Phase Space We recalled in chapter 1 the definition of *Phase Space* in ordinary classical mechanics as the cotangent bundle T^*Q of the classical configuration space Q. We showed that every classical phase space is symplectic trough the natural PoincarÃÍ form. However, every quantization procedure require a modification of this standard symplectic form in order to implement the canonical commutation rules.

This leads us to the abstract formulation for the Hamiltonian systems[**?**]:

Definition 1: Hamiltonian System

$(\mathscr{M}, \omega, H)$ 3.3.1 fomm

Observation 1

In classical mechanics Hamiltonian systems could be seen as a subset of Lagrangian systems.
The key is the definition of the Legendre Map $TL : TQ \to T^*Q$. Only in the case that the Lagrangian$L$ is *hyperegular*, i.e. TL is a diffeomorphism, is possible to push-forward L to give a proper Hamiltonian on $\mathscr{M} = T^*Q$. (see for example [**?**]

Remember that there's an important theorem attributed to Darboux that state that, at least locally, every symplectic form can be coordinated as the ...canonical coordinate

Theorem 0.0.1 (Darboux) *3.2.2 fomm*

Classical Observables Observables in classical mechanics are represented by real valued smooth function on $\mathscr{M}$

Notation fixing

The *Classical Observables* space is denoted as:

$$\mathscr{E} \equiv C^\infty(\mathscr{M}, \mathbb{R})$$

Observation 2

Trivially, the space $C^\infty(\mathscr{M}, \mathbb{R})$ of smooth real valued function on $\mathscr{M}$, inherits the structure of commutive algebra over $\mathbb{R}$ from its codomain $\mathbb{R}$.

The symplectic structure on $\mathscr{M}$ give rise to an alternative algebraic structure on the vector space of observables. At first it is necessary to introduce the Hamiltonian fields:

Definition 2: Hamiltonian field

3.3.1 fomm

from that follows the definition of the bracket:

Definition 3: Poisson Bracket

3.3.11 fomm

Proposition 0.0.1 (Symplectic char representation) *3.3.14 fomm*