CHAPTER 8

IMAGE REGISTRATION

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Image Registration

- Image Registration
- Different Mismatch Or Match Measure
  - Cross-correlation Between Two Image
  - Cauchy – Schwartz Inequality
  - Example

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Image Registration

- Registration is a process which make the pixel in two image precisely coincide to the same point in the scene
- Once registered the image can be fused/combined in a way that improves information extraction

Applications of Image Registration

- Stereo imaging where two image are taken from different position
- Remote Sensing where the image may be taken by different sensor
- Image taken at different instance of time
- Finding a place in a picture where it matches a given pattern or temple. (Automated navigation)

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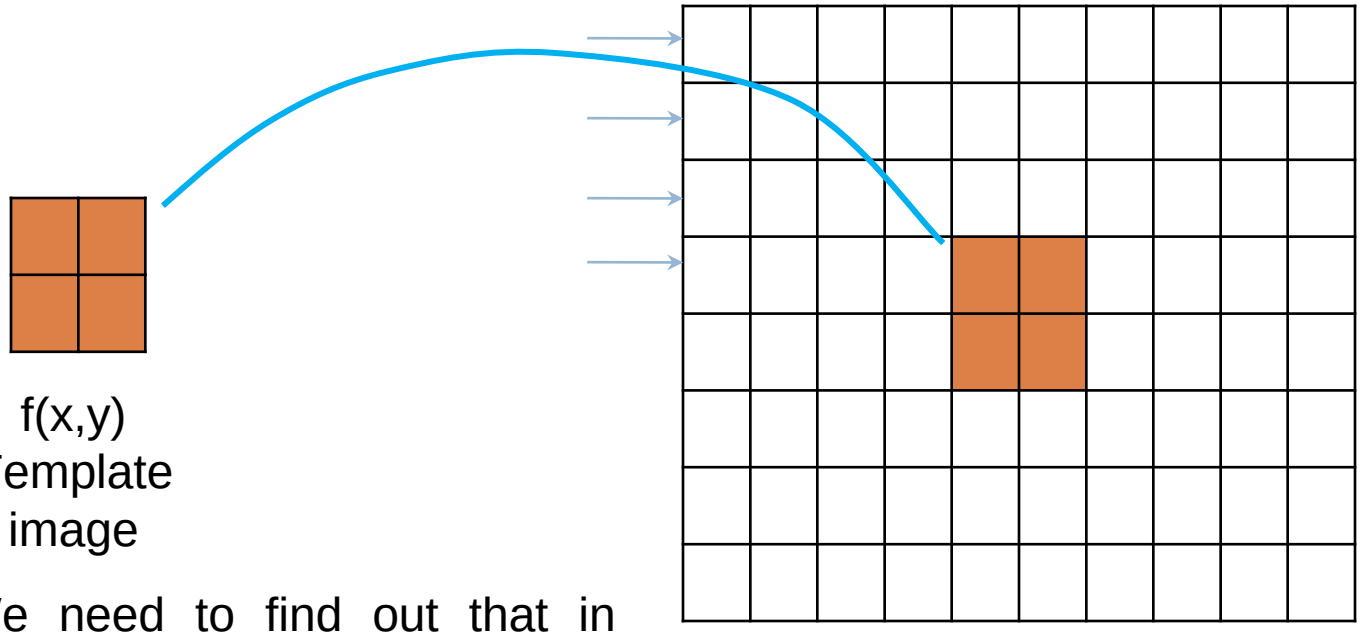
Template Matching

- Template/pattern is a small (clipped image) which need to match in original image



f(x,y)
Template
image

g(x,y)
Bigger image

Aim : We need to find out that in which position the template matches in the original image

Match/Similarity Measure

- Let g → given image
- f → template image
- A similarity measure find over region A
- So the similarity measure is given by

1. max|f-g|
2. $\int \int |f-g|$
3. $\int \int |f-g|^2$

- Here difference means pixel by pixel difference

Match or Mismatch Measure

- In discrete form we can write
- $\int\int_A |f - g| = \sum\sum [f(i,j) - g(i,j)]$ for all i,j ϵ A

