

Marked w/ Alex, Zvet, + Hailey

Problem 1

Jose visits campus every Thursday evening. However, some days the parking garage is full, often due to college events. There are academic events on 35% of evenings, sporting events on 20% of evenings, and no events on 45% of evenings. When there is an academic event, the garage fills up about 25% of the time, and it fills up 70% of evenings with sporting events. On evenings when there are no events, it only fills up about 5% of the time. If Jose comes to campus and finds the garage full, what is the probability that there is a sporting event?

- Draw a tree diagram (marginal probabilities in the primary branch and conditional probabilities in the secondary branches).
- What is the probability that there is a sporting event when Jose finds the garage full?

$$\begin{array}{lcl}
 \text{a) } .35 & \text{Full} & .25 \cdot .35 = .0875 \\
 \text{academic} & \text{Space} & .75 \cdot .35 = .2625 \\
 .2 & \text{Full} & .7 \cdot .2 = .14 \\
 \text{Sporting} & \text{Space} & .3 \cdot .2 = .06 \\
 .45 & \text{Full} & .05 \cdot .45 = .0225 \\
 \text{none} & \text{Space} & .95 \cdot .45 = .4275
 \end{array}$$

$$b) P(\text{Sports} | \text{Full}) = P(\text{Sports} + \text{Full}) / P(\text{Full}) = .14 / (.0875 + .14 + .0225) = .56$$

Problem 2

Edison Research gathered exit poll results from several sources for the Wisconsin recall election of Scott Walker. They found that 53% of the respondents voted in favor of Scott Walker. Additionally, they estimated that of those who did vote in favor for Scott Walker, 37% had a college degree, while 44% of those who voted against Scott Walker had a college degree. Suppose we randomly sampled a person who participated in the exit poll and found that he had a college degree. What is the probability that he voted in favor of Scott Walker.

- Draw a tree diagram (marginal probabilities in the primary branch and conditional probabilities in the secondary branches).
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$$\begin{array}{lcl}
 \text{a) } .53 & \text{degree} & .37 \cdot .53 = .1961 \\
 \text{Vote Yes} & \text{no} & .63 \cdot .53 = .3339 \\
 .47 & \text{degree} & .44 \cdot .47 = .2068 \\
 \text{Vote no} & \text{no} & .66 \cdot .47 = .3102
 \end{array}$$

$$b) P(\text{College} | \text{Yes}) = P(\text{Yes degree}) / P(\text{degree}) = .1961 / (.1961 + .2068) = .4867$$

Problem 3

In a particular region during a 1-year period, there were 1000 deaths. It was observed that 321 people died of a renal failure and 460 people had at least one parent with renal failure. Of these 460 people, 115 died of renal failure. Calculate the probability of a person that he dies of renal failure if neither of his parents had a renal failure

$$\begin{array}{lcl}
 \text{Parent had} & .46 & \begin{array}{l} \text{Failure} .115 \cdot .46 = .0529 \\ \text{no} .885 \cdot .46 = .4071 \end{array} \\
 \text{Parent didn't} & .54 & \begin{array}{l} \text{Failure} .321 \cdot .54 = .1733 \\ \text{no} .679 \cdot .54 = .3666 \end{array}
 \end{array}$$

$$P(\text{die} | \text{didn't}) = .1733 / (.1733 + .3666) = .3209$$

Problem 4

You draw a random sample of 12 orders of Chicken nuggets from the local McDonalds. You find 3 of the 12 have more calories than advertised. What is the probability of drawing 3 or more that fail to meet the published standard in a sample of 12 if the true failure rate is 0.05?

$$\begin{aligned}
 P(X \geq 3) &= 1 - P(X < 3) \\
 &= 1 - [(.05)^0 (.95)^{12} + (.05)^1 (.95)^{11} + (.05)^2 (.95)^{10}] \\
 &= 1 - [.5403 + .0284 + .0015] \\
 &= 1 - .5703 \\
 &= .4297
 \end{aligned}$$

Problem 5

Suppose 1 in 5 college interns turns out to be a great employee.

- How many interns would you need to try out to have a 90% chance of finding at least one great employee? Using the binomial, write the equation defining the solution then solve for n.
- How many interns would you need to try out to have a 90% chance of finding two or more great employees? This turns out to be hard to solve explicitly, but it is still easy to write the simple equation defining the solution implicitly using the binomial. So, work it out up to the point where it is ugly to solve. You can use trial and error or a numerical solution technique to get the actual answer if you want, but that is not required. We will do that later with software.

$$\begin{aligned}
 \text{a) } P(X \geq 1) &= 1 - P(X = 0) \\
 .9 &= 1 - .8^n \\
 .9 + .8^n &= 1 \\
 .8^n &= .1 \\
 n &= 10.3 \\
 \text{Try 11 people}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } P(X \geq 2) &= 1 - [P(X = 0) + P(X = 1)] \\
 .9 &= 1 - [(.2^0)(.8^n) + (.2^1)(.8^{n-1})]
 \end{aligned}$$

Problem 6

Exam scores in Professor Curmudgeon's Calculus 1 class are approximately normally distributed with a mean of 71 and a standard deviation of 13. Exam scores in Professor Pollyanna's Calculus 1 class are approximately normally distributed with a mean of 83 and a standard deviation of 7. Assume each class is made up of random draws from the same underlying student population, and so comparable in principle.

- Compare a student who got a 68 in Curmudgeon's class to one that got a 75 in Pollyanna's by calculating z scores and population percentiles. Use a z-table or excel or whatever software, as you wish, to calculate the percentiles.
- What maintained assumption are you making?

$$\begin{array}{lcl}
 \text{C: } \mu = 71 & \sigma = 13 & X = x \\
 \text{a) } z = (x - 71) / 13 & & P: \mu = 83 \quad \sigma = 7 \\
 z = (68 - 71) / 13 & & z = (x - 83) / 7 \\
 z = -.2307 & & z = (75 - 83) / 7 \\
 \% = .4090 & & z = -1.1429 \\
 = 40.9\% & & \% = .1271 \\
 & & = 12.7\%
 \end{array}$$

b) The assumption is that the students are the same and both classes are at the same level