Monopoly Review – Basic Profit Maximization

A monopolist faces inverse demand of P=4-0.02Q, a per unit variable cost of \$1.5, and fixed costs of F.

a) Find the profit maximizing price and quantity.

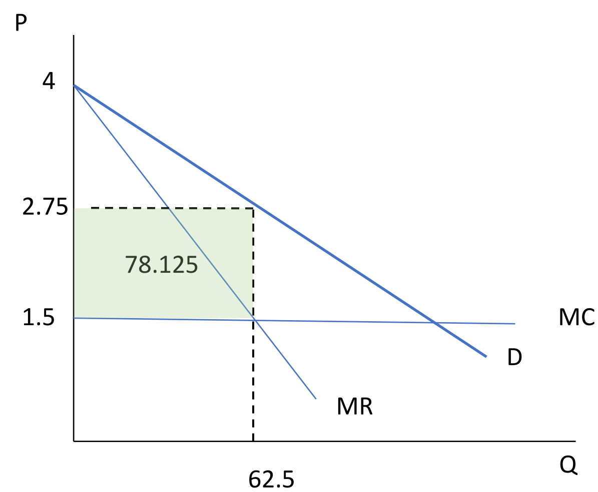
b) Depict the solution to (a) in a figure.

c) At what level of fixed cost would the monopolist decide it was not worth remaining in the market?

a) MR=MC, 4-0.04Q=1.5,
Q=2.5×100/4=62.5, p=4-0.02×62.5=2.75.

b) See figure.

c) π=(2.75-1.5)62.5-F=78.125-F. F can be at most 78.125 or the firm would exit in the long run.



Monopoly Review – Cost and Demand Approximation

A profit maximizing monopolist charges \$20 and sells 100 units. Elasticity of demand is -2.5. The monopolist's cost function is $C(Q)=F+cQ$ where F is a fixed cost and c is the constant per unit variable cost.

a) What is the per unit variable cost?

b) What is the highest fixed cost could be if the monopolist has not chosen to exit the industry?

c) Write a linear approximation of both demand and inverse demand around the current price. Hint, use the formula for point elasticity and the current price and quantity, then rearrange for Q to get demand and P to get inverse demand.

a) $MR=P(1+1/\eta_D)=c$, $20(1-2/5)=c$, $c=12$.

b) $\pi=(20-12)100-F=800-F$. Since the monopolist is still in business, $F\leq 800$.

c) $\eta_D=[(Q-Q_1)/(p-p_1)]\times(p_1/Q_1)$

$-2.5=[(Q-100)/(p-20)]\times(20/100)$

$-2.5\times5\times(p-20)=Q-100$

$-12.5p+250=Q-100$

$350=Q+12.5p$

From there, rearrange for whichever you need.

Demand: $Q=300-12.5p$

Inverse Demand: $p=(350-Q)/12.5=28-0.08p$.

Valuing Direct Supply in a Monopoly Market – Cost and Changes in P and Q Given

Initially, a profit maximizing local monopolist charges \$15 and sells 500 units per week. Per unit variable cost is \$10. Now assume the local government begins to provide 100 units per week at the market price. As a result, the market price falls to \$13 and the quantity sold by the monopolist falls to 430.

a) Find the changes in CS, PS, and GS created by the monopoly.

b) Assume the METB is 0.25. Find the changes in SS.

c) Depict all of this in a diagram. You probably want to sketch the diagram right at the start of the problem for reference as you work, and then to redraw a neat version to submit.

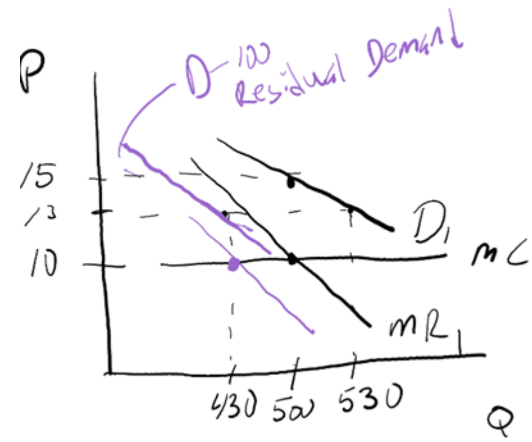
a) ΔCS=(15-13)500+(15-13)(530-500)/2=1030

ΔPS=(13-10)430-(15-10)500=-1210

ΔGS=13×100=1300

b) ΔSS=1030-1210+1.25×1300=1445

c) See figure.



Valuing Direct Supply in a Monopoly Market – Elasticity Given

Initially, a profit maximizing local monopolist charges \$15 and sells 500 units per week. Elasticity of demand is -3. The monopolist's cost function is C(Q)=F+cQ where F is a fixed cost and c is the constant per unit variable cost.

a) What is the per unit cost of the product?

b) What are the demand and inverse demand functions?

