   (f) the Marr-Hildreth method

   Which seems to you to provide the best looking result?

4. Repeat the above exercise, but use the image `tire.tif`.

5. Obtain a grey-scale flower image with:

   ```
   fl=imread('flowers.tif');
   f=im2uint8(rgb2gray(fl));
   ```

   Now repeat question 3.

6. Pick a grey-scale image, and add some noise to it; say with

   ```
   c=imread('cameraman.tif');
   c1=imnoise(c,'salt & pepper',0.1);
   c2=imnoise(c,'gaussian',0,0.02);
   ```

   Now apply the edge finding techniques to each of the "noisy" images `c1` and `c2`.

   Which technique seems to give

   (a) the best results in the presence of noise?

   (b) the worst results in the presence of noise?

Chapter 9

The Fourier Transform

9.1 Introduction

The Fourier Transform is of fundamental importance to image processing. It allows us to perform tasks which would be impossible to perform any other way; its efficiency allows us to perform other tasks more quickly. The Fourier Transform provides, among other things, a powerful alternative to linear spatial filtering; it is more efficient to use the Fourier transform than a spatial filter for a large filter. The Fourier Transform also allows us to isolate and process particular image "frequencies", and so perform low-pass and high-pass filtering with a great degree of precision.

Before we discuss the Fourier transform of images, we shall investigate the one-dimensional Fourier transform, and a few of its properties.

9.2 The one-dimensional discrete Fourier transform

Our starting place is the observation that a periodic function may be written as the sum of sines and cosines of varying amplitudes and frequencies. For example, in figure 9.1 we show a function, and its decomposition into sine functions.

Some functions will require only a finite number of functions in their decomposition; others will require an infinite number. For example, a "square wave", such as is shown in figure 9.2, has the decomposition

$$f(x) = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \frac{1}{9}\sin 9x + \cdots \tag{9.1}$$

In figure 9.2 we take the first four terms only to provide the approximation. The more terms of the series we take, the closer the sum will approach the original function.

When we deal with a *discrete* function, as we shall do for images, the situation changes slightly. Since we only have to obtain a finite number of values, we only need a finite number of functions to do it.

Consider for example the discrete sequence

$$1,\quad 1,\quad 1,\quad 1,\quad -1,\quad -1,\quad -1,\quad -1$$

which we may take as a discrete approximation to the square wave of figure 9.2. This can be expressed as the sum of only *two* sine functions; this is shown in figure 9.3. We shall see below how to obtain those sequences.

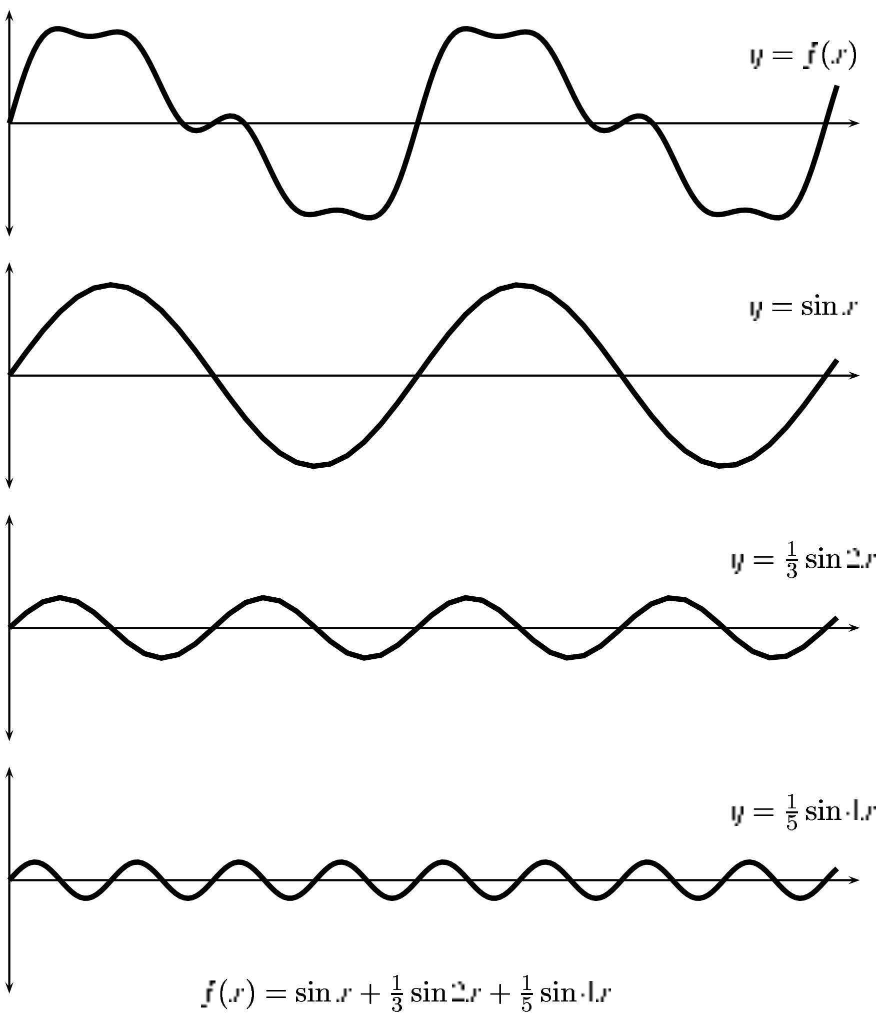


Figure 9.1: A function and its trigonometric decomposition

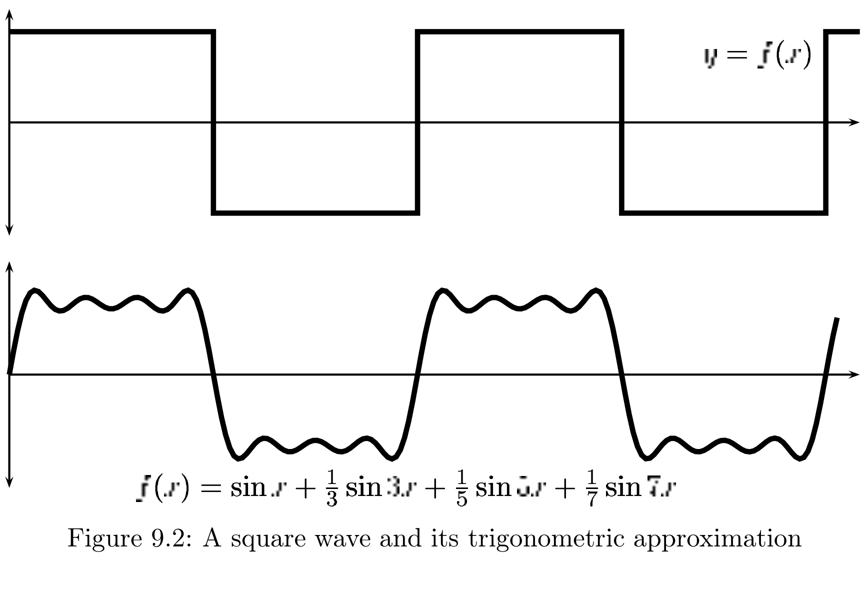


Figure 9.2: A square wave and its trigonometric approximation

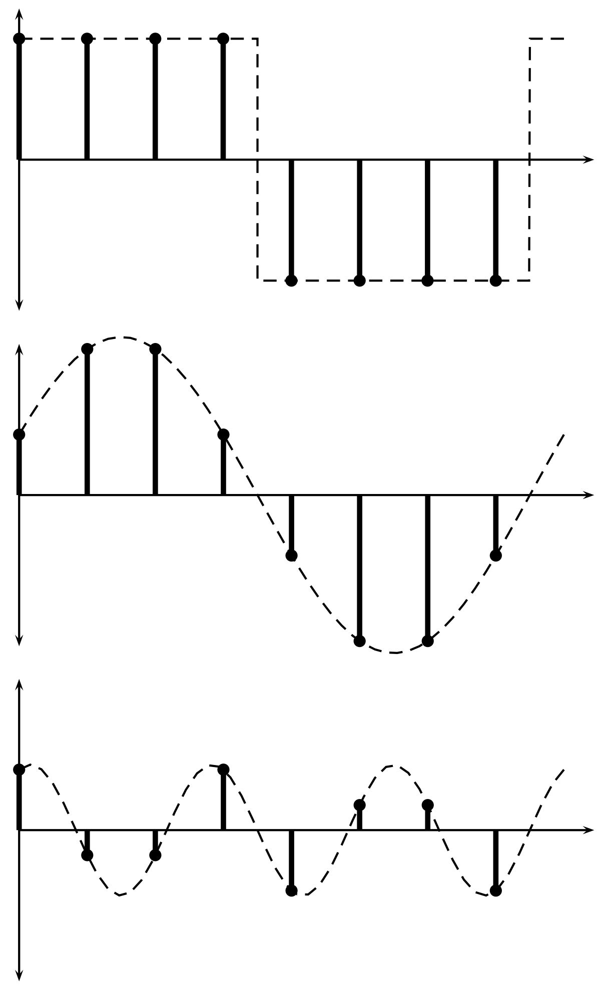


Figure 9.3: Expressing a discrete function as the sum of sines

The Fourier transform allows us to obtain those individual sine waves which compose a given function or sequence. Since we shall bne concerend with discrete sequences, and of course images, we shall investigate only the *discrete Fourier transform*, abbreviated DFT.

9.2.1 Definition of the one dimensional DFT

Suppose

$$\mathbf{f} = [f_0, f_1, f_2, \ldots, f_{N-1}]$$

is a sequence of length N. We define its *discrete Fourier transform* to be the sequence

$$\mathbf{F} = [F_0, F_1, F_2, \ldots, F_{N-1}]$$

where

$$F_u = \frac{1}{N}\sum_{x=0}^{N-1} \exp\left[-2\pi i \frac{xu}{N}\right] f_x. \tag{9.2}$$

The inverse DFT

The formula for the inverse DFT is very similar to the forward transform:

$$x_u = \sum_{x=0}^{N-1} \exp\left[2\pi i \frac{xu}{N}\right] F_u. \tag{9.3}$$

If you compare this equation with equation 9.2 you will see that there are really only two differences:

1. there is no scaling factor $1/N$,
2. the sign inside the exponential function has been changed to positive.

In MATLAB, we can calculate the forward and inverse transforms with `fft` and `ifft`. Here `fft` stands for *Fast Fourier Transform*, which is a fast and efficient method of performing the DFT (see below for details). For example:

```
a =

     1     2     3     4     5     6

>> fft(a')

ans =

  21.0000
  -3.0000 + 5.1962i
  -3.0000 + 1.7321i
  -3.0000
  -3.0000 - 1.7321i
  -3.0000 - 5.1962i
```

We note that to apply a DFT to a single vector in MATLAB, we should use a column vector.

9.3 Properties of the one-dimensional DFT

The one-dimensional DFT satisfies many useful and important properties. We will investigate some of them here. A more complete list can be found in, for example [9].

Linearity. This is a direct consequence of the definition of the DFT as a matrix product. Suppose $\mathtt{f}$ and $\mathtt{g}$ are two vectors of equal length, and μ and ν are scalars, with $\mathtt{h} = \mu\mathtt{f} + \nu\mathtt{g}$. If $\mathtt{F}$, $\mathtt{G}$ and $\mathtt{H}$ are the DFT's of $\mathtt{f}$, $\mathtt{g}$ and $\mathtt{h}$ respectively, we have

$$\mathtt{H} = \mu\mathtt{F} + \nu\mathtt{G}.$$

This follows from the definitions of

$$\mathtt{F} = \mathcal{F}\mathtt{f}, \quad \mathtt{G} = \mathcal{F}\mathtt{g}, \quad \mathtt{H} = \mathcal{F}\mathtt{h}$$

and the linearity of the matrix product.

Shifting. Suppose we multiply each element x_n of a vector $\mathtt{x}$ by $(-1)^n$. In other words, we change the sign of every second element. Let the resulting vector be denoted $\mathtt{x}'$. The the DFT $\mathtt{X}'$ of $\mathtt{x}'$ is equal to the DFT $\mathtt{X}$ of $\mathtt{x}$ with the swapping of the left and right halves.

Let's do a quick MATLAB example:

```
>> x = [2 3 4 5 6 7 8 1];

>> x1=(-1).^[0:7].*x

x1 =

     2    -3     4    -5     6    -7     8    -1

>> X=fft(x')

  36.0000
  -9.6569 + 4.0000i
  -4.0000 - 4.0000i
   1.6569 - 4.0000i
   4.0000
   1.6569 + 4.0000i
  -4.0000 + 4.0000i
  -9.6569 - 4.0000i

>> X1=fft(x1')

X1 =

   4.0000
   1.6569 + 4.0000i
  -4.0000 + 4.0000i
  -9.6569 - 4.0000i
  36.0000
  -9.6569 + 4.0000i
  -4.0000 - 4.0000i
   1.6569 - 4.0000i
```

Notice that the first four elements of `X` are the last four elements of `X1`, and vice versa.

Conjugate symmetry If `x` is real, and of length N, then its DFT `X` satisfies the condition that

$$X_k = \overline{X_{N-k}},$$

where $\overline{X_{N-k}}$ is the complex conjugate of X_{N-k}, for all $k = 1, 2, 3, \dots, N-1$. So in our example of length 8, we have

$$X_1 = \overline{X_7}, \quad X_2 = \overline{X_6}, \quad X_3 = \overline{X_5}.$$

In this case we also have $X_4 = \overline{X_4}$, which means X_4 must be real. In fact if N is even, then $X_{N/2}$ will be real. Examples can be seen above.

The Fast Fourier Transform. Without an efficient method of calculating a DFT, it would remain only of academic interest, and of no practical use. However, there are a number of extremely fast and efficient algorithms for computing a DFT; such an algorithm is called a *fast Fourier transform*, or FFT. The use of an FFT vastly reduces the time needed to compute a DFT.

One FFT method works recursively by dividing the original vector into two halves, computing the FFT of each half, and then putting the results together. This means that the FFT is most efficient when the vector length is a power of 2.

Table 9.1 shows that advantage gained by using the FFT algorithm as opposed to the direct arithmetic definition of equations 9.6 and 9.7 by comparing the number of multiplications required for each method. For a vector of length 2^n, the direct method takes $(2^n)^2 = 2^{2n}$ multiplications; the FFT only $n2^n$. The saving in time is thus of an order of $2^n/n$. Clearly the advantage of using an FFT algorithm becomes greater as the size of the vector increases.

Because of the this computational advantage, any implementation of the DFT will use an FFT algorithm.

2^n	Direct arithmetic	FFT	Increase in speed
4	16	8	2.0
8	84	24	2.67
16	256	64	4.0
32	1024	160	6.4
64	4096	384	10.67
128	16384	896	18.3
256	65536	2048	32.0
512	262144	4608	56.9
1024	1048576	10240	102.4

Table 9.1: Comparison of FFT and direct arithmetic

9.4 The two-dimensional DFT

In two dimensions, the DFT takes a matrix as input, and returns another matrix, of the same size, as output. If the original matrix values are $f(x, y)$, where x and y are the indices, then the output matrix values are $F(u, v)$. We call the matrix F the *Fourier transform of* f and write

$$F = \mathcal{F}(f).$$

Then the original matrix f is the *inverse Fourier transform of* F, and we write

$$f = \mathcal{F}^{-1}(F).$$

We have seen that a (one-dimensional) function can be written as a sum of sines and cosines. Given that an image may be considered as a two-dimensional function $f(x, y)$, it seems reasonable to assume that f can be expressed as sums of "corrugation" functions which have the general form

$$z = a \sin(bx + cy).$$

A sample such function is shown in figure 9.4. And this is in fact exactly what the two-dimensional

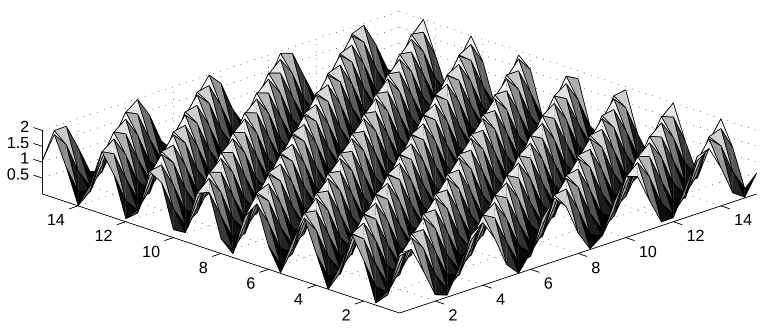


Figure 9.4: A "corrugation" function

Fourier transform does: it rewrites the original matrix in terms of sums of corrugations. We can be more precise here: the magnitude of the fourier transform is a matrix (the same size as the starting matrix) whose elements f_{ij} give the height of the corrugation for which the horizontal and vertical distances between consecutive troughs are $1/i$ and $1/j$. This is shown in figure 9.5.

The definition of the two-dimensional discrete Fourier transform is very similar to that for one dimension. The forward and inverse transforms for an $M \times N$ matrix, where for notational convenience we assume that the x indices are from 0 to $M-1$ and the y indices are from 0 to $N-1$, are:

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp\left[-2\pi i \left(\frac{xu}{M} + \frac{yv}{N}\right)\right]. \tag{9.4}$$

$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp\left[2\pi i \left(\frac{xu}{M} + \frac{yv}{N}\right)\right]. \tag{9.5}$$

These are horrendous looking formulas, but if we spend a bit of time pulling them apart, we shall see that they aren't as bad as they look.

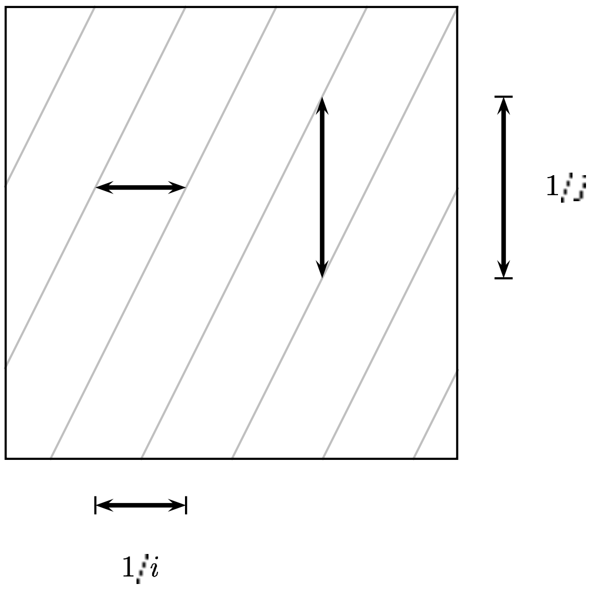


Figure 9.5: Corrugations and their measurements

Before we do this, we note that the formulas given in equations 9.4 and 9.5 are not used by all authors. The main change is the position of the scaling factor $1/MN$. Some people put it in front of the sums in the forward formula. Others put a factor of $1/\sqrt{MN}$ in front of both sums. The point is the sums by themselves would produce a result (after both forward and inverse transforms) which is too large by a factor of MN. So somewhere in the forward-inverse formulas a corresponding $1/MN$ must exist; it doesn't really matter where.

9.4.1 Some properties of the two dimensional Fourier transform

All the properties of the one-dimensional DFT transfer into two dimensions. But there are some further properties not previously mentioned, which are of particular use for image processing.

Similarity. First notice that the forward and inverse transforms are very similar, with the exception of the scale factor $1/MN$ in the inverse transform, and the negative sign in the exponent of the forward transform. This means that the same algorithm, only very slightly adjusted, can be used for both the forward an inverse transforms.

The DFT as a spatial filter. Note that the values

$$\exp\left[\pm 2\pi i\left(\frac{xu}{M}+\frac{yv}{N}\right)\right]$$

are independent of the values f or F. This means that they can be calculated in advance, and only then put into the formulas above. It also means that every value $F(u,v)$ is obtained by multiplying every value of $f(x,y)$ by a fixed value, and adding up all the results. But this is precisely what a linear spatial filter does: it multiplies all elements under a mask with fixed values, and adds them all up. Thus we can consider the DFT as a linear spatial filter which is as big as the image. To deal with the problem of edges, we assume that the image is tiled in all directions, so that the mask always has image values to use.

Separability. Notice that the Fourier transform "filter elements" can be expressed as products:

$$\exp\left[2\pi i\left(\frac{xu}{M}+\frac{yv}{N}\right)\right]=\exp\left[2\pi i\frac{xu}{M}\right]\exp\left[2\pi i\frac{yv}{N}\right].$$

The first product value

$$\exp\left[2\pi i\frac{xu}{M}\right]$$

depends only on x and u, and is independent of y and v. Conversely, the second product value

$$\exp\left[2\pi i\frac{yv}{N}\right]$$

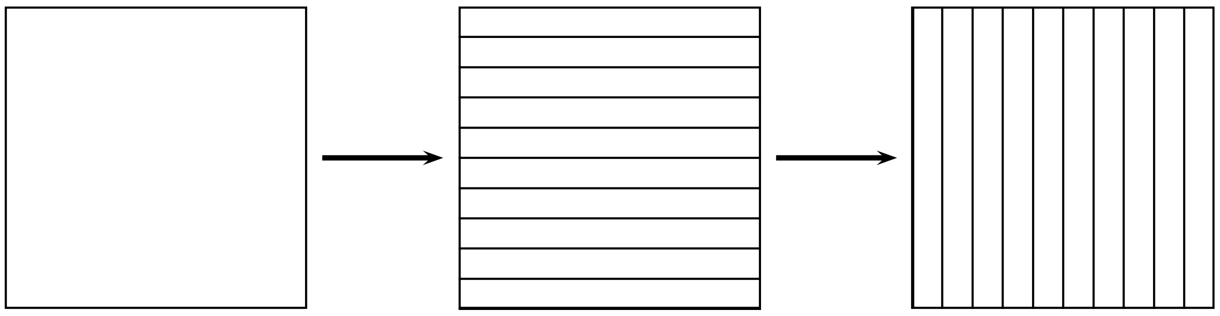
depends only on y and v, and is independent of x and u. This means that we can break down our formulas above to simpler formulas that work on single rows or columns:

$$F(u) = \sum_{x=0}^{M-1} f(x)\exp\left[-2\pi i\frac{xu}{M}\right], \tag{9.6}$$

$$f(x) = \frac{1}{M}\sum_{u=0}^{M-1} F(u)\exp\left[2\pi i\frac{xu}{M}\right]. \tag{9.7}$$

If we replace x and u with y and v we obtain the corresponding formulas for the DFT of matrix columns. These formulas define the *one-dimensional DFT* of a vector, or simply the DFT.

The 2-D DFT can be calculated by using this property of "separability"; to obtain the 2-D DFT of a matrix, we first calculate the DFT of all the rows, and then calculate the DFT of all the columns of the result, as shown in figure 9.6. Since a product is independent of the order, we can equally well calculate a 2-D DFT by calculating the DFT of all the columns first, then calculating the DFT of all the rows of the result.



(a) Original image (b) DFT of each row of (a) (c) DFT of each column of (b)

Figure 9.6: Calculating a 2D DFT

Linearity An important property of the DFT is its linearity; the DFT of a sum is equal to the sum of the individual DFT's, and the same goes for scalar multiplication:

$$\begin{aligned}\mathcal{F}(f+g) &= \mathcal{F}(f)+\mathcal{F}(g)\\ \mathcal{F}(kf) &= k\mathcal{F}(f)\end{aligned}$$

where k is a scalar, and f and g are matrices. This follows directly from the definition given in equation 9.4.

This property is of great use in dealing with image degradation such as noise which can be modelled as a sum:

$$d = f + n$$

where f is the original image; n is the noise, and d is the degraded image. Since

$$\mathcal{F}(d) = \mathcal{F}(f) + \mathcal{F}(n)$$

we may be able to remove or reduce n by modifying the transform. As we shall see, some noise appears on the DFT in a way which makes it particularly easy to remove.

The convolution theorem. This result provides one of the most powerful advantages of using the DFT. Suppose we wish to convolve an image M with a spatial filter S. Our method has been place S over each pixel of M in turn, calculate the product of all corresponding grey values of M and elements of S, and add the results. The result is called the *digital convolution* of M and S, and is denoted

$$M * S.$$

This method of convolution can be very slow, especially if S is large. The *convolution theorem* states that the result $M * S$ can be obtained by the following sequence of steps:

1. Pad S with zeroes so that is the same size as M; denote this padded result by S'.
2. Form the DFT's of both M and S, to obtain $\mathcal{F}(M)$ and $\mathcal{F}(S')$.
3. Form the element-by-element product of these two transforms:

$$\mathcal{F}(M)\cdot\mathcal{F}(S').$$

4. Take the inverse transform of the result:

$$\mathcal{F}^{-1}(\mathcal{F}(M)\cdot\mathcal{F}(S')).$$

Put simply, the convolution theorem states:

$$M * S = \mathcal{F}^{-1}(\mathcal{F}(M)\cdot\mathcal{F}(S'))$$

or equivalently that

$$\mathcal{F}(M * S) = \mathcal{F}(M)\cdot\mathcal{F}(S').$$

Although this might seem like an unnecessarily clumsy and roundabout way of computing something so simple as a convolution, it can have enormous speed advantages if S is large.

For example, suppose we wish to convolve a 512×512 image with a 32×32 filter. To do this directly would require $32^2 = 1024$ multiplications for each pixel, of which there are $512 \times 512 = 262144$. Thus there will be a total of $1024 \times 262144 = 268,435,456$ multiplications needed. Now look at applying the DFT (using an FFT algorithm). Each row requires 4608 multiplications by table 9.1; there are 512 rows, so a total of $4608 \times 512 = 2359296$ multiplications; the same must be done again for the columns. Thus to obtain the DFT of the image requires 4718592 multiplications. We need the same amount to obtain the DFT of the filter, and for the inverse DFT. We also require 512×512 multiplications to perform the product of the two transforms.

Thus the total number of multiplications needed to perform convolution using the DFT is

$$4718592 \times 3 + 262144 = 14,417,920$$

which is an enormous saving compared to the direct method.

The DC coefficient. The value $F(0,0)$ of the DFT is called the *DC coefficient.* If we put $u = v = 0$ in the definition given in equation 9.4 we find that

$$F(0,0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp(0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y).$$

That is, this term is equal to the sum of *all* terms in the original matrix.

Shifting. For purposes of display, it is convenient to have the DC coefficient in the centre of the matrix. This will happen if all elements $f(x,y)$ in the matrix are multiplied by $(-1)^{x+y}$ before the transform. Figure 9.7 demonstrates how the matrix is shifted by this method. In each diagram the DC coefficient is the top left hand element of submatrix A, and is shown as a black square.

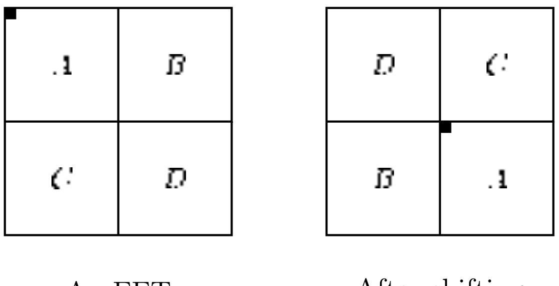


Figure 9.7: Shifting a DFT

Conjugate symmetry An analysis of the Fourier transform definition leads to a symmetry property; if we make the substitutions $u = -u$ and $v = -v$ in equation 9.4 then we find that

$$\mathcal{F}(u,v) = \mathcal{F}^*(-u + pM, -v + qN)$$

for any integers p and q. This means that half of the transform is a mirror image of the conjugate of the other half. We can think of the top and bottom halves, or the left and right halves, being mirror images of the conjugates of each other.

Figure 9.8 demonstrates this symmetry in a shifted DFT. As with figure 9.7, the black square shows the position of the DC coefficient. The symmetry means that its information is given in just half of a transform, and the other half is redundant.

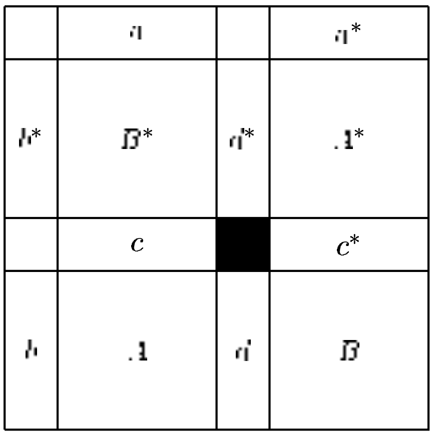


Figure 9.8: Conjugate symmetry in the DFT

Displaying transforms. Having obtained the Fourier transform $F(u,v)$ of an image $f(x,y)$, we would like to see what it looks like. As the elements $F(u,v)$ are complex numbers, we can't view them directly, but we can view their magnitude $|F(u,v)|$. Since these will be numbers of type `double`, generally with large range, we have two approaches

1. find the maximum value m of $|F(u,v)|$ (this will be the DC coefficient), and use `imshow` to view $|F(u,v)|/m$,
2. use `mat2gray` to view $|F(u,v)|$ directly.

One trouble is that the DC coefficient is generally very much larger than all other values. This has the effect of showing a transform as a single white dot surrounded by black. One way of stretching out the values is to take the logarithm of $|F(u,v)|$ and to display

$$\log(1+|F(u,v)|).$$

The display of the magnitude of a Fourier transform is called the *spectrum* of the transform. We shall see some examples later on.

9.5 Fourier transforms in Matlab

The relevant Matlab functions for us are:

- `fft` which takes the DFT of a vector,
- `ifft` which takes the inverse DFT of a vector,
- `fft2` which takes the DFT of a matrix,
- `ifft2` which takes the inverse DFT of a matrix,
- `fftshift` which shifts a transform as shown in figure 9.7.

of which we have seen the first two above.

Before attacking a few images, let's take the Fourier transform of a few small matrices to get more of an idea what the DFT "does".

Example 1. Suppose we take a constant matrix $f(x, y) = 1$. Going back to the idea of a sum of corrugations, we see that in fact *no* corrugations are required to form a constant. Thus we would hope that the DFT consists of a DC coefficient and zeroes everywhere else. We will use the `ones` function, which produces an $n \times n$ matrix consisting of 1's, where n is an input to the function.

```
>> a=ones(8);
>> fft2(a)
```

The result is indeed as we expected:

```
ans =
      64     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
```

Note that the DC coefficient is indeed the sum of all the matrix values.

Example 2. Now we'll take a matrix consisting of a single corrugation:

```
>> a = [100 200; 100 200];
>> a = repmat(a,4,4)

ans =
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200
     100   200   100   200   100   200   100   200

>> af = fft2(a)

ans =

    9600     0     0     0 -3200     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
       0     0     0     0     0     0     0     0
```

What we have here is really the sum of two matrices: a constant matrix each element of which is 150, and a corrugation which alternates -50 and 50 from left to right. The constant matrix alone would produce (as in example 1), a DC coefficient alone of value $64 \times 150 = 9600$; the corrugation a single value. By linearity, the DFT will consist of just the two values.

Example 3. We will take here a single step edge:

```
>> a = [zeros(8,4) ones(8,4)]
  a =

        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
        0     0     0     0     1     1     1     1
```

Now we shall perform the Fourier transform with a shift, to place the DC coefficient in the centre, and since it contains some complex values, for simplicity we shall just show the rounded absolute values:

```
>> af=fftshift(fft2(a));
>> round(abs(af))

ans =

        0     0     0     0     0     0     0     0
        0     0     0     0     0     0     0     0
        0     0     0     0     0     0     0     0
        0     0     0     0     0     0     0     0
        0     9     0    21    32    21     0     9
        0     0     0     0     0     0     0     0
        0     0     0     0     0     0     0     0
        0     0     0     0     0     0     0     0
```

The DC coefficient is of course the sum of all values of `a`; the other values may be considered to be the coefficients of the necessary sine functions required to from an edge, as given in equation 9.1. The mirroring of values about the DC coefficient is a consequence of the symmetry of the DFT.

9.6 Fourier transforms of images

We shall create a few simple images, and see what the Fourier transform produces.

Example 1. We shall produce a simple image consisting of a single edge:

```
>> a=[zeros(256,128) ones(256,128)];
```

This is displayed on the left in figure 9.10. Now we shall take its DFT, and shift it:

```
>> af=fftshift(fft2(a));
```

Now we'll view its spectrum; we have the choice of two commands:

1. `afl=log(1+abs(af));`

   `imshow(afl/afl(129,129))`

   This works because after shifting, the DC coefficient is at position $x = 129$, $y = 129$. We stretch the transform using `log`, and divide the result by the middle value to obtain matrix of type `double` with values in the range 0.0–1.0. This can then be viewed directly with `imshow`.

2. `imshow(mat2gray(log(1+abs(af))))`

   The `mat2gray` function automatically scales a matrix for display as an image, as we have seen in chapter 6

It is in fact convenient to write a small function for viewing transforms. One such is shown in figure 9.9. Then for example

```
>> fftshow(af,'log')
```

will show the logarithm of the absolute values of the transform, and

```
>> fftshow(af,'abs')
```

will show the absolute values of the transform without any scaling.

The result is shown on the right in figure 9.10. We see immediately that the result is similar (although larger) to example 3 in the previous section.

Example 2. Now we'll create a box, and then its Fourier transform:

```
>> a=zeros(256,256);
>> a(78:178,78:178)=1;
>> imshow(a)
>> af=fftshift(fft2(a));
>> figure,fftshow(af,'abs')
```

The box is shown on the left in figure 9.11, and its Fourier transform is is shown on the right.

Example 3. Now we shall look at a box rotated 45°.

```
>> [x,y]=meshgrid(1:256,1:256);
>> b=(x+y<329)&(x+y>182)&(x-y>-67)&(x-y<73);
>> imshow(b)
>> bf=fftshift(fft2(b));
>> figure,fftshow(bf)
```

The results are shown in figure 9.12. Note that the transform of the rotated box is the rotated transform of the original box.

```
function fftshow(f,type)

% Usage:  FFTSHOW(F,TYPE)
%
% Displays the fft matrix F using imshow, where TYPE must be one of
% 'abs' or 'log'.  If TYPE='abs', then then abs(f) is displayed; if
% TYPE='log' then log(1+abs(f)) is displayed.  If TYPE is omitted, then
% 'log' is chosen as a default.
%
%  Example:
%    c=imread('cameraman.tif');
%    cf=fftshift(fft2(c));
%    fftshow(cf,'abs')
%

if nargin<2,
  type='log';
end

if (type=='log')
  fl = log(1+abs(f));
  fm = max(fl(:));
  imshow(im2uint8(fl/fm))
elseif (type=='abs')
  fa=abs(f);
  fm=max(fa(:));
  imshow(fa/fm)
else
  error('TYPE must be abs or log.');
end;
```

Figure 9.9: A function to display a Fourier transform

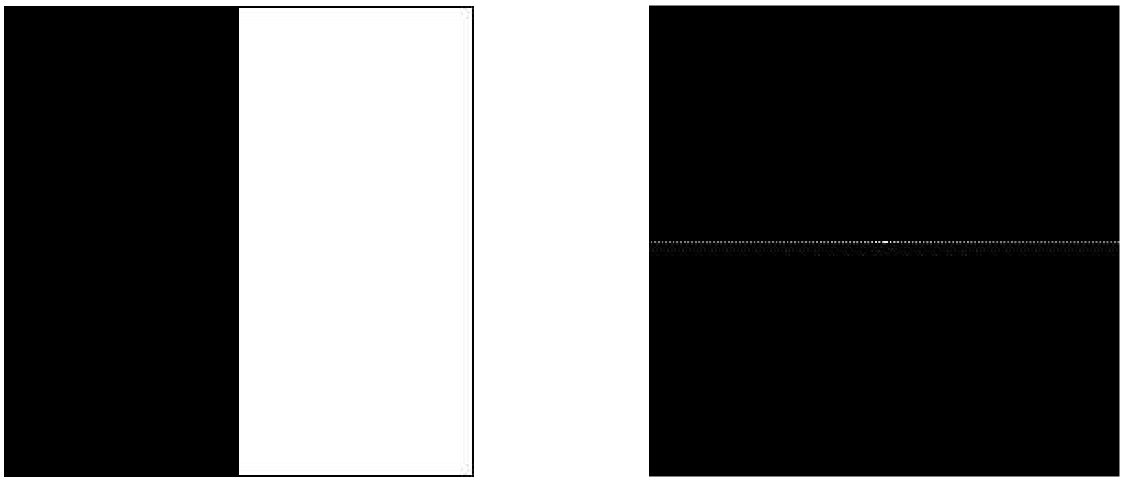


Figure 9.10: A single edge and its DFT

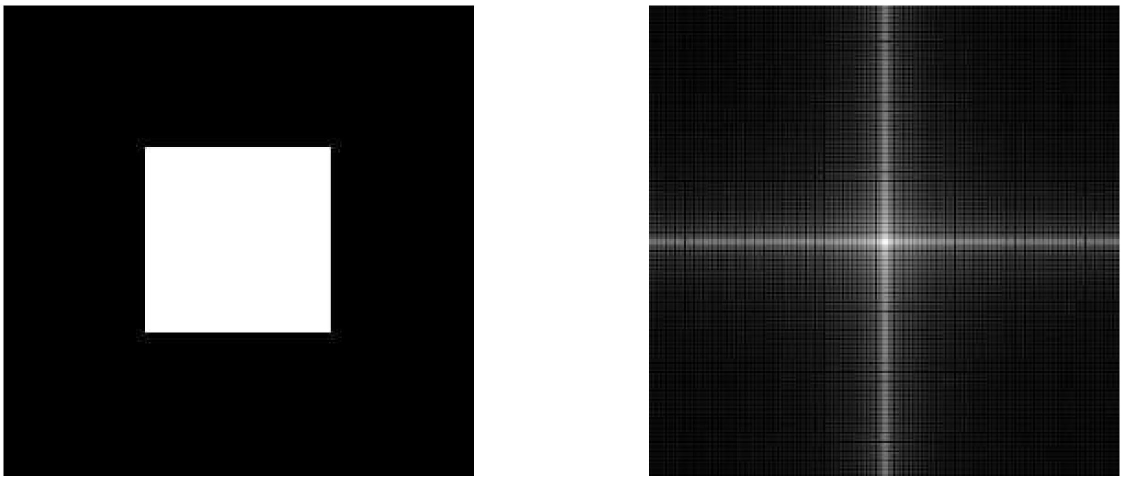


Figure 9.11: A box and its DFT

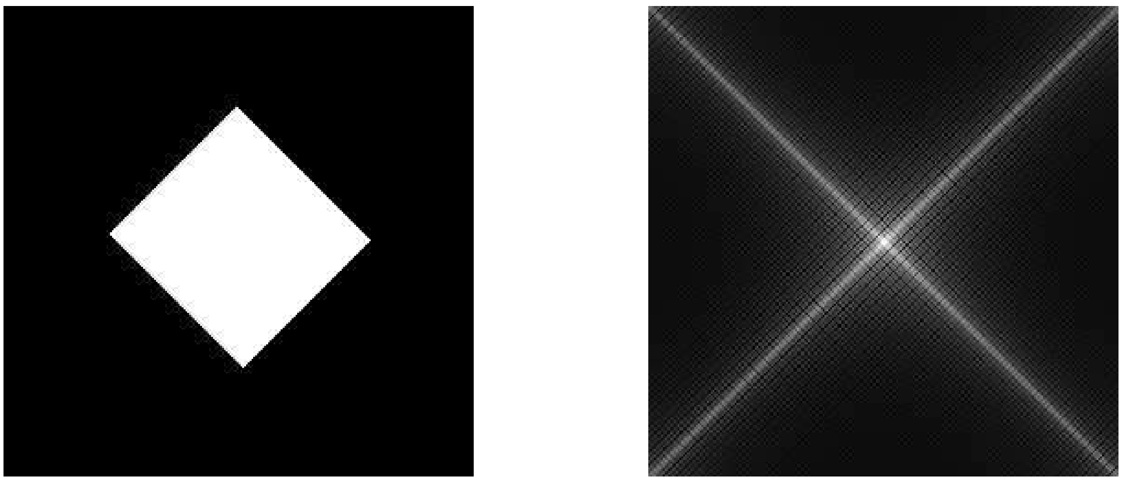


Figure 9.12: A rotated box and its DFT

Example 4. We will create a small circle, and then transform it:

```
>> [x,y]=meshgrid(-128:217,-128:127);
>> z=sqrt(x.^2+y.^2);
   >> c=(z<15);
```

The result is shown on the left in figure 9.13. Now we will create its Fourier transform and display it:

```
>> cf=fft2shift(fft2(z));
>> fftshow(cf,'log')
```

and this is shown on the right in figure 9.13. Note the "ringing" in the Fourier transform. This is an artifact associated with the sharp cutoff of the circle. As we have seen from both the edge and box images in the previous examples, an edge appears in the transform as a line of values at right angles to the edge. We may consider the values on the line as being the coefficients of the appropriate corrugation functions which sum to the edge. With the circle, we have lines of values radiating out from the circle; these values appear as circles in the transform.

