Coordinate system

The following assumes the player to be located at the origin. These coordinates are acquired by subtracting the player's position from the destination's position. The y direction is up/down, x is left/right.

Notation

- x_f, y_f: The final desired x and y position of the projectile.
- t_f: The total flight time.
- g: The (positive) gravitational acceleration in the negative y direction.
- x'_0, y'_0: The initial velocity of the projectile. Its magnitude is the desired speed, and the desired direction is its normalized value.
- M: The maximum desired height of the projectile above the player.

System

The following equations summarize the trajectory of the projectile:

$$y(t) = -gt^2 + y'_0 t$$

$$x(t) = x'_0 t$$

Solution

Because the y position of the projectile is a parabola with respect to time, it's vertex will be the maximum height of the projectile. For $ax^2 + bx + c$, the vertex sits at $x = \frac{-b}{2a}$. Therefore, we have:

$$M = y\left(\frac{-y'_0}{-2g}\right)$$

$$= -g\left(\frac{y'_0}{2g}\right)^2 + y'_0\left(\frac{y'_0}{2g}\right)$$

$$= -g\left(\frac{y'^2_0}{4g^2}\right) + \frac{y'^2_0}{2g}$$

$$= \frac{-y'^2_0}{4g} + \frac{2y'^2_0}{4g}$$

$$= \frac{y'^2_0}{4g}$$

So we can solve for y'_0 given M and g:

$$M = \frac{y'^2_0}{4g}$$

$$4Mg = y_0'^2$$
$$y_0' = 2\sqrt{Mg}$$

Next we will solve for t_f given y_0' and g:

$$y_f = y(t_f) = -gt_f^2 + y_0' t_f$$
$$0 = -gt_f^2 + y_0' t_f - y_f$$

We can use the quadratic equation to find the final time t_f. Note that there are two solutions; the largest, real solution will be the correct value of t_f because it is the second time the y value crosses the correct y_f value (the first being its initial launch into the air!)

$$t_f = \frac{-y_0'}{-2g} \pm \frac{\sqrt{y_0'^2 - 4(-g)y_f}}{-2g}$$
$$= \frac{y_0'}{2g} \pm \frac{\sqrt{y_0'^2 + 4gy_f}}{2g}$$

Finally, we can use t_f and x_f to find x_0':

$$x_f = x(t_f) = x_0' t_f$$
$$x_0' = \frac{x_f}{t_f}$$