5.2 Two Sample T-Distribution

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Dependent Means T-Test

Paired Data

1) use sample as its own control
2) use a control sample

The same dependent variable is measured twice on the same subjects OR on related subjects

Based on the difference score D

↳ $D_{k} = X_{k=2} - X_{k=1}$

Compare the mean difference score to θ given standard error of the mean of the difference score

↳ $t_D = \bar{D} - \theta / (s_D / \sqrt{n})$

$\bar{D}$ = mean of the difference scores
θ = the expected mean under H_0
s_D = std dev of difference scores
n = sample size

For repeated measures, $df = n-1$
For matched samples, df = [number of pairs] - 1

$CI = \bar{D} \pm t \cdot s_{\bar{D}}$

Independent Samples T-Test (difference of 2 means)

- Two separate groups, no interdependence
  ↳ experimental & control
- Same dependent variable measured
- Independence of observations
- Group level

Variance Sum Law: the variance of a sum or difference of two independent variables is equal to the sum of their variances

↳ $\sigma^2_{\bar{x}_1+\bar{x}_2} = \sigma^2_{\bar{x}_1-\bar{x}_2} = \sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_2} = (\sigma_1^2/n_1) + (\sigma_2^2/n_2)$

$t = (\bar{X}_1 - \bar{X}_2) / s(\bar{X}_1 - \bar{X}_2) \approx (\bar{X}_1 - \bar{X}_2) / \sqrt{(s_1^2/n_1) + (s_2^2/n_2)} = t$

$df = n_1 + n_2 - 2$

What if sample sizes are not equivalent?

↳ Pooled variance: $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

↳ s_p = pooled variance

↳ $t = (\bar{x}_1 - \bar{x}_2) \Big/ \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} \cdot \left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}$

Example:

$H_1: \mu_{threat} \neq \mu_{no\ threat}$ $\quad H_0: \mu_{threat} = \mu_{no\ threat}$

Threat: $n_1 = 12$ $\quad \bar{x}_1 = 6.58$ $\quad s_1 = 3.17$
no threat: $n_2 = 11$ $\quad \bar{x}_2 = 9.64$ $\quad s_2 = 3.03$

$s_p^2 = [(12-1)(3.17)^2 + (11-1)(3.03)^2] / (12+11-2) = 9.594^2$

Standard error of the mean difference:

↳ $se_{\bar{x}_1-\bar{x}_2} = \sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}$

$t = (\bar{x}_1 - \bar{x}_2) / se_{\bar{x}_1-\bar{x}_2}$

$CI = (\bar{x}_1 - \bar{x}_2) \pm t_{critical} \cdot s_{(\bar{x}_1-\bar{x}_2)}$

Example cont:

$t = (9.64 - 6.58) \Big/ \sqrt{\dfrac{3.17^2}{11} + \dfrac{3.03^2}{12}} = 2.37$