PW SKILLS

Master's Theorem

Reccurrence Relation

$$\boxed{T(n) = a\,T\left(\frac{n}{b}\right) + \theta\left(n^k \log^p n\right)}$$

Recursive Term

Constraints

- $a >= 1$
  - a — num of subproblems
- $b > 1$, n/b — size of subproblem
- $k >= 0$
- p — Real Number

$n = 4$

array

2	4	6	8
1000	1004	1008	1012

int = 4 Bytes

(1) Recursion — $n/2$: 2, 4

(2) Recursion — $n/2$: 6, 8

Example 1

$$T(n) = 3T(n/5) + 5n^2$$

$a = 3$

$b = 5$

$k = 2$

$p = 0$

$a < b^k = 5^2 = 25$

$T(n) = O(n^k \log^p n)$

$$\boxed{T(n) = O(n^2)}$$

Case 1 $\quad a > b^k$

$$T(n) = O(n^{\log_b a})$$

Case 2 $\quad a = b^k$

2.1 — $p > -1$ — $T(n) = O(n^{\log_b a} \log^{p+1} n)$

2.2 — $p = -1$ — $T(n) = O(n^{\log_b a} \log \log n)$

2.3 — $p < -1$ — $T(n) = O(n^{\log_b a})$

Case 3 $\quad a < b^k$

3.1 — $p >= 0$ — $T(n) = O(n^k \log^p n)$

3.2 — $p < 0$ — $T(n) = O(n^k)$

Example 2

$\theta(n^k \log^p n)$

$$T(n) = 2T\left(\frac{2n}{3}\right) + 1$$

$$\begin{cases} a = 2 \\ b = 3/2 \\ k = 0 \\ p = 0 \end{cases}$$

case 1

$$a > b^k \implies 2 > (3/2)^0$$

$$T(n) = O(n^{\log_b a})$$

$$= O(n^{\log_{3/2} 2})$$

$$= O(n^{\log_{1.5} 2})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \underbrace{n^2}$$

$n^{\log_b a}$

$a = 2$

$b = 2$

$\log_b a = 1$

$n^{\log_b a} = n$

$f(n)$

n^2

$O(n^2)$

standard Method

$a = 2$

$b = 2$

$k = 2$

$p = 0$

$a < b^k$ case 3

$2 < 2^2$

$3.1 \longrightarrow p >= 0$

$O(n^k \log^p n)$

$O(n^2)$