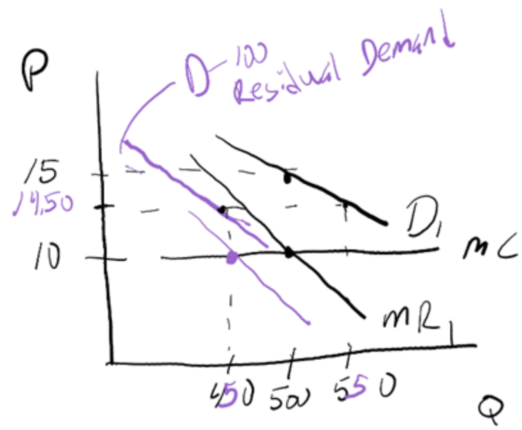
Now assume the local government begins to provide 100 units per week at the market price.

c) What is the residual demand left for the monopolist?

d) Find the new price and the monopolist's quantity and the total market quantity.

e) Assume the METB is 0.25. Find the changes in CS, PS, GS, and SS.

f) Depict all of this in a diagram. You probably want to sketch the diagram right at the start of the problem for reference as you work, and then to redraw a neat version to submit.



a) MR=MC, 15(1-1/3)=MC, MC=10

b) $\eta_D=[(Q-Q_1)/(p-p_1)]\times(p_1/Q_1)$
$-3=[(Q-500)/(p-15)]\times(15/500)$
$-3\times(500/15)\times(p-15)=Q-500$
$1500-100p=Q-500$
$2000=Q+100p$
From there, rearrange for whichever you need.
Demand: $Q=2000-100p$
Inverse Demand: $p=(2000-Q)/100=20-0.01p$

c) Residual Demand
$Q_{RES} = 1900-100p$

d) Inverse Residual Demand
$100p=1900-Q_{RES}$
$p=19-0.01Q_{RES}$
MR=MC
$19-0.02Q_{MON}=10$
$Q_{MON}=450$
$Q=550$
$p=20-0.01(550)=14.50$

e) $\Delta CS=(15-14.5)500+(15-14.5)(550-500)/2=262.5$
$\Delta PS=(14.5-10)450-(15-10)500=-475$
$\Delta GS=14.5\times100=1450$
$\Delta SS=262.5-475+1.25\times1450=1412.5$

f) See figure.

BCA of a Carbon Tax

In this problem you will conduct a BCA of a tax intended to correct a negative externality associated with electricity consumption. It is oversimplified in a number of ways, for example it ignores: i) complexities associated with different production technologies with differing cost structures, ii) differences in types of consumers that pay different prices, and iii) differences in actual and scheduled depreciation which cause regulated prices to differ from marginal long run average production costs. Nevertheless, done correctly, this approximation should give you a reasonable sense of the magnitude of the problem and the potential importance of addressing it.

Suppose the elasticity of demand for electricity is -0.75. The current price per kilowatt hour (KWh) in Florida is $0.12. The price is regulated so that electricity providers make only a normal return. For purposes of this problem, treat the price as approximately equal to the marginal cost of production. Current consumption in Florida is 28 KWh per person per day. Economists estimate that the marginal external cost of carbon emissions is $0.05 per KWh. The METB is 0.25.

Perform a BCA of a carbon tax in Florida equal to the marginal external cost of carbon emissions.

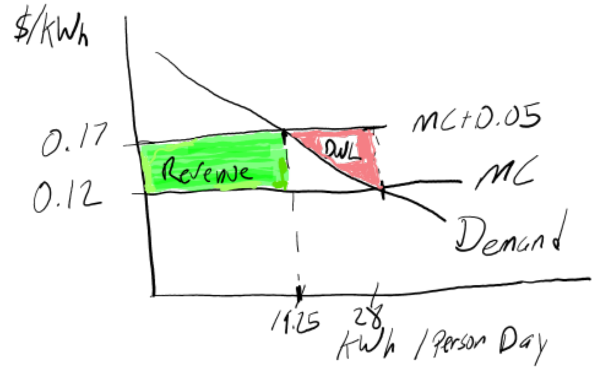
First, draw a figure to illustrate the impacts per Floridian under the assumptions of the problem.

Second, there are about 21.5 million Floridians. What is the annual cost to Florida's electricity consumers? The annual benefit to Florida's taxpayers? The annual benefit of decreased carbon emissions?

Third, if the benefits of carbon reduction are spread evenly over every resident of the world, and there are about 7.6 billion people in the world, what are the benefits of the carbon reduction to Floridians and the rest of the world, respectively?

Fourth, organize this information on annual costs and benefits by group into a neat table. Is it likely carbon, or any other pollutant, will be effectively addressed by policy on a state or local level? Why or why not? If not, what will it take to address it?

BCA of a Carbon Tax - ANSWER



A corrective tax of $0.05/KWh will raise the price to $0.17, assuming constant unit cost, a 42% increase. Thus the percentage change in quantity demanded is -31%, causing per capita consumption to fall to 19.25 KWh. The impacts per Floridian are as follows:

$\Delta CS=-0.05\times19.25-0.05\times8.75/2=-1.18125$
$\Delta PS=0$
$\Delta GS=0.05\times19.25=0.9625$
$\Delta SS=-1.18125+1.25\times0.9625=0.021875$

Since there are 21.5 million Floridians, and 365 days per year, the gain to taxpayers is about:

1.18125 per Person Day×21.5 Million People ×365 Days/Year=$9.44 Billion / Year.

