15 Extra Problems

Friday, October 30, 2020 10:07 AM

Consider a game in which player 1 first selects between I and O. If player 1 selects O, then the game ends with the payoff vector (*x*, 1) (*x* for player 1), where *x* is some positive number. If player 1 selects I, then this selection is revealed to player 2 and then the players play the battle-of-the-sexes game in which they simultaneously and independently choose between A and B. If they coordinate on A, then the payoff vector is (3, 1). If they coordinate on B, then the payoff vector is (1, 3). If they fail to coordinate, then the payoff vector is (0, 0).

(a) Represent this game in the extensive and normal forms.

(b) Find the pure-strategy Nash equilibria of this game.

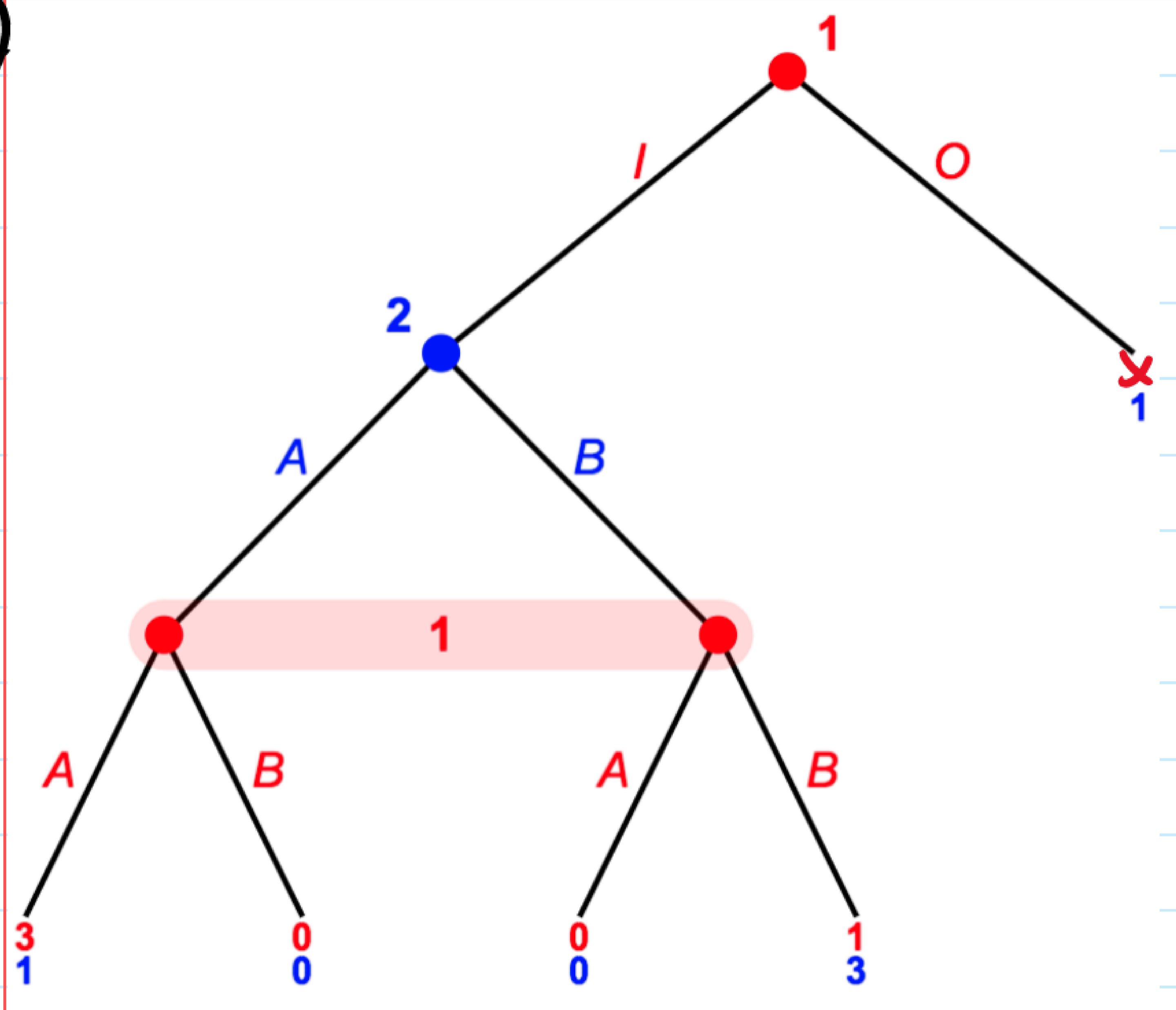
(c) Calculate the mixed-strategy Nash equilibria and note how they depend on *x*.

(d) Represent the proper subgame in the normal form and find its equilibria.

(e) What are the pure-strategy subgame perfect equilibria of the game? Can you find any Nash equilibria that are not subgame perfect?

(f) What are the mixed-strategy subgame perfect equilibria of the game?

a)



1 \ 2	A	B
IA	3, 1	0, 0
IB	0, 0	1, 3
OA	x, 1	x, 1
OB	x, 1	x, 1

b) if $x > 3$: (OA, A) (OA, B) (OB, A) (OB, B)
$x = 3$: $x > 3$ and (IA, A)
$1 < x < 3$: (IA, A) (OA, B) (OB, B)
$x = 1$: $1 < x < 3$ and (IB, B)
$x < 1$: (IA, A) and (IB, B)

d)

1\2	A	B	
A	3, 1	0, 0	p
B	0, 0	1, 3	$1-p$

Pure NE

$\rightarrow 1p + 0(1-p) = 0p + 3(1-p) \rightarrow p = .75$, $q = .25$

Mixed NE

$u_1(.75, .25) \rightarrow 3\frac{1}{4}\cdot\frac{3}{4} + 0 + 0 + 1\frac{1}{4}\cdot\frac{3}{4} = \frac{9}{16} + \frac{3}{16} = \frac{12}{16} = \frac{3}{4}$

$u_2(.25, .75)$

B, B is subgame equilibria

if $x < 1$, IB, B is eq
$x > 1$, OB, B is eq
$x < 3$, IA, A
$x > 3$, OA, A
$x < .75$, I $\sigma_1 = .75$, $\sigma_2 = .25$
$x > .75$, O $\sigma_1 = .75$, $\sigma_2 = .25$