Chapter 1

Lagrangian Systems and Pierels Brackets

1.1 Abstract Lagragian Systems

Definition 1: Lagrangian System

Pair $(E, \mathscr{L})$ composed of:

- $E \xrightarrow{\pi} M$ smooth fiber bundle of typical fiber Q called *"configuration bundle"*.
- $\mathscr{L} : J^r E \to \Omega^m(M)$ function from the r-th Jet Bundle to the top-dimensionial form over the base manifold M called *"Lagrangian density"* or simply *"Lagrangian"* of r-th order.

1.1.1 Kinematics

The configuration bundle encompass all the kinematical structure of the system, the pivotal role is played by the smooth sections which are to be understood as all the possible conformation of the system.

Notation fixing

$$\mathrm{C} = \Gamma^\infty(M, E)$$

Space of kinematic configurations.

A section is not a statical configuration, equivalent to a specific point in the configuration space of ordinary classical systems, but has to be seen as a specific realization of the kinematics in the sense of a complete description of a possible motion.

At this level of abstraction, since no space-time structure has been specified, terms like stasis and motion must be taken with care .The natural physical interpretation should be clearly manifested through the concrete realization of systems with discrete and continuous degree of freedom.

Observation 1: Mathematical structure

Mathematically speaking this set should be regarded as an infinite dimensional Manifold.

This framework provides a geometric characterization of the notion of variations as tangent vectors on the the space of kinematic configurations .[**?**]

Observation 2: Coordinate Representation

The choice of a chart atlas $\mathscr{A}(M)$ on the base space M and $\mathscr{A}(E)$ on the total space E provides a correspondence between each configuration $\gamma \in \mathrm{C}$ and family of smooth real functions $\{f_{\alpha\beta} : A_\alpha \subset \mathbb{R}^m \to \mathbb{R}^q\}$. The process is trivial:

$$\gamma \in \mathrm{C} \mapsto \{f_{A,U} = \psi_U \circ \gamma \circ \psi_A^{-1} | (A, \psi_A) \in \mathscr{A}(M), (U, \psi_U) \in \mathscr{A}(E)\}$$

Since the whole section as a global object is quite difficult to handle is customary in field theory to work in the more practical local representation.

Observation 3: Further specification of the system's kinematics

The general formalism doesn't require any other structure to be carried forward. Additional structure on the fiber and the whole bundle are to be prescribed in order to specify a precise physical model, e.g. the spin structure for the Dirac Field.[**?**]

Figure 1.1: Geometric picture of the basic kinematic structure.

1.1.2 Dynamics

1.1.3 Generalization

1.2 Concrete Realization

1.2.1 Fields on curved Background

1.2.2 Finite Degree systems

1.3 Geometric mechanics of Finite Degree systems

1.3.1 Linear dynamical systems

1.4 Peierls Brackets

1.4.1 Finite Dimensional case

1.5 Eliminata

- quando parlo della cinematica mi piacerebbe dare indicazioni sulla struttura matematica dello spazio delle configurazioni cinematiche:

  1. costituisce una frechet manifold (gli unici risultati che ho trovato sono quelli di Palais di "non linear global analysis"
  2. le curve parametrizzate sono le variazioni
  3. classi di equivalenza definiscono delle variazioni infinitesime che costituiscono lo spazio tangente allo spazio delle configurazioni cinematiche
  4. questo spazio tangente ÃÍ isomorfo allo spazio delle sezioni del pullback rispetto alla sezione $\phi \in C$ del verical bundle (vedere forger romero)
  5. il problema dell'atlante e della rappresentazione delle sezioni in carta locale (da scegliere sia sul total space E che sul base space M)