

CHAPTER 6

IMAGE ENHANCEMENT

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Image Enhancement

- [Necessity of Image Enhancement](#)
- [Spatial Domain Operation](#)

Back to Course Content Page

[Click Here](#)

Image Enhancement

- Image Enhancement: It is processing of enhancement of certain feature of image
 - ▣ Removal of noise
 - ▣ Increasing contrast for a dark image etc.
 - ▣ Choice of image enhancement technique are vary from problem to problem
 - ▣ i.e. Best technique fro enhancement of X-Ray image may not be suitable for enhancement of microscopic images.

Image Enhancement

- There are two image enhancement technique
 - ▣ Spatial Domain Technique
 - Work on image plane itself
 - Discrete manipulation of pixels in an image
 - ▣ Frequency Domain Technique
 - Modify Fourier Transform coefficient of image and taking inverse Fourier Transform of modified image to obtain modified image



Back to the chapter content

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Spatial Domain Operation

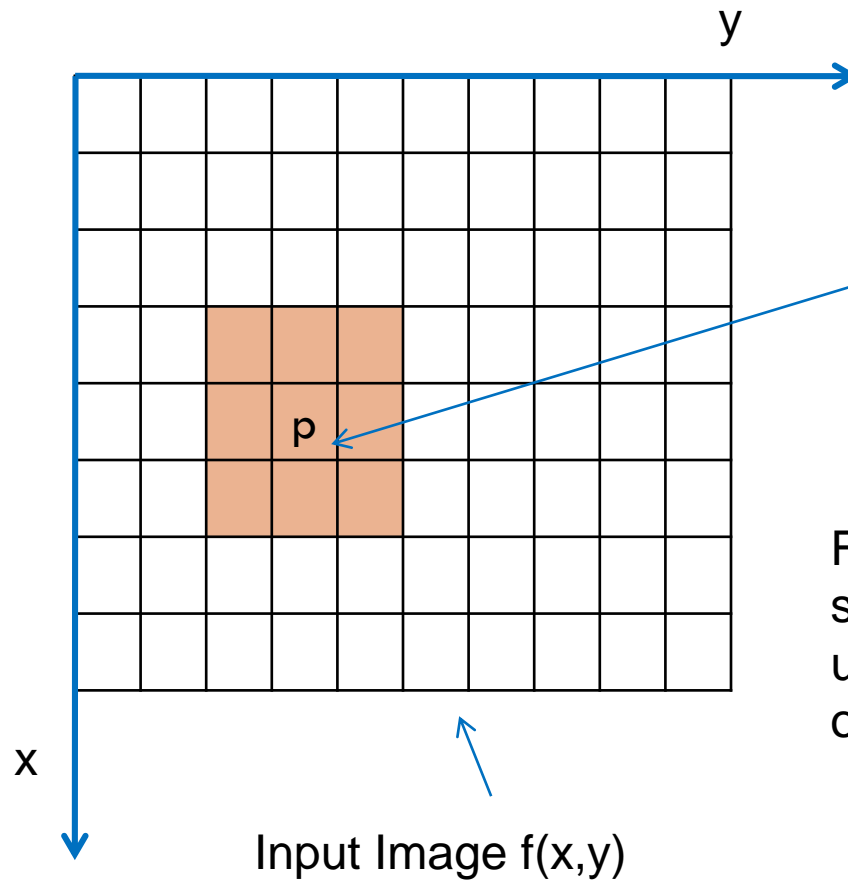
- Let we image $f(x,y)$ then

$$g(x,y) = T[f(x,y)]$$

- Where $T [.]$ is the transfer function which transform the input image $f(x,y)$ (which need to be enhanced) to enhanced image $g(x,y)$
- The operator T operates at point (x,y) considering certain neighborhood of the point (x,y) of the image
- Similarly for a single pixel x we can write

$$g(x) = T[f(x)]$$

Spatial Domain Operation



3 x 3 neighborhood at point (x,y)

For different technique neighborhood size may be different depending upon the type of image and type of operation

Spatial Domain Operation

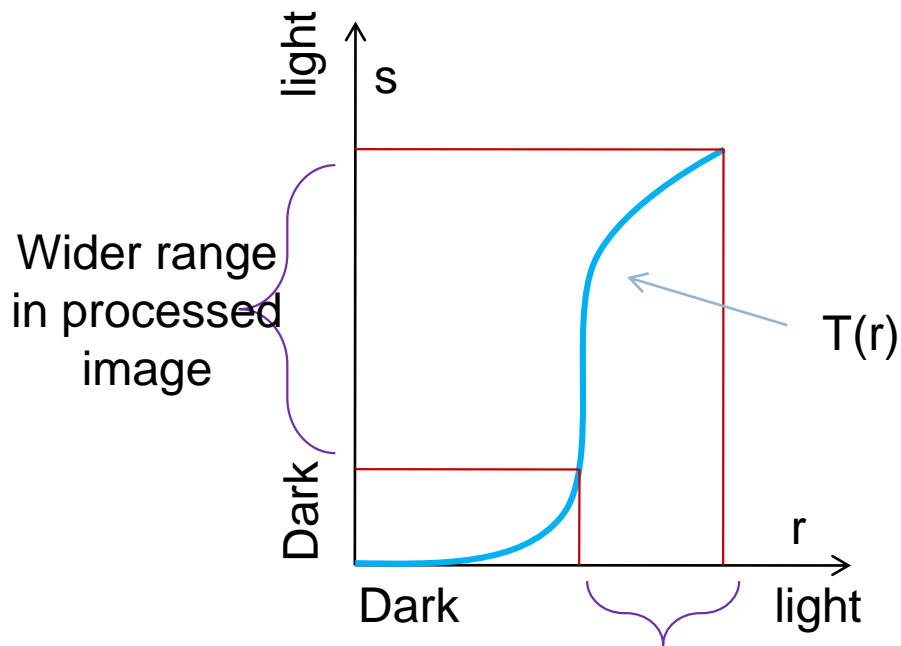
- Spatial domain operation includes
 - ▣ Point Processing Techniques
 - ▣ Histogram Based Technique
 - ▣ Mask Processing Technique

Point Processing Technique

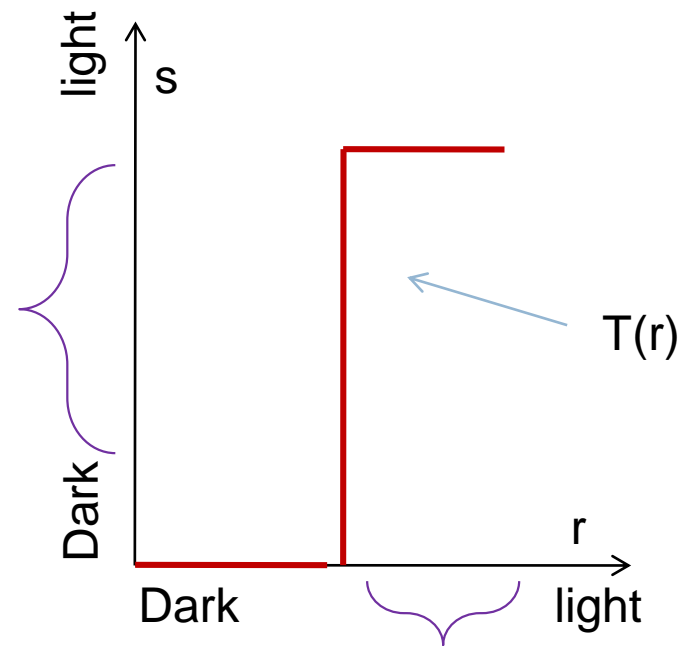
- In case of point processing neighborhood size considered is 1×1
- In point processing we process single pixel at a time
- So Transfer function become
- $S = T(r)$
- Where s is the processed point image
 r is the point to be processed
 T is transfer function

