

Suppose there exists a polynomial  $P(x)$  of degree  $n - 1$ . This polynomial can be expressed in two ways:

① **Coefficient form:**  $a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$ .

② **Point-value form:**  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ .

We can convert from point-value form to coefficient form (and thus evaluate  $P(x)$  at arbitrary points), provided we have  $n$  distinct points, using the following formula:

$$P(x) = \sum_{j=1}^n P_j(x), \quad P_j(x) = y_j \prod_{\substack{k=1 \\ k \neq j}}^n \frac{x - x_k}{x_j - x_k}.$$