A circle with a gentle cutoff, so that its edge appears blurred, will have a transform with no ringing. Such a circle can be made with the command (given `z` above):

```
b=1./(1+(z./15).^2);
```

This image appears as a blurred circle, and its transform is very similar—check them out!

9.7 Filtering in the frequency domain

We have seen in section 9.4 that one of the reasons for the use of the Fourier transform in image processing is due to the convolution theorem: a spatial convolution can be performed by element-wise multiplication of the Fourier transform by a suitable "filter matrix". In this section we shall explore some filtering by this method.

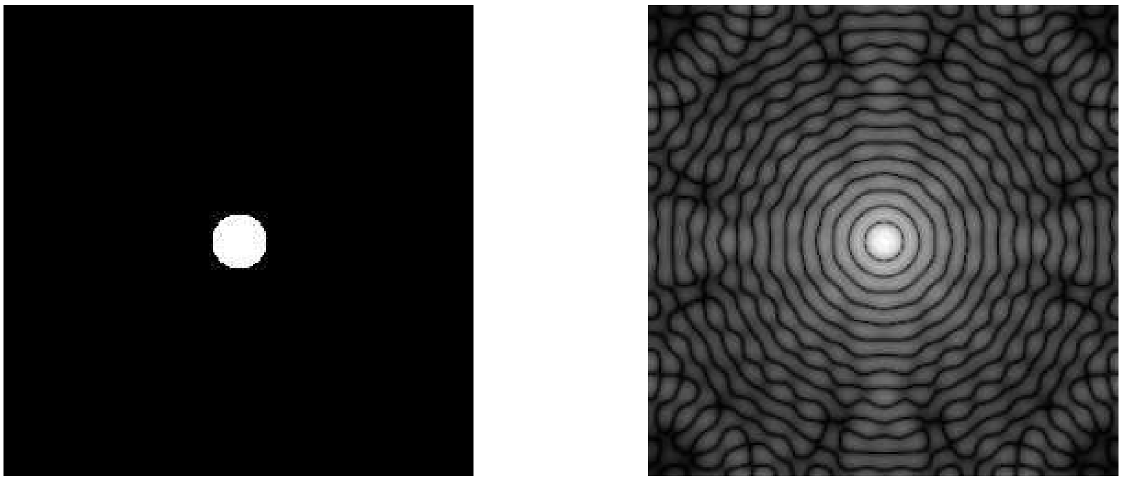


Figure 9.13: A circle and its DFT

9.7.1 Ideal filtering

Low pass filtering

Suppose we have a Fourier transform matrix $\boldsymbol{F}$, shifted so that the DC coefficient is in the centre. Since the low frequency components are towards the centre, we can perform low pass filtering by multiplying the transform by a matrix in such a way that centre values are maintained, and values away from the centre are either removed or minimized. One way to do this is to multiply by an *ideal low-pass matrix*, which is a binary matrix m defined by:

$$m(x,y) = \begin{cases} 1 & \text{if } (x,y) \text{ is closer to the center than some value } D, \\ 0 & \text{if } (x,y) \text{ is further from the center than } D. \end{cases}$$

The circle `c` displayed in figure 9.13 is just such a matrix, with $D = 15$. Then the inverse Fourier transform of the element-wise product of $\boldsymbol{F}$ and m is the result we require:

$$\mathcal{F}^{-1}(\boldsymbol{F} \cdot m).$$

Let's see what happens if we apply this filter to an image. First we obtain an image and its DFT.

```
>> cm=imread('cameraman.tif');
>> cf=fftshift(fft2(cm));
>> figure,fftshow(cf,'log')
```

The cameraman image and its DFT are shown in figure 9.14. Now we can perform a low pass filter by multiplying the transform matrix by the circle matrix (recall that "dot asterisk" is the MATLAB syntax for element-wise multiplication of two matrices):

```
>> cfl=cf.*c;
>> figure,fftshow(cfl,'log')
```

and this is shown in figure 9.15(a). Now we can take the inverse transform and display the result:

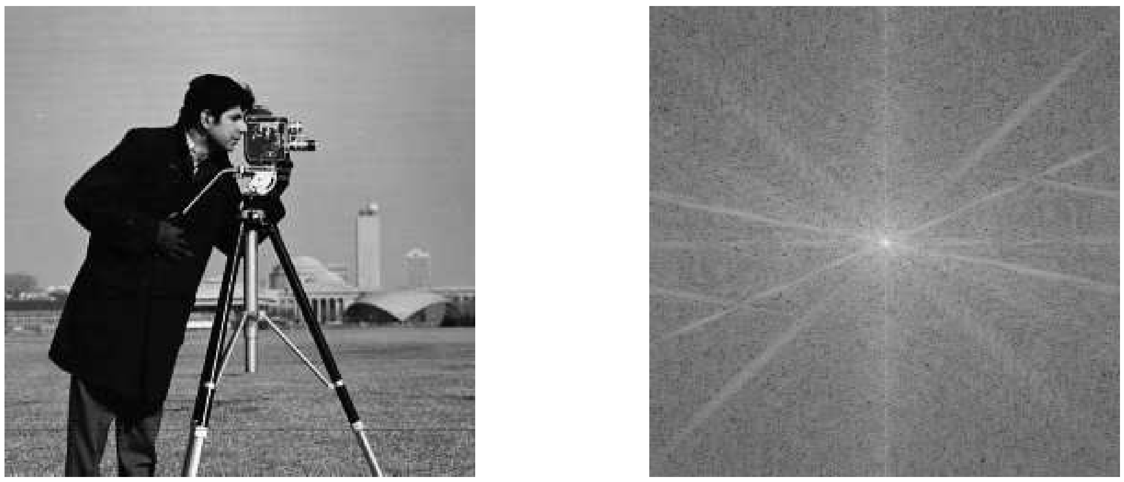
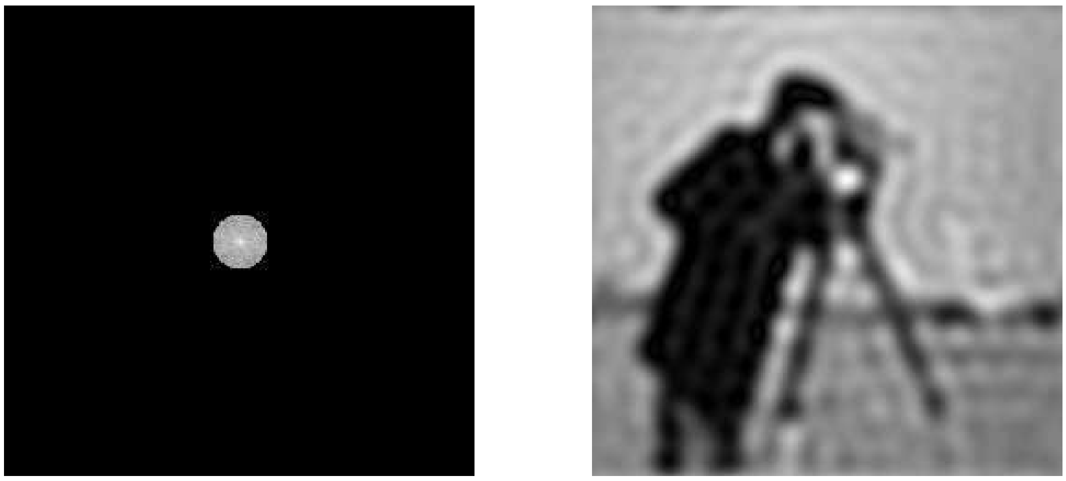


Figure 9.14: The "cameraman" image and its DFT

```
>> cfli=ifft2(cfl);
>> figure,fftshow(cfli,'abs')
```

and this is shown in figure 9.15(b). Note that even though `cfli` is supposedly a matrix of real numbers, we are still using `fftshow` to display it. This is because the `fft2` and `fft2` functions, being numeric, will not produce mathematically perfect results, but rather very close numeric approximations. So using `fftshow` with the `'abs'` option rounds out any errors obtained during the transform and its inverse. Note the "ringing" about the edges in this image. This is a direct result



(a) Ideal filtering on the DFT (b) After inversion

Figure 9.15: Applying ideal low pass filtering

of the sharp cutoff of the circle. The ringing as shown in figure 9.13 is transferred to the image.

We would expect that the smaller the circle, the more blurred the image, and the larger the circle; the less blurred. Figure 9.16 demonstrates this, using cutoffs of 5 and 30. Notice that ringing

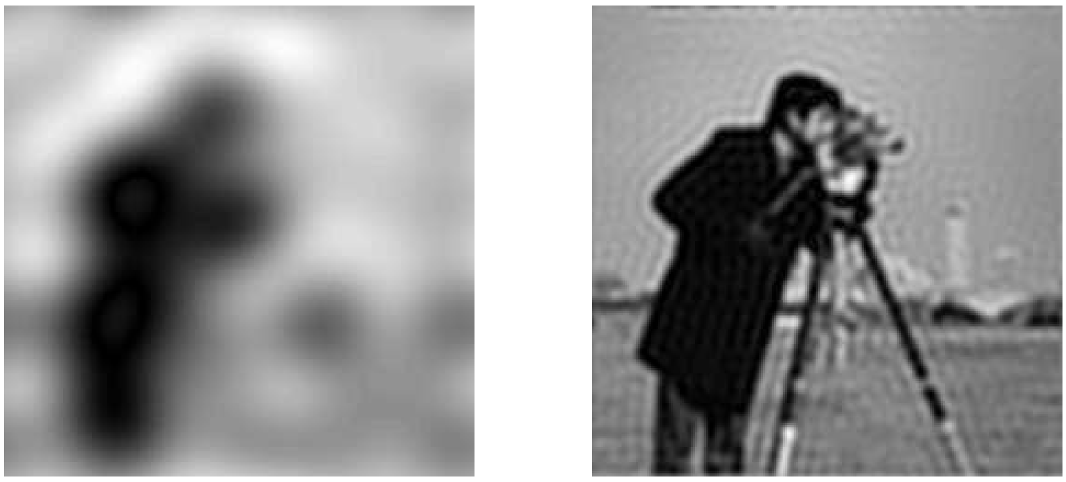

(a) Cutoff of 5 (b) Cutoff of 30

Figure 9.16: Ideal low pass filtering with different cutoffs

is still present, and clearly visible in figure 9.16(b).

High pass filtering

Just as we can perform low pass filtering by keeping the centre values of the DFT and eliminating the others, so high pass filtering can be performed by the opposite: eliminating centre values and keeping the others. This can be done with a minor modification of the preceding method of low pass filtering. First we create the circle:

```
>> [x,y]=meshgrid(-128:127,-128:127);
>> z=sqrt(x.^2+y.^2);
>> c=(z>15);
```

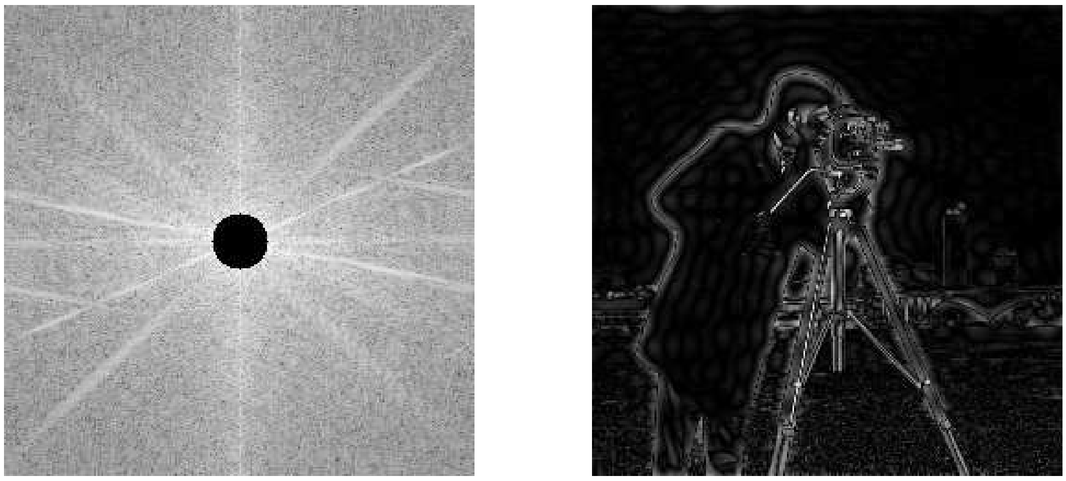
and the multiply it by the DFT of the image:

```
>> cfh=cf.*c;
>> figure,fftshow(cfh,'log')
```

This is shown in figure 9.17(a). The inverse DFT can be easily produced and displayed:

```
>> cfhi=ifft2(cfh);
>> figure,fftshow(cfhi,'abs')
```

and this is shown in figure 9.17(b). As with low pass filtering, the size of the circle influences the information available to the inverse DFT, and hence the final result. Figure 9.18 shows some results of ideal high pass filtering with different cutoffs. If the cutoff is large, then more information is removed from the transform, leaving only the highest frequencies. We see this in figure 9.18(c) and (d); only the edges of the image remain. If we have small cutoff, such as in figure 9.18(a), we



(a) The DFT after high pass filtering (b) The resulting image

Figure 9.17: Applying an ideal high pass filter to an image

are only removing a small amount of the transform. We would thus expect that only the lowest frequencies of the image would be removed. And this is indeed true, as we see in figure 9.18(b); there is some greyscale detail in the final image, but large areas of low frequency are close to zero.

9.7.2 Butterworth filtering

Ideal filtering simply cuts off the Fourier transform at some distance from the centre. This is very easy to implement, as we have seen, but has the disadvantage of introducing unwanted artifacts: ringing, into the result. One way of avoiding this is to use as a filter matrix a circle with a less sharp cutoff. A popular choice is to use *Butterworth filters.*

Before we describe these filters, we shall look again at the ideal filters. As these are radially symmetric about the centre of the transform, they can be simply described in terms of their cross sections. That is, we can describe the filter as a function of the distance x from the centre. For an ideal low pass filter, this function can be expressed as

$$f(x) = \begin{cases} 1 & \text{if } x < D, \\ 0 & \text{if } x \geq D \end{cases}$$

where D is the cutoff radius. Then the ideal high pass filters can be described similarly:

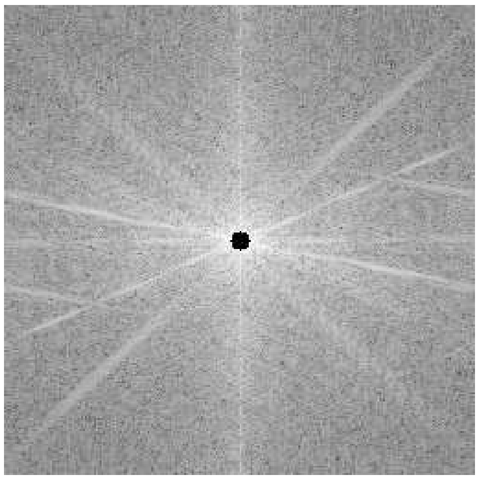
$$f(x) = \begin{cases} 1 & \text{if } x > D, \\ 0 & \text{if } x \leq D \end{cases}$$

These functions are illustrated in figure 9.19. Butterworth filter functions are based on the following functions for low pass filters:

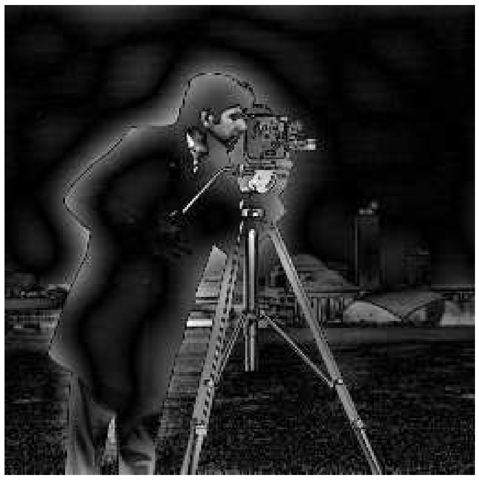
$$f(x) = \frac{1}{1 + (x/D)^{2n}}$$

and for high pass filters:

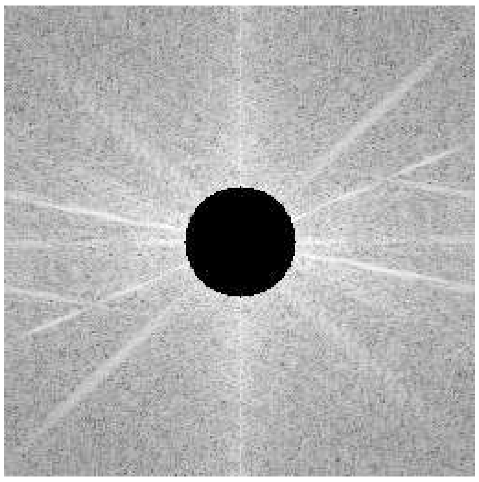
$$f(x) = \frac{1}{1 + (D/x)^{2n}}$$



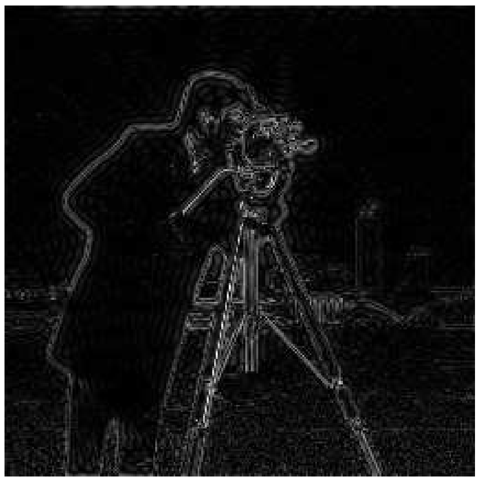
(a) Cutoff of 5



(b) The resulting image

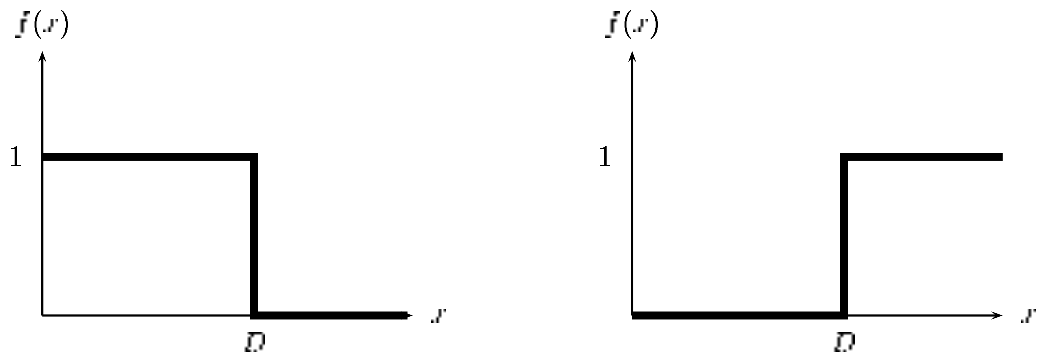


(a) Cutoff of 30



(b) The resulting image

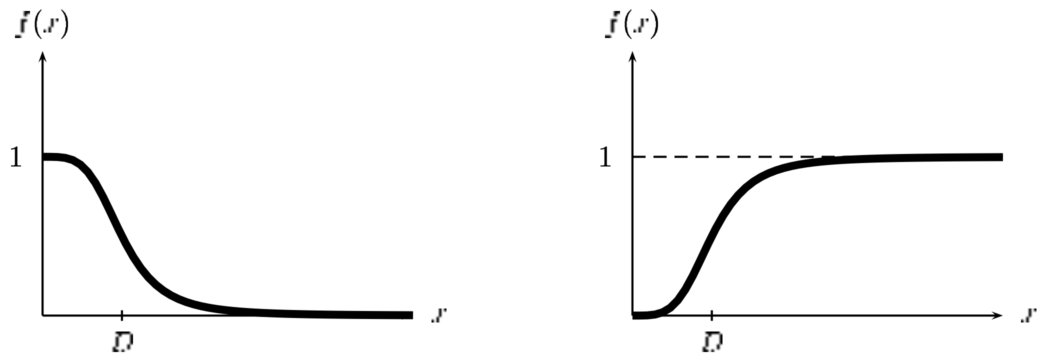
Figure 9.18: Ideal high pass filtering with different cutoffs



(a) Low pass (b) High pass

Figure 9.19: Ideal filter functions

where in each case the parameter n is called the *order* of the filter. The size of n dictates the sharpness of the cutoff. These functions are illustrated in figures 9.20 and 9.21.



(a) Low pass (b) High pass

Figure 9.20: Butterworth filter functions with $n = 2$

It is easy to implement these in MATLAB; here are the commands to produce a Butterworth low pass filter of size 256×256 with $D = 15$ and order $n = 2$:

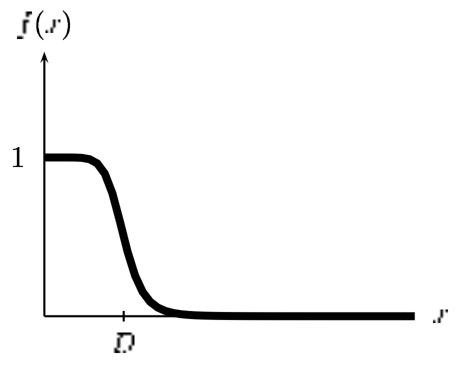
```
>> [x,y]=meshgrid(-128:217,-128:127));
>> bl=1./(1+((x.^2+y.^2)/15).^2);
```

Since a Butterworth high pass filter can be obtained by subtracting a low pass filter from 1, we can write general MATLAB functions to generate Butterworth filters of general sizes. These are shown in figures 9.22 and 9.23.

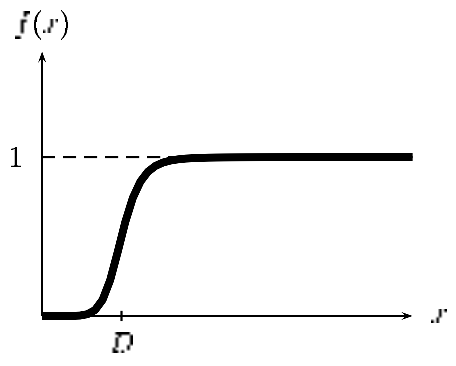
So to apply a Butterworth low pass filter to the DFT of the cameraman image:

```
>> bl=lbutter(c,15,1);
>> cfbl=cf.*bl;
>> figure,fftshow(cfbl,'log')
```

and this is shown in figure 9.24(a). Note that there is no sharp cutoff as seen in figure 9.15; also that



(a) Low pass



(b) High pass

Figure 9.21: Butterworth filter functions with $n = 4$

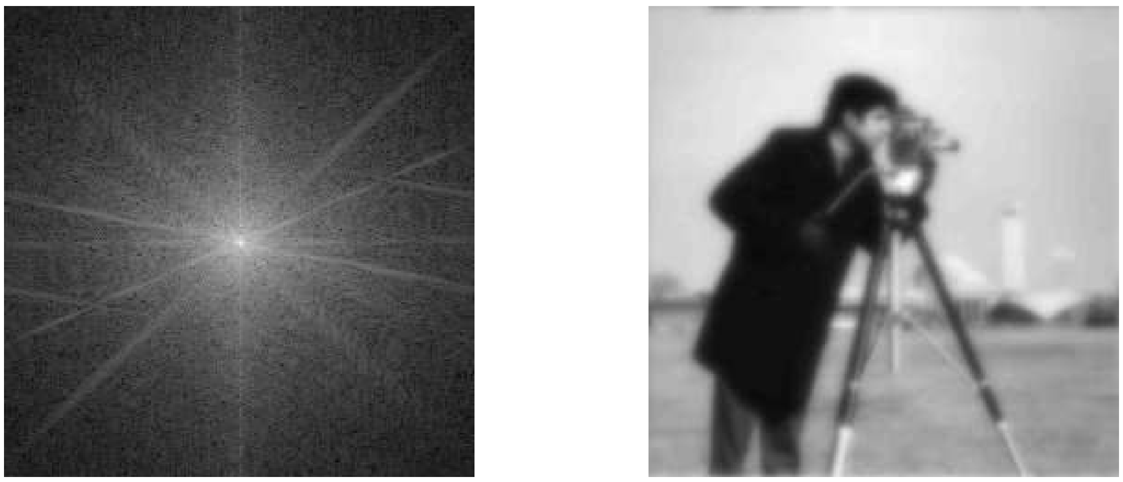
```
function out=lbutter(im,d,n)
% LBUTTER(IM,D,N) creates a low-pass Butterworth filter
% of the same size as image IM, with cutoff D, and order N
%
% Use:
%   x=imread('cameraman.tif');
%   l=lbutter(x,25,2);
%
height=size(im,1);
width=size(im,2);
[x,y]=meshgrid(-floor(width/2):floor((width-1)/2),-floor(height/2): ...
       floor((height-1)/2));
out=1./(1+(sqrt(2)-1)*((x.^2+y.^2)/d^2).^n);
```

Figure 9.22: A function to generate a low pass Butterworth filter

```
function out=hbutter(im,d,n)
% HBUTTER(IM,D,N) creates a high-pass Butterworth filter
% of the same size as image IM, with cutoff D, and order N
%
% Use:
%   x=imread('cameraman.tif');
%   l=hbutter(x,25,2);
%

out=1-lbutter(im,d,n);
```

Figure 9.23: A function to generate a high pass Butterworth filter

the outer parts of the transform are not equal to zero, although they are dimmed considerably. Performing the inverse transform and displaying it as we have done previously produces figure 9.24(b). This is certainly a blurred image, but the ringing seen in figure 9.15 is completely absent. Compare



(a) The DFT after Butterworth low pass filtering (b) The resulting image

Figure 9.24: Butterworth low pass filtering

the transform after multiplying with a Butterworth filter (figure 9.24(a)) with the original transform (in figure 9.14). We see that the Butterworth filter does cause an attenuation of values away from the centre, even if they don't become suddenly zero, as with the ideal low pass filter in figure 9.15.

We can apply a Butterworth high pass filter similarly, first by creating the filter and applying it to the image transform:

```
>> bh=hbutter(cm,15,1);
>> cfbh=cf.*bh;
>> figure,fftshow(cfbh,'log')
```

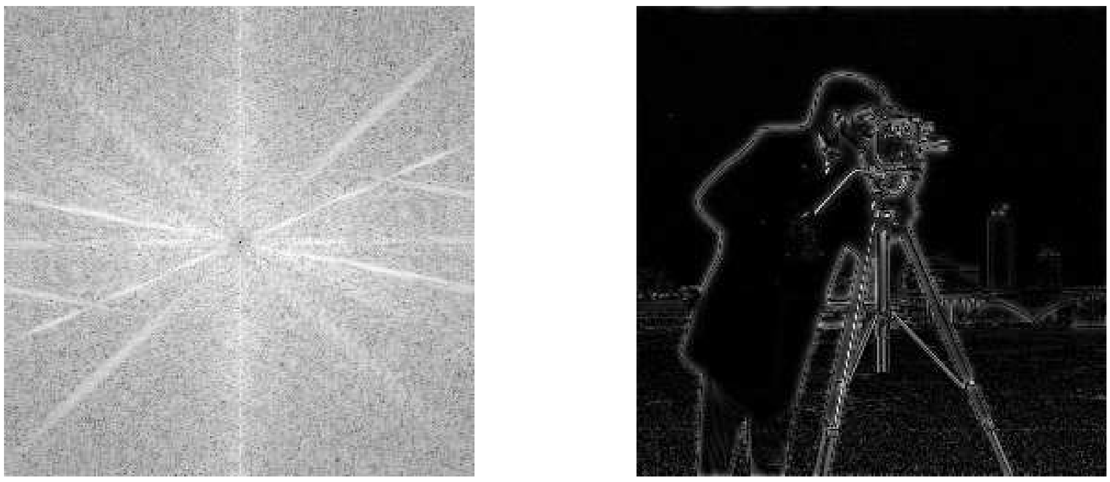
and then inverting and displaying the result:

```
>> cfbhi=ifft2(cfbh);
>> figure,fftshow(cfbhi,'abs')
```

The images are shown in figure 9.25

9.7.3 Gaussian filtering

We have met Gaussian filters in chapter 6, and we saw that they could be used for low pass filtering. However, we can also use Gaussian filters in the frequency domain. As with ideal and Butterworth filters, the implementation is very simple: create a Gaussian filter, multiply it by the image transform, and invert the result. Since Gaussian filters have the very nice mathematical property that a Fourier transform of a Gaussian is a Gaussian, we should get exactly the same results as when using a linear Gaussian spatial filter.



(a) The DFT after Butterworth high pass filtering (b) The resulting image

Figure 9.25: Butterworth high pass filtering

Gaussian filters may be considered to be the most “smooth” of all the filters we have discussed so far, with ideal filters the least smooth, and Butterworth filters in the middle.

We can create Gaussian filters using the `fspecial` function, and apply them to our transform.

```
>> g1=mat2gray(fspecial('gaussian',256,10));
>> cg1=cf.*g1;
>> fftshow(cg1,'log')
>> g2=mat2gray(fspecial('gaussian',256,30));
>> cg2=cf.*g2;
>> figure,fftshow(cg2,'log')
```

Note the use of the `mat2gray` function. The `fspecial` function on its own produces a low pass Gaussian filter with a very small maximum:

```
>> g=fspecial('gaussian',256,10);
>> format long, max(g(:)), format

ans =

   0.00158757552679
```

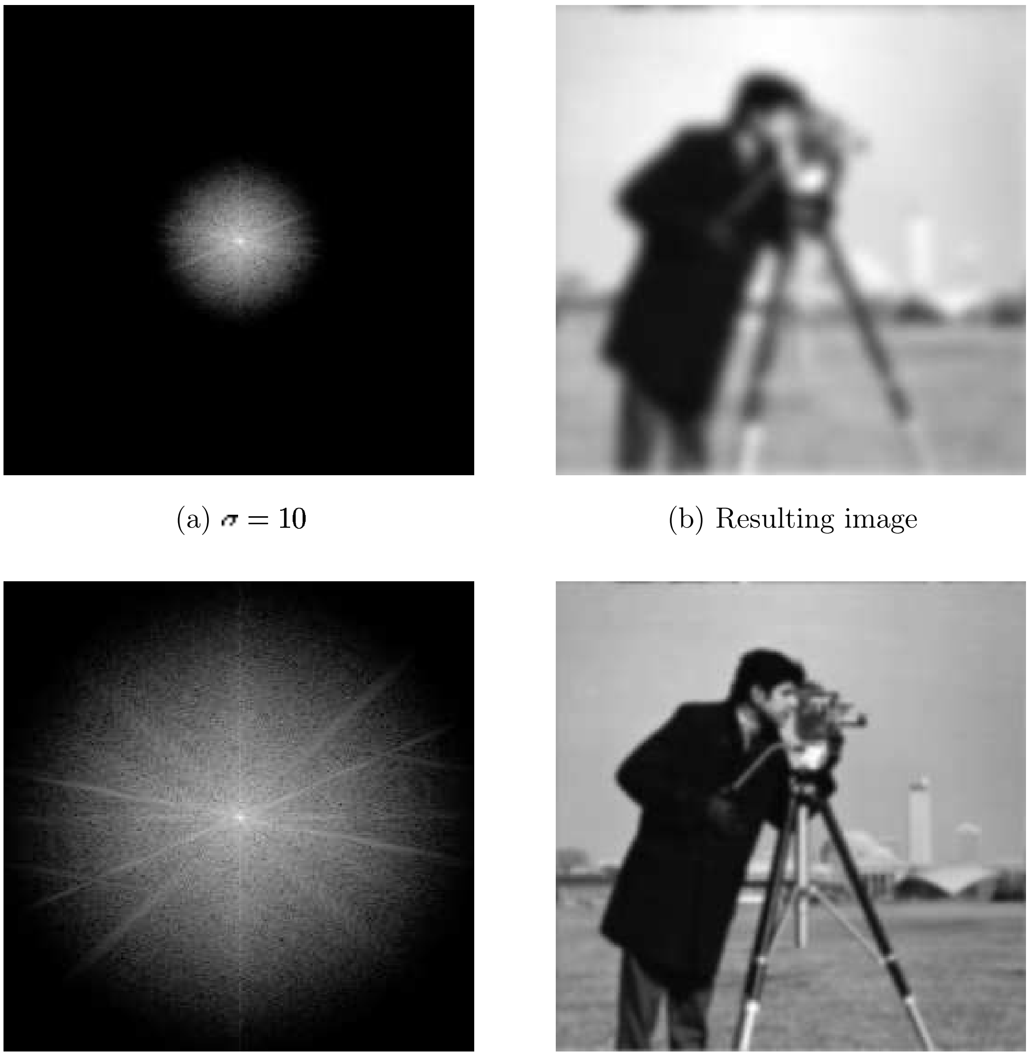
The reason is that `fspecial` adjusts its output to keep the volume under the Gaussian function always 1. This means that a wider function, with a large standard deviation, will have a low maximum. So we need to scale the result so that the central value will be 1; and `mat2gray` does that automatically.

The transforms are shown in figure 9.26(a) and (c). In each case, the final parameter of the `fspecial` function is the standard deviation; it controls the width of the filter. As we see, the larger the standard deviation, the wider the function, and so the greater amount of the transform is preserved.

