Chapter 1

Geodesic Fields

Usually, in the context of differential geometry, a *geodesic curve* It's characterized as self-parallel curve in order to generalize the *straight lines*. Considering a differential manifold M endowed with an affine connection ∇ we define:

Definition 1: Geodesic

A curve ⚠[a] $\gamma : [a,b] \to M$ such that:

$$\nabla_{\dot{\gamma}}\dot{\gamma} = 0 \tag{1.1}$$

where $\dot{\gamma}^\mu := \frac{d\gamma^\mu}{dt}$ is the tangent vector to the curve.

[a]Devo dire smooth o piecewise?

Notation fixing

In local chart the previous equation assume the popular expression:

$$\ddot{\gamma}^i + \Gamma^i_{\ jk}\dot{\gamma}^j\dot{\gamma}^k = 0 \tag{1.2}$$

Where $\Gamma^i_{\ jk}$ is the coordinate representation of the Christoffel symbols of the connection.

In presence of a pseudo-Riemmanian metric is possible to present the geodesic in a metric sense i.e. as the curve which extremizes the *Energy Functional*[1]:

Definition 2: Energy functional

[1]Remember that for arc-length parametrized curves the Energy functional coincide with the length functional.[**?**, Lemma 1.4.2]

$$E(\gamma) := \int_a^b \left\| \frac{d\gamma}{dt}(t) \right\|^2 dt$$

Considering only the proper variation (that keep the end-point fixed), the extremum condition corresponds to equation where ∇ is the unique Levi-Civita connection (torsion-free and metric-compatible).

1.1 Geodesic Problem as a Mechanical Systems

1.1.1 Geodesic Motion

1.1.2 Geodesic Field

1.2 Peierls Bracket of the Geodesic field

1.2.1 Example: Geodesic field on FRW space-time.

1.3 Algebraic quantization of the Geodesic Field

1.3.1 Peierls Approach

1.3.2 Inital data Approach

1.4 Interpretations??????