HW 10.15

Sunday, October 18, 2020 4:58 PM

15. An island has two reefs that are suitable for fishing, and there are twenty fishers who simultaneously and independently choose at which of the two reefs (1 or 2) to fish. Each fisher can fish at only one reef. The total number of fish harvested at a single reef depends on the number of fishers who choose to fish there. The total catch is equally divided between the fishers at the reef. At reef 1, the total harvest is given by $f_1(r_1) = 8r_1 - \frac{r_1^2}{2}$, where r_1 is the number of fishers who select reef 1. For reef 2, the total catch is $f_2(r_2) = 4r_2$, where r_2 is the number of fishers who choose reef 2. Assume that each fisher wants to maximize the number of fish that he or she catches.

(a) Find the Nash equilibrium of this game. In equilibrium, what is the total number of fish caught?

$dw/dR_1 = 8 - R_1/2$ $\quad dw/dR_2 = 4$

$4 = 8 - R_1/2$
$8 - 4 = R_1/2$
$4 = R_1/2$
$8 = R_1$

$R_2 = 20 - 8$
$R_2 = 12$

$f_1(8) = 64 - 64/2 = 32$
$f_2(12) = 4 \cdot 12 = 48$

Total = 80

(b) The chief of the island asks his economics advisor whether this arrangement is efficient (i.e., whether the equilibrium allocation of fishers among reefs maximizes the number of fish caught). What is the answer to the chief's question? What is the efficient number of fishers at each reef?

NO. You can maximize more

$8R_1 - R_1^2/2 + 4(20 - R_1)$

$dw/dR_1 = 8 - R_1 - 4 \Rightarrow R_1 = 4 \Rightarrow R_2 = 16$

$8(4) - 16/2 = 32 - 8 = 24$
$4 \cdot 16 = 64$

$24 + 64 = 88$ Fish

(c) The chief decides to require a fishing license for reef 1, which would require each fisher who fishes there to pay the chief x fish. Find the Nash equilibrium of the resulting location-choice game between the fishers. Is there a value of x such that the equilibrium choices of the fishers results in an efficient outcome? If so, what is this value of x?

$R_1 \Rightarrow 8 - R_1/2 - x = 4$
$4 = R_1/2 + x$
$x = 2$