Passed Solution review

Two firms engage in price competition. They sell somewhat differentiated products (think Nike and adidas), which are substitutes but not perfect substitutes. The demand for firm $i \in \{1,2\}$ is $q_i = 0.5 - p_i + 0.5p_{-i}$. For simplicity, assume per unit operating costs are 0, or equivalently, that price is measured in terms of its difference from cost.

Here is the intuition for the strategic market situation captured by these demand curves. If both firms set prices of 0, total sales are 1, so we are scaling units so 1 is the largest reasonable number of sales. An increase in a firm's price leads customers to purchase less of the firm's product—some drop out of the market and others switch brands. In this case half switch and half drop out.

a) Find the Nash Equilibrium prices, and associated payoffs, assuming prices are chosen simultaneously and independently. Draw the reaction functions and label the equilibrium.

$C_1 = C_2 = 0$

$q_1 = \frac{1}{2} - P_1 + \frac{1}{2}P_2 \qquad q_2 = 1/2 - P_2 + \frac{1}{2}P_1$

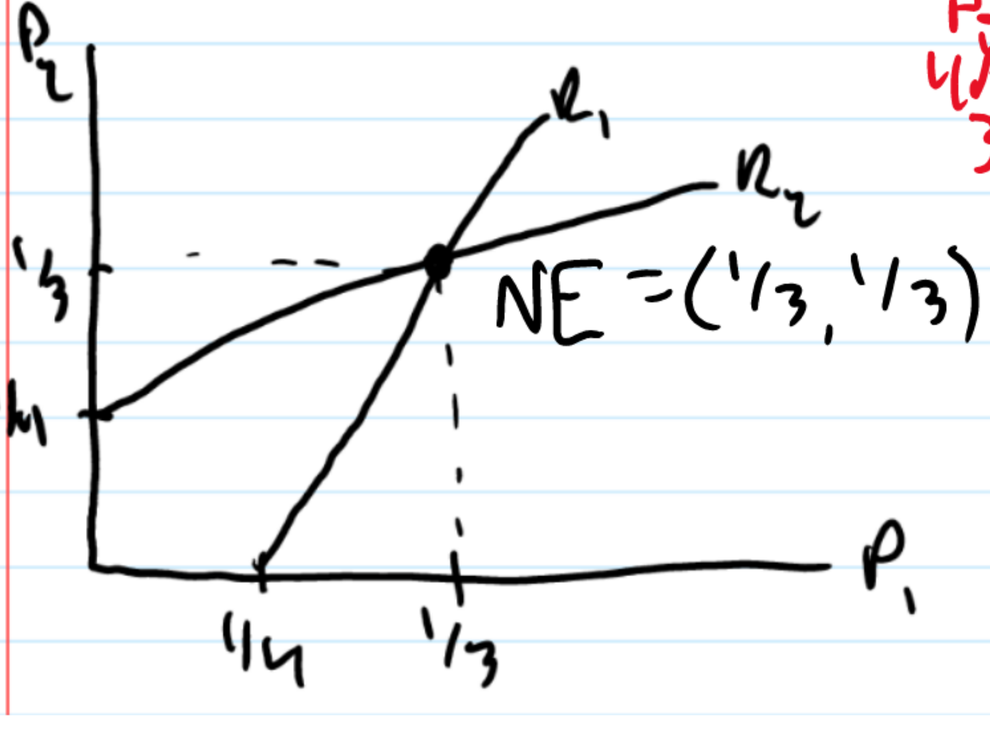
$u_1 = (P_1 - C_1)q_1 = P_1(\frac{1}{2} - P_1 + \frac{1}{2}P_1) \rightarrow \frac{du}{dP_1} = \frac{1}{2} - \cancel{P_1} + \cancel{\frac{1}{2}P_2} - P_1 = 0 \rightarrow \cancel{P_1 = 1/4 + P_2/4}$

