PW SKILLS

Recurrence Relation

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

↳ Recursive Term

Recurrence Tree Method

↳ more than one Recursive Term in Recurrence Relation

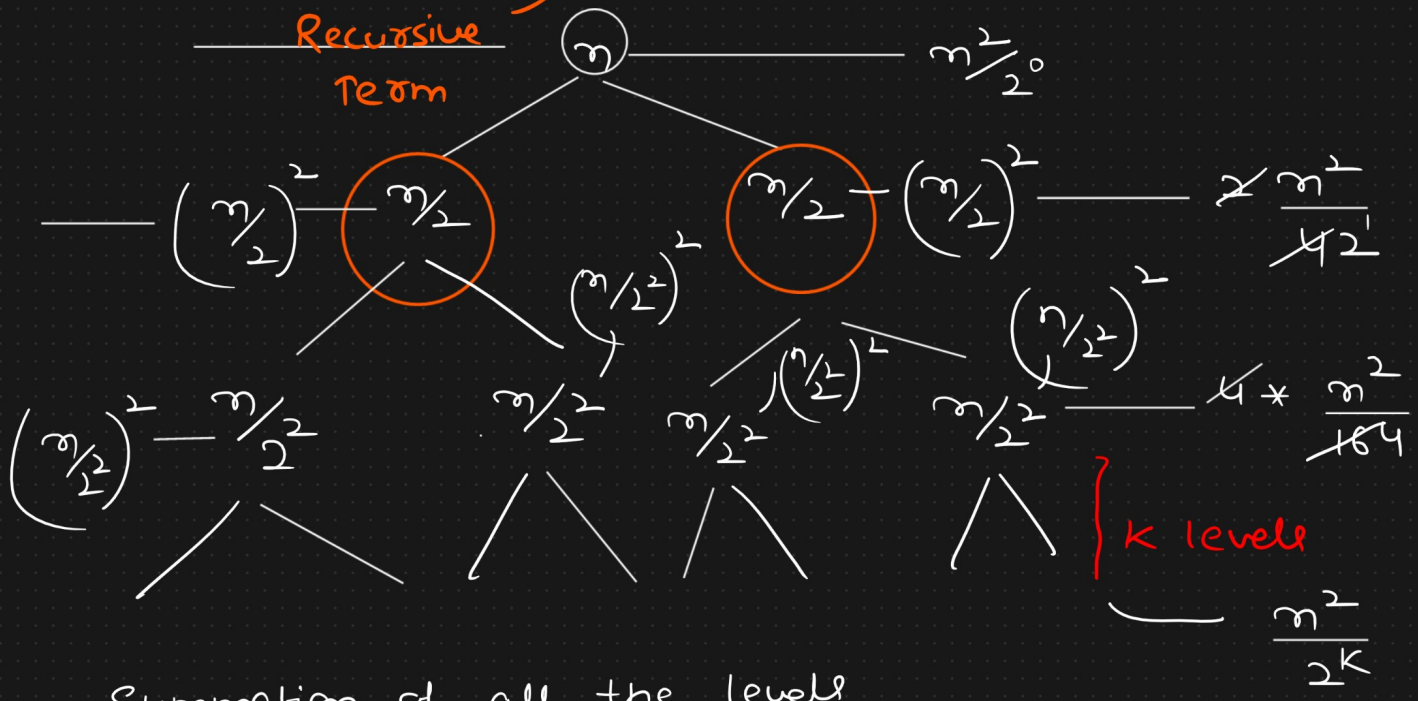
Example 1

(1) (2)

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n^2$$

↳ Cost

Recursive Term



Summation of all the levels

$$\frac{n^2}{2^0} + \frac{n^2}{2^1} + \frac{n^2}{2^2} + \cdots + \frac{n^2}{2^k}$$

$$n^2\left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^k}\right)$$

Sum of GP Series

$$r = 1/2 \qquad \frac{a}{1-r} = \frac{1}{1-1/2} = 2$$

Levels in Recursive Tree

$$\frac{n}{2^k} = 1$$

$$k = \log_2 n$$

$$\Rightarrow n^2 * 2$$

$$\Rightarrow O(n^2)$$

Example 2

Left part — Division by 3
Right Part — Division by 4

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{n}{4}\right) + kn$$

(1) (2)

Recurrence Tree

0th Level: n — $k \cdot n$

1st level: $\frac{n}{3}$ — $k\left(\frac{n}{3}\right)$; $\frac{n}{4}$ — $k\left(\frac{n}{4}\right)$ — $k\left(\frac{n}{3}+\frac{n}{4}\right)$

2nd Level: $\frac{n}{3^2}$ — $k\left(\frac{n}{3^2}\right)$; $\frac{n}{3.4}$ — $k\left(\frac{n}{12}\right)$; $\frac{n}{4.3}$ — $k\left(\frac{n}{12}\right)$; $\frac{n}{4^2}$ — $k\left(\frac{n}{4^2}\right)$; $k\left(\frac{4n+3n}{12}\right)$; $k\left(\frac{7n}{12}\right)$

$$kn\left(\frac{7}{12}\right)$$

$$nk\left(\frac{1}{9} + \frac{1}{12} + \frac{1}{12} + \frac{1}{16}\right) = \left(\frac{7}{12}\right)^2 kn$$

$$T(n) = kn\left(\underbrace{1 + \left(\frac{7}{12}\right)^1 + \left(\frac{7}{12}\right)^2 + \cdots + \left(\frac{7}{12}\right)^i}_{i \text{ Level}}\right)$$

$r = 7/12$

Left side

$\frac{n}{3^i} = 1$

$i_L = \log_3 n$

↳ Higher level

Right side

$\frac{n}{4^i} = 1$

$i_R = \log_4 n$

Lower the base in Logarithm → Higher the value

constant $kn\left(\frac{1}{1 - 7/12}\right)$ constant

Sum of GP series $= \frac{a}{1-r}$

$O(n)$