22.3 Aside on Annuities and Perpetuities

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Paid V@ $t=1, 2, 3, \ldots, T$

Perpetuity V@ $t=1, 2, \ldots, \infty$

$V_A = \delta V + \delta^2 V + \ldots + \delta^T V = v(\delta + \delta^2 + \ldots \delta^T)$

$V_A = v a(\delta, T)$

$a(\delta, T) = \delta + \delta^2 + \ldots + \delta^T$

$a - \delta a = \delta - \delta^{T+1} = \delta(1 - \delta^T)$

$a(1-\delta) = \delta(1-\delta)^T$

$a = (\delta / 1-\delta)(1-\delta)^T$

$0 < \delta < 1 \quad T \to \infty, \delta^T \to 0 \to a \to \delta / 1-\delta$

$V_{Perpetuity} = \delta / 1-\delta$

NPV of $V + \delta V + \delta^2 V + \ldots = V/(1-\delta)$

↑ Payment now

↳ Perpetuity starts at t=1