HW 13.12

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Passed Solution review

12. Consider the following two-player team production problem. Each player i chooses a level of effort $a_i \geq 0$ at a personal cost of a_i^2. The players select their effort levels simultaneously and independently. Efforts a_1 and a_2 generate revenue of $r = 4(a_1 + a_2)$. There is limited verifiability in that the external enforcer (court) can verify only the revenue generated by the players, not the players' individual effort levels. Therefore, the players are limited to revenue-sharing contracts, which can be represented by two functions $f_1 : [0, \infty) \rightarrow [0, \infty)$ and $f_2 : [0, \infty) \rightarrow [0, \infty)$. For each player i, $f_i(r)$ is the monetary amount given to player i when the revenue is r. We require $f_1(r) + f_2(r) \leq r$ for every r.

Call a contract *balanced* if, for every revenue level r, it is the case that $f_1(r) + f_2(r) = r$. That is, the revenue is completely allocated between the players. A contract is *unbalanced* if $f_1(r) + f_2(r) < r$ for some value of r, which means that some of the revenue is destroyed or otherwise wasted.

(a) What are the efficient effort levels, which maximize the joint value $4(a_1 + a_2) - a_1^2 - a_2^2$?

$dI = 4 - 2a_i \rightarrow 0 = 4 - 2a_i$ a_2 is the same

$a_1 = 2$

Total $= 4(2+2) = 4 \cdot 4 = 16$

(b) Suppose that the players have a revenue-sharing contract specifying that each player gets half of the revenue. That is, player i gets $(r/2) - a_i^2 = 2(a_1 + a_2) - a_i^2$. What is the Nash equilibrium of the effort-selection game?

$r/2 = 2a_1 + 2a_2 - a_i^2 + a_i^2$

$r/2 = 2a_1 + 2a_2$?

$r = 4a_1 + 4a_2$

max $2a_1 + 2a_2 - a_i^2$

$dI = 2 - 2a_i = 0$

$2 = 2a_i$

$a_i = 1$ $a_1 = a_2$

(c) Next consider more general contracts. Can you find a balanced contract that would induce the efficient effort levels as a Nash equilibrium? If so, describe such a contract. If not, see if you can provide a proof of this result. $\frac{df_1}{dr}\, 4 = 2a_1$, $\frac{df_2}{dr}\, 4 = 2a_2$ for individual choice

It's symmetric so they should each get half. This is examined in part b and is not efficient

Efficiency: $4 = 2a_1$ and $4 = 2a_2$

Combining: $\frac{df_1}{dr} = 1$ and $\frac{df_2}{dr} = 1$

Balancing: $f_1(r) + f_2(r) = r$, thus $\frac{df_1}{dr} + \frac{df_2}{dr} = 1$

You can't combine and balance

(d) Can you find an unbalanced contract that would induce the efficient effort levels as a Nash equilibrium? If so, describe such a contract. If not, provide a proof as best you can. to be efficient, $r = 16$

Players get $r - x$

max $r - x - a_i^2$ so $a_i = 2$

$f_i = \begin{cases} 8 & r = 16 \\ 0 & \text{otherwise} \end{cases}$

But no one would sue because they'd get zero

$2(r - x) < r \rightarrow r < 2x$

So long as someone gets more than half, you're good

(e) Would the issues of balanced transfers matter if the court could verify the players' effort levels? Explain.

If effort was verifiable, they could be punished accordingly

No because the contract would specify effort, not revenue