Suppose M is a smooth manifold, denote by $\Omega(M)$ the algebra of differential forms on M and by $\mathfrak{X}(M)$ the $C^\infty(M)$-module of vector fields.

	$f \in C^\infty(M)$	$\mathrm{d}x^\mu \in \Omega^1(M)$	$\omega^{(k)} \wedge \beta$	$T_1 \otimes T_2$	$\partial_\mu \in \mathfrak{X}(M)$
d	$\left(\dfrac{\partial f}{\partial x^\nu}\right) \mathrm{d}x^\nu$	0	$(\mathrm{d}\omega) \wedge \beta + (-)^k \omega \wedge (\mathrm{d}\beta)$	-	-
$\mathcal{L}_X$	$X(f) = X^\nu \left(\dfrac{\partial f}{\partial x^\nu}\right)$	$\mathcal{L}_X \mathrm{d}x^\mu = \mathrm{d}(X^\mu) = \left(\dfrac{\partial X^\mu}{\partial x^\nu}\right) \mathrm{d}x^\nu$	$(\mathcal{L}_X\omega) \wedge \beta + \omega \wedge (\mathcal{L}_X\beta)$	$(\mathcal{L}_X T_1) \otimes T_2 + T_1 \otimes (\mathcal{L}_X T_2)$	$\mathcal{L}_X \partial_\mu = [X, \partial_\mu]$
ι_X	0	$\iota_X \mathrm{d}x^\mu = \mathrm{d}x^\mu(X) = X^\mu$	$(\iota_X\omega) \wedge \beta + (-)^k \omega \wedge (\iota_X\beta)$	$(\iota_X T_1) \otimes T_2 + T_1 \otimes (\iota_X T_2)$	0
g^*	$g^*(f) = g \circ f$	$g^*(\mathrm{d}x^\mu) = \mathrm{d}(x^\mu \circ f)$	$g^*(\omega) \wedge g^*(\beta)$	$g^*(T_1) \otimes g^*(T_2)$	$(g^{-1})_* \partial_\mu$ [a]

The *Cartan calculus* consists of the following three *graded derivations* on $\Omega(M)$

- the *exterior derivative* d;
- the space of *Lie derivative operators* $\mathcal{L}_X$, where $X \in \mathfrak{X}(M)$;
- the space of *contraction operators* ι_X, where $X \in \mathfrak{X}(M)$.

$$\begin{tikzcd}
\cdots \arrow[r, bend left, "\mathrm{d}"] & \Omega^k(M) \arrow[l, bend left, "\iota_X"] \arrow[loop above, "\mathcal{L}_X"] \arrow[r, bend left, "\mathrm{d}"] & \Omega^{k+1}(M) \arrow[l, bend left, "\iota_X"] \arrow[loop above, "\mathcal{L}_X"] \arrow[r, bend left, "\mathrm{d}"] & \cdots \arrow[l, bend left, "\iota_X"]
\end{tikzcd}$$

Together with the following identities:

$$\mathrm{d}^2 = 0, \tag{1}$$
$$\mathrm{d}\mathcal{L}_X - \mathcal{L}_X d = 0, \tag{2}$$
$$\mathrm{d}\iota_X + \iota_X d = \mathcal{L}_X, \tag{3}$$
$$\mathcal{L}_X\mathcal{L}_Y - \mathcal{L}_Y\mathcal{L}_X = \mathcal{L}_{[X,Y]}, \tag{4}$$
$$\mathcal{L}_X\iota_Y - \iota_Y\mathcal{L}_X = \iota_{[X,Y]}, \tag{5}$$
$$\iota_X\iota_Y + \iota_Y\iota_X = 0, \tag{6}$$

A *graded derivation* of $\Omega(M)$ is a degree k linear operator A on $\Omega(M)$ such that

$$A(\omega \wedge \eta) = A(\omega) \wedge \eta + (-1)^{kp} \omega \wedge A(\eta) \qquad \forall \omega \in \Omega^k(M),\ \eta \in \Omega^{\cdot}(M) \tag{7}$$

$$\Omega(M) = \left(\bigoplus_{k=0}^{m} \Omega^k(M), \wedge \right) \qquad \textit{Grassmann Algebra} \text{ on M}$$

is a graded commutative graded algebra over ring $C^\infty(M)$.

$$\mathcal{T}(M) = \left(\bigoplus_{l,k=0}^{\infty} T^k_l(M), \otimes \right) \qquad \textit{Tensor Algebra} \text{ on M}$$

is a commutative graded algebra over ring $C^\infty(M)$.

[a] g has to be a diffeo

How to compute brackets:

$$[\partial_i, \partial_j] = 0 \tag{8}$$
$$[f\partial_i, g\partial_j] = f \cdot (\partial_i g) \cdot \partial_j - g \cdot (\partial_j f) \cdot \partial_i \tag{9}$$
$$[fX, gY] = f \cdot g \cdot [X, Y] + f \cdot X(g) \cdot Y - g \cdot Y(f) \cdot X \tag{10}$$
$$[X, Y] = \left(X^i \partial_i Y^j - Y^i \partial_i X^j \right) \partial_j \tag{11}$$

$(\mathfrak{X}(M), [-, -])$ form a Lie algebra over $\mathbb{R}$:

$$[X, Y] = -[Y, X]$$
$$[aX + bY, Z] = a[X, Z] + b[Y, Z]$$
$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0$$

An (I)-graded vector space $'V$ is a vector space that can be written as a direct sum of subspaces indexed by elements $i \in I$:

$$V = \bigoplus_{i \in I} V_i$$

TODO: glossary:

- Graded vector space
- Graded algebra
- linear operator of degree k
- substitute $\partial_i f$ with $\dfrac{\partial f}{\partial x^i}$ in order to avoid confusion between natural basis vector and usual partial derivative operator acting on function f