10 Extra Problems

Monday, October 19, 2020 10:22 AM

10. Consider a game that has a continuum of players. In particular, the players are uniformly distributed on the interval [0, 1]. (See Appendix A for the definition of uniform distribution.) Each $x \in [0, 1]$ represents an individual player; that is, we can identify a player by her location on the interval [0, 1]. In the game, the players simultaneously and independently select either F or G. The story is that each player is choosing a type of music software to buy, where F and G are the competing brands. The players have different values of the two brands; they also have a preference for buying what other people are buying (either because they want to be in fashion or they find it easier to exchange music with others who use the same software). The following payoff function represents these preferences. If player x selects G, then her payoff is the constant g. If player x selects F, then her payoff is $2m - cx$, where c is a constant and m is the fraction of players who select F. Note that m is between 0 and 1.

(a) Consider the case in which $g = 1$ and $c = 0$. What are the rationalizable strategies for the players? Is there a symmetric Nash equilibrium, in which all of the players play the same strategy? If so, describe such an equilibrium.

(b) Next, consider the case in which $g = 1$ and $c = 2$. Calculate the rationalizable strategy profiles and show your steps. (Hint: Let $\overline{m}$ denote an upper bound on the fraction of players who rationally select F. Use this variable in your analysis.)

(c) Describe the rationalizable strategy profiles for the case in which $g = -1$ and $c = 4$. (Hint: Let $\overline{m}$ denote an upper bound on the fraction of players who rationally select F and let $\underline{m}$ denote a lower bound on the fraction of players who rationally select F.)

b) $g=1$ $c=2$

$m=0$
← fraction buying F

x, $G>F$
$x>\hat{x}$, $G>F$
$\hat{x}$ ~~m~~ G ~~m~~ F
$x>\hat{x}$ $G>F$
$x<\hat{x}$ $G<F$

0 — 1/2 — 1 x

$x \in [0,1]$
G, F
g, $2m-cx$

$v(F)=2m-2x$
$1=2m-2x$

c) $g=-1$, $c=4$

$-1=2\hat{m}-4\hat{x}$
$-1=2\hat{x}-4\hat{x}$
$\hat{x}=1/2$

$m=0$
$-1=2\cdot 0-4x$
$x=1/4$

$m=1/4$
$-1=2\cdot 1/4-4x$
$4x=3/2$
$m=3/8$

$m=1$
$-1=2-4x$
$4x=3$
$x=3/4$
$m=3/4$
$x=5/8$

a) $g=1$ $c=0$

$m \in [0,1]$

$1>2m$ $G>F$
$<$ $F>G$

NE $m=0$
NE $m=1$

$v(F)=0<1$
$v(F)=2\cdot 1=2>1$