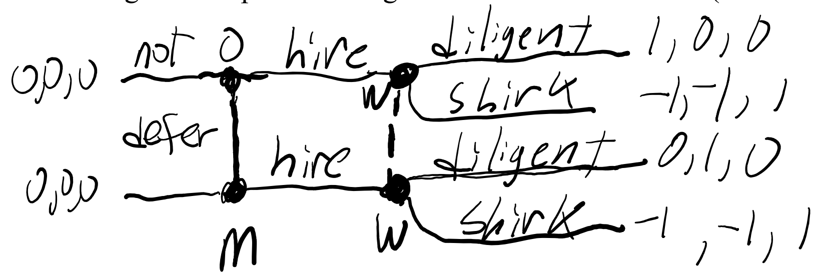
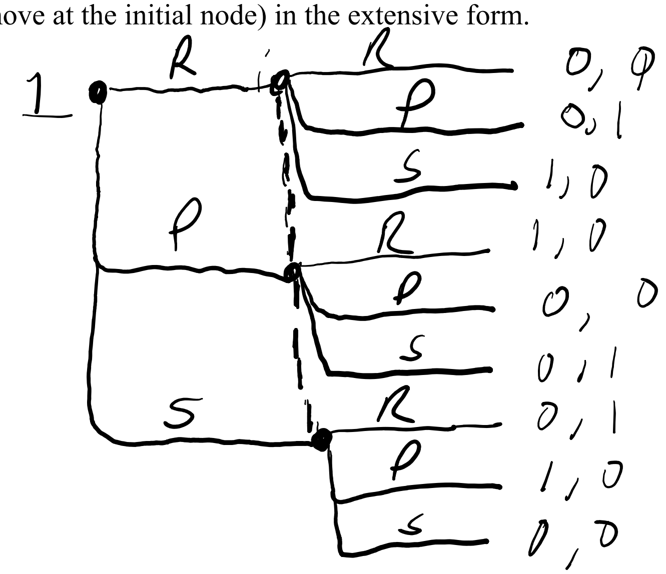
Game Theory Solutions – Ch 2 – Ch 6

2.2**.** Consider the following strategic situation concerning the owner of a firm (O), the manager of the firm (M), and a potential worker (W). The owner first decides whether to hire the worker, to refuse to hire the worker, or to let the manager make the decision. If the owner lets the manager make the decision, then the manager must choose between hiring the worker or not hiring the worker. If the worker is hired, then he or she chooses between working diligently and shirking. Assume that the worker does not know whether he or she was hired by the manager or the owner when he or she makes this decision. If the worker is not hired, then all three players get a payoff of 0. If the worker is hired and shirks, then the owner and manager each get a payoff of −1, whereas the worker gets 1. If the worker is hired by the owner and works diligently, then the owner gets a payoff of 1, the manager gets 0, and the worker gets 0. If the worker is hired by the manager and works diligently, then the owner gets 0, the manager gets 1, and the worker gets 1. Represent this game in the extensive form (draw the game tree).