Point Processing Technique

Type of Transfer function



Narrow range in
original image



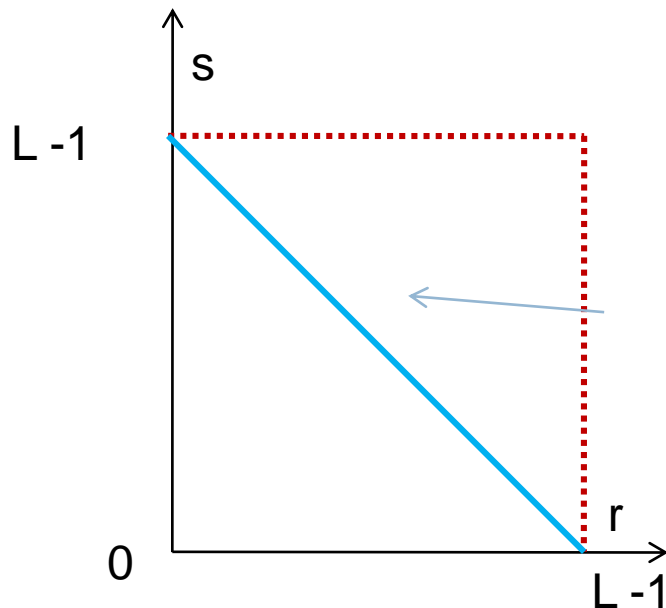
Thresholding Transfer
function

Point Processing Technique

- Point Processing Technique Includes
 - ▣ Image Negative
 - ▣ Contrast Stretching
 - ▣ Gray Level Slicing

Image Negative

- Light image converted to darker image
- Darker image converted to lighter image



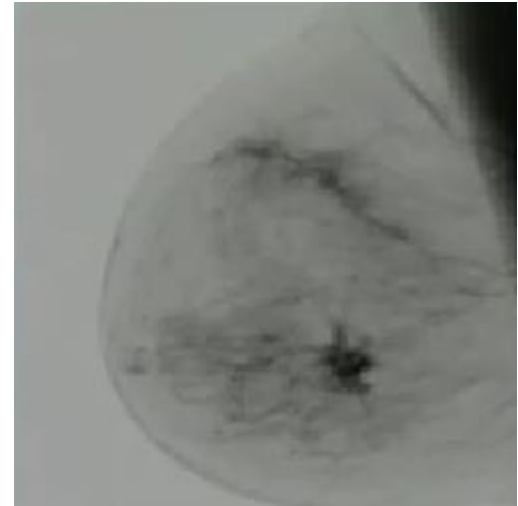
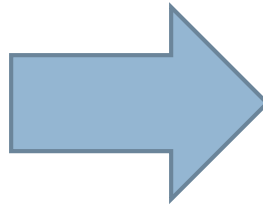
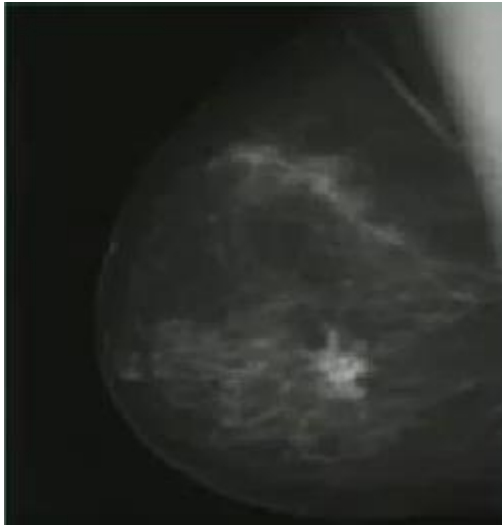
$$s = T(r) = L-1-r$$

Concept:

max intensity \rightarrow min intensity
min intensity \rightarrow max intensity

This kind of Transfer function is very useful in medical science

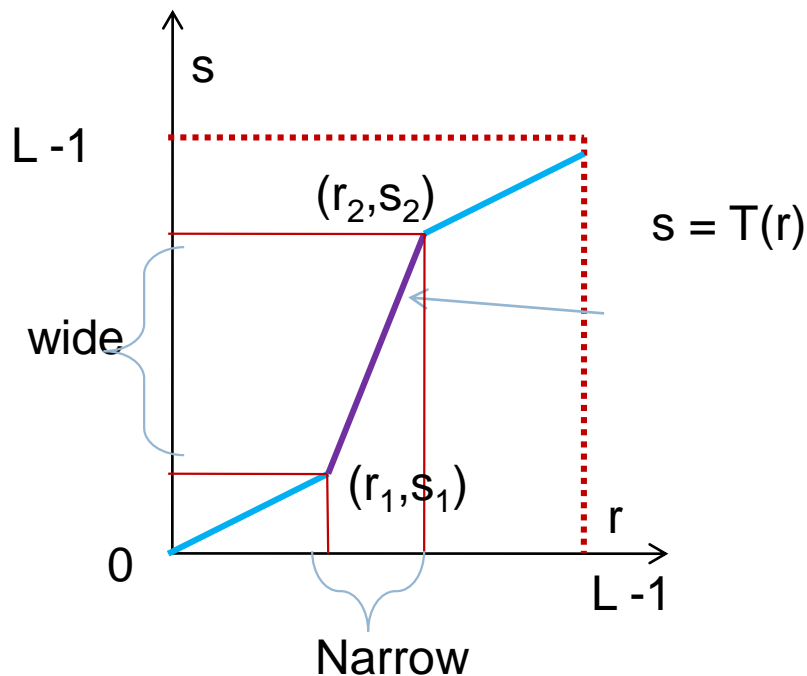
Image Negative Result



Cancer region detection

Contrast Stretching

- Contrast image is process of increasing low contrast image to high contrast image



Concept:

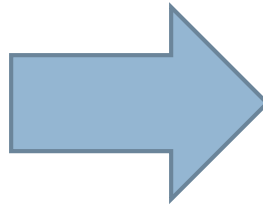
Narrow intensity range in original image is converted wide intensity range in processed image

For enhancement operation

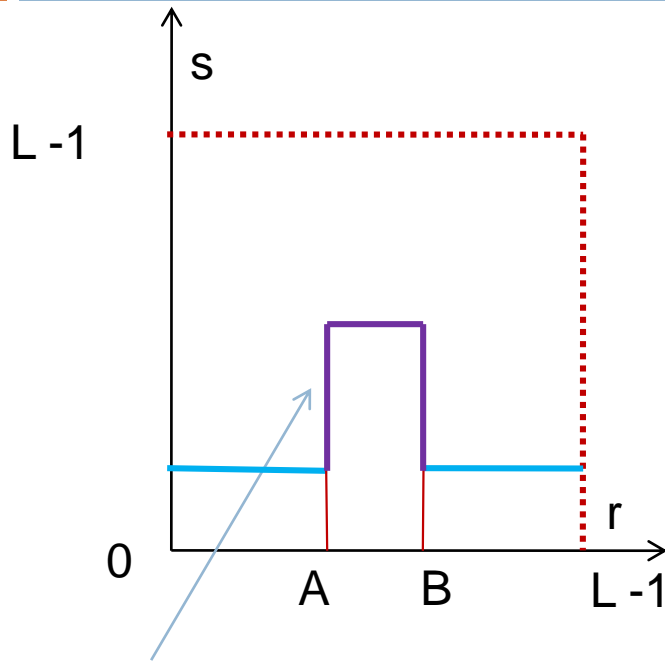
$$r_1 \leq r_2 \text{ and } s_1 \leq s_2$$

For $r_1 = r_2$, $s_1 = 0$ and $s_2 = L-1$ the transfer function become thresholding operation