The results of the transform on the original image can be produced using the usual sequence of commands:

```
>> cgi1=ifft2(cg1);
>> cgi2=ifft2(cg2);
>> fftshow(cgi1,'abs');
>> fftshow(cgi2,'abs');
```

and the results are shown in figure 9.26(b) and (d)


(a) $\sigma = 10$ (b) Resulting image

(c) $\sigma = 30$ (d) Resulting image

Figure 9.26: Applying a Gaussian low pass filter in the frequency domain

We can apply a high pass Gaussian filter easily; we create a high pass filter by subtracting a low pass filter from 1.

```
>> h1=1-g1;
>> h2=1-g2;
>> ch1=cf.*h1;
>> ch2=cf.*h2;
>> ch1i=ifft2(ch1);
>> chi1=ifft2(ch1);
>> chi2=ifft2(ch2);
>> fftshow(chi1,'abs')
>> figure,fftshow(chi2,'abs')
```

and the images are shown in figure 9.27. As with ideal and Butterworth filters, the wider the high

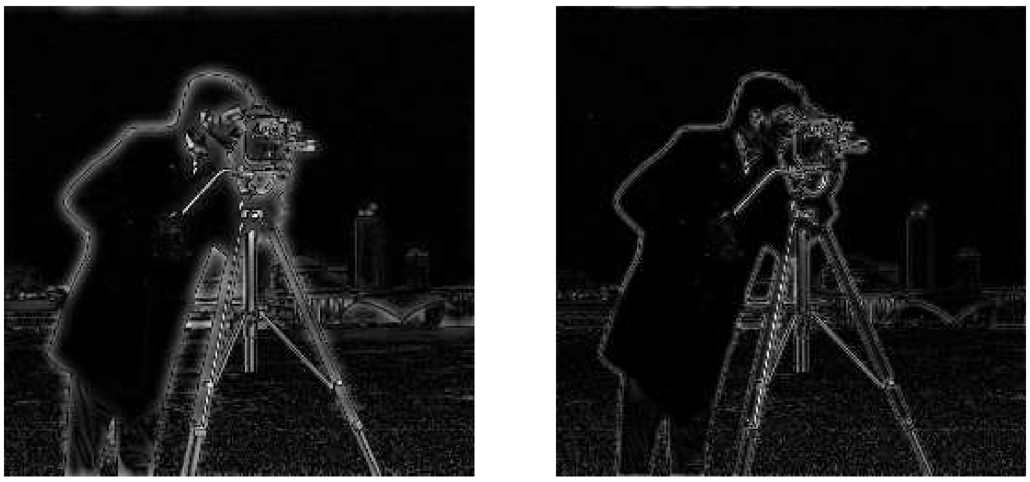

(a) Using $\sigma = 10$ (b) Using $\sigma = 30$

Figure 9.27: Applying a Gaussian high pass filter in the frequency domain

pass filter, the more of the transform we are reducing, and the less of the original image will appear in the result.

9.8 Removal of periodic noise

In chapter 7 we investigated a number of different noise types which can affect images. However, we did not look at *periodic noise*. This may occur if the imaging equipment (the acquisition or networking hardware) is subject to electronic disturbance of a repeating nature, such as may be caused by an electric motor. We can easily create periodic noise by overlaying an image with a trigonometric function:

```
>> [x,y]=meshgrid(1:256,1:256);
>> s=1+sin(x+y/1.5);
>> cp=(double(cm)/128+s)/4;
```

where `cm` is the cameraman image from previous sections. The second line simply creates a sine function, and adjusts its output to be in the range 0–2. The last line first adjusts the cameraman image to be in the same range; adds the sine function to it, and divides by 4 to produce a matrix of type `double` with all elements in the range 0.0–1.0. This can be viewed directly with `imshow`, and it is shown in figure 9.28(a). We can produce its DFT:

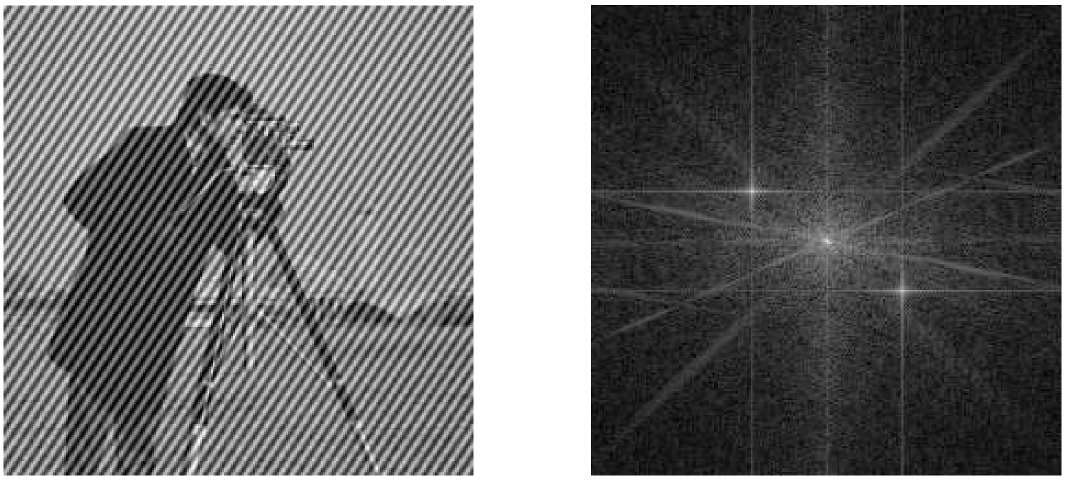

(a) (b)

Figure 9.28: The cameraman image (a) with periodic noise, and (b) its transform

```
>> cpf=fftshift(fft2(cp));
```

and this is shown in figure 9.28(b). The extra two "spikes" away from the centre correspond to the noise just added. In general the tighter the period of the noise, the further from the centre the two spikes will be. This is because a small period corresponds to a high frequency (large change over a small distance), and is therefore further away from the centre of the shifted transform.

We will now remove these extra spikes, and invert the result. If we put `pixval on` and move around the image, we find that the spikes have row, column values of $(156, 170)$ and $(102, 88)$. These have the same distance from the centre: 49.0918. We can check this by

```
>> z(156,170)
>> z(102,88)
```

There are two methods we can use to eliminate the spikes, we shall look at both of them.

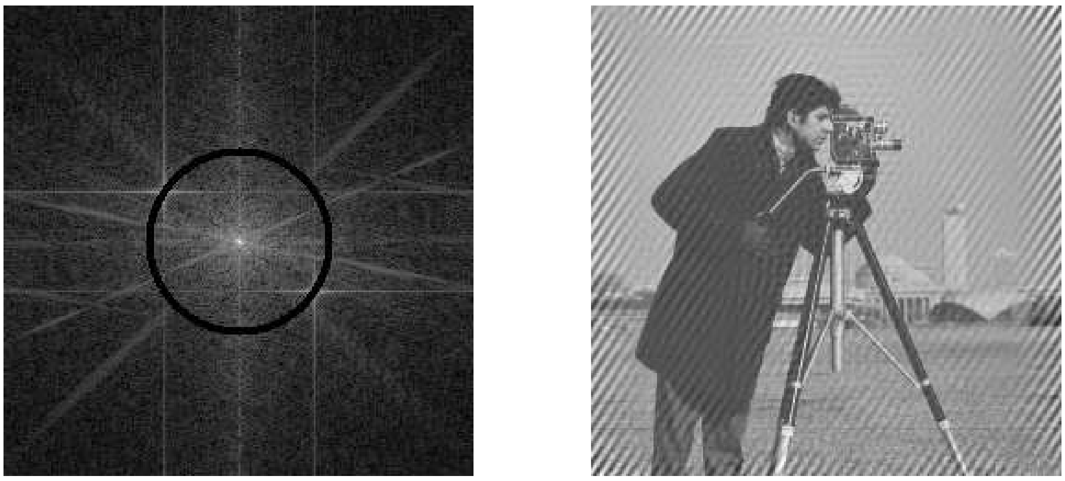
Band reject filtering. We create a filter consisting of ones with a ring of zeroes; the zeroes lying at a radius of 49 from the centre:

```
>> br=(z < 47 | z > 51);
```

where `z` is the matrix consisting of distances from the origin. This particular ring will have a thickness large enough to cover the spikes. Then as before, we multiply this by the transform:

```
>> cpfbr=cpf.*br
```

and this is shown in figure 9.29(a). The result is that the spikes have been blocked out by this filter. Taking the inverse transform produces the image shown in figure 9.29(b). Note that not all the noise has gone, but a significant amount has, especially in the centre of the image.



(a) A band-reject filter (b) After inversion

Figure 9.29: Removing periodic noise with a band-reject filter

Notch filtering. With a notch filter, we simply make the rows and columns of the spikes zero:

```
>> cpf(156,:)=0;
>> cpf(102,:)=0;
>> cpf(:,170)=0;
>> cpf(:,88)=0;
```

and the result is shown in figure 9.30(a). The image after inversion is shown in figure 9.30(b). As before, much of the noise in the centre has been removed. Making more rows and columns of the transform zero would result in a larger reduction of noise.

9.9 Inverse filtering

We have seen that we can perform filtering in the Fourier domain by multiplying the DFT of an image by the DFT of a filter: this is a direct use of the convolution theorem. We thus have

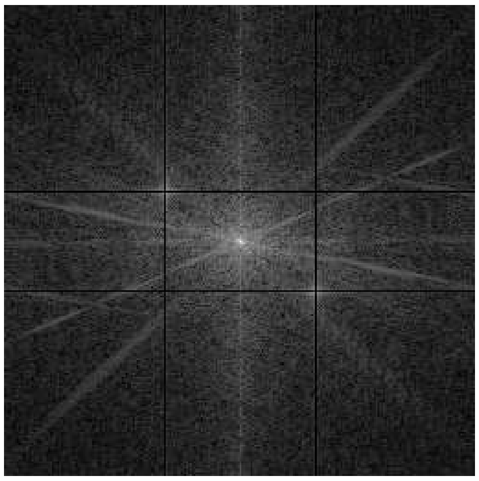
$$Y(i,j) = X(i,j)F(i,j)$$

where X is the DFT of the image; F is the DFT of the filter, and Y is the DFT of the result. If we are given Y and F, then we should be able to recover the (DFT of the) original image X simply by dividing by F:

$$X(i,j) = \frac{Y(i,j)}{F(i,j)}. \tag{9.8}$$

Suppose, for example we take the wombats image `wombats.tif`, and blur it using a low-pass Butterworth filter:

```
>> w=imread('wombats.tif');
>> wf=fftshift(fft2(w));
```


(a) A notch filter

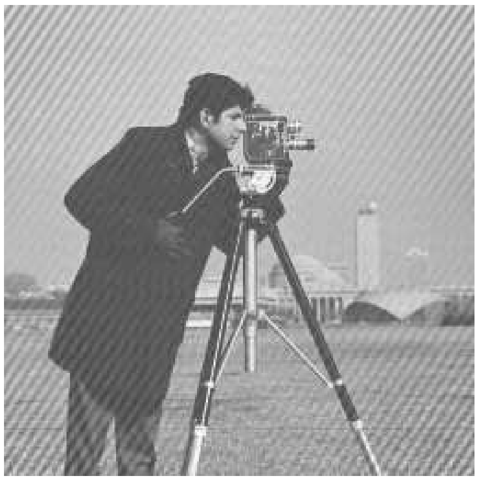

(b) After inversion

Figure 9.30: Removing periodic noise with a notch filter

```
>> b=lbutter(w,15,2);
>> wb=wf.*b;
>> wba=abs(ifft2(wb));
>> wba=uint8(255*mat2gray(wba));
>> imshow(wba)
```

The result is shown on the left in figure 9.31. We can attempt to recover the original image by dividing by the filter:

```
>> w1=fftshift(fft2(wba))./b;
>> w1a=abs(ifft2(w1));
>> imshow(mat2gray(w1a))
```

and the result is shown on the right in figure 9.31. This is no improvement! The trouble is that some elements of the Butterworth matrix are very small, so dividing produces very large values which dominate the output. We can deal with this problem in two ways:

1. Apply a low pass filter L to the division:

   $$X(i,j) = \frac{Y(i,j)}{F(i,j)} L(i,j).$$

   This should eliminate very low (or zero) values.

2. "Constrained division": choose a threshold value d, and if $|F(i,j)| < d$, we don't perform a division, but just keep our original value. Thus:

   $$X(i,j) = \begin{cases} \dfrac{Y(i,j)}{F(i,j)} & \text{if } |F(i,j)| \geq d, \\ Y(i,j) & \text{if } |F(i,j)| < d. \end{cases}$$

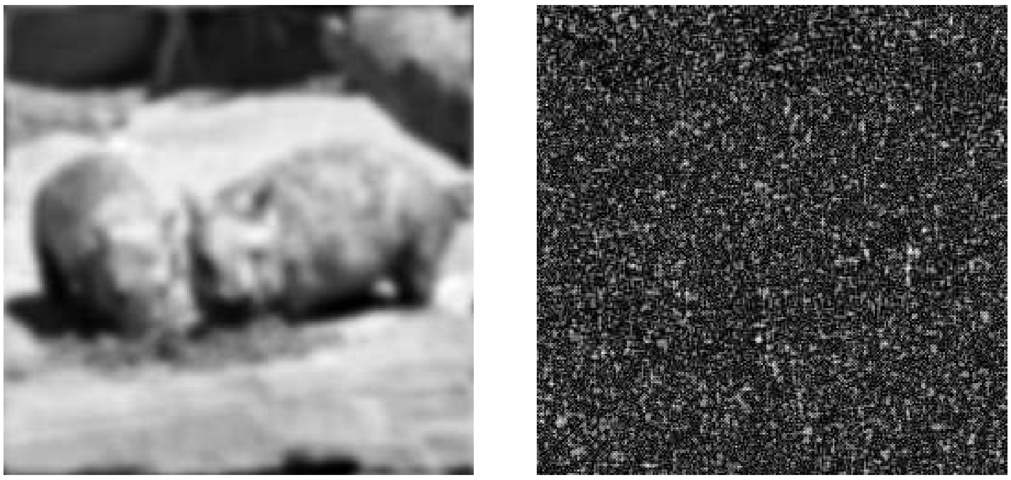


Figure 9.31: An attempt at inverse filtering

We can apply the first method by multiplying a Butterworth low pass filter to the matrix `c1` above:

```
>> wbf=fftshift(fft2(wba));
>> w1=(wbf./b).*lbutter(w,40,10);
>> w1a=abs(ifft2(w1));
>> imshow(mat2gray(w1a))
```

Figure 9.32 shows the results obtained by using a different cutoff radius of the Butterworth filter each time: (a) uses 40 (as in the Matlab commands just given); (b) uses 60; (c) uses 80, and (d) uses 100. It seems that using a low pass filter with a cutoff round about 60 will yield the best results. After we use larger cutoffs, the result degenerates.

We can try the second method; to implement it we simply make all values of the filter which are too small equal to 1:

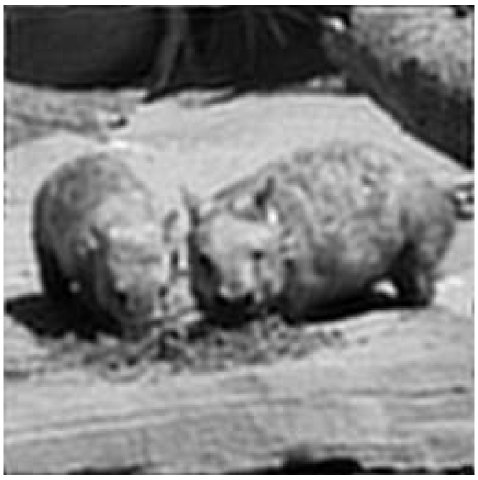
```
>> d=0.01;
>> b=lbutter(w,15,2);b(find(b<d))=1;
>> w1=fftshift(fft2(wba))./b;
>> w1a=abs(ifft2(w1));
>> imshow(mat2gray(w1a))
```

Figure 9.33 shows the results obtained by using a different cutoff radius of the Butterworth filter each time: (a) uses $d = 0.01$ (as in the Matlab commands just given); (b) uses $d = 0.005$; (c) uses $d = 0.002$, and (d) uses $d = 0.001$. It seems that using a threshold d in the range $0.002 \leq d \leq 0.005$ produces reasonable results.

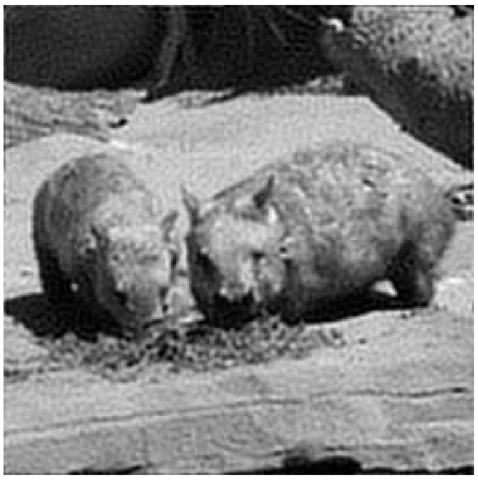
Motion deblurring

We can consider the removal of blur caused by motion to be a special case of inverse filtering. Suppose we take an image and blur it by a small amount.

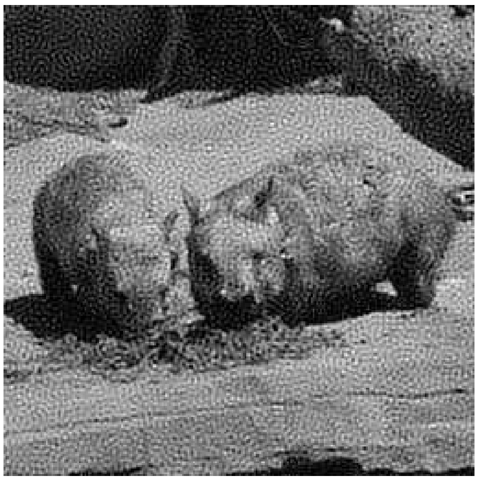
```
>> bc=imread('board.tif');
>> bg=im2uint8(rgb2gray(bc));
```



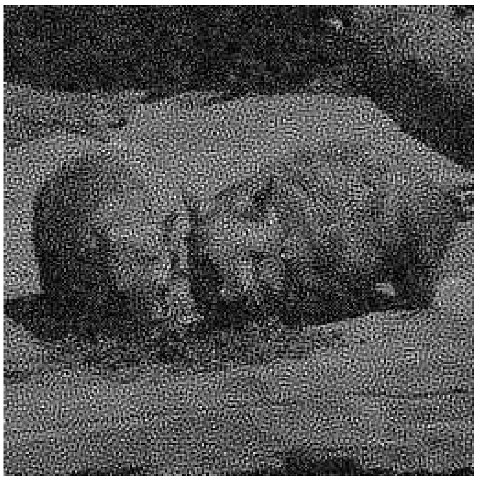
(a)



(b)



(c)



(d)

Figure 9.32: Inverse filtering using low pass filtering to eliminate zeros

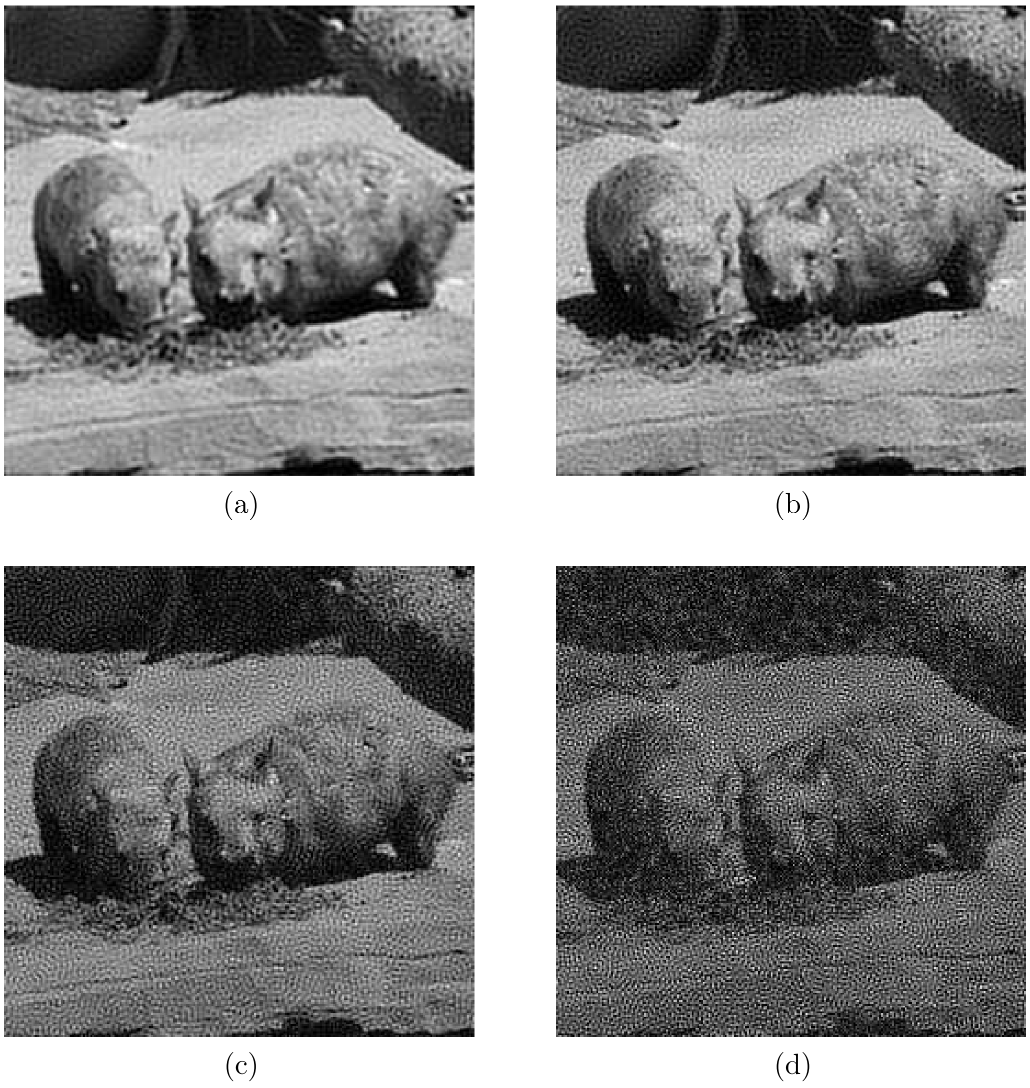


Figure 9.33: Inverse filtering using constrained division

```
>> b=bg(100:355,50:305);
>> imshow(b)
```

These commands simply take the colour image of a circuit board (the image `board.tif`), makes a greyscale version of data type `uint8`, and picks out a square subimage. The result is shown as figure 9.34(a). To blur it, we can use the `blur` parameter of the `fspecial` function.

```
>> m=fspecial('motion',7,0);
>> bm=imfilter(b,m);
>> imshow(bm)
```

and the result is shown as figure 9.34(b). The result of the blur has effectively obliterated the text

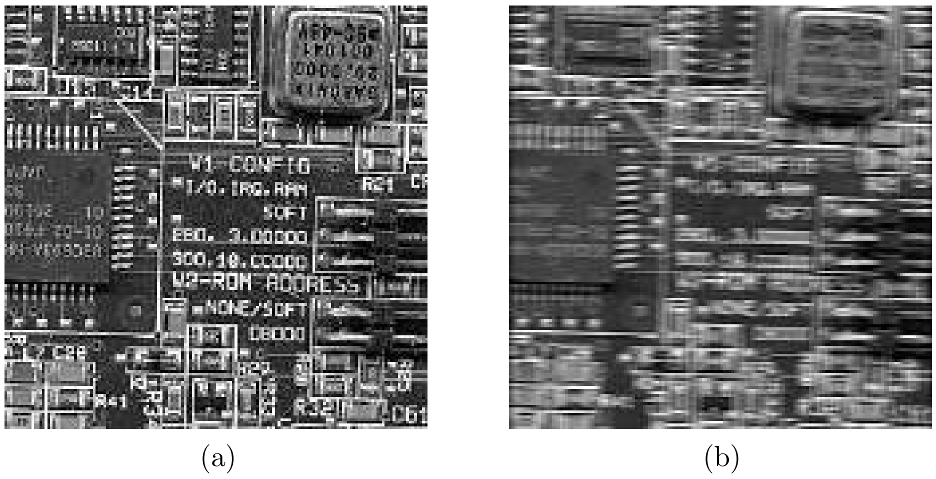


Figure 9.34: The result of motion blur

on the image.

To deblur the image, we need to divide its transform by the transform corresponding to the blur filter. This means that we first must create a matrix corresponding to the transform of the blur:

```
>> m2=zeros(256,256);
>> m2(1,1:7)=m;
>> mf=fft2(m2);
```

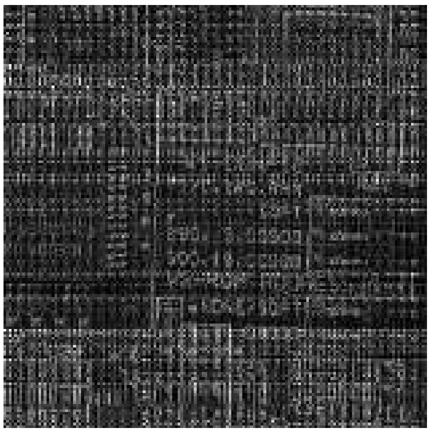
Now we can attempt to divide by this transform.

```
>> bmi=ifft2(fft2(bm)./mf);
>> fftshow(bmi,'abs')
```

and the result is shown in figure 9.35(a). As with inverse filtering, the result is not particularly good, because the values close to zero in the matrix `mf` have tended to dominate the result. As above, we can constrain the division by only dividing by values which are above a certain threshold.

```
>> d=0.02;
>> mf=fft2(m2);mf(find(abs(mf)<d))=1;
>> bmi=ifft2(fft2(bm)./mf);
>> imshow(mat2gray(abs(bmi))*2)
```

where the last multiplication by 2 just brightens the result, which is shown in figure 9.35(b). The


(a) Straight division

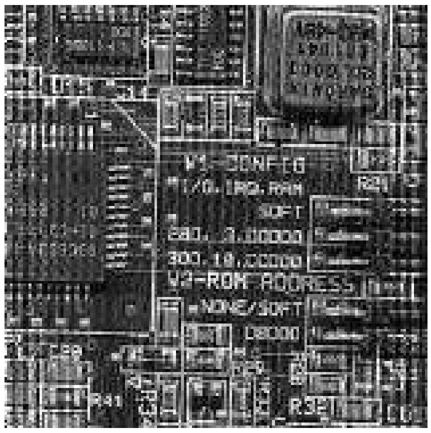

(b) Constrained division

Figure 9.35: Attempts at removing motion blur

writing, especially in the centre of the image, is now quite legible.

Exercises

1. By hand, compute the DFT of each of the following sequences:

   (a) $[2,\ \ 3,\ \ 4,\ \ 5]$ (b) $[2,\ \ -3,\ \ 4,\ \ -5]$ (c) $[-9,\ \ -8,\ \ -7,\ \ -6]$

   (d) $[-9,\ \ 8,\ \ -7,\ \ 6]$

   Compare your answers with those given by Matlab's `fft` function.

2. For each of the transforms you computed in the previous question, compute the inverse transform by hand.

3. By hand, verify the convolution theorem for each of the following pairs of sequences:

   (a) $[2,\ \ 4,\ \ 6,\ \ 8]$ and $[-1,\ \ 2\ \ -3,\ \ 4]$ (b) $[4,\ \ 5,\ \ 6,\ \ 7]$ and $[3,\ \ 1\ \ 5,\ \ -1]$

4. Using Matlab, verify the convolution theorem for the following pairs of sequences:

   (a) $[2,\ \ -3,\ \ 5,\ \ 6,\ \ -2,\ \ -1,\ \ 3,\ \ 7]$ and $[-1,\ \ 5,\ \ 6,\ \ 4,\ \ -3,\ \ -5,\ \ 1,\ \ 2]$

   (b) $[7,\ \ 6,\ \ 5,\ \ 4,\ \ -4,\ \ -5,\ \ -6,\ \ -7]$ and $[2,\ \ 2,\ \ -5,\ \ -5,\ \ 6,\ \ 6,\ \ -7,\ \ -7]$

5. Consider the following matrix:

$$\begin{bmatrix} 4 & 5 & -9 & -5 \\ 3 & -7 & 1 & 2 \\ 6 & -1 & -6 & 1 \\ 3 & -1 & 7 & -5 \end{bmatrix}$$

   Using Matlab, calculate the DFT of each row. You can do this with the commands:

```
>> a=[4 5 -9 -5;3 -7 1 2;6 -1 -6 1;3 -1 7 -5];
>> a1=zeros(4,4);
>> for i=1:4,a1(i,:)=fft(a(i,:));end
```

Now use similar commands to calculate the DFT of each column of `a1`.

Compare the result with the output of the command `fft2(a)`.

6. Perform similar calculations as in the previous question with the matrices produced by the commands `magic(4)` and `hilb(6)`.

7. Open up the image `engineer.tif`:

   ```
   >> en=imread('engineer.tif');
   ```

   Experiment with applying the Fourier transform to this image and the following filters:

   (a) ideal filters (both low and high pass),
   (b) Butterworth filters,
   (c) Gaussian filters.

   What is the smallest radius of a low pass ideal filter for which the face is still recognizable?

8. If you have access to a digital camera, or a scanner, produce a digital image of the face of somebody you know, and perform the same calculations as in the previous question.

9. Add the sine waves to the engineer face using the same commands as for the cameraman:

   ```
   >> [x,y]=meshgrid(1:256,1:256);
   >> s=1+sin(x+y/1.5);
   >> ep=(double(en)/128+s)/4;
   ```

   Now attempt to remove the noise using band-reject filtering or notch filtering. Which one gives the best result?

10. For each of the following sine commands:

    (a) `s=1+sin(x/3+y/5);`
    (b) `s=1+sin(x/5+y/1.5);`
    (c) `s=1+sin(x/6+y/6);`

    add the sine wave to the image as shown in the previous question, and attempt to remove the resulting periodic noise using band-reject filtering or notch filtering.

    Which of the three is easiest to "clean up"?

11. Apply a 3×3 blurring filter to the cameraman image with `imfilter`. Attempt to deblur the result using inverse filtering with constrained division. Which threshold gives the best results?

12. Repeat the previous question using a 7×7 blurring filter.

13. Work through the motion deblurring example, experimenting with different values of the threshold. What gives the best results?

Chapter 10

The Hough and Distance Transforms

10.1 The Hough transform

As we have mentioned in chapter 5, an image *transform* is an operation which takes an image, or the array of its pixel values, and "transforms" it into a different form. This new form is then processed to provide the result we require. Sometimes it is necessary to apply an inverse transform after the processing, to obtain the processed image. In this chapter we investigate two important transforms, with many different applications.

The *Hough transform*[1] is designed to find lines in images, but it can be easily varied to find other shapes. The idea is simple. Suppose (x, y) is a point in the image (which we shall assume to be binary). We can write $y = ax + b$, and consider all pairs (a, b) which satisfy this equation, and plot them into an "accumulator array". The (a, b) array is the "transform array".

For example, take $(x, y) = (1, 1)$. Since the equation relating a and b is

$$1 = a.1 + b$$

we can write

$$b = -a + 1.$$

Thus the line $b = -a + 1$ consists of all pairs of points relating to the single point $(1, 1)$. This is shown in figure 10.1.

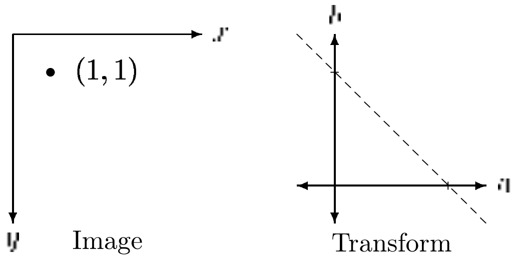


Figure 10.1: A point in an image and its corresponding line in the transform

Each point in the image is mapped onto a line in the transform. The points in the transform corresponding to the greatest number of intersections correspond to the strongest line in the image.

[1]"Hough" is pronounced "Huff".

For example, suppose we consider an image with five points: $(1,0)$, $(1,1)$, $(2,1)$, $(4,1)$ and $(3,2)$. Each of these points corresponds to a line as follows:

$$\begin{aligned}
(1,0) &\;\rightarrow\; b=-a\\
(1,1) &\;\rightarrow\; b=-a+1\\
(2,1) &\;\rightarrow\; b=-2a+1\\
(4,1) &\;\rightarrow\; b=-4a+1\\
(3,2) &\;\rightarrow\; b=-3a+2.
\end{aligned}$$

Each of these lines appears in the transform as shown in figure 10.2.

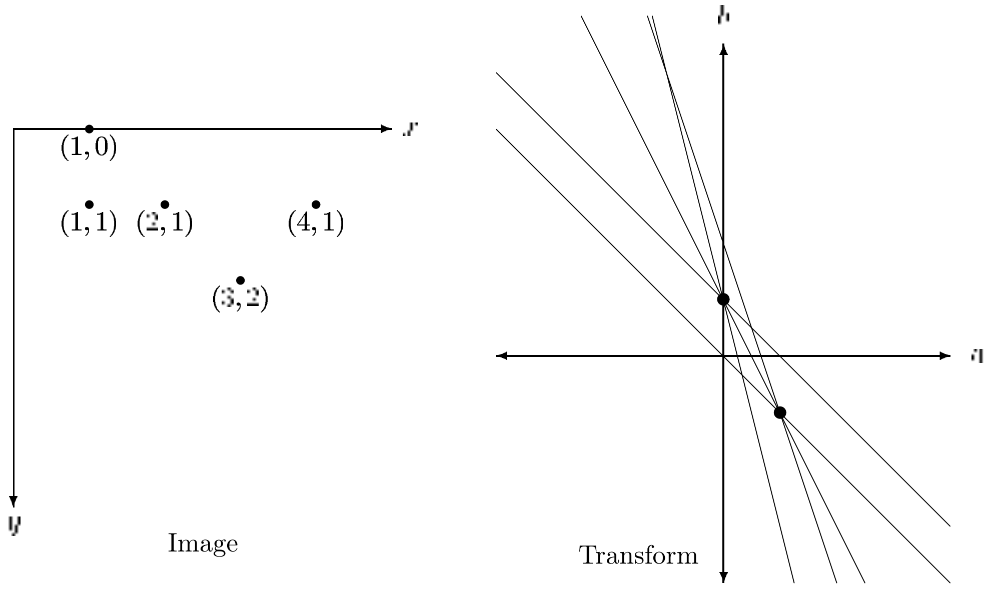


Figure 10.2: An image and its corresponding lines in the transform

The dots in the transform indicate places where there are maximum intersections of lines: at each dot three lines intersect. The coordinates of these dots are $(a,b)=(1,0)$ and $(a,b)=(1,-1)$. These values correspond to the lines

$$y=1.x+0$$

and

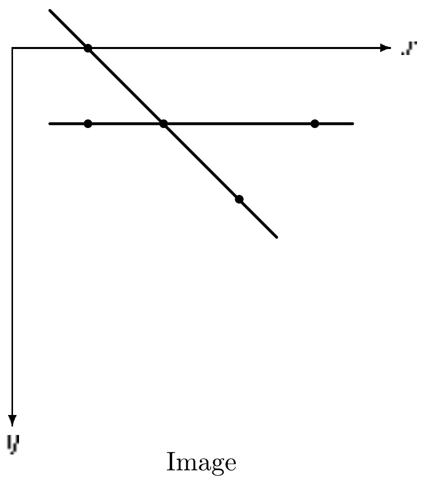
$$y=1.x+(-1)$$

or $y=x$ and $y=x-1$. These lines are shown on the image in figure 10.3.

We see that these are indeed the "strongest" lines in the image in that they contain the greatest number of points.

There is a problem with this implementation of the Hough transform, and that is that it can't find vertical lines: we can't express a vertical line in the form $y=mx+c$, as m represents the gradient, and a vertical line has infinite gradient. We need another parameterization of lines.

Consider a general line, as shown in figure 10.4. Clearly any line can be described in terms of the two parameters r and θ: r is the perpendicular distance from the line to the origin, and θ is the



Image

Figure 10.3: Lines found by the Hough transform

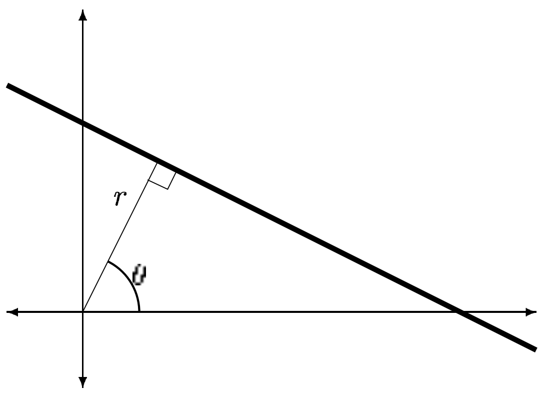


Figure 10.4: A line and its parameters

angle of the line's perpendicular to the x-axis. In this parameterization, vertical lines are simply those which have $\theta = 0$. If we allow r to have negative values, we can restrict θ to the range

$$-90 < \theta \leq 90.$$

Given this parameterization, we need to be able to find the equation of the line. First note that the point (p, q) where the perpendicular to the line meets the line is $(p, q) = (r\cos\theta, r\sin\theta)$. Also note that the gradient of the perpendicular is $\tan\theta = \sin\theta/\cos\theta$. Now let (x, y) be any point on then line. The gradient of the line is

$$\begin{aligned}\frac{\text{rise}}{\text{run}} &= \frac{y-q}{x-p}\\ &= \frac{y-r\sin\theta}{x-r\cos\theta}.\end{aligned}$$

But since the gradient of the line's perpendicular is $\tan\theta$, the gradient of the line itself must be

$$-\frac{1}{\tan\theta} = -\frac{\cos\theta}{\sin\theta}.$$

Putting these two expressions for the gradient together produces:

$$\frac{y-r\sin\theta}{x-r\cos\theta} = -\frac{\cos\theta}{\sin\theta}.$$

If we now multiply out these fractions we obtain:

$$y\sin\theta - r\sin^2\theta = -x\cos\theta + r\cos^2\theta$$

and this equation can be rewritten as

$$\begin{aligned}y\sin\theta + x\cos\theta &= r\sin^2\theta + r\cos^2\theta\\ &= r(\sin^2\theta + \cos^2\theta)\\ &= r.\end{aligned}$$

We finally have the required equation for the line as:

$$x\cos\theta + y\sin\theta = r.$$

The Hough transform can then be implemented as follows: we start by choosing a discrete set of values of r and θ to use. For each pixel (x, y) in the image, we compute

$$x\cos\theta + y\sin\theta$$

for each value of θ, and place the result in the appropriate position in the (r, θ) array. At the end, the values of (r, θ) with the highest values in the array will correspond to strongest lines in the image.

