Chapter 7

<u>Exercise 1:</u> (2 points)

Determine the set of rationalizable strategies for each of the following games.

a)

1 \ 2	X	Y
U	(0,4)	(4,0)
M	(3,3)	(3,3)
D	(4,0)	(0,4)

The rationalizable strategy set is

1 \ 2	X	Y
U	(0,4)	(4,0)
M	(3,3)	(3,3)
D	(4,0)	(0,4)

b)

1 \ 2	X	~~Y~~	Z
U	(2,0)	~~(1,1)~~	(4,2)
M	(3,4)	~~(1,2)~~	(2,3)
~~D~~	~~(1,3)~~	~~(0,2)~~	~~(3,0)~~

Note: for player 1 strategy D is dominated by a mix strategy of 0.75U and 0.25M

The rationalizable strategy set is

1 \ 2	X	Z
U	(2,0)	(4,2)
M	(3,4)	(2,3)

c)

1 \ 2	X	~~Y~~	~~Z~~
U	(6,3)	~~(5,1)~~	~~(0,2)~~
~~M~~	~~(0,1)~~	~~(4,6)~~	~~(6,0)~~
~~D~~	~~(2,1)~~	~~(3,5)~~	~~(2,8)~~

Note: for player 1 strategy D is dominated by a mix strategy of 0.5U and 0.5M

The rationalizable strategy set is {(U, X)}

d)

1 \ 2	X	Y	~~Z~~
U	(8,6)	(0,1)	~~(8,2)~~
M	(1,0)	(2,6)	~~(5,1)~~
~~D~~	~~(0,8)~~	~~(1,0)~~	~~(4,4)~~

Note: for player 1 strategy D is dominated by a mix strategy of 0.5U and 0.5M

The rationalizable strategy set is

1 \ 2	X	Y
U	(8,6)	(0,1)
M	(1,0)	(2,6)

e)

1 \ 2	X	Y
A	(2,2)	(0,0)
B	(0,0)	(3,3)

The rationalizable strategy set is

1 \ 2	X	Y
A	(2,2)	(0,0)
B	(0,0)	(3,3)

f)

1 \ 2	X	Y
A	(8,10)	(4,1)
B	(6,4)	(8,5)

The rationalizable strategy set is

1 \ 2	X	Y
A	(8,10)	(4,1)
B	(6,4)	(8,5)

g)

1 \ 2	X	Y
~~U~~	~~(3,10)~~	~~(4,1)~~
D	(6,4)	(8,5)

The rationalizable strategy set is {(8, 5)}

Exercise 2: (1 point)

Suppose that you manage a firm and are engaged in a dispute with one of your employees. The process of dispute resolution is modeled by the following game, where your employee chooses either to "settle" or to "be tough in negotiation," and you choose either to "hire an attorney" or to "give in". In the cells of the matrix, your payoff is listed second; x is a number that both you and the employee know. Under what conditions can you rationalize selection of "give in"? Explain what you must believe for this to be the case.

Employee \ you	Give in	Hire attorney
Settle	(1,2)	(0,1)
Be though	(3,0)	(x,1)

When x is bigger or equal to 0, for employee strategy "Be though" dominate strategy "Settle". Then you will only choose "Hire attorney" because 1 is bigger than 0, which is not the case mentioned in the question.

When x is smaller than 0, there is no dominate strategies for either of the players. If you choose "Hire attorney" the expected payoff is always 1 no matter which strategy employee choose. If you choose "Give in", your expected payoff is $2 \times p + 0 \times (1 - p) = 2p$ if we denote the probability of employee choosing "Settle" by p. In this case, the only reason you will rationalize selection of "give in" is x is smaller than 0 **AND** you believe that the probability that employee play "Settle" is greater than 0.5 ($2p > 1 \rightarrow p > 0.5$).

Note: Since x=0 is the critical value, we won't deduct points if you include 0.

Chapter 9

Exercise 1: (1 points)

Consider the normal-form game pictured here:

1 \ 2	a	b	c
w	(5,2)	(3,4)	(8,4)
x	(6,2)	(2,3)	(8,8)
y	(1,1)	(0,1)	(9,2)

a) What are the Nash equilibria of this game?
b) Which of these equilibria are efficient?

1 \ 2	~~a~~	b	c
w	~~(5,2)~~	(3,4)	(8,4)
~~x~~	~~(6,2)~~	~~(2,3)~~	~~(8,8)~~
y	~~(1,1)~~	(0,1)	(9,2)

a) After eliminated the strictly dominated strategies, we find Nash Equilibrium are (w, b) and (y, c)
b) The strategy is (y, c) is efficient because no other solution at least better for one agent and not worse for the others.

Exercise 3: (2 points)

Find the Nash equilibria of the games in Exercise 1 of Chapter 7
a)

1 \ 2	X	Y

U	(0,4)	(4,0)
M	(3,3)	(3,3)
D	(4,0)	(0,4)

The rationalizable strategy set is

1 \ 2	X	Y
U	(0,4)	(4,0)
M	(3,3)	(3,3)
D	(4,0)	(0,4)

There is no pure Nash equilibria strategy in this game.

b)

1 \ 2	X	~~Y~~	Z
U	(2,0)	~~(1,1)~~	(4,2)
M	(3,4)	~~(1,2)~~	(2,3)
~~D~~	~~(1,3)~~	~~(0,2)~~	~~(3,0)~~

Note: For player 1 strategy D is dominated by a mix strategy of 0.5U and0.5M

The rationalizable strategy set is

1 \ 2	X	Z
U	(2,0)	(4,2)
M	(3,4)	(2,3)

The Nash equilibrium are (M, X) and (U, Z)

c)

