Probability

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Probability

- Chapter 3 from Textbook

Random Variables

- A function that assigns a numerical value to each possible outcome
- A numeric quantity whose value depends on the outcome of a random event.
  - Denoted using a Capital letter like X.
  - Values of random variable are denoted in lower case i.e. x
  - For example P(X=x)

Random Variables

- Discrete Random Variables
  - Takes integer values only
  - # of credit hours
  - Outcomes of a die roll
- Continuous Random Variables
  - Takes real values
  - Cost of books
  - Tuition cost
  - Speed of car
  - Amount of alcohol in a person’s blood

Expectation of a Discrete Random Variable

- Average outcome of a random variable
- Called the expected value (mean)
- Weighted average of all possible outcomes

$$\mu = E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$$

Expectation of a Discrete Random Variable

- The expected value of a constant is the constant itself

$$E(c) = c$$

- The expected value of constant times random variable is the product of constant and the expected value of the random variable

$$E(cX) = cE(X)$$

- The expected value of random variable plus constant is the sum of expected value of the random variable plus the constant value

$$E(X + c) = E(X) + c$$

Variance of a Discrete Random Variable

$$\sigma^2 = Var(X) = \sum_{i=1}^{k}(x_i - E(X))^2 P(X = x_i)$$

$$\sigma = SD(X) = \sqrt{Var(X)}$$

Variance of a Discrete Random Variable

- The variance of a constant is the constant itself

$$\mathrm{Var}(c) = c$$

- The variance of constant times the random variable is the product of square of the constant and the variance of the random variable variable.

$$\mathrm{Var}(cX) = c^2\mathrm{Var}(X)$$

Expectation of a Discrete Random Variable

- Example

  In a game of cards you win \$1 if you draw a heart (not an ace), \$5 if you draw an ace (including the ace of hearts), \$10 if you draw the king of spades and nothing for any other card you draw. Write the probability model for your winnings and calculate your expected winning.

  Hint: always helps with these problems to set up a table…

Expectation of a Discrete Random Variable

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Hint: always helps with these problems to set up a table…

Event	X	$P(X)$	$X\,P(X)$
Heart (not ace)	1	$\frac{12}{52}$	$\frac{12}{52}$
Ace	5	$\frac{4}{52}$	$\frac{20}{52}$
King of spades	10	$\frac{1}{52}$	$\frac{10}{52}$
All else	0	$\frac{35}{52}$	0
Total			$E(X) = \frac{42}{52} \approx 0.81$

Variance of a Discrete Random Variable

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X	$P(X)$	$X\,P(X)$	$(X - E(X))^2$	$P(X)\,(X - E(X))^2$
1	$\frac{12}{52}$	$1 \times \frac{12}{52} = \frac{12}{52}$	$(1 - 0.81)^2 = 0.0361$	$\frac{12}{52} \times 0.0361 = 0.0083$
5	$\frac{4}{52}$	$5 \times \frac{4}{52} = \frac{20}{52}$	$(5 - 0.81)^2 = 17.5561$	$\frac{4}{52} \times 17.5561 = 1.3505$
10	$\frac{1}{52}$	$10 \times \frac{1}{52} = \frac{10}{52}$	$(10 - 0.81)^2 = 84.4561$	$\frac{1}{52} \times 84.0889 = 1.6242$
0	$\frac{35}{52}$	$0 \times \frac{35}{52} = 0$	$(0 - 0.81)^2 = 0.6561$	$\frac{35}{52} \times 0.6561 = 0.4416$
		$E(X) = 0.81$		$V(X) = 3.4246$ $SD(X) = \sqrt{3.4246} = 1.85$

Linear Combinations of Random Variables

- If X and Y are random variables and their linear combination is given by $Z = aX + bY$

$$E(Z) = a*E(X) + b*E(Y)$$

$$Var(Z) = a^2*Var(X) + b^2*Var(Y) \text{ only if X and Y are independent}$$

Example

- A contractor has 3 independent jobs. The average job takes 36 man hours (H). The standard deviation is 3 man hours. What are the mean and standard deviation of total time?

Examples of Discrete Distributions

- Bernoulli
- Binomial
- Geometric
- Negative Binomial
- Poisson
- Hyper-geometric etc.