An example will clarify this: consider the image shown in figure 10.5.

We shall discretize θ to use only the values

$$-45^\circ, \quad 0^\circ, \quad 45^\circ, \quad 90^\circ.$$

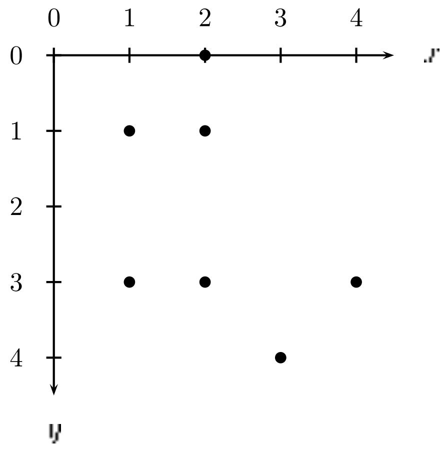


Figure 10.5: A small image

We can start by making a table containing all values $x\cos\theta + y\sin\theta$ for each point, and for each value of θ:

(x,y)	$-45°$	$0°$	$45°$	$90°$
$(2,0)$	1.4	2	1.4	0
$(1,1)$	0	1	1.4	1
$(2,1)$	0.7	2	2.1	1
$(1,3)$	−1.4	1	2.8	3
$(2,3)$	−0.7	2	3.5	3
$(4,3)$	0.7	4	4.9	3
$(3,4)$	−0.7	3	4.9	4

The accumulator array contains the number of times each value of (r,θ) appears in the above table:

	−1.4	−0.7	0	0.7	1	1.4	2	2.1	2.8	3	3.5	4	4.9
$-45°$	1	2	1	2		1							
$0°$					2		3			1		1	
$45°$						2		1	1		1		2
$90°$			1		2					3		2	

In practice this array will be very large, and can be displayed as an image. We see that the two equal largest values occur at $(r,\theta) = (2, 0°)$ and $(r,\theta) = (3, 90°)$. The lines then are

$$x\cos 0 + y\sin 0 = 2$$

or $x = 2$, and

$$x\cos 90 + y\sin 90 = 3$$

or $y = 3$. These lines are shown in figure 10.6

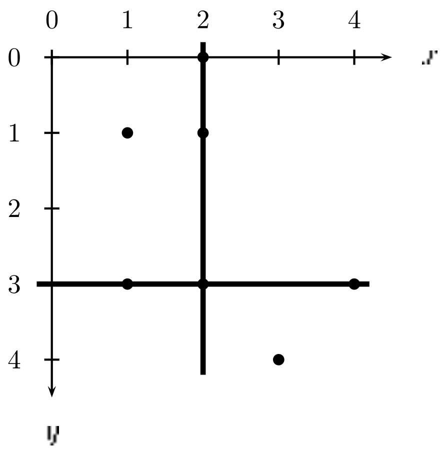


Figure 10.6: Lines found by the Hough transform

10.2 Implementing the Hough transform in Matlab

The Image Processing Toolbox does not contain a `hough` routine, and although the Hough transform can be obtained by other means, it is a pleasant programming exercise to write our own. Following the procedure outlined above, we will:

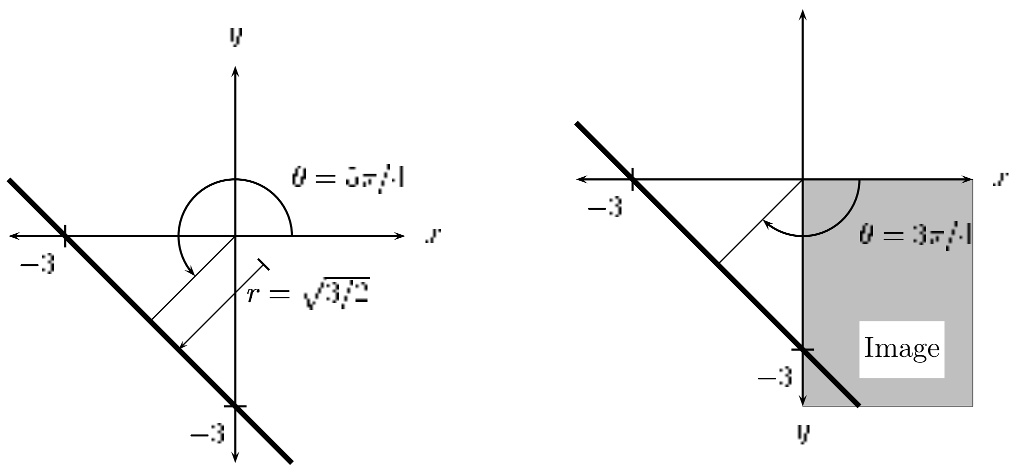
1. decide on a discrete set of values of θ and r to use,
2. for each foreground pixel (x, y) in the image calculate the values of $r = x\cos\theta + y\sin\theta$ for all of our chosen values of θ,
3. create an accumulator array whose sizes are the numbers of angles θ and values r in our chosen discretizations from step 1,
4. step through all of our r values updating the accumulator array as we go.

We shall consider each step in turn:

Discretizing r and θ. We observe that even though a particular pair (r, θ) corresponds to only one line, a given line can be parameterized in different ways. For example, the line $x + y = -3$ can be parameterized using $\theta = 5\pi/4$ and $r = \sqrt{3/2}$ as shown in figure 10.7(a).

However, the same line can be parameterized using $\theta = \pi/4$ and $r = -\sqrt{3/2}$. So we have a choice: we may restrict the values of θ, say to the range $-\pi/2 < \theta \leq \pi/2$, and let r have both positive and negative values, or we may let θ take any value chosen from the range $0 \leq \theta < 2\pi$ and restrict r to non-negative values. Since we are going to use the values of θ and r as indices into the accumulator array, it is simplest to take the second option. However, consider figure 10.7(a) slightly redrawn to take into account the directions of x and y as column and row indices of a matrix, as shown in (b). Since our image, for which both x and y will be positive, will sit in the bottom right quadrant, we will only have positive r values for $-90 \leq \theta \leq 180$. For if θ is outside that range, the perpendicular will point into the second (upper left) quadrant, which would require a negative value of r if the line was to intersect the image.

We can choose any discrete set of values we like, but let us take all the integer degree values in the given range, but convert them to radians for the purposes of calculation:



(a) Using ordinary Cartesian axes (b) Using matrix axes

Figure 10.7: A line parameterized with r and θ

```
>> angles=[-90:180]*pi/180;
```

Calculating the r values. Suppose that the image is binary, then we can find the positions of all the foreground pixels with a simple application of the `find` function:

```
>> [x,y]=find(im);
```

If the image is not binary, we can create a binary edge image by use of the `edge` function. Now we can calculate all the r values with one simple command:

```
>> r=floor(x*cos(angles)+y*sin(angles));
```

where we use `floor` so as to obtain only integer values.

Forming the accumulator array. If the image is of size $m \times n$ pixels, then supposing the origin to be at the upper left corner, the maximum value of r will be $\sqrt{m^2 + n^2}$. So the size of the accumulator array can be set at $\sqrt{m^2 + n^2} \times 270$. However, we are only interested in positive values of r: given our choice of range of θ, negative values can be discarded. So we can first find the largest positive value of r, and use that as one dimension of the array:

```
>> rmax=max(r(find(r>0)));
>> acc=zeros(rmax+1,270);
```

The `rmax+1` allows us to have values of $r = 0$.

Updating the accumulator array. We now have to step through the array of r values, and for each value (r, θ), increase the corresponding accumulator value by one. First notice that the array r, as define above, has size $N \times 360$, where N is the number of foreground pixels. Hence the second index of both r and acc corresponds to the angle.

This can be done with nested loops, at the heart of which will be the command

```
  if r(i,j)>=0, acc(r(i,j)+1,i)=acc(r(i,j)+1,i)+1;
```

Note that this is not a very efficient method of creating the accumulator array, but it does have the advantage of following the theory very closely. The entire program is shown in figure 10.8.

```
function res=hough(image)

%
% HOUGH(IMAGE) creates the Hough transform corresponding to the image IMAGE
%

if ~isbw(image)
  edges=edge(image,'canny');
else
  edges=image;
end;
[x,y]=find(edges);
angles=[-90:180]*pi/180;
r=floor(x*cos(angles)+y*sin(angles));
rmax=max(r(find(r>0)));
acc=zeros(rmax+1,270);
for i=1:length(x),
  for j=1:270,
    if r(i,j)>=0
      acc(r(i,j)+1,j)=acc(r(i,j)+1,j)+1;
    end;
  end;
end;
res=acc;
```

Figure 10.8: A simple MATLAB function for implementing the Hough transform

An example

Let us take the cameraman image, and apply our Hough transform procedure to it:

```
>> c=imread('cameraman.tif');
>> hc=hough(c);
```

This last command may take some time, because of the inefficient nested loops. The first thing we may do is to view the result:

```
>> imshow(mat2gray(hc)*1.5)
```

where the extra 1.5 at the end is just to brighten up the image. The result is shown in figure 10.9.

What we are viewing here is the accumulator array. And it is what we should expect: a series of curves, with some bright points indicating places of maximum intersection. Because of the use of

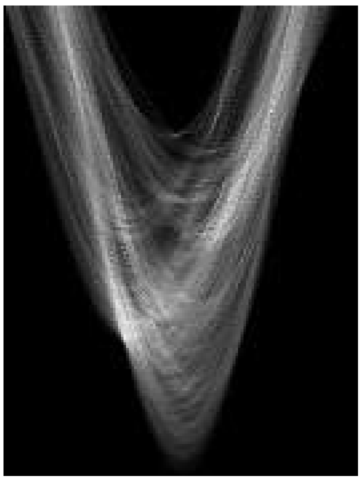


Figure 10.9: The result of a Hough transform

sines and cosines in the construction of the transform, we can expect the curves to be of a sinusoidal nature.

What now? Let us find the maximum value of the transform:

```
>> max(hc(:))

ans =

    91
```

We can now find the r and θ values corresponding to the maximum:

```
>> [r,theta]=find(hc==91)

r =

   138


theta =

   181
```

However, since we are using the pixel coordinates as cartesian coordinates, our x and y axes have been rotated counter-clockwise by 90°. Thus we are measuring θ in a clockwise direction from the left vertical edge of our image. Figure 10.10 shows how this works:

We have also got the problem that We can easily create a small function to draw lines for us, given their perpendicular distance from the origin, and the angle of the perpendicular from the x axis. The tool for this will be the `line` function, which in the form

```
>> line([x1,x2],[y1,y2])
```

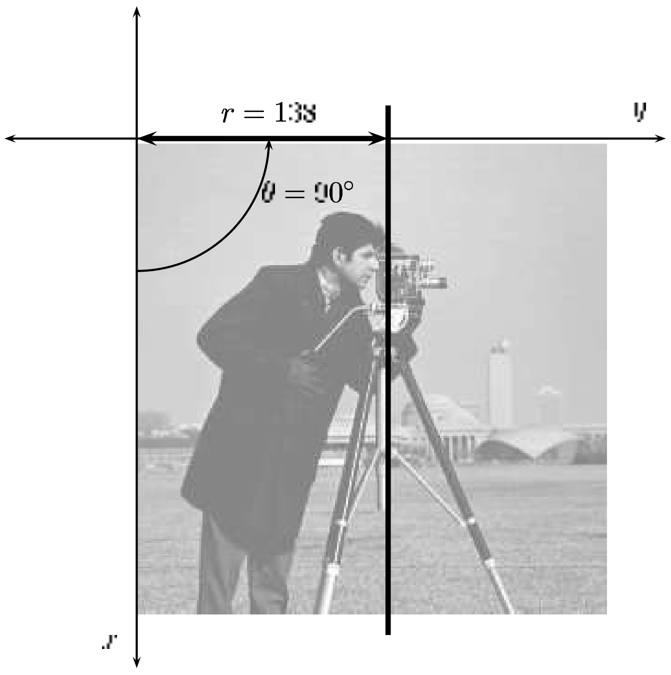


Figure 10.10: A line from the Hough transform

draws a line between coordinates (x1,y1) and (x2,y2) over the current image. The only difficulty here is that `line` takes the x axis to be on the top, and the y axis going down on the left. This is easily dealt with by replacing θ with $\pi/2 - \theta$, and a simple function for drawing lines is shown in figure 10.11.

The only thing to notice is the expression `181-theta` in the line

```
angle=pi*(181-theta)/180;
```

This can be interpreted as $180+1-\theta$; the initial 180 swaps between cartesian and matrix coordinates, and the extra 1 counters the accumulation of `r+1` in the function `hough.m`.

We can use this as follows:

```
>> c2=imadd(imdivide(cm,4),192);
>> imshow(c2)
>> houghline(c2,r,theta)
```

The idea of using `c2` is simply to lighten the image so as to show the line more clearly. The result is shown on the left in figure 10.12. To draw other lines, we can extract some more points from the transform:

```
>> [r,theta]=find(hc>80)

r =

   148
   138
   131
```

```
function houghline(image,r,theta)
%
% Draws a line at perpendicular distance R from the upper left corner of the
% current figure, with perpendicular angle THETA to the left vertical axis.
% THETA is assumed to be in degrees.
%
[x,y]=size(image);
angle=pi*(181-theta)/180;
X=[1:x];
if sin(angle)==0
  line([r r],[0,y],'Color','black')
else
  line([0,y],[r/sin(angle),(r-y*cos(angle))/sin(angle)],'Color','black')
end;
```

Figure 10.11: A simple MATLAB function for drawing lines on an image

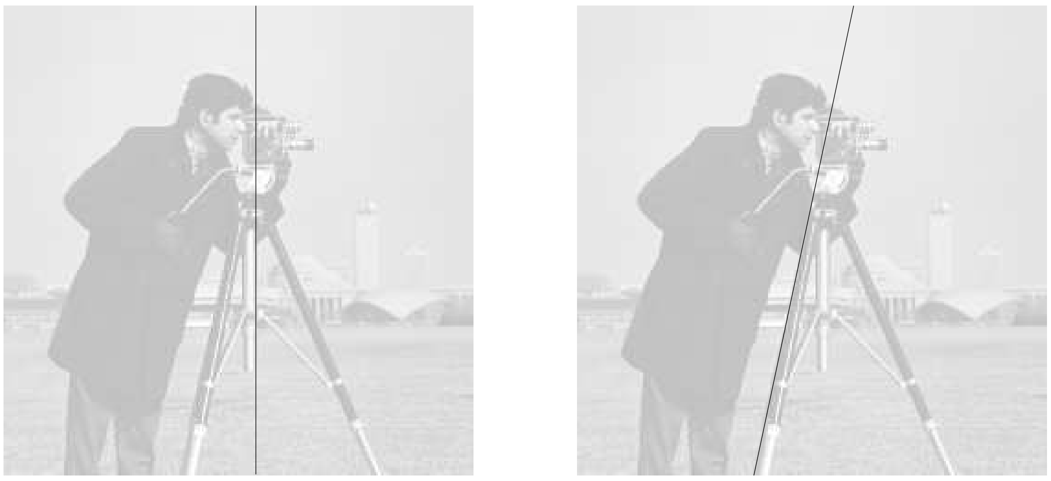


Figure 10.12: The `houghline` function in action

```
    85
    90

theta =

   169
   181
   182
   204
   204
```

Then the command

```
>> houghline(c,r(1),theta(1))
```

will place the line as shown on the right in figure 10.12. Clearly we can use these functions to find any lines we like.

10.3 The distance transform

In many applications, it is necessary to find the distance of every pixel from a region R. This can be done using the standard Euclidean distance:

$$d(i,j) = \min_{(p,q)\in R} \sqrt{(i-p)^2 + (j-q)^2}.$$

This means that to calculate the distance of (i,j) to R, we find all possible distances from (i,j) to pixels in R, and take the smallest value as our distance. This is very computationally inefficient: if R is large, then we have to compute many square roots. A saving can be obtained by noting that since the square root function is increasing, we can write the above definition as

$$d(i,j) = \sqrt{\min_{(p,q)\in R} \left((i-p)^2 + (j-q)^2\right)}$$

which involves only one square root. But even this definition is slow to compute. There is a great deal of arithmetic involved, and the finding of a smallest value.

The *distance transform* is a computationally efficient method of finding such distance. We shall describe it in a sequence of steps:

Step 1. Attach to each pixel (x,y) in the image a label $d(x,y)$ giving its distance from R. Start with labelling each pixel in R with 0, and each pixel not in R with ∞.

Step 2. We now travel through the image pixel by pixel. For each pixel (x,y) replace its label with

$$\min\{d(x,y), d(x+1,y)+1, d(x-1,y)+1, d(x,y-1)+1, d(x,y+1)+1\}$$

using $\infty + 1 = \infty$.

Step 3. Repeat step 2 until all labels have been converted to finite values.

To give some examples of step 2, suppose we have this neighbourhood:

$$\begin{array}{ccc} \infty & \infty & \infty \\ 2 & \infty & \infty \\ \infty & 3 & \infty \end{array}$$

We are only interested in the centre pixel (whose label we are about to change), and the four pixels above, below, and to the left and right. To these four pixels we add 1:

$$\begin{array}{ccc} & \infty & \\ 3 & \infty & \infty \\ & 4 & \end{array}$$

The minimum of these five values is 3, and so that is the new label for the centre pixel.

Suppose we have this neighbourhood:

$$\begin{array}{ccc} 2 & \infty & \infty \\ \infty & \infty & \infty \\ 3 & \infty & 5 \end{array}$$

Again, we add 1 to each of the four pixels above, below, to the left and right, and keep the centre value:

$$\begin{array}{ccc} & \infty & \\ \infty & \infty & \infty \\ & \infty & \end{array}$$

The minimum of these values is ∞, and so at this stage the pixel's label is not changed.

Suppose we take an image whose labels after step 1 are given below

$$\begin{array}{cccccc} \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{array}$$

Step 1

$$\begin{array}{cccccc} \infty & \infty & 1 & 1 & \infty & \infty \\ \infty & 1 & 0 & 0 & 1 & \infty \\ \infty & \infty & 1 & 0 & 1 & \infty \\ \infty & \infty & 1 & 0 & 1 & \infty \\ \infty & \infty & 1 & 0 & 0 & 1 \\ \infty & \infty & \infty & 1 & 1 & \infty \end{array}$$

Step 2 (first pass)

$$\begin{array}{cccccc} \infty & 2 & 1 & 1 & 2 & \infty \\ 2 & 1 & 0 & 0 & 1 & 2 \\ \infty & 2 & 1 & 0 & 1 & 2 \\ \infty & 2 & 1 & 0 & 1 & 2 \\ \infty & 2 & 1 & 0 & 0 & 1 \\ \infty & \infty & 2 & 1 & 1 & 2 \end{array}$$

Step 2 (second pass)

$$\begin{array}{cccccc} 3 & 2 & 1 & 1 & 2 & 3 \\ 2 & 1 & 0 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ \infty & 3 & 2 & 1 & 1 & 2 \end{array}$$

Step 2 (third pass)

$$\begin{array}{cccccc} 3 & 2 & 1 & 1 & 2 & 3 \\ 2 & 1 & 0 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 4 & 3 & 2 & 1 & 1 & 2 \end{array}$$

Step 2 (final pass)

At this stage we stop, as all label values are finite.

An immediate observation is that the distance values given are not in fact a very good approximation to "real" distances. To provide better accuracy, we need to generalize the above transform. One way to do this is to use the concept of a "mask"; the mask used above was

$$\begin{array}{ccc} & 1 & \\ 1 & 0 & 1 \\ & 1 & \end{array}$$

Step two in the transform then consists of adding the corresponding mask elements to labels of the neighbouring pixels, and taking the minimum. To obtain good accuracy with simple arithmetic, the mask will generally consist of integer values, but the final result may require scaling.

Suppose we apply the mask

$$\begin{array}{ccc}
4 & 3 & 4 \\
3 & 0 & 3 \\
4 & 3 & 4
\end{array}$$

to the above image. Step 1 is as above; steps 2 are:

$$\begin{array}{cccccc}
\infty & 4 & 3 & 3 & 4 & \infty \\
\infty & 3 & 0 & 0 & 3 & \infty \\
\infty & 4 & 3 & 0 & 3 & \infty \\
\infty & \infty & 3 & 0 & 3 & \infty \\
\infty & \infty & 3 & 0 & 0 & 3 \\
\infty & \infty & 4 & 3 & 3 & 4
\end{array}
\qquad
\begin{array}{cccccc}
7 & 4 & 3 & 3 & 4 & 7 \\
6 & 3 & 0 & 0 & 3 & 6 \\
7 & 4 & 3 & 0 & 3 & 6 \\
8 & 6 & 3 & 0 & 3 & 6 \\
\infty & 6 & 3 & 0 & 0 & 3 \\
\infty & 7 & 4 & 3 & 3 & 4
\end{array}
\qquad
\begin{array}{cccccc}
7 & 4 & 3 & 3 & 4 & 7 \\
6 & 3 & 0 & 0 & 3 & 6 \\
7 & 4 & 3 & 0 & 3 & 6 \\
8 & 6 & 3 & 0 & 3 & 6 \\
9 & 6 & 3 & 0 & 0 & 3 \\
10 & 7 & 4 & 3 & 3 & 4
\end{array}$$

Step 2 (first pass) Step 2 (second) Step 2 (third)

at which point we stop, and divide all values by three:

$$\begin{array}{cccccc}
2.3 & 1.3 & 1 & 1 & 1.3 & 2.3 \\
2 & 1 & 0 & 0 & 1 & 2 \\
2.3 & 1.3 & 1 & 0 & 1 & 2 \\
2.7 & 2 & 1 & 0 & 1 & 2 \\
3 & 2 & 1 & 0 & 0 & 1 \\
3.3 & 2.3 & 1.3 & 1 & 1 & 1.3
\end{array}$$

These values are much closer to the Euclidean distances than those provided by the first transform.

Even better accuracy can be obtained by using the mask

$$\begin{array}{ccccc}
 & 11 & & 11 & \\
11 & 7 & 5 & 7 & 11 \\
 & 5 & 0 & 5 & \\
11 & 7 & 5 & 7 & 11 \\
 & 11 & & 11 &
\end{array}$$

and dividing the final result by 5:

$$\begin{array}{cccccc}
11 & 7 & 5 & 5 & 7 & 11 \\
10 & 5 & 0 & 0 & 5 & 10 \\
11 & 7 & 5 & 0 & 5 & 10 \\
14 & 10 & 5 & 0 & 5 & 7 \\
16 & 10 & 5 & 0 & 0 & 5 \\
16 & 11 & 7 & 5 & 5 & 7
\end{array}
\qquad
\begin{array}{cccccc}
2.2 & 1.4 & 1 & 1 & 1.4 & 2.2 \\
2 & 1 & 0 & 0 & 1 & 2 \\
2.2 & 1.4 & 1 & 0 & 1 & 2 \\
2.8 & 2 & 1 & 0 & 1 & 1.4 \\
3.2 & 2 & 1 & 0 & 0 & 1 \\
3.2 & 2.2 & 1.4 & 1 & 1 & 1.4
\end{array}$$

Result of transform After division by 5

The method we have described can in fact be very slow; for a large image we may require many passes before all distance labels become finite. A quicker method requires *two* passes only: the first pass starts at the top left of the image, and moves left to right, and top to bottom. The second pass starts at the bottom right of the image, and moves right to left, and from bottom to top. For this method we break the mask up into two halves; one half inspects only values to the left and above

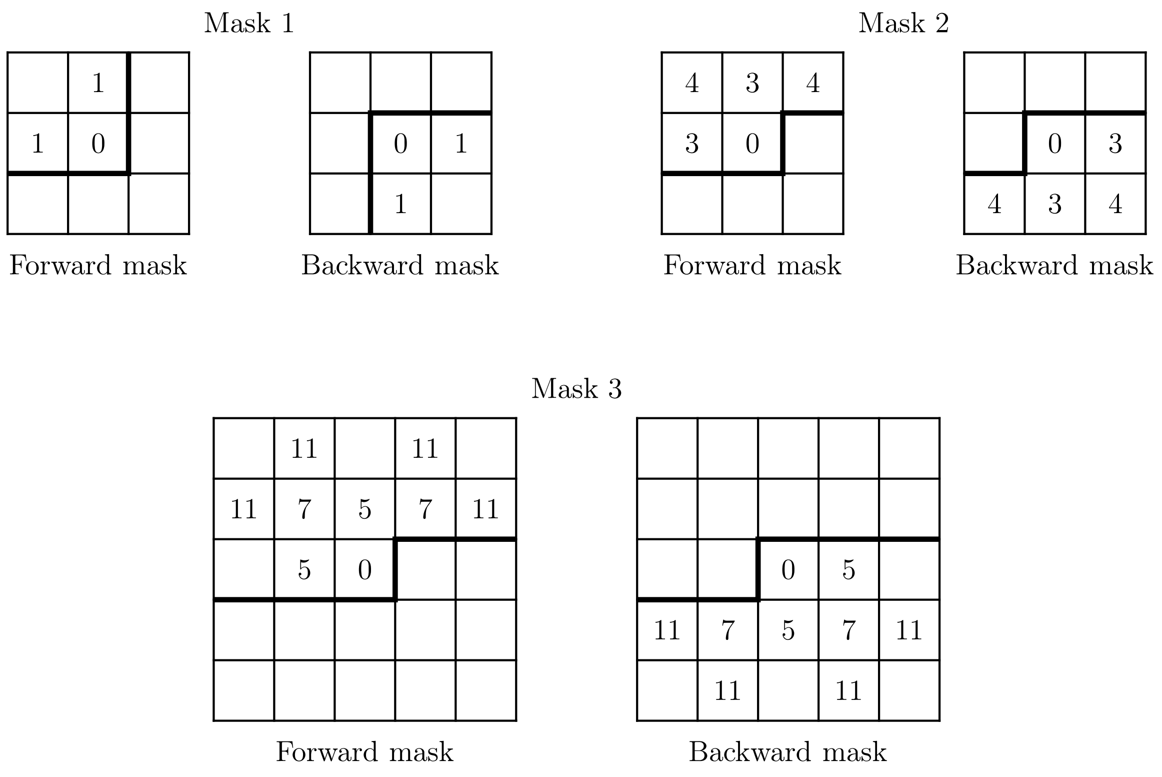


Figure 10.13: Pairs of masks for the two pass distance transform

(this is used for the first pass), and the other half inspects only values to the right and below (this is used for the second pass).

Such pairs of masks are shown in figure 10.13, and the solid lines show how the original mask is broken up into its two halves.

We apply these masks as follows: first surround the image with zeros (such as for spatial filtering), so that at the edges of the image the masks have values to work with. For the forward pass, for each pixel at position (i, j), add the values given in the forward mask to its neighbours and take the minimum, and replace the current label with this minimum. This is similar to step 2 in the original algorithm, except that we are using less pixels. We do the same for the backward pass, except that we use the backwards mask, and we start at the bottom right.

If we apply forward masks 1 and 2 to our image, the results of the forward passes are:

$$\begin{array}{cccccc}
\infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 0 & 0 & 1 & 2 \\
\infty & \infty & 1 & 0 & 1 & 2 \\
\infty & \infty & 2 & 0 & 1 & 2 \\
\infty & \infty & 3 & 0 & 0 & 1 \\
\infty & \infty & 4 & 1 & 1 & 2
\end{array}
\qquad\qquad
\begin{array}{cccccc}
\infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 0 & 0 & 3 & 6 \\
\infty & 4 & 3 & 0 & 3 & 6 \\
8 & 7 & 5 & 0 & 3 & 6 \\
11 & 8 & 4 & 0 & 0 & 3 \\
12 & 8 & 4 & 3 & 3 & 4
\end{array}$$

Use of mask 1 Use of mask 2

After applying the backward masks, we will obtain the distance transforms as above.

Implementation in Matlab

We can easily write a function to perform the distance transform using the second method as above. Our function will implement the transform as follows:

1. Using the size of the mask, pad the image with an appropriate number of zeros.
2. Change each zero to infinity, and each one to zero.
3. Create forward and backwards masks.
4. Perform a forward pass: replace each label with the minimum of its neighbourhood plus the forward mask.
5. Perform a backwards pass: replace each label with the minimum of its neighbourhood plus the backwards mask.

Let's consider each of these steps separately:

1. Suppose the image is of size $r \times c$, and the mask is of size $r_m \times c_m$, where both mask dimensions are odd numbers. We need to add, on each side of the image, a number of columns equal to $(c_m - 1)/2$, and we need to add, on both top and bottom of the image, a number of rows equal to $(r_m - 1)/2$. In other words we can embed the image in a larger array of size $(r + r_m - 1) \times (c + c_m - 1)$:

```
>> [r,c]=size(image);
>> [mr,mc]=size(mask);
>> nr=(mr-1)/2;
>> nc=(mc-1)/2;
>> image2=zeros(r+mr-1,c+mc-1);
>> image2(nr+1:r+nr,nc+1:c+nc)=image;
```

2. This can be easily done using the `find` function: first change all zeros to infinity, and next change all ones to zeros:

```
>> image2(find(image2)==0)=Inf;
>> image2(find(image2)==1)=0;
```

3. Suppose we are given a forward mask. We can do this most simply by making all blank entries in the masks shown in figure 10.13 infinity; this means that pixels in these positions will have no effect on the final minimum. The backwards mask can be obtained by two $90°$ rotations. For example:

```
>> mask1=[Inf 1 Inf;1 0 Inf;Inf Inf Inf];
>> backmask=rot90(rot90(mask1));
```

4. We can implement the forward mask with nested loops:

```
>> for i=nr+1:r+nr,
     for j=nc+1:c+nc,
       image2(i,j)=min(min(image2(i-nr:i+nr,j-nc:j+nc)+mask));
     end;
   end;
```

5. The backward pass is done similarly:

```
>> for i=r+nr:-1:nr+1,
     for j=c+nc:-1:nc+1,
       image2(i,j)=min(min(image2(i-nr:i+nr,j-nc:j+nc)+backmask));
     end;
   end;
```

The full function is shown in figure 10.14.

Let's try it out, first creating our image, and the three masks:

```
>> im=[0 0 0 0 0 0;...
0 0 1 1 0 0;...
0 0 0 1 0 0;...
0 0 0 1 0 0;...
0 0 0 1 1 0;...
0 0 0 0 0 0]

im =

     0     0     0     0     0     0
     0     0     1     1     0     0
     0     0     0     1     0     0
     0     0     0     1     0     0
     0     0     0     1     1     0
     0     0     0     0     0     0
```

```
function res=disttrans(image,mask)
%
% This function implements the distance transform by applying MASK to
% IMAGE, using the two step algorithm with "forward" and "backwards" masks.
%
backmask=rot90(rot90(mask));
[mr,mc]=size(mask);
if ((floor(mr/2)==ceil(mr/2)) | (floor(mc/2)==ceil(mc/2))) then
     error('The mask must have odd dimensions.')
     end;
[r,c]=size(image);
nr=(mr-1)/2;
nc=(mc-1)/2;
image2=zeros(r+mr-1,c+mc-1);
image2(nr+1:r+nr,nc+1:c+nc)=image;
%
% This is the first step; replacing R values with 0 and other values
% with infinity
%
image2(find(image2==0))=Inf;
image2(find(image2==1))=0;
%
% Forward pass
%
for i=nr+1:r+nr,
  for j=nc+1:c+nc,
    image2(i,j)=min(min(image2(i-nr:i+nr,j-nc:j+nc)+mask));
  end;
end;
%
% Backward pass
%
for i=r+nr:-1:nr+1,
  for j=c+nc:-1:nc+1,
    image2(i,j)=min(min(image2(i-nr:i+nr,j-nc:j+nc)+backmask));
  end;
end;

res=image2(nr+1:r+nr,nc+1:c+nc);
```

Figure 10.14: A function for computing the distance transform

```
>> mask1=[Inf 1 Inf;1 0 Inf;Inf Inf Inf]

mask1 =

   Inf     1   Inf
     1     0   Inf
   Inf   Inf   Inf

>> mask2=[4 3 4;3 0 Inf;Inf Inf Inf]

mask2 =

     4     3     4
     3     0   Inf
   Inf   Inf   Inf

>> mask3=[Inf 11 Inf 11 Inf;...
11 7 5 7 11;...
Inf 5 0 Inf Inf;...
Inf Inf Inf Inf Inf;...
Inf Inf Inf Inf Inf]

mask3 =

   Inf    11   Inf    11   Inf
    11     7     5     7    11
   Inf     5     0   Inf   Inf
   Inf   Inf   Inf   Inf   Inf
   Inf   Inf   Inf   Inf   Inf
```

Now we can apply the transform:

```
>> disttrans(im,mask1)

ans =

     3     2     1     1     2     3
     2     1     0     0     1     2
     3     2     1     0     1     2
     3     2     1     0     1     2
     3     2     1     0     0     1
     4     3     2     1     1     2

>> disttrans(im,mask2)

ans =
     7     4     3     3     4     7
     6     3     0     0     3     6
     7     4     3     0     3     6
     8     6     3     0     3     4
     9     6     3     0     0     3
    10     7     4     3     3     4

>> disttrans(im,mask3)

ans =

    11     7     5     5     7    11
    10     5     0     0     5    10
    11     7     5     0     5    10
    14    10     5     0     5     7
    15    10     5     0     0     5
    16    11     7     5     5     7
```

Now the results of the transforms usings masks 2 and 3 should be divided by appropriate values to get aproxiations to the true distances.

We can of course apply the distance transform to a large image; say `circles.tif`:

```
>> c=~imread('circles.tif');
>> imshow(c)
>> cd=disttrans(c,mask1);
>> figure,imshow(mat2gray(cd))
```

Note that we invert the circles image to produce black circles on a white background. This means that our transform will find distances towards the inside of the original circles. The only reason for doing this is that it is easier to make sense of the resulting image. The circles image is shown on the leftt in figure 10.15 and the distance transform on the right.

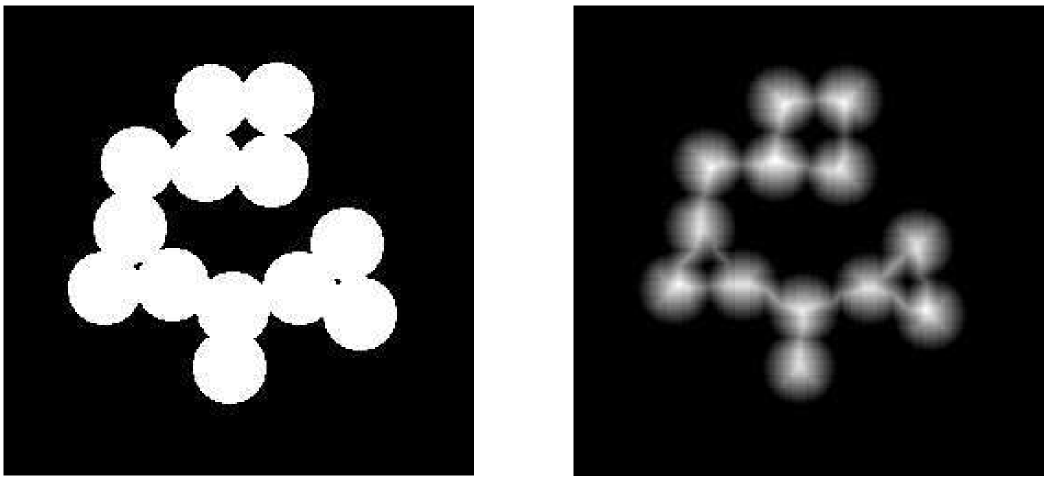


Figure 10.15: An example of a distance transform

Application to skeletonization

The distance transform can be used to provide the skeleton of a region R. We apply the distance transform, using mask 1, to the image negative, just as we did for the circle above. Then the skeleton consist of those pixels (i,j) for which

$$d(i,j) \geq \max\{d(i-1,j), d(i+1,j), d(i,j-1), d(i,j+1)\}.$$

For example, suppose we take a small region concisting of a single rectangle, and find the distance transform of its negative:

```
>> c=zeros(7,9);c(2:6,2:8)=1

c =

     0     0     0     0     0     0     0     0     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     0     0     0     0     0     0     0     0

>> cd=disttrans(~c,mask1)

cd =

     0     0     0     0     0     0     0     0     0
     0     1     1     1     1     1     1     1     0
     0     1     2     2     2     2     2     1     0
     0     1     2     3     3     3     2     1     0
     0     1     2     2     2     2     2     1     0
     0     1     1     1     1     1     1     1     0
     0     0     0     0     0     0     0     0     0
```

We can obtain the skeleton using a double loop:

```
>> skel=zeros(size(c));
>> for i=2:6,
     for j=2:8,
       if cd(i,j)>=max([cd(i-1,j),cd(i+1,j),cd(i,j-1),cd(i,j+1)])
         skel(i,j)=1;
       end;
     end;
   end;

>> skel

skel =
     0     0     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     1     0
     0     0     1     0     0     0     1     0     0
     0     0     0     1     1     1     0     0     0
     0     0     1     0     0     0     1     0     0
     0     1     0     0     0     0     0     1     0
     0     0     0     0     0     0     0     0     0
```

In fact we can produce the skeleton more efficiently by using MATLAB's `ordfilt2` function, which we met in chapter 7. This can be used to find the largest value in a neighbourhood, and the neighbourhood can be very precisely defined:

```
>> cd2=ordfilt2(cd,5,[0 1 0;1 1 1;0 1 0])

cd2 =

     0     1     1     1     1     1     1     1     0
     1     1     2     2     2     2     2     1     1
     1     2     2     3     3     3     2     2     1
     1     2     3     3     3     3     3     2     1
     1     2     2     3     3     3     2     2     1
     1     1     2     2     2     2     2     1     1
     0     1     1     1     1     1     1     1     0

>> cd2<=cd

ans =

     1     0     0     0     0     0     0     0     1
     0     1     0     0     0     0     0     1     0
     0     0     1     0     0     0     1     0     0
     0     0     0     1     1     1     0     0     0
     0     0     1     0     0     0     1     0     0
     0     1     0     0     0     0     0     1     0
     1     0     0     0     0     0     0     0     1
```

We can easily restrict the image so as not to obtain the extra 1's in the corners of the result. Now let's do the same thing with our circles image:

```
>> c=~imread('circles.tif');
>> cd=disttrans(c,mask1);
>> cd2=ordfilt2(cd,5,[0 1 0;1 1 1;0 1 0]);
>> imshow((cd2<=cd)&~c)
```

and the result is shown in figure 10.16. The use of the command `(cd2<=cd)&~c` blocks out the outside of the circles, so that just the skeleton is left. We will see in chapter 11 how to thicken this skeleton, and also other ways of obtaining the skeleton.

