

Master's Theorem

Recurrence Relation

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$$

constraints

Recursive Term

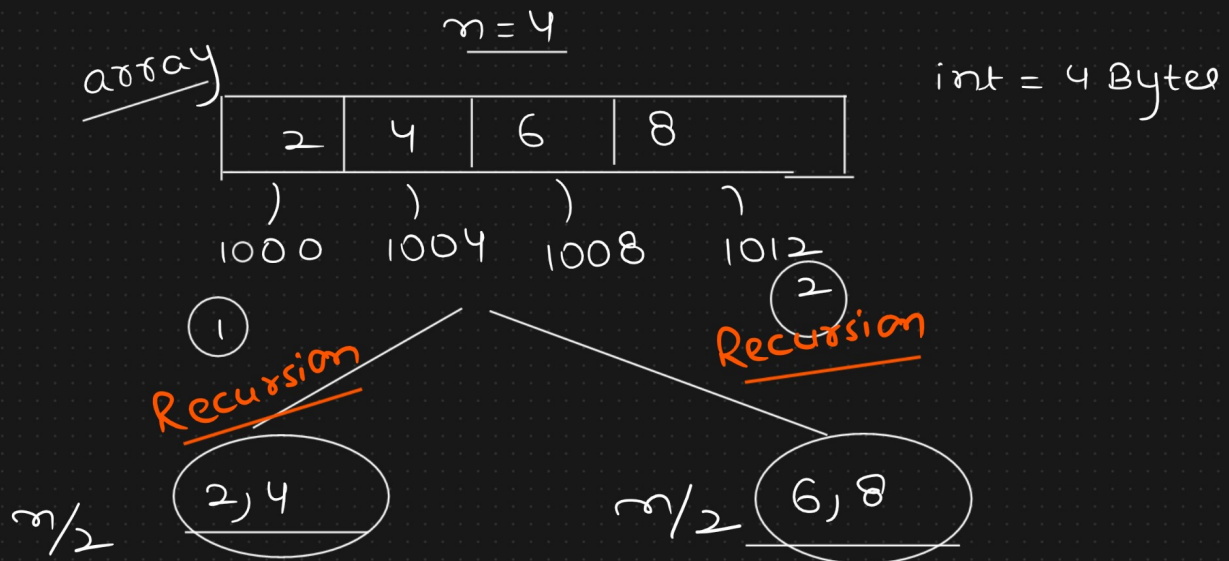
$$a \geq 1$$

$a$  — num of subproblems

$b > 1$   $n/b$  — size of subproblem

$$k \geq 0$$

$p$  — Real number



Example 1

$$T(n) = 3T(n/5) + 5n^2$$

$$\begin{cases} a = 3 \\ b = 5 \\ k = 2 \\ \underline{\underline{p = 0}} \end{cases}$$

$$a < b^k = 5^2 = 25$$

$$T(n) = O(n^k \log^p n)$$

$$T(n) = O(n^2)$$

Case 1

$$a > b^k$$

$$\underline{\underline{T(n) = O(n^{\log_b a})}}$$

Case 2

$$a = b^k$$

(2.1)

$$p > -1$$

$$T(n) = O(n^{\log_b a} \log^{p+1} n)$$

(2.2)

$$p = -1$$

$$T(n) = O(n^{\log_b a} \log \log n)$$

(2.3)

$$p < -1$$

$$T(n) = O(n^{\log_b a})$$

Case 3

$$a < b^k$$

(3.1)

$$\underline{p \geq 0}$$

$$T(n) = O(n^k \log^p n)$$

(3.2)

$$\underline{p < 0}$$

$$T(n) = O(n^k)$$

### Example 2

$$\tau(n) = 2\tau\left(\frac{n}{3}\right) + 1 - \Theta(n^k \log^p n)$$

$$\left\{ \begin{array}{l} a=2 \\ b=3/2 \\ k=0 \\ p=0 \end{array} \right.$$

Case 1

$$\underline{a > b^k} \Rightarrow 2 > \left(\frac{3}{2}\right)^0$$

$$\rightarrow T(n) = O(n^{\log_a b})$$

$$= O(n^{\log_{3/2}^2})$$

$$= O(n^{\log_{1.5}^2})$$

$$T(n) = 2T(n/2) + \underline{n^2}$$

$f(n)$

$n^{\log_b a}$

$$a = 2$$

$$b = 2$$

$$\log_b a = 1$$

$$n^{\log_b a} = n$$

$n^2$

$O(n^2)$

standard  
Method

$$a = 2$$

$$b = 2$$

$$k = 2$$

$$p = 0$$

$$a < b^k \quad \text{Case 3}$$

$$2 < 2^2$$

$$3.1 \rightarrow p \geq 0$$

$$O(n^k \log^p n)$$

$O(n^2)$