Hypothesis Testing

QMB 3200

09/22/2020

Review

- Descriptive vs Inferential Statistics
- Measurement Scales – Nominal, Ordinal, Interval and Ratio
- Correlation vs Causation
- Types of Variables
- Measures of Central tendency
- Measures of Variation
- Concepts of Probability
- Z-Scores and Normal Distribution

Hypothesis Testing

- Hypothesis
  - Specific ***prediction,*** often based on a theory, previous research, informal observation, that is tested in a research study
  - Example:
    - Five-year old children should outperform four-year old children in the Tower of Hanoi

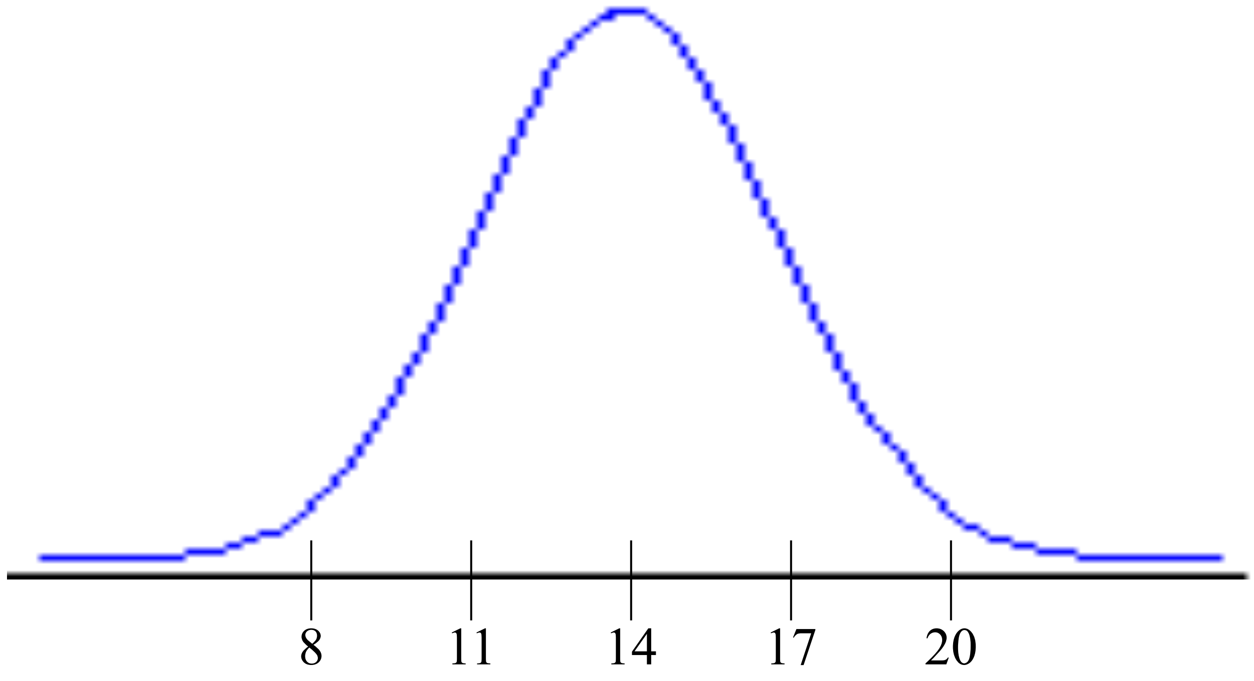
Basic Definitions

- Hypothesis Testing
  - Procedure for deciding whether the outcome of a study (based on a sample) support or refute a specific hypothesis
  - Example:
    - Five-year old children should outperform four-year old children in the Tower of Hanoi

Logic of Hypothesis Testing

Example

- The age when infants begin to walk can be described by a normal distribution with a mean of 14 months and a standard deviation of 3 months.



Example

- A team of researchers creates a motor skills exercise program for infants designed to make them walk earlier.
- Assume this motor skills intervention was given to a *single* child.
- If the intervention is effective, then what can we expect regarding when this child will walk?

Hypothesis Testing

- Standard way of examining research questions
- However, it has a backwards way of doing it.
- The backwards way of doing it is based on the information we know (as opposed to information we don't or can't know)

Back to our Example

- We know:
  - The distribution of ages when infants first walk (normally distributed with a mean of 14 and a standard deviation of 3)
- We don’t know:
  - The distribution of ages when infants first walk if they engage in this specific motor intervention

So…

- We can compare the score (age when the child first walks) for the infant who had the motor intervention and determine the probability that this infant is part of the distribution we know (ages when infants first walk)

Then…

- If the infant starts walking earlier than expected (based on the distribution we know), then we can *reject* the hypothesis that this intervention infant is part of that distribution
  - Thus, accept that the *motor intervention was effective*

- If the infant starts walking similarly to when children normally begin to walk, then we can *retain (fail to reject)* the hypothesis that this intervention infant is part of that distribution
  - Thus, claim that the *motor intervention was not effective*

Hypothesis Testing Procedure

Hypothesis Testing

- Hypothesis testing helps us determine whether or not our theory about an event we observe is likely to occur in the population
- Create two mutually exclusive hypotheses (Null & Alternative (Research) Hypotheses)
- Comparing two hypotheses in light of sample evidence (i.e., data) is called a ***statistical test***
- The competing hypotheses often are stated in terms of one or more parameters of the population distribution (e.g., mean)

Basic Definitions

- Research Question
    - The question we wish to answer
    - At the *population* level
  - Example:
    - Do babies who engage in motor intervention begin to walk earlier than babies who do not?
  - Populations
    - Babies who engage in motor intervention
    - Babies (in general) – babies who do not engage in motor intervention

Basic Definitions

- Research (Alternative) Hypothesis (H_1)
  - The formal (statistical) hypothesis we want to test based on our *Research Question*
  - Statement in hypothesis regarding the predicted relation between populations (often about *means*)

- Example:
  - The population of babies who engage in motor intervention will *on the average* walk earlier than the population of babies who do not
  - Population mean walking age for babies who engage in motor intervention *is lower* than the population mean walking age for babies who do not
  - Population mean walking age for babies who engage in motor intervention *is different* than the population mean walking age for babies who do not
  - H_1: $\mu_{motor} < \mu_{babies}$
  - H_1: $\mu_{motor} \neq \mu_{babies}$

Basic Definitions

- Null Hypothesis (H_0)
  - Statement about a relation between populations that is the opposite of the research hypothesis; statement that in the population there is no difference
- Example:
  - The population of babies who engage in motor intervention will on the average **not** walk earlier than the population of babies who do not
  - Population mean walking age for babies who engage in motor intervention is **no different** than the population mean walking age for babies who do not
  - H_0: $\mu_{motor} = \mu_{babies}$ or $\mu_{motor} \geq \mu_{babies}$

Null & Research Hypotheses

- Null and Research Hypotheses are complete opposites
- Taken together, the Null and Research Hypotheses must be
  - Mutually Exclusive
    - Propositions that logically cannot both be true
    - The occurrence of one automatically implies the non-occurrence of the other
    - For our purposes, the null and alternative hypotheses cannot both be true
  - Exhaustive
    - All possibilities are accounted for

Mutually Exclusive and Exhaustive Hypotheses

$$H_0 \geq 54 \,\&\, H_1 \leq 54$$

1. Mutually Exclusive? 2. Exhaustive?

$$H_0 \geq 54 \,\&\, H_1 \leq 52$$

1. Mutually Exclusive? 2. Exhaustive?

$$H_0 \geq 54 \,\&\, H_1 < 54$$

1. Mutually Exclusive? 2. Exhaustive?

$$H_0 = 54 \,\&\, H_1 \neq 54$$

1. Mutually Exclusive? 2. Exhaustive?

Research Process

1. Research Question
2. Specify Research & Null Hypotheses based on research question
3. Collect data from the appropriate samples
4. Determine characteristics of the comparison distribution (under the Null Hypothesis)
5. Transform observed data to z-score based on the characteristics of the comparison distribution (under the null hypothesis)
6. Determine probability of observing such data under the assumption that the null hypothesis is correct
7. Note significance level ($p < .05$ or $p < .01$) and whether we are doing a one-tailed or two-tailed significance test
8. On the basis of probability, make a decision regarding the Null Hypothesis
    - Reject the Null Hypothesis OR Retain (Fail to Reject) Null Hypothesis
9. Interpret result based on your decision

Back to our Example
Step #1: Research Question

- Do babies who engage in motor intervention begin to walk at a different time than babies who do not?
- Do babies who engage in motor intervention begin to walk at a different time than babies who do not?