  Sum of absolute difference
- $\int\int_A |f - g| = \sum\sum [f(i,j) - g(i,j)]^2$ for all i,j ϵ A

  Sum of difference square
- $\int\int_A |f - g|^2 = \int\int_A f^2 + \int\int_A g^2 - 2\int\int_A f.g$

  fixed as it is the template image

  fixed as it is the bigger image
- Where $\int\int_A |f - g|^2$ is mismatch. Hence only its minimum value can give best matching
- **$\iint_A$ f.g → match measure and only for maximum value match the value of mismatch $\iint_A |f - g|^2$ will be minimum.**

Cauchy – Schwartz – Inequality

$$\int\int_A \mathbf{f.g \le \sqrt{(\int\int f^2 . \int\int g^2)}}$$

$$\int\int_A f.g = \sqrt{(\int\int f^2 . \int\int g^2)} \text{ when } g = cf$$

where c is some constant

- The Cauchy – Schwartz – Inequality says that whenever f (Template image) is similar to a region of the given image g within with a multiplicative factor of constant c then the $\iint f.g$ will take on maximum value otherwise it will less.
- In digital case

$$\sum\sum_{i,j \in A} f(i,j).\ g(i,j) \le \sqrt{\sum\sum_{i,j \in A} f^2(i,j).\ g^2(i,j)}$$

$$g(i,j) = cf(i,j)$$

Cauchy – Schwartz – Inequality

- Let u, v is the pixel by pixel shift of template in the image g

$$\iint_A f(x,y).g(x+u,y+v)dxdy \leq$$

$$\iint_A f^2(x,y)\,dxdy + \iint_A g^2(x+u,y+v)dxdy$$

- LHS $\iint_A f(x,y).g(x+u,y+v)dxdy$ is the cross-correlation between f & g
- As RHS is not fixed so we can not use cross-correlation directly in the similarity match
- We need to normalized cross-correlation

Normalized Cross-Correlation

- Taking integration over entire image

$$\iint_{-\infty \text{ to } \infty} f(x,y).g(x+u, y+v)dxdy = cfg$$

- Normalized correlation is given

$$cfg/[\iint_A g^2(x+u, y+v)\, dxdy]^{1/2}$$

- Max value of cross-correlation cfg can have is

$$[\iint_A f^2(x, y)\, dxdy]^{1/2}$$

- for a particular value of u and v for which g = cf

Example

1	**1**	**1**	**1**	**1**	**1**
1	**20**	**2**	**2**	**2**	**1**
1	**2**	**3**	**3**	**2**	**1**
1	**2**	**3**	**3**	**2**	**1**
1	**2**	**2**	**2**	**2**	**1**
1	**1**	**1**	**1**	**1**	**1**

Given image g(x,y)

3	**3**	**2**
3	**3**	**2**
2	**2**	**2**

Template image f(x,y)

Q: Find the matching of template in the given image g(x,y)

Example

3	3	2				
3	3 1	2 1	1	1	1	1
2	2 1	2 20	2	2	2	1
	1	2	3	3	2	1
	1	2	3	3	2	1
	1	2	2	2	2	1
	1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image
f(x,y)

Cross-correlation (cfg)

cfg = 3x0 + 3x0 + 2x0 + 3x0
+ 3x1 + 2x1 + 2x0 + 2x1
+ 2x20

= 47

Example

3	3	2			
3 1	3 1	2 1	1	1	1
2 1	2 2 0	2 2	2	2	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

	3	3	2		
1	31	31	21	1	1
1	20	22	22	2	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

		3	3	2	
1	1	31	31	21	1
1	20	22	22	22	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

			3	3	2
1	1	1	31	31	21
1	20	2	22	22	21
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

				3	3	2
1	1	1	1	31	31	2
1	20	2	2	22	21	2
1	2	3	3	2	1	
1	2	3	3	2	1	
1	2	2	2	2	1	
1	1	1	1	1	1	

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

3	3 1	2 1	1	1	1	1
3	3 1	2 2 0	2	2	2	1
2	2 1	2 2	3	3	2	1
	1	2	3	3	2	1
	1	2	2	2	2	1
	1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

