HW 10.4

Sunday, October 18, 2020 2:44 PM

4. Consider the game between a criminal and the government described in this chapter.

Here is a game that illustrates how the government balances the social cost of crime with law-enforcement costs and how criminals balance the value of illegal activity with the probability of arrest. The game has two players: a criminal (C) and the government (G). The government selects a level of law enforcement, which is a number $x \geq 0$. The criminal selects a level of crime, $y \geq 0$. These choices are made simultaneously and independently. The government's payoff is given by $u_G = -xc^4 - y^2/x$ with the interpretation that $-y^2/x$ is the negative effect of crime on society (moderated by law enforcement) and c^4 is the cost of law enforcement, per unit of enforcement. The number c is a positive constant. The criminal's payoff is given by $u_C = y^{1/2}/(1 + xy)$, with the interpretation that $y^{1/2}$ is the value of criminal activity when the criminal is not caught, whereas $1/(1 + xy)$ is the probability that the criminal evades capture. Exercise 4 of this chapter asks you to compute the Nash equilibrium of this game.

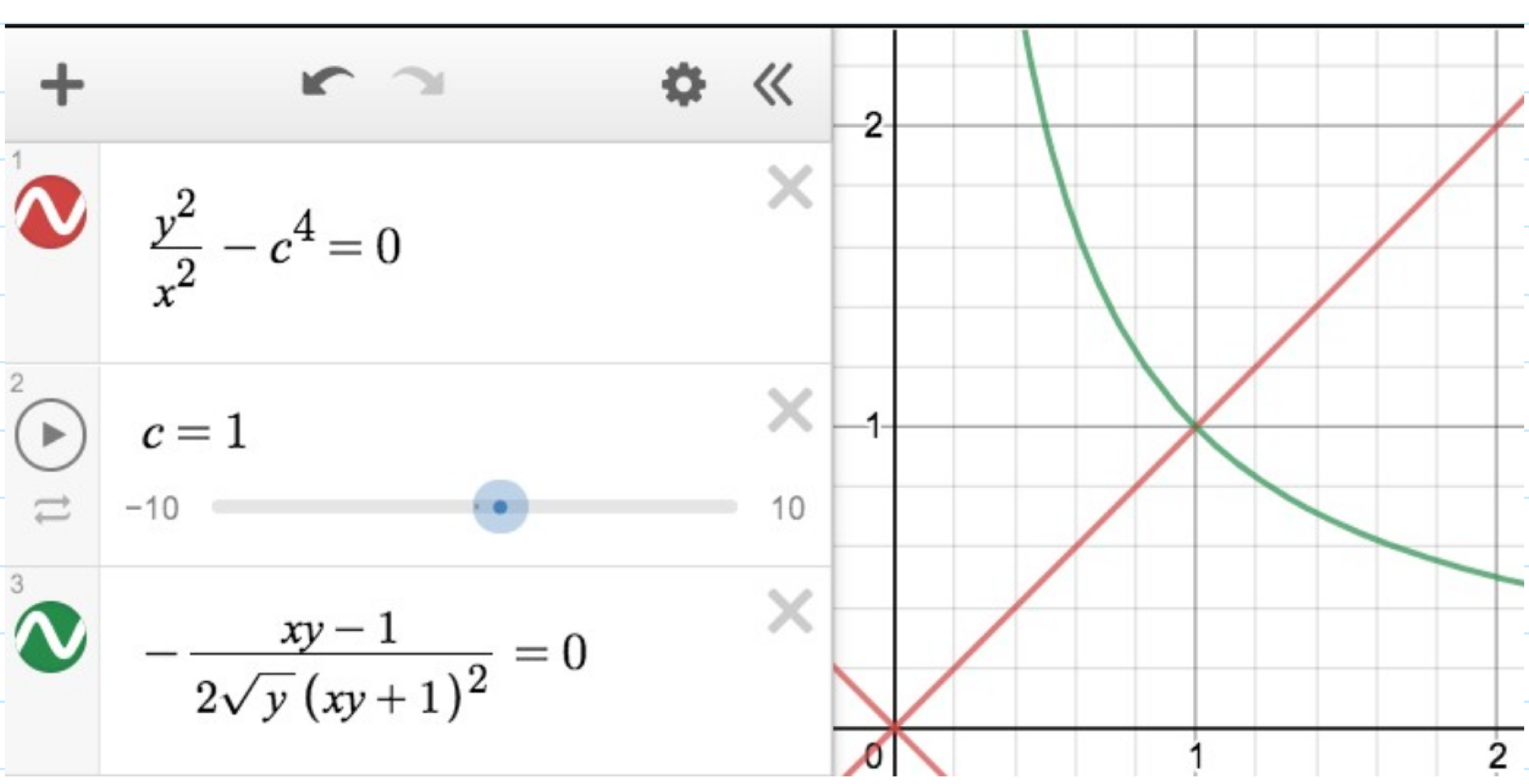
(a) Write the first-order conditions that define the players' best-response functions and solve them to find the best-response functions. Graph the best-response functions.

$G = -xc^4 - y^2/x$

$du/dG = y^2/x^2 - c^4 = 0$

$C = y^{1/2}/(1+xy)$

$du/dC = -xy-1 / [2\sqrt{y}(xy+1)^2] = 0$



(b) Compute the Nash equilibrium of this game.

$du/dG \rightarrow x = y/c^2$

$y = xc^2$

$y = 1/y\, c^2$

$x = 1/c$

$du/dC \rightarrow y = 1/x$

$x = 1/y$

$y = c$

(c) Explain how the equilibrium levels of crime and enforcement change as c increases.

As costs c increase, enforcement x decreases and criminal activity y increases