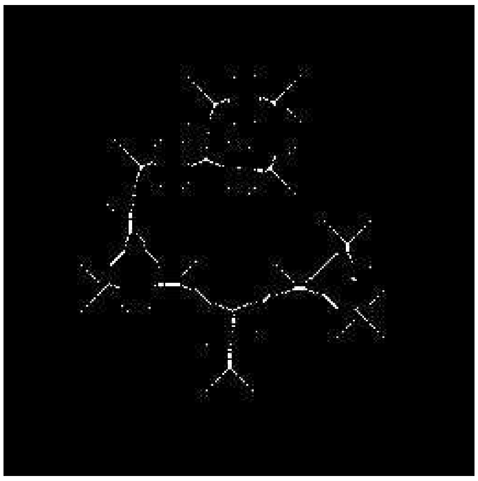
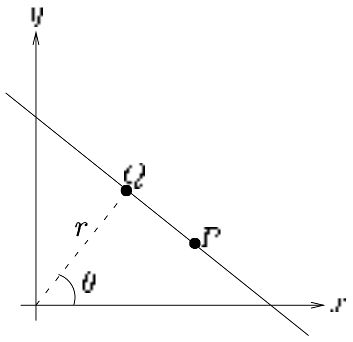


Figure 10.16: Skeletonization using the distance transform

Exercises

1. In the diagram shown $P = (x, y)$ is any point on the line l and Q is the point on the line closest to the origin.



   (a) Find the coordinates of Q in terms of r and θ.

   (b) Hence find the vector $\vec{OP}$.

   (c) Also find the vector $\vec{QP}$.

   (d) What is known about the dot product (inner product) of these two vectors?

   (e) Deduce that (x, y) satisfies $x \cos\theta + y \sin\theta = r$.

2. We define r and θ as above.

   (a) A straight line has $r = \sqrt{2}$ and $\theta = \pi/4$. What is the equation of the line?

   (b) Find r and θ for the lines

$$\sqrt{3}x + y = 4, \qquad x = 2, \qquad y = 0.$$

3. Use the Hough transform to detect the strongest line in the binary image shown below. Use the form $x\cos\theta + y\sin\theta = r$ with θ in steps of 45° from −45° to 90° and place the results in an accumulator array.

					x			
		−3	−2	−1	0	1	2	3
	−3	0	0	0	0	0	1	0
	−2	0	0	0	0	0	0	0
	−1	0	1	0	1	0	1	0
y	0	0	0	1	0	0	0	0
	1	0	0	0	0	0	0	0
	2	1	0	0	0	0	1	0
	3	0	0	0	0	0	0	0

4. Repeat the above question with the images:

					x			
		−3	−2	−1	0	1	2	3
	−3	0	0	0	0	1	0	0
	−2	0	0	0	0	0	0	0
	−1	0	0	1	0	0	0	1
y	0	0	1	0	0	1	0	0
	1	1	0	0	0	0	0	1
	2	0	0	0	0	1	0	0
	3	0	0	0	1	1	0	0

					x			
		−3	−2	−1	0	1	2	3
	−3	0	0	0	1	0	0	0
	−2	1	0	0	0	1	0	0
	−1	0	0	0	0	0	0	0
y	0	0	0	1	0	0	1	0
	1	0	1	0	1	0	0	0
	2	1	0	0	0	0	0	1
	3	0	0	1	0	0	1	0

5. Find some more lines on the cameraman image, and plot them with `houghline`.

6. Read and display the image `alumgrns.tif`.

   (a) Where does it appear that the "strongest" lines will be?

   (b) Using `hough` and `houghline`, plot the five strongest lines.

7. Experiment with the two routines by changing the initial edge detection of `hough`. Can you affect the lines found by the Hough transform?

8. For each of the following images:

0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0
0	1	1	1	1	0	0	0
0	0	1	1	1	1	0	0
0	0	0	1	1	1	1	1
0	1	1	1	1	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	1	1	1	1	0	0
0	1	1	1	1	1	1	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	0
0	0	0	0	0	0	0	0

apply the distance transform to approximate distances from the region containing 1's to all other pixels in the image, using the masks:

$$\text{(i)}\quad \begin{array}{ccc} & 1 & \\ 1 & 0 & 1 \\ & 1 & \end{array} \qquad \text{(ii)}\quad \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array} \qquad \text{(iii)}\quad \begin{array}{ccc} 4 & 3 & 4 \\ 3 & 0 & 3 \\ 4 & 3 & 4 \end{array}$$

$$\text{(iv)}\quad \begin{array}{ccccc} & 11 & & 11 & \\ 11 & 7 & 5 & 7 & 11 \\ & 5 & 0 & 5 & \\ 11 & 7 & 5 & 7 & 11 \\ & 11 & & 11 & \end{array}$$

and applying any necessary scaling at the end.

9. Apply the distance transform and use it to find the skeleton in the images `circlesm.tif` and `nicework.tif`.

Chapter 11

Morphology

11.1 Introduction

Morphology, or *mathematical morphology* is a branch of image processing which is particularly useful for analysing shapes in images. Although morphology can be applied to grey-scale images, we shall only investigate binary morphology. MATLAB has many tools for binary morphology in the image processing toolbox; most of which can be used for greyscale morphology as well.

11.2 Basic ideas

Translation

Suppose that A is a set of pixels in a binary image, and $w = (x, y)$ is a particular coordinate point. Then A_w is the set A "translated" in direction (x, y). That is

$$A_x = \{(a, b) + (x, y) : (a, b) \in A\}.$$

For example, in figure 11.1, A is the cross shaped set, and $w = (2, 2)$. The set A has been shifted

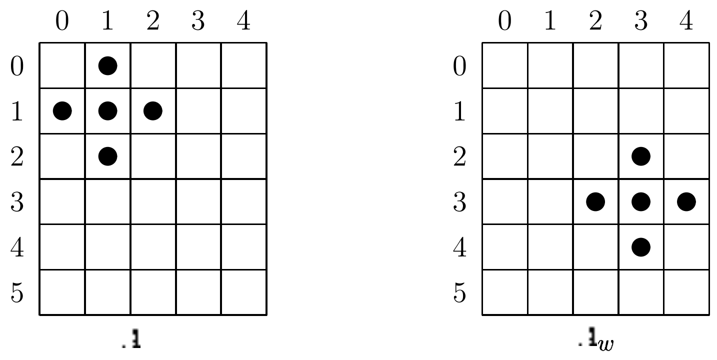


Figure 11.1: Translation

in the x and y directions by the values given in w. Note that here we are using matrix coordinates, rather than Cartesian coordinates, so that the origin is at the top left, x goes down and y goes across.

Reflection

If A is set of pixels, then its *reflection*, denoted $\hat{A}$, is obtained by reflecting A in the origin:

$$\hat{A} = \{(-x, -y) : (x, y) \in A\}.$$

For examples, in figure 11.2, the open and closed circles form sets which are reflections of each other.

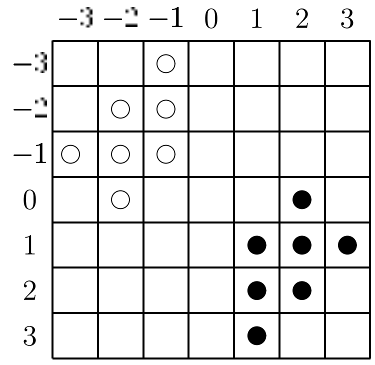


Figure 11.2: Reflection

11.3 Dilation and erosion

These are the basic operations of morphology, in the sense that all other operations are built from a combination of these two.

11.3.1 Dilation

Suppose A and B are sets of pixels. Then the *dilation of A by B*, denoted $A \oplus B$, is defined as

$$A \oplus B = \bigcup_{x \in B} A_x.$$

What this means is that for every point $x \in B$, we translate A by those coordinates. Then we take the union of all these translations.

An equivalent definition is that

$$A \oplus B = \{(x, y) + (u, v) : (x, y) \in A, (u, v) \in B\}.$$

From this last definition, we see that dilation is commutative; that

$$A \oplus B = B \oplus A.$$

An example of a dilation is given in figure 11.3. In the translation diagrams, the grey squares show the original position of the object. Note that $A_{(0,0)}$ is of course just A itself. In this example, we have

$$B = \{(0, 0), (1, 1), (-1, 1), (1, -1), (-1, -1)\}$$

and those these are the coordinates by which we translate A.

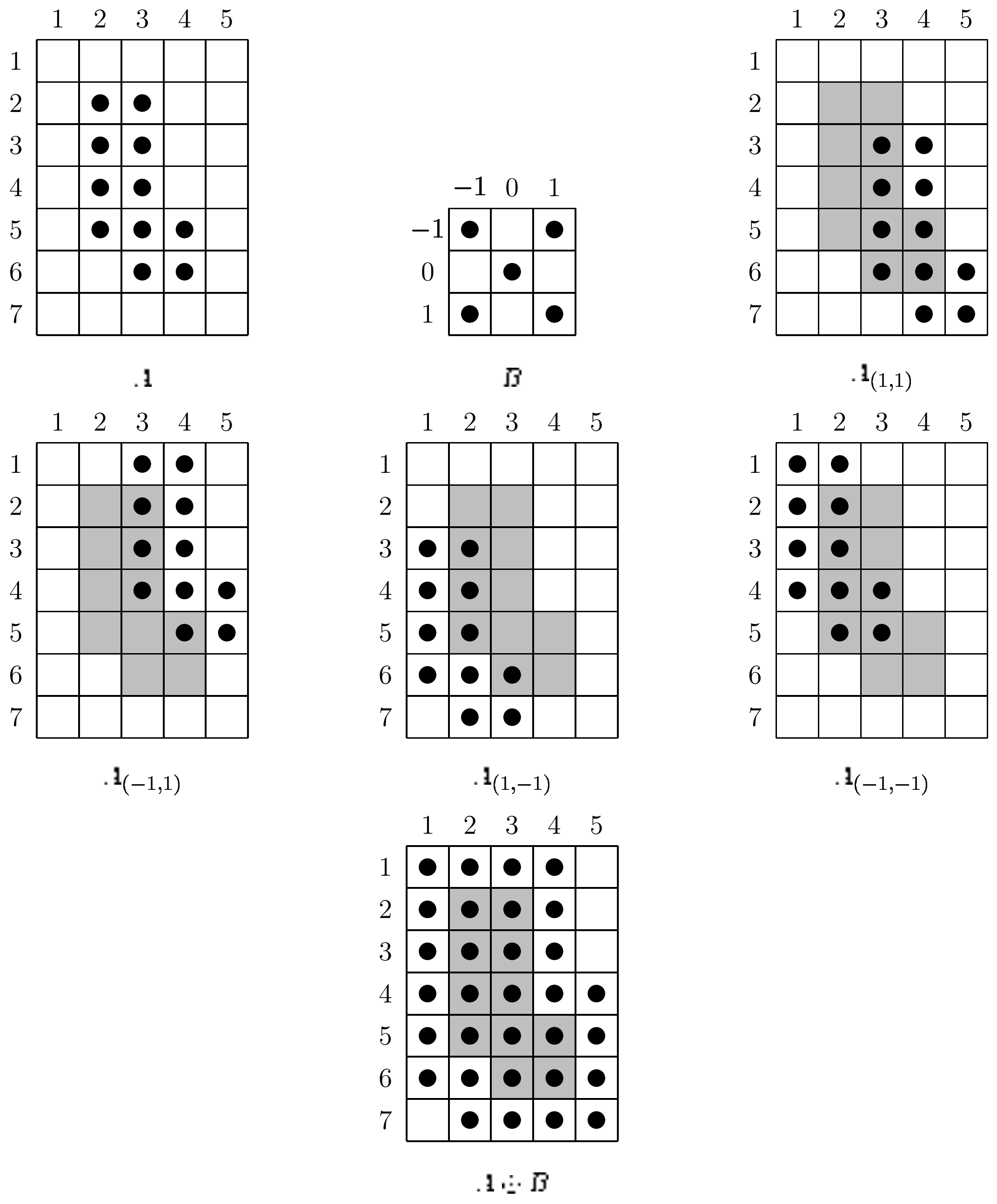


Figure 11.3: Dilation

In general, $A \oplus B$ can be obtained by replacing every point (x, y) in A with a copy of B, placing the $(0, 0)$ point of B at (x, y). Equivalently, we can replace every point (u, v) of B with a copy of A.

As you see in figure 11.3, dilation has the effect of increasing the size of an object. However, it is not necessarily true that the original object A will lie within its dilation $A \oplus B$. Depending on the coordinates of B, $A \oplus B$ may end up quite a long way from A. Figure 11.4 gives an example of this: A is the same as in figure 11.3; B has the same shape but a different position. In this figure, we have

$$B = \{(7,3), (6,2), (6,4), (8,2), (8,4)\}$$

so that

$$A \oplus B = A_{(7,3)} \cup A_{(6,2)} \cup A_{(6,4)} \cup A_{(8,2)} \cup A_{(8,4)}.$$

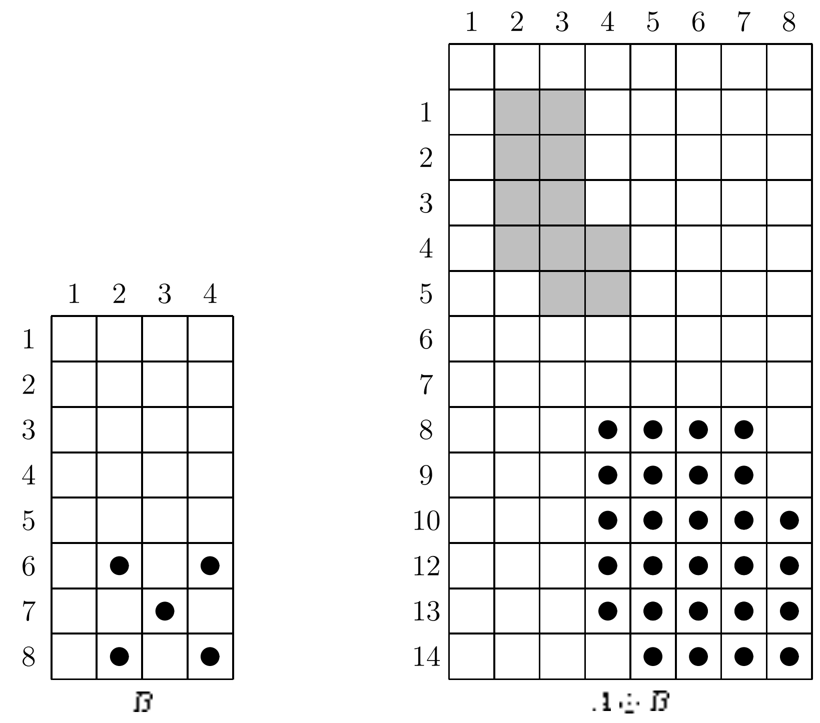


Figure 11.4: A dilation for which $A \not\subseteq A \oplus B$

For dilation, we generally assume that A is the image being processed, and B is a small set of pixels. In this case B is referred to as a *structuring element* or as a *kernel*.

Dilation in MATLAB is performed with the command

```
>> imdilate(image,kernel)
```

To see an example of dilation, consider the commands:

```
>> t=imread('text.tif');
>> sq=ones(3,3);
>> td=imdilate(t,sq);
>> subplot(1,2,1),imshow(t)
>> subplot(1,2,2),imshow(td)
```

The result is shown in figure 11.5. Notice how the image has been "thickened". This is really what dilation does; hence its name.

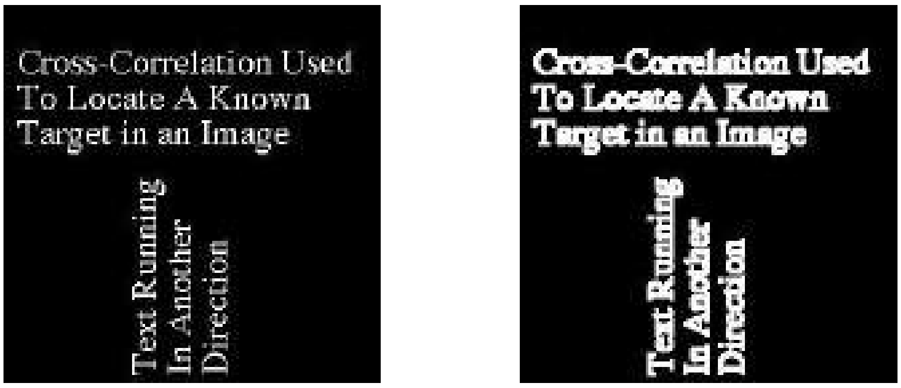


Figure 11.5: Dilation of a binary image

11.3.2 Erosion

Given sets A and B, the *erosion of A by B*, written $A \ominus B$, is defined as:

$$A \ominus B = \{w : B_w \subseteq A\}.$$

In other words the erosion of A by B consists of all points $w = (x, y)$ for which B_w is in A. To perform an erosion, we can move B over A, and find all the places it will fit, and for each such place mark down the corresponding $(0, 0)$ point of B. The set of all such points will form the erosion.

An example of erosion is given in figures 11.6.

Note that in the example, the erosion $A \ominus B$ was a subset of A. This is not necessarily the case; it depends on the position of the origin in B. If B contains the origin (as it did in figure 11.6), then the erosion will be a subset of the original object.

Figure 11.7 shows an example where B does not contain the origin. In this figure, the open circles in the right hand figure form the erosion.

Note that in figure 11.7, the *shape* of the erosion is the same as that in figure 11.6; however its *position* is different. Since the origin of B in figure 11.7 is translated by $(-4, -3)$ from its position in figure 11.6, we can assume that the erosion will be translated by the same amount. And if we compare figures 11.6 and 11.7, we can see that the second erosion has indeed been shifted by $(-4, -3)$ from the first.

For erosion, as for dilation, we generally assume that A is the image being processed, and B is a small set of pixels: the structuring element or kernel.

Erosion in Matlab is performed with the command

```
>> imerode(image,kernel)
```

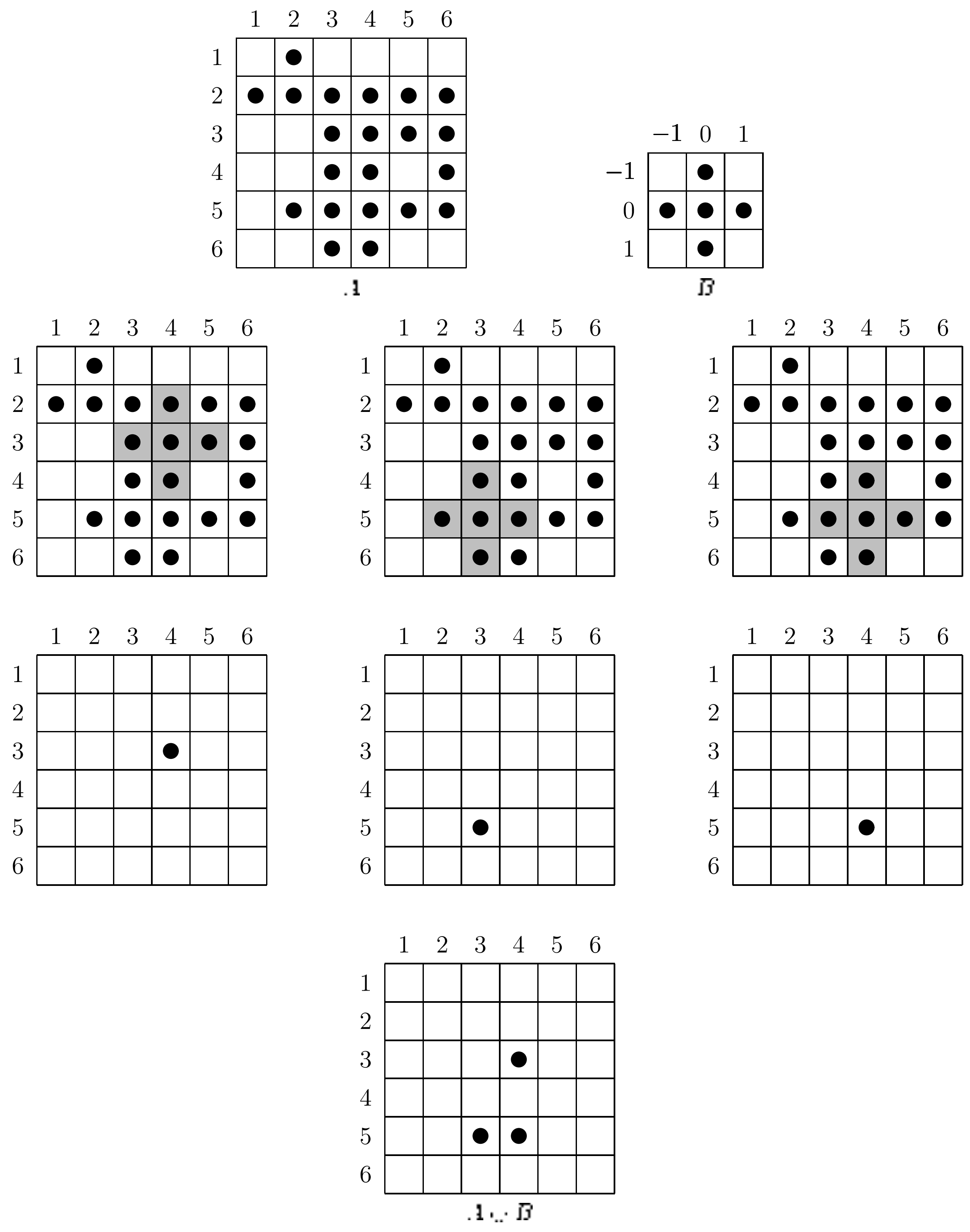


Figure 11.6: Erosion with a cross-shaped structuring element

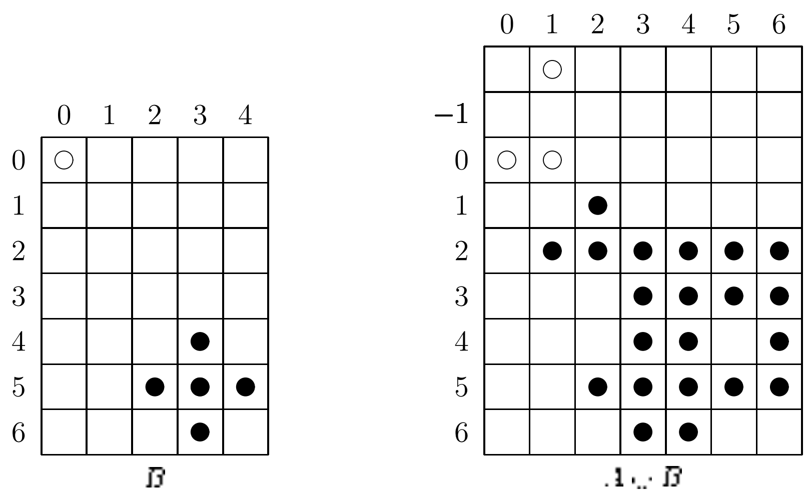


Figure 11.7: Erosion with a structuring element not containing the origin

We shall give an example; using a different binary image:

```
>> c=imread('circbw.tif');
>> ce=imerode(c,sq);
>> subplot(1,2,1),imshow(c)
>> subplot(1,2,2),imshow(ce)
```

The result is shown in figure 11.8. Notice how the image has been "thinned". This is the expected result of an erosion; hence its name. If we kept on eroding the image, we would end up with a completely black result.

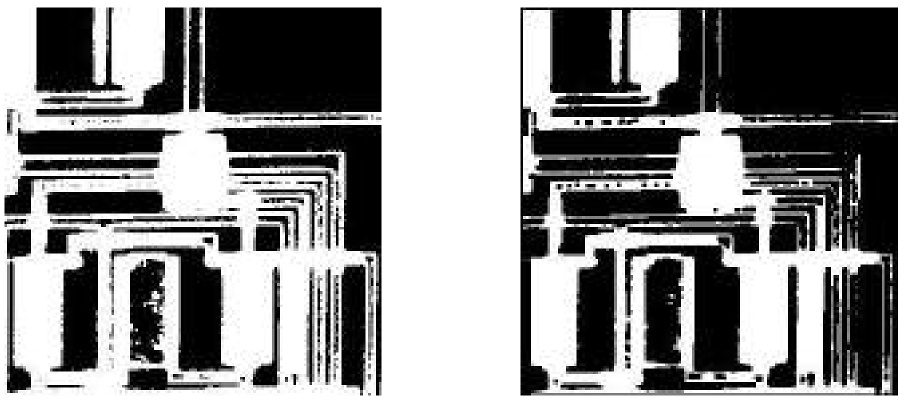


Figure 11.8: Erosion of a binary image

11.3.3 Relationship between erosion and dilation

It can be shown that erosion and dilation are "inverses" of each other; more precisely, the complement of an erosion is equal to the dilation of the complement. Thus:

$$\overline{A \ominus B} = \overline{A} \oplus \hat{B}.$$

To show this, see the proof in the box below.

First note that, by its definition:

$$\overline{A \ominus B} = \overline{\{w : B_w \subseteq A\}}.$$

We also know that from elementary properties of subsets, that if $B_w \subseteq A$, then $B_w \cap \overline{A} = \emptyset$. Thus:

$$\overline{A \ominus B} = \overline{\{w : B_w \cap \overline{A} = \emptyset\}}. \tag{11.1}$$

But the complement of $\{w : B_w \cap \overline{A} = \emptyset\}$ is clearly the set

$$\{w : B_w \cap \overline{A} \neq \emptyset\}.$$

Recall from the definition of dilation that

$$\begin{aligned} A \oplus B &= \cup_{w \in B} A_w \\ &= \{a + b : a \in A, \quad b \in B\}. \end{aligned}$$

So if $x \in A \oplus B$, then $x = a + b$ for some $a \in A$ and $b \in B$. Then $x - b = a$. Thus:

$$\begin{aligned} A \oplus B &= \{x : x - b = a, b \in B, \ a \in A\}, \\ &= \{x : -b + x = a, b \in B, \ a \in A\}, \\ &= \{x : \hat{B}_x \cap A \neq \emptyset\}. \end{aligned}$$

Replacing A with $\overline{A}$ in the last equation produces

$$\overline{A} \oplus B = \{x : \hat{B}_x \cap \overline{A} \neq \emptyset\}. \tag{11.2}$$

Now compare equations 11.1 and 11.2, and use the fact that $\hat{\hat{B}} = B$. Then

$$\overline{A \ominus B} = \overline{A} \oplus \hat{B}$$

as required.

It can be similarly shown that the same relationship holds if erosion and dilation are interchanged; that

$$\overline{A \oplus B} = \overline{A} \ominus \hat{B}.$$

We can demonstrate the truth of these using MATLAB commands; all we need to know is that the complement of a binary image

```
b
```

is obtained using

```
>> ~b
```

and that given two images `a` and `b`; their equality is determined with

```
>> all(a(:)==b(:))
```

To demonstrate the equality

$$\overline{A \ominus B} = \overline{A} \oplus \hat{B}.$$

pick a binary image, say the text image, and a structuring element. Then the left hand side of this equation is produced with

```
>> lhs=~imerode(t,sq);
```

and the right hand side with

```
>> rhs=imdilate(~t,sq);
```

Finally, the command

```
>> all(lhs(:)==rhs(:))
```

should return 1, for true.

11.3.4 An application: boundary detection

If A is an image, and B a small structuring element consisting of point symmetrically places about the origin, then we can define the boundary of A by any of the following methods:

(i)	$A - (A \ominus B)$	" internal boundary"
(ii)	$(A \oplus B) - A$	"external boundary"
(iii)	$(A \oplus B) - (A \ominus B)$	"morphological gradient"

In each definition the minus refers to set difference. For some examples, see figure 11.9. We see that the internal boundary consists of those pixels in A which are at its edge; the external boundary consists of pixels outside A which are just next to it, and that the morphological gradient is a combination of both the internal and external boundaries.

To see some examples, choose the image `rice.tif`, and threshold it to obtain a binary image:

```
>> rice=imread('rice.tif');
>> r=rice>110;
```

Then the internal boundary is obtained with:

```
>> re=imerode(r,sq);
>> r_int=r&~re;
>> subplot(1,2,1),imshow(r)
>> subplot(1,2,2),imshow(r_int)
```

The result is shown in figure 11.10.
The external boundary and morphological gradients can be obtained similarly:

```
>> rd=imdilate(r,sq);
>> r_ext=rd&~r;
>> r_grad=rd&~re;
>> subplot(1,2,1),imshow(r_ext)
>> subplot(1,2,2),imshow(r_grad)
```

The results are shown in figure 11.11.

Note that the external boundaries are larger than the internal boundaries. This is because the internal boundaries show the outer edge of the image components; whereas the external boundaries show the pixels just outside the components. The morphological gradient is thicker than either, and is in fact the union of both.

11.4 Opening and closing

These operations may be considered as "second level" operations; in that they build on the basic operations of dilation and erosion. They are also, as we shall see, better behaved mathematically.

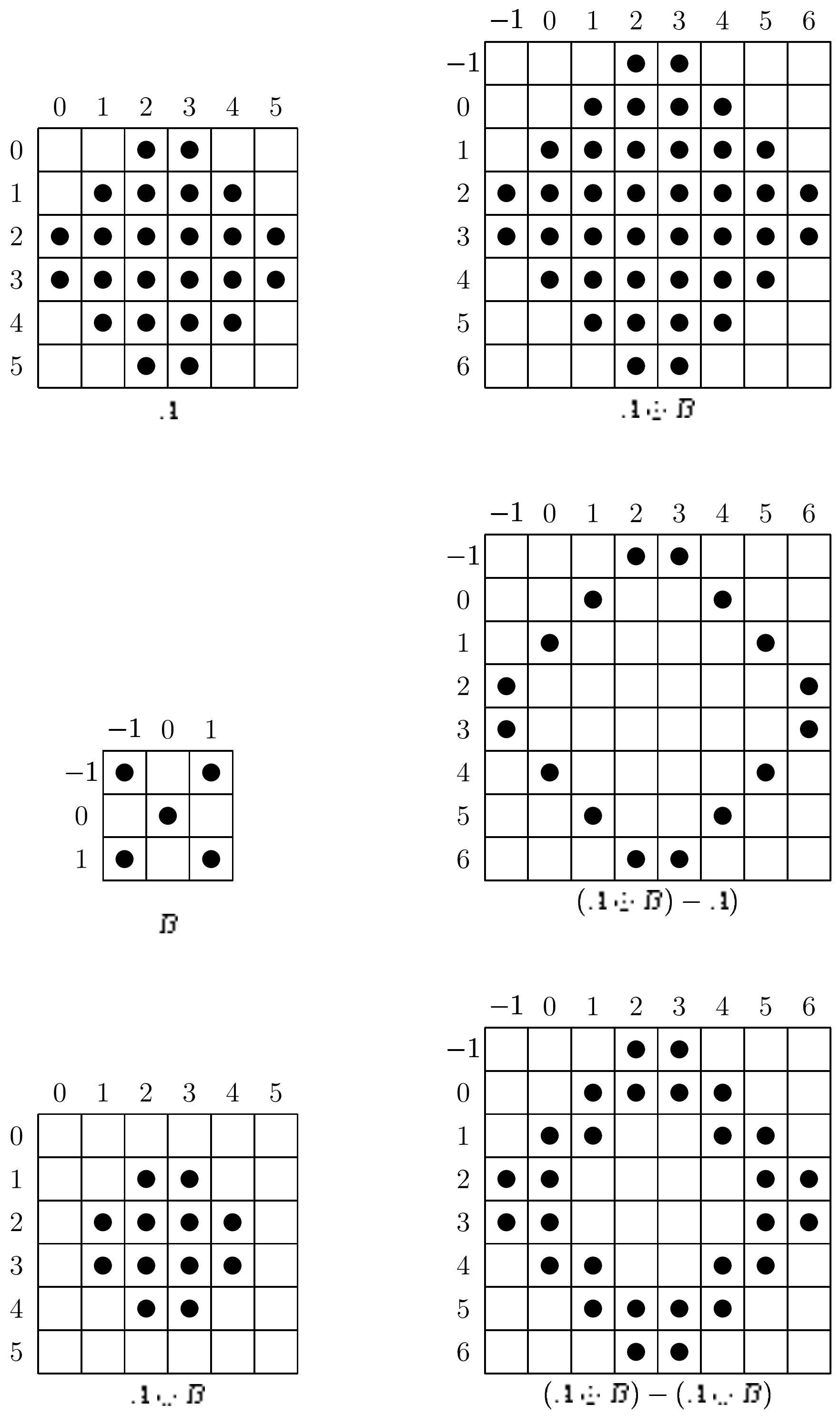


Figure 11.9: Boundaries

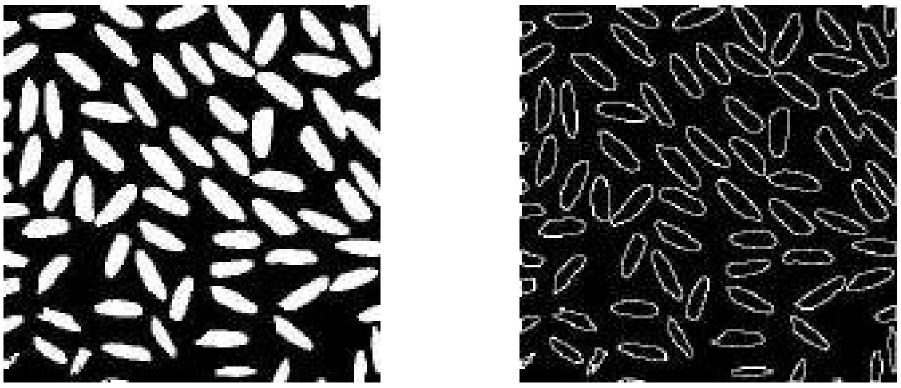


Figure 11.10: “Internal boundary” of a binary image

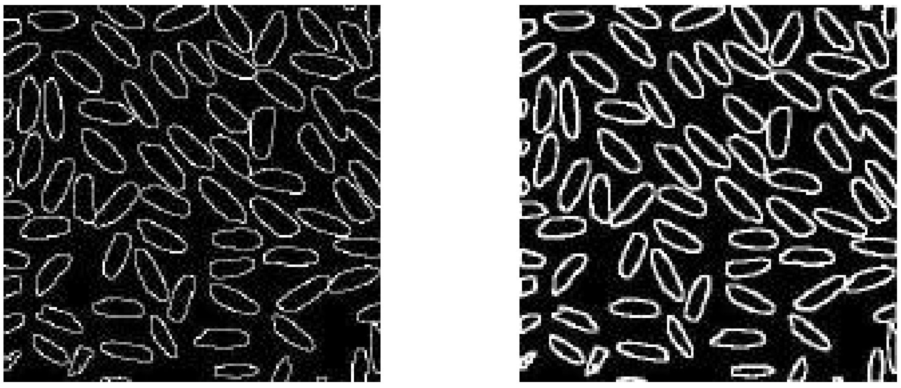


Figure 11.11: “External boundary” and the morphological gradient of a binary image

11.4.1 Opening

Given A and a structuring element B, the *opening of A by B*, denoted $A \circ B$, is defined as:

$$A \circ B = (A \ominus B) \oplus B.$$

So an opening consists of an erosion followed by a dilation. An equivalent definition is

$$A \circ B = \cup\{B_w : B_w \subseteq A\}.$$

That is, $A \circ B$ is the union of all translations of B which fit inside A. Note the difference with erosion: the erosion consists only of the $(0,0)$ point of B for those translations which fit inside A; the opening consists of all of B. An example of opening is given in figure 11.12.

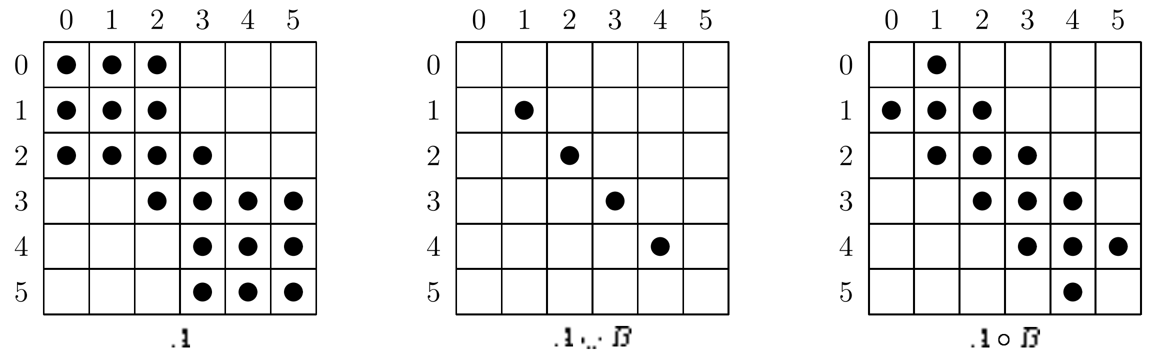


Figure 11.12: Opening

The opening operation satisfies the following properties:

1. $(A \circ B) \subseteq A$. Note that this is not the case with erosion; as we have seen, an erosion may not necessarily be a subset.
2. $(A \circ B) \circ B = A \circ B$. That is, an opening can never be done more than once. This property is called *idempotence*. Again, this is not the case with erosion; you can keep on applying a sequence of erosions to an image until nothing is left.
3. If $A \subseteq C$, then $(A \circ B) \subseteq (C \circ B)$.
4. Opening tends to "smooth" an image, to break narrow joins, and to remove thin protrusions.