Contrast Stretching Result

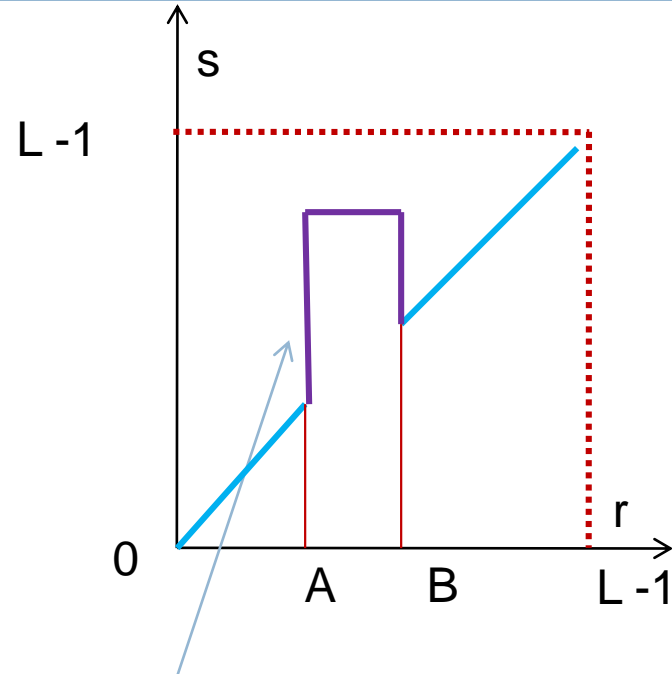


Gray Level Slicing



Concept:

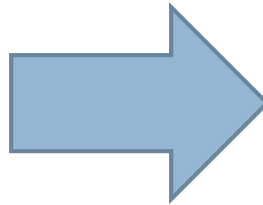
Here it shows that only in the range $A - B$ the image is enhanced and for all other pixel value are suppressed



Concept:

Here it shows that only in the range $A - B$ the image is enhanced and all other pixel value are retained

Gray Level Slicing Results



Histogram Based Technique

- Histogram Definition
- A image having $(0, L-1)$ discrete intensity level
- Then Histogram:

$$h(r_k) = n_k$$

- Where r_k : Intensity level
- n_k : No. of pixel having intensity level r_k
- The plot of distinct intensity level against all possible intensity level is known as Histogram

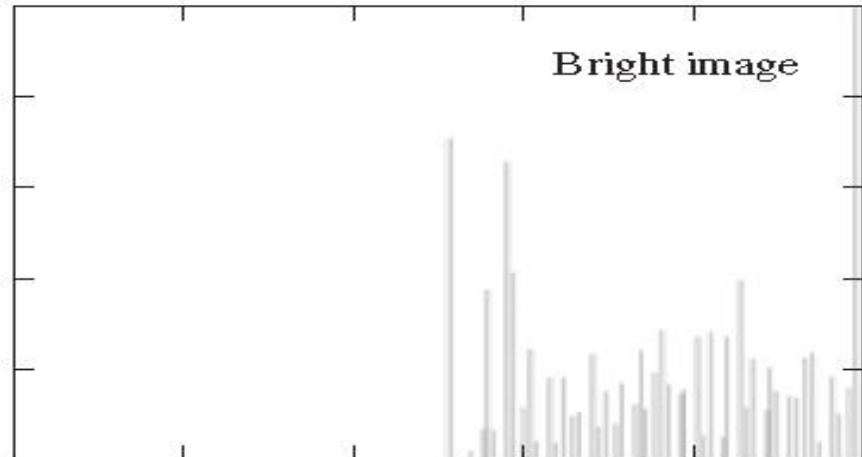
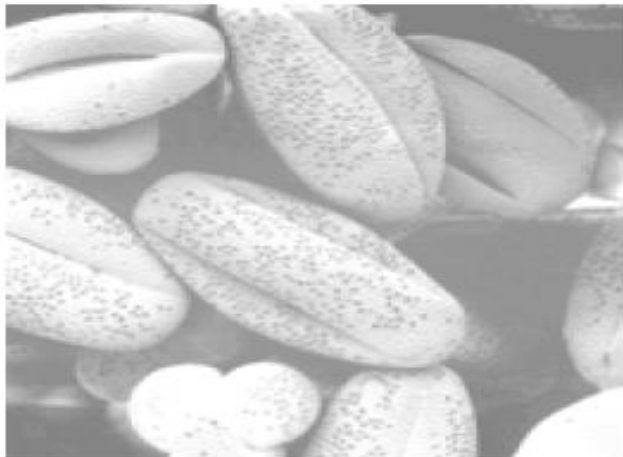
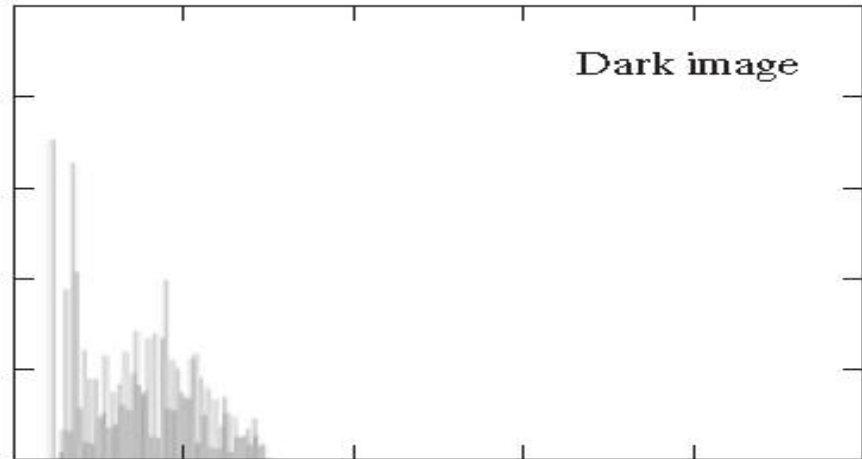
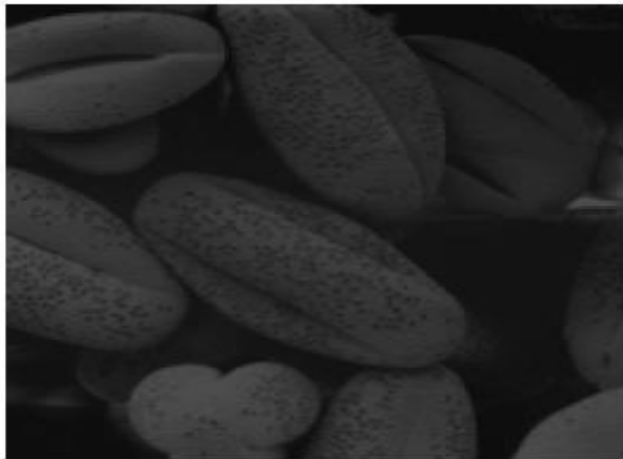
Histogram Based Technique

- Normalized Histogram:

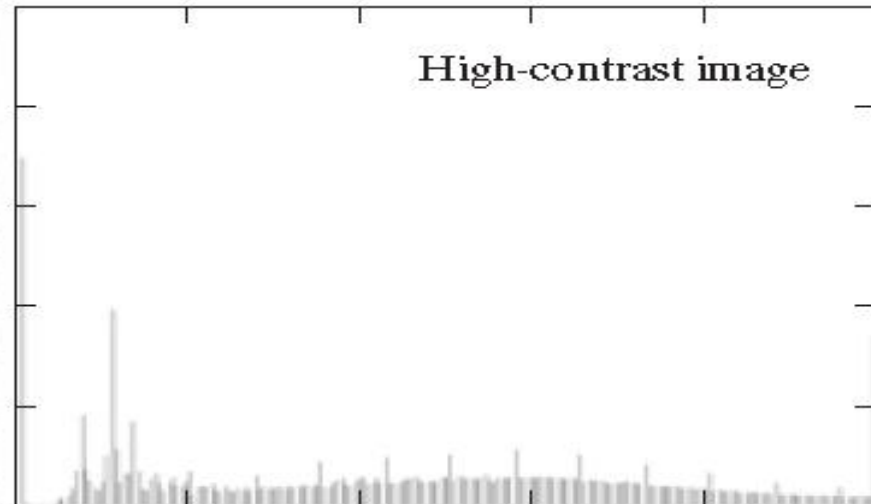
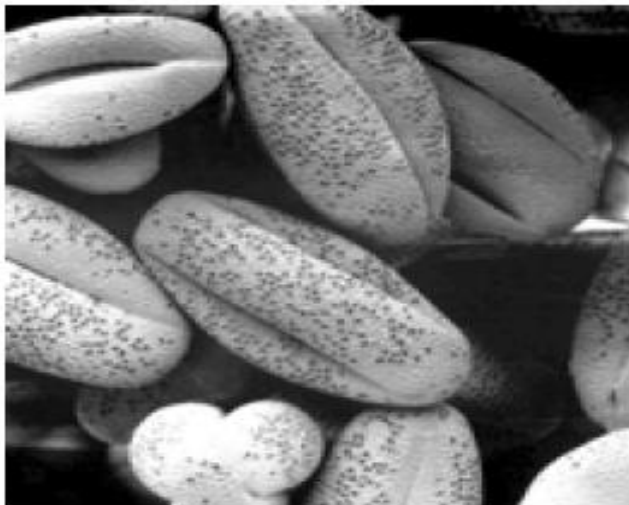
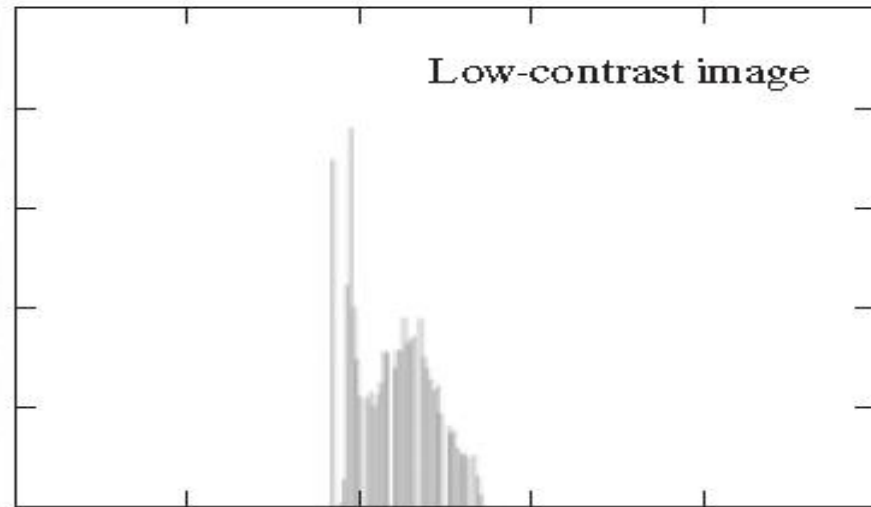
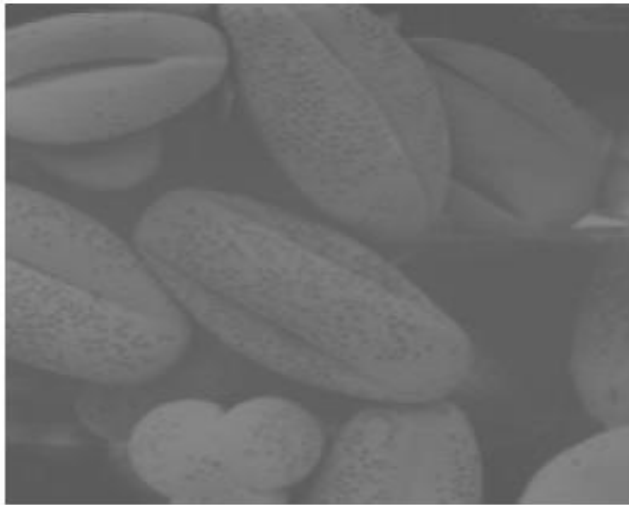
$$p(r_k) = n_k / n$$

- Where n is the total no. of pixel in the image
- $P(r_k)$ indicate the probability of occurrence of intensity of level r_k in the image
- Histogram Techniques includes
 - ▣ Histogram equalization
 - ▣ Histogram specification
 - ▣ Image Subtraction
 - ▣ Image Averaging

Histogram Based Technique



Histogram Based Technique



Histogram Equalization

- Let r represents gray level in an image
- Assume $[0,1]$ is the normalized pixel in the image where $0 \rightarrow$ black pixel and $1 \rightarrow$ white pixel

$$s = T(r)$$

- where r is the intensity in original image and s is the intensity in the processed image
- We assume the transfer function satisfy the following condition
 1. $T(r)$ must be a single valued & monotonically increasing in the range $0 \leq r \leq 1$ i.e. Darker pixel remains darker in the processed image
 2. $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$ i.e. any pixel intensity value may not be larger than the maximum intensity level.

Histogram Equalization

- Inverse Transfer

$$r = T^{-1}(s)$$

- It should also satisfy the above condition
- For continuous domain
- $p_r(r) \rightarrow$ pdf of r
- $p_s(s) \rightarrow$ pdf of s
- For elementary probability theory we know that if $p_r(r)$ and $p_s(s)$ are known and $T^{-1}(s)$ is single valued monotonically increasing function
- Then we can obtained

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|_{\text{at } r = T^{-1}(s)}$$

Histogram Equalization

- Now we take transfer function

- $s = T(r) = \int_{0 \rightarrow r} p_r(w) dw$

- From this we can compute

$$ds/dr = p_r(r)$$

- $p_s(s) = p_r(r) |dr/ds|$

- $= p_r(r) \cdot 1/p_r(r) = 1$

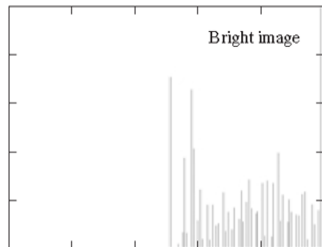
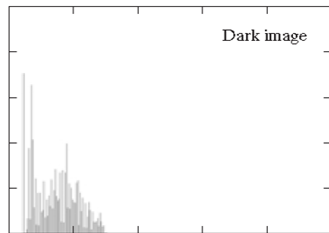
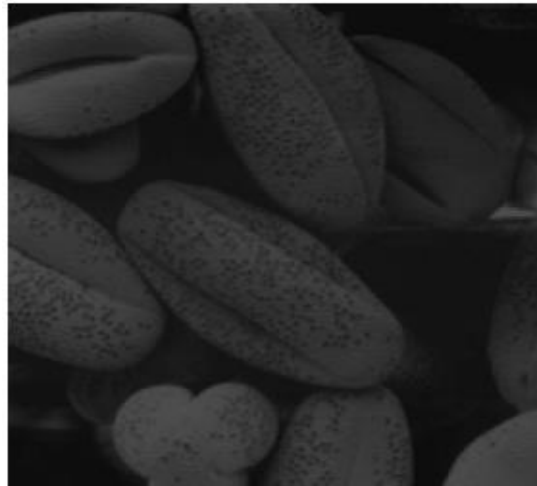
- Discrete Formulation

- $p_r(r_k) = n_k/n$

- Histogram equalization become

$$\begin{aligned} s_k = T(r_k) &= \sum_{i=0-k} p_r(r_i) \\ &= \sum_{i=0-k} n_i/n \end{aligned}$$