  - Population #1: Babies who engage in motor intervention
  - Population #2: Babies in general (babies who do not engage in motor intervention)

Step #2: Hypotheses

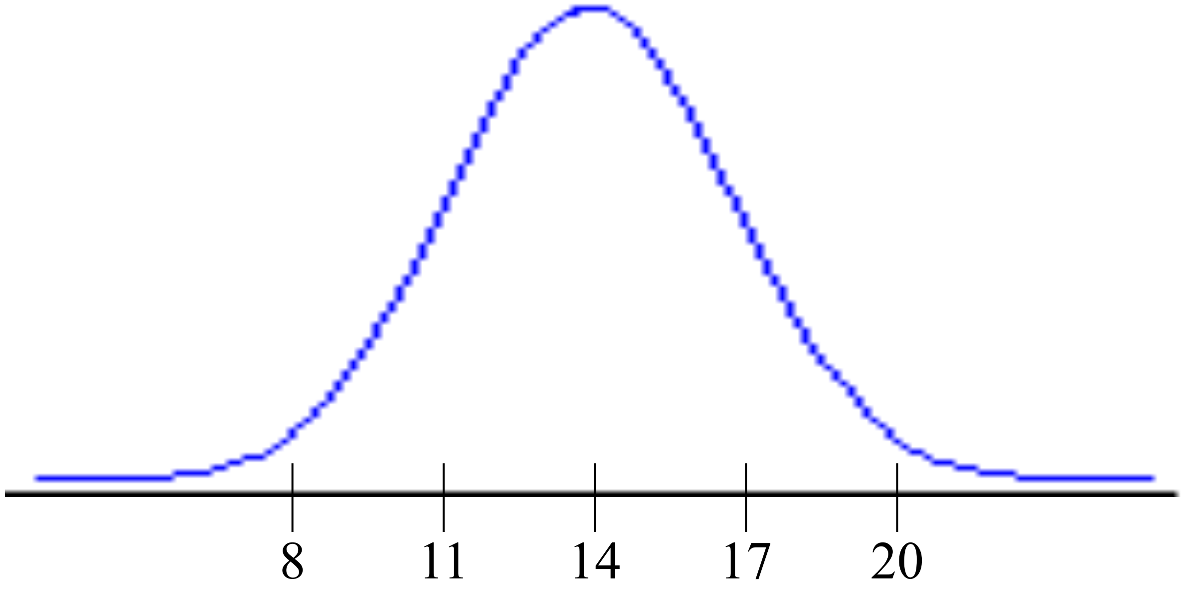
- Null Hypothesis (H_0)
  - Population mean walking age for babies who engage in motor intervention is **no different** than the population mean walking age for babies who do not engage in motor intervention
  - H_0: $\mu_{motor} = \mu_{babies}$
- Research Hypothesis (H_1)
  - Population mean walking age for babies who engage in motor intervention is different than the population mean walking age for babies who do not engage in motor intervention
  - H_1: $\mu_{motor} \neq \mu_{babies}$

Step #3: Collect Data

- For now, we collect data from one child who engaged in the motor intervention
  - Child walked at 8 months

Step #4: Comparison Distribution

- Based on the Null Hypothesis (intervention babies are just like non-intervention babies), what would we expect the distribution of scores to look like?



Normally Distributed with a mean of 14 and a standard deviation of 3

Step #5: Transform observed data to z-score based on the characteristics of the comparison distribution

$$z_i = \frac{X_i - \overline{X}}{s}$$

$$z_i = \frac{8 - 14}{3}$$

$$z_i = -2$$

Step #6: Determine probability of observing data **this extreme** under the assumption that the null hypothesis is correct

- Based on the z-table

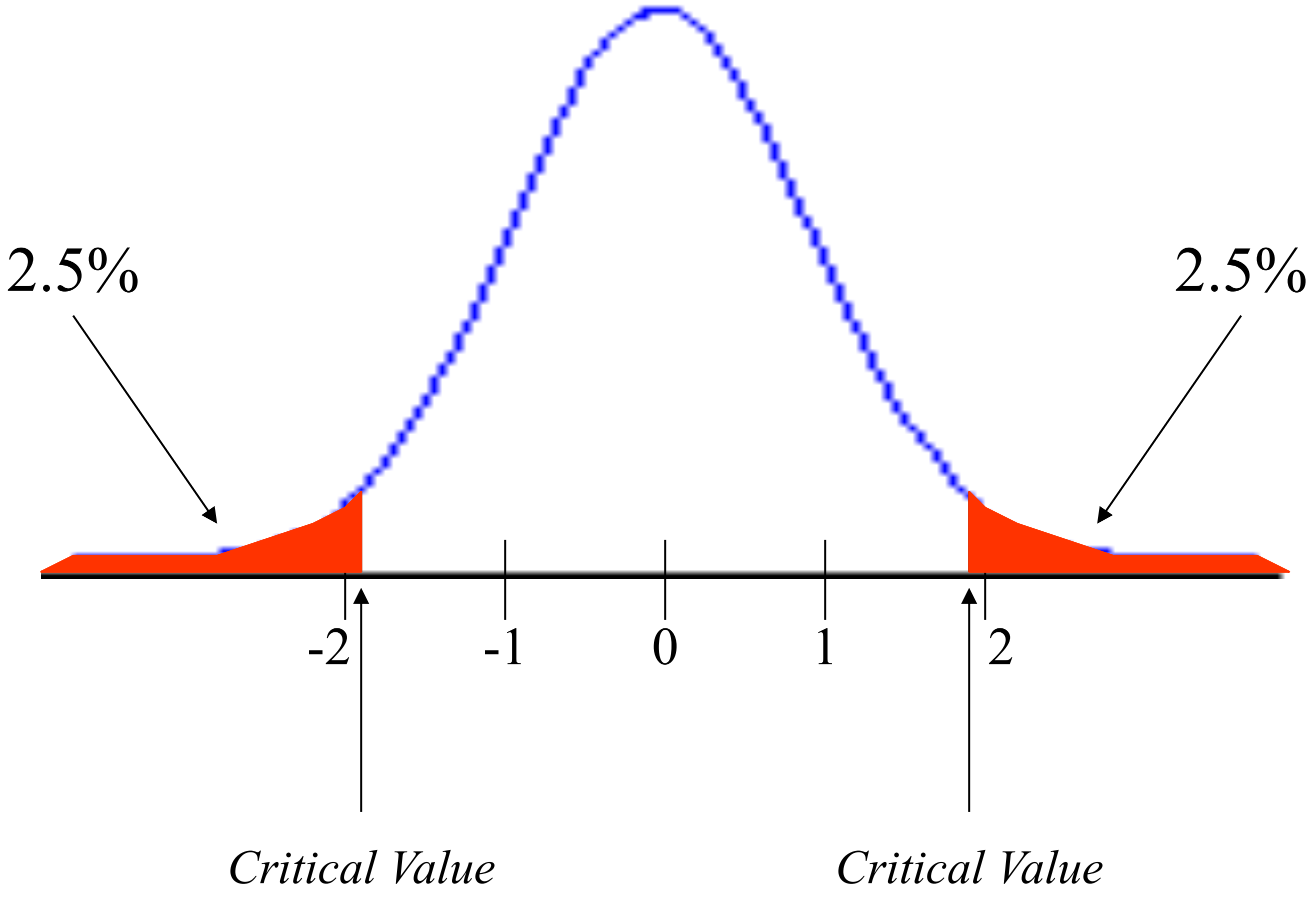
z	% Mean to z	% in Tail
2.00	47.72	2.28

The probability of observing a child walking at 8 months or earlier is .0228 (one-tailed p-value).