11.4.2 Closing

Analogous to opening we can define *closing*, which may be considered as a dilation followed by an erosion, and is denoted $A \bullet B$:

$$A \bullet B = (A \oplus B) \ominus B.$$

Another definition of closing is that $x \in A \bullet B$ if *all* translations B_w which contain x have non-empty intersections with A. An example of closing is given in figure 11.13. The closing operation satisfies the following properties:

1. $A \subseteq (A \bullet B)$.

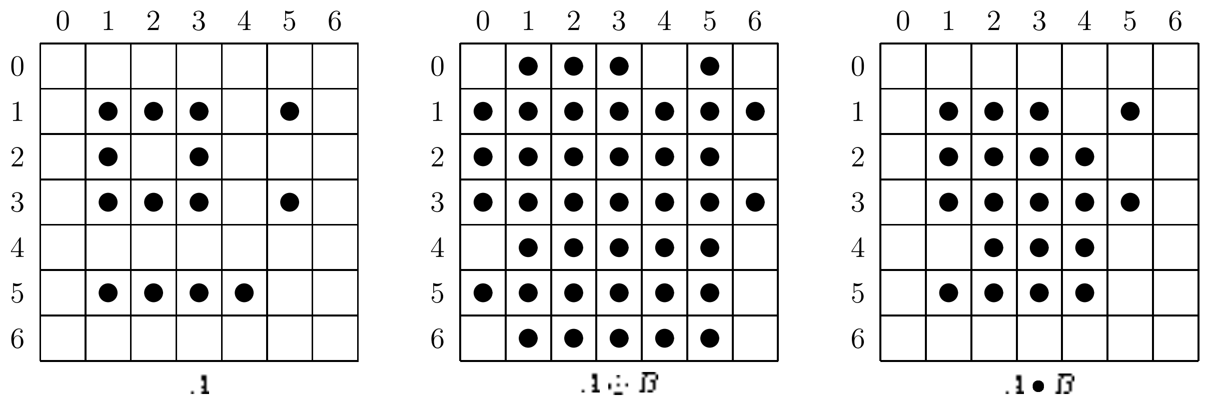


Figure 11.13: Closing

2. $(A \bullet B) \bullet B = A \bullet B$; that is, closing, like opening, is idempotent.
3. If $A \subseteq C$, then $(A \bullet B) \subseteq (C \bullet B)$.
4. Closing tends also to smooth an image, but it fuses narrow breaks and thin gulfs, and eliminates small holes.

Opening and closing are implemented by the `imopen` and `imclose` functions respectively. We can see the effects on a simple image using the square and cross structuring elements.

```
>> cr=[0 1 0;1 1 1;0 1 0];
>> >> test=zeros(10,10);test(2:6,2:4)=1;test(3:5,6:9)=1;test(8:9,4:8)=1;test(4,5)=1

test =

     0     0     0     0     0     0     0     0     0     0
     0     1     1     1     0     0     0     0     0     0
     0     1     1     1     0     1     1     1     1     0
     0     1     1     1     1     1     1     1     1     0
     0     1     1     1     0     1     1     1     1     0
     0     1     1     1     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     0     0     0     0     0     0     0

>> imopen(test,sq)

ans =

     0     0     0     0     0     0     0     0     0     0
     0     1     1     1     0     0     0     0     0     0
     0     1     1     1     0     1     1     1     1     0
     0     1     1     1     0     1     1     1     1     0
     0     1     1     1     0     1     1     1     1     0
     0     1     1     1     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0

>> imopen(test,cr)

ans =

     0     0     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0
     0     1     1     1     0     1     1     1     0     0
     0     1     1     1     1     1     1     1     1     0
     0     1     1     1     0     1     1     1     0     0
     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
```

Note that in each case the image has been separated into distinct components, and the lower part has been removed completely.

```
>> imclose(test,sq)

ans =

     1     1     1     1     0     0     0     0     0     0
     1     1     1     1     0     0     0     0     0     0
     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     1     1     1     1     1     0     0

>> imclose(test,cr)

ans =

     0     0     1     0     0     0     0     0     0     0
     0     1     1     1     0     0     0     0     0     0
     1     1     1     1     1     1     1     1     1     0
     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0     0
     0     0     1     1     1     1     1     0     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     1     1     1     1     1     0     0
     0     0     0     0     1     1     1     0     0     0
```

With closing, the image is now fully "joined up". We can obtain a joining-up effect with the text image, using a diagonal structuring element.

```
>> diag=[0 0 1;0 1 0;1 0 0]

diag =

     0     0     1
     0     1     0
     1     0     0

>> tc=imclose(t,diag);
>> imshow(tc)
```

The result is shown in figure 11.14.

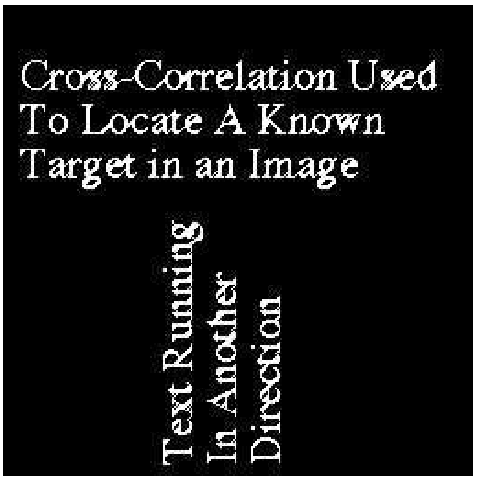


Figure 11.14: An example of closing

An application: noise removal

Suppose A is a binary image corrupted by impulse noise—some of the black pixels are white, and some of the white pixels are back. An example is given in figure 11.15. Then $A \ominus B$ will remove the single black pixels, but will enlarge the holes. We can fill the holes by dilating twice:

$$((A \ominus B) \oplus B) \oplus B.$$

The first dilation returns the holes to their original size; the second dilation removes them. But this will enlarge the objects in the image. To reduce them to their correct size, perform a final erosion:

$$(((A \ominus B) \oplus B) \oplus B) \ominus B.$$

We see that the inner two operations constitute an opening; the outer two operations a closing. Thus this noise removal method is in fact an opening followed by a closing:

$$(A \circ B) \bullet B).$$

This is called *morphological filtering.*

Suppose we take an image and apply 10% shot noise to it:

```
>> c=imread('circles.tif');
>> x=rand(size(c));
>> d1=find(x<=0.05);
>> d2=find(x>=0.95);
>> c(d1)=0;
>> c(d2)=1;
>> imshow(c)
```

The result is shown as figure 11.15(a). The filterWe have discussed the Fourier transform and its uses in chapter 9. However, asing process can be implemented with

```
>> cf1=imclose(imopen(c,sq),sq);
>> figure,imshow(cf1)
>> cf2=imclose(imopen(c,cr),cr);
>> figure,imshow(cf2)
```

and the results are shown as figures 11.15(b) and (c). The results are rather "blocky"; although less so with the cross structuring element.

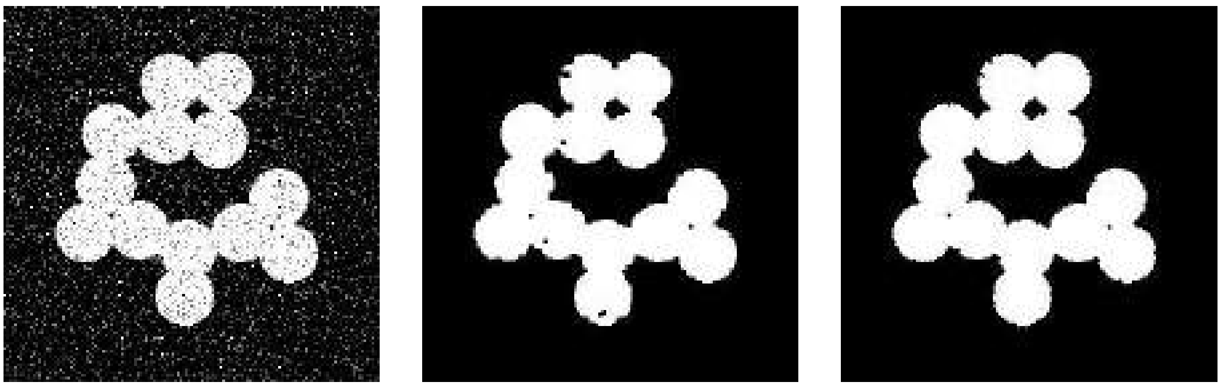

(a) (b) (c)

Figure 11.15: A noisy binary image and the result after morphological filtering.

11.4.3 Relationship between opening and closing

Opening and closing share a relationship very similar to that of erosion and dilation: the complement of an opening is equal to the closing of a complement, and complement of an closing is equal to the opening of a complement. Specifically:

$$\overline{A \bullet B} = \overline{A} \circ \hat{B}$$

and

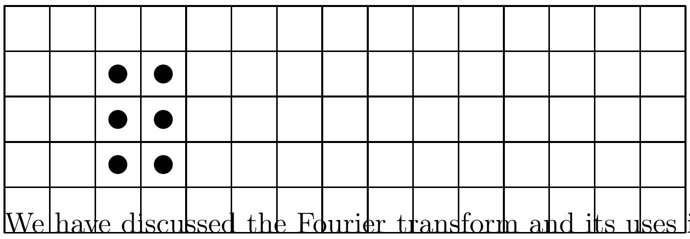
$$\overline{A \circ B} = \overline{A} \bullet \hat{B}.$$

These can be demonstrated using similar techniques to those in section 11.3.3.

11.5 The hit-or-miss transform

This is a powerful method for finding shapes in images. As with all other morphological algorithms, it can be defined entirely in terms of dilation and erosion; in this case, erosion only.

Suppose we wish to locate 3×3 square shapes, such as is in the centre of the image A in figure 11.16.



We have discussed the Fourier transform and its uses in chapter 9. However, as

Figure 11.16: An image A containing a shape to be found

If we performed an erosion $A \ominus B$ with B being the square structuring element, we would obtain the result given in figure 11.17.

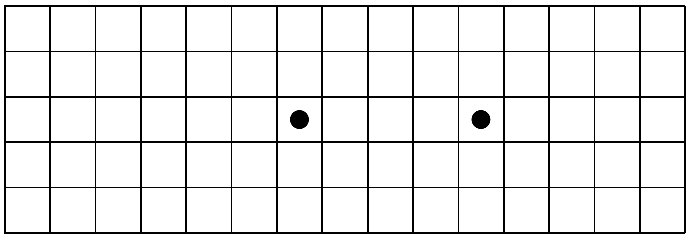


Figure 11.17: The erosion $A \ominus B$

The result contains two pixels, as there are exactly two places in A where B will fit. Now suppose we also erode the complement of A with a structuring element C which fits exactly around the 3×3 square; $\overline{A}$ and C are shown in figure 11.18. (We assume that $(0, 0)$ is at the centre of C.)

If we now perform the erosion $\overline{A} \ominus C$ we would obtain the result shown in figure 11.19.

The intersection of the two erosion operations would produce just one pixel at the position of the centre of the 3×3 square in A, which is just what we want. If A had contained more than one

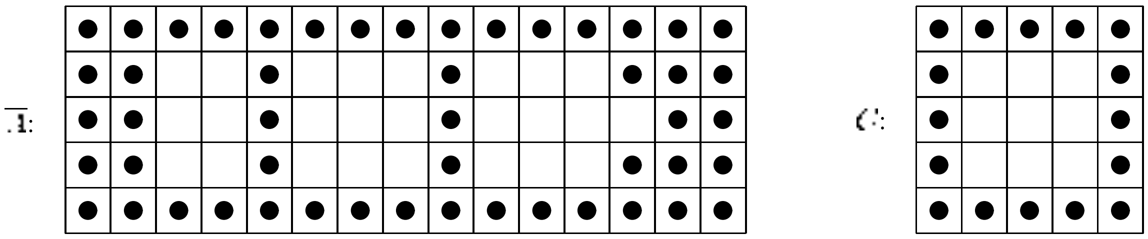


Figure 11.18: The complement and the second structuring element

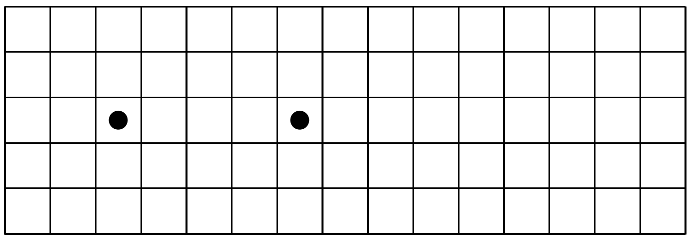


Figure 11.19: The erosion $\overline{A} \ominus C$

square, the final result would have been single pixels at the positions of the centres of each. This combination of erosions forms the hit-or-miss transform.

In general, if we are looking for a particular shape in an image, we design two structuring elements: B_1 which is the same shape, and B_2 which fits around the shape. We then write $B = (B_1, B_2)$ and

$$A \circledast B = (A \ominus B_1) \cap (\overline{A} \ominus B_2)$$

for the hit-or-miss transform.

As an example, we shall attempt to find the hyphen in "Cross-Correlation" in the text image shown in figure 11.5. This is in fact a line of pixels of length six. We thus can create our two structuring elements as:

```
>> b1=ones(1,6);
>> b2=[1 1 1 1 1 1 1 1;1 0 0 0 0 0 0 1; 1 1 1 1 1 1 1 1];
>> tb1=erode(t,b1);
>> tb2=erode(~t,b2);
>> hit_or_miss=tb1&tb2;
>> [x,y]=find(hit_or_miss==1)
```

and this returns a coordinate of $(41, 70)$, which is right in the middle of the hyphen. Note that the command

```
>> tb1=erode(t,b1);
```

is not sufficient, as there are quite a few lines of length six in this image. We can see this by viewing the image `tb1`, which is given in figure 11.20.

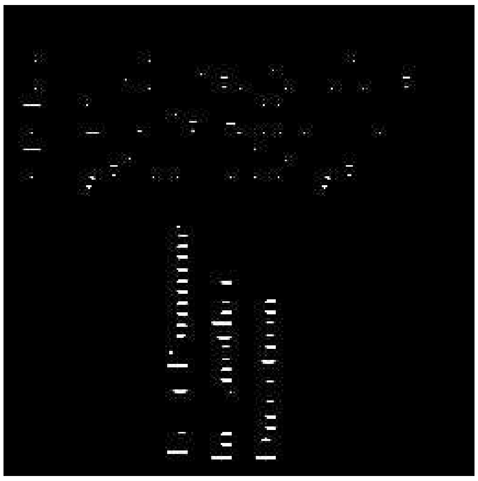


Figure 11.20: Text eroded by a hyphen-shaped structuring element

11.6 Some morphological algorithms

In this section we shall investigate some simple algorithms which use some of the morphological techniques we have discussed in previous sections.

11.6.1 Region filling

Suppose in an image we have a region bounded by an 8-connected boundary, as shown in figure 11.21.

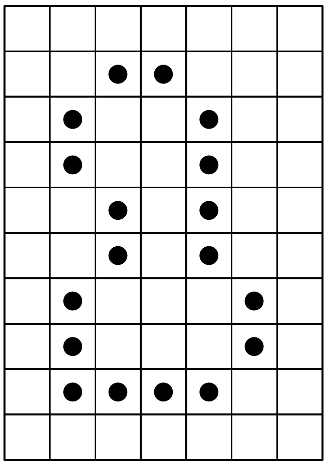


Figure 11.21: An 8-connected boundary of a region to be filled

Given a pixel p within the region, we wish to fill up the entire region. To do this, we start with p, and dilate as many times as necessary with the cross-shaped structuring element B (as used in figure 11.6), each time taking an intersection with $\overline{A}$ before continuing. We thus create a sequence of sets:

$$\{p\} = X_0, X_1, X_2, \dots, X_k = X_{k+1}$$

for which

$$X_n = (X_{n-1} \oplus B) \cap \overline{A}.$$

Finally $X_k \cup A$ is the filled region. Figure 11.22 shows how this is done.

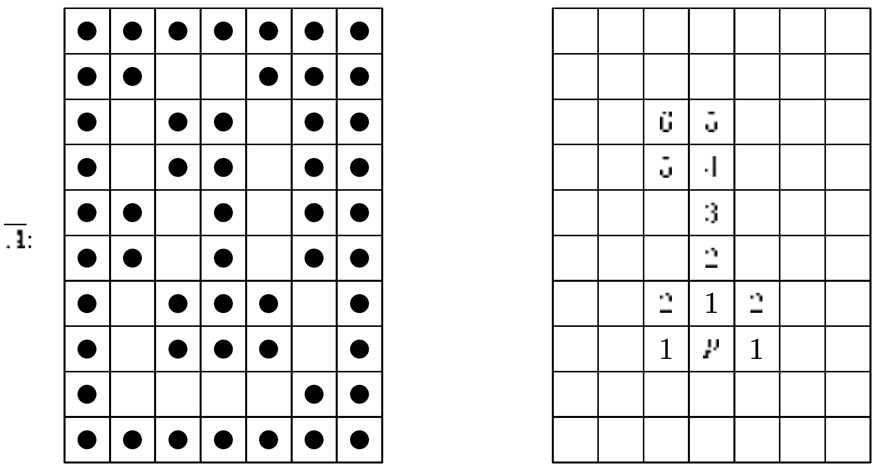


Figure 11.22: The process of filling a region

In the right hand grid, we have

$$X_0 = \{p\}, X_1 = \{p, 1\}, X_2 = \{p, 1, 2\}, \dots$$

Note that the use of the cross-shaped structuring element means that we never cross the boundary.

11.6.2 Connected components

We use a very similar algorithm to fill a connected component; we use the cross-shaped structuring element for 4-connected components, and the square structuring element for 8-connected components. Starting with a pixel p, we fill up the rest of the component by creating a sequence of sets

$$X_0 = \{p\}, X_1, X_2, \dots$$

such that

$$X_n = (X_{n-1} \oplus B) \cap A$$

until $X_k = X_{k-1}$. Figure 11.23 shows an example.
In each case we are starting in the centre of the square in the lower left. As this square is itself a 4-connected component, the cross structuring element cannot go beyond it.

Both of these algorithms can be very easily implemented by MATLAB functions. To implement region filling, we keep track of two images: `current` and `previous`, and stop when there is no

●	●		●	●	
●	●	●		●	
			●	●	●
●	●	●			
●	●	●			
●	●	●			

2	1	2			
1	p	1			
2	1	2			

Using the cross

5	4		4	4	
5	4	3		3	
			2	3	4
1	1	1			
1	p	1			
1	1	1			

Using the square

Figure 11.23: Filling connected components

difference between them. We start with `previous` being the single point p in the region, and `current` the dilation $(p \oplus B) \cap \overline{A}$. At the next step we set

$$\begin{aligned} \texttt{previous} &\leftarrow \texttt{current}, \\ \texttt{current} &\leftarrow (\texttt{current} \oplus B) \cap \overline{A}. \end{aligned}$$

Given B, we can implement the last step in MATLAB by

```
imdilate(current,B)&~A.
```

The function is shown in figure 11.24 We can use this to fill a particular region delineated by a

```
function out=regfill(im,pos,kernel)
% REGFILL(IM,POS,KERNEL) performs region filling of binary image IMAGE,
% with kernel KERNEL, starting at point with coordinates given by POS.
%
% Example:
%          n=imread('nicework.tif');
%          nb=n&~imerode(n,ones(3,3));
%          nr=regfill(nb,[74,52],ones(3,3));
%
current=zeros(size(im));
last=zeros(size(im));
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&~im;
while any(current(:)~=last(:)),
  last=current;
  current=imdilate(last,kernel)&~im;
end;
out=current;
```

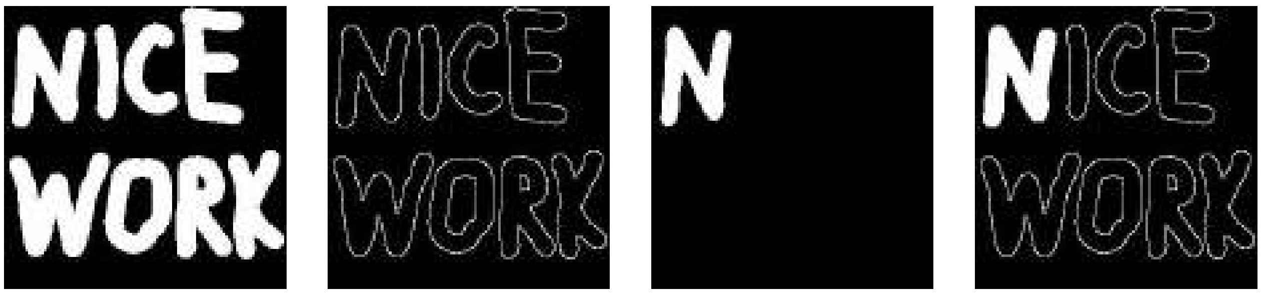
Figure 11.24: A simple program for filling regions

boundary.

```
>> n=imread('nicework.tif');
>> imshow(n),pixval on
>> nb=n&~imerode(n,sq);
>> figure,imshow(nb)
>> nf=regfill(nb,[74,52],sq);
>> figure,imshow(nf)
```

The results are shown in figure 11.25. Image (a) is the original; (b) the boundary, and (c) the result of a region fill. Figure (d) shows a variation on the region filling, we just include all boundaries. This was obtained with

```
>> figure,imshow(nf|nb)
```



(a) (b) (c) (d)

Figure 11.25: Region filling

The function for connected components is almost exactly the same as that for region filling, except that whereas for region filling we took an intersection with the *complement* of our image, for connected components we take the intersection with the image itself. Thus we need only change one line, and the resulting function is shown in 11.26 We can experiment with this function with

```
function out=components(im,pos,kernel)
% COMPONENTS(IM,POS,KERNEL) produces the connected component of binary image
% IMAGE which nicludes the point with coordinates given by POS, using
% kernel KERNEL.
%
% Example:
%          n=imread('nicework.tif');
%          nc=components(nb,[74,52],ones(3,3));
%
current=zeros(size(im));
last=zeros(size(im));
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&im;
while any(current(:)~=last(:)),
  last=current;
  current=imdilate(last,kernel)&im;
end;
out=current;
```

Figure 11.26: A simple program for connected components

the "nice work" image. We shall use the square structuring element, and also a larger structuring element of size 11×11.

```
>> sq2=ones(11,11);
>> nc=components(n,[57,97],sq);
>> imshow(nc)
>> nc2=components(n,[57,97],sq2);
>> figure,imshow(nc2)
```

and the results are shown in figure 11.27. Image (a) uses the 3×3 square; image (b) uses the 11×11 square.

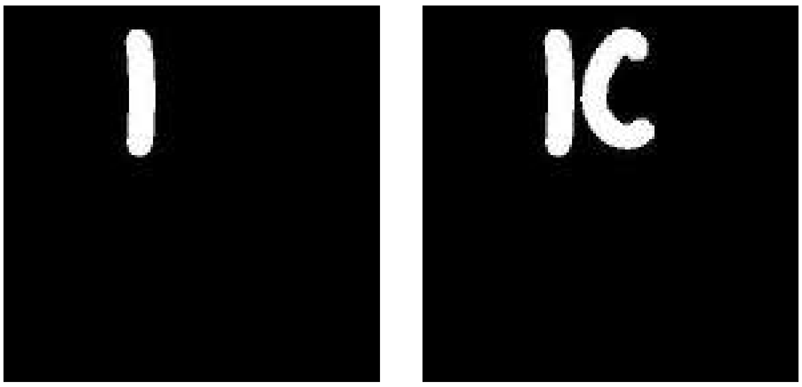

(a) (b)

Figure 11.27: Connected components

11.6.3 Skeletonization

Recall that the *skeleton* of an object can be defined by the "medial axis transform"; we may imagine fires burning in along all edges of the object. The places where the lines of fire meet form the skeleton. The skeleton may be produced by morphological methods.

Consider the table of operations as shown in table 11.1.

Erosions	**Openings**	**Set differences**
A	$A \circ B$	$A - (A \circ B)$
$A \ominus B$	$(A \ominus B) \circ B$	$(A \ominus B) - ((A \ominus B) \circ B)$
$A \ominus 2B$	$(A \ominus 2B) \circ B$	$(A \ominus 2B) - ((A \ominus 2B) \circ B)$
$A \ominus 3B$	$(A \ominus 3B) \circ B$	$(A \ominus 3B) - ((A \ominus 3B) \circ B)$
$\vdots$	$\vdots$	$\vdots$
$A \ominus kB$	$(A \ominus kB) \circ B$	$(A \ominus kB) - ((A \ominus kB) \circ B)$

Table 11.1: Operations used to construct the skeleton

Here we use the convention that a sequence of k erosions using the same structuring element B is denoted $A \ominus kB$. We continue the table until $(A \ominus kB) \circ B$ is empty. The skeleton is then obtained by taking the unions of all the set differences. An example is given in figure 11.28, using the cross structuring element.

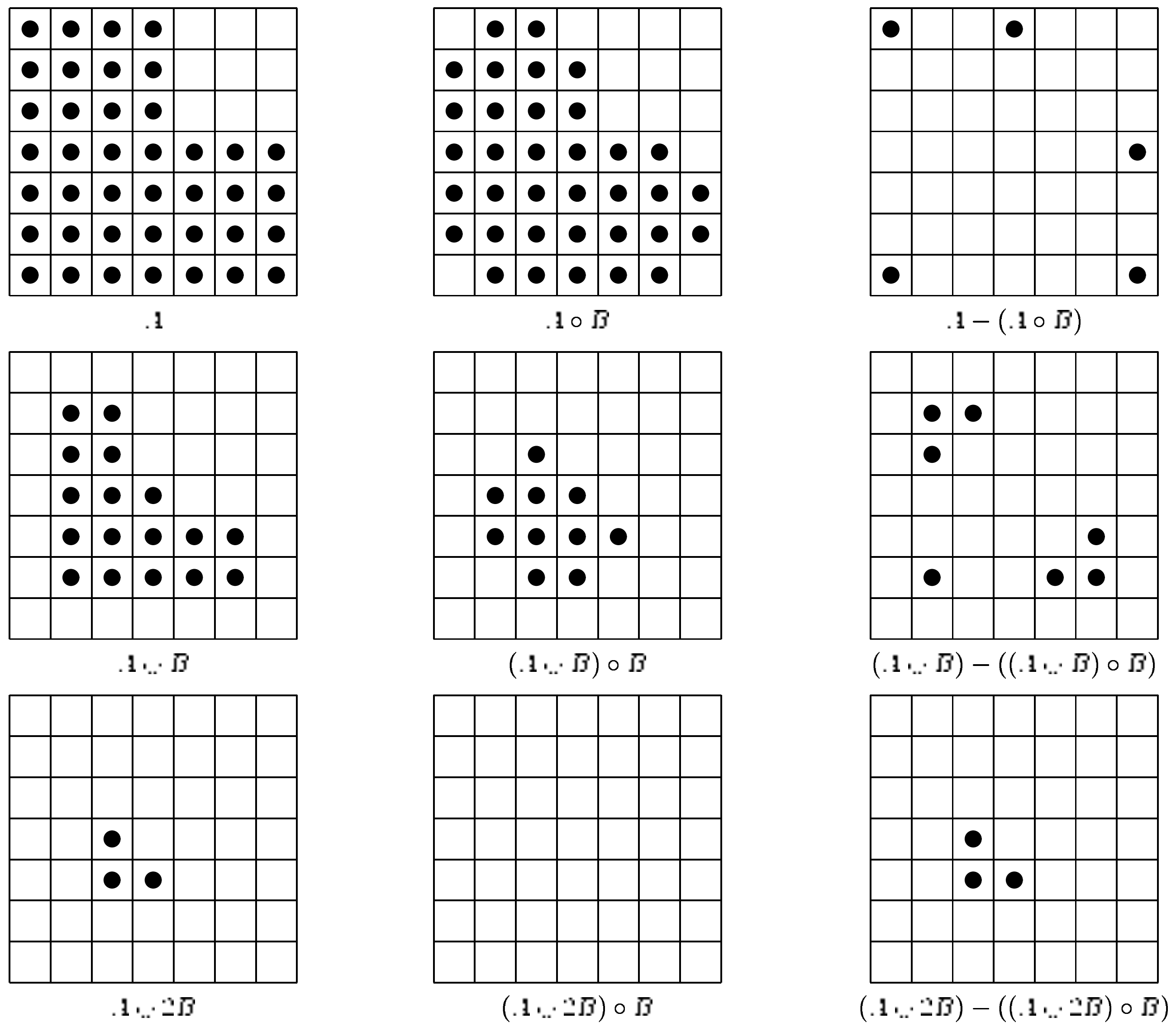


Figure 11.28: Skeletonization

Since $(A \ominus 2B) \circ B$ is empty, we stop here. The skeleton is the union of all the sets in the third column; it is shown in figure 11.29.

This algorithm again can be implemented very easily; a function to do so is shown in figure 11.30. We shall experiment with the nice work image.

```
>> nk=imskel(n,sq);
>> imshow(nk)
>> nk2=imskel(n,cr);
```

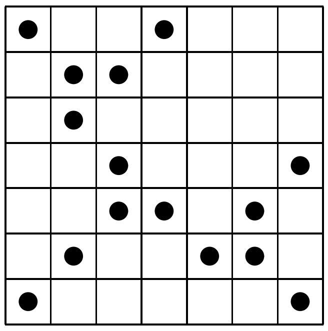


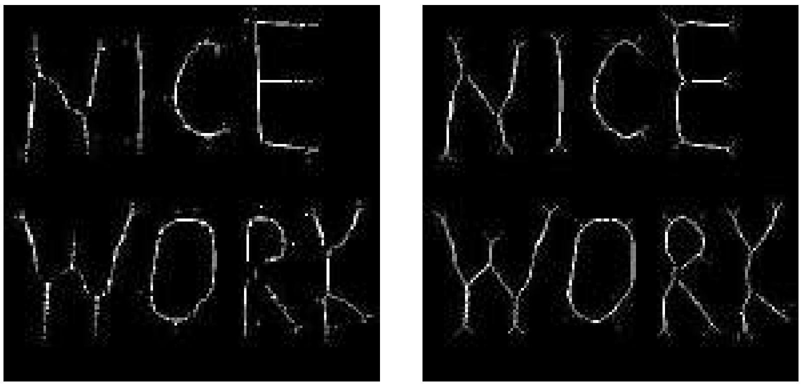
Figure 11.29: The final skeleton

```
function skel = imskel(image,str)
% IMSKEL(IMAGE,STR) - Calculates the skeleton of binary image IMAGE using
% structuring element STR.  This function uses Lantejoul's algorithm.
%
skel=zeros(size(image));
e=image;
while (any(e(:))),
    o=imopen(e,str);
    skel=skel | (e&~o);
    e=imerode(e,str);
end
```

Figure 11.30: A simple program for computing skeletons

```
>> figure,imshow(nk2)
```

The result is shown in figure 11.31. Image (a) is the result using the square structuring element; Image (b) is the result using the cross structuring element.



(a) (b)

Figure 11.31: Skeletonization of a binary image

Exercises

1. For each of the following images A and structuring elements B:

   $A =$

```
0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0
0 0 0 1 1 1 1 0    0 1 1 1 1 1 1 0    0 0 0 0 0 1 1 0
0 0 0 1 1 1 1 0    0 1 1 1 1 1 1 0    0 1 1 1 0 1 1 0
0 1 1 1 1 1 1 0    0 1 1 0 0 1 1 0    0 1 1 1 0 1 1 0
0 1 1 1 1 1 1 0    0 1 1 0 0 1 1 0    0 1 1 1 0 1 1 0
0 1 1 1 1 0 0 0    0 1 1 1 1 1 1 0    0 1 1 1 0 0 0 0
0 1 1 1 1 0 0 0    0 1 1 1 1 1 1 0    0 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0
```

   $B =$

```
0 1 0    1 1 1    1 0 0
1 1 1    1 1 1    0 0 0
0 1 0    1 1 1    0 0 1
```

   calculate the erosion $A \ominus B$, the dilation $A \oplus B$, the opening $A \circ B$ and the closing $A \bullet B$.

   Check your answers with MATLAB.

2. Using the binary images `circbw.tif`, `circles.tif`, `circlesm.tif`, `logo.tif` and `testpat2.tif`, view the erosion and dilation with both the square and the cross structuring elements.

   Can you see any differences?

3. Read in the image `circlesm.tif`.
    (a) Erode with squares of increasing size until the image starts to split into disconnected components.
    (b) Using `pixval on`, find the coordinates of a pixel in one of the components.
    (c) Use the `components` function to isolate that particular component.
4. (a) With your disconnected image from the previous question, compute its boundary.
    (b) Again with `pixval on`, find a pixel inside one of the boundaries.
    (c) Use the `regfill` function to fill that region.
    (d) Display the image as a boundary with one of the regions filled in.
5. Using the 3×3 square structuring element, compute the skeletons of
    (a) a 7 square,
    (b) a 5×9 rectangle,
    (c) an L shaped figure formed from an 8×8 square with a 3×3 square taken from a corner,
    (d) an H shaped figure formed from a 15×15 square with 5×5 squares taken from the centres of the top and bottom,
    (e) a cross formed from an 11×11 square with 3×3 squares taken from each corner.

    In each case check your answer with MATLAB
6. Repeat the above question but use the cross structuring element.
7. For the images listed in question 2, obtain their skeletons by both the `bwmorph` function, and by using the function given in figure 11.30. Which seems to provide the best result?
8. Use the hit-or-miss transform with appropriate structuring elements to find the dot on the " i" in the word " in" in the image `text.tif`.

Chapter 12

Colour processing

For human beings, colour provides one of the most important descriptors of the world around us. The human visual system is particularly attuned to two things: edges, and colour. We have mentioned that the human visual system is not particularly good at recognizing subtle changes in grey values. In this section we shall investigate colour briefly, and then some methods of processing colour images

12.1 What is colour?

Colour study consists of

1. the physical properties of light which give rise to colour,
2. the nature of the human eye and the ways in which it detects colour,
3. the nature of the human vision centre in the brain, and the ways in which messages from the eye are perceived as colour.

Physical aspects of colour

Visible light is part of the *electromagnetic spectrum*: radiation in which the energy takes the form of waves of varying wavelength. These range from cosmic rays of very short wavelength, to electric power, which has very long wavelength. Figure 12.1 illustrates this. The values for the wavelengths of blue, green and red were set in 1931 by the CIE (Commission Internationale d'Eclairage), an organization responsible for colour standards.

Perceptual aspects of colour

The human visual system tends to perceive colour as being made up of varying amounts of red, green and blue. That is, human vision is particularly sensitive to these colours; this is a function of the cone cells in the retina of the eye. These values are called the *primary colours.* If we add together any two primary colours we obtain the *secondary colours*:

$$\begin{aligned} \text{magenta (purple)} &= \text{red} + \text{blue}, \\ \text{cyan} &= \text{green} + \text{blue}, \\ \text{yellow} &= \text{red} + \text{green}. \end{aligned}$$

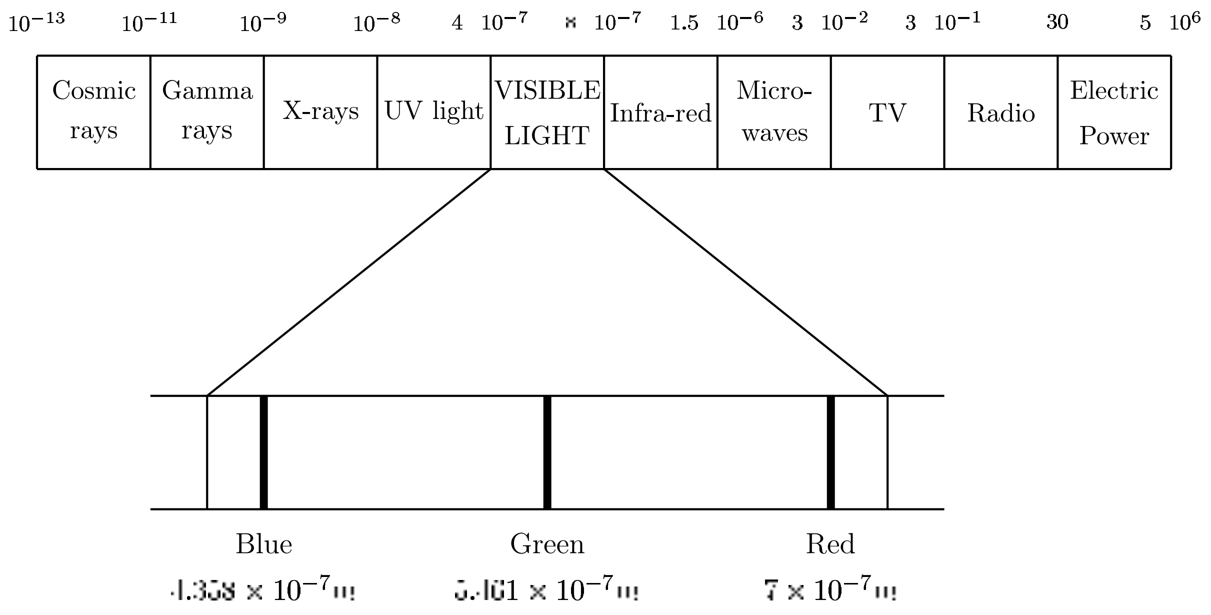


Figure 12.1: The electromagnetic spectrum and colours

The amounts of red, green, and blue which make up a given colour can be determined by a *colour matching experiment*. In such an experiment, people are asked to match a given colour (a *colour source*) with different amounts of the additive primaries red, green and blue. Such an experiment was performed in 1931 by the CIE, and the results are shown in figure 12.2. Note that for some wavelengths, various of the red, green or blue values are *negative*. This is a physical impossibility, but it can be interpreted by adding the primary beam to the colour source, to maintain a colour match.