Histogram Equalization



Histogram Specification/Matching

- **Target Histogram**

- Let $r \rightarrow$ given image and $z \rightarrow$ target area in the image

- Hence $p_z(z) \rightarrow$ target histogram

- $s = T(r) = \int_{0 \rightarrow r} p_r(w) dw$

- Similarly we have function $G(z)$ instead $T(r)$ for target histogram

- $G(z) = \int_{0 \rightarrow z} p_z(t) dt$

- Discrete formulation

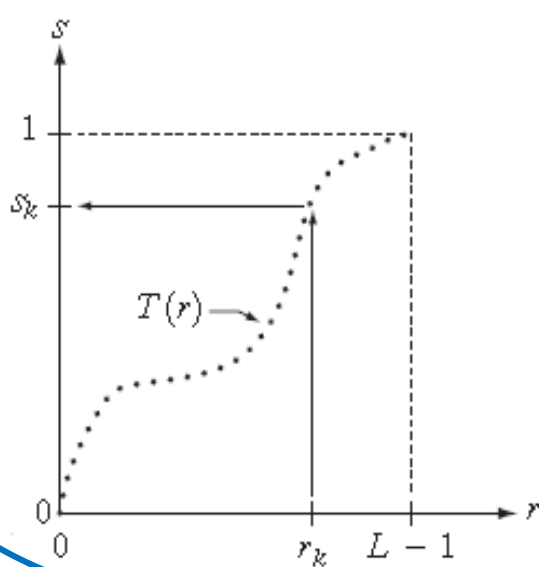
$$s_k = T(r_k) = \sum_{i=0-k} n_i/n = \sum_{i=0-k} p_r(r_i)$$

Let $p_z(z)$ is the **specified target histogram**

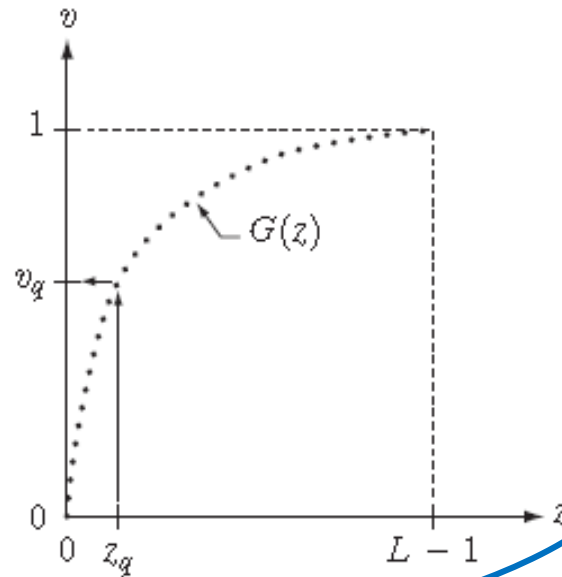
$$V_k = G(z_k) = \sum_{i=0-k} p_z(z_i) = s_k$$

$$Z_k = G^{-1} [T(r_k)] = G^{-1}(s_k)$$

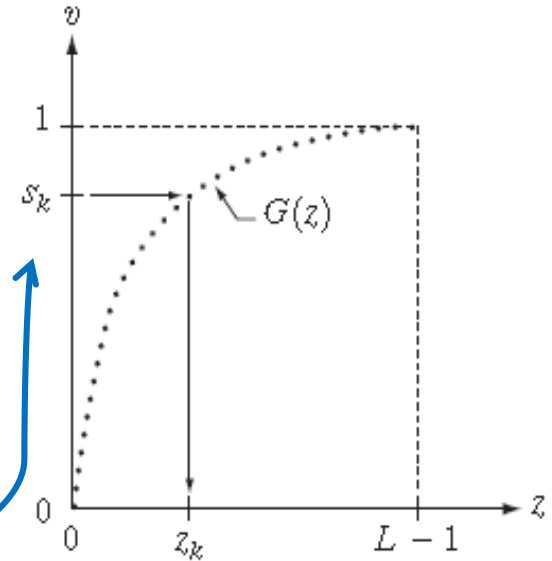
Histogram Specification/Matching



Graphical interpretation
of mapping from r_k to s_k
via $T(r)$.



Mapping of z_q to its
corresponding value
 v_q via $G(z)$.



Inverse mapping from s_k
to its corresponding
value of z_k

Histogram Exercise

- For digital image
- Min Intensity = 0
- Max Intensity (r_{L-1}) = 255
- The Mapping function

$$s' = \text{Int} \left[\frac{s - s_{\min}}{1 - s_{\min}} \times (L - 1) + 0.5 \right]$$

- Where $0 \leq s_k \leq 1$ to $0 \leq s'_k \leq 255$

Problem Histogram Equalization

- Given that
- $r \rightarrow 0, 1, 2, \dots, 7$ input image
- $s \rightarrow 0, 1, 2, \dots, 7$ processed image
- And given probability of occurrence of r_k
- $p_r(0) = 0.0$, $p_r(1) = 0.1$
- $P_r(2) = 0.1$, $p_r(3) = 0.3$
- $P_r(4) = 0.0$, $p_r(5) = 0.0$
- $P_r(6) = 0.4$, $p_r(7) = 0.1$
- Perform Histogram Equalization

Problem Histogram Equalization

| r | $p_r(r)$ | $s_k = T(r_k) = \sum_{i=0-k} p_r(r_i)$ | s' |
|-----|----------|--|------|
| 0 | 0 | 0 | 0 |
| 1 | 0.1 | 0.1 | 1 |
| 2 | 0.1 | 0.2 | 2 |
| 3 | 0.3 | 0.3 | 4 |
| 4 | 0 | 0.5 | 4 |
| 5 | 0 | 0.5 | 4 |
| 6 | 0.4 | 0.9 | 7 |
| 7 | 0.1 | 1.0 | 7 |

For $r = 3$ $s' = 4$ $\longrightarrow s' = \text{Int} \left[\frac{0.5 - 0}{1 - 0} \times (7) + 0.5 \right] = 4$

For $r = 6$ $s' = 7$ $\longrightarrow s' = \text{Int} \left[\frac{0.9 - 0}{1 - 0} \times (7) + 0.5 \right] = 7$

Histogram Specification

- Histogram equalization is not suitable for interactive histogram processing but histogram specification is
- **Problem**
- $p_r(0) = 0.0$, $p_r(1) = p_r(2) = 0.1$, $p_r(3) = 0.3$
- $p_r(4) = p_r(5) = 0.0$ $p_r(6) = 0.4$, $p_r(7) = 0.1$
- And
- $p_z(0) = 0.0$, $p_z(1) = 0.1$ $p_z(2) = 0.2$, $p_z(3) = 0.4$
- $p_z(4) = 0.2$, $p_z(5) = 0.1$ $p_r(6) = p_r(7) = 0.1$
- **Perform Histogram Specification**

Histogram Specification

| Input Image | | | Processed Image | Target Image | | |
|-------------|----------|--|-----------------|--------------|----------|--------------------------------|
| r | $p_r(r)$ | $s_k = T(r_k) = \sum_{i=0-k} p_r(r_i)$ | z' | z | $P_z(z)$ | $G(z) = \sum_{i=0-k} p_z(z_i)$ |
| 0 | 0 | 0 | 0 | 0 | 0.0 | 0 |
| 1 | 0.1 | 0.1 | 1 | 1 | 0.1 | 0.1 |
| 2 | 0.1 | 0.2 | 2 | 2 | 0.2 | 0.3 |
| 3 | 0.3 | 0.3 | 3 | 3 | 0.4 | 0.7 |
| 4 | 0 | 0.5 | 3 | 4 | 0.2 | 0.9 |
| 5 | 0 | 0.5 | 3 | 5 | 0.1 | 1.0 |
| 6 | 0.4 | 0.9 | 4 | 6 | 0.0 | 1.0 |
| 7 | 0.1 | 1.0 | 5 | 7 | 0.0 | 1.0 |