The probability of observing a score this extreme is .0456 (two-tailed p-value).

Standard Normal Distribution

(probability of observing value as extreme < .05 – Two-Tailed Test)

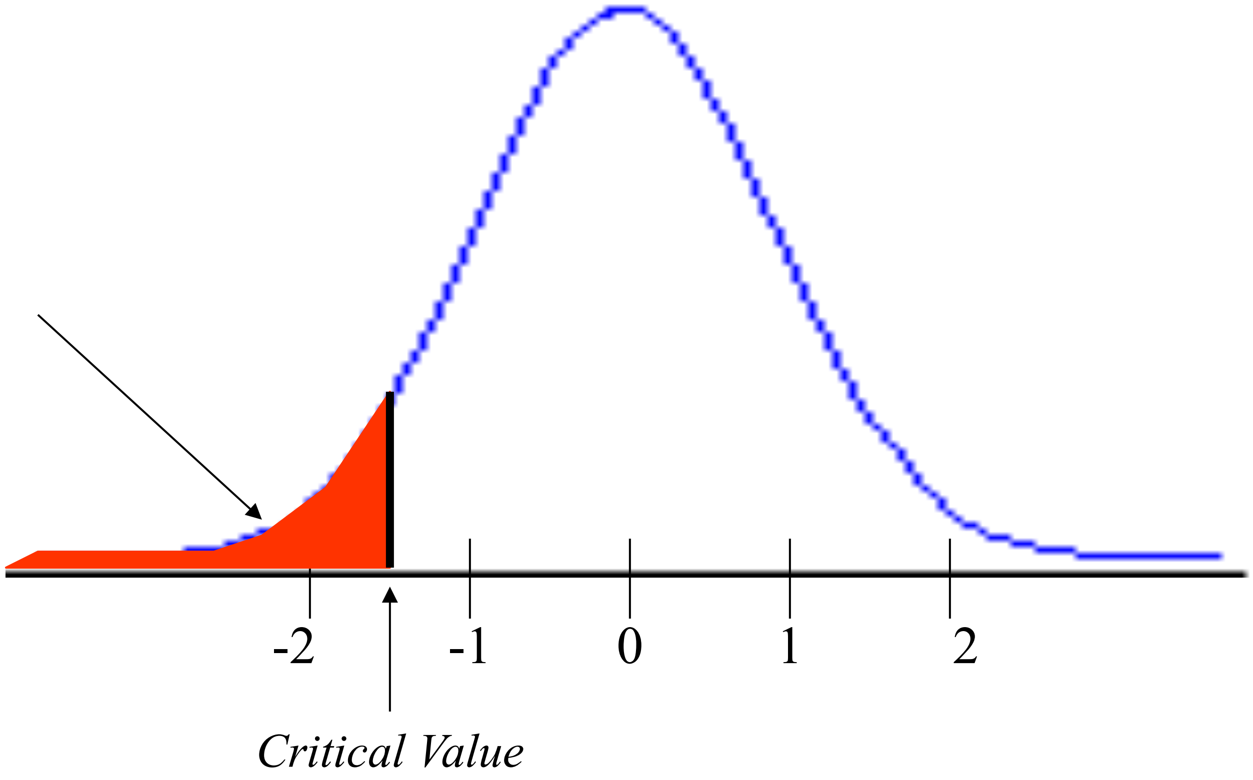


Z-value for p < .05 (Two-Tailed Test)

z	% Mean to z	% in Tail
1.96	47.50	2.50

Standard Normal Distribution

(probability of observing value as extreme in this direction < .05 – One-Tailed Test)*



* *Must make sure direction of effect is consistent with direction of hypothesis.*

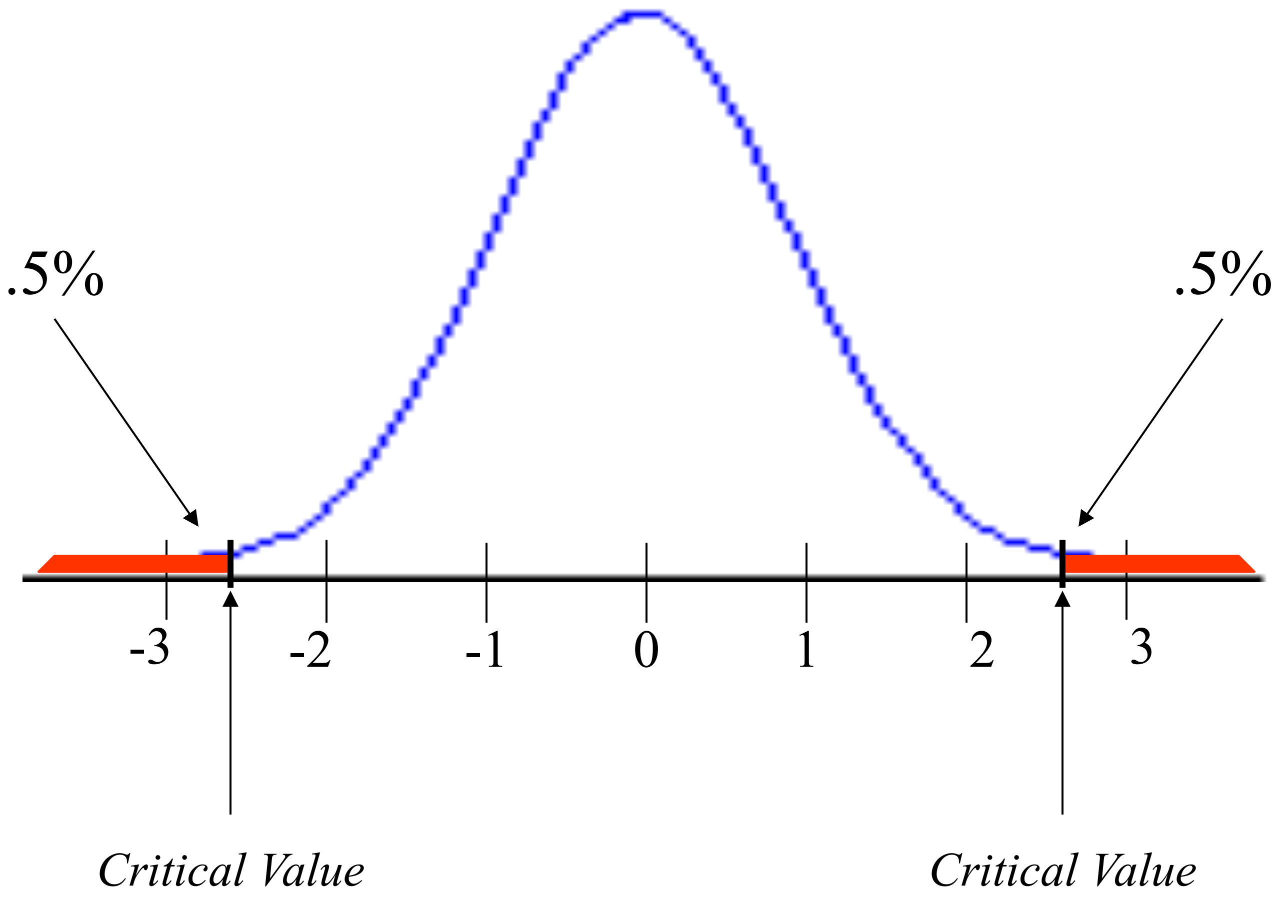
Z-value for $p < .05$ (One-Tailed Test)

z	% Mean to z	% in Tail
1.65	45.05	4.95

** If effect is in opposite direction, then p-value equals 1 - % in tail*

Standard Normal Distribution

(probability of observing value as extreme < .01 – Two-Tailed Test)



Step #7: Note significance level ($p < .05$ or $p < .01$) and whether we're using a one- or two-tailed test

- In this class, we will use a p-value (significance value) of .05, UNLESS a different p-value is specified.