1 \ 2	X	~~Y~~	~~Z~~
U	(6,3)	(5,1)	(0,2)
~~M~~	~~(0,1)~~	~~(4,6)~~	(6,0)
~~D~~	~~(2,1)~~	~~(3,5)~~	~~(2,8)~~

The rationalizable strategy set is {(U, X)}

The Nash equilibria is (U, X)

d)

1 \ 2	X	Y	~~Z~~
U	(8,6)	(0,1)	(8,2)
M	(1,0)	(2,6)	(5,1)
~~D~~	~~(0,8)~~	~~(1,0)~~	~~(4,4)~~

The rationalizable strategy set is

1 \ 2	X	Y
U	(8,6)	(0,1)
M	(1,0)	(2,6)

The Nash equilibrium are (U, X) and (M, Y)

e)

1 \ 2	X	Y
A	(2,2)	(0,0)
B	(0,0)	(3,3)

The rationalizable strategy set is

1 \ 2	X	Y
A	(2,2)	(0,0)
B	(0,0)	(3,3)

The Nash equilibrium are (A, X) and (B, Y)

f)

1 \ 2	X	Y
A	(8,10)	(4,1)
B	(6,4)	(8,5)

The rationalizable strategy set is

1 \ 2	X	Y
A	(8,10)	(4,1)
B	(6,4)	(8,5)

The Nash Equilibrium are (A, X) and (B, Y)

g)

1 \ 2	X	Y
~~U~~	~~(3,10)~~	~~(4,1)~~
D	(6,4)	(8,5)

The rationalizable strategy set is {(8, 5)}

The Nash Equilibria is (D, Y)

Chapter 11

Exercise 4: (2 points)

Compute the mix-strategy equilibria of the following games:

1 \ 2	A	B	
A	(2,4)	(0,0)	p
B	(1,6)	(3,7)	1-p
	q	1-q	

Let p denote the probability that player 1 plays strategy A and q denote the probability that player 2 plays strategy A.

For player 1: $q \times 2 + 0 \times (1-q) = 1 \times q + 3 \times (1-q) \Rightarrow q = 0.75$

For player 2: $p \times 4 + 6 \times (1-p) = 0 \times p + 7 \times (1-p) \Rightarrow p = 0.2$

The mix-strategy Nash equilibria is ((0.2A, 0.8B), (0.75A, 0.25B))

1 \ 2	~~L~~	M	R	
U	~~(8,3)~~	(3,5)	(6,3)	p
C	~~(3,3)~~	(5,5)	(4,8)	1-p
~~D~~	~~(5,2)~~	~~(3,7)~~	~~(4,9)~~	
		q	1-q	

After eliminated dominated strategy L and D, let p denote the probability that player 1 plays strategy U and q denote the probability that player 2 plays strategy M.

For player 1: $q \times 3 + 6 \times (1-q) = 5 \times q + 4 \times (1-q) \Rightarrow q = 0.5$

For player 2: $p \times 5 + 5 \times (1-p) = 3 \times p + 8 \times (1-p) \Rightarrow p = 0.6$

The mix-strategy Nash equilibria is ((0.6U, 0.4C, 0D), (0, 0.5M, 0.5R))

Exercise 6: (2 points)

Determine all the Nash equilibria (pure-strategy and mixed strategy equilibria) of the following games.

a)

1 \ 2	H	T	
H	(1,-1)	(-1,1)	p
T	(-1,1)	(1,-1)	1-p

	q	1-q

Let p denote the probability that player 1 plays strategy H and q denote the probability that player 2 plays strategy H.

For player 1: $q \times 1 + (-1) \times (1-q) = -1 \times q + 1 \times (1-q) \Rightarrow q = 0.5$

For player 2: $p \times (-1) + 1 \times (1-p) = 1 \times p + (-1) \times (1-p) \Rightarrow p = 0.5$

There is no pure-strategy Nash equilibria, and there is a mix-strategy Nash equilibria is ((0.5H, 0.5T), (0.5H, 0.5T))

b)

1 \ 2	C	D
C	(2,2)	(0,3)
D	(3,0)	(1,1)

After eliminated dominated strategy C and D, there is one pure-strategy Nash equilibria (D, D), and there is no mix-strategy Nash equilibria.

c)

1 \ 2	H	D	
H	(2,2)	(3,1)	p
D	(3,1)	(2,2)	1-p
	q	1-q	

Let p denote the probability that player 1 plays strategy H and q denote the probability that player 2 plays strategy H.

For player 1: $q \times 2 + 3 \times (1-q) = 3 \times q + 2 \times (1-q) \Rightarrow q = 0.5$

For player 2: $p \times 2 + 1 \times (1-p) = 1 \times p + 2 \times (1-p) \Rightarrow p = 0.5$

There is no pure-strategy Nash equilibria, and there is a mix-strategy Nash equilibria is ((0.5H, 0.5D), (0.5H, 0.5D))

d)

1 \ 2	A	B	
A	(1,4)	(2,0)	p

B	(0,8)	(3,9)	1-p
	q	1-q	

Let p denote the probability that player 1 plays strategy A and q denote the probability that player 2 plays strategy A.

For player 1: $q \times 1 + 2 \times (1 - q) = 0 \times q + 3 \times (1 - q) \Rightarrow q = 0.5$

For player 2: $p \times 4 + 8 \times (1 - p) = 0 \times p + 9 \times (1 - p) \Rightarrow p = 0.2$

There are 2 pure-strategy Nash equilibria (A, A) and (B, B), and there is a mix-strategy Nash equilibria is ((0.2A, 0.8B), (0.5A, 0.5B))