3 1	3 1	2 1	1	1	1
3 1	3 2 0	2 2	2	2	1
2 1	2 2	2 3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	31	31	21	1	1
1	30	32	22	2	1
1	22	23	23	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	1	31	31	21	1
1	20	32	32	22	1
1	2	23	23	22	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	1	1	31	31	21
1	20	2	32	32	21
1	2	3	23	22	21
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	1	1	1	31	31	2
1	20	2	2	32	31	2
1	2	3	3	22	21	2
1	2	3	3	2	1	
1	2	2	2	2	1	
1	1	1	1	1	1	

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

	1	1	1	1	1	1
3	3 1	2 2 0	2	2	2	1
3	3 1	2 2	3	3	2	1
2	2 1	2 2	3	3	2	1
	1	2	2	2	2	1
	1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	1	1	1	1	1
31	320	22	2	2	1
31	32	23	3	2	1
21	22	23	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	1	1	1	1	1
1	3 2 0	3 2	2 2	2	1
1	3 2	3 3	2 3	2	1
1	2 2	2 3	2 3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Given image g(x,y)

3	3	2
3	3	2
2	2	2

Template image f(x,y)

Cross-correlation (cfg) computed

Example

1	1	1	1	1	1
1	20	2	2	2	1
1	2	3 3	3 3	2 2	1
1	2	3 3	3 3	2 2	1
1	2	2 2	2 2	2 2	1
1	1	1	1	1	1

3	3	2
3	3	2
2	2	2

Cross-correlation (cfg)

cfg = 3x3 + 3x3 + 2x2 +
3x3 + 3x3 + 2x1 +
2x2 + 2x2 + 2x2

= 56

This way we can continue to find all cfg value for entire shift u,v

Example

1	1	1	1	1	1
1	20	2	2	2	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

3	3	2
3	3	2
2	2	2

Cross-correlation (cfg)

47	54	56	20	18	12
54	87	94	40	34	21
56	90	107	54	44	24
20	40	54	56	44	24
18	34	43	44	37	22
13	21	24	24	22	15

Maximum value for matching at marked position but in real it is **not matched** so we need to go for normalzed cross correlation

Example

Normalized cross-correlation

$$\mathbf{cfg/[\sum\sum_{i,j \in A} g^2(x+u,y+v)]^{1/2}}$$

So compute

$$\mathbf{[\sum\sum_{i,j \in A} g^2(x+u,y+v)]^{1/2}}$$

Example

3	3	2				
3	3 1	2 1	1	1	1	1
2	2 1	2 20	2	2	2	1
	1	2	3	3	2	1
	1	2	3	3	2	1
	1	2	2	2	2	1
	1	1	1	1	1	1

Given image g(x,y)

Computing
$[\sum\sum_{i,j \in A} g^2(x+u,y+v)]^{1/2}$

3	3	2
3	3	2
2	2	2

Template image f(x,y)

$[\sum\sum_{i,j \in A} g^2(x+u,y+v)]^{1/2} =$

$[20^2 + 1^2 + 1^2 + 1^2]^{1/2}$

$= 20.07$

Example

1	1	1	1	1	1
1	20	2	2	2	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

3	3	2
3	3	2
2	2	2

Computing
$[\sum\sum_{i,j \in A} g^2(x+u,y+v)]^{1/2}$
for entire shift u,v for the given image

20.07	20.19	20.30	3.82	3.40	2.46
20.19	20.54	20.80	6.08	5.09	3.46
20.17	20.75	21.26	7.48	6.08	3.87
3.87	5.91	7.48	7.48	6.08	3.87
3.46	5.09	6.08	6.08	5.09	3.46

Example

1	1	1	1	1	1
1	20	2	2	2	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

3	3	2
3	3	2
2	2	2

Template matched

2.34	2.67	2.75	5.16	5.2	4.54
2.67	4.2	4.51	6.5	6.87	6.06
2.76	4.37	5.03	7.2	7.23	6.2
5.16	6.76	7.21	7.48	7.23	6.2
8.2	6.67	7.07	7.23	7.26	6.35
4.54	6.06	6.20	6.20	6.35	5.68

Normalized cross-correlation

$$cfg/[\sum\sum_{i,j \in A} g^2(x+u,y+v)]^{1/2}$$

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