Similarly, the cost to consumers is about:

1.25×0.9625 per Person Day×21.5 Million People ×365 Days/Year=$9.27 Billion / Year.

The benefits to the world of reduced carbon emissions is about:

0.05 per KWh×8.75 KWh per Person Day×21.5 M People ×365 Days/Year=$3.43B/Year.

The share of the benefits of the reduced emissions captured by Floridians is only 21.5/7600, or, 0.283%, or $9.7M/Year.

All of this is summarized in the table to the right.

Benefit or Cost	**Value ($B)**
Benefits	
Reduced Emissions - Floridians	0.0097
Reduced Emissions – Rest of World	3.4236
To Florida Taxpayers	9.4415
Subtotal	12.8738
Cost to Florida Consumers	9.2699
Net Impact to Floridians	0.1813
Net Impact to the Rest of the World	3.4236
Net Benefit	3.6049

The only reason there is a net gain to Floridians has only to do with the ability to reduce other taxes which have a METB of 0.25 per dollar raised. If the METB were lower, there would be a net cost to Floridians, and the small value of the reduced emissions captured by residents of the state would not make much difference. This, the small value of reduced emissions to Floridians, coupled with political opposition to a tax increase, even if for a good reason, means a state level carbon tax will have very little support. Generally, problems which spill over jurisdictional boundaries require solutions from a higher level, to address the impacts of the spillovers.

Opportunity Cost 1

Suppose a project needs to hire 10 electricians. At the initial wage of 60K/year (including benefits) 480 are currently employed in the local area and 20 are estimated to be unemployed. The MEBT is 0.2. Estimate the opportunity cost of hiring the workers. Explain, and defend any assumptions you make.

ANSWER

Since there is low unemployment, the labor market appears to be functioning efficiently, and we will value worker time at the market wage rate. Since the number hired is small relative to market size, assume hiring does not significantly alter the wage. The social cost is then $1.2 \times 60 \times 10 = 720$.

Opportunity Cost 2

Suppose a project needs to hire 50 electricians. At the initial wage of 60K/year (including benefits) 480 are currently employed in the local area and 20 are estimated to be unemployed. The MEBT is 0.2. After electricians are hired for the project, the local wage increases to 65K/year and the number employed locally increases to 510. Estimate the opportunity cost of hiring the workers. Draw a diagram to illustrate. Explain, and defend any assumptions you make.

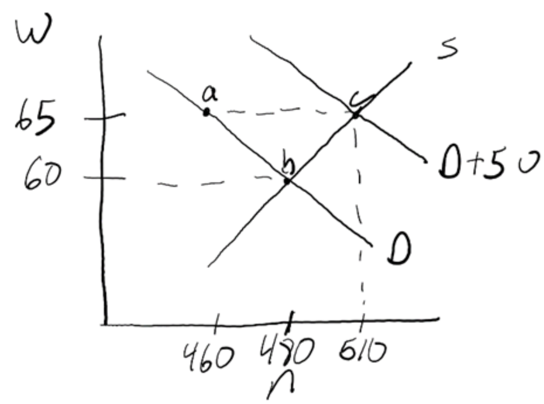
ANSWER

Since there is low unemployment, the labor market appears to be functioning efficiently, and we will value worker time at the market wage rate. Since the number hired is large relative to market size, we assume the wage is significantly changed by the program, complicating the analysis. See the figure.

ΔCS=-(65-60)460-(65-60)(480-460)/2=-2350
ΔPS=(65-60)480+(65-60)(510-480)/2=2475
ΔGS=-65×50=-3250
ΔSS=-2350+2475-1.2×3250=-3775

The opportunity cost is 3.775 $M/year.

*The increase in producer surplus exceeds the drop in consumer surplus by the area of the triangle abc, 125 $K/year.



Opportunity Cost of Inputs 3

Imagine a rural area with a current wage rate of $20 per hour at which 1500 workers are employed and 500 are unemployed. A government project will hire 50 workers at the going wage rate. Assume reservation wages for those hired are more or less uniformly distributed between $2 and $20. The METB is 0.2. Estimate the expected opportunity cost of project labor.

ANSWER

Since unemployment is so high, the market wage is not the correct measure of the opportunity cost of labor. A reasonable approximation is to assume the opportunity cost of the additional workers are randomly drawn from a uniform distribution between $2 and $20. No one with an opportunity cost over $20 will accept the job. (One could argue for using $0 rather than $2 as the lower bound.) The average opportunity cost is then $11 per hour. With the METB, the cost is 0.2 times the public funds used to pay the wage, 0.2*20=4, plus the OC of the labor itself, or $11+$4=$15 per hour per worker, or $750 per project hour.

Opportunity Cost of Inputs 4

A project requires signing buying 500 cubic yards of concrete per week for the next year from the only local provider. The currently price is $100 per yard and the provider sells 5,000 yards per week. Assuming marginal cost is constant, elasticity of demand at the current price is -1.5, and using a linear demand approximation, estimate the opportunity cost of the weekly government purchase. The METB is 0.2. Note you will need to find the original demand, the new demand, the new price, and the new quantity purchased by those other than the government in the process.

