AP Calculus AB Take-Home Final

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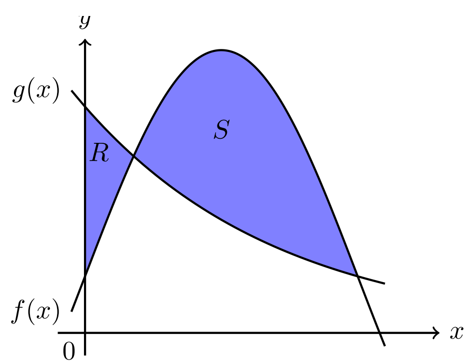

$$f(x) = \frac{1}{4} + \sin \pi x, \quad g(x) = 4^{-x}$$

Finding the area of R:

First, we find the first intersection of $f(x)$ and $g(x)$. We'll call the x value of the intersection k. We can then use a calculator approximate the integration of the difference between $g(x)$ and $f(x)$ from 0 until k to obtain the area R.

$$R = \int_0^k [g(x) - f(x)] dx \approx 0.064$$

Finding the area of S

Next, we can find the second intersection of $f(x)$ and $g(x)$ (which we will call j. We can then integrate similarly to get the area; Note that we reverse the order of $g(x)$ and $f(x)$ because $f(x)$ has a higher value of y.

$$S = \int_k^j [f(x) - g(x)] dx \approx 0.410$$

Revolving S:

To revolve S around the horizontal line $y = -1$, we have to adjust $f(x)$ and $g(x)$ by $+1$, and then shroud them in the circle area equation πr^2, finally integrating the resulting difference between k and j to obtain volume.

$$S_{vol} = \pi \int_k^j \left[(f(x)+1)^2 - (g(x)+1)^2\right] dx \approx 4.56$$

Sorry there's no graphic for the revolve, it's *hard.*

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$$f(0) = 2, \;\; f'(0) = -4, \;\; f''(0) = 3$$

2.1 Part A

$$g(x) = e^{ax} + f(x)$$

First, we need to know the derivatives of $g(x)$, so we evaluate them as such:

$$\frac{d}{dx} g(x) = g'(x) = ae^{ax} + f'(x)$$
$$\frac{d}{dx} g'(x) = g''(x) = e^{ax} a^2 + f''(x)$$

Next, we can evaluate the derivatives of $g(x)$ in line with the pre-defined derivatives of $f(x)$ like so:

$$g'(0) = ae^{a(0)} + f'(0) = ae^0 - 4 = a - 4$$
$$g''(0) = e^{a(0)} a^2 + f''(0) = a^2 e^0 + 3 = a^2 + 3$$

2.2 Part B

$$h(x) = \cos(kx) f(x)$$

First, we derive $h(x)$:

$$\frac{d}{dx} h(x) = h'(x) = -k \sin(kx) f(x) + \cos(kx) f'(x)$$

This gave us the slope $(h'(0))$, but we still need to find the y value of $h(0)$ and the slope at $h'(0)$

$$y = h(0) = \cos[k(0)] f(0) = \cos(0)(2) = (1)(2) = 2$$
$$m = h'(0) = -k \sin(k0) f(x) + \cos(k0) f'(0) = 0 + \cos(1)(-4) = -4$$

We can finally find the tangent equation:

$$y - 2 = -4(x - 0)$$