Z' = Minimum value of z for which $G(z) - s \geq 0$

At $r = 3$, $Z' = 3$ because for value

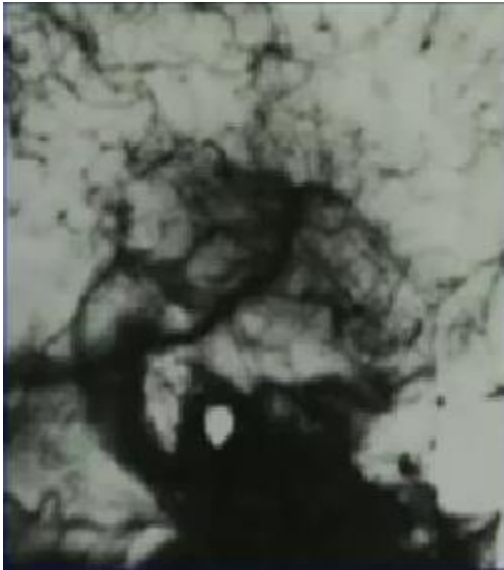
$\mathbf{z = 2}$ $G(2) - 0.5 = -0.2 \leq 0$; $\mathbf{z = 3}$ $G(3) - 0.5 = 0.2 \geq 0$; $\mathbf{z = 4}$ $G(4) - 0.5 = 0.2 \geq 0$

Image Differencing

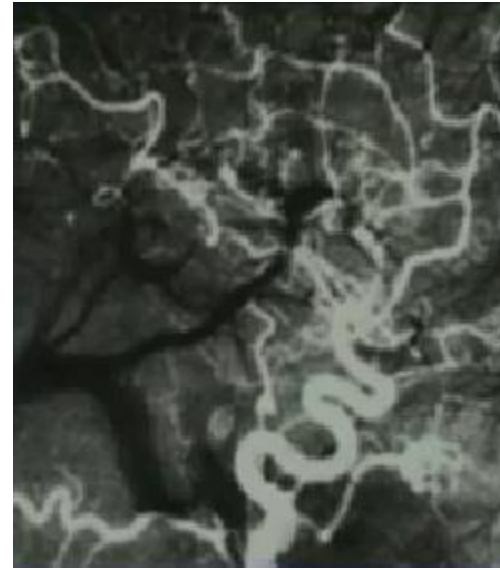
$$g(x,y) = f(x,y) - h(x,y)$$

- where $h(x,y)$ is the image subtracted from image $f(x,y)$
- Application of Image Subtraction
 - ▣ Mask mode radiography: (It is a image differencing preprocess for showing oriental diseases.)
- *In this case $h(x, y)$, the mask, is an X-ray image of a region of a patient's body captured by an intensified TV camera (instead of traditional X-ray film) located opposite an X-ray source. The procedure consists of injecting a contrast medium into the patient's bloodstream, taking a series of images of the same anatomical region as $h(x, y)$, and subtracting this mask from the series of incoming images after injection of the contrast medium. The net effect of subtracting the mask from each sample in the incoming stream of TV images is that the areas that are different between $f(x, y)$ and $h(x, y)$ appear in the output image as enhanced detail.*

Application Image Differencing



Mask image.



An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out

Image Averaging

$$g(x,y) = f(x,y) + \eta(x,y)$$

Error (noise must be zero mean)

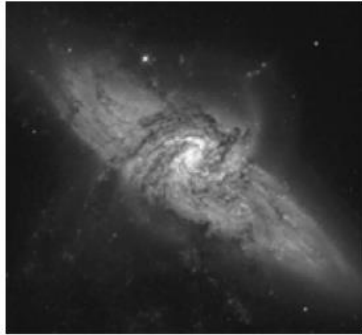
$$g'(x,y) = 1/k \sum_{i=1}^K g_i(x,y)$$

average of k frames

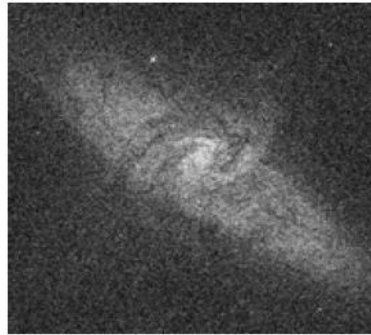
$$E [g'(x,y)] = f(x,y)$$

- Application in Astronomical Field
- Noise reduction by image averaging

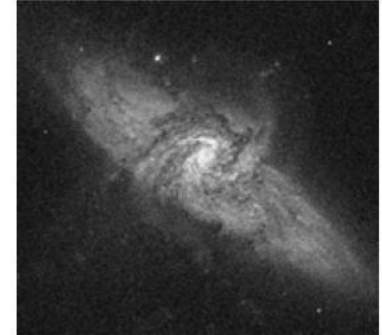
Image Averaging Result



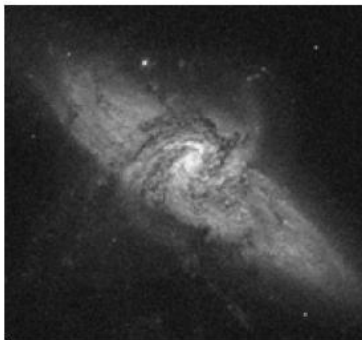
a



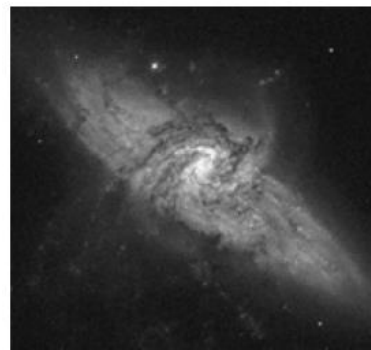
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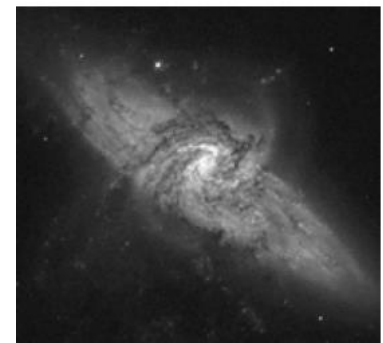
c