Alpha Level or Significance Level

- The probability that we’re willing to accept that the observed result is due to chance (sampling variation)

- The degree of uncertainty you’re willing to accept
  - Alpha = 0.05 means you’re willing to accept the 5 in 100 chance of incorrectly Rejecting the Null Hypothesis

Significance Level

- Based on conventional rules in statistics, we specify that 5% of the area of the *Comparison Distribution* under the *Null Hypothesis* is considered unusual enough for us to *Reject the Null Hypothesis*
- *95%* of the comparison distribution under the null hypothesis is associated with random sampling

Step #8: Decision Making

- The probability of observing a score at least as small as 8 months ($Z = -2.00$) based on our sampling distribution is .0228 (2.28%)

- Is this probability low enough for us to consider it unlikely and state that this intervention child is part of the distribution of walking ages for all children?
  - Should we *Reject the Null Hypothesis*?

    OR

  - Should we *Retain the Null Hypothesis*?
    - Fail to Reject the Null Hypothesis

Step #9: Interpretation

- The intervention child’s walking age is far enough away from 14 months for us to *Reject the Null Hypothesis*. Therefore, we state that it is unlikely that this child belongs to this distribution. The motor intervention significantly affected the child’s walking age. The intervention was effective in making the child walk earlier than expected.

Summary of Research Process

1. Research Question
2. Specify Research & Null Hypotheses based on research question
3. Collect data from the appropriate samples
4. Determine characteristics of the comparison distribution (under the Null Hypothesis)
5. Transform observed data to z-score based on the characteristics of the comparison distribution (under the null hypothesis)
6. Determine probability of observing such data under the assumption that the null hypothesis is correct
7. Note significance level ($p < .05$ or $p < .01$) and whether we are doing a one-tailed or two-tailed significance test
8. On the basis of probability, make a decision regarding the Null Hypothesis
    - Reject the Null Hypothesis OR Retain (Fail to Reject) Null Hypothesis
9. Interpret result based on your decision

Type I & II Errors

Correct Decisions

1. Retain the Null Hypothesis when the Null Hypothesis is TRUE

    There is no difference in school readiness due to attending preschool and we determine there is no difference

2. Reject the Null Hypothesis when the Null Hypothesis is FALSE

    There is a difference in school readiness due to attending preschool and we declare there is a difference

Incorrect Decisions

1. Retain the Null Hypothesis when the Null Hypothesis is FALSE

    There is a difference in school readiness due to attending preschool and we determine there is no difference

2. Reject the Null Hypothesis when the Null Hypothesis is TRUE

    There is no difference in school readiness due to attending preschool and we declare there is a difference

Type I Error

- *Rejecting the Null Hypothesis* when the *Null Hypothesis* is TRUE

- α (alpha) level: The probability of making a Type I Error

Type II Error

- *Retaining the Null Hypothesis* when the *Null Hypothesis* is FALSE

- β (beta) level: The probability of making a Type II Error

Court System Example

- Did the defendant commit the crime he is charged with
  - Null Hypothesis (H_0): the defendant is innocent (did not commit the crime)
  - Alternate Hypothesis (H_1): the defendant committed the crime
- Jury must make a decision based on a limited amount of information

Types of Errors

- Mistake #1 (Type I)
  - Innocent person is sent to jail
  - Worst kind of error in our legal system

- Mistake #2 (Type II)
  - Guilty person is released
  - Bad but our legal system is biased towards making sure that the innocent are not unjustly punished
    - Innocent until proven guilty

Power

- Power (1 - β)
  - Probability of *Rejecting the Null Hypothesis* when it is *FALSE*

Review

- Comparing a sample mean to a population and the population mean and standard deviation are **<u>known</u>**
  - z-test
- Comparing a sample mean to a population and the population mean is **<u>known</u>**, but the standard deviation is **<u>unknown</u>**
  - *t*-test (one sample, two sample (dependent sample, independent samples))
- Confidence intervals are used to give a sense of how uncertain we are about the population mean. Confidence interval will shrink as *n* increases (holding everything else constant)

Z and T Test Statistics

- *z*-value

$$z = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{\overline{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$$

- *t*-value

$$t = \frac{\overline{X} - \mu_{\overline{X}}}{s_{\overline{X}}} = \frac{\overline{X} - \mu}{\dfrac{s}{\sqrt{n}}}$$

Review - Central Limit Theorem

Given a population with mean μ and variance σ^2, the sampling distribution of the mean (the distribution of sample means) will have a mean equal to μ (i.e., $\mu_{\overline{X}} = \mu$), a variance ($\sigma^2_{\overline{X}}$) equal to σ^2/n, and a standard deviation ($\sigma_{\overline{X}}$) equal to $\sigma/\sqrt{n}$. The distribution will approach the normal distribution as *n*, the *sample size*, increases.

Review

- A researcher wants to examine the effect low-birth weight (LBW) has on psychomotor abilities. The population mean on the Bayley Scales of Infant Development (BSID) is 100. The population standard deviation is 8.
  - The BSID was collected from 15 low-birth weight babies. The sample has a mean of 95.
  - Based on this information, do low-birth weight babies differ from the population in their development of psychomotor abilities?

Review

- Hypotheses:
  - H_0: $\mu_{LBW} = 100$
  - H_1: $\mu_{LBW} \neq 100$

- Sampling Distribution of the mean
  - $\mu_{\overline{X}} = \mu = 100$
  - $s_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{15}} = 2.07$

Review

- Calculate z-value:

$$z_{\overline{X}} = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{95 - 100}{2.07} = -2.42$$

- Decision

|-2.42| is greater than the critical *z*-value (1.96)

Reject the Null Hypothesis

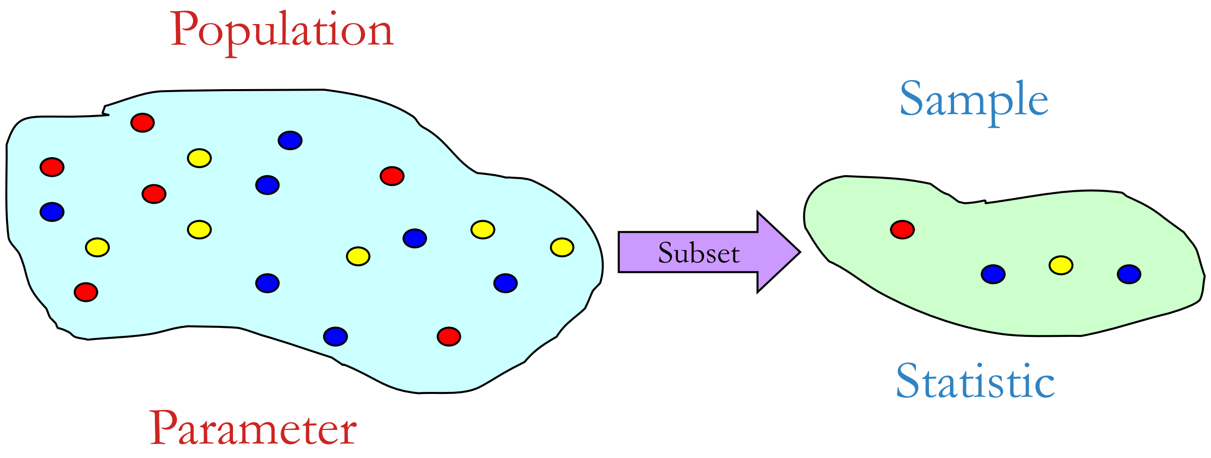
- Conclusion

LBW babies have significantly lower psychomotor abilities, as measured by the BSID, than the population of infants.

