HW 22.10

Wednesday, November 11, 2020   4:07 PM

Passed solution review

10. Consider a repeated game between a supplier (player 1) and a buyer (player 2). These two parties interact over an infinite number of periods. In each period, player 1 chooses a quality level $q \in [0, 5]$ at cost q. Simultaneously, player 2 decides whether to purchase the good at a fixed price of 6. If player 2 purchases, then the stage-game payoffs are $6 - q$ for player 1 and $2q - 6$ for player 2. Here, player 2 is getting a benefit of $2q$. If player 2 does not purchase, then the stage-game payoffs are $-q$ for player 1 and 0 for player 2. Suppose that both players have discount factor δ.

(a) Calculate the efficient quality level under the assumption that transfers are possible (so you should look at the sum of payoffs).

$q = (6-q) + (2q-6)$ $\rightarrow U_1 + U_2 = 2q - q = q$

$q = 6 - q + 2q - 6$

$q = q$

max $q = 5$

(b) For sufficiently large δ, does this game have a subgame perfect Nash equilibrium that yields the efficient outcome in each period? If so, describe the equilibrium strategies and determine how large δ must be for this equilibrium to exist.

$\frac{6-q}{1-\delta} \geq 6 + \frac{0 \cdot \delta}{1-\delta}$ $q = 5 \rightarrow \frac{1}{1-\delta} \geq 6 \rightarrow \delta \geq 5/6$

SPE if $\delta \geq 5/6$

Not a complete answer

$q = 0$ is NE of stage game + payoffs $\rightarrow 0, 0$

If $q = 5$, $U_1 = 1$, $U_2 = 4$ ...

Seller: $1 \cdot 1/1-\delta \geq 6 + 0\,\delta/1-\delta$

$1 \geq 6 - 6\delta$

$6\delta \geq 5$

$\delta \geq 5/6$

Buyer: $6\,1/1-\delta \geq 0 + 0\,\delta/1-\delta$

Any δ works

If $\delta \geq 5/6$, buy at $t=1$ at t if $q_{t-k} = 5$ $\forall k$ else do not buy and $q = 5$ at $t = 1$ & if the buyer has always purchased, else $q_t = 0$ is a NE