Answer

First, guess MC using p and elasticity:

p=MC(η/(1+η))

100=MC(-1.5/(1-1.5))

MC=33.33

Second, find initial demand:

$-1.5 = [(q-q_0)/(p-p_0)](p_0/q_0)$

-1.5 = [(q-5000)]/(p-100)](100/5000)

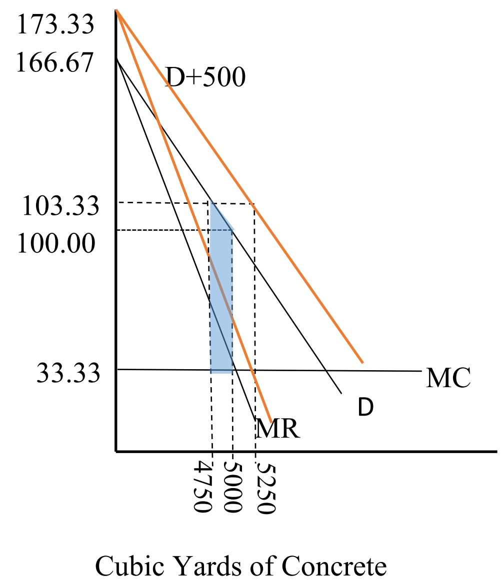
Solving gives q=12500-75p

Let us also find marginal revenue…

Rearranging gives p=166.67-(1/75)q

Marginal Revenue is then MR=166.67-(2/75)q

Third, find new demand by adding 500 (government purchases) to old demand: q=13000-75p



Fourth, find the new marginal revenue.

What is the new choke point? 0=13000-75A, A=173.33. So MR=173.33-(2/75)q

Fifth, find the new quantity: MR=MC, 173.33-(2/75)q=33.33, q=5250. Private quantity is 4750.

Sixth, find the new price: 5250=13000-75p, p=103.33

Seventh, calculate the surplus changes

ΔCS=-3.33×4750-3.33×250/2=-16234
ΔPS=70×5250-66.67×5000=34150
ΔGS=-103.33×500=-51665
ΔSS=-16233.8+34150-1.2×51665=-44082

Simple Discounting

a. A project costs \$10 up front and has net benefits of -\$1 at the end of the first year and \$15 at the end of the second year. The discount rate is 0.05. What is the NPV?

$NPV = -10 - 1 \times 1.05^{-1} + 15 \times 1.05^{-2}$

b. At what discount rate would the NPV be 0? (You may just want to use a spreadsheet, trying different rates until you find the right one, to solve this. But, write out the equation that defines what you are solving for in your answer.)

$NPV = -10 - 1 \times (1+d)^{-1} + 15 \times (1+d)^{-2} = 0$

Discount Rate		0.176
t	NB	PV
0	-10	-10.000
1	-1	-0.851
2	15	10.851
NPV		0.000

More Discounting

A project has benefits and costs as shown in the table to the right. You will probably want to do these calculations in a spreadsheet. Assuming you do, include a copy of the relevant portion of the spreadsheet, neatly formatted, labeled, and explained, with your answers.

a. Assuming time 0 is right now and every benefit and cost is received at exactly the time indicated, calculate the NPV if i=0.06.

$NPV=\Sigma_t(B_t-C_t)(1.06)^{-t}$ (Calculations below)

b. Continuing from (a), what discount rate makes the NPV 0?

$NPV=\Sigma_t(B_t-C_t)(1+d)^{-t}=0$ (Calculations below)

c. Assume time 0 is right now and that benefits and costs benefits and costs are spread more or less evenly over the period following where they are listed. For example the cost of 25 at time 0 represents expenditures spread out evenly between t=0 and t=1. Estimate the NPV if i=0.06.

The key is to approximate by assuming all costs and benefits occur in the middle of the period, essentially splitting the difference. This is just a reasonable guesstimate. Time 0 values are discounted half a year, time 1 values 1.5 years, and so on.

$NPV=\Sigma_t(B_t-C_t)(1.06)^{-(t+0.5)}=0$ (Calculations below)

d. Continuing from (c), at what discount rate would the NPV be 0?

$NPV=\Sigma_t(B_t-C_t)(1+d)^{-(t+0.5)}=0$ (Calculations below)

	Discount Rate				0.060	0.116		0.060	0.116
Year	C	B	NB	t(a,b)	NPVa	NPVb	t(c,d)	NPVb	NPVb
0	25	0	-25	0	-25.000	-25.000	0.5	-24.282	-23.662
1	40	0	-40	1	-37.736	-35.832	1.5	-36.652	-33.914
2	10	10	0	2	0.000	0.000	2.5	0.000	0.000
3	5	25	20	3	16.792	14.377	3.5	16.310	13.607
4	5	45	40	4	31.684	25.758	4.5	30.774	24.379
5	5	35	30	5	22.418	17.306	5.5	21.774	16.379
6	5	25	20	6	14.099	10.335	6.5	13.694	9.782
7	20	5	-15	7	-9.976	-6.944	7.5	-9.689	-6.572
		NPV			12.281	0.000		11.929	0.000

Annuity Formulae

You may want to use a spreadsheet to actually find the solutions to some of these problems. But make sure to write out the equations you need to solve in your answers.

a. What is the present value of 15 annual payments of $100, with the first payment one year from now, if the discount rate is 0.05?