$\cancel{4P_1 = 1/4}$

$P_i = (1 + P_{-i})/4$

$4P = 1 + P$

$3P = 1 \rightarrow P = 1/3$



$V_i = (1/2 - 1/6)1/3 = 1/9$

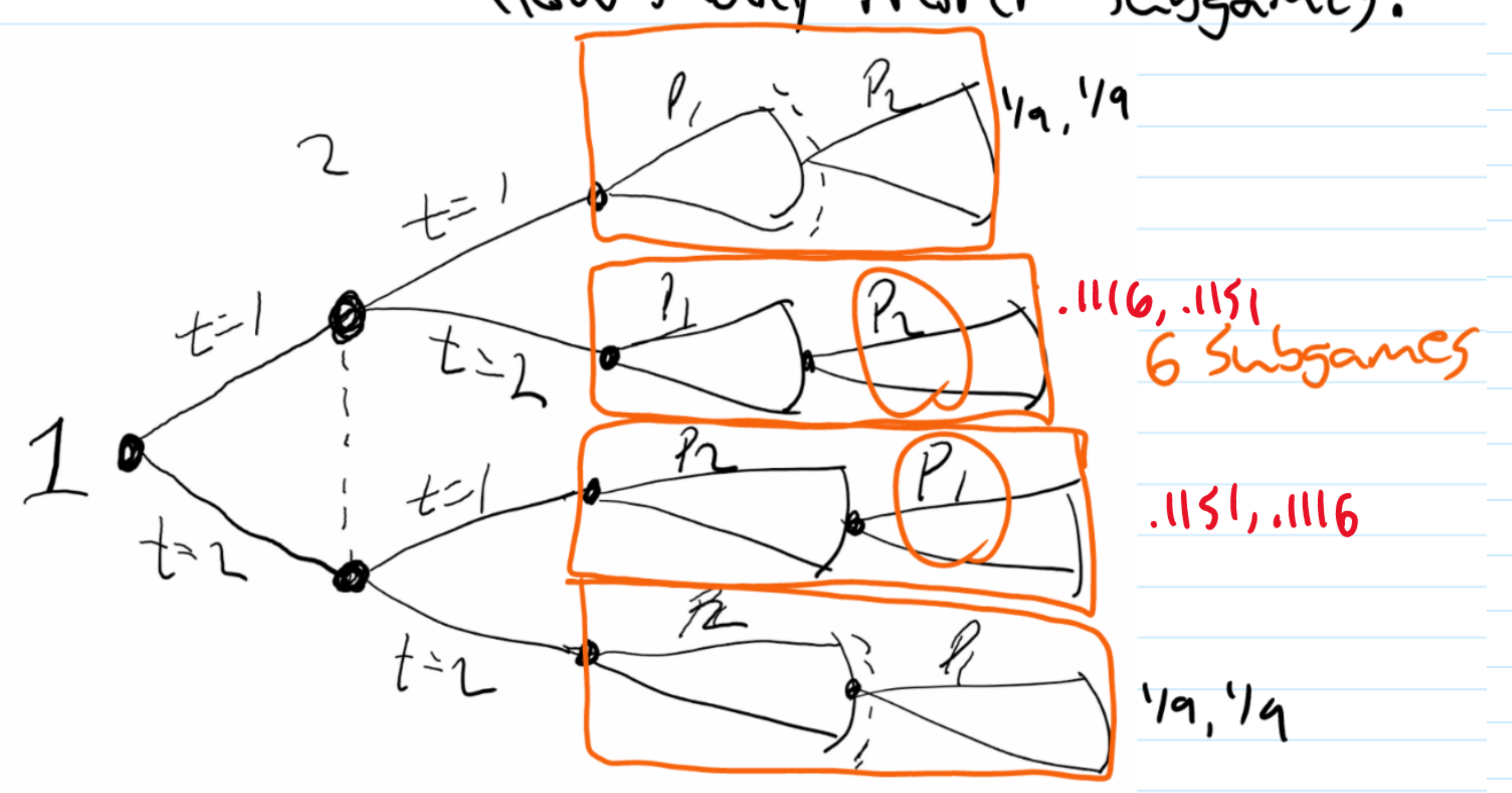
1\2	1st	2nd
1st	1/9, 1/9	
2nd		1/9, 1/9

1 moves first, $P_1 = 1/3 + C$
$P_2 = 1/3 + C - \varepsilon$, ε = tiny #

b) Find the SPE prices, and associated payoffs, assuming firm 1 chooses price first in a sequential game. Label this outcome in the reaction function diagram, and, intuitively explain the nature of the difference form the simultaneous move game.

$u_1 = V_2 = \frac{1}{3}(\frac{1}{2} - \frac{1}{3} + \frac{1}{6}) = \frac{1}{9}$ ← if simultaneous moves

How many proper subgames?