Testing a Sample Mean When σ is Unknown

- If we don't know the population standard deviation (σ), we need to estimate it using the standard deviation of our sample (s)
- Since we need to estimate s, we can't use the *z-test*. Instead, we need to use a *t-test*.
  - The switch to the *t-test* is because we know less information, so we should be more cautious.
  - Also, s^2, the sample variance, has a sampling distribution, so our sample variance (s^2) is pulled from a *sampling distribution* of the population variance.

Sampling Distribution



The sampling distribution of a statistic is its probability distribution.

Sampling distribution mean is **expected value.**

The **standard error** of the statistic is the standard deviation of its sampling distribution.

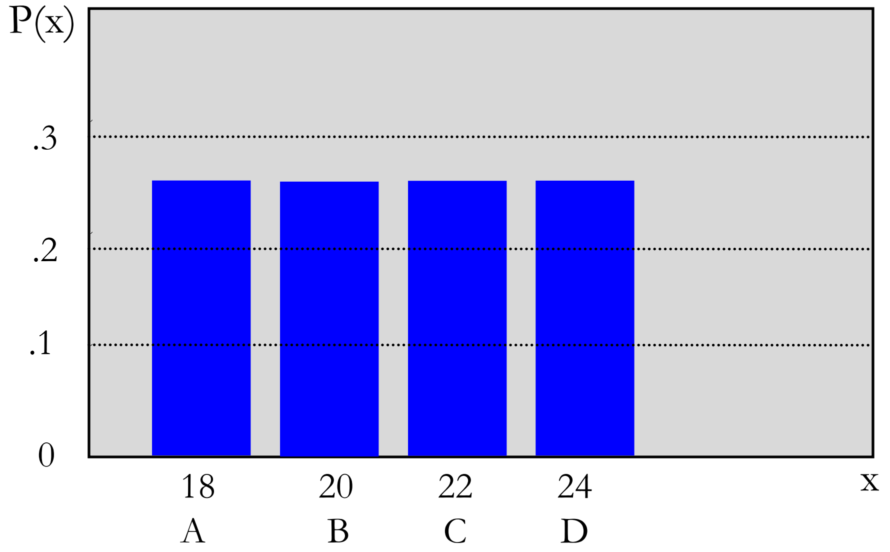
Developing a Sampling Distribution

- **Assume there is a population …**
- Population size N=4
- Random variable, X, is age of individuals
- Values of X: 18, 20, 22, 24 (years)

$$\mu = \frac{\sum X_i}{N}$$

$$= \frac{18 + 20 + 22 + 24}{4} = 21$$

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}} = 2.236$$



Developing a Sampling Distribution

Now consider all possible samples of size n=2

1st Obs	2nd Observation			
	18	20	22	24
18	18, 18	18, 20	18, 22	18, 24
20	20, 18	20, 20	20, 22	20, 24
22	22, 18	22, 20	22, 22	22, 24
24	24, 18	24, 20	24, 22	24, 24

16 possible samples (sampling with replacement)

1st Obs	2nd Observation			
	18	20	22	24
18	18	19	20	21
20	19	20	21	22
22	20	21	22	23
24	21	22	23	24

16 Sample Means

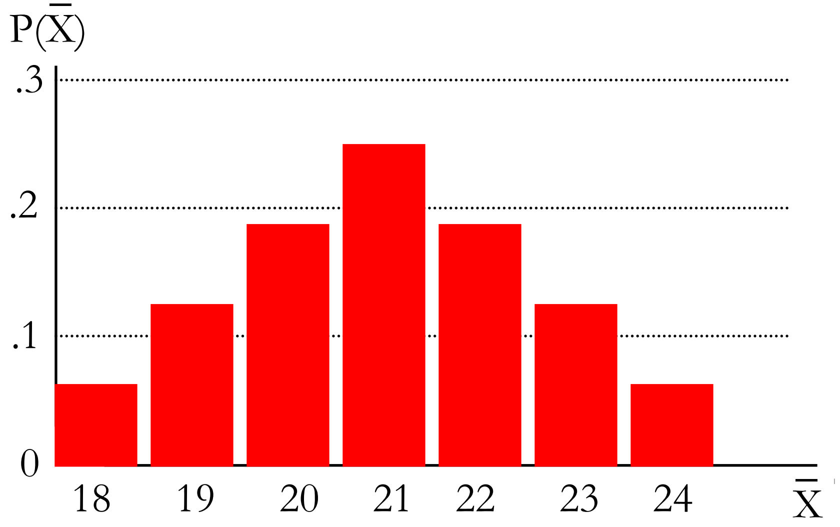
Developing a Sampling Distribution

Sampling Distribution of All Sample Means

16 Sample Means

1st Obs	2nd Observation			
	18	20	22	24
18	18	19	20	21
20	19	20	21	22
22	20	21	22	23
24	21	22	23	24

Sample Means Distribution

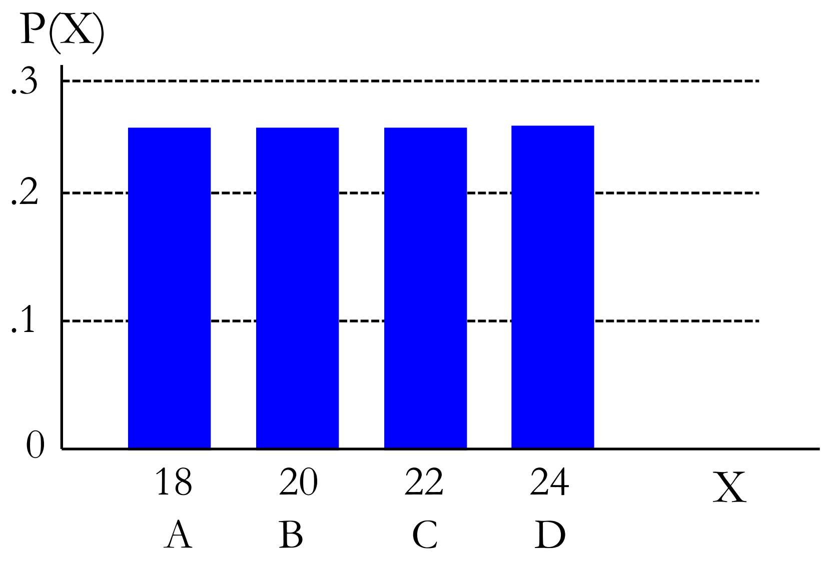


(no longer uniform)