$NPV=100\Sigma_1^{15}1.05^{-t}=100(1-1.05^{-15})[(1/1.05)/(1-1/(1.05))]=100(1-1.05^{-15})/0.05=1038$

b. What is the present value of 15 annual payments of $100, with the first payment right now, if the discount rate is 0.05?

$NPV=100\Sigma_0^{14}1.05^{-t}=100+100(1-1.05^{-14})/[(1/1.05)/(1-1/(1.05))]=100+100(1-1.05^{-14})/0.05=1090$

c. What is the present value of 15 annual payments of $100, with the first payment five years from now, if the discount rate is 0.05?

This is just the answer from (a) discounted 5 additional years, so $1090/1.05^5=854$

d. At what discount rate would the present value of 15 annual payments of $100, with the first payment right now, be 0?

Take the expression for the answer from a, but insert d instead of 0.05 for the discount rate, set it equal to 0 and solve it for d: $100+100(1-(1+d)^{-14})/d=0$. What you will find is that as $d\to\infty$, so the future does not matter at all, the value approaches $100, the value of the payment received right now. So, even when the future does not matter at all, this is still worth $100, there is no discount rate to make it equal 0.

e. How many annual payments of $100, with the first payment right now, would it take to be worth more than $1,000, if the discount rate is 0.05?

Take the expression for the answer from a, but insert T-1 instead of the year of the last payment, set it equal to 1,000 and solve it for T: $100+100(1-1.05^{-(T-1)})/0.05=1000$. For this one, we know it will be at least 11. Solving gives T=14 payments, at which the NPV is 1039.

f. What is the value of 15 annual payments which begin at $100 one year from now and increase at 2% per year thereafter, if the discount rate is 0.05?

In this case, $\delta=1.02/1.05\approx0.9714$. Using the formula above:

$NPV=(1-(1.02/1.05)^{15})[(1.02/1.05)/(1-(1.02/1.05))]=1199$

Equivalent Annual Net Benefit

Which of the following projects has the larger Equivalent Annual Net Benefit if the interest rate is 4%?

	Time	1	2	3	4	5	6	7	8
Project A	NB	-20	-10	5	10	10	10	10	5
Project B	NB	-30	10	15	15	5			

$NPV_A=-20/1.04-10/1.04^2+5/1.04^3+10/1.04^4+10/1.04^5+10/1.04^6+10/1.04^7+5/1.04^8=11.8917$
$NPV_B=-30/1.04+10/1.04^2+15/1.04^3+15/1.04^4+5/1.04^5=10.6661$

Delta=$1/1.04\approx 0.9615$
$a_A=(1-.9615^8)(0.9615/(1-0.9615))\approx 6.7327$
$a_B=(1-.9615^5)(0.9615/(1-0.9615))\approx 4.4518$

$11.8917\approx 6.7327EANBA$
$EANBA\approx 1.7663$

$10.6661\approx 4.4518EANBB$
$EANBB\approx 2.3959$

Project B has the larger EANB.

It is fine to shift all of this so you start in period 1 not 0. The comparison would be the same though the numbers would be slightly different.

Inflation 1

a. What is the value in today's dollars of $300 in 2024 if average annual compound rate of inflation is 0.025 between now and then? $300/1.025^4=271.79$

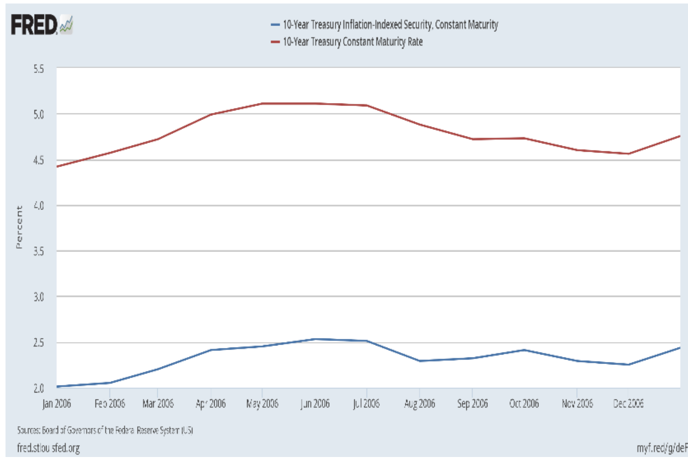
b. What is the value of $200 today expressed in 2030 dollars if the average annual compound rate of inflation is 0.05 between now and then? $200\times 1.05^{10}=325.78$

Inflation 2

a. Go to the Federal Reserve Economic Database (FRED) maintained by the St. Louis Fed at https://fred.stlouisfed.org/ and calculate/estimate/retrieve the 2006 average of the *10-Year Treasury Constant Maturity Rate* and the *10-Year Treasury Inflation-Indexed Security, Constant Maturity*, both series maintained in FRED. Based on these, what was the expected annual rate of inflation from 2006 to 2016?

*Note for grading: It is possible the series have been updated since I pulled the data. So, this should be the approach, but the numbers might be very slightly different.

