

Passed Solution review

13. Consider the following three-player team production problem. Simultaneously and independently, each player chooses between exerting effort (E) or not exerting effort (N). Exerting effort imposes a cost of 2 on the player who exerts effort. If two or more of the players exert effort, each player receives a benefit of 4 regardless of whether she herself exerted effort. Otherwise, each player receives zero benefit. The payoff to each player is her realized benefit less the cost of her effort (if she exerted effort). For instance, if player 1 selects N and players 2 and 3 both select E, then the payoff vector is $(4, 2, 2)$. If player 1 selects E and players 2 and 3 both select N, then the payoff vector is $(-2, 0, 0)$.

(a) Is there a pure-strategy equilibrium in which all three players exert effort? Explain why or why not.

$(0, 0, 0) = (N, N, N)$ is the pure NE. I'm having a hard time with words, but it makes sense given first problems

No. Any player could unilaterally improve from $U = 4 - 2$ to $U = 4 - 0$ by not exerting effort.

I'm dumb. This makes so much sense. There's no incentive for the third person to exert effort if they get 4 no matter what

(b) Find a symmetric mixed-strategy Nash equilibrium of this game. Let p denote the probability that an individual player selects N.

I'm honestly not sure and don't have the group to walk this through. I think it's $(1/3, 1/3, 1/3)$

~~$$U = 4 - 2p$$
$$p = 1/3$$~~

$$U_i(E) = U_i(N)$$

$$4(1-p^2) + 0p^2 - 2 = 4(1-p)^2 + 0(1-(1-p)^2)$$

$$2 - 4p^2 = 4(1-p)^2$$

$$2 - 4p^2 = 4(1 - 2p + p^2)$$

$$2 - 4p^2 = 4 - 8p + 4p^2$$

$$-4p^2 - 4p^2 + 8p + 2 - 4 = 0$$

$$-8p^2 + 8p - 2 = 0$$

$$4p(1-p) = 1$$

$$p = 1/2$$

Duh. Just needed to set equal then plug and chug