Developing a Sampling Distribution

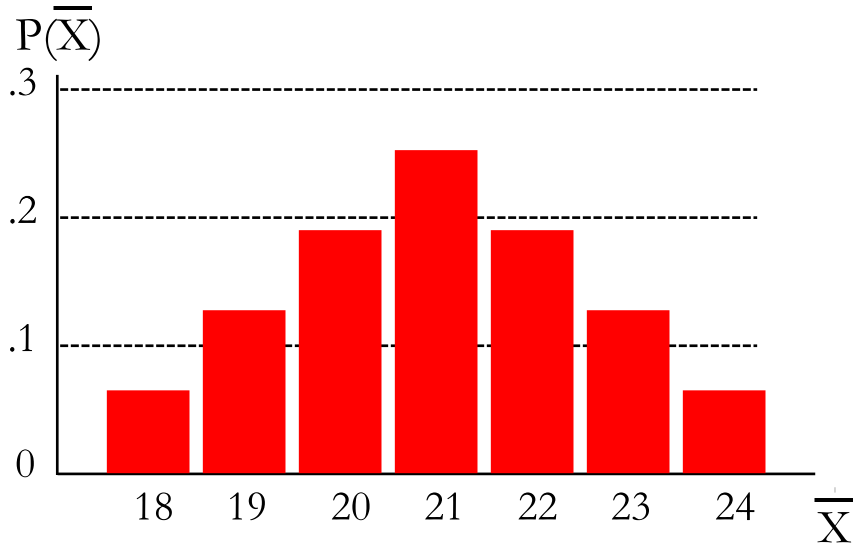
Population

$\mu = 21 \quad \sigma = 2.236$

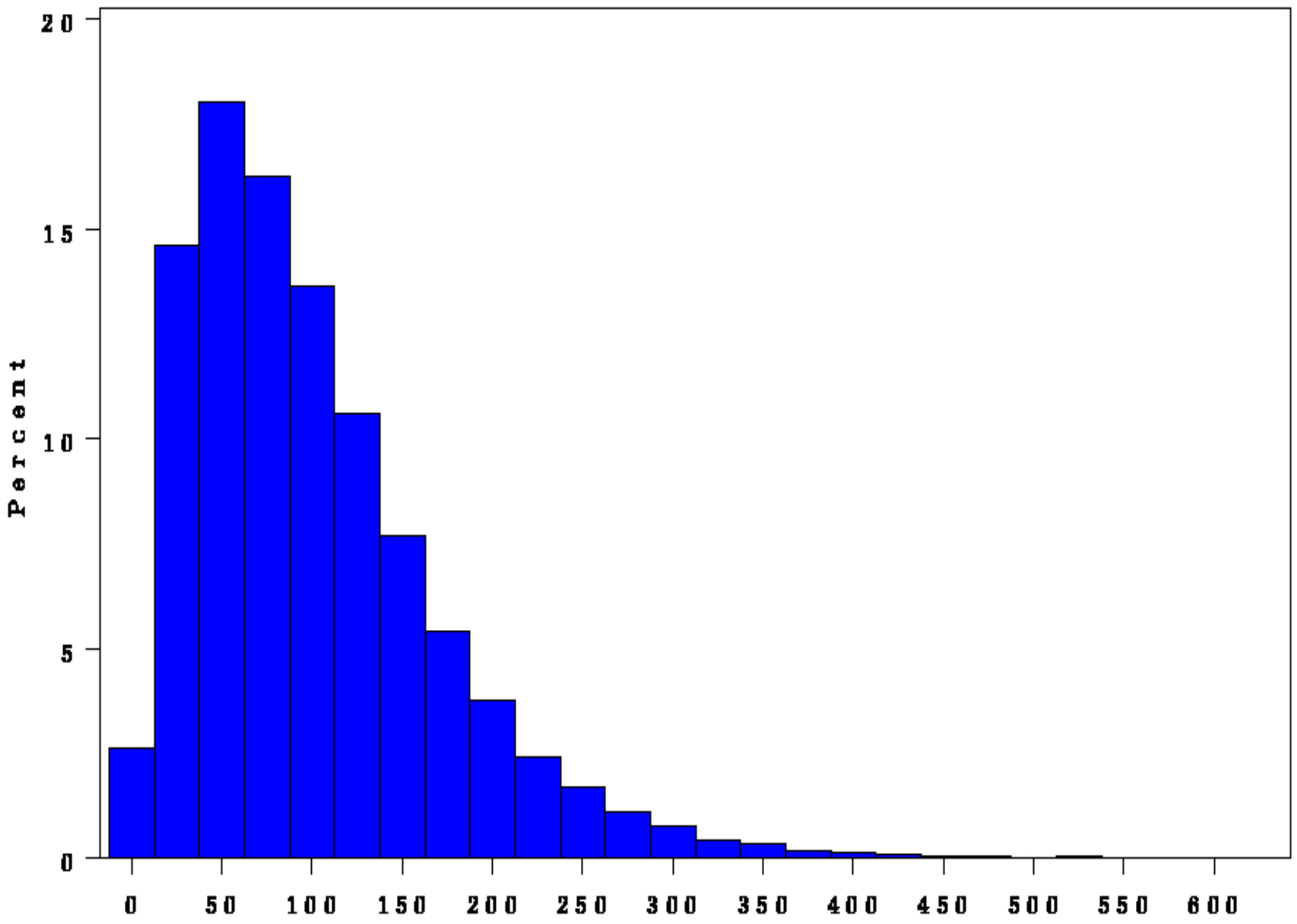


Sample Means Distribution

$\mu_{\overline{X}} = 21 \quad \sigma_{\overline{X}} = 1.58$



Empirical Sampling Distribution of s^2



Sampling Distribution of s^2

- The sampling distribution of s^2 is positively skewed
  - Therefore, s^2 is more likely to underestimate $\boldsymbol{\sigma}^2$ than to overestimate $\boldsymbol{\sigma}^2$
  - The resulting *t-value* is likely to be larger than the corresponding *z-value*, especially if our sample size is small
  - If our sample size is large (i.e., $N > 120$) then our *t-value* approaches the corresponding *z-value*

Normal Distribution vs t-Distribution

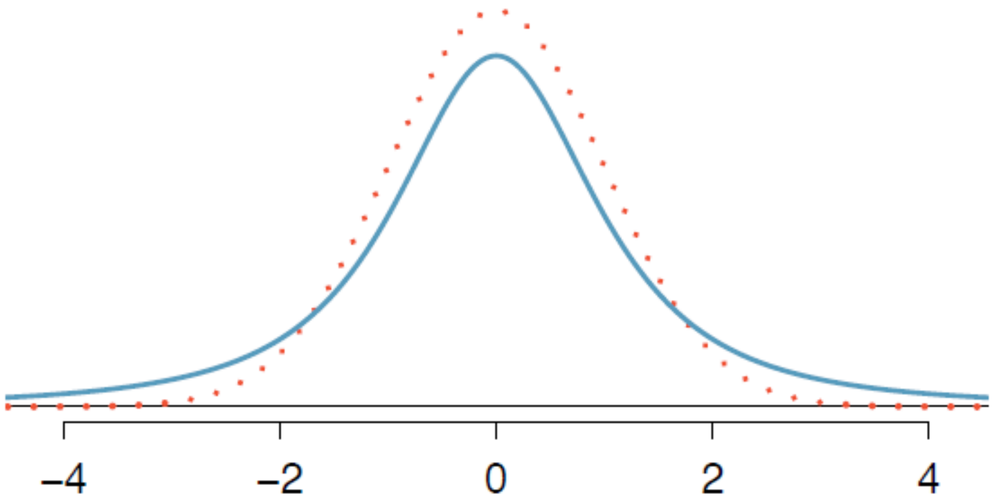


Figure 5.1: Comparison of a t-distribution (solid line) and a normal distribution (dotted line).