The figure shows the monthly averages of the two series. The averages of the monthly averages were i=4.8 and r=2.3, so inflation was expected to average about m-4.8-2.3=2.5%. [Note: you could try to be more precise, r=(i-m)/(1+m), 0.023=(0.048-m)/(1+m), which gives m=0.0244, essentially the same thing.]



b. Obtain the Consumer Price Index for 2006 and 2016 (FRED has it, or you can get it from the Bureau of Labor Statistics) and calculate the average annual rate of inflation for those 10 years. (Remember, it gets compounded each year to produce the full difference between the 2016 and 2006 price levels.) What do your calculations suggest about the ability to forecast inflation?

In January 2006 the CPI was 199.3 (base 1984, all urban consumers). In December 216 it was 242.8. The average rate of inflation solves $242.8=199.3(1+\pi)^{16}$, giving $\pi=0.012$, or about 1.2%. Over the past decade, inflation was only half what was expected a decade ago. It is hard to accurately predict inflation. [Note, I pulled these numbers in 2017—they could have been revised since, but any difference would be small.]

Horizon Values - 1

A project involved initial construction costs of \$2.5 million. The annual rate of economic depreciation for that construction class is 0.008. The project is expected to terminate in 25 years. The expected annual rate of inflation is 0.025.

a. Estimate the horizon value at time t=25 owing to the remaining value of this asset.

In today's dollars, the asset will be worth an estimated 0.992^{25}\$2.5M, or \$2.045M. If you want to know what that would be in the futures dollars, multiply by 1.025^{25}, to get \$3.792M.

b. If the real discount rate is 0.035, what is the present value of the horizon value from (a)?

Working with the real discount rate, we want to work with the real salvage value, that is the value in today's dollars, of \$2.045M, and discount it, so NPV=\$2.045M/1.035^{25}=\$0.865M.

Horizon Values - 2

A project involved initial construction costs of \$1.75 million. After 15 years, the useful life of that construction will be over and the facility will be demolished, involving sensitive environmental protections and cleanup. You estimate that 25% of the cost of the facility represents items that could be sold for scrap at 30% of their initial construction cost. You estimate the proper demolition cost of such a facility to be \$0.9M.

a. What is the NPV of the horizon value?

In today's dollars, $H_{15} = -\$(0.9-0.3\times 1.75/4)M = -768.8K$. Since it was not specified, just state your assumption on the discount rate. I will use a real discount rate of 0.035, so the NPV is $-\$768.8K/1.035^{15}=-458.9K$

b. If the expected annual rate of inflation is 0.02, what is the nominal horizon value in 15 years?

$-\$768.8K\times 1.02^{15}=-1.034M$

Horizon Values – 3

A program, if implemented, will operate for 10 years for certain. Your best guess is that after year 10 and following each year thereafter there will be a 0.02 probability the program will end. Real net benefits are \$25/year in year 1 and are expected to grow 1% per year as long as the program is in operation. The real discount rate is 3.5%. What is the NPV of the horizon value of net benefits following year 10?

I should have been a bit more precise in the wording of this question regarding timing. Best given this wording is to assume net benefits are \$25 at time t=0 but that for the first year the program operates, they will be 25×1.01, for the second year 25×1.01^2 and so on. But, if you assumed 25 was the value in one year, and did all calculations correct for that assumption, that is fine too.

This question puts lots of things together to push you to synthesize what we have been studying. The value of net benefits in year 10 is $25\times 1.01^{10}=27.62$. Since the probability of operating in year 11 is 0.98, in year 12 is 0.98^2, and so on, since net benefits grow 1% annually if still operating, and since the discount rate is 3.5%, looking forward from year 10 $H_{10}=25\times 1.01^{10}a(\infty,\delta_{10})$ where $\delta_{10}=0.98\times 1.01/1.035\approx 0.9563$. Thus $H_{10}=25\times 1.01^{10}(0.9563/(1-0.9563))=\$604.73$. Discount this back 10 years to get its present value at time 0, $\$604.73/1.035^{10}=\$428.71$.

Simple ENPV

a. A project costs \$10 up front and has net benefits of \$15 with probability 0.8 at the end of the second year and otherwise returns nothing. The discount rate is 0.035. What is the NPV?

b. At what probability of returning \$15 after year 2 would the ENPV be 0?

$ENPV = -\$10 + 0.8 \times \$15/1.035^2 = \$1.20$

$0 = -\$10 + f \times \$15/1.035^2$ $\qquad f = 1.035^2 \times 2/3$ $\qquad f = 0.71$

A Simple Normal Form Game against Nature

A project faces three potential states of the world with a varying number of users (n): 10, 20, or 80, with probabilities 0.3, 0.5, and 0.2. Three options are available with values per user, v, of \$2, \$5, and \$10, with total costs of 10, 50, and 200, respectively.

a. Set up the normal form representation.

b. Find the expected value of each option. Without additional information, which option is best?

c. What would be the value of perfect information about the state of the world before the decision is made?

d. Suppose for a price the decision maker could purchase information that would reveal, prior to making the decision, whether the number of users will be more than 10 but provide no other insight. What is this information worth?

a) See table.

