BCA Solutions – Discounting up to (not including) Games Against Nature

Simple Discounting

a. A project costs \$10 up front and has net benefits of -\$1 at the end of the first year and \$15 at the end of the second year. The discount rate is 0.05. What is the NPV?

NPV = $-10 - 1\times1.05^{-1} + 15\times1.05^{-2}$

b. At what discount rate would the NPV be 0? (You may just want to use a spreadsheet, trying different rates until you find the right one, to solve this. But, write out the equation that defines what you are solving for in your answer.)

NPV = $-10 - 1\times(1+d)^{-1} + 15\times(1+d)^{-2} = 0$

Discount Rate		0.176
t	NB	PV
0	-10	-10.000
1	-1	-0.851
2	15	10.851
NPV		0.000

More Discounting

A project has benefits and costs as shown in the table to the right. You will probably want to do these calculations in a spreadsheet. Assuming you do, include a copy of the relevant portion of the spreadsheet, neatly formatted, labeled, and explained, with your answers.

a. Assuming time 0 is right now and every benefit and cost is received at exactly the time indicated, calculate the NPV if i=0.06.

$NPV=\Sigma_t(B_t-C_t)(1.06)^{-t}$ (Calculations below)

b. Continuing from (a), what discount rate makes the NPV 0?

$NPV=\Sigma_t(B_t-C_t)(1+d)^{-t}=0$ (Calculations below)

c. Assume time 0 is right now and that benefits and costs benefits and costs are spread more or less evenly over the period following where they are listed. For example the cost of 25 at time 0 represents expenditures spread out evenly between t=0 and t=1. Estimate the NPV if i=0.06.

The key is to approximate by assuming all costs and benefits occur in the middle of the period, essentially splitting the difference. This is just a reasonable guesstimate. Time 0 values are discounted half a year, time 1 values 1.5 years, and so on.

$NPV=\Sigma_t(B_t-C_t)(1.06)^{-(t+0.5)}=0$ (Calculations below)

d. Continuing from (c), at what discount rate would the NPV be 0?

$NPV=\Sigma_t(B_t-C_t)(1+d)^{-(t+0.5)}=0$ (Calculations below)

	Discount Rate				0.060	0.116		0.060	0.116
Year	C	B	NB	t(a,b)	NPVa	NPVb	t(c,d)	NPVb	NPVb
0	25	0	-25	0	-25.000	-25.000	0.5	-24.282	-23.662
1	40	0	-40	1	-37.736	-35.832	1.5	-36.652	-33.914
2	10	10	0	2	0.000	0.000	2.5	0.000	0.000
3	5	25	20	3	16.792	14.377	3.5	16.310	13.607
4	5	45	40	4	31.684	25.758	4.5	30.774	24.379
5	5	35	30	5	22.418	17.306	5.5	21.774	16.379
6	5	25	20	6	14.099	10.335	6.5	13.694	9.782
7	20	5	-15	7	-9.976	-6.944	7.5	-9.689	-6.572
		NPV			12.281	0.000		11.929	0.000

Annuity Formulae

You may want to use a spreadsheet to actually find the solutions to some of these problems. But make sure to write out the equations you need to solve in your answers.

a. What is the present value of 15 annual payments of \$100, with the first payment one year from now, if the discount rate is 0.05?

$NPV=100\Sigma_1^{15}1.05^{-t}=100(1-1.05^{-15})[(1/1.05)/(1-1/(1.05))]=100(1-1.05^{-15})/0.05=1038$

b. What is the present value of 15 annual payments of \$100, with the first payment right now, if the discount rate is 0.05?

$NPV=100\Sigma_0^{14}1.05^{-t}=100+100(1-1.05^{-14})/[(1/1.05)/(1-1/(1.05))]=100+100(1-1.05^{-14})/0.05=1090$

c. What is the present value of 15 annual payments of \$100, with the first payment five years from now, if the discount rate is 0.05?

This is just the answer from (a) discounted 5 additional years, so $1090/1.05^5=854$

d. At what discount rate would the present value of 15 annual payments of \$100, with the first payment right now, be 0?