To remove negative values from colour information, the CIE introduced the XYZ colour model. The values of X, Y and Z can be obtained from the corresponding R, G and B values by a linear transformation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.431 & 0.342 & 0.178 \\ 0.222 & 0.707 & 0.071 \\ 0.020 & 0.130 & 0.939 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}.$$

The inverse transformation is easily obtained by inverting the matrix:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.063 & -1.393 & -0.476 \\ -0.969 & 1.876 & 0.042 \\ 0.068 & -0.229 & 1.069 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}.$$

The XYZ colour matching functions corresponding to the R, G, B curves of figure 12.2 are shown in figure 12.3. The matrices given are not fixed; other matrices can be defined according to the definition of the colour white. Different definitions of white will lead to different transformation matrices.

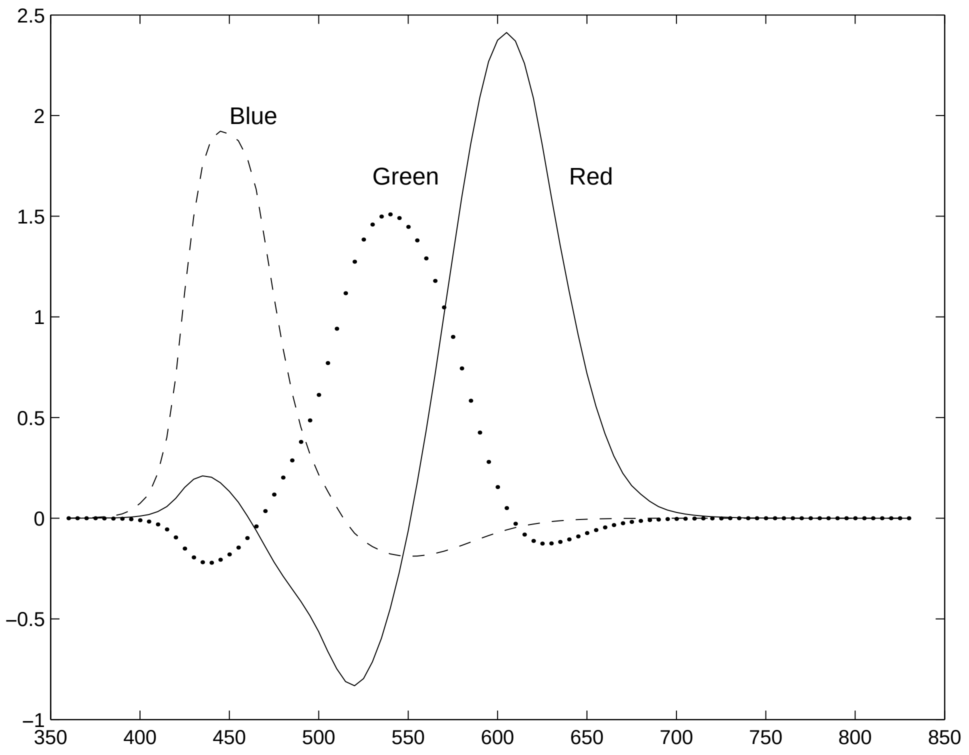


Figure 12.2: RGB colour matching functions (CIE, 1931)

A note on the diagrams

The diagrams shown in figures 12.3 and 12.3 were obtained from the text file `ciexyz31.txt`[1] which lists the XYZ values corresponding to colours of different wavelengths. Figure 12.4 shows a simple function which takes the relevant information out of this file and plots it. Figure 12.5 shows the similar function for plotting the RGB values (obtained from their XYZ counterparts by the linear transformation above).

The CIE required that the Y component corresponded with *luminance*, or perceived brightness of the colour. That is why the row corresponding to Y in the first matrix (that is, the second row) sums to 1, and also why the Y curve in figure 12.3 is symmetric about the middle of the visible spectrum.

In general, the values of X, Y and Z needed to form any particular colour are called the *tristimulus values*. Values corresponding to particular colours can be obtained from published tables. In order to discuss colour independent of brightness, the tristimulus values can be normalized by dividing by $X+Y+Z$:

$$\begin{aligned} x &= \frac{X}{X+Y+Z} \\ y &= \frac{Y}{X+Y+Z} \\ z &= \frac{Z}{X+Y+Z} \end{aligned}$$

[1]This file can be obtained from the *Colour & Vision Research Laboratories* web page `http://www.cvrl.org`.

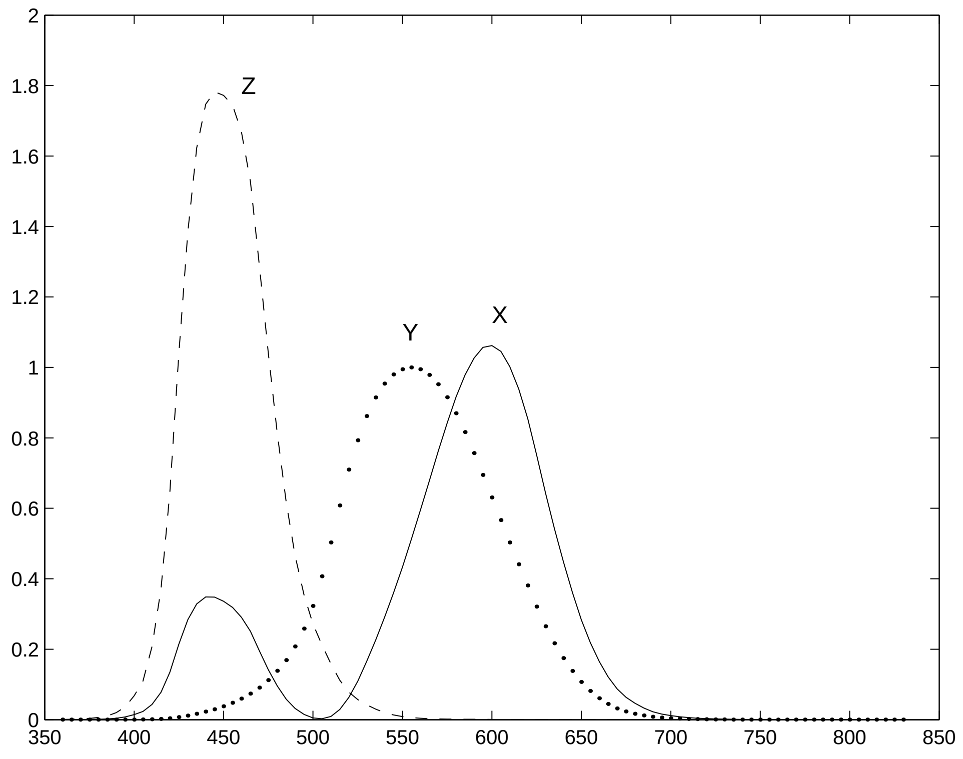


Figure 12.3: XYZ colour matching functions (CIE, 1931)

```
function plotxyz()
%
% This function simply plots the colour matching curves for CIE XYZ (1931),
% obtaining the data from the file ciexyz31.txt
%
wxyz=load('ciexyz31.txt');
w=wxyz(:,1);
x=wxyz(:,2);
y=wxyz(:,3);
z=wxyz(:,4);
figure,plot(w,x,'-k',w,y,'.k',w,z,'--k')
text(600,1.15,'X','Fontsize',12)
text(550,1.1,'Y','Fontsize',12)
text(460,1.8,'Z','Fontsize',12)
```

Figure 12.4: A function for plotting the XYZ curves

```
function plotrgb()
%
% This function simply plots the colour matching curves for CIE RGB (1931),
% obtaining the original XYZ data from the file ciexyz31.txt
%
wxyz=load('ciexyz31.txt');
x2r=[3.063 -1.393 -0.476;-0.969 1.876 0.042;0.068 -0.229 1.069];
xyz=wxyz(:,2:4)';
rgb=x2r*xyz;
w=wxyz(:,1);
figure,plot(w,rgb(1,:)','-k',w,rgb(2,:)','.k',w,rgb(3,:)','--k')
text(450,2,'Blue','Fontsize',12)
text(530,1.7,'Green','Fontsize',12)
text(640,1.7,'Red','Fontsize',12)
```

Figure 12.5: A function for plotting the RGB curves

and so $x + y + z = 1$. Thus a colour can be specified by x and y alone, called the *chromaticity coordinates*. Given x, y, and Y, we can obtain the tristimulus values X and Z by working through the above equations backwards:

$$\begin{aligned} X &= \frac{x}{y}Y \\ Z &= \frac{1-x-y}{y}Y. \end{aligned}$$

We can plot a chromaticity diagram, again using the ciexyz31.txt file of XYZ values:

```
>> z=zeros(size(xyz));
>> xy=xyz./(sum(xyz')'*[1 1 1]);
>> x=xy(:,1)';
>> y=xy(:,2)';
>> figure,plot([x x(1)],[y y(1)]),axis square
```

Here the matrix xyz consists of the second, third and fourth columns of the data, and plot is a function which draws a polygon with vertices taken from the x and y vectors. The extra x(1) and y(1) ensures that the polygon joins up. The result is shown in figure 12.6. The values of x and y which lie within the horseshoe shape in figure 12.6 represent values which correspond to physically realizable colours.

12.2 Colour models

A *colour model* is a method for specifying colours in some standard way. It generally consists of a three dimensional coordinate system and a subspace of that system in which each colour is represented by a single point. We shall investigate three systems.

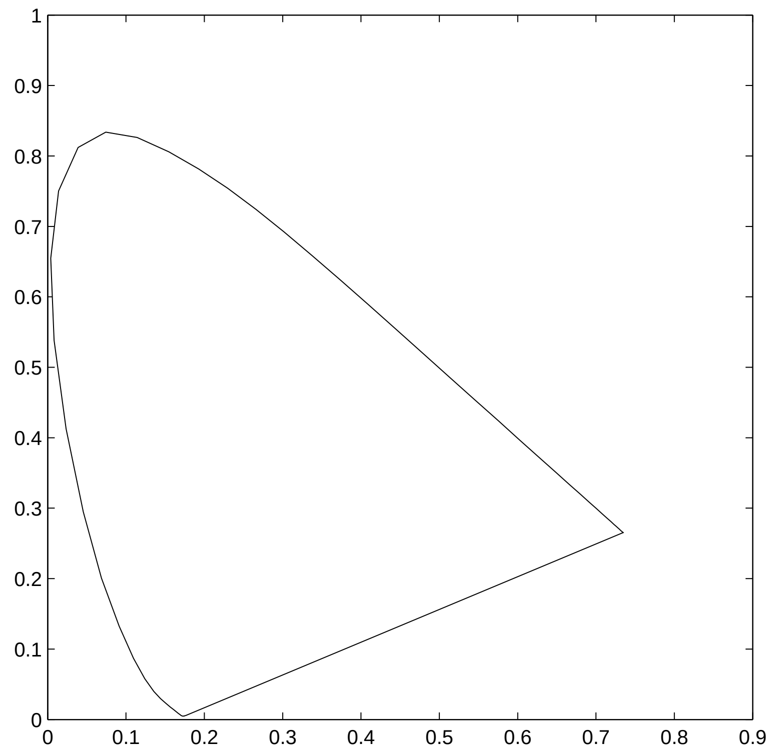
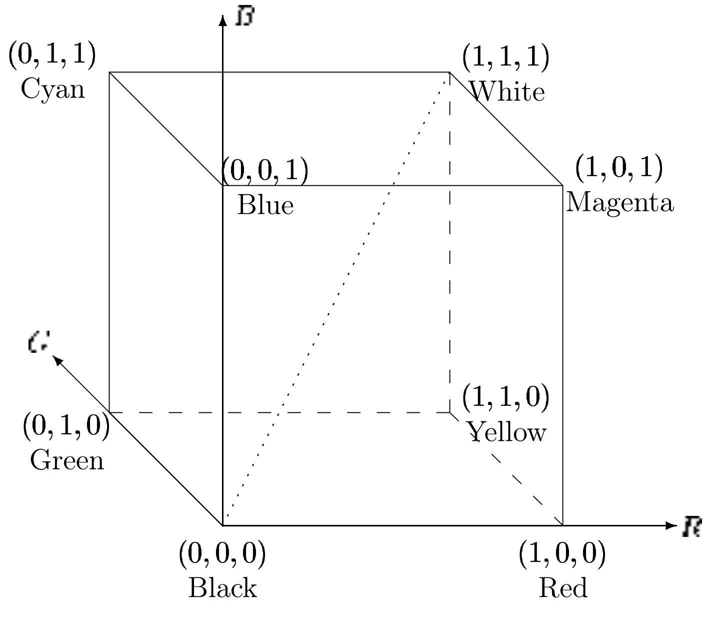


Figure 12.6: A chromaticity diagram

12.2.1 RGB

In this model, each colour is represented as three values R, G and B, indicating the amounts of red, green and blue which make up the colour. This model is used for displays on computer screens; a monitor has three independent electron "guns" for the red, green and blue component of each colour. We may imagine all the colours sitting inside a "colour cube" of side 1:



The colours along the black-white diagonal, shown in the diagram as a dotted line, are the points of the space where all the R, G, B values are equal. They are the different intensities of grey.

RGB is the standard for the *display* of colours: on computer monitors; on TV sets. But it is not a very good way of *describing* colours. How, for example, would you define light brown using RGB?

Note also from figure 12.2 that some colours require negative values of R, G or B. These colours are not realizable on a computer monitor or TV set, on which only positive values are possible. The colours corresponding to positive values form the *RGB gamut*; in general a colour "gamut" consists of all the colours realizable with a particular colour model. We can plot the RGB gamut on a chromaticity diagram, using the xy coordinates obtained above. To define the gamut, we shall create a $100 \times 100 \times 3$ array, and to each point (i, j) in the array, associate an XYZ triple defined by $(i/100, j/100, 1 - i/100 - j/100)$. We can then compute the corresponding RGB triple, and if any of the RGB values are negative, make the output value white. This is easily done with the simple function shown in figure 12.7.

```
function res=gamut()

global cg;
x2r=[3.063 -1.393 -0.476;-0.969 1.876 0.042;0.068 -0.229 1.069];
cg=zeros(100,100,3);
for i=1:100,
  for j=1:100,
    cg(i,j,:)=x2r*[j/100 i/100 1-i/100-j/100]';
    if min(cg(i,j,:))<0,
      cg(i,j,:)=[1 1 1];
    end;
  end;
end;
res=cg;
```

Figure 12.7: Computing the RGB gamut

We can then display the gamut inside the chromaticity figure by

```
>> imshow(cG),line([x' x(1)],[y' y(1)]),axis square,axis xy,axis on
```

and the result is shown in figure 12.8.

12.2.2 HSV

HSV stands for Hue, Saturation, Value. These terms have the following meanings:

Hue: The "true colour" attribute (red, green, blue, orange, yellow, and so on).

Saturation: The amount by which the colour as been diluted with white. The more white in the colour, the lower the saturation. So a deep red has high saturation, and a light red (a pinkish colour) has low saturation.

Value: The degree of brightness: a well lit colour has high intensity; a dark colour has low intensity.

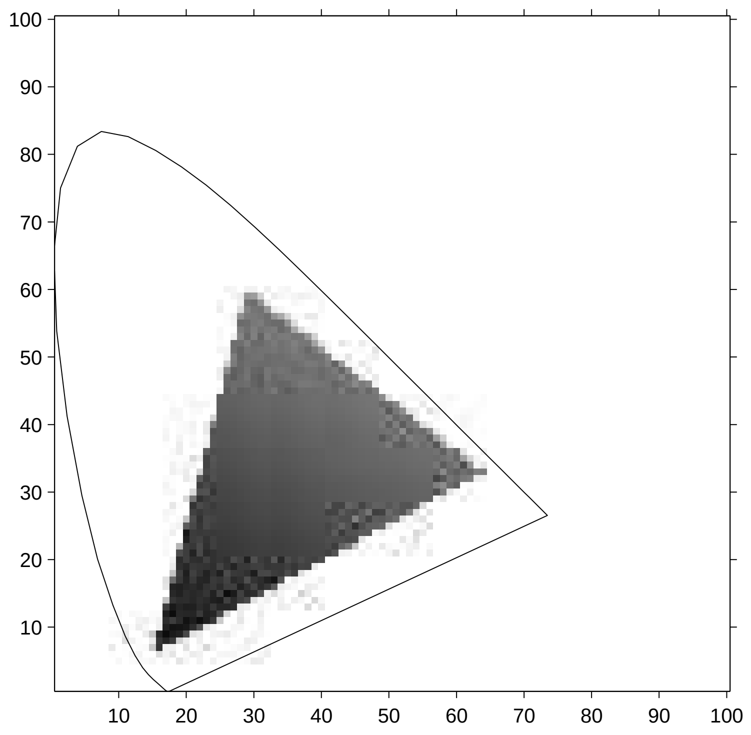


Figure 12.8: The RGB gamut

This is a more intuitive method of describing colours, and as the intensity is independent of the colour information, this is a very useful model for image processing. We can visualize this model as a cone, as shown in figure 12.9.

Any point on the surface represents a purely saturated colour. The saturation is thus given as the relative distance to the surface from the central axis of the structure. Hue is defined to be the angle measurement from a pre-determined axis, say red.

12.2.3 Conversion between RGB and HSV

Suppose a colour is specified by its RGB values. If all the three values are equal, then the colour will be a grey scale; that is, an intensity of white. Such a colour, containing just white, will thus have a saturation of zero. Conversely, if the RGB values are very different, we would expect the resulting colour to have a high saturation. In particular, if one or two of the RGB values are zero, the saturation will be one, the highest possible value.

Hue is defined as the fraction around the circle starting from red, which thus has a hue of zero. Reading around the circle in figure 12.9 produces the following hues:

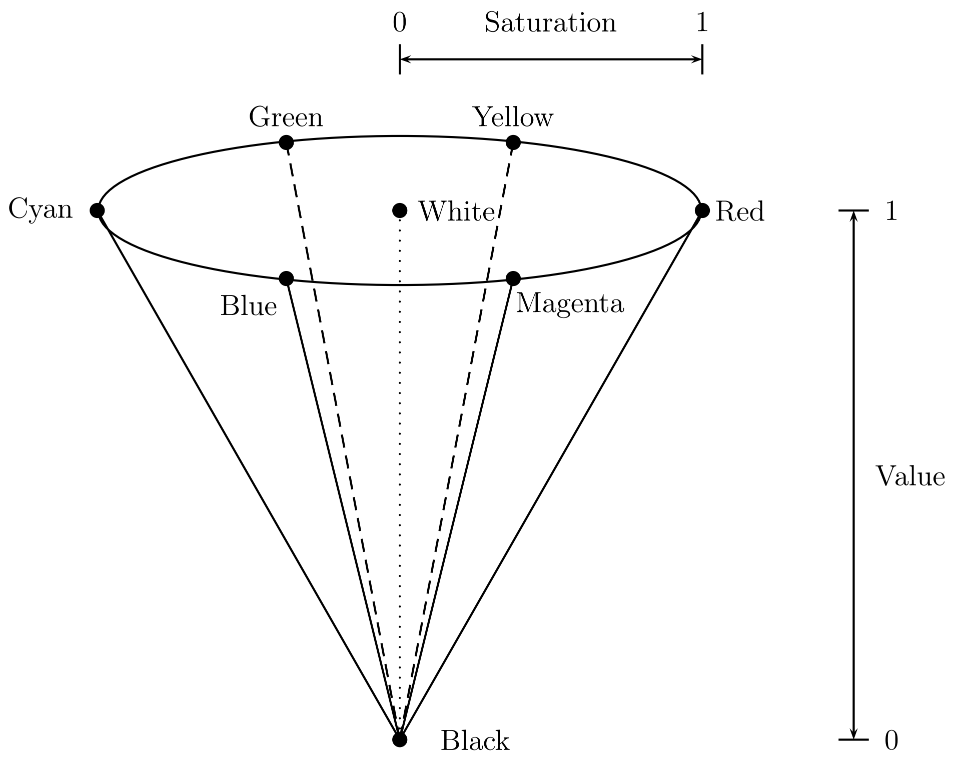


Figure 12.9: The colour space HSV as a cone

Colour	Hue
Red	0
Yellow	0.1667
Green	0.3333
Cyan	0.5
Blue	0.6667
Magenta	0.8333

Suppose we are given three R, G, B values, which we suppose to be between 0 and 1. So if they are between 0 and 255, we first divide each value by 255. We then define:

$$\begin{aligned} V &= \max\{R, G, B\} \\ \delta &= V - \min\{R, G, B\} \\ S &= \frac{\delta}{V} \end{aligned}$$

To obtain a value for Hue, we consider several cases:

1. if $R = V$ then $H = \dfrac{1}{6}\dfrac{G-B}{\delta}$,

2. if $G = V$ then $H = \dfrac{1}{6}\left(2 + \dfrac{B-R}{\delta}\right)$,

3. if $B = V$ then $H = \dfrac{1}{6}\left(4 + \dfrac{R-G}{\delta}\right)$.

If H ends up with a negative value, we add 1. In the particular case $(R, G, B) = (0, 0, 0)$, for which both $V = \delta = 0$, we define $(H, S, V) = (0, 0, 0)$.

For example, suppose $(R, G, B) = (0.2, 0.4, 0.6)$ We have

$$\begin{aligned} V &= \max\{0.2, 0.4, 0.6\} = 0.6 \\ \delta &= V - \min\{0.2, 0.4, 0.6\} = 0.6 - 0.2 = 0.4 \\ S &= \frac{0.4}{0.6} = 0.6667 \end{aligned}$$

Since $B = G$ we have

$$H = \frac{1}{6}\left(4 + \frac{0.2 - 0.4}{0.4}\right) = 0.5833.$$

Conversion in this direction is implemented by the `rgb2hsv` function. This is of course designed to be used on $m \times n \times 3$ arrays, but let's just experiment with our previous example:

```
>> rgb2hsv([0.2 0.4 0.6])

ans =

    0.5833    0.6667    0.6000
```

and these are indeed the H, S and V values we have just calculated.

To go the other way, we start by defining:

$$\begin{aligned} H' &= \lfloor 6H \rfloor \\ F &= 6H - H' \\ P &= V(1 - S) \\ Q &= V(1 - SF) \\ T &= V(1 - S(1 - F)) \end{aligned}$$

Since H' is a integer between 0 and 5, we have six cases to consider:

H'	R	G	B
0	V	T	P
1	Q	V	P
2	P	V	T
3	P	Q	V
4	T	P	V
5	V	P	Q

Let's take the HSV values we computed above. We have:

$$\begin{aligned} H' &= \lfloor 6(0.5833) \rfloor = 3 \\ F &= 6(0.5833) - 3 = 0.5 \\ P &= 0.6(1 - 0.6667) = 0.2 \\ Q &= 0.6(1 - (0.6667)(0.5)) = 0.4 \\ T &= 0.6(1 - 0.6667(1 - 0.5)) = 0.4 \end{aligned}$$

Since $H' = 3$ we have

$$(R, G, B) = (P, Q, V) = (0.2, 0.4, 0.6).$$

Conversion from HSV to RGB is implemented by the `hsv2rgb` function.

12.2.4 YIQ

This colour space is used for TV/video in America and other countries where NTSC is the video standard (Australia uses PAL). In this scheme Y is the "luminance" (this corresponds roughly with intensity), and I and Q carry the colour information. The conversion between RGB is straightforward:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

and

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.000 & 0.956 & 0.621 \\ 1.000 & -0.272 & -0.647 \\ 1.000 & -1.106 & 1.703 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

The two conversion matrices are of course inverses of each other. Note the difference between Y and I:

$$\begin{aligned} I &= 0.333R + 0.333G + 0.333B \\ Y &= 0.299R + 0.587G + 0.114B \end{aligned}$$

This reflects the fact that the human visual system assigns more intensity to the green component of an image than to the red and blue components. Since YIQ is a linear transformation of RGB, we can picture YIQ to be a parallelepiped (a rectangular box which has been skewed in each direction) for which the Y axis lies along the central $(0, 0, 0)$ to $(1, 1, 1)$ line of RGB. Figure 12.10 shows this.

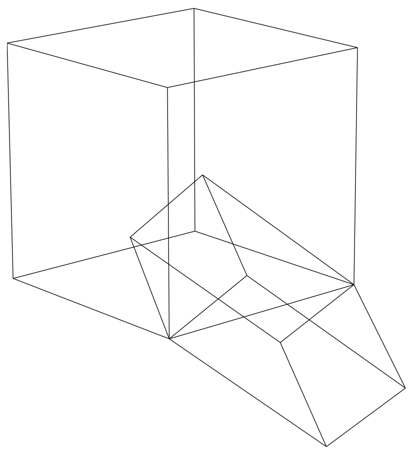


Figure 12.10: The RGB cube and its YIQ transformation

That the conversions are linear, and hence easy to do, makes this a good choice for colour image processing. Conversion between RGB and YIQ are miplemented with the MATLAB functions `rgb2ntsc` and `ntsc2rgb`.

12.3 Colour images in Matlab

Since a colour image requires three separate items of information for each pixel, a (true) colour image of size $m \times n$ is represented in Matlab by an array of size $m \times n \times 3$: a three dimensional array. We can think of such an array as a single entity consisting of three separate matrices aligned vertically. Figure 12.11 shows a diagram illustrating this idea. Suppose we read in an RGB image:

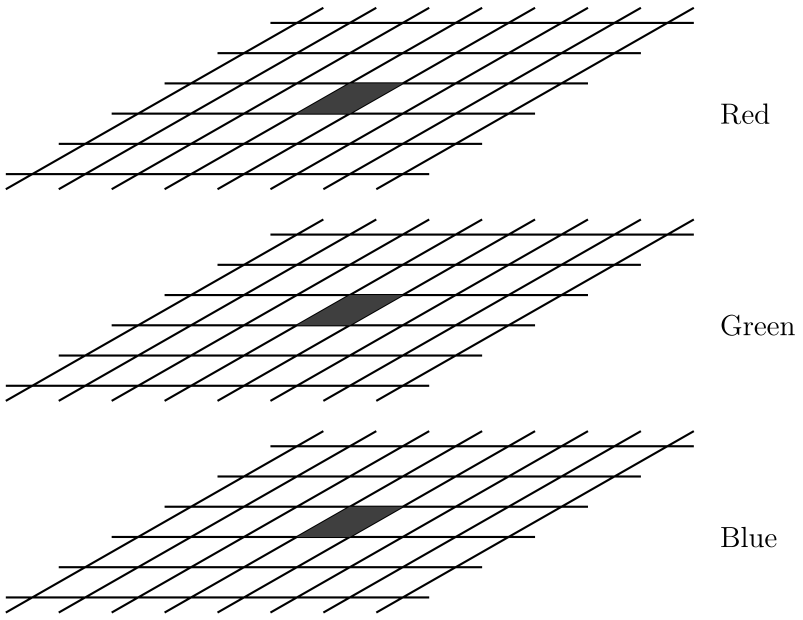


Figure 12.11: A three dimensional array for an RGB image

```
>> x=imread('lily.tif');
>> size(x)

ans =

   186   230     3
```

We can isolate each colour component by the colon operator:

`x(:,:,1)`	The first, or red component
`x(:,:,2)`	The second, or green component
`x(:,:,3)`	The third, or blue component

These can all be viewed with `imshow`:

```
>> imshow(x)
>> figure,imshow(x(:,:,1))
>> figure,imshow(x(:,:,1))
>> figure,imshow(x(:,:,2))
```

These are all shown in figure 12.12. Notice how the colours with particular hues show up with

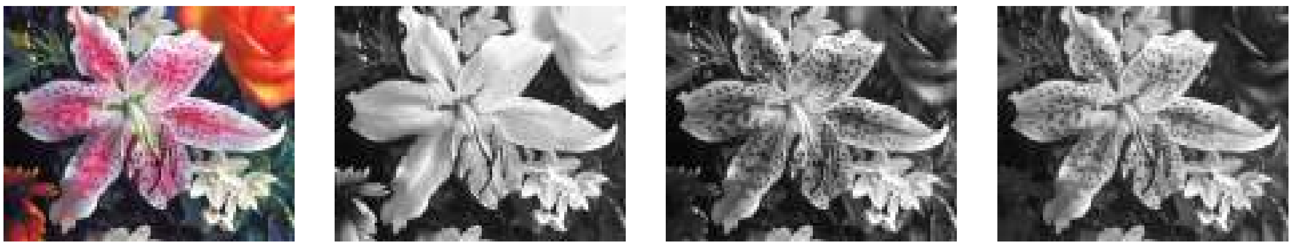


Figure 12.12: An RGB colour image and its components

high intensities in their respective components. For the rose in the top right, and the flower in the bottom left, both of which are predominantly red, the red component shows a very high intensity for these two flowers. The green and blue components show much lower intensities. Similarly the green leaves—at the top left and bottom right—show up with higher intensity in the green component than the other two.

We can convert to YIQ or HSV and view the components again:

```
>> xh=rgb2hsv(x);
>> imshow(xh(:,:,1))
>> figure,imshow(xh(:,:,2))
>> figure,imshow(xh(:,:,3))
```

and these are shown in figure 12.13. We can do precisely the same thing for the YIQ colour space:

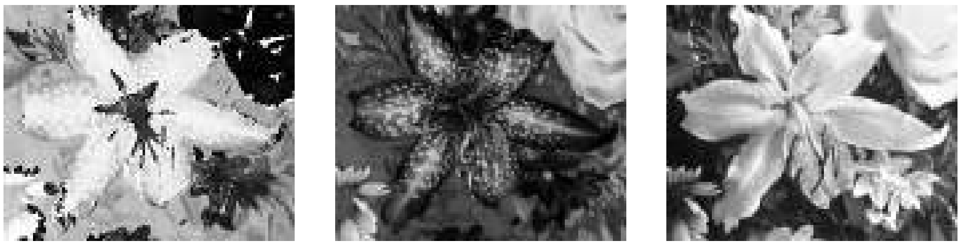


Figure 12.13: The HSV components

```
>> xn=rgb2ntsc(x);
>> imshow(xn(:,:,1))
>> figure,imshow(xn(:,:,2))
>> figure,imshow(xn(:,:,3))
```

and these are shown in figure 12.14. Notice that the Y component of YIQ gives a better greyscale version of the image than the value of HSV. The top right rose, in particular, is quite washed out in figure 12.13 (Value), but shows better contrast in figure 12.14 (Y).

We shall see below how to put three matrices, obtained by operations on the separate components, back into a single three dimensional array for display.

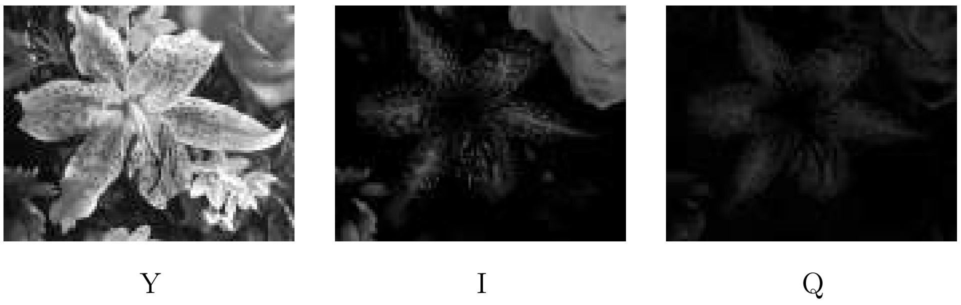


Figure 12.14: The YIQ components

12.4 Pseudocolouring

This means assigning colours to a grey-scale image in order to make certain aspects of the image more amenable for visual interpretation—for example, for medical images. There are different methods of pseudocolouring.

12.4.1 Intensity slicing

In this method, we break up the image into various grey level ranges. We simply assign a different colour to each range. For example:

grey level:	0–63	64–127	128–191	192–255
colour:	blue	magenta	green	red

We can consider this as a mapping, as shown in figure 12.15.

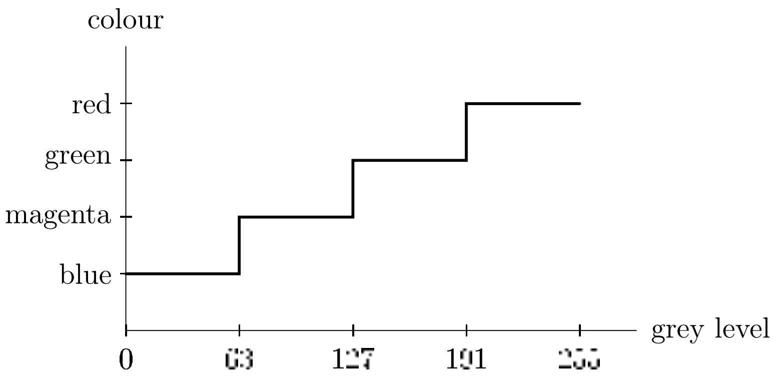
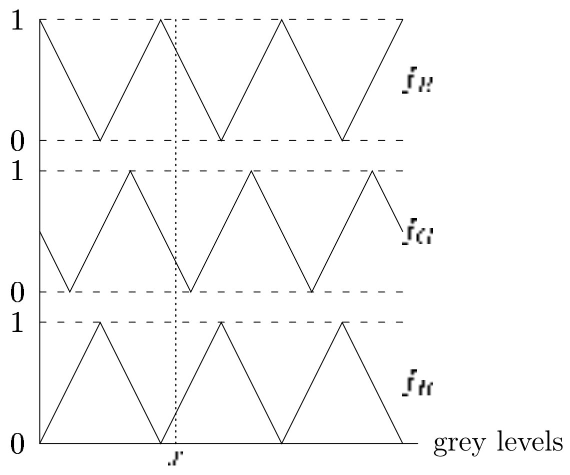


Figure 12.15: Intensity slicing as a mapping

12.4.2 Grey—Colour transformations

We have three functions $f_R(x)$, $f_G(x)$, $f_B(x)$ which assign red, green and blue values to each grey level x. These values (with appropriate scaling, if necessary) are then used for display. Using an appropriate set of functions can enhance a grey-scale image with impressive results.



The grey level x in the diagram is mapped onto red, green and blue values of 0.375, 0.125 and 0.75 respectively.

In MATLAB, a simple way to view an image with added colour is to use `imshow` with an extra `colormap` parameter. For example, consider the image `blocks.tif`. We can add a colourmap with the `colormap` function; there are several existing colourmaps to choose from. Figure 12.16 shows the children's blocks image (from figure 1.4) after colour transformations. We created the colour

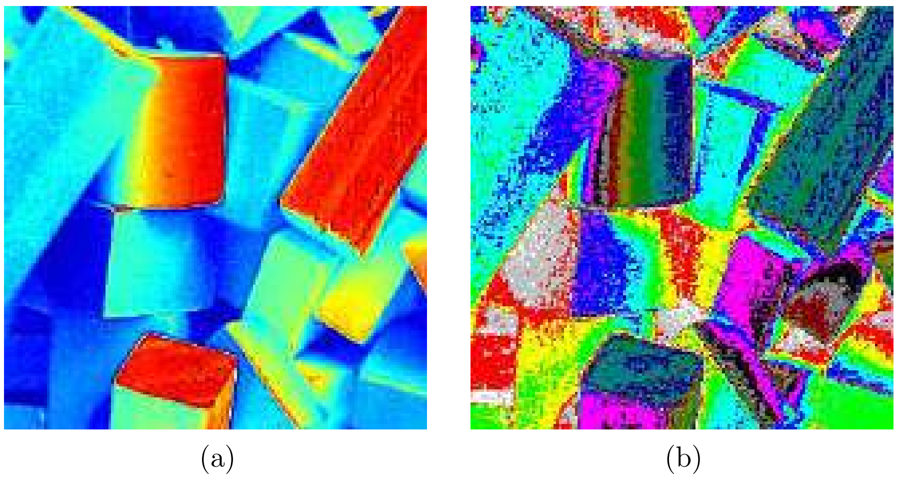


Figure 12.16: Applying a colour map to a greyscale image

image (a) with:

```
>> b=imread('blocks.tif');
>> imshow(b,colormap(jet(256))
```

However, a bad choice of colourmap can ruin an image. Image (b) in figure 12.16 is an example of this, where we apply the `vga` colourmap. Since this only has 16 rows, we need to reduce the number of greyscales in the image to 16. This is done with the `grayslice` function:

```
>> b16=grayslice(b,16);
>> figure,imshow(b16,colormap(vga))
```

The result, although undeniably colourful, is not really an improvement on the original image. The available colourmaps are listed in the help file for `graph3d`:

```
v        - Hue-saturation-value color map.
t        - Black-red-yellow-white color map.
ay       - Linear gray-scale color map.
ne       - Gray-scale with tinge of blue color map.
pper     - Linear copper-tone color map.
nk       - Pastel shades of pink color map.
ite      - All white color map.
ag       - Alternating red, white, blue, and black color map.
nes      - Color map with the line colors.
lorcube  - Enhanced color-cube color map.
a        - Windows colormap for 16 colors.
t        - Variant of HSV.
ism      - Prism color map.
ol       - Shades of cyan and magenta color map.
tumn     - Shades of red and yellow color map.
ring     - Shades of magenta and yellow color map.
nter     - Shades of blue and green color map.
mmer     - Shades of green and yellow color map.
```

There are help files for each of these colourmaps, so that

```
>> help hsv
```

will provide some information on the `hsv` colourmap.

