Conclusions and outlook

We shall now briefly summarize our results.
In Chapter 1 we have reviewed the basic mathematical structures which underlie to the rigorous treatment of any field ~~theories~~ on curved background. The smooth fiber bundles are the natural objects to encompass the kinematics of a system, ~~the imposition of the globally hyperbolic condition on the base spacetime~~ permits to depict a deterministic classical dynamics reconstructable by the choice of an appropriate set of initial data ~~while~~ ~~the~~ Green-hyperbolic operators are an abstract general class of equations which ~~strictly resemble the~~ wave-like propagation.
In Chapter 2 ~~we've~~ dealt with the problem of the ~~Peierls brackets construction~~ for theories ~~not~~ necessarily linear . In the first instance we have identified a general class of systems for which the original Peierls' algorithm applies without ~~incisive~~ changes. We have shown that this class is not a mere academic exercise but ~~include~~ all ~~the~~ classical Lagrangian systems regardless of the cardinality of the degrees of freedom. For such class we revived the Peierls' procedure in details, adapting all its step to the language currently in use in the modern algebraic theory of fields.
We have devoted Chapter 3 to ~~review~~ the algebraic quantization procedure describing two different realization. We paid particular attention to the different "pre-quantum" structures which can be attributed to the classical theory to be quantized. In the first procedure we have stressed how the 2-form on the classical symplectic space, which in most part of our references tends to be postulated, may instead be derived from the more general Peierls brackets. Regarding the second, non covariant, construction of *quantization by initial data*, we showed how it can be compared with the first procedure and how, in rather important ~~case as~~ the scalar field theory, it can be proved to be equivalent.
In the last Chapter we have realized the Peierls brackets and the two ~~procedure of~~ algebraic quantization to the case of ~~Geodesic~~ motion. We brought down the explicit calculation of the Peierls' symplectic form in the special case of Jacobi fields along the isotropic, homogeneous, free-falling observers in De Sitter spacetimes. This simple example made clear that under the rather elegant expression $\{\chi,\omega\} := (\chi, E\omega)$ lie nontrivial calculations, in particular regarding the explicit expression of the Green operators for the ordinary differential equations of Jacobi. At last we proved the equivalence between the algebraic quantization by Peierls bracket and by initial data.

To close, let us discuss some possible extensions of our work. In the courses of this thesis we have tried to keep our mathematical formalization not excessively sophisticated. Currently are under examinations further extensions of the Peierls construction to non Lagrangian fields or to systems with Gauge freedom (see for example [**?**]), all of these are based on the variational bicomplex formalism [**?**]. We have preferred to keep the level of our discussion to an introductory level since the current lack of bibliography on the theme are hinders the recognition of the role of the Peierls brackets among the schemes of Algebraic quantization.

It is not new in the literature the idea of realizing the equivalent of geometric mechanics for systems with continuous degrees of freedom. Such formalism is currently based on the concept of *"covariant phase space"* which is usually defined as the (infinite-dimensional) space of solutions of the equations of motion. This geometric theory fits neatly into the philosophy underlying the symplectic formalism in general; in particular, it admits a natural definition of the Poisson bracket through the Peierls construction. The main advantage of this approach, would be to make clear the parallelism between the geometrical mechanics of ordinary finite dimensional systems and fields. Its main drawback is the lack of mathematical rigor, since it is often restricted to the formal extrapolation of techniques from ordinary calculus on manifolds to the infinite-dimensional setting: transforming such formal results into mathematical theorems is a separate problem, often highly complex and difficult. The application of the modern results in non-linear global analysis in this topic are currently not extensively investigated.