1\2	t=2	t=3
t=2	.1111, .1111	.1116, .1151
t=3	.1151, .1116	.1111, .1111

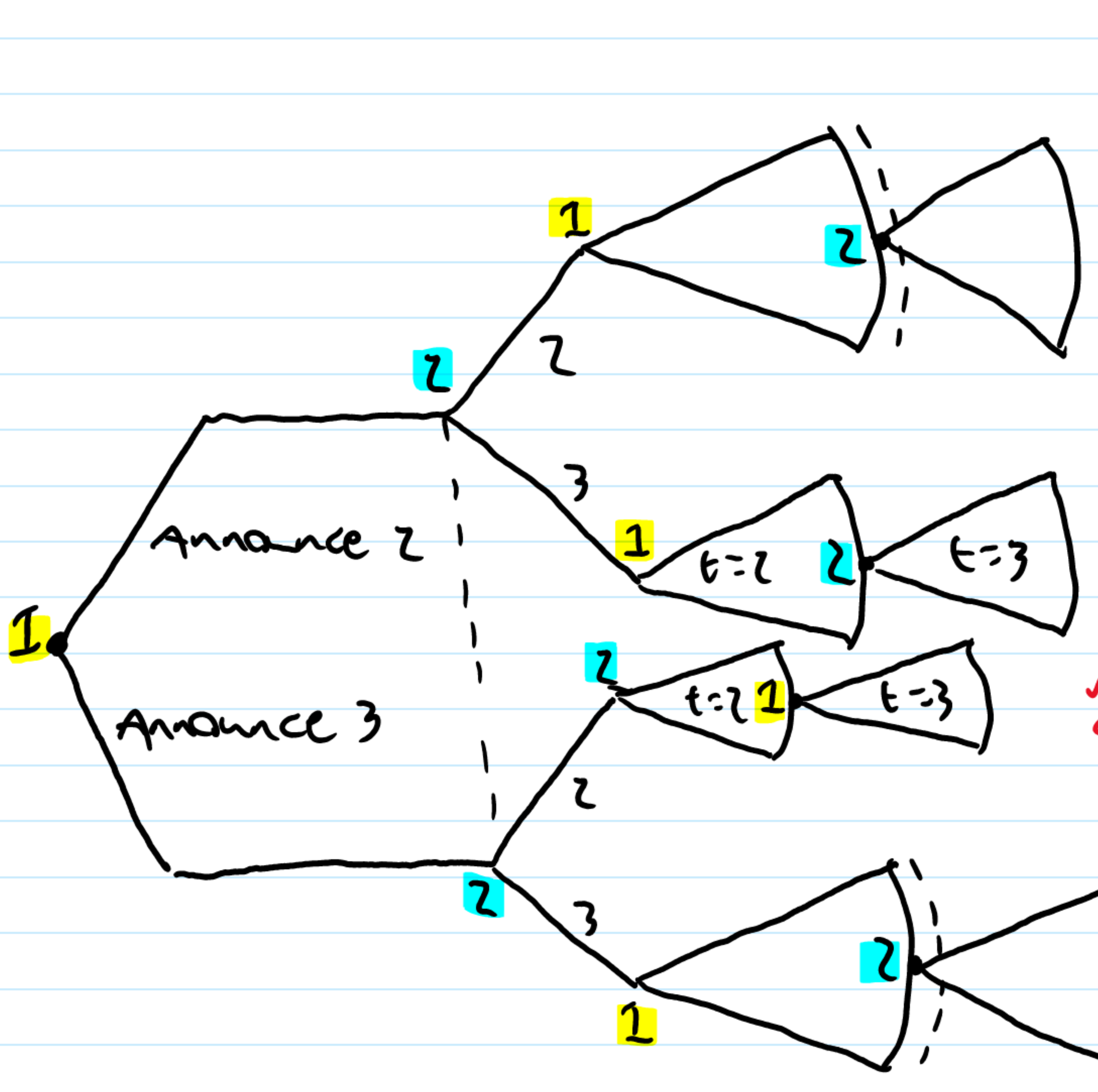
no NE where they announce simultaneously

$.1111P + .1116(1-P) = .1151P + .1111(1-P) \rightarrow P = 1/9$ and it's symmetric

$V_i = \frac{1}{81} \cdot .1111 + \frac{64}{81} \cdot .1111 \rightarrow V_i = .1115$

symmetric mixed NE is worse than either of the 2 Pure NE

c) Assume the game spans four discrete time periods. In the first period each firm simultaneously and independently announces whether they will post their price in period 2 or period 3. Once this announcement is made the firms are committed to that timing, as they are to their prices once posted. In the fourth period, sales and payoffs are realized at the posted prices. Draw the extensive form of this game and find the pure strategy SPE. Is there an equilibrium where players strategies regarding when to announce prices are mixed? If so, what is it? Comment on anything interesting about the timing in this game and how it related to first and second mover advantages.



Timing only matters if a player announces 2 and has a lower price than the other who announces at 3

Wait. NVM. Prices are announced together

Both want to move last, but it's better to move first than at the same time