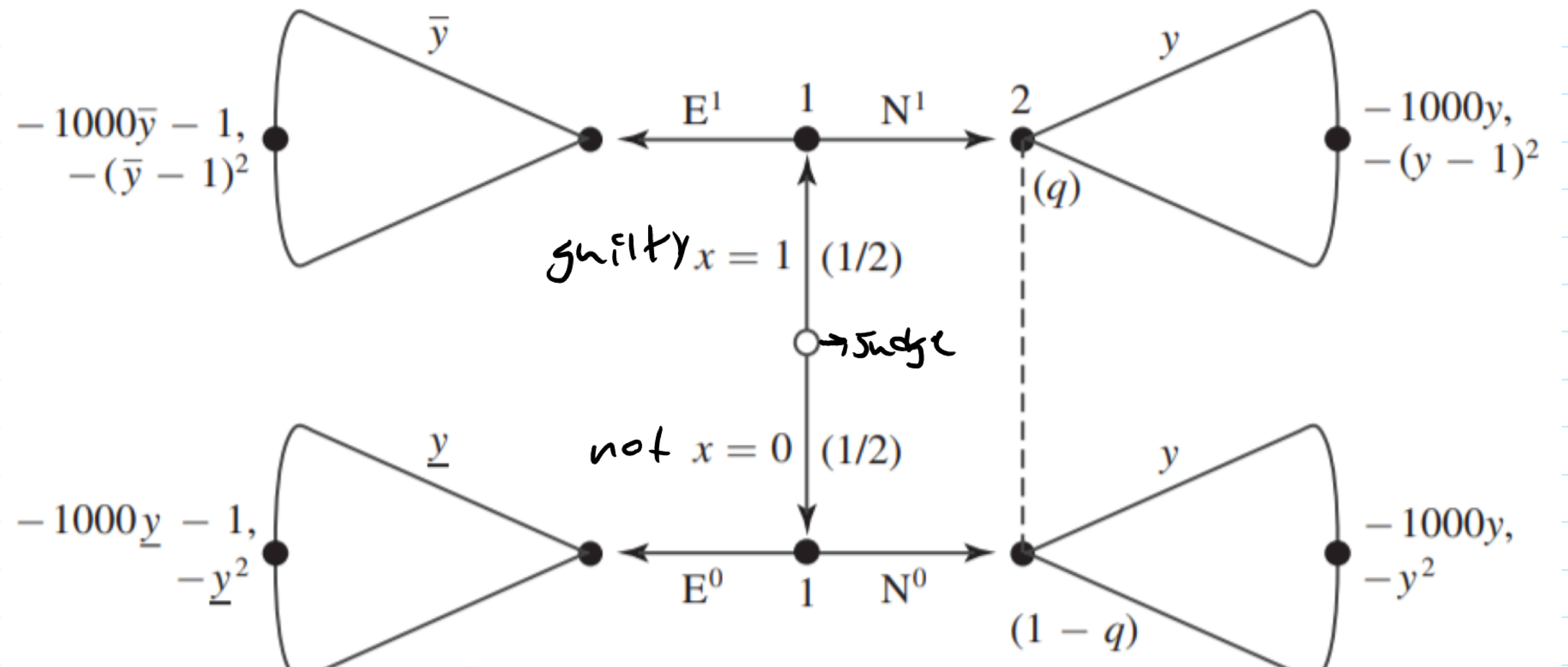
Passed solution review

5. A defendant in a court case appears before the judge. Suppose the actual harm to the plaintiff caused by the defendant is equal to $1000x$ dollars, where either $x = 0$ or $x = 1$. That is, the defendant is either innocent ($x = 0$) or guilty of 1000 dollars of damage ($x = 1$). The defendant knows x and has evidence to prove it. The judge does not observe x directly; she only knows that $x = 1$ with probability 1/2 and $x = 0$ with probability 1/2.

The judge and defendant interact as follows: First, the defendant has an opportunity to provide his evidence of x. He freely chooses whether or not to provide the evidence; the court cannot force him to do it. Providing evidence to the court costs the defendant one dollar (for photocopying). If the defendant chooses to provide the evidence, then it reveals x to the judge. Whether or not evidence is provided, the judge then decides the level of damages y (in thousands of dollars) that the defendant must pay. The judge prefers to select y "fairly"; she would like y to be as close as possible to x.

The defendant wishes to minimize his monetary loss. These preferences and the players' interaction are summarized by the extensive-form diagram that follows. Note that "E" stands for "provide evidence" and N stands for "do not provide evidence."



Player 2 = Judge

(a) This game has a unique perfect Bayesian equilibrium. Find and report it. (Hint: Start by showing that it is optimal for the judge to set y equal to the expected value of x, given her belief.)

1 \ 2	$x=1$
Ee	$[-1001-1]/2 = -501$,
En	
Ne	
Nn	

This doesn't make sense. All of player 2's payoffs are negative. They should be $0 \le y \le$ whatever damages are awarded

No evidence given... $U_J = -q(y-1)^2 - (1-q)(y^2) \to \frac{dU_J}{dy} = -2(y-1)q - 2(1-q)y = 0$
$= y = q$

$E(Y) = 1q + 0(1-q) = q$

Consider E^0N^1; $\underline{y} = 0$, $\bar{y} = 1000$, $y = 1000$, $q = 1$
- given E^0N^1, $q = 1$ so $y = 1000$ is BR
- guilty pays 1000 either way and $-1000y - 1001$ so no evidence
- innocent always provides evidence

(b) In one or two sentences, explain why the result of part (a) is interesting from an economic standpoint.

Innocents provide evidence while guilties don't. Thus, the Judge can deduce the guilt

(c) Consider a version of the game in which x is an integer between 0 and K, inclusive, with each of these values equally likely. Compute the perfect Bayesian equilibrium of this game. (Hint: Use your intuition from part (a).)

Is guilt now a range? Is $x = 2$ double guilty?

SPE is $E^0E^1 \ldots E^{k-1}N^k$, $y^i = 1000i$ $i < k$, $y = 1000k$ if no evidence given, $P(i = k \mid N) = 1$

All $i < k$ provide evidence to avoid $y = 1000k$, k does not to save \$1, thus $P(i = k \mid N) = 1$ is correct

$U_J = -q(y-1)^2 - (1-q)(y^2)$

$dU_J/dy = -2q(y-1) - 2(1-q)y = 0 \to y = q \cdot 1 \quad = q \cdot 1000$

Min MSE

$\underline{y} = 0$, $\bar{y} = 1$, $q = 1$, $y = 1$