Problem 1

In a 30 day month – 4 weeks + 2 days

The two days can be any of the following combinations

1. Sun-Mon
2. Mon-Tue
3. Tue-Wed
4. Wed-Thu
5. Thu-Fri
6. Fri-Sat
7. Sat-Sun

Probability of getting a 5_{th} Tuesday = number of outcomes / Total number of possible outcomes = 2/7

Problem 2

Rolling a six sided dice 3 times

Concept of independence applies here. The outcome on the first roll does not influence the outcome on the second roll. Hence multiplication is the way to go

Probability of rolling 6, 6, 6 $= \frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \frac{1}{216}$

Probability of rolling 2, 3, 3 = $\frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \frac{1}{216}$

Probability of rolling all evens

Possible even numbers are 2, 4, 6. Since these are disjoint outcomes you add the individual probabilities

Probability of rolling a 2 or 4 or 6 = $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

Probability of rolling all evens = $\frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$

Problem 3

1.1 What is the probability a defendant randomly selected from among those convicted in their first trial was convicted in their second trial?

		Convicted at 1st Trial		
		Yes	No	Total
Convicted at 2nd Trial	Yes	0.4	0.3	0.7
	No	0.1	0.2	0.3
	Total	0.5	0.5	1.0

P(C2=Yes|C1=Yes)=0.4/0.5=0.8.

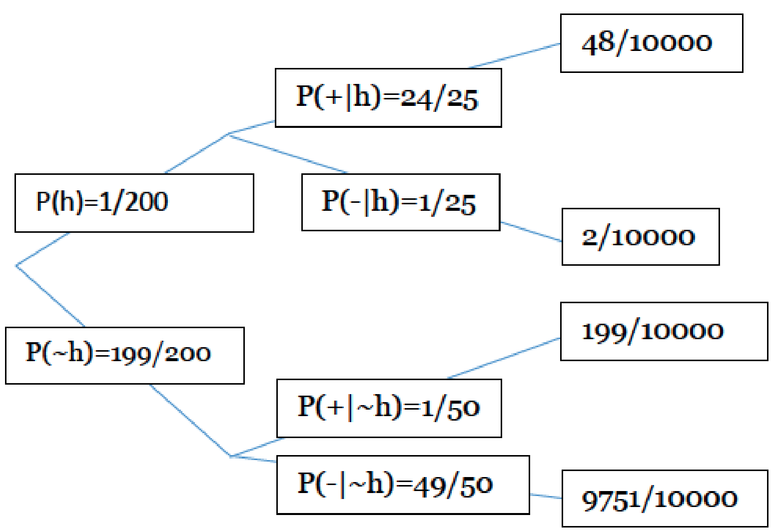
1.2 Is the outcome of the second trial independent of the outcome of the first? Explain.

P(C2=Yes|C1=No)=0.3/0.5=0.6; P(C2=Yes)=0.7.

No, those convicted at the first trial are more likely to be convicted at the second trial than those not convicted at their first trial.

Problem 4

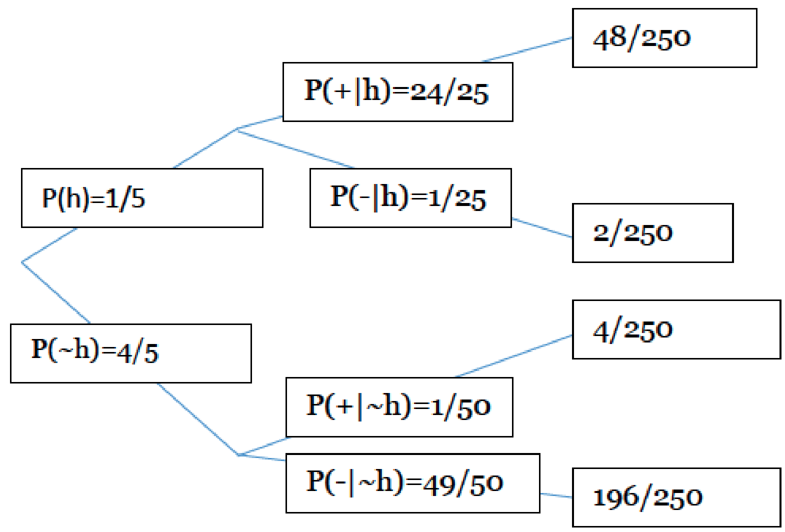
h means the student has the problem, + means they screen positive for it, - means they do not.