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Bernoulli Distribution

If an experiment satisfies the following conditions:

- It consists of a sequence of 1 trial.
- The trial results in one of two possible outcomes
  - success (*S*)
  - failure (*F*).
- probability of success is p and
- Probability of failure is 1-p=q

- If X denotes success or failure

$$P(X = x) = p^{x}\,(1-p)^{1-x}, \quad x = 0 \text{ or } 1$$

Characteristics of Bernoulli Distribution

- Mean $\mu = p$
- Variance = $\sigma^2 = p(1-p)$

$$\mu = E(X) = \sum xp(x) = (1)\,p(1) + (0)\,p(0) = p$$

$$\sigma^2 = E(X^2) - \mu^2 = \sum x^2 p(x) - \mu^2 = (1)^2 p(1) + (0)^2 p(0) - \mu^2$$
$$= p(1) - \mu^2 = p - p^2 = p(1 - p)$$

Binomial Distribution

- If an experiment satisfies the following conditions:
  - Consists of a sequence of **n** trials.
  - The trials are identical and **independent**
  - The trials can result in one of the same two possible outcomes
    - Success (S) or Failure (F)
  - The probability of success is **constant** from trial to trial and is denoted by p.

$$P(X = x) = p(x) = \binom{n}{x} p^{x} (1 - p)^{n - x}, \ x = 0,1,2,\ldots,n.$$

$$\binom{n}{x} = \frac{n!}{x!(n - x)!}$$

x = the number of *successes* among n trials

Properties of Binomial Distribution

Mean

$$\mu = E(x) = np$$

Variance

$$\sigma^2 = np(1 - p)$$

Standard Deviation

$$\sigma = \sqrt{np(1 - p)}$$

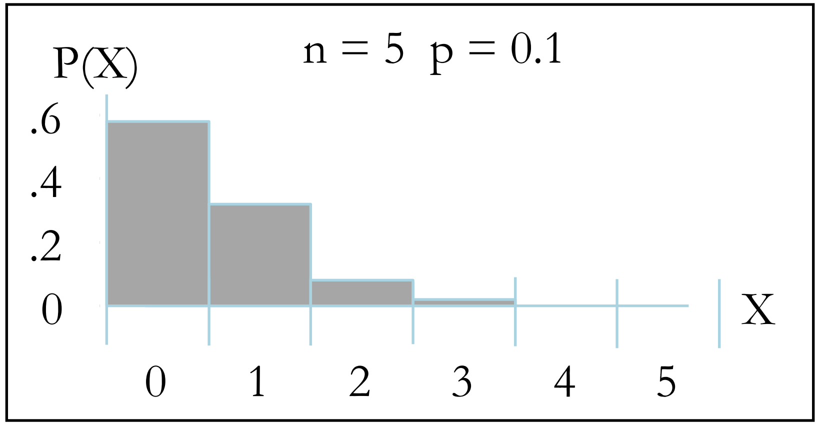
n = number of trials or sample size

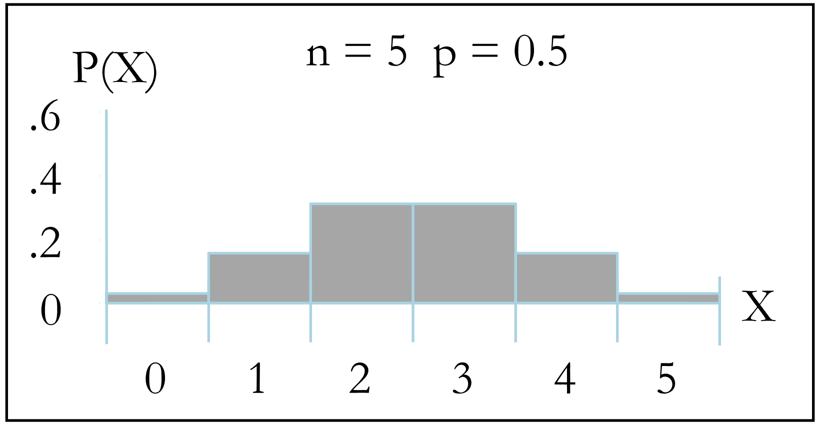
p = probability of success

(1 – p) = probability of failure

Binomial Distribution

- Values of p and n determine the shape of the binomial distribution





Example

What is the probability of one success in five observations if the probability of success is .1?

