HW 23.6

Wednesday, November 11, 2020 5:29 PM

Passed Solution review

6. This exercise addresses the notion of goodwill between generations. Consider an "overlapping generations" environment, whereby an infinite-period game is played by successive generations of persons. Specifically, imagine a family comprising an infinite sequence of players, each of whom lives for two periods. At the beginning of period t, player t is born; he is young in period t, he is old in period $t + 1$, and he dies at the end of period $t + 1$. Thus, in any given period t, both player t and player $t - 1$ are alive. Assume that the game starts in period 1 with an old player 0 and a young player 1.

When each player is young, he starts the period with one unit of wealth. An old player begins the period with no wealth. Wealth cannot be saved across periods, but each player t can, while young, give a share of his wealth to the next older player $t - 1$. Thus, consumption of the old players depends on gifts from the young players. Player t consumes whatever he does not give to player $t - 1$. Let x_t denote the share of wealth that player t gives to player $t - 1$. Player t's payoff is given by $(1 - x_t) + 2x_{t+1}$, meaning that players prefer to consume when they are old.

(a) Show that there is a subgame perfect equilibrium in which each player consumes all of his wealth when he is young; that is, $x_t = 0$ for each t.

If a young player expects nothing when they're old, they'll leave nothing to the older generation

Suppose player k gives $q_k > 0$ to player k-1.
$U_k = (1-q) + 2(0) = 1-q$

Suppose player k gives 0 to player k-1
$U_k = 1 > 1-q \rightarrow$ this is NE

(b) Show that there is a subgame perfect equilibrium in which the young give all of their wealth to the old. In this equilibrium, how is a player punished if he deviates? mostly there but missing part of the complete answer

If t-1 gives $X_{t-1}=1$, then t gives $X_t=1$. If not, no one gets anything. Thus each young player gives \$1 to the old people

$X_t = 1$ if $X_{t-1} = 1$ and $X_t = 0$ if $X_{t-1} > 1$
if coop: $U_t = 0+2$
if defect and $q<1$: $U_t = q+0 < 1 \rightarrow 2 > q$ this is an eq

(c) Compare the payoffs of the equilibria in parts (a) and (b).

Players get 1 in part A and 2 in part B. Full balance of wealth is best.

$2 > 1$