PW SKILLS

$0! = 1$ Base case
$1! = 1$

Recursion → $5! = 5 \times 4 \times 3 \times 2 \times 1$

$n \geq 0$

Method Name

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fact(n):
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Recursive code

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if n <= 1:
    return 1
else:
    return n * fact(n-1)
```

Method name

fact(5) → 120

$5 \times$ fact(4) → 24

$4 \times$ fact(3) → 6

$3 \times$ fact(2) → 2

$2 \times$ fact(1) → 1

Recurrence Relation — Master's Theorem

- Substitution Method
- Recursive Tree Method

⇓

{ Substitute the recursive term

Substitution Method

$$T(n) = 2T\left(\frac{n}{2}\right) + 4n \quad \text{——— (1)}$$

Recursive Term

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + 4\left(\frac{n}{2}\right)$$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + 4\left(\frac{n}{2}\right)\right) + 4n$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 4n + 4n$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2*(4n) \quad \text{——— (2)}$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + 4\left(\frac{n}{2^2}\right)$$

$$T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + 4\left(\frac{n}{2^2}\right)\right) + 2*4n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 4n + 2*(4n)$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right)^{(1)} + 3*(4n) \quad \text{——— (3)}$$

Base case condition

$T(1) = 1$

k times

$$\frac{n}{2^k} = 1 \Rightarrow n = 2^k$$

$$\log_2 n = k \log_2 2$$

$$T(n) = 2^{k} T\left(\frac{n}{2^{k}}\right) + k * (4n)$$

$$= 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n * (4n)$$

$$= n^{\log_2 2} T\left(\frac{n}{n^{\log_2 2}}\right) + (4n) * \log_2 n$$

$$a^{\log_2 b} = b^{\log_2 a}$$

$$= n\, T\left(\frac{n}{n}\right) + (4n) * \log_2 n$$

$$\log_a a = 1$$

$$= n\, T(1) + (4n) * \log_2 n$$

$$= n + (4n) * \log_2 n$$

$$= O(n \log_2 n)$$

$n \ll n \log_2 n$

Recurrence Relation Series

1, 7, 31, 127, 511

1 2 3 4 5

Series

$T(1) = 1$

$T(2) = 4 * T(n-1) + 3$

$= 4 * T(1) + 3$

$= 4 * 1 + 3$

$= 7$

$T(3) = 4 * T(n-1) + 3$

$= 4 * T(2) + 3$

$= 4 * 7 + 3$

$= 28 + 3$

$= 31$

$T(4) = 4 * T(3) + 3$

$= 4 * 31 + 3$

$= 124 + 3$

$= 127$

$$T(n) = 4 * T(n-1) + 3$$

Recursive Term

$T(5) = 4 * T(4) + 3$

$= 4 * 127 + 3$

$= 508 + 3$

$= 511$