9 Book Notes

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Discounting future impacts and handling inflation

9.1 The Basics of Discounting

9.1.1 Projects w/ lives of 1 year

9.1.2 Future value analysis

Compare project or investment benefits

Future Value (FV) = $X(1+i)$
↳ i = interest rate
↳ X = initial investment

9.1.3 Present value analysis

$PV = Y/(1+i)$
↳ Y = amount received in one year

9.1.4 Net present value analysis

NPV = PV(Benefits) - PV(costs)

9.2 Compounding and Discounting over Multiple Years

9.2.1 Future value over multiple years

Simple vs compounded interest

$FV = X(1+i)^n$ → n = years

9.2.2 Present value over multiple years

$PV = Y/(1+i)^n$

$1/(1+i)^n$ = present value factor or discount factor

$PV = FV/(1+i)^n$ and $FV = PV(1+i)^n$

if B = benefits received in time t
↳ $PV(B) = \sum_{t=0}^{n} B_t/(1+i)^t$

Can also use C instead for costs

9.2.3 Net Present Value of a Project

$NPV = PV(B) - PV(C)$

$NSB_t = B_t - C_t$

$NPV = \sum_{t=0}^{n} NSB_t/(1+i)^t$

9.2.4 Annuities and Perpetuities

Annuity is an equal, fixed amount received or paid every period

Perpetuity is an indefinite annuity

A = annuity

$PV = \sum_{t=1}^{n} A/(1+i)^t = A\sum_{t=1}^{n} 1/(1+i)^t = Aa_i^n$

a_i^n = annuity factor = PV of A of \$1 for n years at interest rate i

in perpetuity, $PV = A/i$ if $i > 0$

9.2.5 Continuous Compounding

9.3 Timing of Benefits and Costs

ordinary annuity = regular annuity → paid at year's end

deferred annuity = annuity due → paid at year's start

with continuous annuity, $NPV = -C_0 + \sum_{t=1}^{n} NSB_t/(1+i)^{t-.5}$

Appendix 9A
Formulas for Calculating the Present Value of Annuities and Perpetuities

Present value of an annuity

$PV = \sum_{t=1}^{n} A/(1+i)^t$

$PV = Aa_i^n$

$a_i^n = \frac{1-(1+i)^{-n}}{i}$

PV of A that grows or declines at a constant rate

$B_t = B_1(1+g)^{t-1}$ g = rate at which benefits grow

$PV(B) = B_1/(1+g) \cdot a_{i_0}^n$ → $i_0 = \frac{i-g}{1+g}$ for $i > g$

PV of perpetuity that grows or declines at a constant rate

$NPV = NSB_1/(i-g)$ if $i > g$

9.4 Comparing Projects w/ different time frames

rollover method
repeat short project X times so total time matches long project

Equivalent annual net benefit method (EANB)

$EANB = NPV/a_i^n$

9.5 Inflation and Real vs Nominal Dollars

Nominal dollars = current dollars

Real dollars = Constant dollars

deflator

9.5.1 Problems w/ indices based on consumer prices

9.5.2 Discounting using real or nominal dollars

$r = \frac{i-m}{1+m}$ (r = real, i = nominal, m = inflation)

9.5.6 Estimates of future inflation

9.6 Relative price changes

9.7 Terminal Values and Fixed-Length Projects

$NPV = -C_0 + \sum_{t=1}^{n} RNSB/(1+i)^{t-.5} + T_n/(1+i)^n$

C_0 = initial cost
$RNSB_t$ = regular annual net social benefits
T_n = terminal value at end of year n
i = discount rate

9.8 Terminal (or horizon) Values and Long-Lived Projects

If benefits or costs are indefinite $NPV = \sum_{t=0}^{\infty} NSB_t/(1+i)^t$

discounting horizon   horizon value = terminal value

at horizon value... $NPV = \sum_{t=0}^{k} NSB_t/(1+i)^t + H_k/(1+i)^k$

where... $H_k = \sum_{t=k+1}^{\infty} NSB_t/(1+i)^t$

9.8.1 Horizon value estimated directly

9.8.2 Horizon value based on depreciated value of the asset(s)

Economic value   economic depreciation

decline at geometric rate

accounting depreciation is not a cost in BCA

9.8.3 Reprise of horizon value

Sensitivity analysis w/ different horizon values

9.9 Time-decline Discounting

9.10 Sensitivity analysis in Discounting

Internal Rate of return (breakeven discount rate)
↳ IRR
↳ yields NPV of 0

9.10.1 The internal Rate of return as a decision Rule