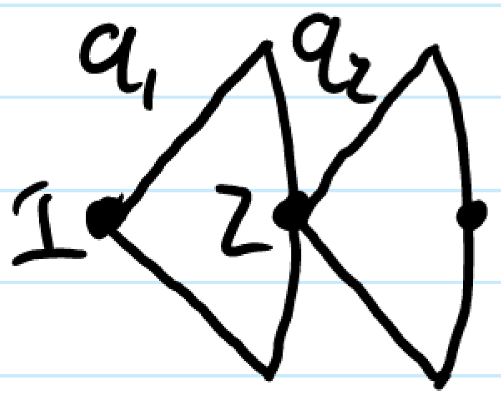
Passed solution review

6. Consider a variant of the game described in Exercise 4. Suppose that the firms move sequentially rather than simultaneously. First, firm 1 selects its quantity q_1, and this is observed by firm 2. Then, firm 2 selects its quantity q_2, and the payoffs are determined as in Exercise 4, so that firm i's payoff is $(12 - q_i - q_j)q_i$. As noted in Exercise 6 of Chapter 3, this type of game is called the *Stackelberg duopoly model*. This exercise asks you to find some of the Nash equilibria of the game. Further analysis appears in Chapter 15.

Note that firm 1's strategy in this game is a single number q_1. Also note that firm 2's strategy can be expressed as a function that maps firm 1's quantity q_1 into firm 2's quantity q_2. That is, considering $q_1, q_2 \in [0, 12]$, we can write firm 2's strategy as a function $s_2 : [0, 12] \to [0, 12]$. After firm 1 selects a specific quantity q_1, firm 2 would select $q_2 = s_2(q_1)$.

(a) Draw the extensive form of this game.



$(12-q_1-q_2)q_1 \ , (12-q_1-q_2)q_2$

(b) Consider the strategy profile (q_1, s_2), where $q_1 = 2$ and s_2 is defined as follows:

$$s_2(q_1) = \begin{cases} 5 & \text{if } q_1 = 2 \\ 12 - q_1 & \text{if } q_1 \neq 2 \end{cases}.$$

That is, firm 2 selects $q_2 = 5$ in the event that firm 1 chooses $q_1 = 2$; otherwise, firm 2 picks the quantity that drives the price to zero. Verify that these strategies form a Nash equilibrium of the game. Do this by describing the payoffs players would get from deviating.

$u_1 = 2(12-2-5) = 10 \to q_1(12-q_1-(12-q_1)) \to = 0$
$u_2 = 5(12-2-5) = 25 \to q_2(12-2-q_2) \to \max \to 10-2q_2 = 0$
$10 = 2q_2$
$q_2 = 5$

No gains from deviating so that is NE

(c) Show that for any $x \in [0, 12]$, there is a Nash equilibrium of the game in which $q_1 = x$ and $s_2(x) = (12 - x)/2$. Describe the equilibrium strategy profile (fully describe s_2) and explain why it is an equilibrium.

$$s = \begin{cases} (12-x)/2 & q_1 = x \\ 12-x & q_1 \neq x \end{cases}$$

$x(12-x)/2 = (12-x)^2/4 \to (12x - x^2)/2 = (12-x)^2/4$

No gains from deviating so it's NE

(d) Are there any Nash equilibria (q_1, s_2) for which $s_2(q_1) \neq (12 - q_1)/2$? Explain why or why not.

No. $(12-q_1)/2$ always maximizes P 2's payoff