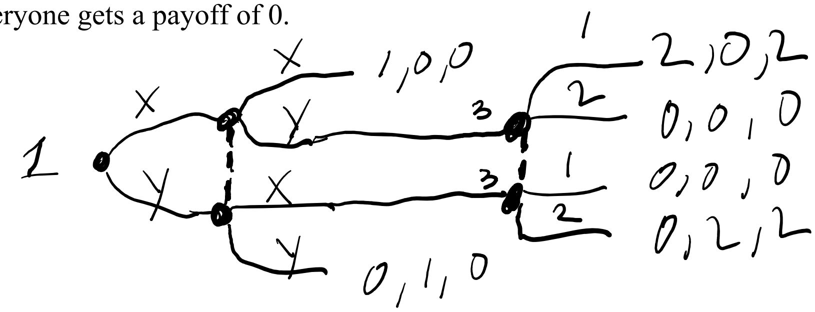


2.4. The following game is routinely played by youngsters—and adults as well—throughout the world. Two players simultaneously throw their right arms up and down to the count of "one, two, three." (Nothing strategic happens as they do this.) On the count of three, each player quickly forms his or her hand into the shape of either a rock, a piece of paper, or a pair of scissors. Abbreviate these shapes as R, P, and S, respectively. The players make this choice at the same time. If the players pick the same shape, then the game ends in a tie. Otherwise, one of the players wins and the other loses. The winner is determined by the following rule: rock beats scissors, scissors beats paper, and paper beats rock. Each player obtains a payoff of 1 if he or she wins, −1 if he or she loses, and 0 if he or she ties. Represent this game in the extensive form. Also discuss the relevance of the order of play (which of the players has the move at the initial node) in the extensive form.



If the game was sequential, the second mover would always win.

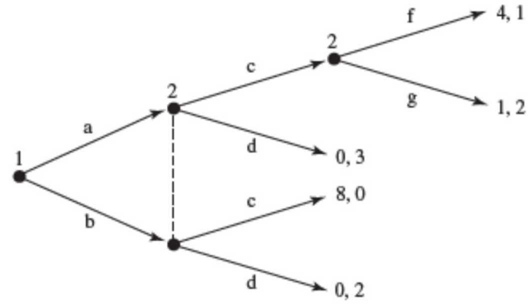
2.6 Represent the following game in the extensive form. There are three players, numbered 1, 2, and 3. At the beginning of the game, players 1 and 2 simultaneously make decisions, each choosing between "X" and "Y." If they both choose "X," then the game ends and the payoff vector is (1, 0, 0); that is, player 1 gets 1, player 2 gets 0, and player 3 gets 0. If they both choose "Y," then the game ends and the payoff vector is (0, 1, 0); that is, player 2 gets 1 and the other players get 0. If one player chooses "X" while the other chooses "Y," then player 3 must guess which of the players selected "X"; that is, player 3 must choose between "1" and "2." Player 3 makes his selection knowing only that the game did not end after the choices of players 1 and 2. If player 3 guesses correctly, then he and the player who selected "X" each obtains a payoff of 2, and the player who selected "Y" gets 0. If player 3 guesses incorrectly, then everyone gets a payoff of 0.



3.2 Suppose a manager and a worker interact as follows. The manager decides whether to hire or not hire the worker. If the manager does not hire the worker, then the game ends. When hired, the worker chooses to exert either high effort or low effort. On observing the worker's effort, the manager chooses to retain or fire the worker. In this game, does "not hire" describe a strategy for the manager? Explain.

No: A strategy must specify what to choose in all possible information sets, wheter or not they will be reached if that strategy is played with fidelity.

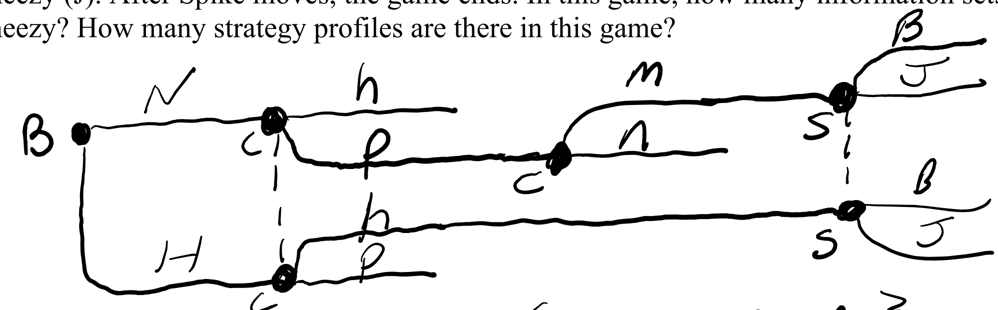
3.4 In the extensive-form game that follows, how many strategies does player 2 have?



cf, cg, dF dg

Four.