	Probability		0.3	0.5	0.2	
Option	n		10	20	80	ENPV
	v	Cost	Net Benefits			
A	2	10	**10**	30	150	48
B	5	50	0	**50**	350	**95**
C	10	200	-100	0	**600**	90

b) Without knowledge of the state of the world, option B, with ENPV of 95, is best.

c) With knowledge of the state of the world, the best choices would be A, B, and C if n were 10, 20, or 80. So, obtaining perfect information before choosing would make the expected value $0.3 \times 10 + 0.5 \times 50 + 0.2 \times 600 = 148$. The value of perfect information is therefore $148 - 95 = 53$.

d) If the state of the world is known to be n=10, A is chosen. Otherwise:

$$E(V|B,N>10) = (0.5/0.7)50 + (0.2/0.7)350 = 136$$

$$E(V|C,N>10) = (0.5/0.7)0 + (0.2/0.7)600 = 171$$

Thus C would be chosen. The expected value is then

$$E(V|Info) = 0.3 \times 10 + 0.7 \times 171 = 123.$$

The value of this information is then $123 - 95 = 28$.

A More Complex Normal Form Game against Nature

A project faces uncertainty about the number of users of the service to be provided. It might be 10, 20, or 40 with probabilities 0.3, 0.5, and 0.2, respectively. There are two options, A and B. Under option A, the value per user, v, is \$10. Under option B, the value per user is \$5 with probability 0.4 and otherwise \$20. Assume the number of users and the value per user under option A are independent. Option A costs \$100 and option B costs \$250.

a. Set up the normal form representation.

b. What are the expected values for each option?

c. What is the value of perfect information before the decision is made?

d. Suppose the decision maker could, for a price, determine whether value per user would be 5 or 20 under option B, though they would learn nothing about the number of users. What is the value of that information?

a) See the table. b) See table.

Option	n v	10 5	20 5	40 5	10 10	20 10	40 10	10 20	20 20	40 20	EV
A	P(n,v)	0	0	0	0.3	0.5	0.2	0	0	0	
	NB				0	100	300				110
B	P(n,v)	0.12	0.2	0.08	0	0	0	0.18	0.3	0.12	
	NB	-200	-150	-50				-50	150	550	94

c) If demand is known to be 10, either option A or simply doing nothing is best (both have a payoff of 0). If demand is known to be 20 or 40, A is best if v would be 5 with option B and B is best if v would be 20. So

$E(V|Info)=0.3\times0+0.5(0.4\times100+0.6\times150)+0.2(0.4\times300+0.6\times550)=155.$

The value of this info is then 155-110-45.

d) If they learn v=5 under B, they still choose A. But, if they learn v=20 under B:

$E(V|B)=0.3\times0+0.5\times150+0.2\times550=185$

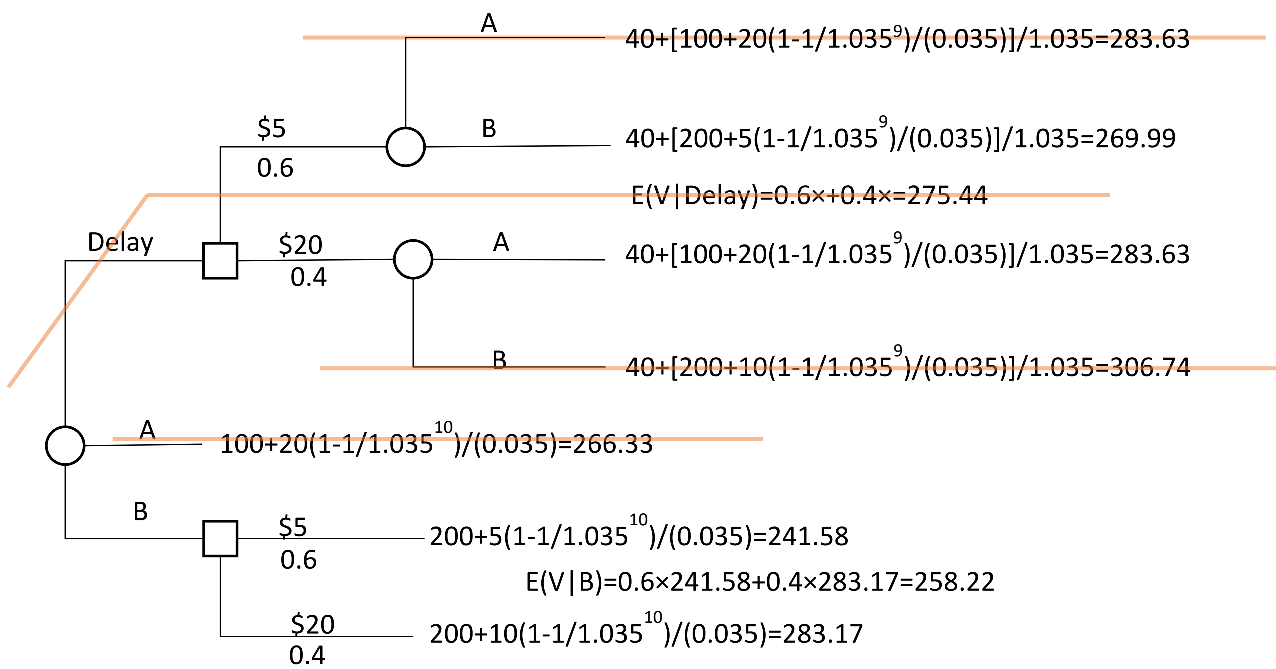
The value of the information is then 185-110=75.

