HW 22.11

Wednesday, November 11, 2020 4:22 PM

Passed Solution review

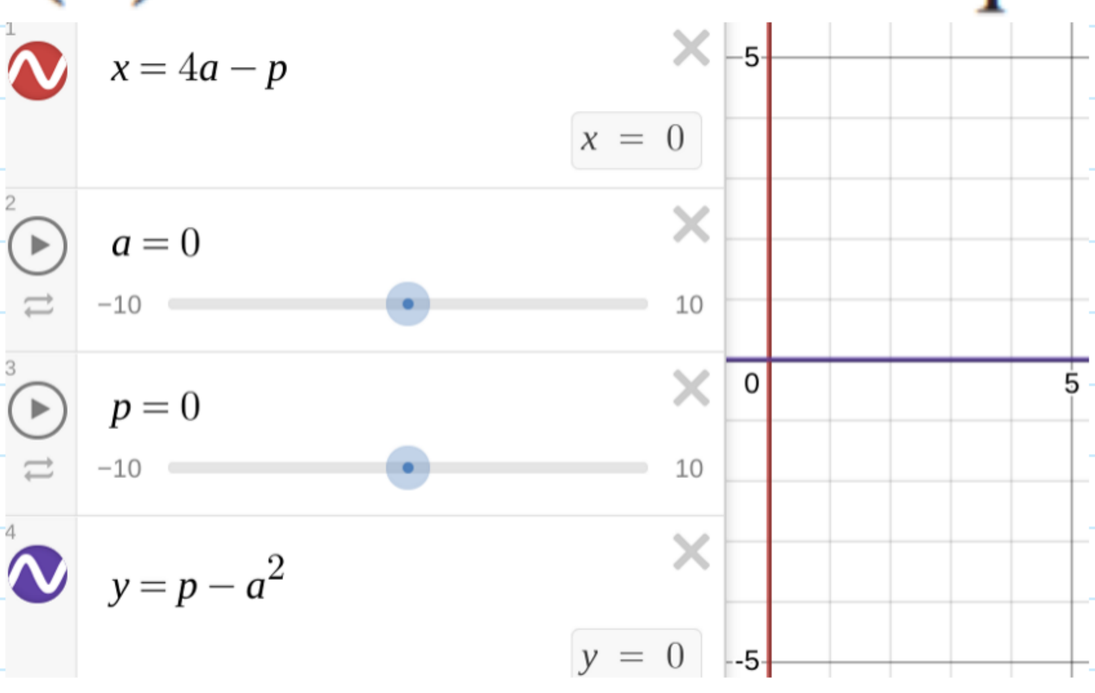
**11.** This is an extension of the previous exercise. Consider the following stage game between a manager (also called the "Principal") and a worker (the "Agent"). Let the manager be player 1 and the worker be player 2. Simultaneously, the manager chooses a bonus payment $p \in [0, \infty)$ and the worker chooses an effort level $a \in [0, \infty)$. The stage-game payoffs are $u_1(p, a) = 4a - p$ and $u_2(p, a) = p - a^2$.

