AP Calculus Cheat Sheet

v1.6

Contents

1 Derivatives

1.1 Using limits to calculate derivatives

$$\frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

1.2 Utilities

Power rule:

$$\frac{d}{dx}x^n = nx^{n-1} \tag{2}$$

Chain rule:

$$\frac{d}{dx}(f(x) \circ g(x)) = f'(g(x)) * g'(x) \tag{3}$$

Derivatives of inverses

$$\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))} \tag{4}$$

Derivatives of logbase

$$\frac{d}{dx}\log_b x = \frac{1}{x} * \frac{1}{\ln b} \tag{5}$$

Derivative of a multiplier

$$\frac{d}{dx}cx = c \tag{6}$$

Derivative of natural log

$$\frac{d}{dx}\ln x = \frac{1}{x} \tag{7}$$

Derivative of multiplication

$$\frac{d}{dx}fg = f'g + fg' \tag{8}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2} \tag{9}$$

Tangent line equation

$$y - y_1 = m(x - x_1) \tag{10}$$

Definition of euler's number

$$e = \lim_{b \to \infty} (1 + \frac{1}{b})^b \tag{11}$$

Derivative of e^x

$$\frac{d}{dx} e^x = e^x \tag{12}$$

Derivative of constant

$$\frac{d}{dx} c = 0 \tag{13}$$

Derivative of addition

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x) \tag{14}$$

Derivative of function multiplied by a constant

$$\frac{d}{dx} c * f(x) = c * f'(x) \tag{15}$$

2 Identities

Trig Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \tag{16}$$

Derivative of

$$\frac{d}{dx} \tag{17}$$

Derivative of cosine

$$\frac{d}{dx} \cos x = -\sin x \tag{18}$$

Derivative of sine

$$\frac{d}{dx} \sin x = \cos x \tag{19}$$

Derivative of tangent

$$\frac{d}{dx} \tan x = \sec^2 x \tag{20}$$

Derivative of cotangent

$$\frac{d}{dx} \cot x = \csc^2 x \tag{21}$$

Derivative of secant

$$\frac{d}{dx}\sec x = \sec x \tan x \tag{22}$$

Derivative of cosecant

$$\frac{d}{dx}\csc x = -\csc x \cot x \tag{23}$$

Derivative of inverse sine

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \tag{24}$$

Derivative of inverse tangent

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2} \tag{25}$$

Pythagorean theorem with sin/cos

$$\sin^2(x) + \cos^2(x) = 1 \tag{26}$$

Rules of logarithms

$$log_b(b^x) = xb^{(log_b\ x)} = xlog_b\ a^n = n \cdot log_b\ a \tag{27}$$

Rules of exponents

$$\begin{aligned} \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\ (x^a)^b &= x^{ab} \\ x^a \cdot x^b &= x^{a+b} \\ x^{-n} &= \frac{1}{x^n} \\ \frac{a}{x} \cdot \frac{b}{x} \cdot \frac{c}{x} \cdot ... &= (a \cdot b \cdot c \cdot ...)^{1/x} \end{aligned} \tag{28}$$

3 Domain No-nos

1. Divide by 0
2. $\sqrt{-x}$
3. $log\ 0$ or $ln\ 0$ or $log_x\ 0$
4. $log\ -x$
5. $sin^{-1}(2)$

4 Factoring reminder

$$\frac{3450}{1080} = \frac{2*5*5*3*23}{2*5*2*3*3*2*3} = \frac{5*23}{2*3*2*3} = \frac{115}{36} \tag{29}$$

5 Limit solving examples

By factoring and cancelling (Only applies in limits)

$$\frac{\sqrt{x}-4}{x-16} = \frac{\sqrt{x}-4}{(\sqrt{x}+4)(\sqrt{x}-4)} = \frac{1}{\sqrt{x}+4} \tag{30}$$

Reduction

$$
\begin{gathered}
\lim_{x\to\infty} \sqrt{9x^2+x} - 3x \\
\lim_{x\to\infty} \frac{\sqrt{9x^2+x} - 3x}{1} \\
\lim_{x\to\infty} \frac{\sqrt{9x^2+x} - 3x}{1} * \frac{\sqrt{9x^2+x} + 3x}{\sqrt{9x^2+x} + 3x} \\
\lim_{x\to\infty} \frac{9x^2+x-9x^2}{\sqrt{9x^2+x} + 3x} \\
\lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x} + 3x} * \frac{\frac{1}{x}}{\frac{1}{x}} \\
\lim_{x\to\infty} \frac{1}{\frac{\sqrt{9x^2+x}}{x} + 3} \\
\lim_{x\to\infty} \frac{1}{\sqrt{\frac{9x^2+x}{x^2}} + 3} \\
\lim_{x\to\infty} \frac{1}{\sqrt{\frac{9x^2}{x^2} + \frac{x}{x^2}} + 3} \\
\lim_{x\to\infty} \frac{1}{\sqrt{9 + \frac{1}{\infty}} + 3} \\
\lim_{x\to\infty} \frac{1}{\sqrt{9+0} + 3} \\
\lim_{x\to\infty} \frac{1}{3+3} \\
\lim_{x\to\infty} \frac{1}{6} = \frac{1}{6}
\end{gathered} \tag{31}
$$

Solve by IVT

$$
\begin{aligned}
f(x) &= 5x^4 - 3x^2 + 2x - 1 \\
\text{show } f(c) &= 0 \text{ for some } c \in \mathbb{R} \\
f(0) &= -1 \text{ and } f(1) = 3 \\
\text{thus } \exists\ c \in (0,1) \text{ such that } f(c) &= 0 \text{ by IVT}
\end{aligned}
\tag{32}
$$

Squeeze Theorem: Short notes because I'm too tired; Basically, use inequalities on both sides to shrink down the space until you get it to a point that both are equal then you've solved it.