Take the expression for the answer from a, but insert d instead of 0.05 for the discount rate, set it equal to 0 and solve it for d: $100+100(1-(1+d)^{-14})/d=0$. What you will find is that as $d\rightarrow\infty$, so the future does not matter at all, the value approaches \$100, the value of the payment received right now. So, even when the future does not matter at all, this is still worth \$100, there is no discount rate to make it equal 0.

e. How many annual payments of \$100, with the first payment right now, would it take to be worth more than \$1,000, if the discount rate is 0.05?

Take the expression for the answer from a, but insert T-1 instead of the year of the last payment, set it equal to 1,000 and solve it for T: $100+100(1-1.05^{-(T-1)})/0.05=1000$. For this one, we know it will be at least 11. Solving gives T=14 payments, at which the NPV is 1039.

f. What is the value of 15 annual payments which begin at \$100 one year from now and increase at 2% per year thereafter, if the discount rate is 0.05?

In this case, $\delta=1.02/1.05\approx0.9714$. Using the formula above:

$NPV=(1-(1.02/1.05)^{15})[(1.02/1.05)/(1-(1.02/1.05))]=1199$

Equivalent Annual Net Benefit

Which of the following projects has the larger Equivalent Annual Net Benefit if the interest rate is 4%?

	Time	1	2	3	4	5	6	7	8
Project A	NB	-20	-10	5	10	10	10	10	5
Project B	NB	-30	10	15	15	5			

$NPV_A = -20/1.04 - 10/1.04^2 + 5/1.04^3 + 10/1.04^4 + 10/1.04^5 + 10/1.04^6 + 10/1.04^7 + 5/1.04^8 = 11.8917$

$NPV_B = -30/1.04 + 10/1.04^2 + 15/1.04^3 + 15/1.04^4 + 5/1.04^5 = 10.6661$

Delta=1/1.04≈0.9615

$a_A = (1-.9615^8)(0.9615/(1-0.9615)) \approx 6.7327$

$a_B = (1-.9615^5)(0.9615/(1-0.9615)) \approx 4.4518$

11.8917≈6.7327EANBA

EANBA≈1.7663

10.6661≈4.4518EANBB

EANBB≈2.3959

Project B has the larger EANB.

It is fine to shift all of this so you start in period 1 not 0. The comparison would be the same though the numbers would be slightly different.

Inflation 1

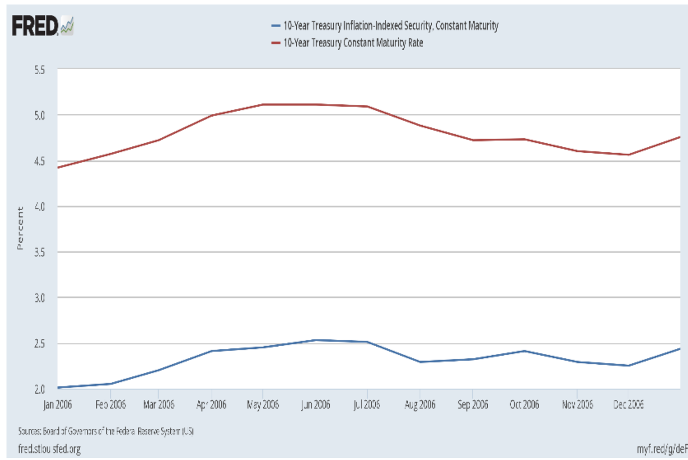
a. What is the value in today's dollars of $300 in 2024 if average annual compound rate of inflation is 0.025 between now and then? $300/1.025^4 = 271.79$

b. What is the value of $200 today expressed in 2030 dollars if the average annual compound rate of inflation is 0.05 between now and then? $200 \times 1.05^{10} = 325.78$

Inflation 2

a. Go to the Federal Reserve Economic Database (FRED) maintained by the St. Louis Fed at https://fred.stlouisfed.org/ and calculate/estimate/retrieve the 2006 average of the *10-Year Treasury Constant Maturity Rate* and the *10-Year Treasury Inflation-Indexed Security, Constant Maturity*, both series maintained in FRED. Based on these, what was the expected annual rate of inflation from 2006 to 2016?

*Note for grading: It is possible the series have been updated since I pulled the data. So, this should be the approach, but the numbers might be very slightly different.