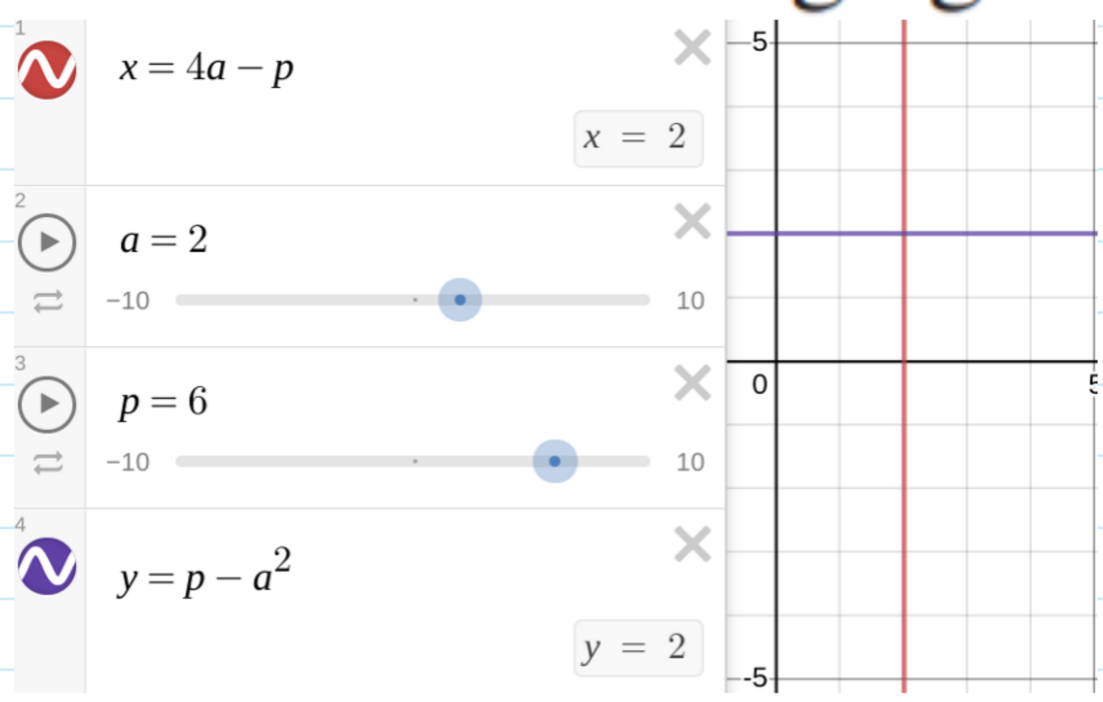
**(a)** Determine the efficient effort level for the worker.

$\max = 4a - p + p - a^2 = 4a - a^2$

$dy/da = 4 - 2a = 0 \rightarrow a = 2$

**(b)** Find the Nash equilibrium of the stage game.




NE is $a = p = 0$. otherwise input does not equal output

**(c)** Suppose the stage game is to be played twice (a two-period repeated game) and there is no discounting. Find all of the subgame perfect equilibria.

**4.** If its stage game has exactly one Nash equilibrium, how many subgame perfect equilibria does a two-period, repeated game have? Explain. Would your answer change if there were $T$ periods, where $T$ is any finite integer?

Because of this the subgame will also play $p = a = 0$

In the second period, you can't have subgame perfection without playing the NE of the stage game. If there's only one NE in the stage game, period two won't affect period one. The only SPE is the NE in both periods. This carries for T periods and doesn't change.

**(d)** Suppose the stage game is to be played infinitely many times in succession (an infinitely repeated game) and assume that the players share the discount factor $\delta < 1$. Find conditions on the discount factor under which there is a subgame perfect equilibrium featuring selection of the efficient effort level in each period (on the equilibrium path).

$a = 2 \quad p = 4$

$4 \geq (4 \cdot 2 - 6)(\delta / 1 - \delta)$

$4 \geq 2\delta / 1 - \delta$

$\delta \geq 2/3$

$\delta \geq 2/3$

$2^2 \leq (6 - 2^2)(\delta / 1 - \delta)$

$4 \geq 2\delta / 1 - \delta$

$\delta \geq 2/3$

Used Wrong Numbers

Principal: $(6 - p)(1 / 1 - \delta) \geq 6 + 0(\delta / 1 - \delta)$

$6 - p \geq 6 - 6\delta$

$6\delta \geq p$

Agent: $(p - 4)(1 / 1 - \delta) \geq p + 0(\delta / 1 - \delta)$

$p - 4 \geq p - p\delta$

$p\delta \geq 4$

$p \geq 4 / \delta$

$\delta\delta \geq p \geq 4/\delta$

$\delta^2 \geq 1/2$

$\delta \geq \sqrt{1/2}$