

Regression w/ binary variable

$$y = \beta_0 + \beta_1 x$$

x is binary

$$E(y|x) = \beta_0 + \beta_1 x + \epsilon$$

$$E(y|x=0) = \beta_0$$

$$E(y|x=1) = \beta_0 + \beta_1$$

$$\begin{aligned}\beta_1 &= E(y|x=1) - \beta_0 \\ &= E(y|x=1) - E(y|x=0)\end{aligned}$$

β_1 = difference in average value of y over subpopulations w/ $x=1$ and $x=0$

Relation between score in QMB and gender

$$\text{score} = \beta_0 + \beta_1(\text{gender}) \quad \text{Male} = 1, \text{Female} = 0$$

$$E(\text{score}|\text{gender} = \text{Female}) = \beta_0$$

$$E(\text{score}|\overset{\uparrow}{\text{given}} \text{gender} = \text{male}) = \beta_0 + \beta_1$$

$$\begin{aligned}\beta_1 &= E(\text{score}|\text{male}) - \beta_0 \\ &= E(\text{score}|\text{male}) - E(\text{score}|\text{female})\end{aligned}$$

β_1 = diff in male and female scores

Population model

$$y = \beta_0 + \beta_1 x + \epsilon$$

↳ slope
↳ intercept

x and y are random

Population model is not known. Use sample x and y data to estimate β_0 and β_1 .

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \epsilon_i$$

ϵ_i = error term that contains unobservable factor other than x that affect y

ex:

$$y = \{5, 10, 15, 2, 1\} \quad x = \{2, 2, 2, 2, 2\} \leftarrow \text{bad}$$

2: zero conditional mean
 $E(\epsilon|x) = 0$

3: x and y are randomly distributed

Estimate $\hat{\beta}_1$ and $\hat{\beta}_0$ from the sampled data using OLS (ordinary least squares) estimation

$\hat{\beta}_1$ and $\hat{\beta}_0$ are unbiased estimators of β_0 and β_1

If the sample is typical and representative of population, then $\hat{\beta}_0$ and $\hat{\beta}_1$ should be near (or) close to population values β_0 and β_1 .

Homoskedasticity \rightarrow constant variance

$$\begin{aligned}\text{Var}(\epsilon|x) &= \text{constant} \\ &= \sigma^2\end{aligned}$$

variance of error term (or) unobservable factors conditional on x is constant

$$\text{Var}(y|x) = \text{constant}$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

$$\text{Residual} = \underset{y}{\text{observed}} - \underset{\hat{y}}{\text{fitted}}$$