HW 10.1

Tuesday, October 13, 2020 6:48 PM

1. Consider a more general Cournot model than the one presented in this chapter. Suppose there are n firms. The firms simultaneously and independently select quantities to bring to the market. Firm i's quantity is denoted q_i, which is constrained to be greater than or equal to zero. All of the units of the good are sold, but the prevailing market price depends on the total quantity in the industry, which is $Q = \sum_{i=1}^{n} q_i$. Suppose the price is given by $p = a - bQ$ and suppose each firm produces with marginal cost c. There is no fixed cost for the firms. Assume $a > c > 0$ and $b > 0$. Note that firm i's profit is given by $u_i = p(Q)q_i - cq_i = (a - bQ)q_i - cq_i$. Defining Q_{-i} as the sum of the quantities produced by all firms except firm i, we have $u_i = (a - bq_i - bQ_{-i})q_i - cq_i$. Each firm maximizes its own profit.

(a) Represent this game in the normal form by describing the strategy spaces and payoff functions.

$p = a - bQ$ $\quad u_i = (a - bq_i - bQ_{-i})q_i - cq_i \rightarrow aq_i - bq_i^2 - bQ_{-i}q_i - cq_i$

Firm i \ firm -i

$S_i = [0, \infty)$

$u_i = (a - bq_i - bQ_{-i})q_i - cq_i$

$du/dq_i = -2bq_i - bQ_{-i} - c = 0$

$2bq_i = -bQ_{-i} - c \rightarrow q_i = (-bQ_{-i} - c)/2b$

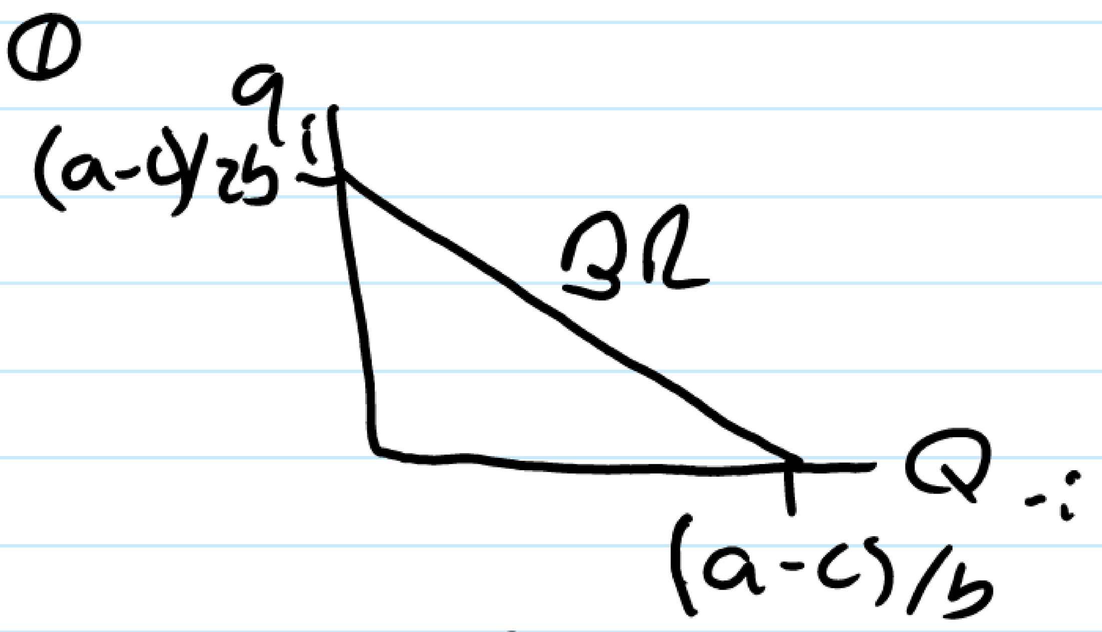
(b) Find firm i's best-response function as a function of Q_{-i}. Graph this function.

$du/dq_i = a - bQ_{-i} - c - 2bq_i = 0$

$2bq_i = a - bQ_{-i} - c$

$q_i = (a - bQ_{-i} - c)/2b$

$q_i = (a - 0 - c)/2b$



$0 = (a - bQ_{-i} - c)/2b \rightarrow$ wolframalpha to solve

$c = a - bq$

$bq = a - c$

$q = (a-c)/b$

(c) Compute the Nash equilibrium of this game. Report the equilibrium quantities, price, and total output. (Hint: Summing the best-response functions over the different players will help.) What happens to the equilibrium price and the firm's profits as n becomes large?

$Q^* = nq^*$

$Q^*_{-i} = (n-1)q^*$

$q^* = (a-c)/(n+1)(b)$

$Q^* = (n)(a-c)/(n+1)(b)$

Plug in

$p^* = a - bn(a-c)/(n+1)(b) \rightarrow (a+cn)/(n+1)$

$u^* = p^*q^* - cq^* \rightarrow$ Plug in $\rightarrow 2(a-c)/(2b)(n+1)$

(d) Show that for the Cournot duopoly game ($n = 2$), the set of rationalizable strategies coincides with the Nash equilibrium.

$q^* = 2(a-c)/(2b)(3)$

$= 2(a-c)/6b$

$a - c/3b$

$R = (a-c)/3b$ for $i = 1, 2$