d



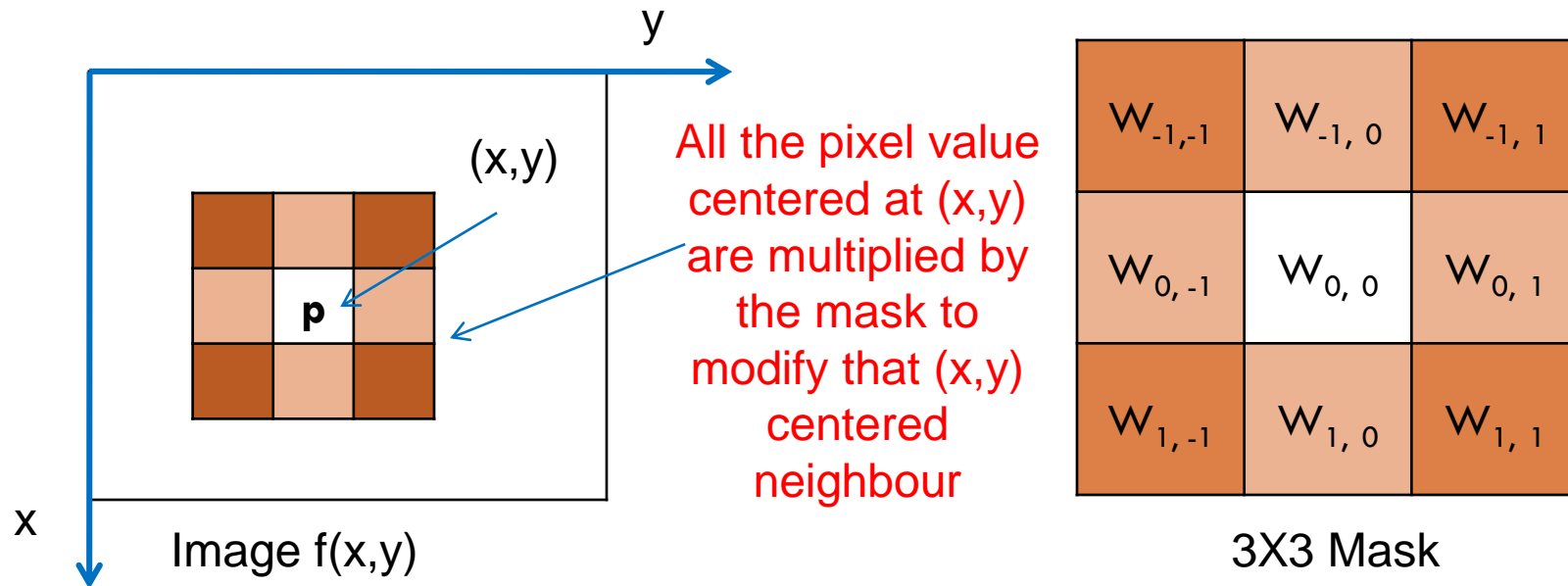
e



f

(a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K=8, 16, 64$, and 128 noisy images.

Mask Processing Techniques



$$g(x,y) = \sum \sum w_{ij} f(x+i, y+i) \quad \text{for } i = -1 \text{ to } 1 \text{ \& } j = -1 \text{ to } 1$$

Where w_{ij} is the mask coefficient and this mask coefficient is different for different techniques

For mask processing neighbourhood must be greater than 1

Operations: Image sharpening, Averaging etc.

Mask Processing Techniques

- The mask processing technique includes
 - ▣ Linear Smoothing Filter
 - ▣ Median Filter (non linear)
 - ▣ Sharpening Filter

Linear Smoothing Filter

- Performing averaging on an image means smoothening an image
- If an image is smoothen much more it become blurred
- Averaging Filter

$1/9 \times$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$$g(x,y) = 1/9 \sum \sum w_{ij} f(x+i, y+i) \\ \text{for } i = -1 \text{ to } 1 \text{ \& } j = -1 \text{ to } 1$$

Linear Smoothing Filter

□ Weighted Averaging

1/16 x

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

Weight reduces when point are away from the center and increases when comes closer to center

$$g(x,y) = 1/16 \sum \sum w_{ij} f(x+i, y+i) \\ \text{for } i = -1 \text{ to } 1 \text{ \& } j = -1 \text{ to } 1$$

In general form :

$$g(x,y) = \frac{\sum_{i=-a}^a \sum_{j=-b}^b w_{ij} f(x+i, y+i)}{\sum_{i=-a}^a \sum_{j=-b}^b w_{ij}}$$

For mask size MxN where $M = 2a + 1$ & $N = 2b + 1$

Results



(a) Original image



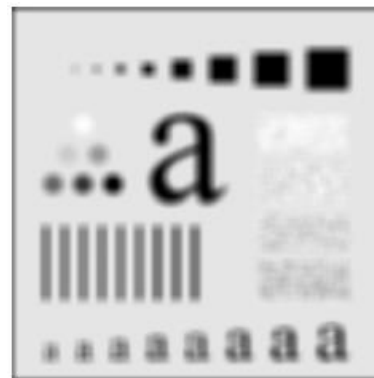
(b)



(c)



(d)



(e)



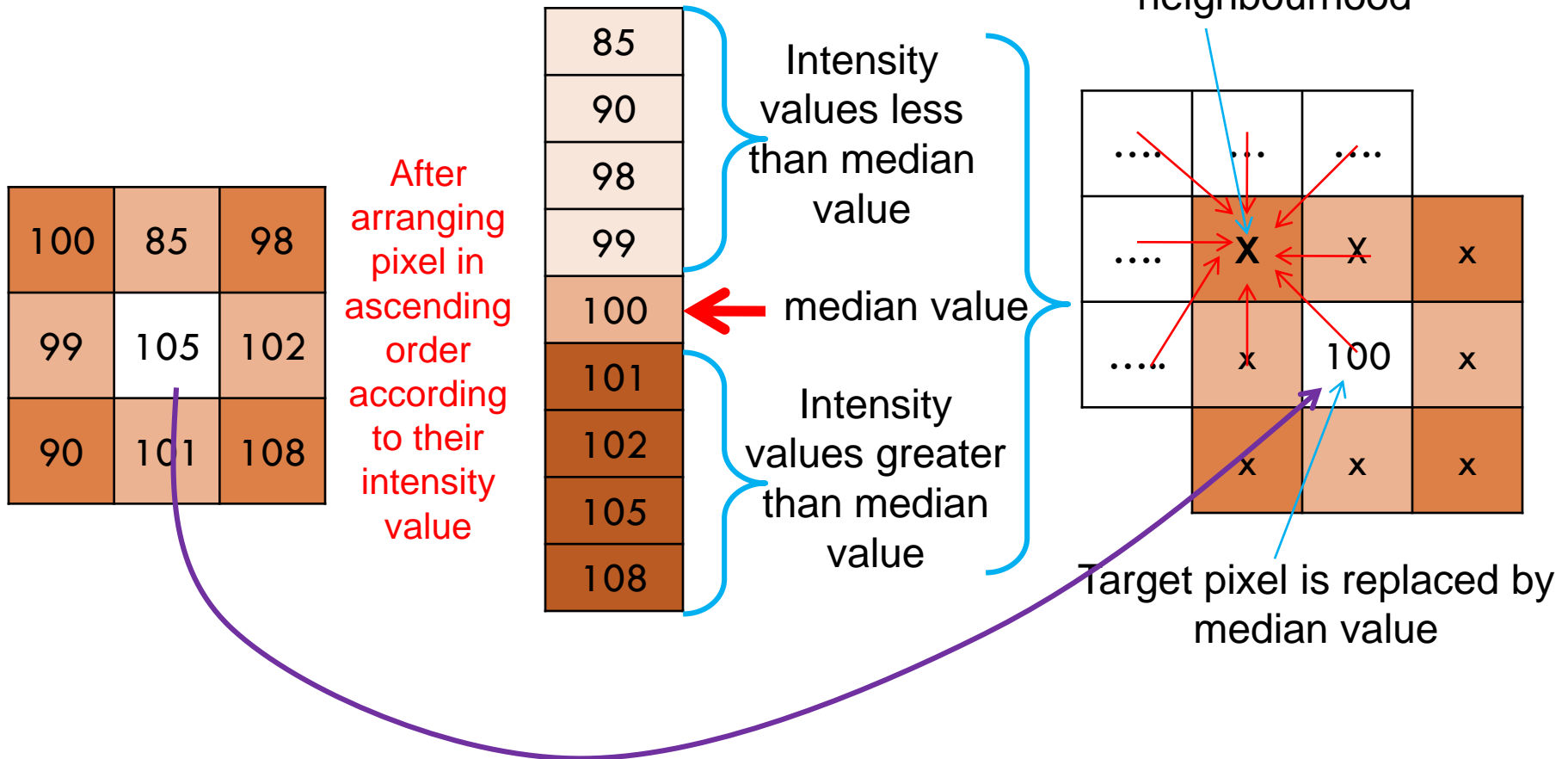
(f)

(a) Original image, of size 500*500 pixels (b)–(f) Results of smoothing with square averaging filter masks of sizes $n=3, 5, 9, 15$, and 35 , respectively.