$X = 1$, $n = 5$, and $p = 0.1$

$$P(X = 1) = ?$$

Example

If the probability of a student successfully passing a course is 0.82, find the probability that given 8 students

a. all 8 pass.

b. none pass.

c. at least 6 pass

Example

If the probability of a student successfully passing a course is 0.82, find the probability that given 8 students

a. all 8 pass. $\binom{8}{8}(0.82)^8(0.18)^0$ $\boxed{\approx 0.2044}$

b. none pass. $\binom{8}{0}(0.82)^0(0.18)^8$ $\boxed{\approx 0.0000011}$

c. at least 6 pass

$$\binom{8}{6}(0.82)^6(0.18)^2+\binom{8}{7}(0.82)^7(0.18)^1+\binom{8}{8}(0.82)^8(0.18)^0$$

$$\approx 0.2758+0.3590+0.2044 \qquad \boxed{=0.8392}$$

Example

A multiple choice test consists of 7 questions with five choices each. If a student guesses on all questions, what is the probability that a student gets exactly 3 correct answers ?

Example

An oil drilling company ventures into various locations, and their success or failure is independent from one location to another. Suppose the probability of a success at any specific location is 0.1. If a driller drills 5 locations, find the probability that there will be at least one successes.

Let X be the number of successful drilling of oils in 5 location…

Geometric Distribution

- Probability of success is p. If X is the number of trials until the first success occurs, then X is called a geometric random variable.

- The probability of finding first success in the xth trial

$$p(x) = (1 - p)^{x-1} p \quad (x = 1, 2.....)$$

$$X = \text{Number of trials the first success is observed}$$

- The first (x-1) trials should all result in failures and the x^{th} trial should result in success.

Geometric Distribution Example

Let X = the number of rolls until we get the first 6.

Possible values of X:{1, 2, 3, …}

The probability distribution of X is given by:

$$P(X = x) = \left(\frac{5}{6}\right)^{x-1}\left(\frac{1}{6}\right), \text{ for } \quad x = 1, 2, \ldots$$

Properties of the Geometric Distribution

Mean

$$\mu = \frac{1}{p}$$

Standard deviation

$$\sigma^2 = \frac{1-p}{p^2}$$

Example

A manufacturer uses electrical fuses in an electronic system. The fuses are purchased in large lots and tested sequentially until the first defective fuse is observed. Assume that the lot contains 10% defective fuses. What is the probability that the

- First defective fuse is observed on the first test?
- First defective fuse is observed on the second test?
- First defective fuse is observed on the third test?

Example

A manufacturer uses electrical fuses in an electronic system. The fuses are purchased in large lots and tested sequentially until the first defective fuse is observed. Assume that the lot contains 10% defective fuses. What is the probability that the

- First defective fuse is observed on the first test?

  0.1

- First defective fuse is observed on the second test?

  $(0.9)(0.1)$

- First defective fuse is observed on the third test?

  $(0.9)(0.9)(0.1)=(0.9)^2(0.1)$

Example

Data collected by the Substance Abuse and Mental Health Services Administration (SAMSHA) suggests that 69.7% of 18-20 year olds consumed alcoholic beverages in 2008.

- Suppose a random sample of 10, 18-20 year olds is taken. Is the use of binomial distribution appropriate for calculating the probability that exactly six consumed alcoholic beverages?
- Calculate the probability that exactly 6 out of 10 randomly sampled 18-20 year olds consumed an alcoholic beverage?
- What is the probability that at most 2 out of 5 randomly sampled 18-20 year olds have consumed alcoholic beverages?
- What is the probability that at least 3 out of 5 randomly sampled 18-20 year olds have consumed alcoholic beverages?
- How many people would you expect to have consumed alcoholic beverages if you randomly sampled 10 18-20 year olds? What is the standard deviation?

Example

In a multiple choice quiz there are 5 questions and 4 choices for each question. Robin has not studied for the quiz at all and decides to randomly guess the answers. Find the following probabilities

- The first questions she gets right is the 3rd question
- She gets exactly 3 or exactly 4 questions right
- She gets the majority of the questions right

Example

There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next 2 bits transmitted. $P(X=0) = 0.5$, $P(X=1) = 0.3$, $P(X=2) = 0.2$. Please calculate $E(X)$ and $Var(X)$.