We can easily create our own colourmap: it must by a matrix with 3 columns, and each row consists of RGB values between 0.0 and 1.0. Suppose we wish to create a blue, magenta, green, red colourmap as shown in figure 12.15. Using the RGB values:

Colour	Red	Green	blue
Blue	0	0	1
Magenta	1	0	1
Green	0	1	0
Red	1	0	0

we can create our colourmap with:

```
>> mycolourmap=[0 0 1;1 0 1;0 1 0;1 0 0];
```

Before we apply it to the blocks image, we need to scale the image down so that there are only the four greyscales 0, 1, 2 and 3:

```
>> b4=grayslice(b,4);
>> imshow(b4,mycolourmap)
```

and the result is shown in figure 12.17.

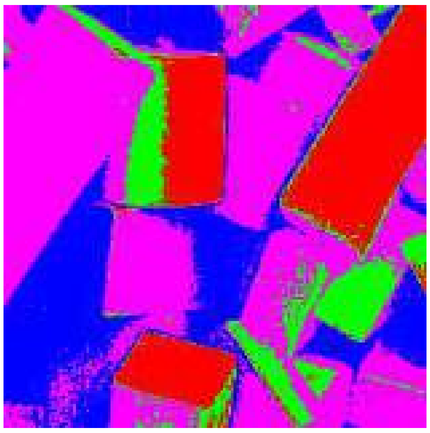


Figure 12.17: An image coloured with a "handmade" colourmap

12.5 Processing of colour images

There are two methods we can use:

1. we can process each R, G, B matrix separately,
2. we can transform the colour space to one in which the intensity is separated from the colour, and process the intensity component only.

Schemas for these are given in figures 12.18 and 12.19.

We shall consider a number of different image processing tasks, and apply either of the above schema to colour images.

Contrast enhancement

This is best done by processing the intensity component. Suppose we start with the image `cat.tif`, which is an indexed colour image, and convert it to a truecolour (RGB) image.

```
>> [x,map]=imread('cat.tif');
>> c=ind2rgb(x,map);
```

Now we have to convert from RGB to YIQ, so as to be able to isolate the intensity component:

```
>> cn=rgb2ntsc(c);
```

Now we apply histogram equalization to the intensity component, and convert back to RGB for display:

```
>> cn(:,:,1)=histeq(cn(:,:,1));
>> c2=ntsc2rgb(cn);
>> imshow(c2)
```

The result is shown in figure 12.20. Whether this is an improvement is debatable, but it has had its contrast enhanced.

But suppose we try to apply histogram equalization to each of the RGB components:

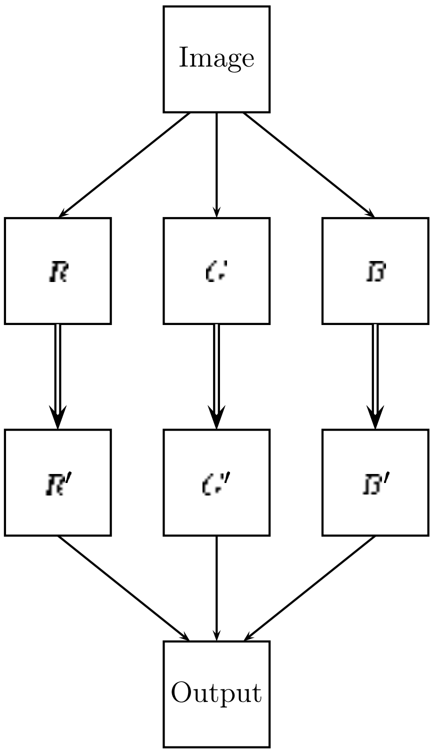


Figure 12.18: RGB processing

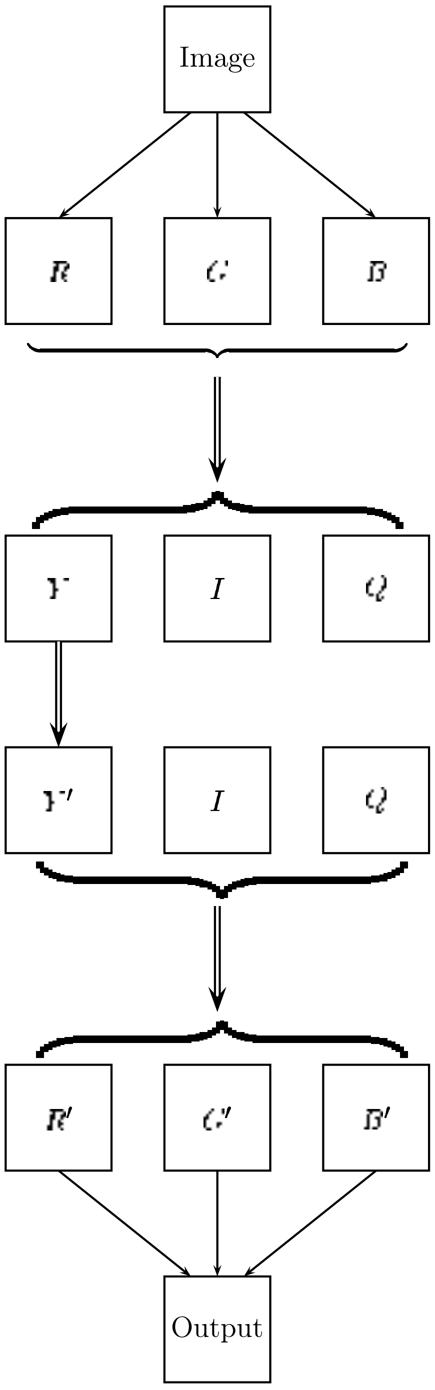


Figure 12.19: Intensity processing

```
>> cr=histeq(c(:,:,1));
>> cg=histeq(c(:,:,2));
>> cb=histeq(c(:,:,3));
```

Now we have to put them all back into a single 3 dimensional array for use with `imshow`. The `cat` function is what we want:

```
>> c3=cat(3,cr,cg,cb);
>> imshow(c3)
```

The first variable to `cat` is the dimension along which we want our arrays to be joined. The result is shown for comparison in figure 12.20. This is not acceptable, as some strange colours have been introduced; the cat's fur has developed a sort of purplish tint, and the grass colour is somewhat washed out.

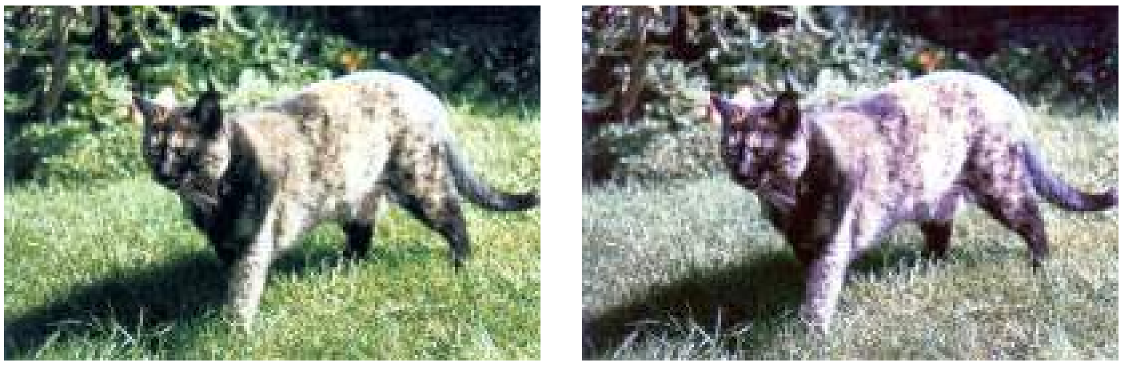


Figure 12.20: Histogram equalization of a colour image

Spatial filtering

It very much depends on the filter as to which schema we use. For a low pass filter, say a blurring filter, we can apply the filter to each RGB component:

```
>> a15=fspecial('average',15);
>> cr=filter2(a15,c(:,:,1));
>> cg=filter2(a15,c(:,:,2));
>> cb=filter2(a15,c(:,:,3));
>> blur=cat(3,cr,cg,cb);
>> imshow(blur)
```

and the result is shown in figure 12.21. We could also obtain a similar effect by applying the filter to the intensity component only. But for a high pass filter, for example an unsharp masking filter, we are better off working with the intensity component only:

```
>> cn=rgb2ntsc(c);
>> a=fspecial('unsharp');
>> cn(:,:,1)=filter2(a,cn(:,:,1));
>> cu=ntsc2rgb(cn);
>> imshow(cu)
```

and the result is shown in figure 12.21. In general, we will obtain reasonable results using the

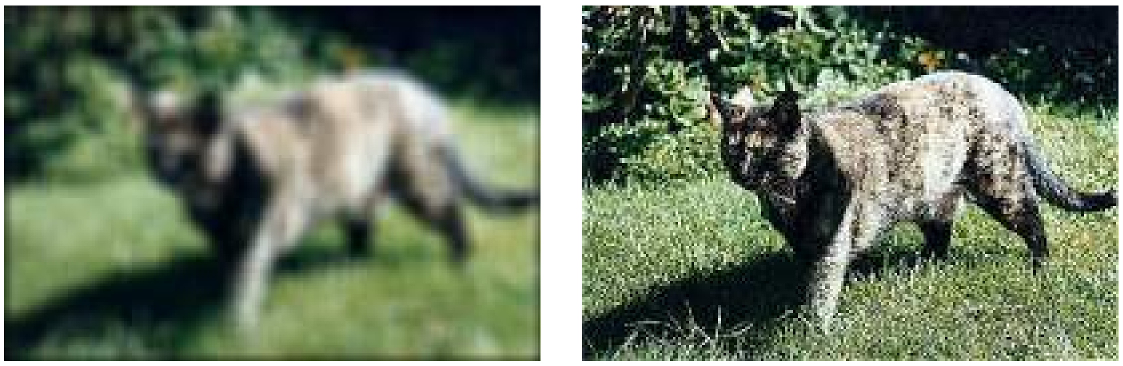


Figure 12.21: Spatial filtering of a colour image

intensity component only. Although we can sometimes apply a filter to each of the RGB components, as we did for the blurring example above, we cannot be guaranteed a good result. The problem is that any filter will change the values of the pixels, and this may introduce unwanted colours.

Noise reduction

As we did in chapter 7, we shall use the image `twins.tif`: but now in full colour!

```
>> tw=imread('twins.tif');
```

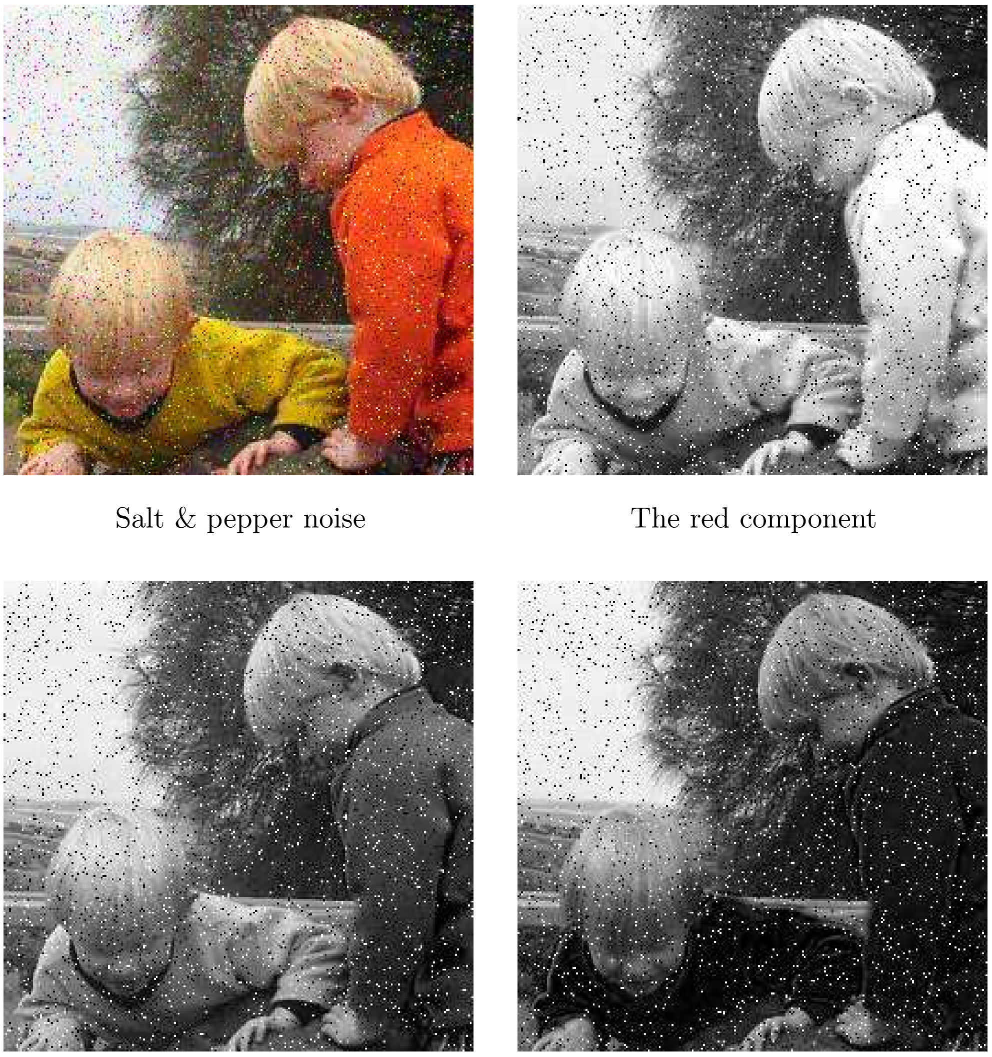
Now we can add noise, and look at the noisy image, and its RGB components:

```
>> tn=imnoise(tw,'salt & pepper');
>> imshow(tn)
>> figure,imshow(tn(:,:,1))
>> figure,imshow(tn(:,:,2))
>> figure,imshow(tn(:,:,3))
```

These are all shown in figure 12.22. It would appear that we should apply median filtering to each of the RGB components. This is easily done:

```
>> trm=medfilt2(tn(:,:,1));
>> tgm=medfilt2(tn(:,:,2));
>> tbm=medfilt2(tn(:,:,3));
>> tm=cat(3,trm,tgm,tbm);
>> imshow(tm)
```

and the result is shown in figure 12.23. We can't in this instance apply the median filter to the intensity component only, because the conversion from RGB to YIQ spreads the noise across all the YIQ components. If we remove the noise from Y only:



Salt & pepper noise

The red component

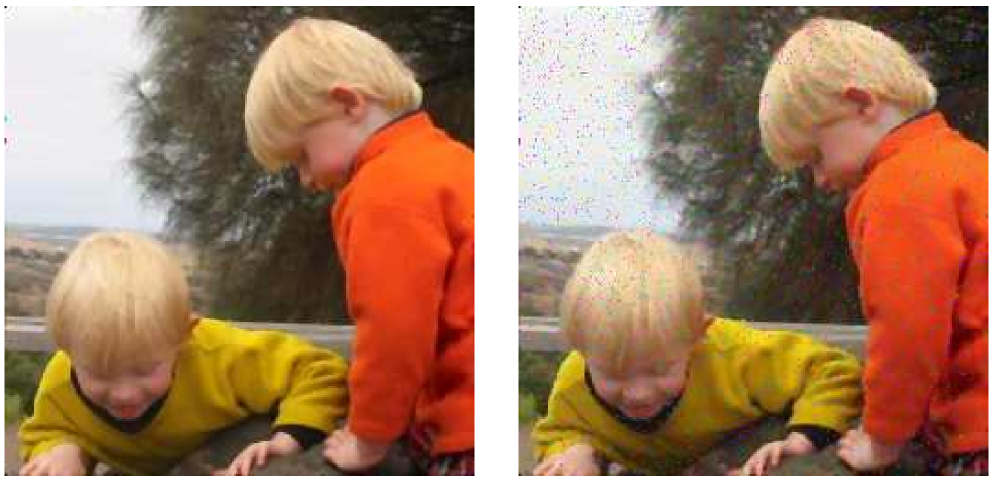
The green component

The blue component

Figure 12.22: Noise on a colour image

```
>> tnn=rgb2ntsc(tn);
>> tnn(:,:,1)=medfilt2(tnn(:,:,1));
>> tm2=ntsc2rgb(tnn);
>> imshow(tm2)
```

we see, as shown in figure 12.23 that the noise has been slightly diminished, but it is still there.



Denoising each RGB component   Denoising Y only

Figure 12.23: Attempts at denoising a colour image

Edge detection

An edge image will be a binary image containing the edges of the input. We can go about obtaining an edge image in two ways:

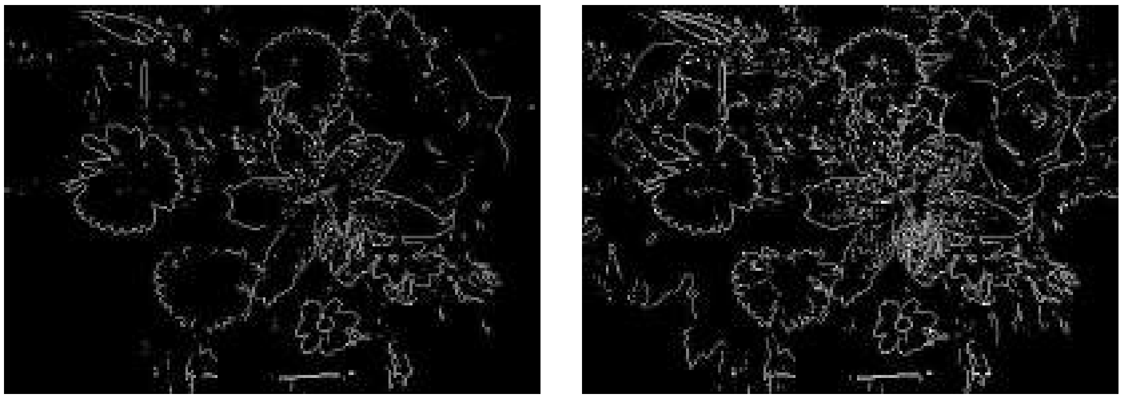
1. we can take the intensity component only, and apply the `edge` function to it,
2. we can apply the `edge` function to each of the RGB components, and join the results.

To implement the first method, we start with the `rgb2gray` function:

```
>> fg=rgb2gray(f);
>> fe1=edge(fg);
>> imshow(fe1)
```

Recall that `edge` with no parameters implements Sobel edge detection. The result is shown in figure 12.24. For the second method, we can join the results with the logical "or":

```
>> f1=edge(f(:,:,1));
>> f2=edge(f(:,:,2));
>> f3=edge(f(:,:,3));
>> fe2=f1 | f2 | f3;
>> figure,imshow(fe2)
```



`fe1`: Edges after `rgb2gray` `fe2`: Edges of each RGB component

Figure 12.24: The edges of a colour image

and this is also shown in figure 12.24. The edge image `fe2` is a much more complete edge image. Notice that the rose now has most of its edges, where in image `fe1` only a few were shown. Also note that there are the edges of some leaves in the bottom left of `fe2` which are completely missing from `fe1`.

Exercises

1. By hand, determine the saturation and intensity components of the following image, where the RGB values are as given:

$(0,1,1)$	$(1,2,3)$	$(7,7,7)$	$(5,1,2)$	$(1,1,7)$
$(2,1,2)$	$(1,7,7)$	$(2,0,2)$	$(3,3,2)$	$(5,5,0)$
$(4,4,4)$	$(4,6,7)$	$(4,5,6)$	$(1,5,7)$	$(3,6,7)$
$(3,0,3)$	$(5,2,2)$	$(1,1,1,$	$(6,6,0)$	$(2,2,2)$
$(1,2,1)$	$(0,4,4)$	$(3,1,6)$	$(3,3,3)$	$(2,4,6)$

2. Suppose the intensity component of an HSI image was thresholded to just two values. How would this affect the appearance of the image?

3. By hand, perform the conversions between RGB and HSV or YIQ, for the values:

R	G	B	H	S	V
0.5	0.5	0			
0	0.7	0.7			
0.5	0	0.5			
			0.33	0.5	1
			0.67	0.7	0.7
			0	0.2	0.8

R	G	B	Y	I	Q
0.3	0.3	0.7			
0.7	0.9	0			
0.8	0.8	0.7			
			1	0.3	0.3
			0.5	0.5	0.5
			0	1	1

You may need to normalize the RGB values.

4. Check your answers to the conversions in question 3 by using the MATLAB functions `rgb2hsv`, `hsv2rgb`, `rgb2ntsc` and `ntsc2rgb`.
5. Threshold the intensity component of a colour image, say `flowers.tif`, and see if the result agrees with your guess from question 2 above.
6. The image `spine.tif` is an indexed colour image; however the colours are all very close to shades of grey. Experiment with using `imshow` on the index matrix of this image, with varying colourmaps of length 64.

   Which colourmap seems to give the best results? Which colourmap seems to give the worst results?
7. View the image `autumn.tif`. Experiment with histogram equalization on:
   (a) the intensity component of HSV,
   (b) the intensity component of YIQ.

   Which seems to produce the best result?
8. Create and view a random "patchwork quilt" with:

```
>> r=uint8(floor(256*rand(16,16,3)));
>> r=imresize(r,16);
>> imshow(r),pixval on
```

   What RGB values produce (a) a light brown colour? (b) a dark brown colour?

   Convert these brown values to HSV, and plot the hues on a circle.
9. Using the flowers image, see if you can obtain an edge image from the intensity component alone, that is as close as possible to the image `fe2` in figure 12.24. What parameters to the `edge` function did you use? How close to `fe2` could you get?
10. Add Gaussian noise to an RGB colour image `x` with

```
  >> xn=imnoise(x,'gaussian');
```

   View your image, and attempt to remove the noise with
   (a) average filtering on each RGB component,
   (b) Wiener filtering on each RGB component.

Chapter 13

Image coding and compression

We have seen that image files can be very large. It is thus important for reasons both of storage and file transfer to make these file sizes smaller, if possible. In section 1.8 we touched briefly on the topic of compression; in this section we investigate some standard compression methods.

13.1 Lossless compression

This refers to the compression keeping all information, so that the image can be decompressed with no loss of information. This is preferred for images of legal or scientific significance, where loss of data—even of apparent insignificance, could have considerable consequences. Unfortunately this style tends not to lead to high compression ratios. However, lossless compression is used as part of many standard image formats.

Huffman coding

The idea of Huffman coding is simple. Rather than using a fixed length code (8 bits) to represent the grey values in an image, we use a variable length code, with smaller length codes corresponding to more probable grey values.

A small example will make this clear. Suppose we have a 2-bit greyscale image with only four grey levels: 0, 1, 2, 3, with the probabilities 0.2, 0.4, 0.3 and 0.1 respectively. That is, 20% of pixels in the image have grey value 50; 40% have grey value 100, and so on. The following table shows fixed length and variable length codes for this image:

Grey value	Probability	Fixed code	Variable code
0	0.2	00	000
1	0.4	01	1
2	0.3	10	01
3	0.1	11	001

Now consider how this image has been compressed. Each grey value has its own unique identifying code. The average number of bits per pixel can be easily calculated as the expected value (in a probabilistic sense):

$$(0.2 \times 3) + (0.4 \times 1) + (0.3 \times 2) + (0.1 \times 3) = 1.7.$$

Notice that the longest codewords are associated with the lowest probabilities. Although this average is only slightly smaller than 2, it is smaller by a significant amount.

To obtain the Huffman code for a given image we proceed as follows:

1. Determine the probabilities of each grey value in the image.
2. Form a binary tree by adding probabilities two at a time, always taking the two lowest available values.
3. Now assign 0 and 1 arbitrarily to each branch of the tree from its apex.
4. Read the codes from the top down.

To see how this works, consider the example of a 3-bit greyscale image (so the grey values are 0–7) with the following probabilities:

Grey value	0	1	2	3	4	5	6	7
Probability	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

We can now combine probabilities two at a time as shown in figure 13.1.

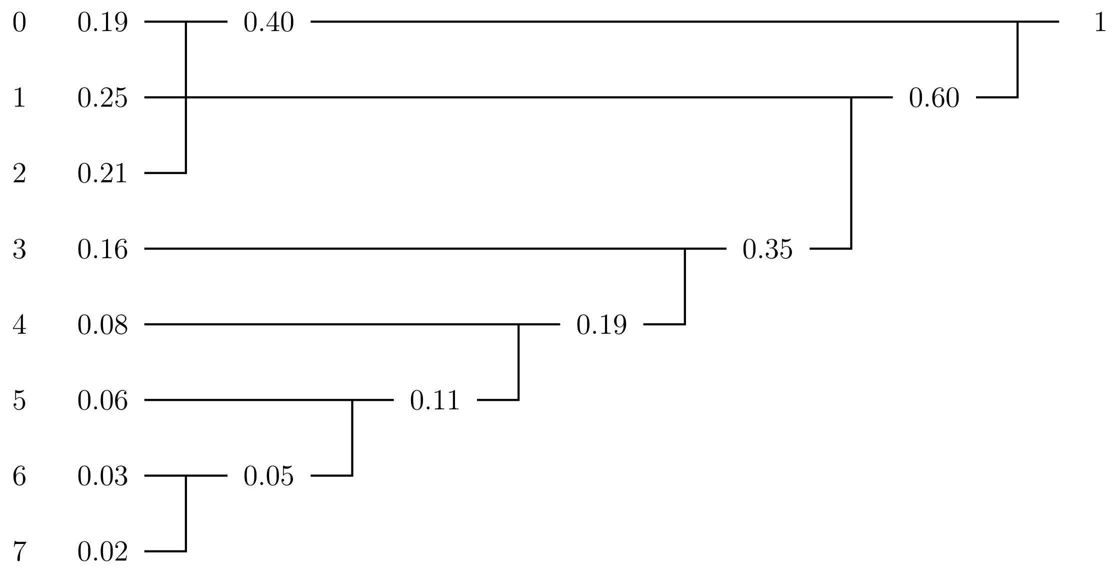


Figure 13.1: Forming the Huffman code tree

Note that if we have a choice of probabilities we choose arbitrarily. The second stage consists of arbitrarily assigning 0's and 1's to each branch of the tree just obtained. This is shown in figure 13.2.

To obtain the codes for each grey value, start at the 1 on the top right, and work back towards the grey value in question, listing the numbers passed on the way. This produces:

Grey value	Huffman code
0	00
1	10
2	01
3	110
4	1110
5	11110
6	111110
7	111111

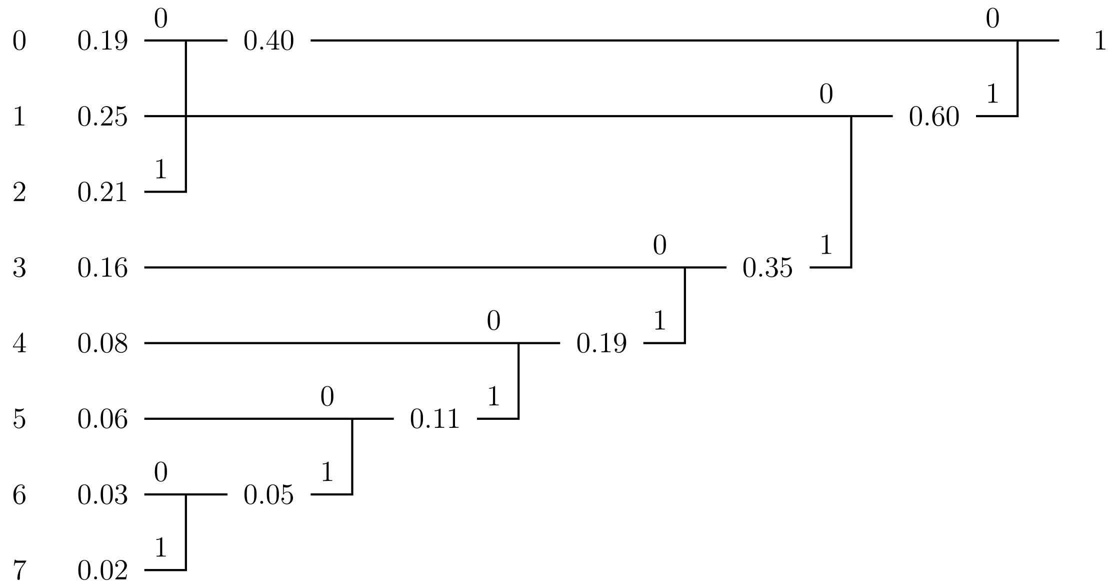


Figure 13.2: Assigning 0's and 1's to the branches

As above, we can evaluate the average number of bits per pixel as an expected value:

$$\begin{aligned}&(0.19\times 2)+(0{,}25\times 2)+(0.21\times 2)+(0.16\times 3)+\\&\quad(0.08\times 4)+(0.06\times 5)+(0.03\times 6)+(0.02\times 6)=2.7\end{aligned}$$

which is a significant improvement over 3 bits per pixel.

Huffman codes are *uniquely decodable*, in that a string can be decoded in only one way. For example, consider the string

$$1\;1\;0\;1\;1\;1\;0\;0\;0\;0\;0\;1\;0\;0\;1\;1\;1\;1\;1\;0$$

to be decoded with the Huffman code generated above. There is no code word 1, or 11, so we may take the first three bits 110 as being the code for grey value 3. Notice also that no other code word begins with this string. For the next few bits, we find that 1110 is a code word; no other begins with this string, and no other smaller string is a codeword. So we can decode this string as grey level 4. Continuing in this way we find:

$$\underbrace{1\;1\;0}_{3}\;\;\underbrace{1\;1\;1\;0}_{4}\;\;\underbrace{0\;0}_{0}\;\;\underbrace{0\;0}_{0}\;\;\underbrace{1\;0}_{1}\;\;\underbrace{0\;1}_{2}\;\;\underbrace{1\;1\;1\;1\;0}_{5}$$

as the decoding for this string.

For more information about Huffman coding, and its limitations and generalizations, see [5, 17].

Run length encoding

Run length encoding (RLE) is based on a simple idea: to encode strings of zeros and ones by the number of repetitions in each string. RLE has become a standard in facsimile transmission. For a binary image, there are many different implementations of RLE; one method is to encode each line separately, starting with the number of 0's. So the following binary image:

```
0 1 1 0 0 0
```

```
0 0 1 1 1 0
1 1 1 0 0 1
0 1 1 1 1 0
0 0 0 1 1 1
1 0 0 0 1 1
```

would be encoded as

$$(123)(231)(0321)(141)(33)(0132)$$

Another method [20] is to encode each row as a list of pairs of numbers; the first number in each pair given the starting position of a run of 1's, and the second number its length. So the above binary image would have the encoding

$$(22)(33)(1361)(24)(43)(1152)$$

Greyscale images can be encoded by breaking them up into their *bit planes*; these were discussed in chapter 4.

To give a simple example, consider the following 4-bit image and its binary representation:

$$\begin{array}{cccc} 10 & 7 & 8 & 9 \\ 11 & 8 & 7 & 6 \\ 9 & 7 & 5 & 4 \\ 10 & 11 & 2 & 1 \end{array} \rightarrow \begin{array}{cccc} 1010 & 0111 & 1000 & 1001 \\ 1011 & 1000 & 0111 & 0110 \\ 1001 & 0111 & 0101 & 0100 \\ 1010 & 1011 & 0010 & 0001 \end{array}$$

We may break it into bit planes as shown:

$$\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \multicolumn{4}{c}{\text{0th plane}} \end{array} \qquad \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ \multicolumn{4}{c}{\text{1st plane}} \end{array} \qquad \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ \multicolumn{4}{c}{\text{2nd plane}} \end{array} \qquad \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \multicolumn{4}{c}{\text{3rd plane}} \end{array}$$

and then each plane can be encoded separately using our chosen implementation of RLE.

However, there is a problem with bit planes, and that is that small changes of grey value may cause significant changes in bits. For example, the change from value 7 to 8 causes the change of all four bits, since we are changing the binary strings 0111 to 1000. The problem is of course exacerbated for 8-bit images. For RLE to be effective, we should hope that long runs of very similar grey values would result in very good compression rates for the code. But as we see, this may not be the case. A 4-bit image consisting of randomly distributed 7's and 8's would thus result in uncorrelated bit planes, and little effective compression.

To overcome this difficulty, we may encode the grey values with their binary *Gray codes.* A Gray code is an ordering of all binary strings of a given length so that there is only one bit change between a string and the next. So a 4-bit Gray code is:

15	1	0	0	0
14	1	0	0	1
13	1	0	1	1
12	1	0	1	0
11	1	1	1	0
10	1	1	1	1
9	1	1	1	0
8	1	1	0	0
7	0	1	0	0
6	0	1	0	1
5	0	1	1	1
4	0	1	1	0
3	0	0	1	0
2	0	0	1	1
1	0	0	0	1
0	0	0	0	0

See [17] for discussion and detail. To see the advantages, consider the following 4-bit image with its binary and Gray code encodings:

$$\begin{array}{cccc} 8 & 8 & 7 & 8 \\ 8 & 7 & 8 & 7 \\ 7 & 7 & 8 & 7 \\ 7 & 8 & 7 & 7 \end{array} \longrightarrow \begin{array}{cccc} 1000 & 1000 & 0111 & 1000 \\ 1000 & 0111 & 1000 & 0111 \\ 0111 & 0111 & 1000 & 0111 \\ 0111 & 1000 & 0111 & 0111 \end{array} \qquad \begin{array}{cccc} 1100 & 1100 & 0100 & 1100 \\ 1100 & 0100 & 1100 & 0100 \\ 0100 & 0100 & 1100 & 0100 \\ 0100 & 1100 & 0100 & 0100 \end{array}$$

where the first binary array is the standard binary encoding, and the second array the Gray codes. The binary bit planes are:

$$\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ \multicolumn{4}{c}{\text{0th plane}} \end{array} \qquad \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ \multicolumn{4}{c}{\text{1st plane}} \end{array} \qquad \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ \multicolumn{4}{c}{\text{2nd plane}} \end{array} \qquad \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \multicolumn{4}{c}{\text{3rd plane}} \end{array}$$

and the bit planes corresponding to the Gray codes are:

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \multicolumn{4}{c}{\text{0th plane}} \end{array} \qquad \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \multicolumn{4}{c}{\text{1st plane}} \end{array} \qquad \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \multicolumn{4}{c}{\text{2nd plane}} \end{array} \qquad \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \multicolumn{4}{c}{\text{3rd plane}} \end{array}$$

Notice that the Gray code planes are highly correlated except for one bit plane, whereas all the binary bit planes are uncorrelated.

Exercises

1. Construct a Huffman code for each of the probability tables given:

grey scale		0	1	2	3	4	5	6	7
probability	(a)	.07	.11	.08	.04	.5	.05	.06	.09
	(b)	.13	.12	.13	.13	.12	.12	.12	.13
	(c)	.09	.13	.15	.1	.14	.12	.11	.16

   In each case determine the average bits/pixel given by your code.

2. From your results of the previous question, what do think are the conditions of the probability distribution which give rise to a high compression rate using Huffman coding?

3. Encode each of the following binary images using run length encoding:

(a)

1	0	0	1	1	1
0	1	0	1	1	1
1	0	0	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
0	1	1	1	1	0

(b)

1	0	1	0	0	0
0	0	1	1	0	1
1	1	0	0	0	0
0	0	0	0	1	1
1	1	1	1	0	0
1	1	1	0	0	0

4. Using run length encoding, encode each of the following 4-bit images:

(a)

1	1	3	3	1	1
1	7	10	10	7	1
6	13	15	15	13	6
6	13	15	15	13	6
1	7	10	10	7	1
1	1	3	3	1	1

(b)

0	0	0	6	12	12	1	9
1	1	1	6	12	11	9	13
2	2	2	6	11	9	13	13
8	10	15	15	7	5	5	5
14	8	10	15	7	4	4	4
14	14	5	10	7	3	3	3

5. Decode the following run length encoded binary image, where each line has been encoded separately:

```
1  2  3  2  2  1  3  1  3  2  2  4  2  2  3  1  1
0  1  2  1  1  1  2  1  1  2  1  2  2  1  2  1  1  1  4  1  2  1  1  2  1
0  1  4  1  4  1  1  1  1  1  5  1  1  1  4  1  2  1  2  1  1
1  2  2  1  4  1  1  1  1  1  4  1  2  3  2  1  2  1  2  1  1
3  1  1  1  4  1  3  1  3  1  6  1  1  1  2  1  2  1  1
0  1  2  1  1  1  2  1  1  1  3  1  2  1  4  1  2  1  1  1  2  1  2  1  1
1  2  3  2  2  1  3  1  2  4  2  2  3  2  2  3
```

   Can you describe the resulting image?

6. The following are the run-length encodings for a 4×4 4-bit image from most to least important bit-planes:

```
3  1  2  2  1  4  1  2
1  2  1  2  1  2  1  2  1  3
2  1  2  1  2  2  1  5
0  3  1  3  2  3  1  2  1
```

Construct the image.

7. (a) Given the following 4-bit image:

0	4	4	4	4	4	6	7
0	4	5	5	5	4	6	7
1	4	5	5	5	4	6	7
1	4	5	5	5	4	6	7
1	4	4	4	4	4	6	7
2	2	8	8	8	10	10	11
2	2	9	9	9	12	13	13
3	3	9	9	9	15	14	14

transform it to a 3-bit image by removing the least most significant bit plane. Construct a Huffman code on the result and determine the average number of bits/pixel used by the code.

(b) Now apply Huffman coding to the original image and determine the average number of bits/pixel used by the code.

(c) Which of the two codes gives the best rate of compression?

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