3.8 Consider the following strategic setting involving a cat named Baker, a mouse named Cheezy, and a dog named Spike. Baker's objective is to catch Cheezy while avoiding Spike; Cheezy wants to tease Baker but avoid getting caught; Spike wants to rest and is unhappy when he is disturbed. In the morning, Baker and Cheezy simultaneously decide what activity to engage in. Baker can either nap (N) or hunt (H), where hunting involves moving Spike's bone. Cheezy can either hide (h) or play (p). If nap and hide are chosen, then the game ends. The game also will end immediately if hunt and play are chosen, in which case Baker captures Cheezy. On the other hand, if nap and play are chosen, then Cheezy observes that Baker is napping and must decide whether to move Spike's bone (m) or not (n). If he chooses to not move the bone, then the game ends. Finally, in the event that Spike's bone was moved (either by Baker choosing to hunt or by Cheezy moving it later), then Spike learns that his bone was moved but does not observe who moved it; in this contingency, Spike must choose whether to punish Baker (B) or punish Cheezy (J). After Spike moves, the game ends. In this game, how many information sets are there for Cheezy? How many strategy profiles are there in this game?



$\{N, H\} \times \{B, J\} \times \{hm, hn, pm, pn\}$

$2 \times 2 \times 4$

Cheesy has 2 information sets.
There are 16 strategy profiles.

4.2 Suppose we have a game where S_1 = {H, L} and S_2 = {X, Y}. If player 1 plays H, then her payoff is z regardless of player 2's choice of strategy; player 1's other payoff numbers are u_1(L, X) = 0 and u_1(L, Y) = 10. You may choose any payoff numbers you like for player 2 because we will only be concerned with player l's payoff.

(a) Draw the normal form of this game.

1 \ 2	X	Y
H	Z,	Z,
L	0,	10,

(b) If player 1's belief is θ_2 = (1/2, 1/2), what is player 1's expected payoff of playing H? What is his expected payoff of playing L? For what value of z is player 1 indifferent between playing H and L?

$u_1(H|\theta_2) = Z \qquad u_1(L|\theta_2) = 1/2 \cdot 0 + 1/2 \cdot 10 = 5$

Indifferent if $Z = 5$

(c) Suppose θ_2 = (1/3, 2/3). Find player l's expected payoff of playing L.

$u_1(L|\theta_2) = 1/3 \cdot 0 + 2/3 \cdot 10 = 20/3$

4.3 Evaluate the following payoffs for the game pictured here:

1 \ 2	I	O
OA	2, 2	2, 2
OB	2, 2	2, 2
IA	4, 2	1, 3
IB	3, 4	1, 3

(a) $u_1(\sigma_1, I)$ for σ_1 = (1/4, 1/4, 1/4, 1/4)

$1/4 \cdot 2 + 1/4 \cdot 2 + 1/4 \cdot 4 + 1/4 \cdot 3 = 11/4$

(b) $u_2(\sigma_1, O)$ for σ_1 = (1/8, 1/4, 1/4, 3/8)

$1/8 \cdot 2 + 1/4 \cdot 2 + 1/4 \cdot 3 + 3/8 \cdot 3 = 21/8$

(c) $u_1(\sigma_1, \sigma_2)$ for σ_1 = (1/4, 1/4, 1/4, 1/4), σ_2 = (1/3, 2/3)

$1/4(1/3 \cdot 2 + 2/3 \cdot 2) + 1/4(1/3 \cdot 2 + 2/3 \cdot 2) + 1/4(1/3 \cdot 4 + \frac{2}{3} \cdot 1) + 1/4(\frac{1}{3} \cdot 3 + \frac{2}{3} \cdot 1)$

$= 23/12$

(d) $u_1(\sigma_1, \sigma_2)$ for σ_1 = (0, 1/3, 1/6, 1/2), σ_2 = (2/3, 1/3)

$0 + 1/3(2/3 \cdot 2 + 1/3 \cdot 2) + 1/6(2/3 \cdot 4 + 1/3 \cdot 1) + \frac{1}{2}(2/3 \cdot 3 + 1/3 \cdot 1) = 7/3$

4.4. For each of the classic normal-form games (see Figure 3.4), find $u1(\sigma 1, \sigma 2)$ and $u2(\sigma 1, s2)$ for $\sigma 1 = (1/2, 1/2)$ and $\sigma 2 = (1/2, 1/2)$.

