HW 23.1

Wednesday, November 11, 2020 4:58 PM

Passed Solution review

1. Consider the Bertrand oligopoly model, where n firms simultaneously and independently select their prices, $p_1, p_2, \ldots, p_n$, in a market. (These prices are greater than or equal to 0.) Consumers observe these prices and only purchase from the firm (or firms) with the lowest price p, according to the demand curve $Q = 110 - p$, where $p = \min\{p_1, p_2, \ldots, p_n\}$. That is, the firm with the lowest price gets all of the sales. If the lowest price is offered by more than one firm, then these firms equally share the quantity demanded Q. Assume that firms must supply the quantities demanded of them and that production takes place at a constant cost of 10 per unit. (That is, the cost function for each firm is $c(q) = 10q$.) Determining the Nash equilibrium of this game was the subject of a previous exercise.

(a) Suppose that this game is infinitely repeated. (The firms play the game each period, for an infinite number of periods.) Define δ as the discount factor for the firms. Imagine that the firms wish to sustain a collusive arrangement in which they all select the monopoly price $p^M = 60$ each period. What strategies might support this behavior in equilibrium? (Do not solve for conditions under which equilibrium occurs. Just explain what the strategies are. Remember, this requires specifying how the firms punish each other. Use the Nash equilibrium price as punishment.)

$p = 60$ until someone defects, then $p = 10$

(b) Derive a condition on n and δ that guarantees that collusion can be sustained.

$q = (110-60)/n = 50/n$ $\quad$ $P = \frac{50}{n} \cdot 60 - \frac{50}{n} \cdot 10 = \frac{3000}{n} - \frac{500}{n} = 2500/n$

$\left(\frac{2500}{n}\right)\left(1/1-\delta\right) \geq 2500 \rightarrow \delta \geq 1 - 1/n$

(c) What does your answer to part (b) imply about the optimal size of cartels?

The smaller the cartel, the better