P(h|-)=2/9753=0.0002

2.3 Based on these two answers, do you see any problem? If so, can you think of a reasonable solution? If so, what? After implementing your solution, what is the probability you will have incorrectly concluded a student has the problem when they in fact do not?

Four in five students that screen positive do not have the problem, so the screening will not do a good job of identifying the group at which to target the intervention. It would be reasonable to perform additional testing on the group that tests positive to eliminate the false positives. If we assume one false positive does not make another more likely for a given student, we could re-administer the screening to those who tested positive the first time. Doing so is represented in the figure below.



P(h|++)=48/52=0.92; P(h|+-)=196/198=0.99

After a second round, we are reasonably sure those that tested positive the first round but negative the second do not have the problem (0.99), and also that those that tested positive both times do have the problem (0.92).

Problem 5

of your winnings.

Outcome	p	w	p*w	$(w-\mu)^2$	$p(w-\mu)^2$
G>S,G<B	0.33	1	0.33	0.11	0.037
G<S,G>B	0.17	2	0.33	1.78	0.296
G>S,G>B	0.17	6	1.00	28.44	4.741
G<S,G<B	0.33	-3	-1.00	13.44	4.481
		E(w)	0.67	VAR(w)	9.556
				SD(w)	3.091

3.2. How can both you and your friend think this bet is a good deal?

The two have different beliefs about the probabilities.

Problem 6
(a) No, 0.18 of respondents fall into this combination.
(b) P(earth is warming or liberal Democrat) =
= P(earth is warming) + P(liberal Democrat) - P(earth is warming and liberal Democrat)
= 0.60 + 0.20 - 0.18 = 0.62
(c) P(earth is warming j liberal Democrat) = 0:18 - 0:20 = 0:9
(d) P(earth is warming j conservative Republican) = 0:11/0:33 = 0:33
(e) No, the two appear to be dependent. The percentages of conservative Republicans and liberal Democrats who believe that there is solid evidence that the average temperature on earth has been getting warmer over the past few decades are very different.
(f) P(moderate/liberal Republican j not warming) = 0:06/0:34 = 0:18

Problem 7

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

1. Probability of a randomly selected individual is both male and graduate. From the table, 19 students are both male and graduates. Probability (male and graduate) = 19/100. This is joint probability

2. Probability that randomly selected individual is male =
$$\frac{total\,number\,of\,males}{total\,number\,of\,individuals} = \frac{60}{100} = \frac{3}{5}$$

3. Probability that randomly selected individual is graduate =
$$\frac{total\,number\,of\,graduates}{total\,number\,of\,individuals} = \frac{31}{100}$$

4. Probability that a randomly selected person is a female given that the selected person is a post graduate – conditional probability

$$P(female \mid post\,graduate) = \frac{P(female\,and\,post\,graduate)}{P(post\,graduate)} = \frac{\frac{28}{100}}{\frac{69}{100}} = \frac{28}{69}$$

Problems 8 – 10

4. A factory has 22 identical machines. The expected number of break downs for each machine is 1.8 per year, with a standard deviation of 1.2.

4.1. What are the mean and standard deviation of the total number of breakdowns each year?

$\mu=22(1.8)=39.6$; $\sigma^2=22(1.2^2)=31.7$, $\sigma^2=22(1.2^2)=31.7$; $\sigma=5.6$

4.2. If each repair costs \$1,000, what are the mean and standard deviation of total annual repair costs?

$\mu=22(1000*1.8)=\$39,600$; $\sigma^2=22(1000^2*1.2^2)=31.7M\$^2$; $\sigma=\$5,629$

5. The population mean score on a statistics exam is 72, with a standard deviation of 12. The population average score on the class project is 95, with a standard deviation of 4. If the exam is 70% of the final grade, and the project 30%, what are the mean and standard deviation of the distribution of final grades?

$\mu=0.7*72+0.3*95=$; $\sigma^2=0.7^2*12^2+0.3^2*4^2=72$; $\sigma=8.5$

6. The population mean score for a particular exam is 74 and the standard deviation is 9. What are the mean and standard deviation of the class average score for classes composed of 36 students randomly drawn from the population?

$$\mu_{\bar{x}}=\sum_{i=1}^{36}\frac{\mu}{36}=36\times\frac{74}{36}=74$$

$$\sigma_{\bar{x}}^2=\sum_{i=1}^{36}\left(\frac{1}{36}\right)^2\sigma^2=36\left(\frac{1}{36}\right)^2 81=\frac{81}{36},\ \sigma_{\bar{x}}=\sqrt{\frac{81}{36}}=\frac{9}{6}=1.5$$