Passed Solution review

5. Consider a differentiated duopoly market in which firms compete by selecting prices and produce to fill orders. Let p_1 be the price chosen by firm 1 and let p_2 be the price of firm 2. Let q_1 and q_2 denote the quantities demanded (and produced) by the two firms. Suppose that the demand for firm 1 is given by $q_1 = 22 - 2p_1 + p_2$, and the demand for firm 2 is given by $q_2 = 22 - 2p_2 + p_1$. Firm 1 produces at a constant marginal cost of 10 and no fixed cost. Firm 2 produces at a constant marginal cost of c and no fixed cost. The payoffs are the firms' individual profits.

(a) The firms' strategies are their prices. Represent the normal form by writing the firms' payoff functions.

$$\pi = (P - c)q$$

$$U_1 = (P_1 - 10)(22 - 2P_1 + P_2) = 42P_1 + P_1 P_2 - 2P_1^2 - 220 - 10P_2$$
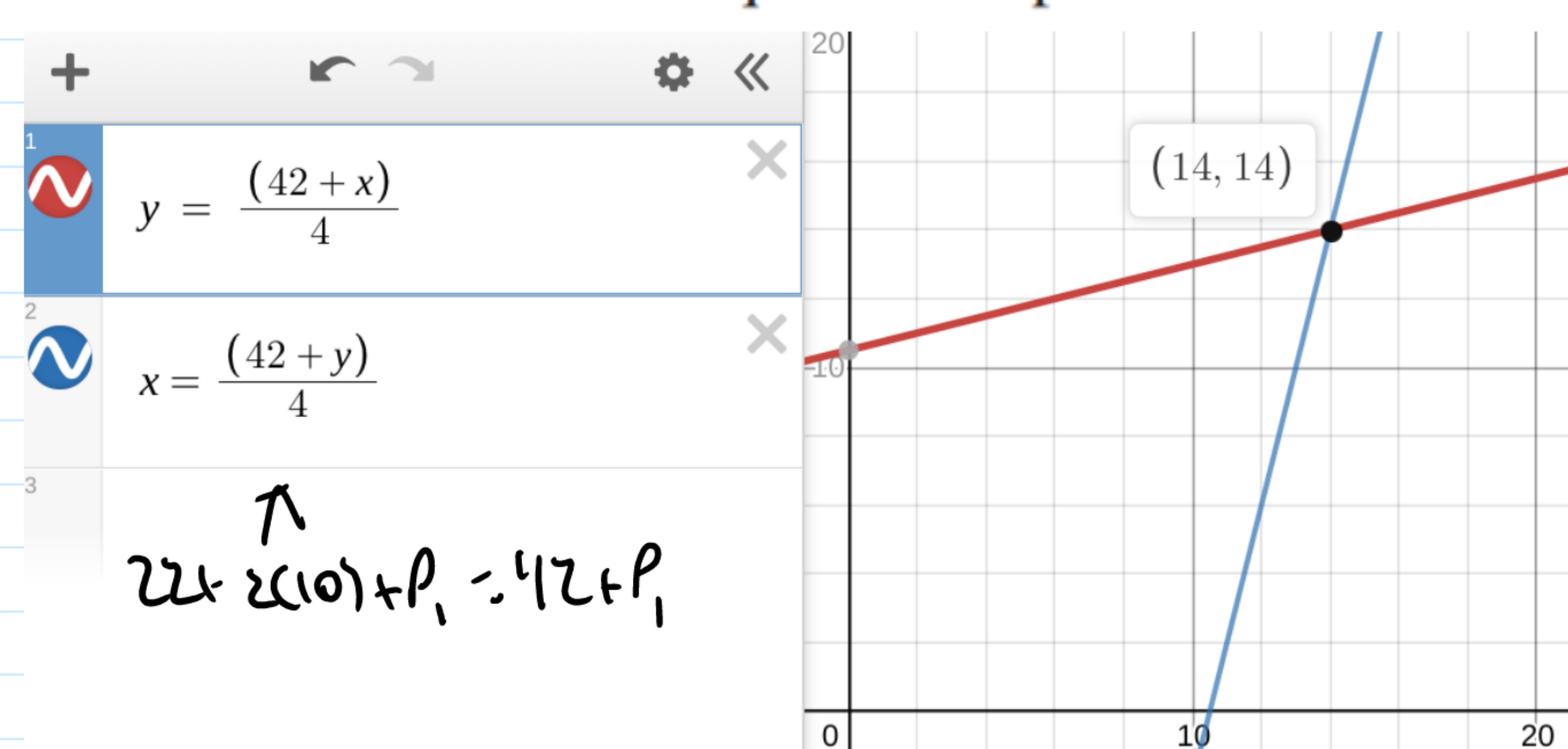$$U_2 = (P_2 - c)(22 - 2P_2 + P_1) = 22P_2 - 2P_2^2 + P_1 P_2 - 22c + 2P_2 c - P_1 c$$

(b) Calculate the firms' best-response functions.

$$BR_1(P_2) = 42 + P_2 / 4$$
$$BR_2(P_1) = 22 + 2c + P_1 / 4$$

(c) Suppose that $c = 10$ so the firms are identical (the game is symmetric). Calculate the Nash equilibrium prices.

$$y = \frac{(42 + x)}{4}$$
$$x = \frac{(42 + y)}{4}$$

$$22 + 2(10) + P_1 = 42 + P_1$$

(d) Now suppose that firm 1 does not know firm 2's marginal cost c. With probability $1/2$ nature picks $c = 14$, and with probability $1/2$ nature picks $c = 6$. Firm 2 knows its own cost (that is, it observes nature's move), but firm 1 only knows that firm 2's marginal cost is either 6 or 14 (with equal probabilities). Calculate the best-response functions of player 1 and the two types ($c = 6$ and $c = 14$) of player 2 and calculate the Bayesian Nash equilibrium quantities.

$$\tfrac{1}{2}q_6 H + \tfrac{1}{2}q_2 L$$

3 players: $P_1 \quad P_{2L} \quad P_{2H}$

where 3 lines meet

$BR_{P_1} =$ Plane, not line

$$U_1 = (P_1 - 10)(22 - P_1 - \tfrac{1}{2}P_{26} - \tfrac{1}{2}P_{214})$$
Solve: $\begin{cases} P_1 = 10.5 + P_{26}/8 + P_{214}/8 \\ P_{26} = 8.5 + P_1/4 \\ P_{214} = 12.5 + P_1/4 \end{cases}$

$\rightarrow P_1 = 14 \quad P_{26} = 12 \quad P_{214} = 16$

$$U_1 = (P_1 - 10)(22 - 2P_1 + P_2) = 42P_1 + P_1 P_2 - 2P_1^2 - 220 - 10P_2$$
$$U_{2L} = (P_2 - 6)(22 - 2P_2 + P_1) = -2P_2^2 + P_1 P_2 + 34P_2 - 6P_1 - 132$$
$$U_{2H} = (P_2 - 14)(22 - 2P_2 + P_1) = -2P_2^2 + P_1 P_2 + 50P_2 - 14P_1 - 308$$

$$BR_1 = 42 + P_2 / 4$$
$$BR_{2L} = -4P_2 + P_1 + 34$$
$$BR_{2H} = -4P_2 + P_1 + 50$$

$$y = \frac{(42 + x)}{4}$$
$$x = -4y - x + 34$$
$$x = -4y + x + 50$$