HW 5 Cost and Demand Approximation

Tuesday, September 22, 2020 5:37 PM

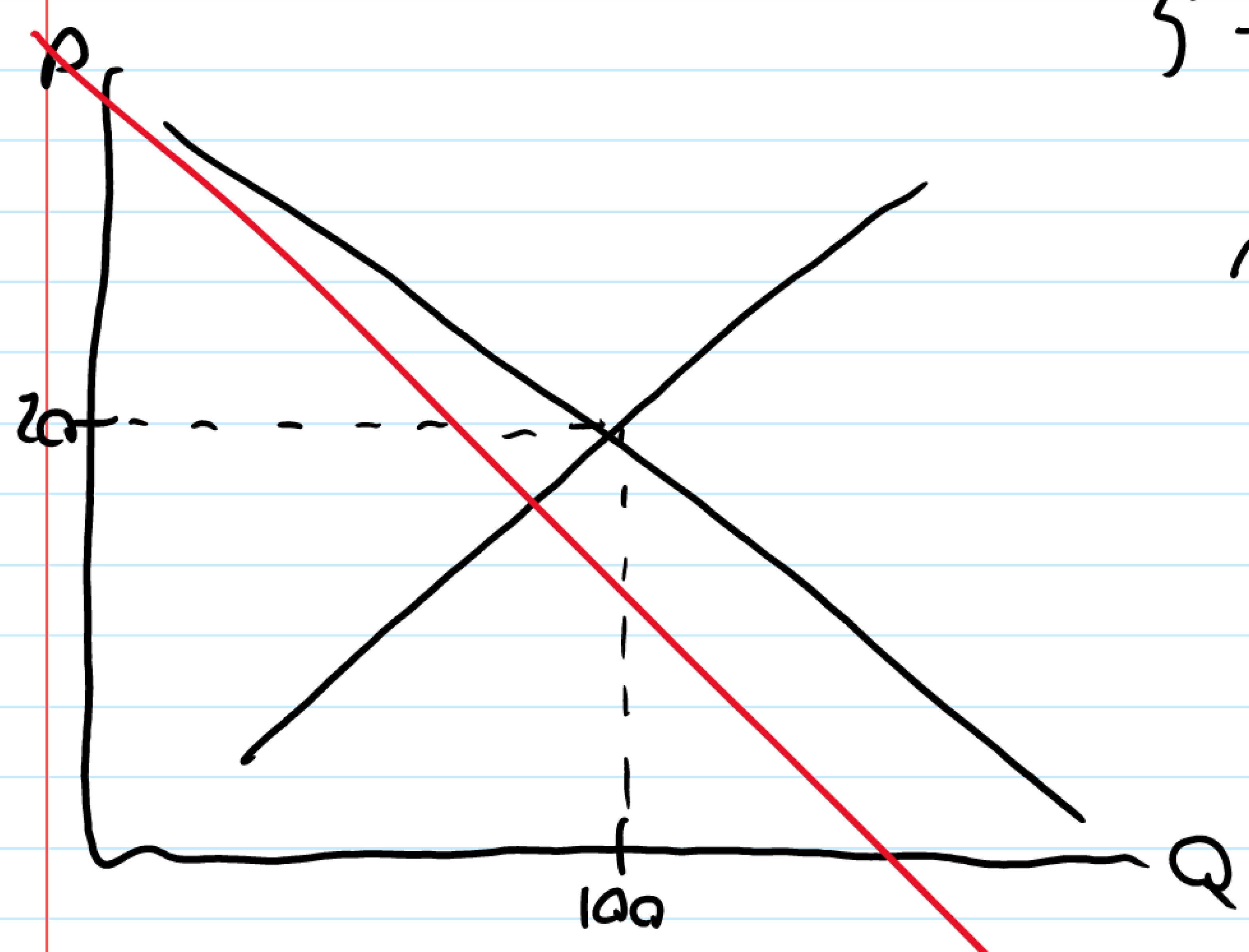
A profit maximizing monopolist charges \$20 and sells 100 units. Elasticity of demand is -2.5. The monopolist's cost function is C(Q)=F+cQ where F is a fixed cost and c is the constant per unit variable cost.

a) What is the per unit variable cost?

b) What is the highest fixed cost could be if the monopolist has not chosen to exit the industry?

c) Write a linear approximation of both demand and inverse demand around the current price. Hint, use the formula for point elasticity and the current price and quantity, then rearrange for Q to get demand and P to get inverse demand.

$\eta^D = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$



$MR = P \cdot (1 + (1/\eta^D))$

a) 20/100 = .2 dollars per unit

b) $\eta^D = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} \to -2.5 = \frac{Q_1 - Q_0}{(F+cQ) - 20} \cdot \frac{20}{100} = \frac{Q_1 - 100}{F + (.2)(100) - 20} \cdot \frac{20}{100}$

If F = 20, then no profit can be made. But so long as F < 20, then at least some profit can be made

c) $\eta^D = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$

$-2.5 = \frac{Q_1 - Q_0}{P_1 - P_0} \cdot \frac{P_0}{Q_0} \to -2.5 = \frac{Q_1 - 100}{P_1 - 20} \cdot \frac{20}{100} \to -2.5 = \frac{Q - 100}{P - 20} \cdot .2$

$-12.5 = \frac{Q - 100}{P - 20} \to -12.5(P - 20) = Q - 100 \to -12.5P + 250 + 100 = Q$

$Q = -12.5P + 350$

$Q - 350 = -12.5P \to P = \frac{Q - 350}{-12.5} \to P = 28 - .08Q$

Restarted w/ Austin + River

a) ~~$y = mx + b$~~

~~$P = \eta^D Q + b$~~

~~$20P = -2.5Q + 100$~~

~~$P = -.125Q + 5$~~

$MR = P(1 + 1/\eta^D) = MC$

$MR = 20(1 + 1/-2.5) = MC$

$MR = 20 + 20/-2.5 = MC$

$MR = 12 = MC$

variable cost = \$12

b) π = revenue - cost

$0 = (20 \cdot 100) - (F + cQ)$

$0 = 2000 - F + (12 \cdot 100)$

$0 = 2000 - F + 1200$

$F = 800$

c) It turns out I got this right by myself!

$\eta^D = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$

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