Student’s t distribution

William Gosset, who worked for the Guinness Brewing Company, and wrote under the pseudonym *Student*, showed that data sampled from a normal distribution, using s^2 instead of σ^2, would lead to a particular sampling distribution now called *Student’s t distribution*

The t statistic

- z-value

$$z = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- *t*-value

$$t = \frac{\overline{X} - \mu_{\overline{X}}}{s_{\overline{X}}} = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

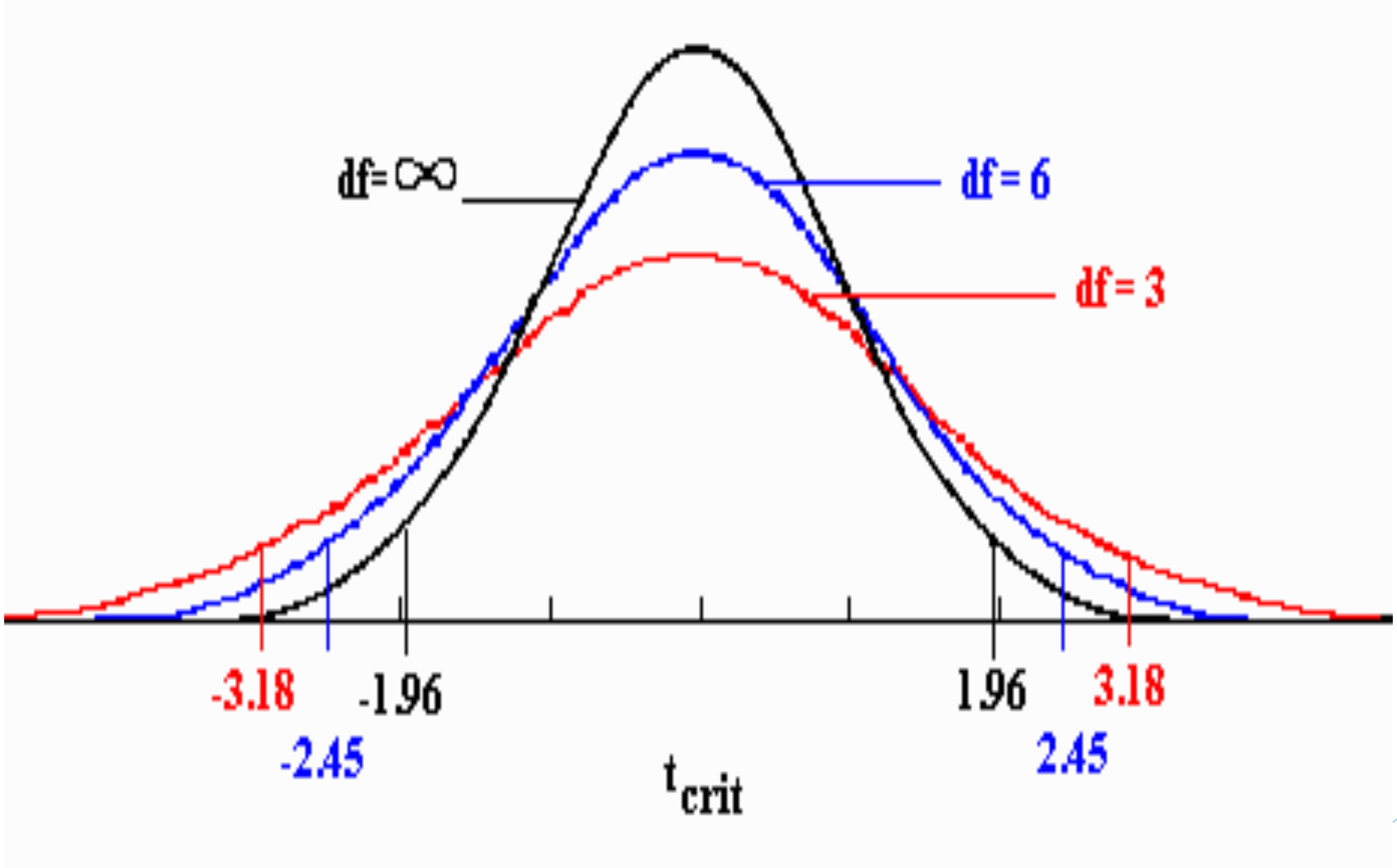
Degrees of Freedom (df)

- The *t* distribution is actually a family of distributions based on *degrees of freedom*
  - *df* is the number of independent pieces of information free to vary.
  - We have *n* scores in our sample so we have *n* pieces of information
  - In the one-sample *t* test we estimate a sample mean so we have $n - 1$ *df*
- There is a *t* distribution for every number of *df*

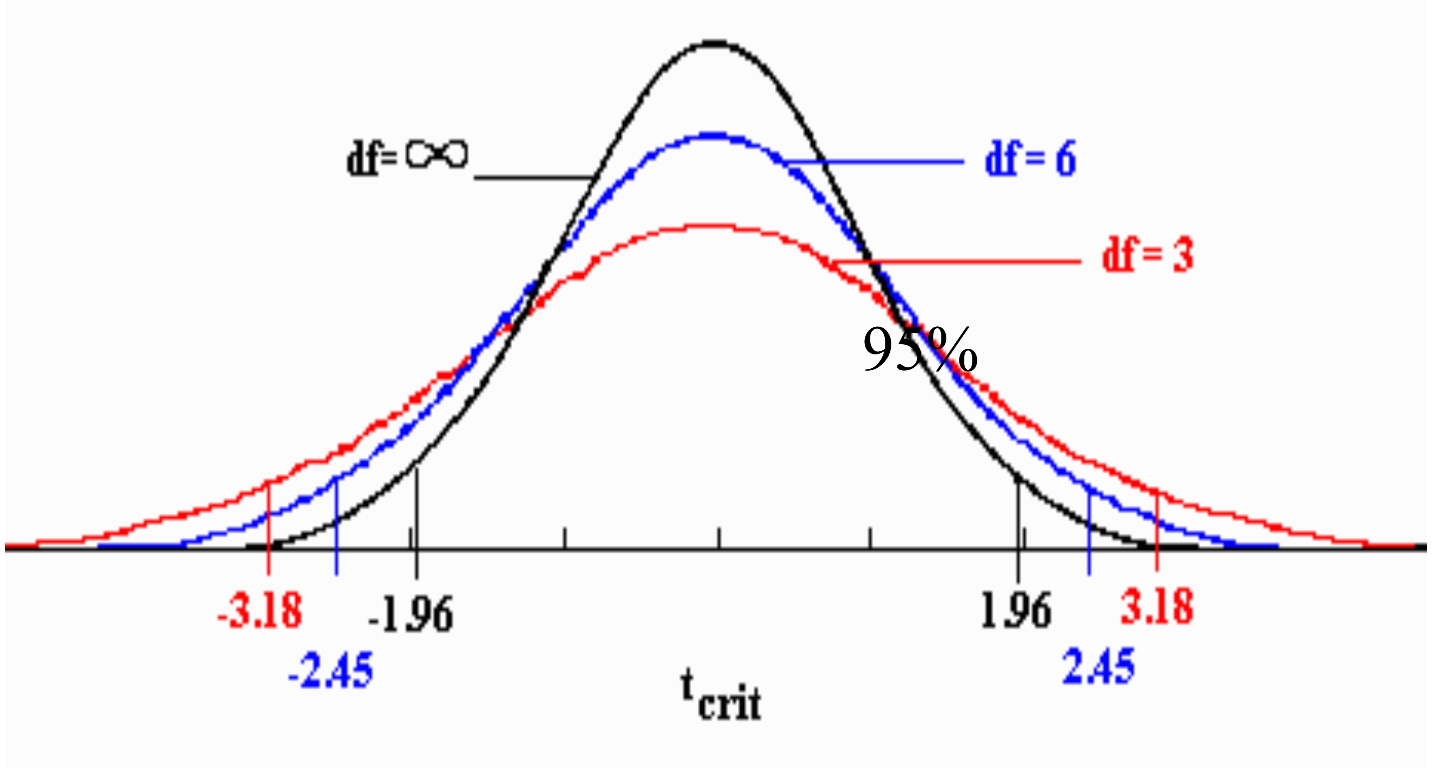
The *t* distribution

- The *t* distribution has larger tails than the standard normal distribution (z)
- Small *df* means large tails
- Large *df* means small tails
- As *df* becomes large (>120) the *t* distribution approaches the *z* distribution

t Distributions



Two tailed statistical significance at α = .05



n large (>120)	$t > \mid 1.96 \mid$
$n = 7$	$t > \mid 2.45 \mid$
$n = 4$	$t > \mid 3.18 \mid$

