HW 8.8

Sunday, September 27, 2020 3:30 PM

Passed solution review

**8.** Finish the analysis of the "social unrest" model by showing that for any $\alpha > 2$, the only rationalizable strategy profile is for all players to protest. Here is a helpful general step: Suppose that is it common knowledge that all players above $y$ will protest, so $x \geq 1 - y$. Find a cutoff player number $f(y)$ with the property that given $x \geq 1 - y$, every player above $f(y)$ strictly prefers to protest.

stay home: $u_i(H, x) = 4x - 2$

protest: $u_i(P, x) = \gamma x - 4 + \alpha_i$

$\gamma \in (\gamma, 1]$

$\gamma(1-\gamma) - 4 + \alpha_i \geq 4(1-\gamma) - 2$

$\gamma - \gamma\gamma - 4 + \alpha_i \geq 4 - 4\gamma - 2$

$4 - \gamma\gamma + \alpha_i \geq 2 - 4\gamma$

$2 - \gamma\gamma + \alpha_i \geq -4\gamma$

$2 + \alpha_i \geq 4\gamma$

$\alpha_i \geq 4\gamma - 2$

$\gamma \geq (4\gamma - 2)/\alpha$

if $\alpha = 2$ and $\gamma \geq 1$, people will protest.

$i$ protests if $\gamma x - 4 + \alpha_i \geq 4x - 2$

$\alpha_i \geq 2 - 4x$

Suppose $\epsilon(x) = \bar{x} = 0$

This becomes $i \geq 2/\alpha$

if $\alpha > 2$ $2/\alpha < 1$ so someone will protest even if $\bar{x} = 0$

therefore $\bar{x} = 0$ is not rationalizable

suppose $\bar{x} = 1 - 2/\alpha$ which is the least rationalizable belief $\sim R^1$

now $i$ protests if $\alpha_i \geq 2 - 4(1 - 2/\alpha)$

$\alpha_i \geq -2 + 8/\alpha$

$i \geq -2/\alpha + 8/\alpha^2$

the least rationalizable belief is now

$\bar{x} = 1 + 2/\alpha - 8/\alpha^2$

$= 1 - 2/\alpha(4/\alpha - 1) > 1 - 2/\alpha$

each round $x$ grows until

$\alpha(1 - x) = 2 - 4x$

$\therefore x = \alpha - 2/\alpha - 4 > 1 \rightarrow$ so $x = 1$