An Extensive Form Game against Nature

The sheriff's department is upgrading its information management system. They have a choice between system A and system B. System A will cost them \$100 up front and \$20 per year over 10 years. System B will cost \$200 up front and then each year for 10 years will cost \$5 with probability 0.6 or else \$20. Alternatively, they could pay \$40 to keep their current system working one more year, at which time it would be known whether the annual cost of system B is \$5 or \$20. If they delay one year, the up-front cost would still be the same, but they would only get 9 years of service from the new system.

a) Draw the extensive form game against nature.
b) What should they do at what time?

a) See figure.



b) Choosing B immediately has the lowest expected cost.

Another Extensive Form Game against Nature

A city is in the running for a major sports franchise, with a 30% chance (officials think) of being selected. They will not know for 1 year. A road not far from the stadium cite is in need of repair following flooding from ruptured water pipelines. The road was in need of upgrades anyway, and will need improvement more urgently if the franchise is awarded.

- Benefits accrue for 20 years beginning one year from incurring the cost of improvements.
- The horizon value is 0.
- The real discount rate is 0.035.
- The cost of a minor upgrade is $10M and benefits are $4M/year if the franchise has not been awarded. If and after the franchise is awarded, the cost is $15M (more traffic disruption) and benefits are $8M/year.
- The cost of a major upgrade is $20M and benefits are $6M/year if the franchise has not yet been awarded. If and after the franchise is awarded, the cost is $30M (more traffic disruption) and benefits are $10M/year.
- The city could implement minimum necessary repairs for $5M now and delay deciding on improvements. Benefits are $0/year. (This restores the status quo level of service.)
- The city could undertake the minor improvement now and the major improvement later.

a) Set up the extensive form.

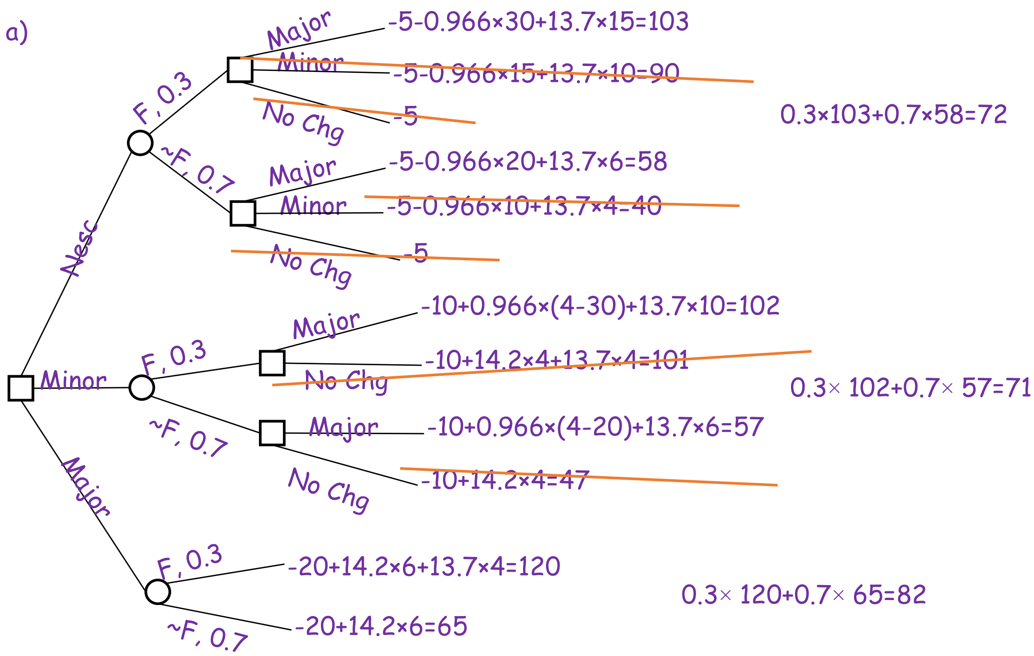
b) Find the solution with the highest ENPV.

c) What are the best and worst case scenarios for the option with the highest ENPV?

d) What is the value of perfect information on whether the franchise will be awarded?

For a flow of benefits starting in one year and continuing 20 years at a discount rate of 0.035, the annuity factor is 14.2. If the benefits only start to flow two years from the present, you have to discount for another year, bringing 14.2 down to 13.7. With that, the decision tree, or extensive form game, is shown below.

a)



b) In this case, the long lifespan make the major improvement the dominant strategy—that is no matter where you are in the tree, you want to do the major upgrade when you get the chance. So, go major right away.

c) In this case, there is only one source of uncertainty, so given the choice to do the major upgrade at the outset, the worst case is $65M, the best is $120M, and the expected value is $82M.

d) Even perfect information has no impact at all, and therefore no value, because the best decision is to build no matter what.