Median Filter (non linear)

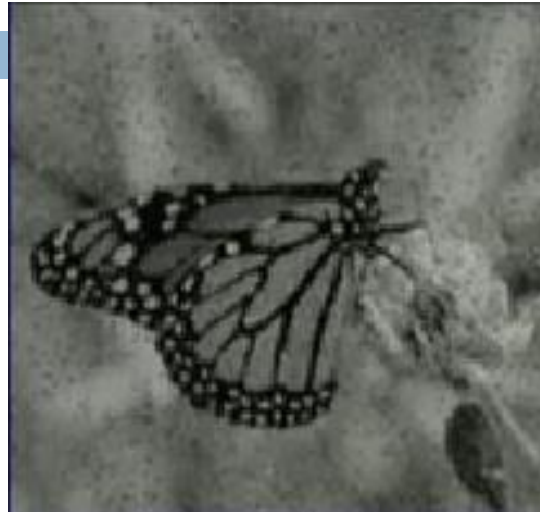
- It is also known as Order-statistics filters
- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
- Median filter is a well known nonlinear filter

Median Filter (non linear)



Results

Original Image



Result of
Averaging using
mask size 5x5



Result of Median
filtering



Superiority of
median filter
over averaging
filter

Sharpening Filter

- In the smoothening filters (linear & non linear) the image are smoothen and as a result image loses its detailing.
- Image sharpening is a process to enhance an image such that it can extract various details of an image
- Averaging is basically an integration operation whereas sharpening uses derivative operation over image
- Two types of derivative operation are First Order Derivative & Second Order Derivative

Desirable Response of Derivative Filters

- First Order Derivative Filter.
 - ▣ Must be Zero in area of constant gray level
 - ▣ Non Zero at the onset of grey level step or ramp
 - ▣ Non Zero along ramp
- Second Order Derivative Filter
 - ▣ Zero in flat area
 - ▣ Non Zero at onset and end of a gray level step or ramp
 - ▣ Zero along ramp of constant slope

Sharpening Filter

- First Order Derivative Filter.

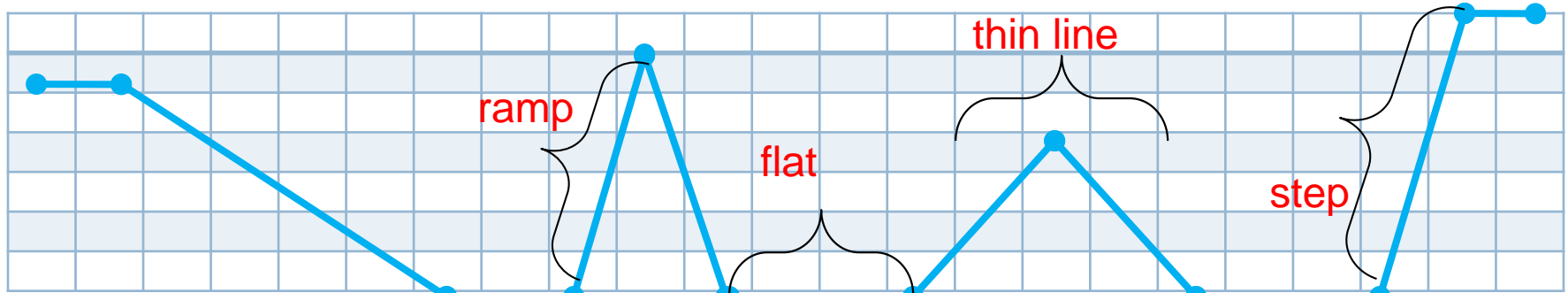
$$\frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{df(x)}{dx} = f(x + 1) - f(x)$$

- Second Order Derivative Filter

$$\frac{d^2f(x)}{dx^2} = f(x + 1) + f(x - 1) - 2f(x)$$

Choice between 1st & 2nd Order Derivative



| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5 | 4 | 3 | 2 | 1 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 1 | 3 | 1 | 0 | 0 | 0 | 0 | 7 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Non Zero ramp

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|----|----|----|----|----|---|---|---|----|---|---|---|---|---|---|----|----|---|---|---|---|---|---|
| 0 | -1 | -1 | -1 | -1 | -1 | 0 | 0 | 6 | -6 | 0 | 0 | 0 | 0 | 1 | 2 | -2 | -1 | 0 | 0 | 0 | 7 | 0 | 0 |
|---|----|----|----|----|----|---|---|---|----|---|---|---|---|---|---|----|----|---|---|---|---|---|---|

Zero ramp

| | | | | | | | | | | | | | | | | | | | | | | | |
|----|---|---|---|---|---|---|---|-----|---|---|---|---|---|---|---|----|---|---|---|---|---|----|---|
| -1 | 0 | 0 | 0 | 0 | 1 | 0 | 6 | -12 | 6 | 0 | 0 | 0 | 0 | 1 | 1 | -4 | 1 | 1 | 0 | 0 | 7 | -7 | 0 |
|----|---|---|---|---|---|---|---|-----|---|---|---|---|---|---|---|----|---|---|---|---|---|----|---|

Non Zero onset at end of the ramp

Response of 2nd order derivative of isolated point is much stronger than 1st order similarly thin line

Almost same for step

Observation

- 1st Order derivation generally produce thicker edge in an image
- 2nd Order derivative gives stronger response to fine details such as thin line and isolated point
- 1st order derivative have stronger response to gray level step
- 2nd order derivative produce a double response at step edge

Second Order derivative are better suitable for image enhancement

- Discrete formulation of 2nd Order derivative filter should be isotropic in nature (The response of filter should be independent of the orientation of the discontinuity in the image)

Laplacian Operator

- Popularly known 2nd order derivative operator is Laplacian Operator , it is Isotropic in nature
- Laplacian Operator in continuous domain

$$\nabla^2 f = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$

$$\frac{d^2 f}{dx^2} = f(x+1) + f(x-1) - 2f(x)$$

For image $f(x,y)$

$$\frac{d^2 f}{dx^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{d^2 f}{dy^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Laplacian Operator

- Adding equations we get Laplacian Operator in discrete domain

$$\nabla^2 f = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

| | | |
|---|----|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

Laplacian Operator for
Vertical & Horizontal direction

| | | |
|---|----|---|
| 1 | 1 | 1 |
| 1 | -8 | 1 |
| 1 | 1 | 1 |

Laplacian Operator allow
diagonal direction also

Laplacian Operator Results

(a)



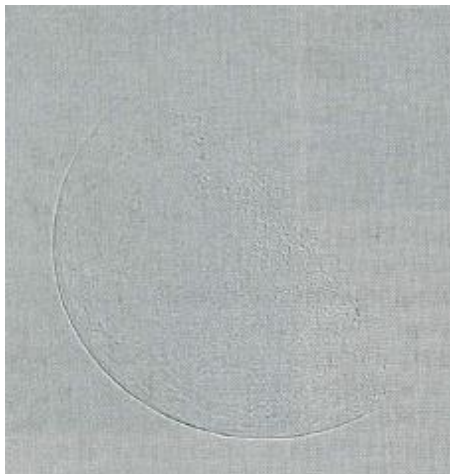
(b)



(a) Original Image

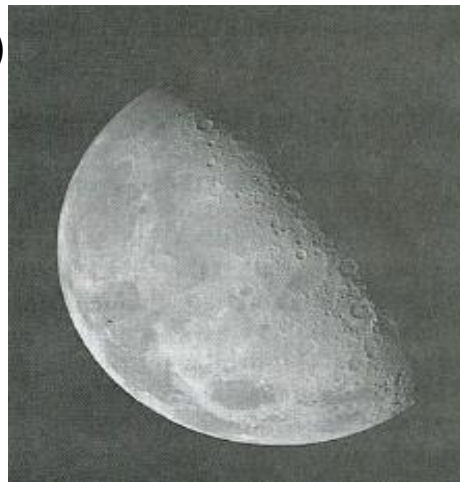
(b) Laplacian
filtered iage

(c)



(c) Laplacian
scaled image

(d)



(d) Enhanced
image using
laplacian
operator



Back to the chapter content

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