Conditions for using t-distribution

- Independence of observations
    - Collect a simple random sample from less than 10% of the population
    - Ensure observations from an experiment are independent
- Observations should be from a nearly normal distribution
    - Look at plot of the data
    - Consider previous experience

t-Table

- Each row represents a t-distribution with different degrees of freedom
- Columns correspond to tail probabilities (one tailed or two tailed for various significance levels)
- Similar to normal distribution, t-distributions are symmetric

Examples

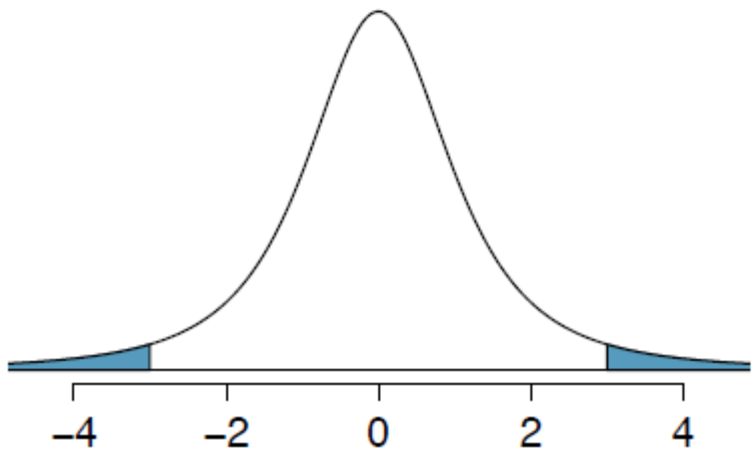
- What proportion of t-distribution with 18 degrees of freedom falls below -2.10?

Examples

- Estimate the proportion of distribution falling above 1.65 for a t-distribution with 20 degrees of freedom.

Examples

- A t-distribution with 2 degrees of freedom is shown in figure. Estimate the proportion of the distribution falling more than 3 units from the mean



Example

- A researcher wants to test whether college students differ from the normal population on depression. She knows that for the general population, the mean depression score is 39. That is, μ = 39.
- Hypotheses:

H_0: $\mu_{\text{college student depression}} = 39$

H_1: $\mu_{\text{college student depression}} \neq 39$

Example

- Obtain a sample of n = 8 college students and administer the measure of depression
  - Sample Mean = 32, Sample Variance = 100

- Test our Null Hypothesis
  - What's the probability of observing a sample mean as extreme as ours ($\bar{X}$ = 32) when the population has a mean of 39?

Given Information

$\mu = 39$

$\sigma = \text{Unknown}$

$s^2 = 100$

$\overline{X} = 32$

$n = 8$

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

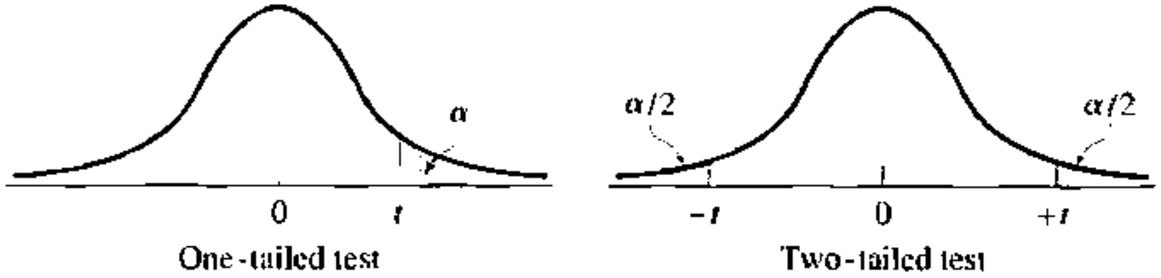
Calculate the *t*-value

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{32 - 39}{\frac{10}{\sqrt{8}}} = -1.98$$

Is this *t*-value unusual given our *Null Hypothesis*?

- t_{OBS} = -1.98
- Use *t* distribution

Appendix t: Percentage Points of the t Distribution



	Level of Significance for One-Tailed Test								
	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test								
df	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.620
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587

Using the *t*-distribution

- *t*-table presents critical values at various levels of *a* for the series of *t*-distributions based on *df*
  - *Questions:*
    - *How Many Degrees of Freedom do we have?*

      7
    - *What's our alpha level?*

      .05
    - *What is the critical value?*

      2.365
    - *Is our observed t-value greater than the critical t-value?*

      No

Conclusion

- The observed *t*-value (-1.98) was not more extreme than the critical *t*-value (2.365).
  - *Retain the Null Hypothesis*

- Conclusion:
  - College students are no more or less depressed than the general population.

Additional Example

- Collect depression data from an additional 10 college students. Now our sample has 18 college students (n = 18). The sample mean and the sample variance remain 32 and 100, respectively.

- Research question:
  - Do college students differ from the normal population (μ = 39) on their level of depression?

Given Information

$\mu = 39$

$\sigma = \text{Unknown}$

$s^2 = 100$

$\overline{X} = 32$

$n = 18$

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

Calculate the *t*-value

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{32 - 39}{\frac{10}{\sqrt{18}}} = -2.97$$

Conclusion

- The observed t-value (-2.97) was more extreme than the critical t-value (2.110).
  - *Reject the Null Hypothesis*

- Conclusion:
  - College students are less depressed than the general population.

Confidence Intervals and Limits

Confidence Interval on μ

- We have made a <u>point estimate</u> of the population mean for college students' level of depression ($X = 32$).
- How confident are we that the true population mean is 32?
- *Confidence Intervals* help to answer this question

95% Confidence Interval on μ

- Based on our sample of $n = 8$
- Based on our sample of $n = 18$
- Use the t-distribution to calculate an upper and lower *raw score* value, such that we are 95% confident the interval contains the population value.

95% Confidence Interval on μ (n = 8)

(A) t-value equation

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

(B) Substitute Critical Value for t

$$\pm 2.365 = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

(C) Substitute Statistics based on the Sample

$$\pm 2.365 = \frac{32 - \mu}{3.5355}$$

95% Confidence Interval on μ (n = 8)

(D) Solve for μ

$$\mu = 32 \pm 2.365 \cdot (3.5355)$$

(E) Calculate Upper and Lower Limits

$$\mu_{upper} = 40.36$$

$$\mu_{lower} = 23.64$$

95% Confidence Intervals

- We are 95% confident the interval 23.64 to 40.36 contains the population mean of the level of depression of college students
  - This is not to say the probability is .95 that the true mean is between 23.64 and 40.36
  - The true mean is a constant and either is or is not in the interval

- Importantly $23.64 \leq \mu_{College\ Students\ Depression} \leq 40.36$ *does* include 39, the average depression score for the general population

95% Confidence Interval on μ (n = 18)

(A) t-value equation

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

(B) Substitute Critical Value for t

$$\pm 2.110 = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

(C) Substitute Statistics based on the Sample

$$\pm 2.110 = \frac{32 - \mu}{2.3570}$$

95% Confidence Interval on μ (n = 18)

(D) Solve for μ

$$\mu = 32 \pm 2.110 \cdot (2.3570)$$

(E) Calculate Upper and Lower Limits

$$\mu_{upper} = 36.97$$

$$\mu_{lower} = 27.03$$

95% Confidence Intervals

- Now, we are 95% confident the interval 27.03 to 36.97 contains the population mean of the level of depression of college students

- Importantly $27.03 \leq \mu_{College\ Students\ Depression} \leq 36.97$ *does not* include 39, the average depression score for the general population