Note: Only do Matching pennies, Battle of the Sexes, and Hawk Dove.

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

Matching Pennies

$u_1 = \frac{1}{4}(1-1+1-1) = 0$

$u_2 = \frac{1}{4}(-1+1-1+1) = 0$

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Battle of the Sexes

$u_1 = \frac{1}{4}(2+0+0+1) = 3/4$

$u_2 = \frac{1}{4}(1+0+0+2) = 3/4$

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

Hawk-Dove/Chicken

$u_1 = \frac{1}{4}(0+3+1+2) = 3/2$

$u_2 = \frac{1}{4}(0+1+3+2) = 3/2$

6.2 For the game in Exercise 1 of Chapter 4, determine the following sets of best responses.

1 \ 2	L	C	R
U	10, 0	0, 10	3, 3
M	2, 10	10, 2	6, 4
D	3, 3	4, 6	6, 6

(a) $BR1(\theta 2)$ for $\theta 2 = (1/3, 1/3, 1/3)$

$U_1(U) = 13/3$ $U_1(M) = 18/3$

$U_1(D) = 13/3$ $BR(\theta_2)\ \{M\}$

(b) $BR2(\theta 1)$ for $\theta 1 = (0, 1/3, 2/3)$

$U_2(L) = 16/3$ $U_2(C) = 14/3$ $U_2(R) = 16/3$

$BR_2(\theta_1) = \{L, R\}$

(c) $BR1(\theta 2)$ for $\theta 2 = (5/9, 4/9, 0)$

$U_1(U) = 50/9$ $U_1(M) = 50/9$ $U_1(R) = 39/9$

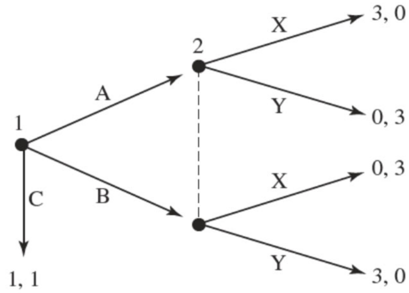
$BR_1(\theta_2) = \{U, M\}$

(d) $BR2(\theta 1)$ for $\theta 1 = (1/3, 1/6, 1/2)$

$U_2(L) = 19/6$ $U_2(C) = 40/6$ $U_2(R) = 28/6$

$BR_2(\theta_1) = \{C\}$

6.6 In the game pictured here, is it ever rational for player 1 to select strategy C? Why?



Let p denote player 1's belief about the probability of X.

$u_1(C,p) \geq u_1(A,p)$

$1 \geq 3p + 0(1-p)$

$1/3 \geq p$

$u_1(C,p) \geq u_1(B,p)$

$1 \geq 0p + 3(1-p)$

$1 \geq 3 - 3p$

$p \geq 2/3$

No, because p cannot satisfy both inequalities.

6.7 In the normal-form game pictured below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief to which M is a best response.

1 \ 2	X	Y
K	9, 2	1, 0
L	1, 0	6, 1
M	3, 2	4, 2

$\sigma_1 = (1/3, 2/3, 0)$

$u_1(\sigma_1, X) = \frac{1}{3}\cdot 9 + \frac{2}{3}\cdot 1 = 11/3 > 3$

$u_1(\sigma_1, Y) = \frac{1}{3}\cdot 1 + \frac{2}{3}\cdot 6 = 13/3 > 4$

M is Dominated by $\sigma_1 = (1/3, 2/3, 0)$