The figure shows the monthly averages of the two series. The averages of the monthly averages were i=4.8 and r=2.3, so inflation was expected to average about m-4.8-2.3=2.5%. [Note: you could try to be more precise, r=(i-m)/(1+m), 0.023=(0.048-m)/(1+m), which gives m=0.0244, essentially the same thing.]

b. Obtain the Consumer Price Index for 2006 and 2016 (FRED has it, or you can get it from the Bureau of Labor Statistics) and calculate the average annual rate of inflation for those 10 years. (Remember, it gets compounded each year to produce the full difference between the 2016 and 2006 price levels.) What do your calculations suggest about the ability to forecast inflation?

In January 2006 the CPI was 199.3 (base 1984, all urban consumers). In December 216 it was 242.8. The average rate of inflation solves $242.8=199.3(1+\pi)^{16}$, giving $\pi=0.012$, or about 1.2%. Over the past decade, inflation was only half what was expected a decade ago. It is hard to accurately predict inflation. [Note, I pulled these numbers in 2017—they could have been revised since, but any difference would be small.]

Horizon Values - 1

A project involved initial construction costs of \$2.5 million. The annual rate of economic depreciation for that construction class is 0.008. The project is expected to terminate in 25 years. The expected annual rate of inflation is 0.025.

a. Estimate the horizon value at time t=25 owing to the remaining value of this asset.

In today's dollars, the asset will be worth an estimated 0.992^{25}\$2.5M, or \$2.045M. If you want to know what that would be in the futures dollars, multiply by 1.025^{25}, to get \$3.792M.

b. If the real discount rate is 0.035, what is the present value of the horizon value from (a)?

Working with the real discount rate, we want to work with the real salvage value, that is the value in today's dollars, of \$2.045M, and discount it, so NPV=\$2.045M/1.035^{25}=\$0.865M.

Horizon Values - 2

A project involved initial construction costs of \$1.75 million. After 15 years, the useful life of that construction will be over and the facility will be demolished, involving sensitive environmental protections and cleanup. You estimate that 25% of the cost of the facility represents items that could be sold for scrap at 30% of their initial construction cost. You estimate the proper demolition cost of such a facility to be \$0.9M.

a. What is the NPV of the horizon value?

In today's dollars, $H_{15} = -\$(0.9-0.3\times1.75/4)M = -768.8K$. Since it was not specified, just state your assumption on the discount rate. I will use a real discount rate of 0.035, so the NPV is $-\$768.8K/1.035^{15}=-458.9K$

b. If the expected annual rate of inflation is 0.02, what is the nominal horizon value in 15 years?

$-\$768.8K\times1.02^{15}=-1.034M$

Horizon Values – 3

A program, if implemented, will operate for 10 years for certain. Your best guess is that after year 10 and following each year thereafter there will be a 0.02 probability the program will end. Real net benefits are \$25/year in year 1 and are expected to grow 1% per year as long as the program is in operation. The real discount rate is 3.5%. What is the NPV of the horizon value of net benefits following year 10?

I should have been a bit more precise in the wording of this question regarding timing. Best given this wording is to assume net benefits are \$25 at time t=0 but that for the first year the program operates, they will be 25×1.01, for the second year 25×1.01^2 and so on. But, if you assumed 25 was the value in one year, and did all calculations correct for that assumption, that is fine too.

This question puts lots of things together to push you to synthesize what we have been studying. The value of net benefits in year 10 is $25\times 1.01^{10}=27.62$. Since the probability of operating in year 11 is 0.98, in year 12 is 0.98^2, and so on, since net benefits grow 1% annually if still operating, and since the discount rate is 3.5%, looking forward from year 10 $H_{10}=25\times 1.01^{10}a(\infty,\delta_{10})$ where $\delta_{10}=0.98\times 1.01/1.035\approx 0.9563$. Thus $H_{10}=25\times 1.01^{10}(0.9563/(1-0.9563))=\$604.73$. Discount this back 10 years to get its present value at time 0, $\$604.73/1.035^{10}=\$428.71$.

Simple ENPV

a. A project costs \$10 up front and has net benefits of \$15 with probability 0.8 at the end of the second year and otherwise returns nothing. The discount rate is 0.035. What is the NPV?

b. At what probability of returning \$15 after year 2 would the ENPV be 0?

$ENPV=-\$10+0.8\times\$15/1.035^2=\$1.20$

$0=-\$10+f\times\$15/1.035^2$ $\quad f=1.035^2\times 2/3$ $\quad f=0.71$